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“Maybe "honor thy father and thy mother": uncertain family aid and the design of social long term care insurance”

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Maybe “honor thy father and thy mother”: uncertain  
family aid and the design of social long term care insurance\*

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## Abstract

We study the role and design of private and public insurance programs when informal care is uncertain. Children's degree of altruism is represented by a parameter which is randomly distributed over some interval. The level of informal care on which dependent elderly can count is therefore random. Social insurance helps parents who receive a low level of care, but it comes at the cost of crowding out informal care. Crowding out occurs both at the intensive and the extensive margins. We consider two types of LTC policies. A topping up (*TU*) scheme provides a transfer which is non exclusive and can be supplemented. An opting out (*OO*) scheme is exclusive and cannot be topped up. *TU* will involve crowding out both at the intensive and the extensive margins, whereas *OO* will crowd out solely at the extensive margin. However, *OO* is not necessarily the dominant policy as it may exacerbate crowding out at the extensive margin. Finally, we show that the distortions of both policies can be mitigated by using an appropriately designed mixed policy.

**JEL classification:** H2, H5.

**Keywords:** Long term care, uncertain altruism, private insurance, public insurance, topping up, opting out.

# 1 Introduction

Long-term care (LTC) concerns people who depend on help for daily activities. Due to population aging the number of dependent elderly with cognitive and physical diseases will increase dramatically during the decades to come. Dependency represents a significant financial risk of which only a small part is typically covered by social insurance. Private insurance markets are also thin. Instead, individuals rely on their savings or on informal care provided by family members. This is inefficient and often insufficient, since it leaves individuals who for whatever reason cannot count on family solidarity without proper care.

To understand the issue we are dealing with it is important to stress that LTC is different from (albeit often complementary to) health care, and particularly terminal care or hospice care. Dependent individual do not just need medical care, but they also need help in their everyday life. Providing this assistance is very labor intensive and may be very costly, especially when severe dependence calls for institutional care. While the medical acts are typically reimbursed, at least in part, by health insurance, LTC is usually not covered.

The importance of informal care is likely to decrease during the decades to come because of various societal trends. These include drastic changes in family values and increased female labor force participation. Consequently, the need of private and social LTC insurance will become more pressing.

Irrespective of these long run trends even currently informal care is already subject to many random shocks. First, there are pure demographic factors such as widowhood, the absence or the loss of children. Divorce and migration can also be put in that category. Other shocks are conflicts within the family or financial problems incurred by children, which prevent them from helping their parents. While private insurance markets could potentially provide coverage against the risk of dependency *per se*, the uncertainty associated with the level of informal care appears to be a mostly uninsurable

risk. There exists no good private insurance mechanism to protect individuals against all sorts of default of family altruism in particular because family care is by definition informal and thus largely unobservable. This market failure creates a *potential* role for public intervention, but since it is not likely that public authorities have better information than parents about their prospects to receive informal care, this won't lead to a first-best outcome and the design of second-best policies is not trivial.

We study the role and design of private and public insurance programs in a world in which informal care is uncertain. We summarize this uncertainty by a single parameter  $\beta$  referred to as the child's degree of altruism. One can also think about this as the (inverse of the) cost of providing care. This parameter is not known to parents when they make their savings and insurance decisions. We model the behavior and welfare of one generation of "parents" over their life cycle. When young, they work, consume, and save for their retirement. In retirement, they face a probability of becoming dependent. In case of dependence, parents face yet another uncertainty which pertains to the level of informal care they can expect from their children. This is determined by the parameter  $\beta$  which is randomly distributed over some interval.

A major concern raised by LTC insurance is that of crowding out of informal care.<sup>1</sup> This may make public LTC insurance ineffective for some persons and overall more expensive. Within the context we consider that crowding out may occur both at the intensive and the extensive margins. By intensive margin we refer to the reduction of informal care (possibly on a one by one basis) for parents who continue to receive aid from their children even when social LTC is available. Crowding out at the extensive margin, on the other hand, occurs when social LTC reduces the range of altruism parameters ensuring that children provide care. Put differently, social LTC makes it less likely that individuals receive informal care.

We consider two types of LTC policies. The first one, referred to as "topping up"

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<sup>1</sup>See Cremer, Pestieau and Ponthière (2012).

(*TU*) provides a transfer to dependent elderly which is non exclusive and can be supplemented by informal care and by market care financed by savings. The second one, is an “opting out” scheme (*OO*); it provides care which is exclusive and cannot be topped up. One can think about the former as monetary help for care provided at home, and about the latter as nursing home care. The distinction between *TU* and *OO* has been widely studied in the literature about in-kind transfers.<sup>2</sup> It has been shown to be relevant for instance in the context of education and health both from a normative and a positive perspective.<sup>3</sup> Within the context of LTC it appears to be particularly relevant because the two types of policies can be expected to have quite different implication on informal care. *TU* will involve crowding out both at the intensive and the extensive margins, whereas *OO* will crowd out solely at the extensive margin. One might be tempted to think that this makes *OO* the dominant policy, but we’ll show that this is not necessarily true as this policy may exacerbate the extensive margin crowding out.

After considering the two policies separately we also study a mixed policy which combines both approaches, and lets parents choose between, say, a monetary help for care provided at home (a *TU* scheme) and nursing home care provided on an *OO* basis. Interestingly the policies interact in a nontrivial way. When combined, the crowding out effects induced by each of the policies do not simply add up. Quite the opposite, the policies can effectively be used to neutralize their respective distortions. For instance variations in the policies can be designed so that the marginal level of altruism (above which children provide care) and savings are not affected.

Throughout the paper we concentrate on the single generation of parents. The role of children in our model is limited to their decision with regard to providing assistance to their parents. The welfare of the grown-up children does not figure in the government’s

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<sup>2</sup>For a review of the literature, see Currie and Gahvari (2008).

<sup>3</sup>On the normative side, *OO* regimes generally dominate *TU* ones for redistributive purposes. See for instance Blomquist and Christiansen (1995). From a positive perspective, however, *TU* regimes may emerge from majority voting rules, as showed by Epple and Romano (1996).

objective function; social welfare accounts only for the expected lifecycle utility of parents. Conversely, we require a social LTC policy to be financed by taxes on the parents, and we rule out taxes on children. These are simplifying assumptions which allow us to concentrate on intra-generational issues. Note that including caregivers' utility in social welfare would not affect the fundamental tradeoffs involved in the design of the considered LTC policies.<sup>4</sup> It would simply imply that informal care no longer comes for free, which in turn mitigates the adverse effects of crowding out.<sup>5</sup>

The issue of uncertain altruism has previously been studied by Cremer et al (2012) and (2014). Both of these papers concentrate on the case where altruism is a binary variable. Children are either altruistic at some known degree or not altruistic at all. The first paper considers both  $TU$  and  $OO$ , while the second one concentrates on  $OO$  but accounts for the possibility that the probability that children provide care is endogenous and can be affected by parents' decisions. None of them studies mixed policies. The current paper considers a continuous distribution of the altruism parameter. This is not just a methodological exercise, but has important practical implications for the results and the tradeoffs that are involved. The very distinction between crowding out at the extensive and intensive margins is not meaningful in the binary model, but turns out to play a fundamental role for policy design when the distribution of the altruism parameter is continuous. This is particularly true in the case of mixed policies; the tradeoffs we have identified there are completely obscured in the binary model.

The paper is organized as follows. In Section 2 we present the model and two interesting benchmarks: the *laissez-faire* and the full-information allocations. We characterize the optimal  $TU$  and  $OO$  policies in Sections 3 and 4, respectively. In Section 5 we compare the two policies regimes and provide a sufficient condition for  $OO$  to

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<sup>4</sup>As long as we maintain the assumption that each generation pays for its own LTC insurance.

<sup>5</sup>The extent of this would depend on the exact specification of social welfare. The crucial issue from that perspective is how to account for the altruistic term in welfare. Under a strict utilitarian approach it would be fully included which raises the well known problem of "double counting".

dominate. We discuss the case with private LTC insurance and the mixed regime in Sections 6 and 7. We conclude in Section 8.

## 2 The model

Consider a setup in wherein elderly parents face the risk of becoming dependent. In that case, they may receive informal care from their children depending on their degree of altruism. All parents are identical *ex ante*.

The sequence of events is as follows. In period 0, the government formulates and announces its tax/transfer policy; this is the first stage of our game. The second stage is played in period 1, when young working parents and their single children appear on the scene; parents decide on their saving. Finally, in period 2, parents have grown old, are retired and may be dependent. When parents are healthy the game is over and no further decisions are to be taken. They simply consume their saving. However, when the parents are dependent, we move to stage 3 where the children, who have turned into working adults, decide how much informal care (if any) they want to provide. We will not be concerned with what will happen to the grown-up children when they turn old, nor with any future generations.

Two uncertain events give rise to the problem that we study. One concerns the health of the parents in old age. They may be either “dependent” or “independent”. Denote the probability of dependency by  $\pi$  which we assume to be exogenously given. The second source of uncertainty concerns the degree of altruism  $\beta \geq 0$  of their children.<sup>6</sup> It is not known to parents when they have to make their decisions. The random variable  $\beta$  is distributed according to  $F(\beta)$ , with density  $f(\beta)$ . Children who are not altruistic towards their parents have  $\beta = 0$ . For simplicity, we assume that  $F$  is concave, which implies that  $F(\beta) > \beta f(\beta)$ .<sup>7</sup>

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<sup>6</sup>We rule out  $\beta < 0$ , which represents a case where children would be happier if their parents were worse off.

<sup>7</sup>This condition is sufficient (but not always necessary) for most of the SOC to be satisfied and for

Parents have preferences over consumption when young,  $c \geq 0$ , consumption when old and healthy,  $d \geq 0$ , and consumption, including LTC services, when old and dependent  $m \geq 0$ . There is no disutility associated with working. We assume, for simplicity, that the parents' preferences are quasilinear in consumption when young. Risk aversion is introduced through the concavity of second period state dependent utilities.

The government's policy consists of levying a tax at rate of  $\tau$  on the parents' wage,  $w$ , to finance public provision of dependency assistance,  $g$ . We shall refer to  $g$  as the LTC insurance. We rule out private insurance in a first stage but reintroduce it later. We denote the level of assistance an altruistic child gives his parent by  $a$ , savings by  $s$ , and set the rate of interest on savings at zero.

Before turning to policy design, we study the *laissez-faire* which is an interesting benchmark. We proceed by backward induction and start by studying the last stage of the decision making process.<sup>8</sup> This is when the grown-up children decide on the extent of their help to their parents, if any.

## 2.1 Laissez-faire

In this section we assume that the government does not provide any form of LTC insurance. The parent's expected utility is given by

$$EU = w\bar{T} - s + (1 - \pi)U(s) + \pi E[H(m)], \quad (1)$$

with  $m = s + a^*(\beta, s)$ , where  $s$  is saving, while  $a^*(\beta, s) \geq 0$  is informal care provided by children, which depends on their degree of altruism  $\beta$ . Assume that  $H' > 0$ ,  $H'' < 0$ ,  $H(0) = 0$ ,  $H'(0) = \infty$  and that the same properties hold for  $U$ . Further, for any given  $x$  we have  $U'(x) < H'(x)$ . To understand this condition consider the simplest case where dependency implies a monetary loss of say  $L$  so that  $H(x) = U(x - L)$ .

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some comparative statics results. We shall point out explicitly where and how it used.

<sup>8</sup>In the *laissez-faire* there is no stage 1. But to be consistent with the next section we refer to the relevant stages as 2 and 3.

We shall assume that the grown-up children too have quasilinear preferences represented by

$$u = \begin{cases} y - a + \beta H(m) & \text{if the parent is dependent} \\ y - a & \text{if the parent is not independent} \end{cases} \quad (2)$$

where  $y$  denotes their income. Observe that altruism is relevant only when the child's parent is effectively dependent. Healthy elderly parents consume their savings; they neither give nor receive a transfer.

### 2.1.1 Stage 3: The child's choice

The altruistic children allocate an amount  $a$  of their income  $y$  to assist their dependent parents, given the parents' savings  $s$ . Its optimal level,  $a^*$ , is found through the maximization of equation (2). The first-order condition with respect to  $a$  is, assuming an interior solution,

$$-1 + \beta H'(s + a) = 0.$$

Define  $\beta_0(s)$  such that

$$1 = \beta_0 H'(s). \quad (3)$$

This function represents the minimum level of  $\beta$  for which a positive level of care is provided. Not surprisingly, we have

$$\frac{\partial \beta_0}{\partial s} = -\frac{\beta_0 H''}{H'} > 0,$$

implying that an increase in parents' savings reduces the probability of children helping out in case of dependence. It follows from condition (3) that when  $\beta \geq \beta_0 > 0$ ,  $a^*$  satisfies

$$m = s + a^* = (H')^{-1}\left(\frac{1}{\beta}\right). \quad (4)$$

When  $\beta < \beta_0$ ,  $a^* = 0$  and  $m = s$ . Thus, in the laissez-faire, the consumption of dependent parents is equal to

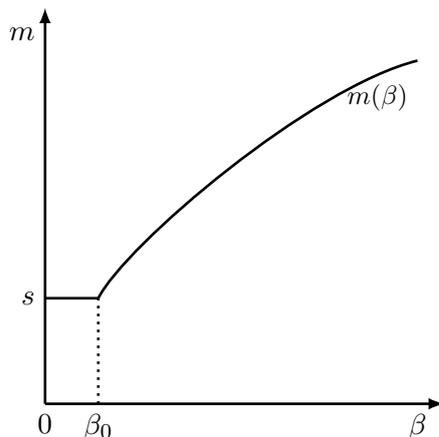
$$m(\beta) \equiv \begin{cases} (H')^{-1}\left(\frac{1}{\beta}\right) & \text{if } \beta \geq \beta_0 \\ m_0 = s & \text{if } \beta < \beta_0 \end{cases} \quad (5)$$

These expressions show that savings crowds out informal care in two ways. First, it makes it less likely that a positive level of care is provided (crowding out at the extensive margin). Second, when informal care is provided it is crowded out on a one to one basis by savings, as long as the solution remains interior (crowding out at the intensive margin). Differentiating (4) yields

$$\frac{\partial m}{\partial \beta} = \begin{cases} \frac{-1}{\beta^2 H''(m)} > 0 & \text{if } \beta \geq \beta_0 \\ 0 & \text{if } \beta < \beta_0 \end{cases}$$

where the first line is positive from the concavity of  $H$ .<sup>9</sup> As expected, a parents total consumption when dependent increases with the degree of altruism of the child. Figure 1, which represents equation (5), illustrates the consumption level of dependent parents as a function of children's altruism.

Figure 1: *Laissez-faire: consumption of dependent parents as a function of children's altruism.*



### 2.1.2 Stage 2: The parent's choice

Recall that the parent may experience two states of nature when retired: dependency with probability  $\pi$  and autonomy with probability  $(1 - \pi)$ . Substituting for  $a^*$  from (4)

<sup>9</sup>The function  $m(\beta)$  is not differentiable at  $\beta = \beta_0$ . To avoid cumbersome notation we use  $\partial m / \partial \beta$  for the right derivative at this point.

in the parent's expected utility function (1), we have

$$\begin{aligned} EU &= w\bar{T} - s + (1 - \pi)U(s) + \pi \left[ \int_0^{\beta_0} H(s) dF(\beta) + \int_{\beta_0}^{\infty} H(m(\beta)) dF(\beta) \right] \\ &= w\bar{T} - s + (1 - \pi)U(s) + \pi \left[ H(s)F(\beta_0) + \int_{\beta_0}^{\infty} H(m(\beta)) dF(\beta) \right]. \end{aligned}$$

Maximizing  $EU$  with respect to  $s$ , and assuming an interior solution the optimal value of  $s$  satisfies<sup>10</sup>

$$(1 - \pi)U'(s) + \pi F(\beta_0)H'(s) = 1. \quad (6)$$

Note that there are no terms involving  $\partial m/\partial s$ . The derivatives with respect to  $\beta_0$  cancel out because  $m(\beta_0) = s$ . Expression (6) simply states that the expected benefits of saving must be equal to its cost, which is equal to 1. Saving provides benefits when the parent is healthy; this is represented by first term on the RHS. The second term corresponds to the benefits enjoyed by the dependent elderly who do not receive informal care. When the dependent parents receive informal care, saving has no benefit because of the crowding out.

Observe that since  $H'(s) > U'(s)$  and  $F(\beta_0) \leq 1$ , equation (6) implies that  $H'(m_0) > 1$  so that as expected the *laissez-faire* leaves dependent individuals who do not receive formal care underinsured.

The SOC is given by

$$(1 - \pi)U''(s) + \pi F(\beta_0)H''(s) + \pi f(\beta_0)H'(s) \frac{\partial \beta_0}{\partial s} < 0,$$

or, substituting for  $\partial \beta_0/\partial s$

$$(1 - \pi)U''(s) + \pi H''(s) [F(\beta_0) - \beta_0 f(\beta_0)] < 0,$$

for which the concavity of  $F(\beta)$  is represents a *sufficient* condition.

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<sup>10</sup>A corner solution at  $s = 0$  can be excluded by the assumption that  $U'(0) = \infty$ . However, a corner solution at  $s = w\bar{T}$ , yielding  $c = 0$  cannot be ruled out. To avoid a tedious and not very insightful multiplication of cases we assume throughout the paper that the constraint  $c \geq 0$  is not binding in equilibrium (even when first period income is taxed to finance social LTC).

## 2.2 Full-information solution

To assess this equilibrium and determine the need for policy intervention let us briefly examine the full-information allocation. We define this as the allocation that maximizes expected utility of the parent *taking the aid behavior as given* but assuming that  $\beta$  is observable. In other words, it is possible to insure individuals both against dependence and the failure of altruism and the payment to dependent parents can be conditioned on  $\beta$  to avoid crowding out. In that case, we maximize

$$EU = w\bar{T} - (1-\pi)d - \pi F(\beta_0)m_0 + (1-\pi)U(d) + \pi \left[ H(m_0)F(\beta_0) + \int_{\beta_0}^{\infty} H(m(\beta))dF(\beta) \right].$$

The FOC with respect to  $d$  and  $m_0$  imply

$$U'(d) = H'(m_0) = 1, \tag{7}$$

so that individuals are “fully insured”.

Differentiating with respect to  $\beta_0$  yields

$$H(m(\beta_0)) = H(m_0),$$

so that dependent parents rely on informal care when  $m(\beta) > m_0$ , where  $m_0 = H'^{-1}(1)$  is the solution to equation (7). Then,  $\beta_0 = 1$  and only children with an altruism parameter greater than one should provide help.

When  $\beta$  is observable, this policy can be easily implemented by a transfer  $m_0 - s$  to all dependent parents whose children’s altruism parameter is lower than one.

However,  $\beta$  is not observable this solution cannot be achieved. We now study three second-best policies. In the first one, referred to as topping up (*TU*), the transfer to the dependent parent is conditional on dependency only. It can be supplemented by informal and market care. Under the second one, referred to as opting out (*OO*), LTC benefits are exclusive and cannot be topped up. Then, we consider a mixed policy which combines cash benefits that can be topped up with in-kind care that is exclusive. In that case dependent parents can choose their preferred regime.

Observe that by restricting our attention to these policies we implicitly assume that informal care,  $a$ , is not observable, except that  $a = 0$  can be enforced to implement an  $OO$  policy. If  $a$  were fully observable, we could of course do better by using a nonlinear transfer scheme  $g(a)$  to screen for the  $\beta$ 's. This would amount to characterizing the optimal incentive compatible mechanism of which  $TU$ ,  $OO$  and the mixed policies are special cases. However, as explained by Norton (2000) family care is by definition informal and thus typically not observable.<sup>11</sup>

### 3 Topping up

In this section we assume that the government provides LTC insurance. The transfer to the dependent elderly,  $g$ , is non-exclusive in the sense that it can be topped up by  $a$  and  $s$ . Parents' expected utility, is now given by

$$EU^{TU} = w(1 - \tau)\bar{T} - s + (1 - \pi)U(s) + \pi E[H(s + g + a^*(\beta, s, g))], \quad (8)$$

where  $a^*(\beta, s, g) \geq 0$  is care provided by children, which as shown in the next subsection now also depends on  $g$ . Preferences of grown-up children continue to be represented by equation (2), with  $m$  redefined as  $m = s + g + a$ .

Once again we proceed by backward induction and start with the last stage.

#### 3.1 Stage 3: The child's choice

The altruistic children allocate an amount  $a$  of their income  $y$  to assist their dependent parents (given the parents' savings  $s$  and the government's provision of  $g$ ). Its optimal level,  $a^*$ , is found through the maximization of equation (2). The first-order condition with respect to  $a$  is, assuming an interior solution,

$$-1 + \beta H'(s + g + a) = 0.$$

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<sup>11</sup>So much that even the statistical knowledge of the significance of informal care is rather imperfect.

Define  $\tilde{\beta}(s+g)$  such that

$$1 = \tilde{\beta}H'(s+g). \quad (9)$$

Comparing (3) with (9), we obtain that  $\tilde{\beta} > \beta_0$  for all  $g > 0$ . It follows from condition (9) that when  $\beta \geq \tilde{\beta}$ ,  $a^*$  satisfies

$$m = s + g + a^* = (H')^{-1}\left(\frac{1}{\beta}\right) = m(\beta). \quad (10)$$

As depicted by the solid line in Figure 2, if  $\beta \geq \tilde{\beta}$ , the consumption of dependent parents  $m(\beta)$  is exactly the same as in the *laissez-faire*. When children's altruism is in that range informal care is fully crowded out by government assistance. For lower levels of altruism, when  $\beta < \tilde{\beta}$ , no formal care is provided,  $a^* = 0$  and  $m = s + g > m(\beta)$ . As usual, crowding out stops when caregivers are brought to a corner solution. For these parents,  $g$  is effectively increasing total care they receive and we have  $\partial m/\partial g = 1$ . Finally, observe that

$$\frac{\partial \tilde{\beta}}{\partial (s+g)} = -\frac{\tilde{\beta}H''}{H'} > 0. \quad (11)$$

In words, as the total amount of formal care increases, the degree of altruism necessary to yield a positive level of informal care increases.

### 3.2 Stage 2: The parent's choice

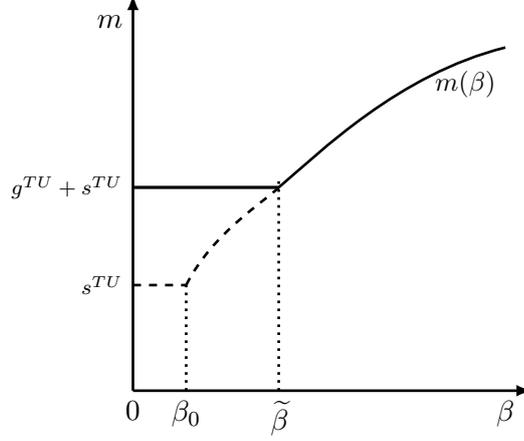
Recall that parents are dependent with probability  $\pi$  and autonomous with probability  $(1-\pi)$ . Substituting for  $a^*$  from (10) in the parent's expected utility function (8), we have

$$\begin{aligned} EU^{TU} &= w(1-\tau)\bar{T} - s + (1-\pi)U(s) + \pi \left[ \int_0^{\tilde{\beta}} H(s+g) dF(\beta) + \int_{\tilde{\beta}}^{\infty} H(m(\beta)) dF(\beta) \right] \\ &= w(1-\tau)\bar{T} - s + (1-\pi)U(s) + \pi \left[ H(s+g)F(\tilde{\beta}) + \int_{\tilde{\beta}}^{\infty} H(m(\beta)) dF(\beta) \right]. \end{aligned}$$

Maximizing  $EU^{TU}$  with respect to  $s$ , and assuming an interior solution, the optimal value of  $s$  satisfies

$$(1-\pi)U'(s) + \pi F(\tilde{\beta})H'(s+g) = 1. \quad (12)$$

Figure 2: *Topping Up*. Consumption of dependent parents as a function of children's altruism



Observe that since by  $m(\tilde{\beta}) = s + g$ , the derivative of  $EU$  with respect to  $\tilde{\beta}$  is zero so that the induced variation in the marginal level of altruism does not appear in (12).

The *SOC* is given by

$$(1 - \pi) U''(s) + \pi F(\tilde{\beta}) H''(s + g) + \pi f(\tilde{\beta}) H'(s + g) \frac{\partial \tilde{\beta}}{\partial (s + g)} < 0,$$

or, substituting for  $\partial \tilde{\beta} / \partial s$

$$(1 - \pi) U''(s) + \pi H''(s + g) [F(\tilde{\beta}) - \tilde{\beta} f(\tilde{\beta})] < 0,$$

which is satisfied when  $F$  is concave.

Denote the solution to equation (12) by  $s^{TU}(g)$ . Substituting  $s^{TU}(g)$  for  $s$  in (12), the resulting relationship holds for all values of  $g$ . Totally differentiating this relationship while making us of (11) and the concavity of  $F$  yields

$$\frac{\partial s^{TU}}{\partial g} = - \frac{\pi H''(s + g) [F(\tilde{\beta}) - \tilde{\beta} f(\tilde{\beta})]}{SOC} < 0.$$

Consequently we obtain that  $s$  decreases with  $g$ . This is not surprising. Savings are useful when the parent is healthy, but also play the role of self-insurance for dependent

parents who do *not* receive formal care. As public LTC becomes available the expected self-insurance benefits associated with  $s$  become less important because more parents will receive formal care.

### 3.3 Stage 1: The optimal policy

Let us now determine the levels of  $\tau$  and  $g$  that maximize  $EU^{TU}$ , as optimized by the parents in stage 2, subject the budget constraint

$$\tau w\bar{T} = \pi g. \quad (13)$$

Substituting for  $\tau$  from (13) into the parents' optimized value of  $EU$ ,  $g$  is then chosen to maximize

$$\begin{aligned} \mathcal{L}^{TU} \equiv & w\bar{T} - \pi g - s(g) + (1 - \pi) U(s^{TU}(g)) + \\ & \pi \left[ \int_{\tilde{\beta}}^{\infty} H(m(\beta)) dF(\beta) + F(\tilde{\beta})H(s^{TU}(g) + g) \right]. \end{aligned}$$

Differentiating  $\mathcal{L}$  with respect to  $g$  yields, using the envelope theorem,

$$\frac{\partial \mathcal{L}^{TU}}{\partial g} = \pi \left[ F(\tilde{\beta})H'(s^{TU}(g) + g) - 1 \right].$$

Let us evaluate the sign of  $\partial \mathcal{L} / \partial g$  at  $g = 0$ ; it is negative if

$$F(\tilde{\beta})H'(s^{TU}(0)) - 1 \leq 0,$$

where  $\tilde{\beta} = \tilde{\beta}(s(0))$ . In this case the solution is given by  $g = \tau = 0$  and no insurance should be provided. This condition is satisfied if  $F[\tilde{\beta}(s(0))]$  is sufficiently small. In that case the probability that individuals receive informal care is so “large” that the benefits of insurance are small and outweighed by its cost in terms of expected crowding out. The *laissez-faire* leaves some individuals (those whose children have  $\beta < \tilde{\beta}$ ) without specific LTC benefits (other than self-insurance). This is inefficient, but the  $TU$  policy we consider here cannot do better.

A different outcome occurs if

$$F(\tilde{\beta})H'(s^{TU}(0)) - 1 > 0.$$

In this case, there will be an interior solution for  $g$ , and  $\tau$ , characterized by

$$H'(s^{TU}(g) + g) = \frac{1}{F(\tilde{\beta})} > 1. \quad (14)$$

Consequently, there is less than full insurance which from (7) would require  $H' = 1$ . Substituting from (14) into (12), it is also the case that

$$U'(d) = U'(s^{TU}(g)) = 1,$$

which implies that the parents' consumption when healthy is at its first-best level.

## 4 Opting out

In this section we assume that  $g$  is exclusive in the sense that it cannot be topped up by  $a$  or  $s$ . The policy is only relevant when  $g \geq s$ ; otherwise, public assistance would be of no use to the parents. The grown-up children have quasilinear preferences represented by

$$u = y - a + \beta H(s + a), \quad (15)$$

if they provide informal care and

$$u = y + \beta H(g), \quad (16)$$

if they decide not to assist their parents who then exclusive rely on public LTC.

### 4.1 Stage 3: The child's choice

If the child provides care, its optimal level  $a^*$  is such that the dependent parents consumption is equal to its laissez faire level,  $m(\beta)$ . This follows from the maximization of (15) while making use of (5) the definition of  $m(\beta)$ . However, children provide care only

if this gives them a higher utility than when their parents rely on exclusive government assistance. They thus compare (15) evaluated at  $a^*$ , with (16), and provide care if

$$\beta [H(m(\beta)) - H(g)] - (m(\beta) - s) > 0. \quad (17)$$

In words, the utility gain from altruism  $\beta [H(m(\beta)) - H(g)]$  must exceed the cost of care  $a^* = (m(\beta) - s)$ . A necessary condition for this inequality to hold is  $g < m(\beta)$ . The LHS is increasing in  $\beta$  for all  $g < m(\beta)$ , so that for each  $g$  and  $s$  there exist a  $\hat{\beta}(g, s)$  such that all children with  $\beta > \hat{\beta}$  provide care, and all children with  $\beta \leq \hat{\beta}$  provide no assistance.<sup>12</sup> This threshold  $\hat{\beta}(g, s)$  is defined by

$$\hat{\beta} [H(m(\hat{\beta})) - H(g)] - (m(\hat{\beta}) - s) = 0.$$

We have

$$\frac{\partial \hat{\beta}}{\partial s} = - \frac{1}{[H(m(\hat{\beta})) - H(g)]} < 0, \quad (19)$$

and

$$\frac{\partial \hat{\beta}}{\partial g} = \frac{\hat{\beta} H'(g)}{[H(m(\hat{\beta})) - H(g)]} > 0.$$

As in the case with topping up, the threshold of  $\beta$  above which the children provide assistance is increasing in  $g$ . However, unlike in the topping up case, this threshold is now *decreasing* in  $s$ . The higher is  $s$ , the higher the incentive for the children to provide assistance (otherwise  $s$  would be wasted). In the case of topping up, the opposite was true, and children were less likely to provide assistance if  $s$  was high.

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<sup>12</sup>The derivative of the LHS with respect to  $\beta$  is

$$[H(m(\beta)) - H(g)] - \frac{\partial m}{\partial \beta} [\beta H'(m(\beta)) - 1]. \quad (18)$$

If  $\beta \leq \beta_0$ , then  $\partial m / \partial \beta = 0$ . If  $\beta > \beta_0$ ,  $\beta H'(m(\beta)) - 1 = 0$ . Thus, equation (18) reduces to

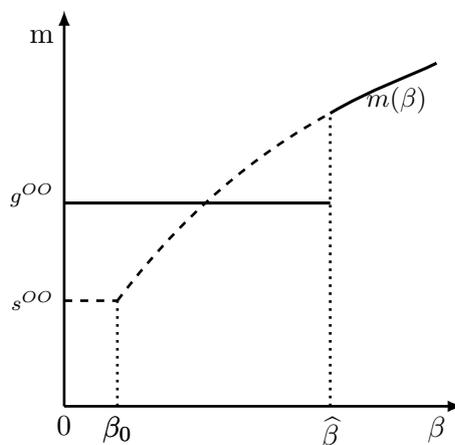
$$[H(m(\beta)) - H(g)].$$

which is positive for all  $g < m(\beta)$ .

Figure 3 illustrates how the level of consumption of dependent parents depends on the degree of children's altruism under opting out (solid line). When  $\beta > \hat{\beta}$ , parents consume  $m(\beta)$ , which is equal to the *laissez-faire* consumption. If the children's level of altruism is lower than  $\hat{\beta}$ , dependent parents will consume  $g^{OO}$ . Unlike in the topping up regime, there is now a discontinuity in the level of  $m$  at  $\hat{\beta}$ . This is because under *OO* children provide care only if  $m(\beta)$  is sufficiently larger than  $g^{OO}$  to make up for the cost of care.

So far we have implicitly assumed that whenever children are willing to provide care, their parents are prepared to accept it, and thus to forego  $g$  and  $s$ . This effectively follows from (17), which requires  $m(\beta) > g$  so that the parent to whom informal care is offered is always better off by opting out of the public LTC system. Intuitively, this does not come as a surprise. Children are altruistic but account for the cost of care, while the latter comes at no cost to parents.

Figure 3: *Opting Out: consumption of dependent parents as a function of children's altruism*



## 4.2 Stage 2: The parent's choice

The parent's expected utility function is

$$\begin{aligned} EU^{OO} &= w(1-\tau)\bar{T} - s + (1-\pi)U(s) + \pi \left[ \int_0^{\hat{\beta}} H(g) dF(\beta) + \int_{\hat{\beta}}^{\infty} H(m(\beta)) dF(\beta) \right] \\ &= w(1-\tau)\bar{T} - s + (1-\pi)U(s) + \pi \left[ H(g)F(\hat{\beta}) + \int_{\hat{\beta}}^{\infty} H(m(\beta)) dF(\beta) \right]. \end{aligned}$$

Maximizing  $EU^{OO}$  with respect to  $s$ , and assuming an interior solution, the optimal value of  $s$  satisfies

$$(1-\pi)U'(s) - \pi f(\hat{\beta}) \left[ H(m(\hat{\beta})) - H(g) \right] \frac{\partial \hat{\beta}}{\partial s} = 1, \quad (20)$$

Note that the derivatives with respect to  $\hat{\beta}$  do *not* cancel out. This is because it's the children with altruism  $\hat{\beta}$  and not their parents who are indifferent between providing care and not providing it. Recall that children do account for the cost of care, while their parents do not. Substituting  $\partial \hat{\beta} / \partial s$  from (19) condition (20) can be written as

$$(1-\pi)U'(s) + \pi f(\hat{\beta}) = 1. \quad (21)$$

The second term in the LHS of (20) and (21) represents the positive effect of  $s$  on the probability that children provide assistance; recall that  $\hat{\beta}$  decreases with  $s$ .

Comparing this expression with (12), we find that for a given level of  $g$ , savings with opting out may be higher or lower than the savings with topping up. On the one hand, opting out reduces the incentives to save, since savings are useful to parents only when dependence does not occur (*i.e.*, with probability  $1-\pi$ ). On the other hand, savings increase the probability that children provide assistance. Since parents are always better off under family assistance than under public assistance ( $H(m(\beta)) - H(g) > 0$  for all  $\beta > \hat{\beta}$ ), this enhances the incentives to save under opting out.

The SOC is given by

$$(1-\pi)U''(s) + \pi f'(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial s} < 0, \quad (22)$$

which we assume to be satisfied. Now the *marginal* benefit of savings via the induced increase in family care may increase or decrease in  $s$ , depending on the slope of the density function  $f(\beta)$ . Unlike for the SOCs considered above, concavity of  $F$  is *not* sufficient. Quite the opposite: with a concave distribution we have  $f' = F'' < 0$  so that the second term of (22) is positive.

Denote the solution to equation (21) by  $s^{OO}(g)$ . Substituting  $s^{OO}(g)$  for  $s$  in (21), the resulting relationship holds for all values of  $g > s$ . Totally differentiating equation (21) yields (assuming concavity of  $F$ )

$$\frac{\partial s^{OO}}{\partial g} = -\frac{\pi f'(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g}}{SOC} < 0. \quad (23)$$

Consequently, like under  $TU$ ,  $s$  decreases with  $g$ . Once again this is due to the fact that the expected self-insurance benefits provided by saving decrease as  $g$  increases. Since  $g$  cannot be topped up,  $s$  does not provide any benefits for parents who receive  $g$ . However, an increase in the level of  $g$  affects the probability that informal care is received and this effect is measured by the numerator of (23). As  $g$  increases, the marginal effect of  $s$  on the probability of informal care decreases (due to the concavity of  $F$ ), which explains the negative sign of  $\partial s^{OO}/\partial g$ .

We now turn to the government's problem which represents stage 1 of our game. Since we consider a subgame perfect equilibrium in which  $s$  is determined by  $g$ , the marginal level of altruism  $\hat{\beta}(g, s)$  now becomes solely a function of  $g$ . Therefore we define

$$\hat{\beta}(g) = \hat{\beta}[g, s^{OO}(g)].$$

For future reference observe that

$$\frac{\partial \hat{\beta}}{\partial g} = \frac{\partial \hat{\beta}}{\partial g} + \frac{\partial \hat{\beta}}{\partial s} \frac{\partial s^{OO}}{\partial g} = \frac{\hat{\beta} H'(g)}{[H(m(\hat{\beta})) - H(g)]} - \frac{\partial s^{OO}}{\partial g} \frac{1}{[H(m(\hat{\beta})) - H(g)]}. \quad (24)$$

This expression accounts for the direct effect of  $g$  and for its indirect impact via the induced variation in  $s$ .

### 4.3 Stage 1: The optimal policy

The government's budget constraint is now given by

$$\tau wT = \pi F(\hat{\beta})(g - s).$$

It differs from (13), its counterpart in the  $TU$  case, in two ways. First,  $g$  is only offered to parent's who do not receive informal care, that is a share  $F(\hat{\beta})$  of the dependent elderly. Second, since  $g$  is exclusive, parents who take up the benefit have to forego their saving. In other words, only  $g - s$  has to be financed. Substituting this budget constraint into the parents' optimized value of  $EU^{OO}$ , we are left with choosing  $g$  to maximize

$$\begin{aligned} \mathcal{L}^{OO} \equiv & w\bar{T} - \pi F(\hat{\beta}) [g - s^{OO}(g)] - s^{OO}(g) + (1 - \pi) U [s^{OO}(g)] + \\ & \pi \left[ \int_{\hat{\beta}}^{\infty} H(m(\beta)) dF(\beta) + F(\hat{\beta})H(g) \right]. \end{aligned}$$

Differentiating  $\mathcal{L}^{OO}$  with respect to  $g$  yields, using the envelope theorem<sup>13</sup>

$$\frac{\partial \mathcal{L}^{OO}}{\partial g} = \pi \left[ \underbrace{F(\hat{\beta})H'(g)}_A - \underbrace{f(\hat{\beta}) [H(m(\hat{\beta})) - H(g)] \frac{\partial \hat{\beta}}{\partial g}}_B - \underbrace{F(\hat{\beta}) \left(1 - \frac{\partial s^{OO}}{\partial g}\right) - (g - s^{OO})f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g}}_C \right].$$

This expression shows that an increase in  $g$  has three different effects, labeled  $A$ ,  $B$  and  $C$ . Term  $A$  measures the expected insurance benefits it provides to parents who receive no informal care. Second,  $g$  affects informal care at the extensive margin: because it increases  $\hat{\beta}$ , it reduces the range of altruism parameters for which care is provided. The cost of this adjustment is measured by term  $B$ . Finally,  $C$  expresses the impact of an increase in  $g$  on first period consumption. It accounts for the induced adjustments in  $s$

<sup>13</sup>The derivative of the parent's objective with respect to  $s$  is zero. Consequently the terms pertaining to the induced variation of  $s$ , including  $\partial \hat{\beta} / \partial g$  vanish for the parent's objective but *not* for the budget constraint. This explains why we have  $\partial \hat{\beta} / \partial g$  in term  $B$  but  $\partial \hat{\beta} / \partial g$  in term  $C$ .

and  $\hat{\beta}$ . Substituting from (19) and (24) and rearranging yields

$$\begin{aligned} \frac{\partial \mathcal{L}^{OO}}{\partial g} &= \pi \left[ \left( F(\hat{\beta}) - f(\hat{\beta})\hat{\beta} \left( 1 + \frac{g - s^{OO}}{H(m(\hat{\beta})) - H(g)} \right) \right) H'(g) - F(\hat{\beta}) \right] \\ &+ \pi \frac{\partial s^{OO}}{\partial g} \left( f(\hat{\beta}) \left( \frac{g - s^{OO}}{H(m(\hat{\beta})) - H(g)} \right) + F(\hat{\beta}) \right). \end{aligned}$$

An interior solution is then characterized by

$$\begin{aligned} &\left[ F(\hat{\beta}) - f(\hat{\beta})\hat{\beta} \left( 1 + \frac{g - s}{H(m(\hat{\beta})) - H(g)} \right) \right] H'(g) \\ &= F(\hat{\beta}) \left( 1 - \frac{\partial s}{\partial g} \right) - \frac{\partial s}{\partial g} f(\hat{\beta}) \left( \frac{g - s}{H(m(\hat{\beta})) - H(g)} \right). \end{aligned}$$

Since  $\partial s / \partial g < 0$ , the RHS of this expression is larger than  $F(\hat{\beta})$  while the term in brackets on the LHS is smaller than  $F(\hat{\beta})$ . Consequently, we have  $H'(g) > 1$ , implying that under an *OO* policy there is less than full insurance for opting-in dependent parents.

## 5 TU vs OO

The previous sections have shown that under both policies  $g$  will crowd out informal care. In the case of *TU* the crowding out occurs both at the intensive margin: for all parents who receive care informal care  $a$  is crowded out by  $g$  on a one by one basis. Crowding out occurs also at the extensive margin but since the informal care provided by the marginal child  $\tilde{\beta}$  is equal to zero, this has no impact on their parents' utility. In the case of *OO*, there is no crowding out at the intensive margin but the crowding out at the extensive margin now does affect parents' utilities. The parents of the marginal children  $\hat{\beta}$  are now strictly better off when they receive informal care.

The precise comparison of the *TU* and *OO* policies is not trivial. To understand the tradeoffs that are involved, we now construct a *sufficient* condition for *OO* to yield a higher welfare than *TU*. To do this, we start from the optimal policy under *TU* and examine under which conditions it can be replicated under *OO*.

Consider the optimal policy under  $TU$ ,  $g^{TU}$ , which yields  $s^{TU}$  and a level of welfare defined by

$$EU^{TU} \equiv w\bar{T} - \pi g^{TU} - s^{TU} + (1 - \pi) U(s^{TU}) + \pi \left[ \int_{\tilde{\beta}}^{\infty} H(m(\beta)) dF(\beta) + F(\tilde{\beta}) H(s^{TU} + g^{TU}) \right]. \quad (25)$$

Let us replace this policy by an  $OO$  policy with  $g^{OO} = g^{TU} + s^{TU}$ . We then have

$$EU^{OO} \geq w\bar{T} - \pi F(\hat{\beta})(g^{OO} - s^{TU}) - s^{TU} + (1 - \pi) U(s^{TU}) + \pi \left[ \int_{\hat{\beta}}^{\infty} H(m(\beta)) dF(\beta) + F(\hat{\beta}) H(g^{OO}) \right],$$

where the inequality sign appears because neither  $g^{OO} = g^{TU} + s^{TU}$  nor  $s^{TU}$  are in general the optimal levels of insurance and savings under  $OO$ . This can be rewritten as

$$EU^{OO} \geq w\bar{T} - \pi F(\hat{\beta})g^{TU} - s^{TU} + (1 - \pi) U(s^{TU}) + \pi \left[ \int_{\hat{\beta}}^{\infty} H(m(\beta)) dF(\beta) + F(\hat{\beta}) H(g^{OO}) \right]. \quad (26)$$

Observe that when  $g^{OO} = g^{TU} + s^{TU}$  and  $s = s^{TU}$  we have  $\hat{\beta} > \tilde{\beta}$ . To see this, evaluate (17) at  $\tilde{\beta}$  which yields

$$\tilde{\beta} \left[ H(m(\tilde{\beta})) - H(g^{OO}) \right] - (m(\tilde{\beta}) - s^{TU}),$$

and from the definition of  $\tilde{\beta}$  we have that  $m(\tilde{\beta}) = g^{TU} + s^{TU}$ , so that this equation can be rewritten as

$$\tilde{\beta} [H(g^{TU} + s^{TU}) - H(g^{TU} + s^{TU})] - (g^{TU} + s^{TU} - s^{TU}) = -g^{TU} < 0.$$

In words, children with  $\beta = \tilde{\beta}$  will not provide aid under  $OO$  so that we must have  $\hat{\beta} > \tilde{\beta}$ . Then, combining (25) and (26) implies that  $EU^{OO} \geq EU^{TU}$  if

$$\begin{aligned} \pi(1 - F(\hat{\beta}))g^{TU} - \pi \int_{\tilde{\beta}}^{\hat{\beta}} H(m(\beta)) dF(\beta) + \pi[F(\hat{\beta}) - F(\tilde{\beta})]H(g^{OO}) = \\ \pi(1 - F(\hat{\beta}))g^{TU} - \pi \int_{\tilde{\beta}}^{\hat{\beta}} [H(m(\beta)) - H(g^{OO})]dF(\beta) \geq 0. \end{aligned}$$

The first term in this expression measures the *benefits* of switching to *OO*; we don't have to pay  $g^{TU}$  for the individuals who receive aid. We also know that for  $\beta > \tilde{\beta}$ ,  $m(\beta) > g^{TU} + s^{TU} = g^{OO}$ . Consequently the second term is negative and it measures the cost of switching to *OO*: the individuals in the interval  $[\tilde{\beta}, \hat{\beta}]$  who receive aid under *TU* but not under *OO* loose. Roughly speaking this requires that the share of children with sufficiently large degrees of altruism is large enough. This makes sense: it is for this population that the intensive margin crowding out induced by the *TU* policy can be avoided by switching to *OO*.

This tradeoff is illustrated in Figure 4. For a given  $s = s^{TU}$  and for  $g^{OO} = s^{TU} + g^{TU}$ , the red line represents consumption in case of dependence under the *TU* regime. The solid black line represents consumption of dependent parents in the *OO* regime. As  $\hat{\beta} > \tilde{\beta}$ , area *A* is the “expected” loss in consumption of dependent parents when the insurance regime switches from *TU* to *OO*. Area *B* represents the savings obtained by switching to *OO*, which depends on the level of public insurance as well as on the number of dependent parents receiving family help under *OO*. The optimal regime will depend on the respective sizes of the two areas.<sup>14</sup> The comparison will depend crucially on the distribution of the altruism parameter,  $F(\beta)$  and on the degree of concavity of the utility function  $H(\cdot)$ .

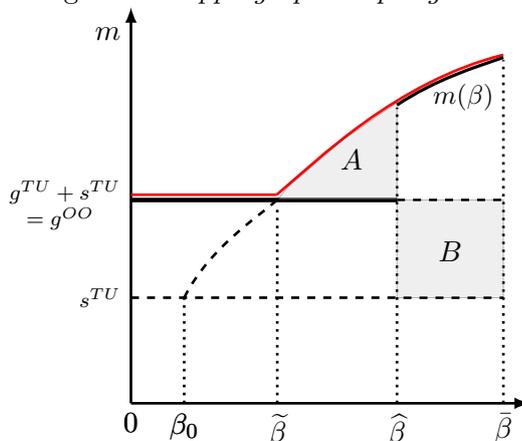
## 6 Private insurance

So far we have ignored private insurance. Assume now that parents can purchase private insurance  $i$  at an actuarially fair premium  $\pi i$ . Private insurance companies cannot enforce an opting out contract which prevents children from helping their parents and forces parents to give away their savings if insurance benefits are claimed.

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<sup>14</sup>This argument is purely illustrative of the tradeoff that is involved. However, the areas cannot directly be compared. First, the area *B* does not account for the distribution of  $\beta$ . To obtain the effective cost savings one has to multiply area *B* by  $[1 - F(\hat{\beta})]$ . Second, area *A* represents the loss in consumption, and not in utility and the sum is not weighted by the density.

Figure 4: *Topping up vs Opting out*



We show in the appendix that in the  $TU$  regime, public LTC insurance is a perfect substitute to fair private insurance. Consequently, when *fair* insurance is available the solution described in Section 3 can be achieved without public intervention. This does not come as a surprise: under  $TU$ , the government is not more efficient than a perfectly competitive insurance markets. Put differently, there is nothing a public insurer can do that markets cannot also accomplish. Then, public and private insurance are equivalent.

This is no longer true under  $OO$ , where public insurance brings about the possibility of preventing children's help and of collecting savings of opting-in dependent parents. Then there *may* be a role for public intervention.

Since  $TU$  public insurance and fair private insurance are equivalent, supplementing private insurance by an  $OO$  policy is effectively a special case of the mixed policies studies in the next section. Consequently, we shall not formally study such a policy at this point. Instead we shall return to it in the following section.

## 7 Mixed policies: opting out and topping up

We now consider the case where the government has two instruments: a transfer to dependent parents that are taken care of by their children, and a transfer to dependent

parents whose children fail to provide assistance. The first transfer, called  $g^{TU}$  can be enjoyed on top of savings and children help. The second transfer, called  $g^{OO}$  is exclusive. One can think about the former as monetary help for care provided at home, and about the latter as nursing home care. We have shown above that  $TU$  is equivalent to fair private insurance in terms of the final allocation. However, it is important to remark that a mixed regime differs for the case where an  $OO$  regime is implemented in addition to fair private insurance purchased by the parents. In a mixed regime, both  $TU$  and  $OO$  transfers are chosen by the social planner. Consequently, the mixed policy (weakly) dominates a combination of fair private insurance and  $OO$ .

### 7.1 Stage 3: The child's choice

If the child decides to provide care, the optimal amount of family assistance  $a^*$  is such that the dependent parents consumption is equal to its laissez faire level,  $m(\beta)$ . However, children provide assistance only if this gives them a higher utility than exclusive government assistance. Thus, there exist a  $\hat{\beta}(g^{OO}, g^{TU}, s)$  such that all parents with children displaying a  $\beta > \hat{\beta}$  opt out, while parents with children displaying  $\beta \leq \hat{\beta}$  receive no assistance and opt in. This threshold  $\hat{\beta}(g^{OO}, g^{TU}, s)$  is defined by

$$\hat{\beta} \left[ H(m(\hat{\beta})) - H(g^{OO}) \right] - (m(\hat{\beta}) - s - g^{TU}) = 0.$$

Observe that

$$\frac{\partial \hat{\beta}}{\partial s} = \frac{\partial \hat{\beta}}{\partial g^{TU}} = - \frac{1}{\left[ H(m(\hat{\beta})) - H(g^{OO}) \right]} < 0, \quad (27)$$

and

$$\frac{\partial \hat{\beta}}{\partial g^{OO}} = \frac{\hat{\beta} H'(g^{OO})}{\left[ H(m(\hat{\beta})) - H(g^{OO}) \right]} > 0. \quad (28)$$

Consequently,  $g^{OO}$  makes family help less likely, while the transfer  $g^{TU}$  provides incentives for children to provide some help. This is due to the fact that the cost of ensuring a consumption level  $m(\beta)$  to parents decreases in  $g^{TU}$ . This property is important for

understanding the respective roles played by the two policies and to comprehend why it may be optimal to combine them. It is also brought out by Figure 5 below. It shows that increasing  $g^{OO}$  makes the no formal care provision more attractive to children. On the other hand, increasing  $g^{TU}$  makes the provision of care more appealing as any given level of total care  $m$  can be achieved at a lower cost to children.

## 7.2 Stage 2: The parent's choice

The parent's expected utility function is

$$EU^M = w(1 - \tau)\bar{T} - s + (1 - \pi)U(s) + \pi \left[ H(g^{OO})F(\hat{\beta}) + \int_{\hat{\beta}}^{\infty} H(m(\beta))dF(\beta) \right].$$

The derivative of  $EU$  with respect to  $s$  is

$$(1 - \pi)U'(s) - \pi f(\hat{\beta}) \left[ H(m(\hat{\beta})) - H(g^{OO}) \right] \frac{\partial \hat{\beta}}{\partial s} - 1. \quad (29)$$

Assuming an interior solution and using (27) and (28) the following expression for the optimal level of savings obtains<sup>15</sup>

$$(1 - \pi)U'(s) + \pi f(\hat{\beta}) = 1. \quad (30)$$

Let us denote by  $s^M(g^{OO}, g^{TU})$  the optimal level of savings as a function of the government transfers. Once again we can then define the critical level of altruism solely as a function of the policy instruments:  $\hat{\beta}(g^{OO}, g^{TU}) = \hat{\beta}(g^{OO}, g^{TU}, s^M(g^{OO}, g^{TU}))$ . Observe that  $s^M(g^{OO}, g^{TU})$  and  $\hat{\beta}(g^{OO}, g^{TU})$  are jointly defined by the system of equations (29)–(30). Differentiating and solving by using Cramer's rule yields

<sup>15</sup>We assume that the second-order condition is satisfied

$$(1 - \pi)U''(s) - \pi \frac{f'(\hat{\beta})}{H(m(\hat{\beta})) - H(g^{OO})} < 0.$$

$$\frac{\partial s^M}{\partial g^{OO}} = \frac{-\pi f'(\hat{\beta})\hat{\beta}H'(g^{OO})}{(1-\pi)U''(s)\Delta H - \pi f'(\hat{\beta})} < 0, \quad (31)$$

$$\frac{\partial s^M}{\partial g^{TU}} = \frac{\pi f'(\hat{\beta})}{(1-\pi)U''(s)\Delta H - \pi f'(\hat{\beta})} > 0, \quad (32)$$

$$\frac{\partial \hat{\beta}}{\partial g^{OO}} = \frac{(1-\pi)U''(s)\hat{\beta}H'(g^{OO})}{(1-\pi)U''(s)\Delta H - \pi f'(\hat{\beta})} > 0, \quad (33)$$

$$\frac{\partial \hat{\beta}}{\partial g^{TU}} = \frac{-(1-\pi)U''(s)}{(1-\pi)U''(s)\Delta H - \pi f'(\hat{\beta})} < 0, \quad (34)$$

where  $\Delta H = H(m(\hat{\beta})) - H(g^{OO})$ . Observe that the effects of an increase in  $g^{OO}$  are similar to the ones obtained under the pure  $OO$  scenario. Conversely, an increase in  $g^{TU}$  now has opposite effects than under the pure  $TU$  scenario. In the pure  $TU$  scenario the public transfer is received by *all* dependent parents, and crowds out informal care at the intensive and the extensive margins. As a consequence, an increase in the transfer reduces savings and increases the threshold above which children provide help. In the mixed regime, however, the  $TU$  transfer is only received by parents that opt out from the exclusive public provision. Consequently, an increase in this transfer makes it *more* attractive to opt out and rely on family care.

Since  $F(\beta)$  is concave, parents also anticipate that the positive effect of  $s$  on  $\hat{\beta}$  increases as  $g^{TU}$  increases. Consequently,  $g^{TU}$  also has a positive effect on savings. Conditions (31)–(34) imply

$$\frac{\partial s^M}{\partial g^{OO}} = -\beta H'(g^{OO}) \frac{\partial s^M}{\partial g^{TU}}, \quad \frac{\partial \hat{\beta}}{\partial g^{OO}} = -\beta H'(g^{OO}) \frac{\partial \hat{\beta}}{\partial g^{TU}}, \quad \frac{\partial \hat{\beta}}{\partial g^{OO}} = -\beta H'(g^{OO}) \frac{\partial \hat{\beta}}{\partial g^{TU}}. \quad (35)$$

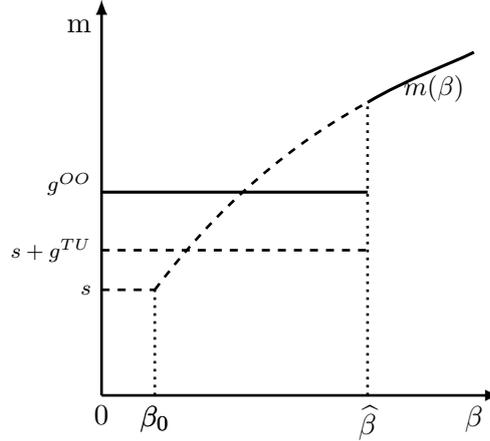
Combining these expression we obtain

$$\hat{\beta}H'(g^{OO}) = -\frac{\frac{\partial s}{\partial g^{OO}}}{\frac{\partial s}{\partial g^{TU}}} = \frac{dg^{TU}}{dg^{OO}} \Big|_s = -\frac{\frac{\partial \hat{\beta}}{\partial g^{OO}}}{\frac{\partial \hat{\beta}}{\partial g^{TU}}} = \frac{dg^{TU}}{dg^{OO}} \Big|_{\hat{\beta}}. \quad (36)$$

This equation gives the “marginal rates of substitution” between  $g^{OO}$  and  $g^{TU}$ , for a given level of  $s$  and a given level of  $\hat{\beta}$  and shows that these two expressions are equal.

A marginal increase in  $g^{OO}$  has to be compensated by an increase  $\widehat{\beta}H'(g^{OO})$  in  $g^{TU}$  to ensure that  $\widehat{\beta}$  and  $s$  are held constant. Consequently, any effect of  $g^{OO}$  on the individual behaviors can be compensated by an appropriate increase in  $g^{TU}$ . The intuition for this result is illustrated in Figure 5. The solid line represents the consumption of dependent parents in the mixed regime, which is not directly affected by  $g^{TU}$ . An increase in  $g^{TU}$  affects consumption only decreasing  $\widehat{\beta}$ . As  $g^{OO}$  increases, the utility of the marginal children (with altruism  $\widehat{\beta}$ ) when they provide no care increases by  $\widehat{\beta}H'(g^{OO})$ . So if  $g^{TU}$  increases by this amount, these children remain indifferent between providing and not providing care. Since savings affect  $m$  only indirectly through  $\widehat{\beta}$ , also  $s$  remains unchanged as long as  $dg^{TU}/dg^{OO} = \widehat{\beta}H'(g^{OO})$ .

Figure 5: *Mixed regime. Consumption of dependent parents as a function of children's altruism*



### 7.3 Stage 1: The optimal policy

The government chooses  $g^{TU}$  and  $g^{OO}$  to maximize

$$\begin{aligned} \mathcal{L}^M \equiv & w\bar{T} - \pi F(\widehat{\beta})(g^{OO} - s^M) - \pi [1 - F(\widehat{\beta})] g^{TU} - s^M + (1 - \pi) U(s^M) + \\ & \pi \left[ \int_{\widehat{\beta}}^{\infty} H(m(\beta)) dF(\beta) + F(\widehat{\beta})H(g^{OO}) \right]. \end{aligned}$$

Differentiating  $\mathcal{L}$  with respect to  $g^{TU}$  and  $g^{OO}$ , and using the envelope theorem, we obtain

$$\frac{\partial \mathcal{L}^M}{\partial g^{OO}} = \pi \left[ F(\hat{\beta}) \left( H'(g^{OO}) - 1 + \frac{\partial s^M}{\partial g^{OO}} \right) - f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g^{OO}} (g^{OO} - s^M - g^{TU}) - f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g^{OO}} \Delta H \right], \quad (37)$$

and

$$\frac{\partial \mathcal{L}^M}{\partial g^{TU}} = \pi \left[ F(\hat{\beta}) \left( 1 + \frac{\partial s^M}{\partial g^{TU}} \right) - 1 - f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g^{TU}} (g^{OO} - s^M - g^{TU}) - f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g^{TU}} \Delta H \right]. \quad (38)$$

Using the conditions in (35), we can rewrite (37) as

$$\begin{aligned} \frac{\partial \mathcal{L}^M}{\partial g^{OO}} &= \pi F(\hat{\beta}) \left( H'(g^{OO}) \left( 1 - \tilde{\beta} \frac{\partial s^M}{\partial g^{TU}} \right) - 1 \right) \\ &\quad + \pi \hat{\beta} H'(g^{OO}) f(\hat{\beta}) \left( \frac{\partial \hat{\beta}}{\partial g^{TU}} (g^{OO} - s^M - g^{TU}) + \frac{\partial \hat{\beta}}{\partial g^{TU}} \Delta H \right). \end{aligned} \quad (39)$$

If the solution is interior, (38) is equal to zero. This yields

$$F(\hat{\beta}) \left( 1 + \frac{\partial s}{\partial g^{TU}} \right) - 1 = f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g^{TU}} (g^{OO} - s - g^{TU}) + f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g^{TU}} \Delta H.$$

Substituting this condition in (39) yields the following condition for an interior  $g^{OO}$

$$F(\hat{\beta}) (H'(g^{OO}) - 1) - \hat{\beta} H'(g^{OO}) (1 - F(\hat{\beta})) = 0. \quad (40)$$

This expression is intuitive. It shows the tradeoff between  $g^{OO}$  and  $g^{TU}$  for given levels of  $\hat{\beta}$  and  $s$ . The optimal policy must satisfy the following (necessary) condition: welfare cannot be increased by a “compensated” variation in  $g^{OO}$  and  $g^{TU}$  that leaves  $\hat{\beta}$  and  $s$  unchanged, that is a variation such that  $dg^{TU} = \hat{\beta} H'(g^{OO}) dg^{OO}$ ; see equation (36).

The first term in (40) represents the net marginal benefit of  $g^{OO}$  for  $\hat{\beta}$  and  $s$  given. An increase in  $g^{OO}$  entails an increase in the utility for dependent parents that do not receive family help, and a marginal cost equal to 1. In the second term,  $(1 - F(\hat{\beta}))$  represents the cost of increasing  $g^{TU}$ , and  $\hat{\beta} H'(g^{OO})$  represents the increase in  $g^{TU}$

ensuring that  $\hat{\beta}$  and  $s$  are held constant as  $g^{OO}$  increases. When choosing  $g^{OO}$ , the social planner takes into account the need to provide insurance to dependent parents with no family help, but also the fact that the insurance provided to parents that get no help from their children,  $g^{TU}$  will have to adjust in order to keep  $\hat{\beta}$  and  $s$  constant.

Condition (40) also implies that there is less than full insurance for the parents that do not receive family help, unless no child provides any help.

Finally, the tradeoff we just described is relevant only when *both* instruments are set by the government. When  $g^{TU}$  is replaced by private insurance, individual coverage is controlled only indirectly. The compensated variation considered in (40) is then no longer feasible.

## 8 Conclusion

This paper has studied the role of social insurance programs in a world in which family assistance is uncertain. It has considered the behavior and welfare of a single generation of “parents” over their life cycle. It has considered social LTC under *TU* and *OO* out regimes.

With *TU*, crowding out occurs both at the intensive and the extensive margins (level of care and share of children who provide care). With *OO* there is no crowding out at the intensive margin, but the one at the extensive margin may be exacerbated. We have provided a sufficient condition under which *OO* dominates *TU*. Roughly speaking this requires that the share of children with sufficiently large degrees of altruism is large enough. This makes sense: its for this population that the intensive margin crowding out induced by the *TU* policy can be avoided by switching to *OO*.

Finally, we have considered a policy combining financial aid on a *TU* basis with public *OO* care provision. We have shown that the policies interact in a nontrivial way. When combined in an appropriate way the policies can effectively be used to neutralize their respective distortions. For instance variations in the policies can be designed so

that the marginal level of altruism (above which children provide care) and savings are not affected. Consequently the mixed policy may be an effective way to provide LTC insurance coverage even when none of the policies is effective if used as sole instrument.

Our results highlight a tradeoff that can inform policy makers considering different schemes for financing long term care. However, the analysis lies on some simplifying assumptions. First, we assume that parents cannot influence the amount of family help, for instance through strategic bequests. Second, we assume that the social planner takes into account only the utility of the parents' generation. Relaxing these assumptions is in our research agenda.

## Appendix

### A Appendix: TU and private insurance

In the case with  $TU$  and actuarially fair private insurance, individuals can purchase an insurance coverage  $i$ , to be received in case of dependence. The fair premium is  $\pi i$ .

The first-order condition of the children's problem with respect to  $a$  is, assuming an interior solution,

$$-1 + \beta H'(s + g + i + a) = 0.$$

Define  $\tilde{\beta}(s + g + i)$  such that

$$1 = \tilde{\beta} H'(s + g + i) \tag{A1}$$

If  $\beta \geq \tilde{\beta}$ , the consumption of dependent parents  $m(\beta)$  is exactly the same as in the laissez-faire. When  $\beta < \tilde{\beta}$ ,  $a^* = 0$  and  $m = s + g + i$ . Finally, observe that

$$\frac{\partial \tilde{\beta}}{\partial (s + g + i)} = -\frac{\tilde{\beta} H''}{H'} > 0.$$

The problem of the parents is to maximize their expected utility with respect to  $s$  and  $i$ , and assuming an interior solution (i.e.  $i > 0$ ), the optimal value of  $s$  and  $i$  satisfy, respectively

$$(1 - \pi) U'(s) + \pi F(\tilde{\beta}) H'(s + g + i) = 1, \tag{A2}$$

and

$$F(\tilde{\beta}) H'(s + g + i) = 1, \tag{A3}$$

which implies

$$U'(s) = F(\tilde{\beta}) H'(s + g + i) = 1. \tag{A4}$$

Consequently,  $s$  does not depend on the level of public LTC insurance, and  $\partial i / \partial g = -1$ .

In stage 1, the government chooses  $g$  to maximize

$$\mathcal{L} \equiv w\bar{T} - \pi g - s - \pi i + (1 - \pi)U(s) + \pi \left[ \int_{\tilde{\beta}}^{\infty} H(m(\beta)) dF(\beta) + F(\tilde{\beta})H(s + g + i) \right]. \quad (\text{A5})$$

Differentiating  $\mathcal{L}$  with respect to  $g$  yields, using the envelope theorem,

$$\frac{\partial \mathcal{L}}{\partial g} = \pi \left[ F(\tilde{\beta})H'(s + g + i(g)) - 1 \right], \quad (\text{A6})$$

which, under (A3), is equal to zero for all  $g$  such that  $F(\tilde{\beta})H'(s + g) \geq 1$ , and negative otherwise, when  $g$  is so large that the constraint that  $i \geq 0$  becomes binding (so that  $i = 0$ )

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