Unbundling in the Presence of Network Externalities

And

Information Complements, Substitutes, and Strategic Product Design

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This document contains two separate papers, both dealing with "two-sided" network effects. This refers to a demand economy of scale that crosses distinct market populations, such as HDTV producers and HDTV consumers, as distinct from a more traditional interpretation of a single homogeneous market for HDTV.

Taking a monopoly perspective, the first paper considers when a firm profits more by bundling (to reduce demand heterogeneity) or by unbundling (to stimulate network effects). In contrast to earlier bundling literature, it shows that a firm frequently chooses to unbundle and subsidize one side of two coupled markets. This explains several examples of free information observed on the Internet and also the reluctance to adopt a format unless both sides are on board.

The second generalizes functional forms of the first, in additive and multiplicative terms, and shows how they are equivalent. It also introduces competition and shows how a firm can use free information to penetrate markets that become competitive upon entry. The initial model setup is necessarily the same.

An efficient reading might be to cover the more general results of the second paper then return to just the section on unbundling in the first.

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ABSTRACT: Why do firms unbundle related products? In particular, why do they give away free goods and services without tying revenues to those same consumers? We show that seemingly anomalous firm behaviors are consistent with previous literature once a novel form of "two-sided" network effect enters the analysis.

Introducing two-sidedness into an externality model precipitates a twist on traditional price discrimination such that one market need not pay positive prices for any good. In fact, this strategy differs from both tying and multi-market price discrimination. Although information examples motivate our understanding, the rationale for unbundling applies to a variety of goods and services characterized by low marginal costs. These include operating systems, games, advertising, auctions, and credit cards.

The main contribution of this paper is to show how to improve price discrimination through product design, specifically by the optimal choice of bundling versus unbundling . This involves methods to compare compound demand curves, extending the previous literature on bundling. We show that at one set of extremes, bundling remains optimal while, contrary to prior literature, at another extreme unbundling becomes optimal. We also develop tests for which market to heavily discount when unbundling dominates.

The insights of this model help to explain several interesting market phenomena including (i) the profitable existence of free information and free services, (ii) when to split closely related products, and (iii) why below marginal cost pricing need not in fact be predatory. It is also suggestive of why the promise of Internet micropayments has so far failed to materialize.

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1 Introduction

The principal question explored in this paper is why should a firm prefer unbundling low marginal cost products such as information. Firms face strong pressures to bundle (Adams & Yellen 1976, Bakos & Brynjolfsson 1999, Nalebuff 1999), yet the fact remains that many successful companies choose component sales. They not only spend resources to cleave products in half, they also acquire related products to give away for free (Lohr 1999).

A related question concerns the level of disaggregation. If firms can successfully boost profits by splitting a product once, then why not twice, three times or more? Technological advancement, coupled with vanishingly small transaction costs could motivate the complete disaggregation of products to whatever unit might be priced independently as, for example, in the case of micropayments.

Building on the recent two-sided network externality literature, we introduce a novel mechanism that (i) explains firms' unbundled component sales strategy, (ii) is distinct from tying and traditional multi-market price discrimination, and (iii) sheds light on the viability of certain industry structures with intermediaries. Of managerial interest is the proposition that discounting an unbundled component can increase profits to the point where negative prices become optimal. Profits increase conditionally, however, on promoting network effects through clever product design. Of legal and economic interest is the implied industry concentration and difficulty applying antitrust tests of predation. Pricing below marginal cost maximizes profits even in the absence of competition while internalizing externalities can improve consumer welfare from the independent firm case

The innovation of this and related articles is a *two-sided* network externality – a demand economy of scale that crosses two heterogeneous markets as distinct from a rescaled demand within a single market. Juxtaposing paired demands results in a product complement modeled as a network effect. A natural intuition then applies. Lowering price on one half of a complementary pair sells more of the other with the novelty that sales accrue to different buyers.

In the classic network externality story, a stock of telephone consumers benefits from incremental telephone consumption as this extends the reach of existing consumers through a larger network. Network effects compound consumer surplus within the same market. In contrast, we consider the matched-market, chicken-and-egg problem of creating music or video for a new format. Producers want consumers and consumers want producers before either readily switches to the new format.

An incumbent firm producing content for the new format probably does not welcome entry by a competing firm producing similar content as there is no profitable direct exchange between them. Buyers in the end-consumer market, however, welcome entry because it increases the prospect of a viable format should the incumbent fail. It also increases variety while possibly lowering prices. This increases both the value to individuals and the number of individuals willing to switch formats. Thus, in the present example, the externality runs from content-creators to end-consumers.

Conversely, consider the end-consumer's original choice of VHS versus Beta. Initially, at least, consumers probably care less about adding another consumer to a new format—it could even bid up prices if supply is limited—than they care about the number and diversity of firms who provide content for that format. Producers, however, care immensely about the size of the consumer market. The existence of a larger consumer base makes production under any given format more attractive. Again, in the present articulation, the externality runs across markets from consumers to providers of content.

Importantly, the externality benefit can run across markets and back again. Both content creators and consumers do value growth in their own markets but this may be mediated by the indirect effect of the inter-network externality. At issue is whether own-market entry expands participation on the other side of each transaction. Content creators may not object to other content providing firms if effective consumer demand rises instead of falls.

Consider finally, a third participant, the focus of our attention here, who produces tools to support both content creators and end-consumers. Examples include Sun, Apple, and Microsoft who support software developers as well as private and business buyers; it includes Sony and Phillips who support the entertainment industry as well as households; and it includes firms like Adobe who produce portable document writers as well as portable document readers. Applied to services, E-Bay coordinates buyers and sellers, while VISA coordinates merchants and card holders. For these firms, the chicken-and-egg profit maximization problem is how to grow both markets. A straightforward solution, widely observed in the Internet economy, is to discount one market in order to grow both or profit more from the other.[1]

We note that the popularity of this product design strategy relies less on nonrivalry than on low cost reproduction. Although fixed costs can be high, near zero marginal costs allow a firm to subsidize an arbitrarily large market with only a one time initial investment. In fact, low marginal costs and inter-network externalities apply to many goods and services

[1] The consequences of mispricing internetwork markets can also be severe. In 2001, the number two auction site, Yahoo, raised seller fees and listings unravelled, dropping 90% (Hansell, 2001). Sellers concluded that for such fees, they preferred the larger number of buyers on E-Bay.

such as those in Table 1.

This model favors an intermediary who, straddling both markets, can set prices more efficiently by internalizing their externalities. Independent firms serving either market separately lose this advantage. Apart from this advantage, the pressures to bundle low marginal cost goods are quite high, particularly if such goods are complementary[2]. Unbundling therefore succeeds to the extent that it can overcome the advantages of bundling as in the case of leveraging inter-network effects. Complete disaggregation, for example, to the level of micropayments, risks losing one major advantage without winning offsetting gains.

Table 1

Product Category	Mkt 1 Product	Intermediary	Mkt 2 Product
Portable Documents	Document Reader ✠	Adobe	Document Writer
Credit Cards	Consumer Credit ✠	Issuing Bank	Merchant Processing
Operating Systems	Complementary Applications	Microsoft, Apple, Sun	Systems Developer Toolkits ✠
Plug-Ins	Applications Software	Microsoft, Adobe	Systems Developer Toolkits ✠
Ladies' Nights	Men's Admission	Bars, Restaurants	Women's Admission ✠
TV Format	color UHF, VHF, HDTV ✠	Sony, Phillips, RCA	Broadcast Equipment
Advertisements	Content ✠	Magazine Publishers, TV, Radio Broadcasters	Advertisers
Computer Games	Game Engine/Player	Games Publishers	Level Editors ✠
Auctions	Buyers ✠	E-Bay, Christie's, Sotheby's	Sellers
Streaming Audio/Video	Content ✠	RealPlayer, Microsoft, Apple	Servers

✠Indicates which market is discounted, free, or subsidized.

To put this pricing strategy in context, the idea differs from traditional multi-market, or third degree, price discrimination in that firms make product offers to markets where they never intend to capture consumer surplus. In fact, they can subsidize one market indefinitely to promote demand in the other. The insight also differs from tying, or second

[2]Exceptions favoring mixed bundling include budget constraints and high variance values (Bakos & Brynjolfsson '99, Chuang & Sirbu '99, Hitt & Chen '01, Mackie-Mason & Riveros '99).

degree price discrimination in that, unlike razors and blades, consumers of one product need never buy the complement. Consumers of broadcast content, portable document readers, and credit cards may, at their discretion, forgo purchasing ads, portable document writers, and merchant transaction processing now and forever. Complementary inter-network goods are more loosely coupled than in the classic case of tying with product lock-in. Finally, unlike traditional network externality pricing, profits need not derive from penetration prices with temporal lock-in where a firm subsidizes a market initially only to exploit it later. Microsoft, for example, has little incentive to limit future compatible OS applications by exploiting developers.

2 Related Literature

This analysis adds to a recent literature on two-sided network effects (Armstrong, 2002; Caillaud & Jullien, 2001, Parker & Van Alstyne 2000; Rochet & Tirole 2001) that makes precise a form of indirect network effect (Katz & Shapiro 1994, Liebowitz & Margolis 1994). Indirect effects are consumption externalities from purchasing compatible products such as hardware and software. The key distinction for two-sidedness is that network effects must cross market populations. When they do not, the same consumer populations choose systems of compatible products only for themselves. These can lead to pecuniary externalities, efficiently handled through the pricing system (Liebowitz & Margolis 1994). In contrast, two-sided networks yield true externalities in which one population chooses a good affecting another population's choice of a different good. As in Farrell & Saloner (1985), coordinating purchases becomes essential to avoid inertia, but in two-sided networks coordination across markets matters. Coordination within markets may have no effect. Rochet and Tirole (2001) model two-sided network effects in a multiplicative framework focusing on competing platforms, such as credit card networks. This nicely captures "multihoming," for example, the decision to carry multiple credit cards from competing networks. Caillaud and Jullien (2001) consider a matchmaking intermediary, as for dating services. They explain why agents register with more than one service and whether fees should hinge on a successful match. A useful survey of these and other results is provided in Armstrong (2002), examining which side of a market is subsidized and whether the outcome is socially efficient.

The contribution of this paper is to examine not industry structure but product design. In many cases, one side of a market has positive demand for the product offered to the other side. Since bundling reduces demand heterogeneity, which typically increases profits for firms with market power, a firm must decide whether the benefit of bundling dominates the benefit of stimulating network effects. This paper extends the framework introduced in Parker and Van Alstyne (2000) that lays out the basic model.[3] DeGraba (1996) presents an alternative model where firms prefer to enter zero profit markets. It begins with a model of tying based on Whinston (1990) and relies on declining average cost curves to favor firms with increased sales. Our model does not use tying and instead relies on reciprocal content provision and consumption to increase sales, yielding different selling and product design strategies. Shapiro and Varian (1999) discuss the possibility of negative pricing to increase profit in complementary markets. We formally model when this is optimal and how a third

[3] At editor and reviewer request, Parker and Van Alstyne (2000) has been split into two shorter manuscripts. A revised version, Parker and Van Alstyne (2001), lays out a general framework, welfare results, and analyses oligopolistic competition. This manuscript presents a brief version of the framework, then develops alternate bundling versus unbundling tradeoff in detail.

party, the content creator, contributes value to consumers. This strategy is similar to a form of product versioning (Varian 1997), with different goods sold to different markets but it includes the benefit of strategic complements modeled after the network externality framework of Katz and Shapiro (1985).

This work also complements the work on bundling (Adams and Yellen, 1976; Bakos and Brynjolfsson, 1999; Nalebuff, 1999; McAfee, McMillan and Whinston, 1989; Salinger, 1995), mixed bundling (Chuang and Sirbu, 1999; Fay and MacKie-Mason, 1999), and consumer choice models of bundling (MacKie-Mason and Riveros, 1999). Bundling offers an entire collection of goods for a flat fee or subscription price in contrast to component sales in which individual items are metered. In mixed bundling, a firm sells both bundles and individual components—a strategy that includes pure bundling and pure component sales as special cases. In consumer choice models, consumers pay a flat fee as if for a bundle but then choose for themselves only K from among N information goods (with $K < N$) sold by the firm.

In these cases, the advantage of bundling is reduced demand heterogeneity. So long as buyer values are not perfectly correlated, individual demands for the sum of components is more uniform than the sum of demands for individual components. This smoothing of idiosyncratic consumer values allows a monopolist to extract more consumer surplus (Salinger, 1995; Bakos and Brynjolfsson, 1999). Rather than seeking to reduce market heterogeneity, the inter-network product design model seeks to leverage it. Articulating the benefit of these competing forces is one contribution of this research. A key insight is that unbundling together with heavy discounting implements a novel form of third degree price discrimination in markets linked by inter-network effects. Of further interest is the method used to show this result. We apply convolution integrals to generate a bundled demand curve for two separate goods. This permits a direct comparison of profits in the two competing cases.

The paper proceeds as follows. First, we develop a model of externality-based complements. We find the conditions for a subsidy market to exist and identify which market receives the subsidy. We then develop mechanisms to contrast bundling with unbundling in the presence of inter-network effects. The product design decision weighs the relative sizes of these effects in determining whether to sell a bundle or separate components. Finally, we consider further unbundling to the level of micropricing and whether this increases profits.

3 A Model of Complements

In this section we consider the question of why a firm should spend resources to create a product it intends to distribute for free. Drawing on complementary goods markets, where there are demand economies of scale, the model shows that participating in a free goods market can be profit maximizing.

3.1 Markets

Starting with standard price p and quantity q terms, we consider two markets for a given information product. The first is the end-user or general consumer market (subscript c). The second market is the joint-producer, developer, or content-creator market (subscript j). This yields prices p_c and p_j for the consumer and content creator markets respectively. Building on Katz and Shapiro (1985), we model each consumer's willingness to pay as a function of their own valuation and the number of consumers in the other market who buy.

The parameter Q represents the maximum market size and V the maximum product value in the absence of externalities. The inter-network effect e_{jc} then determines how much joint-producer purchases affect consumer purchases. More precisely, it is the partial derivative of demand in the C market with respect to demand in the J market or $\frac{\partial q_c}{\partial q_j}$. Conversely, e_{cj} determines how much effect purchases in the consumer market have on the joint-producer market. To facilitate analysis, we assume that potential consumers in each market have a uniform distribution of values, but share externality effects from the complementary market.[4] Together, these terms describe a pair of simultaneous demand curves:

$$q_c = Q_c - p_c \frac{Q_c}{V_c} + e_{jc} q_j, \quad (1)$$

$$q_j = Q_j - p_j \frac{Q_j}{V_j} + e_{cj} q_c. \quad (2)$$

Eliminating q_i from the right-hand-side and setting $M = \frac{1}{1 - e_{cj} e_{jc}}$ yields independent formulae:

$$q_c = M \left[\left(Q_c - p_c \frac{Q_c}{V_c} \right) + e_{jc} \left(Q_j - p_j \frac{Q_j}{V_j} \right) \right], \quad (3)$$

$$q_j = M \left[\left(Q_j - p_j \frac{Q_j}{V_j} \right) + e_{cj} \left(Q_c - p_c \frac{Q_c}{V_c} \right) \right]. \quad (4)$$

The first principal term is the native demand while the second term is the demand stimulated by inter-network effects, with both terms scaled by the market multiplier M. Note that as either externality term shrinks, its cross market demand stimulus vanishes and $M \to 1$. To render the interpretation meaningful, we impose a finite market restriction that the inter-network externality terms contribute only a finite and positive amount to consumer surplus

[4]The results in this section also hold for general demand curves with linear externalities of the form $q_c = D_c(p_c) + e_{jc} q_j$ and similarly for q_j. For clarity of exposition and to ensure tractability in the analysis of bundling, we work with the linear demand form.

thus $0 < M < \infty$. This implies $1 - e_{cj}e_{jc} > 0$.

3.2 Monopoly Choice

Allowing for near zero marginal costs to distribute an information good, we can model the firm's profit for the bundled good as $\pi = pq$. When the firm disaggregates the product for sale to two different markets, this becomes $\pi = p_c q_c + p_j q_j$. Taking first-order conditions gives monopoly choices for p_c and p_j:

$$p_c^* = \frac{(e_{cj}e_{jc} - 2)\, Q_c Q_j V_c^2 V_j + e_{cj} Q_c Q_j V_c V_j^2 - e_{jc} Q_j^2 V_c^2 V_j + e_{cj}^2 Q_c^2 V_c V_j^2}{2\,(e_{cj}e_{jc} - 2)\, Q_c Q_j V_c V_j + e_{cj}^2 Q_c^2 V_j^2 + e_{jc}^2 Q_j^2 V_c^2} \tag{5}$$

$$p_j^* = \frac{(e_{cj}e_{jc} - 2)\, Q_c Q_j V_c V_j^2 + e_{jc} Q_c Q_j V_c^2 V_j - e_{cj} Q_c^2 V_c V_j^2 + e_{jc}^2 Q_j^2 V_c^2 V_j}{2\,(e_{cj}e_{jc} - 2)\, Q_c Q_j V_c V_j + e_{cj}^2 Q_c^2 V_j^2 + e_{jc}^2 Q_j^2 V_c^2} \tag{6}$$

These prices then determine π^* as follows:

$$\pi^* = \frac{Q_c V_c + Q_j V_j + e_{cj} Q_c V_j + e_{jc} Q_j V_c}{4 - \left(\frac{e_{cj}^2 Q_c V_j}{Q_j V_c} + 2 e_{cj} e_{jc} + \frac{e_{jc}^2 Q_j V_c}{Q_c V_j} \right)} \tag{7}$$

In maximizing two prices, the normal assumptions of a positive first derivative and negative second derivative that guarantee concavity become the Hessian conditions. In the present model, the Hessian of

$$\begin{pmatrix} \frac{\partial^2 \pi}{\partial c^2} & \frac{\partial^2 \pi}{\partial c \partial j} \\ \frac{\partial^2 \pi}{\partial j \partial c} & \frac{\partial^2 \pi}{\partial j^2} \end{pmatrix}$$

yields $4 - \left(\frac{e_{cj}^2 Q_c V_j}{Q_j V_c} + 2 e_{cj} e_{jc} + \frac{e_{jc}^2 Q_j V_c}{Q_c V_j} \right) > 0$.

A simple graphic then illustrates how the model functions. Consider first a monopolist's profit maximizing decision in the absence of network externalities, $e_{cj} = e_{jc} = 0$. A straightforward maximization on $\pi_c = p_c(Q_c - \frac{Q_c}{V_c}p_c)$ yields the standard result, $p_c^* = \frac{V_c}{2}$, $q_c^* = \frac{Q_c}{2}$, with profits of $\pi_c^* = \frac{V_c Q_c}{4}$ and similarly for π_j. Figure 1 shows the linkage between markets. The network externality from the J market shifts the demand curve in the C market up

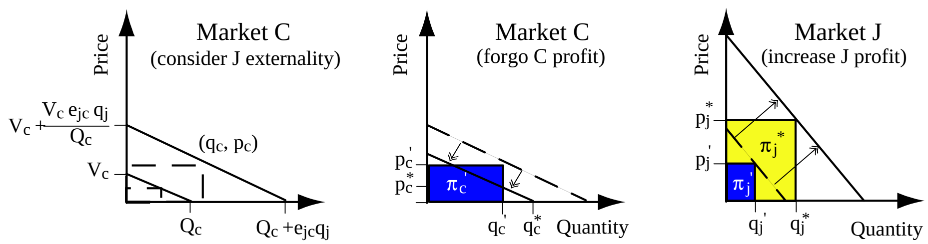

Figure 1: In the leftmost panel, a network externality shifts the demand curve out. The optimal (q_c, p_c) pair shifts from $(\frac{Q_c}{2}, \frac{V_c}{2})$ to amounts given by equations 3 and 5. In the middle panel, a monopolist forgos profits $\pi_c^{'}$ at price $p_c^{'}$ and sets $p_c^* < p_c^{'}$ in order to increase total profit. The increase appears in the J market, the third panel, where the decision to discount the C market is rational whenever net profits rise, $\pi_j^* - \pi_j^{'} > \pi_c^{'} - \pi_c^*$.

and to the right. This models the idea that content-providers stimulate demand among consumers, just as consumers stimulate interest from content-providers.

The number of content-providers who actually enter the market, however, governs the degree of shift. The externality coupling these two markets implies that the more a monopolist can coax consumers into adopting one of his products, the more he can charge for—and sell of—the other. If profit increment on one complementary good exceeds the profit loss on the other, then a discount or even subsidy becomes profit maximizing. Free goods markets can therefore exist whenever the profit maximizing price of zero or less generates inter-network externality benefits greater than intra-market losses. Firms do, in fact, offer subsidized goods when they offer free service and technical expertise. This form of subsidy also addresses an interesting problem of adverse selection arising from an offer to a general audience. Only legitimate consumers request technical support, so the subsidy is never wasted.

3.3 The Choice of a Free Goods Market

Choosing an optimal price pair is easily illustrated using equations 3 and 4 above for reaction curves. Figure 2 shows the four regions that result. In region I, both markets are charged positive prices while region III represents a subsidy to both markets and is never profit maximizing.

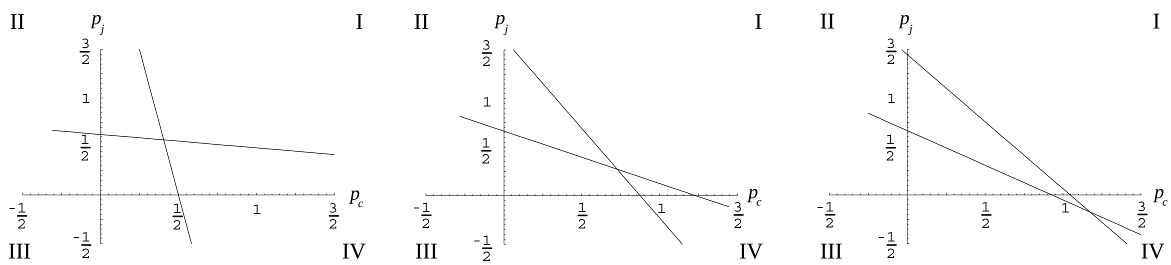

Figure 2: Intersecting price reaction curves show consumer price (p_c) rising and developer price (p_j) falling as the developer-to-consumer externality (e_{jc}) rises from 0 to 1.1 in the linear case with parameters $\{Q_c = Q_j = V_c = V_j = 1 \text{ and } e_{jc} = \frac{1}{3}\}$.

The relative market sizes, valuations, and network externalities, however, can give rise to different convex regions and optimal price choices. With zero marginal costs, negative prices imply that subsidy markets exist in either Region II or Region IV.

Since the basic model spans six degrees of freedom, we introduce a substitution that simplifies proofs and permits useful interpretation. Integrate the demand curves over the range of prices $p \in [0, V]$ to produce surplus representations as follows:

Native Surplus	*2-Sided Externality Surplus*
$S_c \equiv Q_c V_c$	$S_{cj} \equiv e_{cj} Q_c V_j$
$S_j \equiv Q_j V_j$	$S_{jc} \equiv e_{jc} Q_j V_c$

These terms proportionally represent the consumer surplus in each market individually and the consumer surplus created by inter-network externalities in the complementary market[5]. We can then express the optimal prices and profits in surplus space more succinctly as:

$$p_c^* = \frac{V_c\left(2S_cS_j - S_{cj}(S_{jc}+S_{cj}) + S_j(S_{jc}-S_{cj})\right)}{4S_cS_j - (S_{cj}+S_{jc})^2} \tag{8}$$

$$p_j^* = \frac{V_j\left(2S_cS_j - S_{jc}(S_{cj}+S_{jc}) + S_c(S_{cj}-S_{jc})\right)}{4S_cS_j - (S_{cj}+S_{jc})^2} \tag{9}$$

$$\pi^* = \frac{S_cS_j\left(S_c + S_{cj} + S_{jc} + S_j\right)}{4S_cS_j - (S_{cj}+S_{jc})^2} \tag{10}$$

If the markets are independent such that $e_{cj} = e_{jc} = 0$ (implying $S_{cj} = S_{jc} = 0$) then these expressions simplify to $p_c^* = \frac{V_c}{2}$, $p_j^* = \frac{V_j}{2}$, $\pi^* = \frac{S_c+S_j}{4}$ as expected. The surplus space simplification also provides a somewhat more congenial expression for the Hessian restriction and the finite market restriction respectively:[6]

$$4S_cS_j - (S_{cj}+S_{jc})^2 > 0, \quad 1 - \frac{S_{cj}S_{jc}}{S_cS_j} > 0.$$

The decision of which market to subsidize, if any, and which to charge then rests on Proposition 1 below. Let p_i^* be the optimal price in the presence of an inter-network externality and $\hat{p}_i$ be the price in the absence of an inter-network externality.

Proposition 1 *Market C contributes more inter-network externality surplus to market J if and only if the optimal price in the C market falls relative to the no-externality case. Mathematically,* $S_{cj} > S_{jc} \Longleftrightarrow p_c^* < \hat{p}_c$.

Proof. *See Appendix of Proofs.* ■

[5]Total surplus is also calculated as a triangular area from Figure 1 such as $\frac{QV}{2}$. For simplicity, we drop constants of $\frac{1}{2}$; results remain the same.

[6]It can easily be shown that the Hessian condition subsumes the finite market restriction. That is, $4S_cS_j - (S_{cj}+S_{jc})^2 > 0$ yields a tighter bound than $1 - \frac{S_{cj}S_{jc}}{S_cS_j} > 0$.

Thus, the choice of which market to subsidize is given by the relative externality surplus terms. If $S_{cj} > S_{jc}$ then consumers create relatively more benefit for content-providers so discount the consumer good. Two additional points then follow from Proposition 1. First, only one market at a time can reasonably be discounted. Second, firms may set prices myopically when the sizes of the network surplus terms between markets are equal.

Corollary 1 *A discount in one market implies a price premium in the other or* $p_c^* < \hat{p}_c$ *if and only if* $p_j^* > \hat{p}_j$. *This follows from the fact that* $p_c^* < \hat{p}_c$ *implies* $S_{cj} > S_{jc}$ *and that the markets are symmetric. Symmetric restrictions on both prices further imply that if* $S_{cj} = S_{jc}$ *then* $p_c^* = \hat{p}_c$ *and* $p_j^* = \hat{p}_j$.

The corollary states that if inter-network effects across markets are approximately equal in size—the presence of developers is as attractive to consumers as the presence of consumers is to developers—then a monopolist can safely ignore these effects in all pricing decisions. A firm can set prices *as if* the inter-network terms e_{cj} and e_{jc} were both zero. This does *not* imply that network externalities have no effect on quantity sales. In general, output exhibits substantial increases due to the market multiplier $\frac{1}{(1-e_{cj}e_{jc})}$. Taking the ratio of optimal quantities in the presence of network externalities q_c^* to optimal quantities in the absence of network externalities $\hat{q}_c$ provides an expression that reduces to $\frac{S_cS_j+S_{jc}}{S_cS_j-S_{jc}^2}$ with a similar expression for q_j^*. With reasonable assumptions on market sizes, network externalities increase both ratios above one implying that sales in both markets increase. The point is that firms need only consider a joint pricing decision when the difference between network surpluses S_{cj} and S_{jc} is substantial. And, a significant difference can lead to a completely free or even subsidized good.

Having laid out the basic framework, we are now ready to analyze bundling and unbundling in the presence of inter-network externalities.

4 A Model of Bundling vs. Unbundling

In this section, we extend the basic model to address the question of why a firm should spend resources to create multiple versions of a product for sale to different markets as opposed to selling one bundled good to all markets. This requires two steps and hinges on accounting for the possibility that both markets value both products. Step one develops a general demand curve for a bundle, accounting for the possibility that a consumer of one good might have a range of values for the second good. Step two develops the same possibility for the case of selling unbundled goods to both markets.

Several articles have shown that, if consumers' values are not perfectly correlated, bundling goods reduces heterogeneity in their willingness to pay (Adams & Yellen, 1976; Salinger, 1995; Bakos & Brynjolfsson, 1999). This allows a firm to charge a uniform price and extract more consumer surplus. In fact, if values are negatively correlated, such that one consumer values a spreadsheet more than a word processor, say ($80, $120), and another values them at ($120, $80) respectively, then a bundled price of $200 generates two unit sales of both goods extracting the full surplus of $400. In contrast, independent sales at prices of either $80 or $120, sells two or one unit of each good to both consumers for profits of only $360 or $240 respectively. We are interested in the case where positive externalities across two markets lead firms to design complementary products and divide a compound good. Here, unbundling can become a dominant strategy.

4.1 Bundling Model Development

Let the C market value the J good and the J market value the C good. In particular, let consumers value fraction $r_c \in [0,1]$ of the product intended for joint developers such that C value is distributed on $[0, r_c V_j]$. Then quantity sales in the C market of the J product, designated q_{cj}, are given by equation 11 and similarly for developers.

$$q_{cj} = Q_c\left(1 - \frac{p_j}{r_c V j}\right) \tag{11}$$

$$q_{jc} = Q_j\left(1 - \frac{p_c}{r_j V_c}\right) \tag{12}$$

The r_i term represents the fraction of value a consumer places on the primary product of the other market. As C consumers value the J good less and less, $r_c \to 0$ and q_{cj} becomes arbitrarily negative. This cannot occur in the basic case since $p_j < V_j$ while it need not be true that $\frac{p_j}{r_c} < V_j$. Thus for $r_c < \frac{V_j}{p_j}$, we restrict sales to not less than zero or $q_{cj} \equiv Max[0, Q_c(1 - \frac{p_j}{r_c V_j})]$. Also, to remain consistent with earlier sections while using the more specific notation, let q_{cc} (respectively q_{jj}) characterize sales to C consumers of the C product exactly as given by q_c (respectively q_j) in the original equation 1 (respectively 2). Since q_{cc} and q_{jj} are already reciprocally endogenous in the externality terms e_{cj} and e_{jc}, we simplify the analysis by omitting them from equations in this section. Inserting them again adds little to the intuition from analysis while rendering closed form solutions too complex to add much meaning.

The model in Section 3 is equivalent to this more general model under the boundary assumption that $r_c = r_j = 0$. Under these circumstances, each market values only one good and we show below that unbundling always dominates bundling or $\pi_u^* \geq \pi_b^*$. Another possibility is to assume that the markets are identically distributed such that both markets value both goods the same and $r_c = r_j = 1$ with $V_c = V_j$. Under minimal externality assumptions, it is possible to show that bundling always dominates or $\pi_u^* \leq \pi_b^*$. We establish both possibilities in the generalized framework. The first contribution is unique to this model while the second generalizes results of Adams & Yellen, and Bakos & Brynjolfsson. We then illustrate the transition from one set of market circumstances to the other.

In order to develop a fair comparison of bundling to unbundling, we must allow for one type of consumer to place a positive value on the other market's primary good. In this way, each consumer will have a value for the bundle composed of their valuation of their primary good, plus their valuation of the other market's primary good. We model primary demand for the consumer's native market good the same way as in the beginning of the paper, except that we denote primary demand as q_{ii}.

$$q_{cc} = Q_c\left(1 - \frac{p}{V_c}\right) + e_{jc}q_{jj} \tag{13}$$

$$q_{jj} = Q_j\left(1 - \frac{p}{V_j}\right) + e_{cj}q_{cc} \tag{14}$$

For tractability, we constrain the network externality term to contribute surplus across primary markets as opposed to within. We can then develop independent representations for each q_{ii} in terms of price.

$$q_{cc} = M\left[Q_c\left(1 - \frac{p}{V_c}\right) + e_{jc}Q_j\left(1 - \frac{p}{V_j}\right)\right] \tag{15}$$

$$q_{jj} = M\left[Q_j\left(1 - \frac{p}{V_j}\right) + e_{cj}Q_c\left(1 - \frac{p}{V_c}\right)\right] \tag{16}$$

In order to determine the total demand from each type of consumer, we must aggregate their demand functions for the individual goods in the bundle. Note that we replace q_{cc} and q_{jj} with q_c and q_j, the total demand from C consumers for the bundle and the total demand for J consumers for the bundle respectively. To do this, we assume that the bundle is made up of one good of each type. Having assumed linear demand functions, consumer valuations for each piece of the bundle are represented by the following uniform distributions.

$$
\begin{aligned}
f_{cc}(p) &= Uniform[0, V_c + \frac{V_c}{Q_c} e_{jc} q_{jj}] && (17) \\
f_{cj}(p) &= Uniform[0, r_c V_j] && (18) \\
f_{jj}(p) &= Uniform[0, V_j + \frac{V_j}{Q_j} e_{cj} q_{cc}] && (19) \\
f_{jc}(p) &= Uniform[0, r_j V_c] && (20)
\end{aligned}
$$

For example, the C consumer's demand for the bundle is determined by density functions $f_{cc}(z)$, a C consumer's valuation for good C and $f_{cj}(z)$, a C consumer's valuation for good J. In order to determine the sum of the density of these demands, we must convolve the two densities (see, e.g., Papoulis, 1991). To do this, we evaluate the following integral:

$$g_c(z) = \int_0^z f_{cc}(z) f_{cj}(z-y)\, dy. \tag{21}$$

In order to evaluate this integral, we follow Nalebuff (1999) and assume that C demand for the J good is independent of C demand for the C good, and similarly for J. This takes advantage of the fact that externalities appear in only half of the value distributions. For uniform probability densities (linear demand curves), the resulting density is trapezoidal. This density function can be returned to a demand function by integrating over the valid range of prices.

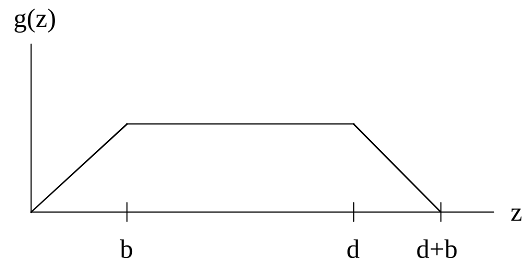

Figure 3: Trapezoidal demand density function results when two uniform density functions are convolved.

$$q(p) = QF(v) = Q\left(1 - \int_0^p g(v)dv\right) \tag{22}$$

Since there are three distinct regions, the result is a three-part demand function.

$$q(p) = \begin{cases} Q\left(1 - \frac{p^2}{2bd}\right) & \text{if} \quad 0 < p < b \\ Q\left(1 + \frac{b}{2d} - \frac{p}{d}\right) & \text{if} \quad b < p < d \\ Q\left(1 + \frac{b^2+d^2}{2bd} - \frac{p(b+d)}{bd} + \frac{p^2}{2bd}\right) & \text{if} \quad d < p < b+d \\ 0 & \text{elsewhere} \end{cases} \tag{23}$$

For C consumers, $b = r_c V_j$, $d = V_c + \frac{V_c}{Q_c} e_{jc} q_j$, and we assume that $r_c V_j < V_c + \frac{V_c}{Q_c} e_{jc} q_j$. The demand of J consumers for the bundle can be found similarly. Figure 4 shows the demand curves for the bundled good for both types of consumers. Note that the externality from the J market to the C market increases the number of C consumers by up to 100%. For this example, we have set the externality from the C to the J market to zero so we see no increase in the number of J market participants over Q_j.

4.2 Unbundled Demand with cross-market participation

We now develop the model of unbundled demand with cross-externalities in order to compare bundling and unbundling. As in the bundling case above, we restrict the externality to the

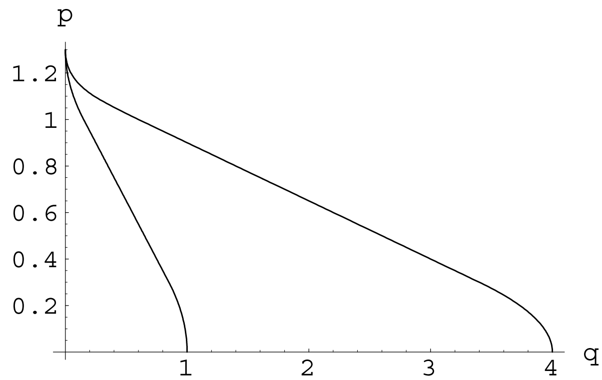

Figure 4: C and J demands for the bundled good are plotted for the following parameter values: $\{e_{cj} = 0,\ e_{jc} = 2, Q_c = 2, Q_j = 1, V_c = 1, V_j = 1, r_c = 0.3, r_j = 0.3\}$

demand from the other market for their own primary product. In this way, independent formulae for each market's demand for their own primary product can be developed.

$$q_{cc} = M[Q_c(1 - \frac{p_c}{V_c}) + e_{jc}Q_j(1 - \frac{p_j}{V_j})] \tag{24}$$

$$q_{jj} = M[Q_j(1 - \frac{p_j}{V_j}) + e_{cj}Q_c(1 - \frac{p_c}{V_c})] \tag{25}$$

Also as in the bundled demand case, let the C types value the J good and the J types value the C good. In particular, let consumers value fraction $r_c \in [0, 1]$ of the product intended for joint developers such that C value for the J good is distributed on $[0, r_cV_j]$. Then quantity sold in each market of the other market's primary product is given by:

$$q_{cj} = Max\left[0, Q_c\left(1 - \frac{p_j}{r_cVj}\right)\right] \tag{26}$$

$$q_{jc} = Max\left[0, Q_j\left(1 - \frac{p_c}{r_jV_c}\right)\right] \tag{27}$$

Combining demands in each market yields total demand for each product.

$$
\begin{aligned}
q_c &= q_{cc} + q_{jc} = M\left[Q_c(1-\frac{p_c}{V_c}) + e_{jc}Q_j(1-\frac{p_j}{V_j})\right] + Max\left[0, Q_c\left(1-\frac{p_j}{r_c Vj}\right)\right] \\
q_j &= q_{jj} + q_{cj} = M\left[Q_j(1-\frac{p_j}{V_j}) + e_{cj}Q_c(1-\frac{p_c}{V_c})\right] + Max\left[0, Q_j\left(1-\frac{p_c}{r_j V_c}\right)\right]
\end{aligned}
$$

Because there are prices at which there will be no cross-market participation, there are four total demand regions to consider. Optimal price pairs for each of these demand regions can readily be found by taking first-order conditions. For specific parameter values, the regions of interest can be determined and profit comparisons under bundling and unbundling can finally be made. Having developed the necessary framework to compare bundling and unbundling, we now proceed with an analysis of the model.

5 Bundling vs. Unbundling Results

We begin by considering the extreme case when there is no demand across markets as in the case of credit card purchases and merchant transaction processing.

Proposition 2 *When demands across markets exhibit positive inter-network externalities and consumers care only for their own good, a monopolist earns greater profit by unbundling a zero-marginal-cost good for sale to each market at independent prices than by selling a bundled product to both markets. That is* $r_c = r_j = 0$ *implies* $\pi_u^* \geq \pi_b^*$.

Proof. *See Appendix of Proofs.* ■

The intuition for Proposition 2, regarding when unbundling dominates bundling, is that, in the context of $r_c = r_j = 0$, neither market values the good targeted at the other market. That is, individual C preferences for the cross-market good targeted at J are distributed in the range of $[0, r_c V_j]$ and similarly for the J market. Individual preferences for the cross-market good have a single point mass of support at 0. Under these conditions, the point mass values for the goods defeats the value of bundling, which averages heterogeneity in tastes.

Prior research has established that bundling a pair of goods dominates individual component sales when values are independent and identically distributed (iid) with no network externalities $e_{cj} = 0$ and $e_{jc} = 0$ (Adams and Yellen, 1976). This has been extended to more general probability distributions (McAfee, McMillan and Whinston, 1989) and to large numbers of information goods (Bakos and Brynjolfsson, 1999). For a pair of goods, we generalize this result to arbitrarily large externalities where $e_{cj} = e_{jc}$.

It should be noted that there are nine possible regions of demand for the bundle (three for each consumer type). This makes finding a closed form solution for optimal prices and profits potentially intractable because the demand function is convex, linear, then concave. Crossing the two demands across markets can lead to a profit equation that is a fifth degree polynomial in prices. However, in a lemma below, we show that the optimal prices of interest fall into the first region, $0 < p < r_c V_j$. Salinger (1995) showed that bundling increases demand elasticity which leads to an optimal price choice below the independent optimum $\frac{V}{2}$. We confirm this result for our demand model in Lemma 1 below.

Lemma 1 (Region 1 Demand) *For bundled demand with symmetric markets* $r_c = r_j = 1$ *and* $V_c = V_j = V$*, demand in the* C *market is* $e_{jc} q_j + Q_c - \frac{Q_c p^2}{2V^2}$*. This is the nonlinear region with optimal price* $0 < p_c^* < r_c V_j = V$*, and symmetrically for* p_j^**.*

Proof. *See Appendix of Proofs.* ■

We now present a proposition that uses the fact that the relevant demand curves for both C and J demand are region 1, where price falls in $0 < p < r_c V_j$ for the C market and in $0 < p < r_j V_c$ for the J market.

Proposition 3 *When consumers in different markets value different goods over the same range, a monopolist earns strictly greater profit by bundling a zero-marginal-cost good. That is, if $r_c = r_j = 1$ with $V_c = V_j$ and $e_{cj} = e_{jc}$ then $\pi_u^* < \pi_b^*$.*

Proof. *See Appendix of Proofs.* ■

In propositions 2 and 3 we present the limiting cases where product managers reach opposite decisions. We now illustrate the transition between the two results. A direct comparison of the unbundling and bundling models is extremely difficult due to the need to convolve endogenously dependent demand functions over multiple regions of interest. Hence, we do not present a proposition that shows when bundling and unbundling is preferred in general. The general intuition is more straightforward, however, and is depicted in Figure 5.

This figure depicts profit maximization strategies under specific parameter values highly favorable to unbundling due to the large difference in externality terms. The dashed line represents the r_j value below which it becomes unprofitable to sell the C good to J consumers given an optimal price p_c^*. That is, below the dashed line q_{jc} goes negative at p_c^*, an unbundled but optimal price for C consumers. Bundling always makes sense above this line as both consumer types value both goods. Unbundling dominates closer to the origin where r_j is smaller. In this region, the J market cares little for the C good.

Even assuming that the upper valuations $r_j V_c$ of the J consumers are relatively low, the reduction in consumer heterogeneity from bundling can offset the network externality

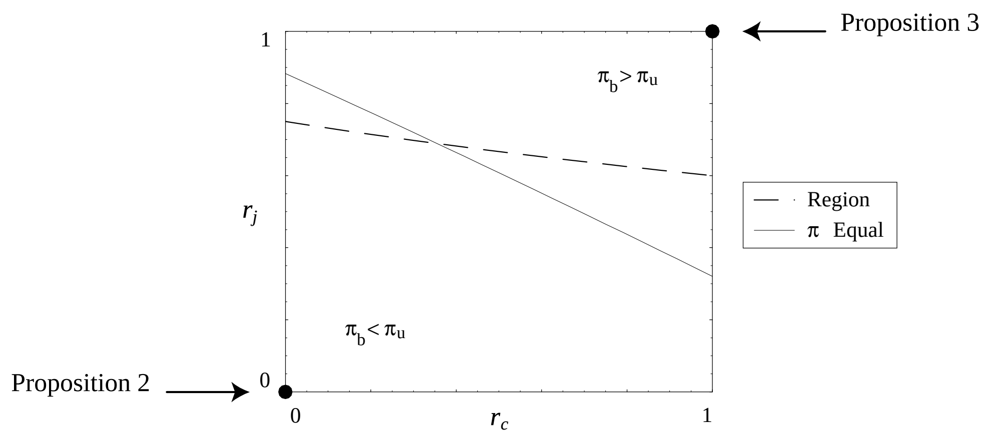

Figure 5: Unbundling dominates in the profit region defined by the lower envelope of the dashed and solid lines. Bundling dominates elsewhere. The image shows the specific case of Consumers: $\{e_{cj} = 0, Q_c = 2, V_c = 1\}$, Developers: $\{e_{jc} = 2, Q_j = 1, V_j = 1\}$.

boost from unbundling and discounting one market. The solid line represents this boundary between π_u^* and π_b^* assuming optimal prices p_c^*, p_j^*, and p_b^*. Above this line, bundling extracts more consumer surplus than unbundling. These boundaries necessarily move with specific parameter values owing to the endogeneity of quantity and price. In this example, unbundling dominates along the lower envelope of the two lines. Reducing the network externality terms e_{jc} and e_{cj} shifts this envelope down and to the left.

The less one market values the good intended for another market, the more likely it is that unbundling dominates. This is consistent with Bakos & Brynjolfsson (1999), who show that the higher the variance in tastes, the lower the benefits of bundling. The chief innovation of the current model is to introduce network externalities into markets with different allowable values for both goods. Bundling dominates when the values are the same. It also tends to dominate when the network surplus contributions across markets are the same. Alternatively, unbundling dominates when the values are different and one market contributes more surplus to the other. In this region, a firm should discount or even subsidize one good. The relative advantages of unbundling and bundling under intermediate conditions then hinge on the exact parameter values.

6 Further Implications

6.1 Relaxing Assumptions

Implications of prior work (Adams & Yellen '76, McAfee, McMillan & Whinston '89, Bakos & Brynjolfsson '99) suggest that, in the presence of inter-network effects, relaxing the assumed uniform distribution of values favors unbundling. That is, the area surrounding Proposition 2 in Figure 5 would generally increase.

The reason is that bundling's consumer homogenization effect rises with more variable demand. Based on the law of large numbers, bundling greater numbers of goods warps the demand distribution toward a Gaussian with decreasing variance (Bakos & Brynjolfsson '99). If demand starts as Gaussian, or any of several other distributions with lower variance than uniform, then bundling has less room to boost profit. In the no variance extreme, for example, point mass distributions of demand at V_c and V_j would permit independent prices of $p_c^* = V_c - \varepsilon$ and $p_j^* = V_j - \varepsilon$ (already well above $\frac{V_c}{2}$ and $\frac{V_j}{2}$) so that bundling could increase profits by no more than $\varepsilon(Q_c + Q_j) \longrightarrow 0$. Since inter-network effects rest on total transactions, not the distribution of values, firms retain their profit leverage in an unbundled discounted product.

A similar logic holds for correlation of values, again based on prior work. Moving from independence to positive correlation weakens bundling while moving to negative correlation strengthens it. The reason is that bundling negatively correlated values more effectively pushes total value toward the center of the joint distribution. In the extreme, with perfect positive correlation, $\rho_{cj} = 1$ implying $p_B^* = \frac{V_c+V_j}{2}$ and still half the market does not buy. Thus, if values of content creators and content consumers are positively correlated rather than independent, then again bundling has less room to boost profit.

In both cases, inter-network effects must be large and skew to affect price discrimination. Otherwise, as shown in Corollary 1, firms can set prices independent of inter-network effects.

6.2 Implications

The inter-network externality framework highlights the value of an intermediary in transactions where demand rises in one market reciprocally with demand increases in another. Internalizing these indirect network effects as true externalities leads to more efficient pricing. In contrast, pecuniary externalities (Liebowitz & Margolis 1994) are consumption externalities mediated through the price system, which do not necessarily lead to inefficient resource allocation. For linear demand curves, internalizing inter-network externalities has the surprising consequence that even the market paying the premium price exhibits increased consumer surplus (Parker & Van Alstyne 2001). The principal reason is that internalizing the externality leads firms to price more efficiently, creating strictly positive welfare gains in both markets. Unless price discrimination is then perfect, consumers retain a fraction of any surplus gains.

As shown in Figure 2 and Proposition 1, this implies that choosing an optimal price pair can easily reduce price in one market and reduce it, in fact, below marginal cost to the level of a subsidy. In antitrust law, this complicates tests of predation because prices that appear anti-competitive by standard tests can be optimal even in the absence of competition. Thus courts may need to account for inter-network effects as well as below marginal cost pricing when determining predation.

Futher implications of an inter-network product design strategy include accounting for externalities and economizing on transaction costs in a fashion that reduces the attractiveness of "micropayments." These are direct payments between consumers and merchants at the sub-penny or even the $\frac{1}{1000}$ of a cent level (cf. The Millicent Protocol) made possible by technological advancement[7]. Reverse payments, from merchant to consumer, suddenly become feasible as in the case of subsidies to view ads. Micropayments offer the promise of allowing a person to rent tiny bits of software for fractions of a second or even to "imagine renting a French spelling checker for one document once" (Metcalfe, 1997).

Despite their appeal, the inventor of one digital payment scheme and the co-founder of one troubled startup, CyberCash, observes:

> CyberCash [is] not alone [in its troubles] ... First Virtual Holdings brought out a system in 1995. That business is now gone... Carnegie-Mellon University created Netbill whose commercial rights were acquired by CyberCash. Digicash in the Netherlands ... filed for bankrupcy and has ceased operation. Digital Equipment Corporation developed its Millicent system, ran some trials and never came to market... IBM developed a system in its labs but has not brought it to market. Mastercard invested in Mondex and worked on bringing Mondex payments to the Internet. That effort ... has dissipated. Visa built and tested Internet micropayment system but has also not yet brought it to market. Many more schemes have been invented ... but none has gained a secure foothold (Crocker 1999).

Several economic principles might account for this. First, any payment scheme that exhibits the properties of a standard must manage network effects. If a market is monolithic,

[7] We grant that others define micropayments at $5 on the basis that this approximates the threshold of feasible credit card transactions, although this seems less in keeping with the true sense of a "micro" payment.

as for a *de jure* currency, then regulation, consumer communication, or penetration pricing may be necessary to cause tipping in favor of a new standard. If, on the other hand, the currency is a private scrip, then markets are coupled: The issuer must persuade merchants to accept it and consumers to carry it. A many-to-one and one-to-many set of transactions that internalizes network effects will price more efficiently than many-to-many transactions that ignore network effects. This parallels Table 1's credit card, auction, operating system, advertising, and computer games markets. Inducements analogous to free VISA cards may be necessary to promote adoption yet most scrip was offered at par.

Second, a desired behavior may fail to occur because there exist opportunity costs of time, resources, or attention for either party. Individuals that ignore pennies on a sidewalk may well ignore fractional pennies on their screens. Trade will not occur when divisible surplus, which is the difference between consumer value and merchant marginal cost, is too small relative to the opportunity costs of action. The smaller the size of a viable micropayment, the larger is any relative opportunity cost.

Third, there are substantial benefits of *unbundling* market pairs but *bundling* within each market separately. Bundling theories are, in fact, quite powerful. Aggregating end-consumers not only reduces the heterogeneity of demand for content, making it easier to sell content or place effective ads, it also implies that the sum of divisible surplus can overcome opportunity costs of merchants. Aggregating merchants also reduces their hetereogeneity of value for tools used to reach consumers.

Fourth, even zero marginal costs do not force average costs to zero. Operating systems, computer games, and credit card processing—like valuable distribution channels—all represent high fixed cost platform investments that every member of one market would necessarily re-incur if trading independently and directly with members of the other market. Aggregating across sellers, advertisers, or developers secures economies of scale in the high first cost of establishing a distribution channel or platform amortized over the low marginal cost of distributing more auctions, ads, or applications over that same channel or platform. If marginal value of any given transaction is negligible, then even moderate fixed costs of acquiring and configuring a new micropayment system could make it unattractive.

As noted in the introduction, a key feature of the inter-network product design strategy is the low marginal costs of incremental transactions. Although this represents the most fertile ground for micropayments, an effective strategy to further unbundle a product or service must account for the power of bundling to reduce heterogeneity, and for limiting both transaction costs and average costs, overcoming opportunity costs, and managing network effects. These suggest that disaggregation has certain limits.

7 Conclusions

Existing theory explains why bundling reduces demand heterogeneity, allowing firms to capture consumer surplus. Our main accomplishment is to introduce a novel theory of inter-network effects to show how unbundling leverages demand heterogeneity to create surplus. We also cast both theories in a general framework to show how these competing forces interact. Product managers prefer bundling when either inter-network effects are small or large but equiproportional. They prefer unbundling when inter-network effects are large and skewed. These insights apply to a number of industries including operating systems, auctions, advertising, games, and credit cards.

The insight, however, gives rise to several additional conslusions. First, a firm can rationally invest in unbundling and supporting a product it gives away in one market even in the absence of competition. The reason is that inter-network effects increase sales in a complementary premium goods market above the costs incurred in the discount goods market. This effect is strong enough that it can resemble predation. Firms can price products below marginal cost and yet still be maximizing profit apart from stifling competition.

Second, we identify distinct matched markets linked by two-sided network effects. As in Table 1, these include several forms of content creators and consumers, applications developers and end-users, and service adopters and processors. We show that either market can be a candidate for discounting or even subsidy. Deciding which market to discount depends on the relative sizes of externality benefits. If the externality is large enough, the market that contributes more to sales of its complement is the market to subsidize with a free good. At lower levels, firms may charge positive prices in both markets but keep one price artificially low.

Third, the model presents intermediaries as an attractive means of internalizing two-sided network effects of coupled markets. This offers one explanation for why micropayment systems have yet to be widely adopted: adopters may need intermediaries to coordinate price pairs, economize on transaction costs, and amortize fixed channel costs. This framework also offers guidance on what hurdles firms must cross in order for these systems to be successful.

Finally, we have shown how to compare bundled and unbundled demands accounting for two-sided network effects. For two goods, this generalizes a prior bundling result by accounting for matched spillovers. Two convenient assumptions regarding uniform demand and independence can be relaxed in ways that strengthen our key findings in favor of unbundling. The low marginal cost assumption, however, is essential to subsidizing arbitrarily large markets. This model contributes a form of price discrimination that is distinct from tying or second degree price discrimination in the sense that consumers need never purchase both goods—unlike razors and blades, the products are stand-alone goods. It also differs from multi-market or third degree price discrimination in the sense that the firm may extract no consumer surplus from one of the two market segments, implying that this market would have traditionally gone unserved. Thus the model introduces one main idea with several different implications.

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8 Appendix of Proofs

8.1 Proof of Proposition 1:

Proof. Recall that for optimal prices, equation 8 gives

$$p_c^* = \frac{V_c\left(2S_cS_j - S_{cj}(S_{jc} + S_{cj}) + S_j(S_{jc} - S_{cj})\right)}{4S_cS_j - (S_{cj} + S_{jc})^2}.$$

We need to show that:

$$p_c^* < \hat{p}_c = \frac{V_c}{2}.$$

Substitution provides an inequality in which all S_i terms are positive by definition.

$$\frac{V_c\left(2S_cS_j - S_{cj}(S_{jc} + S_{cj}) + S_j(S_{jc} - S_{cj})\right)}{4S_cS_j - (S_{cj} + S_{jc})^2} < \frac{V_c}{2}$$

After eliminating V_c, the Hessian condition allows us to preserve the sense of the inequality when cross-multiplying denominators to give:

$$4S_cS_j - 2S_{cj}(S_{jc} + S_{cj}) + 2S_j(S_{jc} - S_{cj}) < 4S_cS_j - (S_{cj} + S_{jc})^2$$

If the difference $(S_{cj} - S_{jc})$ is positive, after some manipulation, we can divide through on both sides by $(S_{cj} - S_{jc})$ to produce

$$-2S_j < (S_{cj} + S_{jc}),$$

which is strictly true, even when the externality surpluses S_{cj} and S_{jc} are zero. If the difference $(S_{cj} - S_{jc})$ is negative, the inequality is reversed, producing a contradiction. Optimal price can be reconstructed by reversing the steps, confirming that the proposition is only if. ■

8.2 Proof of Proposition 2:

Proof. Assuming a single price across two markets for a bundled information good, a monopolist chooses price and earns profit as follows:

$$p_b^* = \frac{\left(\left(1+e_{cj}\right)Q_c+\left(1+e_{jc}\right)Q_j\right)V_cV_j}{2\left(\left(1+e_{cj}\right)Q_cV_j+\left(1+e_{jc}\right)Q_jV_c\right)} \tag{28}$$

$$\pi_b^* = \frac{\left(\left(1+e_{cj}\right)Q_c+\left(1+e_{jc}\right)Q_j\right)^2V_cV_j}{4\left(1-e_{cj}e_{jc}\right)\left(\left(1+e_{cj}\right)Q_cV_j+\left(1+e_{jc}\right)Q_jV_c\right)} \tag{29}$$

Optimal unbundled profit when prices are chosen independently is

$$\pi_u^* = \frac{Q_cV_c+Q_jV_j+e_{cj}Q_cV_j+e_{jc}Q_jV_c}{4-\left(\frac{e_{cj}^2Q_cV_j}{Q_jV_c}+2e_{cj}e_{jc}+\frac{e_{jc}^2Q_jV_c}{Q_cV_j}\right)}.$$

To simplify the equations, we focus on relative market sizes in order to cancel terms. Let

$$V_c=\delta V, V_j=V, Q_c=Q, Q_j=\varepsilon Q, \delta>0, \varepsilon>0.$$

Then,

$$\pi_u^*-\pi_b^*=\frac{1}{4}QV\delta\left(4\frac{\varepsilon\left(\delta+\varepsilon+e_{cj}+\delta\varepsilon e_{jc}\right)}{\delta\varepsilon+\left(e_{cj}+\delta\varepsilon e_{jc}\right)^2}-\frac{\left(1+\varepsilon+e_{cj}+\varepsilon e_{jc}\right)^2}{\left(1-e_{cj}e_{jc}\right)\left(1+\delta\varepsilon+e_{cj}+\delta\varepsilon e_{jc}\right)}\right).$$

All terms are positive by the Hessian condition and the finite market assumption. Unbundling dominates when this expression is positive. This expression simplifies to a perfect square:

$$\left({e_{cj}}^2+e_{cj}\left(1+\epsilon+\left(\delta-1\right)\epsilon e_{jc}\right)-\epsilon\left(2\left(\delta-1\right)+\delta e_{jc}\left(1+\epsilon+\epsilon e_{jc}\right)\right)\right)^2. \tag{30}$$

■

8.3 Proof of Lemma 1:

Proof. Since optimal independent prices for symmetric markets are $\frac{V_c}{2}=\frac{V_j}{2}=\frac{V}{2}$, it follows that if bundling increases elasticity then the sum of these prices is $\leq V$. This places demand in the first demand region $e_{jc}q_j+Q_c-\frac{Q_cp^2}{2V^2}$ when $r=1$. We therefore need to show that elasticity ξ_c increases when moving from independent to bundled markets, evaluated at optimal independent prices $\frac{V_c}{2}$ and $\frac{r_cV_j}{2}$ in the C market. This leads to an elasticity <-1 such that a firm lowers price to increase profits. The demand equation for region 2 above represents independent demand exactly when $r_c=r_j=0$. Evaluating $\frac{p}{q_c}\frac{\partial q_c}{\partial p}$ for this equation yields

$$\xi_c=\frac{-2\,p\,\left(e_{jc}\,Q_j\,V_c+Q_c\,V_j\right)}{e_{jc}\,Q_j\,V_c\,\left(r_j\,V_c+2\,V_j-2\,p\right)+Q_c\,V_j\,\left(2\,V_c+r_c\,V_j-2\,p\right)} \tag{31}$$

Using the substitutions $V_c = V_j = V$ and $r_c = r_j = r$ leads to significant cancellation of terms, producing

$$\frac{-2\,p}{(2+r)\;V - 2\,p} \tag{32}$$

Substituting the sum of optimal independent prices $p = \frac{V_c}{2} + \frac{r_c V_j}{2} = \frac{V + rV}{2}$ then produces:

$$-1 - r. \tag{33}$$

When $r = 0$ such that the markets are independent, ξ_c equals -1 at the optimal prices as it should. It also shows that elasticity becomes increasingly negative as r takes on values in $(0, 1]$. As a further check, substituting $V_c = V_j = V$ and $r_c = r_j = r$ into p_b^* above yields $\frac{V}{2} + \frac{rV}{4}$, which is less than independent prices by $\frac{rV}{4}$ and produces $\xi_c = -1$. ■

8.4 Proof of Proposition 3:

Proof. With $r_c = r_j = 1$, the unbundled profits are given by

$$\pi_u = p_c(q_{cc} + q_{jc}) + p_j(q_{jj} + q_{cj}) \tag{34}$$

$$= p_c(Q_c + e_{jc}q_{jj} - \frac{Q_c}{V_c}p_c + Max[0, Q_j(1 - \frac{p_c}{V_c})]) + p_j(Q_j + e_{cj}q_c - \frac{Q_j}{V_j}p_j + Max[0, Q_c(1 - \frac{p_j}{V_j})]) \tag{35}$$

To get an expression for optimal profit π_u^*, we substitute $V_c = V_j = V, e_{cj} = e_{jc} = e, Q_c = Q, Q_j = \delta Q$ with $\delta > 0$ and take first order conditions, solving for the region where consumers value both goods. The standard maximization yields the following expression.

$$\pi_u^* = \frac{(2 + e - e^2)\,QV\,(1+\delta)\left(1 - (-2 + e^2)\,\delta + \delta^2\right)}{4e^4\delta + 4\,(1+\delta)^2 - 5e^2\,(1+\delta)^2} \tag{36}$$

On the other hand, calculating bundled profit requires convolving the two demands for each market. By Lemma 1 presented above, we can work with bundled demands when $0 < p < r_c V_j$ for the C market and similarly for the J market.

$$q_c(p) = e_{jc}q_j + Q_c - \frac{Q_c p^2}{2r_c\,(V_c V_j)} \tag{37}$$

$$q_j(p) = e_{cj}q_c + Q_j - \frac{Q_j p^2}{2r_j\,(V_c V_j)} \tag{38}$$

Eliminating quantities from the right-hand-side, adding both demands, and multiplying by p yields the bundled profit equation for this price region:

$$\pi_b = p\left(\begin{array}{c} \frac{2\,(Q_c + e_{jc}\,Q_j)\,r_c\,r_j\,V_c\,V_j - p^2\,(e_{jc}\,Q_j\,r_c + Q_c\,r_j)}{2\,(1 - e_{cj}\,e_{jc})\,r_c\,r_j\,V_c\,V_j} + \\ \frac{2\,(Q_j + e_{cj}\,Q_c)\,r_c\,r_j\,V_c\,V_j - p^2\,(e_{cj}\,Q_c\,r_j + Q_j\,r_c)}{2\,(1 - e_{cj}\,e_{jc})\,r_c\,r_j\,V_c\,V_j} \end{array}\right) \tag{39}$$

Optimizing this equation using first order conditions leads to the following profits in this price region:

$$\pi_b^* = \frac{2\sqrt{\frac{2}{3}}\ \left(\left(1+e_{cj}\right)\ Q_c + \left(1+e_{jc}\right)\ Q_j\right)^{\frac{3}{2}}}{3\ \left(1-e_{cj}\ e_{jc}\right)\ \sqrt{\frac{\left(1+e_{jc}\right) Q_j\, r_c + \left(1+e_{cj}\right) Q_c\, r_j}{r_c\, r_j\, V_c\, V_j}}}.$$

To simplify the comparison between $\pi_u^* < \pi_b^*$ we again use the substitution $V_c = V_j = V, e_{cj} = e_{jc} = e, Q_c = Q, Q_j = \delta Q$, with $\delta > 0$. Unbundled profit then becomes:

$$\pi_b^* = \frac{2\sqrt{\frac{2}{3}}\, Q\, V\ (1+\delta)}{3\ (1-e)} \tag{40}$$

Imposing the same substitution on the Hessian restriction yields the following constraint.

$$\frac{e^2\ (1+\delta)^2}{\delta} < 4 \tag{41}$$

Combining the substitutions above with the Hessian constraint leads to a valid inequality $\pi_u^* < \pi_b^*$, that is:

$$\frac{(2-e)\,(1+e)QV\,(1+\delta)\,\left(1-\left(-2+e^2\right)\delta+\delta^2\right)}{4e^4\delta+4\,(1+\delta)^2-5e^2\,(1+\delta)^2} < \frac{2\sqrt{\frac{2}{3}}\, Q\, V\ (1+\delta)}{3\ (1-e)} \tag{42}$$

With $\frac{e^2\,(1+\delta)^2}{\delta} < 4,\ \delta > 0$, and $e > 0$, the following expression holds.

$$\frac{(2-e)\,(1-e)(1+e)\,\left(1-\left(-2+e^2\right)\delta+\delta^2\right)}{4e^4\delta+4\,(1+\delta)^2-5e^2\,(1+\delta)^2} < \frac{2}{3}\sqrt{\frac{2}{3}} \tag{43}$$

■

Information Complements, Substitutes, and Strategic Product Design

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ABSTRACT: Competitive maneuvers in the information economy have raised a pressing question: How can firms profitably give away free products? This paper provides a possible answer and articulates a strategy space for information product design. Free complements can raise a firm's own profits while, in strategic settings, free substitutes can lower profits for competitors.

We introduce a formal model of cross-market externalities based in textbook economics—a mix of Katz & Shapiro network effects, price discrimination, and product differentiation—that leads to novel strategies such as an eagerness to enter into Bertrand price competition. Externality based complements, however, exploit a different mechanism than either tying or lock-in even as they help to explain many recent strategies such as those of firms selling operating systems, Internet browsers, games, music, and video.

The model presented here argues for three simple and intuitive results. First, even in the absence of competition, a firm can rationally invest in a product it intends to give away into perpetuity. Second, we identify distinct markets for content-providers and end-consumers and show that either can be a candidate for a free good. Third, a firm can use strategic product design to overcome first-mover advantage and to penetrate markets that become competitive post-entry.

The model also generates testable hypotheses about the size and direction of network effects and it offers insights to regulators seeking to apply antitrust law to network markets.

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1 Introduction and Literature

This paper seeks to explain pricing behavior in network markets. Given the seemingly anomalous practice of free-pricing, how is it possible that firms are willing to subsidize information and related products apparently expecting neither future consumer exploitation nor tying? Why do firms give away applications development toolkits, portable document readers, and Internet browsers over extended periods without metering tie-ins to those same consumers?

In answer, we find that designing coupled products and underpricing one relative to the independent goods case implements price discrimination in markets joined by network effects. The insight is that characterizing network markets may require not only product standardization, essential to demand economies of scale (Katz & Shapiro, 1985; Farrell & Saloner, 1986), it may also require sharp distinctions between consumer types, essential for managing demand interdependence.

Separating *intra*-market from *inter*-market network effects, we introduce a model of cross market externalities that shows how subsidizing one consumer type can increase sales to a different consumer type. As a special case, we introduce the concept of content-provider, a distinct market segment that adds value for content consumers but that may not be paid directly. This explains, for example, the provision of Real Audio content for Real Audio players and Acrobat articles for Acrobat readers that have nothing to do with the firms Real Audio or Adobe respectively. After distinguishing between coupled creator and consumer markets, we also identify which market to discount in order for a firm to maximize profits across both.

At issue is the chicken-and-egg problem of creating content for a new format.[1] Producers want consumers and consumers want producers before either readily switches to the new format. An incumbent firm producing music content for a new format probably does not welcome entry by

[1] Examples in music include wax cylinders, long play records, cassettes, compact disks, and MP3. In video, they include black & white TV, color, VHS, and HDTV.

another firm producing similar content, as there is no profitable direct exchange between them. Everyone in the end-consumer market, however, welcomes entry because it increases the prospect of a viable format should the incumbent fail. It also increases variety while possibly lowering prices. This increases both the value to individuals and the number of individuals willing to switch formats. Thus, in the present example, the externality runs between content-creators to end-consumers as distinct from affecting only consumers of the same type. While both *intra* and *inter*-market effects are possible, managing the distinction increases profits.

In effect, we introduce a distinct form of product complementarity modeled as a two-sided or "internetwork" externality. The natural intuition then applies. Lowering price on one half of a complementary pair sells more of the other. This parallels Cournot's (1838) observation that discounting zinc sells more copper in the brass market with the novelty that sales accrue to different buyers. Externality based coupling differs from razors and blades in that one market need never consume the complementary good. Consumers of a portable document reader may, at their discretion, forgo the cost of the portable document creator now and forever. Rather, a tying analogy might suggest that placing razors with content developers sells blades to content consumers, who each represent distinct interdependent markets. Thus, our mechanism differs from both second and third degree price discrimination. Firms do not meter coupled products for the same market. They do, however, make product offers to markets where they may never intend to capture consumer surplus. Firms recover product subsidies by using coupled products to stimulate sales across coupled markets.

Firms can also use internetwork product coupling strategically. Stimulating demand through network effects ensures that one firm can not only sustain a subsidy indefinitely, but can even increase its profits when facing competition. Free or subsidized pricing of product complements can raise a firm's own sales while free or subsidized pricing of product substitutes can lower sales for competitors. Lower sales then induce market exit when average cost curves are declining, as they are for information goods and other natural monopolies. The survivor then benefits from reduced competition.

In strategic terms, this undercuts both the conventional wisdom that firms should avoid Bertrand price competition as well as the logic that a follower suffers strategic market share losses in Stackelberg competition. As in the case of the Internet browser wars, our framework illustrates how prices below marginal cost can maximize profits when they serve to stimulate demand across markets. Correspondingly, this may help to explain the ubiquity of free information offered on the Internet.

If externality-based pricing is useful both as a monopoly price discrimination tool as well as an oligopoly market foreclosure tool, it may condition the application of antitrust laws designed to thwart anticompetitive pricing. To the extent that subsidized prices maximize profits even in the absence of competition, these cases may be indistinguishable. Additionally, we show that superior product design improves consumer welfare when firms jointly determine prices on products linked by cross-market externalities. This implies that competition between firms, each with half of a product pair, can reduce both producer *and* consumer surplus, while competition between firms, each with coupled products, reduces only producer surplus.

Empirically, the model has the advantage of suggesting new approaches for estimating network effects. Models of intra-market externalities suffer from endogeneity of demand estimation in that instruments can barely distinguish demand shocks from the network externality ripple effects they create. If, on the other hand, network effects cross markets, then instrumenting demand shocks in one market can proceed while holding demand constant in the coupled market, thus tracing a demand relationship. The proposed framework might therefore simplify estimation of network effects relative to earlier models. Testable propositions include which internetworked market to discount in markets with pricing power, which of several products firms will favor discounting in markets without pricing power, and whether discounting lasts longer than predicted by models of penetration pricing with lock-in.

Network markets are prone to "excess inertia" in which potential adopters fail to purchase new and superior technology because they fear being stranded in minority communities (Farrell & Saloner, 1986). Complete information in the form of news on who will switch allows buyers to coordinate on a solution. An alternative available to the seller, observed in our framework, allows pre-spending of network externality profits to promote adoption. Since our core argument couples product markets, it also touches on systems versus component competition (Farrell, Monroe, & Saloner, 1998). In their model, lower cost suppliers prefer component competition while superior integrators prefer systems competition. In complementary markets, firms prefer systems competition except under very specialized circumstances noted below.

Internetwork externalities resemble traditional complements more than strategic complements (Bulow, Geanakoplos, & Klemperer, 1985), which focus on marginal rather than total profit increase. In our model, independent firms raise both prices while a monopolist never does. An example where firms prefer to enter zero-profit markets is presented in DeGraba (1996). That mechanism, however, differs in that analysis proceeds from a model of tying based on Whinston (1990) and relies on declining average cost curves to favor firms with increased sales. As noted, our model differs from tying and instead manipulates content creation and consumption to increase sales. Chaudhri (1998) finds a similar result for "circulation industries" where advertising sales permit prices below marginal cost in subscription sales. Shapiro and Varian (1999) discuss the possibility of negative pricing to increase profit in complementary markets. Generalizing Parker and Van Alstyne (1999), we formally model when this is optimal, how a third party sells to both content-creators and to end-consumers, and which receives the discount. Rochet and Tirole (2001) model internetwork externalities in a multiplicative framework focusing on competing platforms, such as credit card networks. This nicely models "multihoming" or when consumers carry multiple credit cards. Caillaud and Jullien (2001) consider a matchmaking intermediary, as for dating services. They explain why agents register with more than one service and whether fees should hinge on a successful match. Armstrong (2002) considers competition in two-sided markets with applications to a large number of markets and focuses on the issue of which side is subsidized and social efficiency.

Our main contribution is to explore both bidirectional externalities and their relative size effects. The strategic choice of price in linked markets and the resulting product design depend significantly on differential sized externalities. In fact, markets collapse to independence when these effects are zero. We also develop two distinct models of these size effects that yield remarkably similar results. In closely related research (Parker & Van Alstyne 2001), these results are extended to show when firms prefer to unbundle an information good that prior theory predicts they should bundle.

We note that the popularity of a product design strategy that sets price at zero is aided by the unique properties of information. Because reproduction costs are negligible, a firm can subsidize an arbitrarily large market while incurring only a fixed up-front investment cost. Each additional sale of the free good costs the clever product designer nearly nothing in incremental costs. With increased consumption of information, use of the proposed design strategy may continue to rise.

The paper proceeds as follows. First, we develop a model of externality-based complements in coupled markets. We then find conditions for a subsidy market to exist, identify which market receives the subsidy, and prove uniqueness in a two-market setting. Introducing competition, we then develop a duopoly model with internetwork externalities. Analysis shows how firms can take advantage of either positive or negative externalities to manage product complements and substitutes. Finally, we generalize results by presenting a multiplicative model with nearly identical optimality criteria, then provide further applications of internetwork product design.

2 Markets

This section adapts a standard model to show why a firm spends resources creating a product it distributes for free when internetwork externalities cross market pairs. It also shows which market receives the discount.

Let the first market represent the general consumer or end-user market, C, and let the second market represent the content-creator, developer, or joint-producer market J. Index price p and quantity q terms by market such that p_c and p_j describe prices for consumers and content-creators respectively. Let costs for information goods be negligible with profit given by $\pi = \pi_c + \pi_j = p_c q_c + p_j q_j$. To provide a standard concave profit function, let this be twice differentiable in both choice parameters, have $\frac{\partial^2 \pi}{\partial p_i^2} < 0$ and a positive Hessian determinant. Accordingly, first-order conditions yield prices in both markets.

Demand is also standard but generalizes to a possible cross-market relationship. Each market has a continuum of consumers willing to buy one discrete unit of good. If v is arbitrary willingness to pay, then $D(p)$ is simply $\int_{v \geq p}^{\bar{V}} dDdp$. Like Katz & Shapiro (1985), we require the distribution of values to be bounded above and well defined over negative values to avoid corner solutions. In our model, this also has a natural interpretation of allowing price subsidies. For $i \in \{j, c\}$ we assume D_i is twice continuously differentiable and decreasing in p_i. We do not require concavity or convexity of demand but only restrict it sufficiently to ensure concavity of the profit function via the Hessian. Paralleling Katz and Shapiro, we also model total consumption as the sum of a good's intrinsic demand incremented by a function of network size in the other market.

Our point of departure is having externalities cross markets. The *inter*network externality term e_{jc} measures how much effect purchases in the developer market have on the consumer market.[2]

[2] Although the effect is linear here for ease of analysis, extension section 4 explores a nonlinear model that yields a remarkably similar structure.

More precisely, e_{jc} is the partial derivative of demand in the C market with respect to demand in the J market, or $\frac{\partial q_c}{\partial q_j}$. Conversely, e_{cj} determines how much effect purchases in the consumer market have on the joint-producer market. Together, these terms describe a pair of demand equations:

$$q_c = D_c(p_c) + e_{jc}D_j(p_j) \tag{1}$$

$$q_j = D_j(p_j) + e_{cj}D_c(p_c) \tag{2}$$

Several notational conventions help develop useful intuition. We refer to the marginal cross-price contribution to sales $\frac{\partial q_j}{\partial p_c} = e_{cj}D'_c(p_c)$ as the internetwork externality or simply "spillover" effect. The importance of network spillovers, as given by the ratio $r \equiv \frac{\partial q_j}{\partial p_c}/\frac{\partial q_c}{\partial p_j}$, plays an important role in subsequent analysis and will later draw attention to which market generates relatively more surplus. For convenience, we define p^*, $\hat{p}$, and $\mathring{p}$ as the optimal monopoly, two independent firm, and no externality prices respectively. In general, $p^* \neq \hat{p} \neq \mathring{p}$ unless $e_{jc} = e_{cj} = 0$ and $q_c \neq D_c(p_c)$ unless $e_{jc} = 0$. Also, solving for optimal prices introduces the term $M \equiv \frac{1}{1-e_{cj}e_{jc}}$ which requires $e_{cj}e_{jc} < 1$ to render the markets finite but also provides useful intuition as a "market multiplier." Where the context is clear, we simplify notation by suppressing parameters and write D_c for $D_c(p_c)$.

For illustration, the linear demand curves $D_c(p_c) = Q_c(1 - \frac{p_c}{V_c})$ and $D_j(p_j) = Q_j(1 - \frac{p_j}{V_j})$ yield simple and elegant results in terms of consumer surplus. Let exogenous parameters Q_i and V_i, $i \in \{c, j\}$, represent the maximum market size and maximum product value in the absence of externalities respectively. A consumption externality from J effects an outward shift in C demand, raising value by an amount proportional to $D_c^{-1}(e_{jc}D_j)$ thus $\bar{V}_c$ becomes $V_c\left(1 + e_{jc}\frac{Q_j}{Q_c}(1 - \frac{p_j}{V_j})\right)$. In the C market, interpretation in terms of surplus space can then be found by integrating demand[3] and defining substitutions:

[3] $\int_0^{\bar{V}_c}\left[Q_c(1 - \frac{p_c}{V_c}) + e_{jc}Q_j(1 - \frac{p_j}{V_j})\right]dp_c = \frac{1}{2}Q_cV_c + e_{jc}Q_jV_c(1 - \frac{p_j}{V_j}) + \frac{1}{2Q_cV_c}e_{jc}^2Q_j^2V_c^2(1 - \frac{p_j}{V_j})^2$. The area of the triangles in Figure 1 gives a similar expression.

Native Surplus	*InterNetwork Externality Surplus*
$S_c \equiv Q_c V_c$	$S_{cj} \equiv e_{cj} Q_c V_j$
$S_j \equiv Q_j V_j$	$S_{jc} \equiv e_{jc} Q_j V_c$

Native terms are proportional to consumer surplus in each market individually, while internetwork terms are proportional to the surplus created by consumption externalities in the complementary market.

2.1 Monopoly Choice

A simple linear example then illustrates how the model functions, although linearity is not required. Here, consider first a monopolist's profit maximizing decision in the absence of network externalities, $e_{cj} = e_{jc} = 0$. A straightforward maximization on $\pi_c = p_c Q_c(1 - \frac{p_c}{V_c})$ yields $p_c^* = \frac{V_c}{2}$, $q_c^* = \frac{Q_c}{2}$, with profits of $\pi_c^* = \frac{V_c Q_c}{4} = \frac{S_c}{4}$ and similarly for π_j as in the left-most panel of Figure 1. Optimal prices and quantities are simply half the consumer value and consumer market respectively.

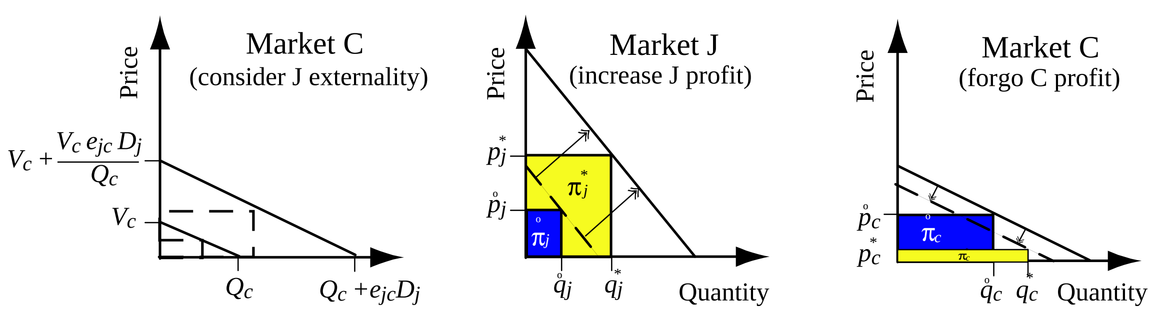

Figure 1: In the left panel, a network externality shifts the demand curve out. The optimal independent pair (q_c, p_c) increases from the no externality case $(\frac{Q_c}{2}, \frac{V_c}{2})$. In the middle panel, a monopolist sets $p_j^* > p_j'$ and reaps profits π_j^*. In the right panel, price in the C market falls $(p_c^* < p_c')$ in order to take profits in the J market. The higher price in the J market then somewhat reduces the total externality impact in the C market. Discounting the C market is rational whenever net profits rise.

Allowing for positive spillover, the J market externality shifts the demand curve in the C market up and to the right relative to the no externality case. Increasing p_j, however, reduces the total outward shift in the C market as shown in the rightmost panel. Taking externality effects into account, the decision of how much to charge in each market is analyzed below. This graphic captures the idea that content-providers stimulate demand among consumers, just as consumers stimulate interest from content-providers.

The number of content-providers who actually enter the market, however, governs the degree of shift. The externality or coupling between these two markets implies that the more a monopolist can coax consumers into adopting one of his products, the more it can charge for—and sell of—the other. If the increment to profit on one complementary good exceeds the lost profit on the other good, then a discount or even subsidy becomes profit maximizing. Free goods markets can therefore exist whenever the profit maximizing price of zero or less generates internetwork externality benefits greater than intra-market losses. Firms do, in fact, offer subsidized goods when they offer free service and technical expertise. Historically, for example, Microsoft offered technical help on applications program interfaces (API) in addition to free systems development toolkits and this may have increased the number of user applications developed for its operating system. Apple did not and has since changed its strategy. This form of subsidy also addresses an interesting problem of adverse selection arising from a subsidy to a general audience. Only developers request API support, so the subsidy is not wasted on non-developers.

In general, the optimal price pair for markets linked by internetwork externalities is not obvious. Figure 2 illustrates that a pricing decision falls into one of four quadrants. Regions II and IV represent a "free" goods market since a firm charges nothing or spends money to place product in the hands of consumers or developers respectively. In region I, both markets are charged positive prices while region III represents a subsidy to both markets and is never profit maximizing.

In principle, the markets are symmetric; in practice, computer games manufacturers charge consumers but give developers free toolkits, while streaming media companies give consumers free

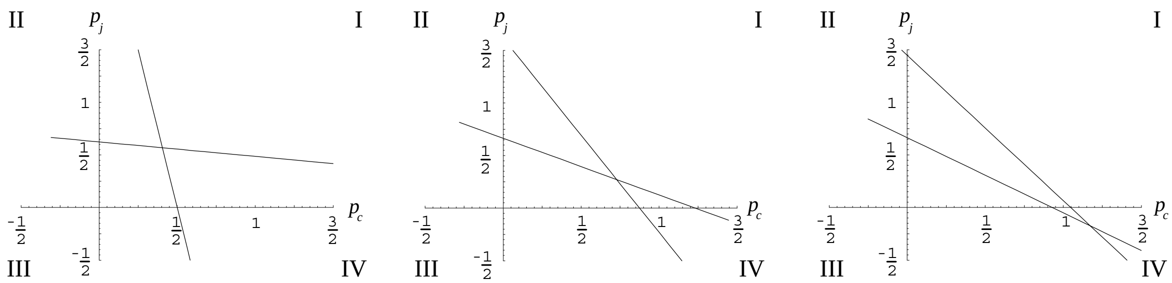

Figure 2: Intersecting price reaction curves show consumer price (p_c) rising and developer price (p_j) falling as the developer-to-consumer externality (e_{jc}) rises from 0 to 1.1 in the linear case with parameters $\{Q_c = Q_j = V_c = V_j = 1$ and $e_{jc} = \frac{1}{3}\}$.

software players but charge developers to create content. Interestingly, banks selling credit card services charge both merchant and consumer fees, illustrating region I. The decision of which market to subsidize, if any, and which to charge then rests on Proposition 2 below. Before analyzing this case, we first develop conditions for optimality.

Proposition 1 *Optimal price p_c^* reflects cross price elasticity of C consumption with respect to J price times the spillover ratio*

$$-1 = \eta_c + \eta_{cj} r. \tag{3}$$

In the J market, the condition is $-1 = \eta_j + \eta_{jc}\frac{1}{r}$. The condition for a free goods market to exist is, subsidy price $p_c^* \leq 0$ which occurs at the point $-1 \geq \eta_{jc} r$. Moreover, relative elasticities always have the structure $e_{cj}e_{jc}\eta_c\eta_j = \eta_{cj}\eta_{jc}$.

Proof. *Definitions for own and cross price elasticities yield $\eta_c = \frac{p_c D_c'}{D_c + e_{jc} D_j}$ and $\eta_{cj} = \frac{e_{jc} p_j D_j'}{D_c + e_{jc} D_j}$. Symmetry in the J market then implies $e_{cj}e_{jc}\eta_c\eta_j = \eta_{cj}\eta_{jc}$. The first order condition on profit in the C market is*

$$\frac{\partial \pi}{\partial p_c} = D_c + e_{jc} D_j + p_c D_c' + e_{cj} p_j D_c' = 0 \tag{4}$$

which rearranges to $-1 = \eta_c + \frac{e_{cj} p_j D_c'}{D_c + e_{jc} D_j}$*. Recall that* $r = \frac{\partial q_j}{\partial p_c} / \frac{\partial q_c}{\partial p_j} = \frac{e_{cj} D_c'}{e_{jc} D_j'}$*. Multiplying the final term by* $1 = \frac{e_{jc} D_j'}{e_{jc} D_j'}$ *and substituting for* η_{cj} *provides equation 3. Then, when prices decreases through zero* $p_c^* \leq 0$ *we have,* $\eta_c \geq 0$ *proving the condition on existence of a free goods market. Note that although* r *is undefined when* $e_{jc} D_j' \to 0$*,* $\eta_{cj} r$ *is still defined as multiplicative terms in* e_{jc} *and* D_j' *cancel leaving the optimum as* $-1 = \eta_c$*.* ■

This suggests that a firm choosing optimal prices considers not just own price effects but the price effects from the other side of the market as influenced by relative spillovers. It is also possible, however, to interpret this in terms of optimal profits created on both sides of the market. Recalling definitions $\pi = \pi_c + \pi_j = p_c q_c + p_j q_j$, we have an alternative test.

Corollary 1 *Optimal price* p_c^* *reflects cross price elasticity of* J *consumption with respect to* C *price times the profit ratio*

$$-1 = \eta_c + \eta_{jc} \frac{\pi_j}{\pi_c}. \tag{5}$$

Proof. *Substitute definitions for elasticities and simplify to confirm that* $\frac{\eta_{jc}}{\eta_{cj}} \frac{\pi_j}{\pi_c} = r$*. Inserting this equality in 1 reduces to the equation above.* ■

This implies that cross-price elasticities matter in both directions. The proposition constrains price through η_{cj} times the spillover ratio while the corollary constrains price through η_{jc} times the profit ratio. The bidirectional nature of spillovers implies that pricing decisions depend heavily on their relative sizes and, in fact, no difference in size can imply no difference in price. This leads to the novel result that prices will be optimal, even if set under the myopic assumption of no network effects, when spillovers between markets are equal.

Proposition 2 *When a monopolist sells to complementary internetwork markets with symmetric* *spillovers, the optimal price is the same regardless of the level of network effects. Mathematically,* $r = 1$ *if and only if* $(p_c^* = \mathring{p}_c$ *and* $p_j^* = \mathring{p}_j)$.

Proof. *First order conditions (eq. 4) indicate* $p_c^* = \frac{D_c + e_{jc}D_j + e_{cj}p_j^* D_c'}{-D_c'}$. *For reference, we also calculate independent firm and no externality prices* $\hat{p}_c$ *and* $\mathring{p}_c$. *In the case of independent firms the final term in the numerator of* p_c^* *is zero so that* $\hat{p}_c = \frac{D_c + e_{jc}D_j}{-D_c'}$ *and in the case of no externalities of either sort* $\mathring{p}_c = \frac{D_c}{-D_c'}$. *For optimal profits, simultaneous solution of* p_c^* *and* p_j^* *yields implicit price*

$$p_c^* = \frac{1}{1 - e_{cj}e_{jc}} \left(\frac{D_c + e_{jc}D_j}{-D_c'} + e_{cj} \frac{D_j + e_{cj}D_c}{-D_j'} \right). \tag{6}$$

with a symmetric expression for p_j^*. *More intuitively, this can be interpreted as the market multiplier times the difference in independent prices* $M(\hat{p}_c - e_{cj}\hat{p}_j)$ *showing that own price generally declines in own externality. To complete the proof, note that* $r = 1$ *requires* $e_{jc}D_j' = e_{cj}D_c'$. *Combining fractions and twice substituting* $e_{jc}D_j'$ *for* $e_{cj}D_c'$ *collapses to* $p_c^* = \frac{D_c}{-D_c'} = \mathring{p}_c$. *Symmetry forces* $p_j^* = \mathring{p}_j$ *which is the required forward result. If both no externality prices are optimal, then reversing each step of the equality requires* $e_{jc}D_j' = e_{cj}D_c'$. ■

The proposition states that if internetwork effects across markets are approximately equal in size—the presence of developers is as attractive to consumers as the presence of consumers is to developers—then a monopolist can safely ignore these effects in its pricing decisions. A firm can set prices *as if* the internetwork terms e_{cj} and e_{jc} were both zero. This does *not* imply that network externalities have no effect on quantity sales. In general, output exhibits substantial increases due to the market multiplier $M = \frac{1}{(1 - e_{cj}e_{jc})}$. This result is unusual in that one might have expected prices to rise with increased demand, but the monopolist takes the gain exclusively in terms of increased sales rather than higher prices. The point is that firms need consider a joint pricing decision that accounts for network externalities only when the difference between the spillover effects on sales,

$e_{cj}D'_c - e_{jc}D'_j$, is substantial.

Building on this analysis, we can now answer the more general question of which market a profit-maximizing firm discounts when spillover levels are unequal.

Corollary 2 *A monopolist that sells to two complementary markets discounts (relative to the no-externality case) the product with the greater spillover effect. That is* $r < 1 \iff p^*_c < \mathring{p}_c$*. In the linear case, the optimal C market price falls relative to the no-externality price if and only if market C contributes more network externality surplus to market J, or* $p^*_c < \mathring{p}_c \iff S_{cj} > S_{jc}$*.*

Proof. *We have that* $r < 1$ *implies* $e_{cj}D'_c = e_{jc}D'_j - \varepsilon$ *for* $\varepsilon > 0$*. An identical sequence of substitutions as those above yields*

$$p^*_c = \frac{D_c}{-D'_c} - \frac{1}{1 - e_{cj}e_{jc}}\left(\frac{\varepsilon}{-D'_c}\right)\left(\frac{D_j + e_{cj}D_c}{-D'_j}\right) \tag{7}$$

Only D'_c*,* $D'_j < 0$ *so that* p^*_c *falls. Alternatively, this can be interpreted as* $p^*_c = \mathring{p}_c - \frac{\varepsilon}{-D'_c}M\hat{p}_j$ *and we note that conclusions are similar for* $e_{cj}D'_c = e_{jc}D'_j + \varepsilon$*. To establish the condition on linear surplus, observe that when* $D_c(p_c) = Q_c(1 - \frac{p_c}{V_c})$*, then* $\frac{\partial q_j}{\partial p_c}$ *is simply* $-e_{cj}\frac{Q_c}{V_c}$ *with a similar expression for* $\frac{\partial q_c}{\partial p_j}$*. Arrange both terms as an inequality. The result,* $-e_{cj}\frac{Q_c}{V_c} < -e_{jc}\frac{Q_j}{V_j}$*, then simplifies to* $e_{cj}Q_cV_j > e_{jc}Q_jV_c$*, which completes the proof. Discounting the C market must create more J market surplus than vice versa for C's price to fall.* ■

A few additional points follow from this analysis. First, a firm can rationally discount only one market at a time. This means that a discount in one market implies a price premium in the other, or $p^*_c < \mathring{p}_c$ if and only if $p^*_j > \mathring{p}_j$. The reason is the fact that $p^*_c < \mathring{p}_c$ implies $e_{cj}D'_c < e_{jc}D'_j$ and the markets are symmetric. An immediate corollary shows that, in the linear case, optimal prices fall in the market that contributes more externality surplus to the other market.

While the preceding analyses compares no externality and optimal prices, it is straightforward to compare independent and optimal prices. Another corollary to Proposition 2 establishes that factoring internetwork externalities into the pricing decision, but failing to coordinate the prices across markets, leads to inefficient price setting. Where p^* is again the optimal price, let $\hat{p}$ represent independent firm price as distinct from the no-externality price $\mathring{p}$.

Corollary 3 *In the case of symmetric spillovers, if independent firms set prices in each market without coordination, then prices vary with the level of spillover and are inefficiently high,* $\hat{p} \geq p^*$.

Proof. *Proposition 2 established that for* $r = 1$ *prices are independent of internetwork effects but for independent prices we have* $\hat{p}_c = \frac{D_c + e_{jc} D_j}{-D_c'}$. *The difference* $\hat{p}_c - \mathring{p}_c$ *is then* $\frac{e_{jc} D_j}{-D_c'}$. *For independent firms, prices are always positive and increasing in the size of cross market network effect.* ■

Without externalities, prices $\hat{p}_c$ and p_c^* are identical and the inefficiency disappears. With externalities, however, independent firms err in direct proportion to their own level of network effect on their market complement. One consequence of corollary 3 is that independent firm prices are strategic substitutes and a price increase by one firm causes a price decrease by the other.

2.2 Consumer Surplus

A firm's ability to manipulate the internetwork externality to increase profits raises a concern for whether its gains represent consumer losses. Does the ability to link markets, particularly when two products are combined under monopoly ownership, hurt consumers?

In the case of symmetric spillovers, proposition 2 and corollary 3 imply that the answer is clearly no. Consumer surplus improves under monopoly ownership. Since two independent firms both price too high, integrated ownership reduces prices even as it increases profits.

Asymmetric spillovers might leave consumer welfare ambiguous since $p_c^* = \frac{1}{1 - e_{cj} e_{jc}} (\hat{p}_c - e_{cj} \hat{p}_j)$ and a careful choice of e_{cj}, e_{jc} can force $p_c^* > \hat{p}_c$. For linear demand, however, welfare analysis of asymmetric spillovers provides a particularly strong result. Welfare exhibits a strict Pareto improvement in the sense that even the market paying the increased premium goods price prefers monopoly control. This result is confirmed in the general case of the extensions section. Let superscripts on consumer surplus $CS^{\hat{}}$ and CS^{*} represent the independent and joint optimization values from the firm's perspective.

Proposition 3 *Consumers benefit when firms merge or coordinate prices in markets with linear demand and internetwork externalities. That is,* $CS_c^* \geq CS_c^{\hat{}}$ *and* $CS_j^* \geq CS_j^{\hat{}}$ *and thus necessarily* $CS_c^* + CS_j^* \geq CS_c^{\hat{}} + CS_j^{\hat{}}$.

Proof. *The upper bound on* $\int_{p_c}^{\bar{V}_c} q_c(r)dr$ *is given by* $\bar{V}_c = D_c^{-1}(q_c - e_{jc}D_j)$*. For* $q_c = 0$*, this becomes* $V_c\left(1 + e_{jc}\frac{Q_j}{Q_c}(1 - \frac{p_j}{V_j})\right)$*. Integrating equation (1) and substituting for* $\bar{V}_c$ *gives consumer surplus in the* C *market as*

(8) $$CS_c = \frac{\left(e_{jc}\, Q_j\, V_c\, \left(p_j - V_j\right) + Q_c\, \left(p_c - V_c\right)\, V_j\right)^2}{2\, Q_c\, V_c\, {V_j}^2}$$

In computing optimal $(p_c^*,\, p_j^*)$ *and independent* $(\hat{p}_c,\, \hat{p}_j)$ *prices, we condense several steps. We also simplify these expressions by converting to surplus notation. This collapses six apparent degrees of freedom to four true degrees. Substituting for* $(p_c^*,\, p_j^*)$ *in (8) yields consumer surplus at optimal prices.*

(9) $$CS_c^* = \frac{\left(2S_c + S_{cj} + S_{jc}\right)^2 \left(S_cS_j - S_{cj}S_{jc}\right)^2}{2S_c\left(\left(S_{cj} + S_{jc}\right)^2 - 4S_cS_j\right)^2}.$$

A similar substitution for $(\hat{p}_c,\, \hat{p}_j)$ *yields consumer surplus at independent prices.*

(10) $$CS_c^{\hat{}} = \frac{S_c(S_{cj}S_{jc} - S_j(2S_c + S_{jc}))^2}{2\left(S_{cj}S_{jc} - 4S_cS_j\right)^2}$$

Note that squares imply these are always positive. Imposing the Hessian condition ($4S_cS_j - (S_{cj} + S_{jc})^2 > 0$) *and simplifying the difference in total surplus* $CS_c^* - CS_c^{\hat{}}$ *proves always non-negative. Equality is only achieved when the markets are independent* ($e_{cj} = e_{jc} = 0$ *implying* $S_{cj} = S_{jc} = 0$)*, in which case,* $CS_c^* = CS_c^{\hat{}} = \frac{1}{8}S_c$*. The case for* CS_j *is symmetric.* ∎

The underlying intuition is that linking the markets allows a firm to manipulate their total size via the internetwork externality. Consumers then benefit to the extent that a self-interested firm sets prices more efficiently but cannot capture all consumer surplus from the efficiency gains. A firm internalizes the market externality but wins only a fraction of this effect.

3 Firm Competition

In this section, we extend our analysis to explore competition in a market linked to another market by an internetwork externality. These results show why a duopolist might voluntarily engage in undifferentiated product, Bertrand price competition that forces profits to zero in one market segment. The product complement serves as a credible threat to enter a market where conditions following entry would argue against that firm's ability to maintain its position. In fact, firms using a complementary free-pricing strategy can even overcome a competitor's first-mover advantage.

The first competitive model extends the previous section results to show how complementary markets provide sufficient reason to seek market share in one market, while reaping a network externality benefit in the other. The second extension introduces interference, modeled as a negative externality, between competing products to capture substitution effects. A similar but distinct intuition emerges in which a firm chooses free-pricing in one market to protect against market share losses in another.

3.1 First Mover Advantage & Defensive Position

Let firms compete according to the standard Hotelling (1929) model of product differentiation with quadratic disutility costs (Tirole, 1988; Lilien, Kotler, & Moorthy, 1992). To make entry by $Firm\,E$ less attractive, assign a first mover advantage to $Firm\,F$, the competitor. Then, *a fortiori*, entry will become more profitable in any more favorable context. We revisit this routine framework to show how free-pricing can overturn the standard results.

Consumers are uniformly distributed over the interval [0, 1], with inelastic demand for one unit of product, and a common willingness to pay v. The cost of purchase rises with $p+td^2$, interpreted as price p, distance d, and transportation cost t. The transportation cost measures consumer disutility for a firm's more remote product offering relative to his or her ideal. It is also small enough that firms fully cover the market. Otherwise, firms could position themselves far enough apart to effectively avoid competition. Duopoly market shares depend on each firm's price and product location disutility curve. With first and second mover locations a and b, the indifferent consumer lies at x with equal surplus if $v - p^F - t(x-a)^2 = v - p^E - t(x-b)^2$. In market 1, the point of indifference is

$$x = \frac{p_1^F - p_1^E + t(a^2 - b^2)}{2t(a-b)} \tag{11}$$

For total market size Q_1, profits for $Firms\,F$ and E are $\pi_1^F = p_1^F Q_1(1-x)$ and $\pi_1^E = p_1^E Q_1 x$ respectively. Since prices are determined endogenously by the choice of location, best response equilibrium prices are

$$p_1^{F*} = \frac{t}{3}(4-a-b)(a-b) \tag{12}$$

$$p_1^{E*} = \frac{t}{3}(2+a+b)(a-b). \tag{13}$$

Maximum product differentiation appears in a baseline case. The unconstrained simultaneous move Nash equilibrium puts firms at opposite ends of the market such that $a = \frac{5}{4}$, $b = \frac{-1}{4}$. With $x = \frac{1}{2}$, market shares and profits both take the ratio 1 : 1. In the sequential move Stackelberg equilibrium, however, the first mover anticipates entry and chooses $a = \frac{1}{2}$ while the second mover's best response is $b = \frac{-1}{2}$ (or $\frac{3}{2}$). With $x = \frac{1}{3}$, $Firm\, F$ enjoys a market share advantage of 2 : 1 and a profit advantage of 4 : 1 (Lilien, Kotler, & Moorthy, 1992). The standard sequential outcome is shown graphically in Figure 3. Interestingly, the monopoly position $a = \frac{1}{2}$ is the same as the

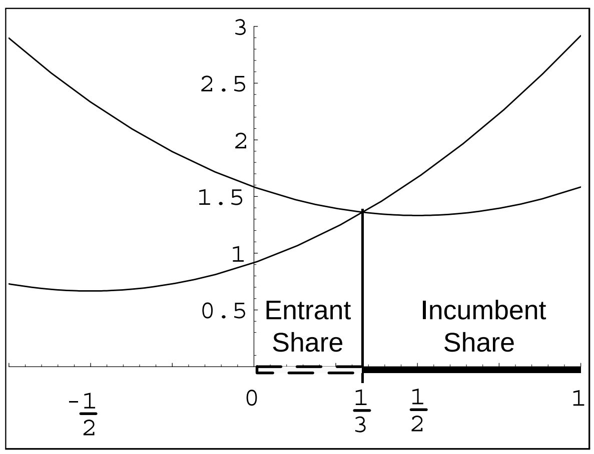

Figure 3: The allocation of market share depends on the choice of location and price. In the sequential move game, the first mover ties up market share with $a = \frac{1}{2}$. The follower chooses $b = \frac{-1}{2}$ and $p^F = 2p^E$.

defensive position anticipating entry. A first mover ties up the most attractive product location, relegating the later entrant to outlying locations with minimal market power. However, as we show below, this situation changes when the entrant links a pair of markets so as to seek market share.

3.2 Complements and Positive Network Externalities

Now provide $Firm\, E$ the option to enter market 1 with an identical product except that, by design, the product complements a second market. Microsoft's willingness to price its Internet browser at zero provides a motivating example. By sharing underlying code and overarching interface design, critics contend that Microsoft's browser displayed significantly greater compatibility with its other products than a competitor could hope to achieve without access to Microsoft's proprietary source code. We again consider two markets for a given information product and we employ the internetwork externality framework introduced in the previous section. Let $Firm\,F$ represent either an incumbent monopoly or a first mover. Then let entrant, $Firm\,E$, couple its product to drive sales in a second market such that $e_{12} > 0$. For simplicity, the second market is not complementary to the first—this would only strengthen the entrant's position—so the second externality is modeled as $e_{21} = 0$ and the market multiplier $\frac{1}{1-e_{12}e_{21}}$ equals 1. We model the network effect on the entrant's complementary product in market 2 to be a function of sales of the entrant's product in market 1 as opposed to total sales in market 1. If the entrant's product strategy were to depend on compatibility with the incumbent, then the incumbent would be at liberty to to change the underlying file format or other product attributes in order to reduce compatibility. In fact, software firms have been observed changing product attributes frequently in order to prevent "piggybacking." We note, however, that similar results may be obtained when both the assumptions of exclusively own-product externality and that of Hotelling's fixed inelastic market are relaxed.

In market 1, firms compute market shares and profits exactly as before, using (11) through (13). In market 2, $Firm\ E$ sells according to

$$q_2^E = D_2(p_2^E) + e_{12}q_1^E. \tag{14}$$

Plugging in market share xQ_1in the first market, $Firm\,E$'s total profits across both markets are given by

$$\pi^E = p_1^E xQ_1 + p_2^E \left[D_2(p_2^E) + e_{12}xQ_1\right]. \tag{15}$$

Since prices in market 1 are determined endogenously by the level of product differentiation, or choice of location, the entrant anticipates the incumbent's price response and uses this to choose location. The number of consumers in the first market then influences consumption in the entrant's second market. Thus we solve for a subgame perfect Nash equilibrium to find $Firm\ E$'s location and $Firm\ F$'s price in market 1 before choosing $Firm\ E$'s optimal price in market 2.

Standard optimization techniques then determine $Firm\ E$'s optimal entry price and location in market 1 after inserting the point of intersection x and the optimal pricing equations 12 and 13 into this profit function. In equilibrium, firms selling complementary goods into one market, when they enjoy pricing power in another, can be unusually tough competitors in the contested market. As we argue below, even if the first mover, $Firm\ F$, anticipates coupling, there is little the firm can do to escape competition from a firm that is seeking market share over profits.

Proposition 4 *A firm may voluntarily enter into Bertrand, or undifferentiated goods, price competition in market* 1 *if it enjoys a monopoly complement in market* 2.

Proof. *We offer a numerical example to illustrate the subgame perfect Nash equilibrium from Proposition 4. Let* $D_2(p_2^E) = Q_2(1 - \frac{p_2^E}{V_2})$ *and to maximize first mover advantage let* $a = \frac{1}{2}$. *Then choose* $t = \frac{1}{4}$, $e_{12} = 1$, $V_2 = 1$, $Q_1 = 1$, $Q_2 = 1$. *Plugging these values into the formulae for optimal price and location leads to*

$$b^* = \frac{1}{2},\ p_2^{E*} = \frac{3}{4},\ \pi^E = \frac{9}{16}. \tag{16}$$

As a check, we note that the price formulae in market 1 *for these parameter values imply* $p_1^E = p_1^F = \pi^F = 0$. *Moreover, two independent firms pricing the complementary goods, or a myopic integrated firm, would have chosen* $b = \frac{-1}{2}$ *and* $p_2^E = \frac{2}{3}$, *leading to combined profits of* $\frac{1}{2}$ *versus* $\frac{9}{16}$ *for the optimal choices.* ■

This proposition implies that a firm will willingly forego profits in one market that it can more than recover from a second market. It further implies that *Firm E* could sustain price competition indefinitely and can, in fact, sustain losses up to the amount of additional profit in market 2. For the incumbent or first mover this could lead to market foreclosure. In the absence of its own complementary good, *Firm F* earns strictly zero profit on sales and loses its sunk costs.[4]

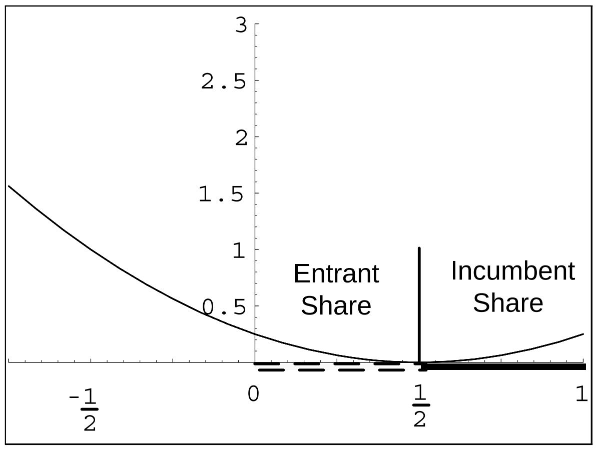

Figure 4: Depending on parameter values, *Firm E* may couple products and seek market share. Choosing $b = a$ forces $p_1^E = p_1^F = 0$.

If *Firm F* anticipates E's product coupling strategy, *Firm F* may not enter the market assuming its product requires an initial investment because it never recovers these costs. *Firm F* could soften price competition by choosing one end of the product spectrum, but this may be insufficient to avoid the effects of a price war. The key point is that *Firm E* is seeking market share, not market power, in the contested market. Regardless of the incumbent's choice of location, the entrant can maximize market share by choosing a location ε to the left or right of *Firm F*. In fact, the usual solution of escaping competition by retreating to one extreme could cede the entire market to the entrant who chooses $b = a \pm \varepsilon$. Whether *Firm F* chooses to enter at all then hinges on the relative parameters that govern avoidance, accommodation, and exit.

[4]This explanation does not exonerate tying as distinct from setting $p_1^E = 0$, a consideration in the Microsoft case. We thank Frank Fisher for this observation.

For information goods, average cost curves are declining due to high first copy and low additional copy costs. Thus, scale is important to remaining profitable. In any case, it should be clear that the existence of a complementary goods market makes E significantly more aggressive. Any incumbent firm faces potential competition from a firm willing to price at zero or below to gain market share.

3.3 Substitutes and Negative Network Externalities

In this section, we introduce the idea of strategic product substitutes via a negative cross-market externality. As a motivating example, we consider the influence that including a runtime environment in Netscape's Internet browser had on demand for Microsoft's operating system. The original intent was to permit "write once, run anywhere" applications development. By providing direct access to the hardware unmediated by the operating system, the runtime environment threatened to allow software to run across diverse and independent platforms (US vs. Microsoft—Findings of Fact. Nov. 5, 1999). Although the browser did not directly substitute for an operating system—and, in fact, required one—the use of Netscape's product freed consumers to choose competing operating systems with less regard for interoperability. Microsoft then had an incentive to limit market share of the indirectly competing product.

We model negative spillovers using the framework of the previous section. Again, we build on Hotelling's model of product competition in which the choice of product locations determines equilibrium prices. Exactly as before, the point of intersection, market shares, and optimal prices in market 1 are given by equations 11 through 13.

Again, $Firm\,E$ enjoys a monopoly position in market 2, but this second product is negatively affected by sales of $Firm\,F$'s product in market 1. That is, we model the influence of sales in market 1 on sales of the competing firm in market 2 as a negative internetwork externality. Let $Firm\ F$'s sales in market 1 reduce $Firm\ E$'s sales in market 2, so that $e_{12}^{FE} < 0$. The last section showed that a firm's own internally managed externality, $e_{12}^{EE} > 0$, can be used to motivate free-pricing in the competitor's market. To highlight the importance of this new effect, we set a firm's own externality $e_{12}^{EE} = 0$ and show that the same willingness to enter into Bertrand competition holds for a fundamentally different reason, namely, mitigating the loss of sales in a favored market. Retaining the positive sense of the externality, $Firm\ F$'s sales in market 1 negatively affect $Firm\ E$'s sales in market 2, yielding:

$$q_2^E = D_2(p_2^E) - e_{12}^{FE} q_1^F. \tag{17}$$

Considering the competitor's market share, $Firm\ E$'s total profit equation becomes

$$\pi^E = p_1^E x Q_1 + p_2^E \left[D_2(p_2^E) - e_{12} Q_1 (1 - x) \right]. \tag{18}$$

$Firm\, E$ again faces a subgame perfect Nash equilibrium decision. By committing to a location in market 1, it knows $Firm\ F$ must respond. It therefore chooses b in market 1 knowing that this influences price and profits in market 2. Substituting for x, and for optimal market 1 best price responses $p_1^{F*}$and p_1^{E*}, $Firm\ E$ selects b^* and p_2^{E*}. The exact formulae can be easily determined by standard techniques. These derivations, in turn, lead to the existence of free-pricing for a different reason—to protect against market share losses.

Proposition 5 *A firm may voluntarily enter into Bertrand, or undifferentiated goods, price competition in market* 1 *if it faces a threatened monopoly in market* 2.

Proof. *To illustrate the indirect threat from the cross-market externality, let* $D_2(p_2^E) = Q_2(1 - \frac{p_2^E}{V_2})$; *again maximize* $Firm\, F$*'s first-mover advantage by setting* $a = \frac{1}{2}$. *Let* $t = \frac{1}{12}$, $e_{12} = 1$, $V_2 = 1$,

$Q_1 = 1$, $Q_2 = 1$. Plugging these values into the formulae for optimal price and location then yields

$$b^* = \frac{1}{2},\ p_2^{E*} = \frac{1}{4},\ \pi^E = \frac{1}{16}. \tag{19}$$

As a check, note that for these parameter values, equilibrium formulae imply $p_1^E = p_1^F = \pi^F = 0$. Moreover, independent profit maximizing firms would have escaped competition by choosing $b = \frac{-1}{2}$ and $p_2^E = \frac{1}{6}$, but would earn combined profits of only $\frac{5}{108}$ versus $\frac{1}{16}$ for the optimal choices. ∎

The illustration in a business setting is straightforward. By including direct links to the microprocessor, Netscape threatened to commoditize operating systems. Even though it was not a direct OS substitute, minimizing Netscape market share became an important goal.

> Microsoft would not have given IE away, nor would it have taken on the high cost of enlisting firms in its campaign to maximize IE's usage share and limit Navigators, had it not been focused on protecting the applications barrier [to its operating system].
> *Judge Thomas Penfield Jackson, Findings of Fact 11-5-99*

The foregoing analyses suggest that we may simultaneously observe a monopolist engaging in fiercely competitive practices while also benefitting consumers. During the Microsoft trial, prosecution argued persuasively that coupling software functionality was anticompetitive. Microsoft countered that coupling benefited consumers. Since Section 3's welfare analysis showed that an internetwork externality can be a significant source of market inefficiency, both claims have merit. Seeking market share in internetworked markets is the crucial insight. In fact, allowing both the first mover and the entrant to exploit both markets yields similar results.

4 Modeling Extensions

In this section, we relax the linear externality assumption maintained in the additive model and find that results are robust to logical changes in formulation. As before, we consider a discrete choice model with the standard assumptions on the distribution of values, properties of demand D_c and D_j, and concavity of profits, only we now allow the cross-market network effects to enter nonlinearly and multiplicatively. Let α denote internetwork effects from the C market to the J market while β denotes the J market externality to the C market.

$$\begin{aligned} q_c &= D_c(p_c)(1 + D_j(p_j))^\beta \\ q_j &= D_j(p_j)(1 + D_c(p_c))^\alpha \end{aligned}$$

For prices to be finite, externalities are again constrained to $\alpha\beta < 1$ with $\alpha \geq 0$, $\beta \geq 0$. Profit in both markets is likewise expressed as $\pi = p_c q_c + p_j q_j$. This multiplicative formulation is reminiscent of the approach taken in Rochet and Tirole (2001). However, consistent with the additive model presented earlier, we allow for asymmetric externalities of arbitrary size across markets. If demand in a complementary market is fractional, for example $0 < D_j < 1$, rising network effects do not shrink native demand. With respect to the additive model, zero native demand is never offset to positive values by spillover effects. Despite these extensions, we find that the multiplicative formulation generates insights that are strikingly consistent with the linear form presented above with minor exceptions noted below.

4.1 Structure of Price Equilibrium

Taking first-order conditions results in well-defined, if somewhat lengthy, solutions in terms of implicit functions.[5] Whereas previously $p_c^* = \mathring{p}_c$ whenever $r = 1$, now $\hat{p}_c = \mathring{p}_c$ always. If two firms set price in each market independently $(\hat{p}_c, \hat{p}_j)$, the pricing decision is the same as if there were no

[5] $p_c^* = \frac{(1+D_j)^{1-\beta}\left((1+D_c)^\alpha D_j^2 \alpha D_c' - D_c(1+D_c)\left(1+D_j\right)^\beta D_j'\right)}{\left(1+D_j+D_c\left(1+D_j(1-\alpha\beta)\right)\right)D_c' D_j'}$, with a symmetric expression for p_j^*.

externalities $(\mathring{p}_c, \mathring{p}_j)$. That is,

$$\hat{p}_c = \mathring{p}_c = \frac{D_c}{-D_c^{'}},\ \hat{p}_j = \mathring{p}_j = \frac{D_j}{-D_j^{'}}.$$

Of substantial interest for assessing the robustness of the model, we find that the price structure at optimality, expressed in terms of elasticities, is exactly the same for the multiplicative formulation as the additive formulation. The following proposition parallels Proposition 1 and Corollary 1.

Proposition 6 *The optimal price in the C market (the J market is symmetric) reflects own price elasticity plus the C to J cross-price elasticity multiplied by the spillover ratio. Further, the optimal price in the C market is the sum of own price elasticity plus the reverse cross-price elasticity multiplied by the optimal profit ratio.*

$$-1 = \eta_c + \eta_{cj} r = \eta_c + \eta_{jc} \frac{\pi_j}{\pi_c}. \tag{20}$$

Proof. *Definitions for elasticities give* $\eta_c = \frac{p_c D_c^{'}}{D_c}$, $\eta_{cj} = \beta \frac{p_j D_j^{'}}{1+D_j}$ *and* $\eta_{jc} = \alpha \frac{p_c D_c^{'}}{1+D_c}$. *The C market first-order condition on profit* $\pi = \pi_c + \pi_j = p_c D_c (1+D_j)^{\beta} + p_j D_j (1+D_c)^{\alpha}$ *is*

$$\frac{\partial \pi}{\partial p_c} = D_c(1+D_j)^{\beta} + p_c D_c^{'}(1+D_j)^{\beta} + \alpha p_j D_j (1+D_c)^{\alpha-1} D_c^{'} = 0. \tag{21}$$

Observing that $r = \frac{\partial q_j}{\partial p_c} / \frac{\partial q_c}{\partial p_j} = \frac{\alpha D_j (1+D_c)^{\alpha-1} D_c^{'}}{\beta D_c (1+D_j)^{\beta-1} D_j^{'}}$, *dividing through by* $D_c(1+D_j)^{\beta}$, *and rearranging terms gives the first result. Given that* $\frac{\pi_j}{\pi_c} = \frac{p_j D_j (1+D_c)^{\alpha}}{p_c D_c (1+D_j)^{\beta}}$, *we note that* $\eta_{cj} r = \eta_{jc} \frac{\pi_j}{\pi_c}$, *which establishes the second result. For completeness, we note that* $\alpha\beta\eta_c\eta_j \left(\frac{D_c D_j}{(1+D_c)(1+D_j)}\right) = \eta_{cj}\eta_{jc}$ *always. Without the need to account for fractional demand,* $q_c = D_c {D_j}^{\beta}$, *and* $\alpha\beta\eta_c\eta_j = \eta_{cj}\eta_{jc}$ *as before.* ■

One result (Proposition 2) developed in the additive formulation is that a price discount in one market, relative to the no-externality case, implies a price increase in the complementary market. In the multiplicative formulation, we find that prices chosen jointly always fall in both markets relative to the case where prices are chosen in each market independently. We demonstrate this in the following proposition.

Proposition 7 *The optimal price, p^*, is less than or equal to the independent or no-externality price, $\hat{p} = \mathring{p}$. That is, $p^* \leq \hat{p} = \mathring{p}$.*

Proof. *We show the case for the C market; the J market is symmetric. Isolating p_c using the first-order condition in equation 21 implies:*

$$p_c^* = \frac{D_c}{-D_c'} - \alpha \frac{p_j D_j (1 + D_c)^\alpha}{(1 + D_j)^\beta (1 + D_c)} = \hat{p}_c - \alpha \frac{\pi_j}{(1 + D_j)^\beta (1 + D_c)}$$

Since all terms in the subtrahend are positive, optimal price falls. ■

The fall in prices for both markets stems from a difference in how the additive and multiplicative formulations increase network demand. The nonlinear multiplicative formulation anchors total value, rotating demand about the upper bound to increase market size. This contrasts with the linear additive formulation that shifts demand outward, increasing market size and maximum value.

In the additive formulation, the relative change in consumption due to cross-market spillovers ($r = \frac{\partial q_j}{\partial p_c} / \frac{\partial q_c}{\partial p_j}$) determines which market receives a price subsidy relative no externality prices. In the multiplicative formulation, both markets are discounted, so we ask instead which market receives the greater proportional discount. That is, what is the relationship between r and $p_c^*/\mathring{p}_c$ and $p_j^*/\mathring{p}_j$?

Corollary 4 *Optimal price declines relatively further in the C market than the J market when the profit ratio exceeds the spillover ratio or $\frac{\pi_c}{\pi_j} > r$. Both markets receive the same relative discount when $\frac{\pi_c}{\pi_j} = r$.*

Proof. *From proposition 7, and given that* $\hat{p}_c = \mathring{p}_c$,

$$p_c^* = \mathring{p}_c - \alpha \frac{\pi_j}{(1+D_j)^\beta (1+D_c)},$$

which implies that

$$\frac{p_c^*}{\mathring{p}_c} = 1 + \alpha \frac{D_c^{'} \pi_j}{D_c (1+D_j)^\beta (1+D_c)}$$

with a symmetric expression for $\frac{p_j^*}{\mathring{p}_j}$*. The C market is discounted more when* $\frac{p_c^*}{\mathring{p}_c} < \frac{p_j^*}{\mathring{p}_j}$*, which is*

$$\alpha \frac{D_c^{'} \pi_j}{D_c (1+D_j)^\beta (1+D_c)} < \beta \frac{D_j^{'} \pi_c}{D_j (1+D_c)^\alpha (1+D_j)}.$$

Cross multiplying and recalling that $r = \frac{\partial q_j}{\partial p_c} / \frac{\partial q_c}{\partial p_j} = \frac{\alpha D_j (1+D_c)^{\alpha-1} D_c^{'}}{\beta D_c (1+D_j)^{\beta-1} D_j^{'}}$ *yields the required result.* ■

Similar to Proposition 2 in the additive model, this result predicts that discounts are equal when the spillover ratio is equal to the profit ratio.

A final corollary examines the implications of coordinated pricing for consumer welfare. In the additive model shown earlier for linear demand (Proposition 3), consumers uniformly prefer monopoly to independent firm prices. We find this is also true more generally for the multiplicative case.

Corollary 5 *Consumers experience a Pareto improvement in welfare when firms merge or coordinates prices across two markets linked by inter-network externalities. That is,* $CS_c^* \geq CS_c^{\hat{}}$ *and thus necessarily* $CS_c^* + CS_j^* \geq CS_c^{\hat{}} + CS_j^{\hat{}}$.

Proof. *Follows immediately from the fact that optimal prices fall* $p_c^* \leq \hat{p}_c = \mathring{p}_c$ *(proposition 7).*

■

As before, this does not imply that consumers are worse off in the presence of competition, but rather implies that consumers are better off when prices for coupled products are coordinated.

4.2 Empirical Implications

It is an open empirical question as to which form of the model best captures the effect of internetwork externalities. The additive formulation predicts higher prices in one market coupled with lower prices in another market. The multiplicative formulation predicts lower prices in both markets. However, both models predict the same elastic structure for optimal prices and also that consumer welfare improves when a single firm coordinates prices across markets.

We point out that both the additive and multiplicative formulations of inter-network externalities offer empirical estimation advantages over standard intra-market models specified with one equation. First, if developers and end-users are, in fact, distinct types, then a one-equation homogeneous market model exhibits bias and inconsistency due to variable omission. Second, having two separate equations to specify demand interaction permits development of instruments for separating demand effects from network effects. This might proceed, for example, through covariance restrictions if the structural disturbances of developers and end-users are uncorrelated. Whether additive or multiplicative, a cross market model creates an opportunity for novel empirical investigation.

5 Applications

Internetwork markets are distinguished from other market types by heterogeneous matched consumers with interdependent demands. Failing to account for externality-based complements might therefore lead to pricing and product design errors. Using a product tying model, for example, firms might meter sales to both consumer types, missing the opportunity to manage network effects between them. Using a lock-in model, they might injudiciously discontinue discounts for the market stimulating cross-market sales, choking network effects. Using uncoupled multi-market price discrimination, they might fail to offer discounts at all. Each model suggests subtle differences in the locus and timing of pricing decisions. Suitable market divisions fall along several different lines based on demand attributes, product features, or time. Practical applications of these divisions follow.

5.1 Tangible Goods & Services

Products targeted for complementary markets can include tangible components—with externalities $e_{it} > 0$ and $e_{ti} > 0$—and need not, in fact, be wholly information goods as in the case of operating systems and applications or games and toolkits. Tangible examples include cameras and scripting languages, radio tuners and broadcasts, and color TV consoles and content.

In fact, broad adoption of color TV was delayed almost a decade past the first color broadcast (Shapiro & Varian, 1998). Developers delayed producing color content until consumers had color TVs, but consumers delayed buying color TVs until there was substantial content. RCA solved the chicken-and-egg problem by subsidizing Disney's Wonderful World of Color, driving consumers to buy TVs. An analogous problem hinders adoption of HDTV. We note also that the high marginal cost tangible good is a less attractive giveaway.

Services can also represent the premium complement since they, like tangibles, have more constrained supply than information. For example, firms that underwrite development of open source or "free" software generally do so under a contract that allows users the same ownership and distribution rights as developers. Developers, however, expand applications in the end-user market, who lack specialized expertise. More widespread adoption in the developer market then raises demand for services in the end-user market. Similarly, low-marginal-cost transactions allow banks to sell credit card services to both merchants and consumers simultaneously (Rochet & Tirole, 2001). Demand interdependence implies that acceptance in either network depends on acceptance in the other.

5.2 Temporal Complements

Time might also mark the boundary of demand heterogeneity allowing the framework to model temporal complements. Let subscripts represent periods such that $e_{12} > 0$. Although $e_{21} = 0$ is more likely, if consumers expect the product to later be successful, they could increase purchases initially.

This recapitulates a form of penetration pricing in which a firm introduces a product at a low price in order to generate future sales. Although temporal lock-in is an obvious traditional interpretation, it may be constructive to view this as distinct market management. For example, vendors of precedent and case law databases commonly subsidize student access while charging premium prices to law firms. The interesting aspect of this strategy is that a "sample" is both full-featured, so there is no need to purchase an upgrade, and not a 30-day trial version, since it runs the full three years of law school. The manner in which this may create a spillover benefit is to save law firms the (re)training costs of having new lawyers learn the command structure and capabilities of a principal research tool. The manner in which it may create a benefit in the other direction is that, ceteris paribus, law students prefer to choose the package adopted by more law firms.

5.3 Upgrades

The same framework also represents a possible design strategy for upgrades and professional versions. Many companies give away an introductory or novice version of their software but charge for a professional or "premium" version, implying $e_{np} > 0$. This gives users the chance to acquire a taste for the good and then to increase their demand for more advanced features. There are good reasons why firms disable functionality in their products in order to create different versions of their goods. Usually, different versions are created to engage in price discrimination, an effort to cause consumers with intrinsically different demand profiles to self-select on the basis of willingness to pay. The standard price discrimination model, however, leaves open the question of why a firm should incur development costs on a good for which it anticipates no revenues. A related argument could apply to the opposite direction, $e_{pn} > 0$, when professionals serve as gatekeepers. Textbook markets can be modeled in the linked market framework, where professors receive free desk copies and students are charged.

5.4 Matchmaking

Dating services and auctions operate at the intersection between markets where intermediaries match couples, merchants and card holders, buyers and sellers (Caillaud & Jullien, 2001; Parker & Van Alstyne 2001). Of particular interest is the fact that each market type strictly prefers greater participation in the cross-market though not necessarily in its own market: daters prefer scarcity on their side, and buyers may bid up each others' prices while sellers bid down each others' receipts. This sets internetwork externalities apart from the standard model in which value grows within a single market (Katz & Shapiro, 1985). It also helps to explain why the same firm will unbundle certain information goods at the same time it bundles others. Although standard theory predicts bundling low marginal cost goods, a firm may prefer to unbundle a developer toolkit to stimulate cross-market effects while it bundles a user office applications suite to reduce demand heterogeneity (Parker & Van Alstyne, 2001).

5.5 Temporal Substitutes

Introducing new products might also be used to foreclose markets for competitors. As a defensive maneuver, Microsoft could have introduced its Internet browser as a preemptive strike against a strategic substitute for its operating system. If a non-Microsoft browser became the standard interface to the Internet, capable of launching other applications, it could have diminished the value and hence the profitability of the operating system. Anticipating entry might therefore be sufficient reason to "picket fence" a market worthy of protection.

5.6 Advertising

Software vendors and web portals offer free consumer content in "sponsored" mode in which consumer market C enjoys free functionality while advertiser market A buys ad placement from the vendor in order to reach the C market. This has also long been an economic model supporting the free information content found in "circulation industries" such as broadcast radio, newspapers, and TV (Chaudri, 1998). Many journals and newspapers also provide evidence of quadrant I in Figure 2 with discounted, but positive prices for the consumer good. Ironically, a larger consumer market increases the attractiveness of buying ads such that $e_{ca} > 0$, but the presence of ads may or may not increase the attractiveness of consuming content. For fashion magazines, consumers may seek the latest information on popular trends such that the internetwork externality term is positive, $e_{ac} > 0$. But for many forms of advertising, consumers may experience clutter costs that interfere with their consumption of content such that $e_{ac} < 0$. Either case can be handled within the proposed framework and firms are better off to the extent that they account for the nature of these mutually complementary markets.

6 Conclusions

The model presented here argues for several simple and intuitive results. First, even in the absence of competition, a firm can rationally invest in a product it intends to give away into perpetuity. The reason is that increased demand in a complementary premium goods market more than covers the cost of investment in the free goods market. In this case, market complementarity arises from an internetwork externality. This strategy also takes advantage of information's near zero marginal cost property as it allows a firm to subsidize an arbitrarily large market at a modest fixed cost.

Second, we reinterpreted network externalities in terms of linked markets and identified distinct submarkets for content-providers and end-consumers. We also showed that either market can be a candidate for subsidy or free-pricing. Deciding which market to subsidize depends on the relative network externality spillovers. At a high level of externality spillover, the market that contributes more to demand for its complement is the market to provide with a free good. At lower levels, firms may charge positive prices in both markets but keep one price artificially low.

Third, a firm can use strategic product design to penetrate a market that becomes competitive post-entry. The threat of entry is credible even when the firm never recovers its sunk costs directly. In fact, a firm may use a complementary good to seek market share, not market power, and the loss from giving away free information can still be profit maximizing. This phenomenon occurs if the free good either boosts sales of the firm's own complementary good or thwarts sales of a competitor's substitute good, enough to offset losses from the investment subsidy.

Fourth, we show that, in general, consumer welfare improves when firms set price across markets with positive complementarities. Firms can manipulate total market size through choice of price in each market. Consumers then benefit to the extent that a self-interested firm sets prices more efficiently but cannot capture all consumer surplus from the efficiency gains. A firm internalizes the market externality but wins only a fraction of this effect.

The modeling contribution is distinct from tying or second degree price discrimination in that consumers need never purchase both goods—unlike razors and blades, the products are stand-alone goods. It also differs from multimarket or third degree price discrimination in the sense that the firm may extract no consumer surplus from one of the two market segments, implying that this market would have traditionally gone unserved. Under various interpretations of the externality terms, the network complements model thus helps to explain several interesting market behaviors such as free goods, temporal complements, tangible complements, and strategic information substitutes. Finally, by separating content creation from content consumption, the model offers new ways to isolate, and thus possibly to measure, demand shocks relative to network effects that are difficult to instrument in models characterized exclusively by intra-market demand economies of scale.

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