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"Valuation of natural capital under uncertain substitutability"

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Valuation of natural capital under uncertain substitutability

Abstract
Natural capital is complex to price notably because of the high uncertainties surrounding the substitutability of its future ecosystem services. We examine a two-tree Lucas economy where both the economic growth and the degree of substitutability are uncertain. We show that the uncertain substitutability raises the expected value of the service and the rate at which it should be discounted. The value effect dominates the discounting effect, so the economic value of natural capital is increased. When the prior beliefs about substitutability are Gaussian, the economic value of future ecosystem services goes to infinity for finite maturities.

Keywords: Asset pricing, CCAPM beta, discounting, bioeconomics.

JEL Codes: G12, Q01.

1. *Introduction*

Our attitude towards the preservation of natural capital such as water, biodiversity, fossil fuels, climate or unspoiled natural sites is determined by the way we price it. But most natural assets generate ecological services that will persist for centuries, and deep uncertainties surround the valuation of these services by future generations. This implies that there is no consensus about how to price natural capital. For this reason, this notion remained up to now a metaphor rather than an instrument (Fenichel and Abbott (2014)). Much of the debate among economists has been focused on the rate at which the flow of future benefits should be discounted. In an economy in which the consumption of manufactured goods increases through time, investing for the future raises intergenerational inequalities. The discount rate can thus be interpreted as the minimum rate of return of the safe investment that compensates for this adverse effect on intergenerational welfare. This is in line with the Ramsey rule (Ramsey (1928)) which states that the consumption discount rate net of the rate of impatience is equal to the product of the growth rate of consumption by the index of relative inequality aversion.

But most natural capital generates environmental services that differ from the consumption of manufactured goods, and that have heterogeneous degrees of substitutability with them. Guesnerie (2004), Hoel and Sterner (2007), Sterner and Persson, (2008), Gollier (2010) and Traeger (2011) have stressed the role of the evolution of relative prices in discounting. In a growing economy, the relative scarcity of the non-substitutable services of natural capital that cannot be expanded will increase, thereby raising their relative value for future generations. Suppose that the elasticity of substitution between manufactured goods and the services of natural capital is constant. If β denotes the inverse of the elasticity of substitution, this means that the relative increase in the value of ecological services in the future will be equal to c^{β}, where c is the relative increase in consumption of manufactured goods. If the elasticity of substitution is small, the increase in value of these services will be large. This dampens the effect of discounting.

The substitutability of scarce environmental goods by manufactured goods is central to any cost-benefit analysis of environmental policies and to the notion of sustainability. In the late 17th century, the French administration expanded oak forests in the perspective of being able to build ships two centuries later to fight the British naval forces, long before realizing that oak would be substituted by steel. More recently, wars have been made to control oil fields before realizing that oil could well be substituted by non-conventional gas reserves and by renewable sources of energy in the near future. Optimistic futurists believe that the need for material goods and disappearing natural capital will be reduced or even eliminated by new technologies.

But none of these things should be taken for granted in advance. In this paper, we take seriously the uncertainty affecting the substitutability between ecological services and manufactured goods. To illustrate, suppose first that β is equal to unity with certainty and that consumption of manufactured goods is expected to be multiplied by a factor 10 within the next century. Assuming that the quantity of ecological services will remain stable in the future, this implies that the value of these services will also be multiplied by a factor 10 within the next century. Suppose alternatively that we are unsure about the degree of substitutability, so that β is either 0 (infinite substitutability) or 2 (weak substitutability) with equal probabilities. In that context, the value of ecological services in 100 years will be either stable, or it will be increased by a factor 100. In expectation, the value of ecological services will be increased by a factor 50. Discounted to the present, this shows that the uncertainty affecting the degree of substitution magnifies the value of the natural asset.[1] In this example, the uncertainty surrounding the substitutability of a natural asset raises its social value by a factor 5.

However, this simple observation should be reconsidered once we recognize that the growth rate of the consumption of manufactured goods is also uncertain. It happens that, when ecological services are stable over time and when parameter β is certain, this parameter is the consumption-based CAPM beta of the natural asset, so that the discounting rule is easily derived from standard asset pricing theory in that case. Said differently, the CAPM beta of a specific natural capital is equal to the inverse of the elasticity of substitution of the service that it provides. But how should

[1] Technically, this is a direct consequence of the fact that function $f(c) = c^{\beta}$ is convex in β.

one discount expected future ecological dividends Ec^{β} when β is uncertain? We show in this paper that in most practical cases, this uncertainty raises the "certainty equivalent" beta of the natural asset, thereby reducing the associated risk-adjusted discount rate and depressing its present value. This implies that this "risk-adjusted discounting" effect goes against the "expected dividend" effect mentioned above. We also show in this paper that the net effect is always positive, i.e., the uncertain substitutability always raises the value of the natural capital. This is in line with the precautionary principle, a general rule favouring the preservation of natural assets in the face of uncertainty. This result is also related to the well-known property that an increase in the volatility of the growth rate of consumption raises aggregate wealth in the standard Lucas-tree economy with constant relative risk aversion.[2]

These new findings are related to some results in the finance literature. Pastor and Veronesi (2003, 2009) show that the uncertainty affecting the growth rate of dividends of an asset increases its market value. In our model, the uncertainty affecting substitutability translates into an uncertain growth rate of natural dividends. But this risk is correlated to the systematic risk, whereas Pastor and Veronesi (2003, 2009) assume an idiosyncratic risk so that they are not concerned by the risk-adjusted discounting effect. This paper is also related to the literature on the impact on asset pricing of learning. Collin-Dufresne, Johannes and Lochstoer (2015) examine the case of learning about the trend of economic growth or about the frequency of macroeconomic catastrophes. Jagannathan and Wang (1996), Lettau and Ludvigson (2001) and Adrian and Franzoni (2008) acknowledge that most assets' betas vary stochastically, and that this uncertainty affects asset pricing. This research is also linked to the recent developments aimed at valuing very distant cash flows (Martin (2012), Barro and Misra (2012)). Using our findings, the high price documented by Giglio, Maggiori, and Stroebel (2015) and Giglio, Maggiori, Stroebel and Weber (2015) for real estate claims maturing in 100 years and more in the United Kingdom and in Singapore could be due to the deep uncertainty affecting the correlation between aggregate consumption and the land rent in the distant future.

[2] Financial data suggests quite the opposite: Equity valuations decline at time of high economic uncertainty (eg., Bansal, Khatchatrian and Yaron (2005)).

2. *The model*

Consider an economy with a representative agent consuming at discrete dates $t = 0,1,2,\ldots$. At any date *t,* the agent consumes c_t units of a numeraire good and x_t units of ecosystem services exogenously generated from some specific natural capital. We consider the standard utilitarian social welfare function *W* with

$$W = \sum_{t=0} e^{-\delta t} EU(x_t, c_t, t), \tag{1}$$

where δ is the rate of pure preference for the present. The expectation is relative to the information set available at date 0. We assume that W exists and is finite. Following Guesnerie (2004), Hoel and Sterner (2007), Sterner and Persson, (2008), Gollier (2010) and Traeger (2011), the instantaneous utility function of the representative consumer is assumed to belong to the CES family, with

$$U(x_t, c_t, t) = \frac{1}{1-\gamma} y_t^{1-\gamma}, \text{ with } y_t = \left[\alpha x_t^{1-\beta_t} + (1-\alpha) c_t^{1-\beta_t}\right]^{\frac{1}{1-\beta_t}}, \tag{2}$$

where y is a measure of aggregate consumption, γ is the aversion to risk on this aggregate good, with $\alpha \in [0,1]$ and $\beta_t \in \mathbb{R}$.[3] Parameter α measures the weight of the services of the natural capital under scrutiny in the aggregate good consumed by the representative agent. Parameter β_t is the inverse of the elasticity of substitution between the numeraire good and ecosystem services at date t.[4]

We contemplate an action today that will increase the flow of the ecosystem services $x_t\big|_{t=0,1,\ldots}$ by $\Delta_t\big|_{t=0,1,\ldots}$, where $\Delta_t \geq 0$ is the sure marginal increase of x_t at date t generated by the action. In order to implement a standard cost-benefit analysis of this action, we characterize P, which denotes

[3] When $\beta = 1$, we get a Cobb-Douglas function with $y = c^{1-\alpha} x^{\alpha}$.

[4] An alternative interpretation of this model is that the consumption good y is produced with two inputs (x, c) through the CES production function. In that case, parameter β_t characterizes the substitutability between the man-made input c and the ecological input x.

the equivalent increase in present consumption that has the same welfare effect than the action itself. This price is given by the marginal rate of substitution:

$$P = \sum_{t=0} e^{-\delta t} E\left[\frac{U_x(x_t, c_t, t)}{U_c(x_0, c_0, 0)}\right]\Delta_t. \tag{3}$$

Without loss of generality, we assume that $c_0 = x_0 = 1$, so that $U_c(x_0, c_0, 0) = 1-\alpha$. Following Barro and Misra (2012), we hereafter assume that the service of the specific natural capital under scrutiny has only a marginal impact on welfare. More specifically, the weight α of the ecosystem under scrutiny in social welfare is assumed to be close to zero. In this context, we have that y_t can be approximated by c_t for all *t,* so that c_t can hereafter be referred to as the aggregate consumption. Moreover, this implies that $U_c(x_t, c_t, t) = (1-\alpha)c_t^{-\gamma}$ and $U_x(x_t, c_t, t) = \alpha x_t^{-\beta_t} c_t^{\beta-\gamma}$. Equation (3) can thus be rewritten as

$$P = F_0 \sum_{t=0} e^{-\rho_t t}\Delta_t, \tag{4}$$

where $F_t = U_x(x_t, c_t, t) / U_c(x_t, c_t, t)$ is the marginal rate of substitution between the ecosystem service and consumption at date *t*, and ρ_t is given by the following equation:

$$\rho_t = \delta - t^{-1}\ln E\left[x_t^{-\beta_t} c_t^{\beta_t-\gamma}\right]. \tag{5}$$

Parameter ρ_t can be interpreted as the ecological discount rate associated to maturity *t.* Indeed, equation (4) tells us that the value of the action is obtained through a sequence of two operations. First, the flow of incremental ecosystem services is discounted at rates $\rho_t\big|_{t=0,1,...}$. This yields the equivalent present increase in ecosystem service, which is in turn monetarized by using the current MRS F_0.

The expectation operator in the RHS of equations (3) and (5) is related to three sources of uncertainties. Both the economic growth and the evolution of the ecosystem services are uncertain. We assume that (x_t, c_t) follows a discrete version of a bivariate geometric Brownian motion. Let $g_{xt} = \ln x_{t+1} / x_t$ and $g_{ct} = \ln c_{t+1} / c_t$ denote the growth rate of respectively the ecosystem services and the numeraire good. We assume that $(g_{xt}, g_{ct})\big|_{t=0,1,...}$ follows a stationary random walk with (g_x, g_c) being normally distributed with mean (μ_x, μ_c), variance (σ_x^2, σ_c^2) and covariance $\rho\sigma_x\sigma_c$ with $\rho \in [-1,1]$. In this paper, we also assume that the elasticity of substitution β_t^{-1} evolves stochastically over time. This means that β_t is a random variable whose probability distribution G_t describes our beliefs about its true value at date 0.

3. *The pricing of natural capital when the elasticity of substitution is known*

Let us define function $\chi(t,z) = \ln E\exp(tz)$, which is the Cumulant-Generating Function (CGF) of random variable z. Most results presented in this paper are derived from the following Lemma, which provides some well-known properties of CGF functions (see Billingsley (1995)).

Lemma 1 : *If it exists, the CGF function* $\chi(t,z) = \ln E\exp(tz)$ *has the following properties:*

- *i.* $\chi(t,z) = \sum_{n=1}^{\infty} \kappa_n(z) t^n / n!$ *where* $\kappa_n(z)$ *is the nth cumulant of random variable z.*
- *ii.* *The most well-known special case is when z is* $N(\mu, \sigma^2)$, *so that* $\chi(t,z) = t\mu + 0.5t^2\sigma^2$.
- *iii.* $t^{-1}\chi(t,z)$ *is increasing in t, from Ez to the supremum of the support of z when t goes from zero to infinity.*

Suppose first that β_t takes value β with certainty.

$$\begin{aligned}\rho_t &= \delta - t^{-1}\ln E\left[\exp\left(-\beta\ln x_t + (\beta-\gamma)\ln c_t\right)\right] \\ &= \delta - t^{-1}\ln E\left[\exp\sum_{\tau=0}^{t-1}\left(-\beta g_{xt} + (\beta-\gamma)g_{ct}\right)\right] \\ &= \delta - t^{-1}\ln\left[E\exp\left(-\beta_t g_x + (\beta_t-\gamma)g_c\right)\right]^t .\end{aligned} \tag{6}$$

The third equality comes from the fact that $(g_{xt}, g_{ct})\big|_{t=0,1,...}$ follows a stationary random walk. Now, observe that $z = -\beta_t g_x + (\beta_t - \gamma)g_c$ is normally distributed. Applying property *ii* of Lemma 1 allows us to rewrite the above equation as $\rho_t = \rho(\beta)$ with

$$\rho(\beta) = \delta - t^{-1}\ln\left[\exp t\left(-\beta\mu_x + (\beta-\gamma)\mu_c + 0.5\left(\beta^2\sigma_x^2 + (\beta-\gamma)^2\sigma_c^2 - 2\beta(\beta-\gamma)\rho\sigma_x\sigma_c\right)\right)\right]$$
$$= \rho_f - a\beta - 0.5b\beta^2, \quad (7)$$

with

$$\rho_f = \delta + \gamma\mu_c - 0.5\gamma^2\sigma_c^2, \quad (8)$$

$$a = \mu_c - \mu_x - \gamma\sigma_c\left(\sigma_c - \rho\sigma_x\right), \quad (9)$$

$$b = Var(g_c - g_x) = \sigma_c^2 + \sigma_x^2 - 2\rho\sigma_x\sigma_c \geq 0. \quad (10)$$

Thus, conditional to $\beta_t = \beta$, the ecological discount rate equals $\rho_f - a\beta - 0.5b\beta^2$. In the special case of perfect substitutability with $\beta = 0$, the future value of ecosystem services is a known constant F_0. From equation (7), incremental ecosystem services should be discounted in that case at the constant rate ρ_f, which can thus be interpreted as the risk free discount rate in this economy. In fact, equation (8) is the well-known extended Ramsey rule (Cochrane (2001)). On the contrary, when the natural capital is only imperfectly substitutable, the ecological discount rates ρ_t must be adapted by subtracting $a\beta + 0.5b\beta^2$ from the risk free rate ρ_f. Observe also that if β is constant over time, the ecological discount rates have a flat term structure.

In this context with no uncertainty on the degree of substitutability of natural capital, one can examine the effect of a change in the degree of substitutability on the ecological discount rates. From equation (7), we see that the ecological discount rate is decreasing in β if and only if $\beta \geq -a/b$. This implies that the ecological discount rate and the value of natural capital are non-monotone in the elasticity of substitution of the ecosystem services.

4. *Uncertain substitutability, ecological discounting and value*

We hereafter consider that β_t is a random variable whose distribution characterizes our current beliefs about the degree of substitutability of the services provided by the natural capital at date *t*. Using the law of iterated expectations, equation (5) can be rewritten as

$$e^{-\rho_t t} = E\left[E\left[x_t^{-\beta_t} c_t^{\beta_t - \gamma} \middle| \beta_t\right]\right] = Ee^{-\rho(\beta_t)t}. \tag{11}$$

When β_t is uncertain, the ecological discount factor equals the expectation of the ecological discount factor conditional to β_t. This is summarized in the following proposition, where equation (11) is rewritten as equation (12).

Proposition 1: *Let β_t denote the inverse of the degree of substitutability of the ecosystem services. The ecological discount rate ρ_t associated to time horizon t is given by the following equation:*

$$\rho_t = \rho_f - t^{-1}\chi\left(t, a\beta_t + 0.5b\beta_t^2\right), \tag{12}$$

where ρ_f, a and b are three scalars defined respectively in equations (8), (9) *and* (10).

In the remainder of this section, we use equation (12) to derive some properties of the impact of the uncertainty affecting the degree of substitutability on the ecological discount rate and on the value of natural capital. It is a direct consequence of the fact that the conditional discount rate $\rho(\beta)$ is non-monotone in β that first-order stochastic changes in the distribution of β_t have an intrinsically ambiguous effect on the ecological discount rate, and therefore on the value of natural capital. However, we show in Proposition 2 that a mean-preserving spread in β_t always raises the value of natural capital. This is the main result of this paper.

Proposition 2: *The ecological discount rate ρ_t is reduced by any mean preserving spread of β_t. In this sense, the uncertainty affecting the degree of substitutability of the ecosystem service always raises the economic value of the natural capital that generates it.*

Proof: We can rewrite equation (12) as follows:

$$\rho_t = \rho_f - t^{-1}\ln Ek(\beta_t), \tag{13}$$

where $k(\beta) = \exp t\left(a\beta + 0.5b\beta^2\right)$. Observe that because *b* is non-negative, function *k* is convex. By Jensen's inequality, $Ek(\beta_t)$ is increased by any mean-preserving spread of β_t. This concludes the proof. □

There are two reasons why an increase in risk about β_t reduces the ecological discount rate. The first reason can be identified in the special case of short maturities. Indeed, we know from property *iii* of Lemma 1 that the limit of $t^{-1}\chi(t, a\beta + 0.5b\beta^2)$ when *t* vanishes is equal to the expectation of $a\beta + 0.5b\beta^2$. This implies that

$$\lim_{t\to 0_+} \rho_t = \rho_f - aE\beta_{0_+} - 0.5bE\beta_{0_+}^2 \leq \rho_f - aE\beta_{0_+} - 0.5b\left(E\beta_{0_+}\right)^2. \tag{14}$$

If β_{0_+} is uncertain, the ecological discount rate for short maturities is reduced by $0.5b\left[E\beta_{0_+}^2 - \left(\beta_{0_+}\right)^2\right]$. The second technical reason of the negative impact of the uncertain β on the ecological discount rate is due to the fact that $t^{-1}\chi(t, a\beta + 0.5b\beta^2)$ is increasing in *t* when β is uncertain.

In Proposition 3, we show that the term structure of ecological discount rates is decreasing when the uncertainty affecting β_t is increasing with maturity, in the sense of Rothschild and Stiglitz (1970).

Proposition 3: *Suppose that, for all $t' > t$, $\beta_{t'}$ is a mean-preserving spread of β_t. It implies that the term structure of ecological discount rates ρ_t is decreasing.*

Proof: Consider any pair (t, t') such that t' is larger than *t*. Because $t^{-1}\chi(t, z)$ is increasing in *t* by property *iii* of Lemma 1, equation (12) implies that

$$\begin{aligned} \rho_{t'} &= \rho_f - t'^{-1}\chi\left(t', a\beta_{t'} + 0.5b\beta_{t'}^2\right) \\ &\leq \rho_f - t^{-1}\chi\left(t, a\beta_{t'} + 0.5b\beta_{t'}^2\right) = \rho_f - t^{-1}\ln Ek(\beta_{t'}), \end{aligned} \tag{15}$$

where $k(\beta) = \exp t\left(a\beta + 0.5b\beta^2\right)$ is convex in β. By Jensen's inequality, it implies that $Ek(\beta_{t'}) \geq Ek(\beta_t)$. Combining these two results implies that

$$\rho_{t'} \leq \rho_f - t^{-1}\ln Ek(\beta_{t'}) \leq \rho_f - t^{-1}\ln Ek(\beta_t) = \rho_t. \tag{16}$$

This concludes the proof. □

The decreasing nature of the term structure of the ecological discount rate says something important about the intrinsic value of natural capital. In a world in which the degree of substitutability of the ecosystem services is uncertain, natural capital is particularly valuable if it can deliver ecological benefits in the distant future. An economic intuition of this central result of this paper can be derived from two observations. First, from equation (11), we know that the ecological discount rate equals the expectation of the ecological discount factors conditional to β_t. Second, the discount factor $\exp(-\rho(\beta)t)$ is a convex function of β, and the degree of convexity of this function is increasing in *t*. These two observations implies that the term structure of the ecological discount rates must be decreasing when the distribution of β_t becomes more dispersed for longer maturities. This result is in line with Weitzman (1998, 2001) who showed that the term structure of the discount rates must be decreasing when the rate at which sure benefits must be discounted in the future is uncertain.

Because of the presence of a term in β^2 is $\rho(\beta)$, there is usually no analytical solution to $\chi(t, a\beta + 0.5b\beta^2)$. However, if one knows the first few cumulants of random variable $a\beta_t + 0.5b\beta_t^2$, one can approximate equation (12) by using property *i* of lemma 1:

$$\rho_t = \rho_f - \sum_{n=1}^{\infty} \kappa_n \left(a\beta_t + 0.5b\beta_t^2 \right) \frac{t^{n-1}}{n!}. \tag{17}$$

For example, if the distribution of β_t is independent of *t* in the neighborhood of $t = 0_+$, then the above equation implies that

$$\lim_{t \to 0_+} \frac{\partial \rho_t}{\partial t} = -0.5Var\left(a\beta_{0_+} + 0.5b\beta_{0_+}^2 \right). \tag{18}$$

One can finally use property *iii* of Lemma 1 to determine the asymptotic value of the ecological discount rate. Suppose that the support of the distribution of β_t is bounded when *t* tends to infinity, with $\lim_{t \to \infty} \text{supp}\, \beta_t = \left[\beta_{\min}, \beta_{\max} \right]$. We know that $t^{-1}\chi(t, x)$ converges to the supremum of the support of $a\beta_t + 0.5b\beta_t^2$. This implies that the ecological discount rates asymptotically tend to

$$\lim_{t\to\infty} \rho_t = \begin{cases} \rho(\beta_{\min}) = \rho_f - a\beta_{\min} - 0.5b\beta_{\min}^2 & if \quad \beta^* \leq -a/b \\ \rho(\beta_{\max}) = \rho_f - a\beta_{\max} - 0.5b\beta_{\max}^2 & if \quad \beta^* > -a/b, \end{cases} \tag{19}$$

where $\beta^* = 0.5(\beta_{\min} + \beta_{\max})$ is the center of the support of β_∞.

5. A financial approach

In the previous two sections, we computed the value of natural capital by discounting its sure incremental flow of ecosystem services at an ecological rate ρ_t. This equivalent immediate increase in ecosystem services is then priced by using the known current value F_0 of these services. Although this method was already recommended by Malinvaud (1953), it remains non-traditional. A more traditional method is obtained by rewriting equation (3) as follows :

$$P = \sum_{t=0} e^{-\delta t} Em_t F_t \Delta_t = \sum_{t=0} e^{-r_t t} EF_t \Delta_t, \tag{20}$$

where

$$F_t = \frac{U_x(x_t, c_t, t)}{U_c(x_t, c_t, t)} = F_0 \left(\frac{c_t}{x_t} \right)^{\beta_t} \tag{21}$$

is the value of ecosystem services at date *t*,

$$m_t = \frac{U_c(x_t, c_t, t)}{U_c(x_0, c_0, 0)} = c_t^{-\gamma} \tag{22}$$

is the standard pricing kernel of the consumption based pricing model (Rubinstein (1976), Lucas (1978)), and

$$r_t = \delta - t^{-1} \ln \frac{Em_t F_t}{EF_t} \tag{23}$$

is the risk-adjusted monetary discount rate. The value P of the incremental flow of ecosystem services is obtained in equation (20) by estimating its flow of expected future monetary benefits

$EF_t\Delta_t\big|_{t=0,1,...}$ and then by discounting this flow by using the risk-adjusted discount rates r_t. This risk adjustment is due to the fact that the value F_t of ecosystem services may be correlated to aggregate consumption c_t, and thus to state prices m_t.

5.1. The risk-adjusted monetary discount rate

Following the same methodology as in the previous section, equation (23) can be rewritten as follows :

$$\begin{aligned} r_t &= \delta + t^{-1}\ln EF_t - t^{-1}\ln Em_tF_t \\ &= \rho_f + t^{-1}\chi\left(t,\left(\mu_c-\mu_x\right)\beta_t + 0.5b\beta_t^2\right) - t^{-1}\chi\left(t, a\beta_t + 0.5b\beta_t^2\right) \end{aligned} \tag{24}$$

In the special case in which β_t is certain and equal to β, this simplifies to

$$r_t = \rho_f + \beta\gamma\sigma_c(\sigma_c - \rho\sigma_x). \tag{25}$$

The intuition of this result is simple if we also assume that $\rho = 0$, i.e., that the growth rates of aggregate consumption and of ecosystem services are independent. In that case, equation (25) is the well-known pricing formula of the consumption-based capital asset pricing model (CCAPM), in which the risk-adjusted discount rate is equal to the risk free rate plus a risk premium proportional to the systematic risk premium $\gamma\sigma_c^2$. The coefficient of proportionality is usually referred to as the CCAPM beta of the asset. Equation (25) shows that the CCAPM beta of natural capital is just equal to β. In this special case, the inverse of the elasticity of substitution can thus be interpreted as the CCAPM beta of natural capital. Although the incremental ecological benefit is certain, its value F_t is not. This value is a function of the relative scarcity of the ecosystem services which is measured by c_t / x_t, as expressed in equation (21). When x_t is positively correlated to c_t, this reduces the correlation between the value of the ecosystem service and aggregate consumption. Therefore, it reduces the risk premium $\pi\beta$, where

$$\pi = \gamma\sigma_c(\sigma_c - \rho\sigma_x) \tag{26}$$

is the unit risk premium associated to natural capital. Notice that this risk premium becomes negative when $\rho\sigma_x$ is larger than σ_c. This is because investing in natural capital hedges the macroeconomic risk in this economy.

Let us now examine the impact of the uncertainty affecting the elasticity of substitution on the risk-adjusted discount rate. Because the two functions $\exp\left[\left(\mu_c-\mu_x\right)\beta_t+0.5b\beta_t^2\right]$ and $\exp\left[a\beta_t+0.5b\beta_t^2\right]$ are two convex functions of β, the impact of the uncertainty affecting β_t on the risk-adjusted discount rate r_t is ambiguous. Similarly, because $t^{-1}\chi(t,z)$ is increasing in *t*, the risk-adjusted discount rate is the difference of two increasing functions of *t*, so that the slope of its term structure has an ambiguous sign. The analysis of the properties of the risk-adjusted discount rate is thus more complex than the analysis of the ecological discount rate.

One can use property *iii* of Lemma 1 to obtain that

$$\begin{aligned}\lim_{t\to 0_+} r_t &= \rho_f + E\left[\left(\mu_c-\mu_x\right)\beta_t+0.5b\beta_t^2\right]-E\left[\left(\mu_c-\mu_x-\pi\right)\beta_t+0.5b\beta_t^2\right] \\ &= \rho_f+\pi E\beta_{0_+}.\end{aligned} \tag{27}$$

This means that, for small maturities, the uncertainty affecting β_t has no effect on the risk-adjusted discount rate. Moreover, if the distribution of β_t is independent of *t* in the neighborhood of $t=0_+$, property *i* of Lemma 1 implies that

$$\begin{aligned}\lim_{t\to 0_+}\frac{\partial r_t}{\partial t} &= 0.5Var\left(\left(\mu_c-\mu_x\right)\beta_{0_+}+0.5b\beta_{0_+}^2\right)-0.5Var\left(\left(\mu_c-\mu_x-\pi\right)\beta_{0_+}+0.5b\beta_{0_+}^2\right) \\ &= \pi\left(\mu_c-\mu_x+bE\beta_{0_+}-0.5\pi\right)Var\left(\beta_{0_+}\right)+0.5b\pi Skew\left(\beta_{0_+}\right).\end{aligned} \tag{28}$$

Finally, suppose that the support of β_t is $\left[\beta_{\min},\beta_{\max}\right]$ when *t* tends to infinity. Then, assuming that π is positive, the risk-adjusted discount rate tends to

$$\lim_{t\to\infty} r_t = \begin{cases} \rho_f+\beta_{\min}\pi & if \quad \mu_c-\mu_x+b\beta^*\le 0 \\ \rho_f+\beta_{\min}\pi+\left(\beta_{\max}-\beta_{\min}\right)\left(\mu_c-\mu_x+b\beta^*\right) & if \quad 0<\mu_c-\mu_x+b\beta^*\le\pi \\ \rho_f+\beta_{\max}\pi & if \quad \pi<\mu_c-\mu_x+b\beta^*, \end{cases} \tag{29}$$

with $\beta^* = 0.5(\beta_{\min} + \beta_{\max})$. In order to extract more information from these equations, consider as a benchmark the case of a constant flow of ecosystem services, so that $\mu_x = \sigma_x = 0, b = \sigma_c^2$ and $\pi = \gamma\sigma_c^2 > 0$. Assume further that the distribution of β_t is symmetric and has a constant positive mean, so that $skew(\beta_t) = 0$ and $\beta^* = E\beta_{0_+} = \overline{\beta}$. It implies from equation (28) that the slope of the term structure for small maturities is positive if and only if $\mu_c - 0.5\gamma\sigma_c^2 + \sigma_c^2\overline{\beta}$ is positive. The asymptotic value of r_t is $\rho_f + \beta_{\max}\pi$ if $\mu_c - \gamma\sigma_c^2 + \sigma_c^2\overline{\beta}$ is positive. A sufficient condition for these two conditions is $\mu_c \geq \gamma\sigma_c^2$. Using historical values in the western world with $\mu_c \approx 2\%$ and $\sigma_c \approx 3\%$, this condition holds whenever relative risk aversion γ is smaller than 22.2. Because it is widely accepted that relative risk aversion is between 1 and 4 for most individuals, we conclude from this calibration exercise that the risk-adjusted discount rates are very likely to be increasing with maturities, from $r_0 = \rho_f + \overline{\beta}\pi$ to $r_\infty = \rho_f + \beta_{\max}\pi$. This result is reinforced if the trend of growth of the ecosystem services is negative, or if the distribution of β_t is positively skewed.

Because both the risk free rate ρ_f and the risk premium π have a flat term structure, the only source of non-constancy of the term structure of the risk-adjusted discount rate r_t comes from the uncertain degree of substitutability. Suppose that the distribution of β_t is independent of *t* in the neighborhood of *t=0*. We can rearrange the RHS of equation (28) to obtain

$$\lim_{t\to 0_+} \frac{\partial r_t}{\partial t} = -0.5\pi^2 Var\left(\beta_{0_+}\right) + \text{cov}\left(\left(\mu_c - \mu_x\right)\beta_{0_+} + 0.5b\beta_{0_+}^2, \pi\beta_{0_+}\right). \tag{30}$$

The two terms in the right side of this equation are hereafter referred to as respectively the "Weitzman effect" and the "correlation effect". The Weitzman effect is best understood by observing that if the beta of natural capital is uncertain, the conditional CCAPM discount rate $r = \rho_f + \beta\pi$ at which future net benefits should be discounted is also uncertain. Following Weitzman (1998, 2001), the discount factor to be used in that case is the expectation of the conditional discount factor $\exp(-(\rho_f + \beta\pi)t)$, which is decreasing and convex in β. In other words, the unconditional discount rate should be equal to $-t^{-1}\chi(t, -\rho_f - \beta\pi)$. By Lemma 1*iii*, this is decreasing in *t* with a slope at *t=0* equaling $-0.5\pi^2 Var(\beta)$. This Weitzman effect corresponds to the first term in the right side of (30). It tends to make the term structure of the risk-adjusted discount rate *decreasing*.

However, the Weitzman argument applies only to safe projects, i.e., to projects whose payoffs are uncorrelated with the future discount rate. But conditional to β, the growth rate of conditional expected net benefit at date *t* is equal to $\left(\mu_c - \mu_x\right)\beta_{0_+} + 0.5b\beta_{0_+}^2$, which is correlated with the conditional rate $\rho_f + \beta_{0_+}\pi$ at which it needs to be discounted. The sign of this correlation is generally ambiguous. But if we assume that π and $\mu_c - \mu_x$ are positive, and that the latter is at least one order of magnitude than b, we can expect that this correlation is positive: The expected value of the ecosystem service is larger when the rate at which it should be discounted is larger. This implies that the Weitzman effect alone overestimates the economic value of the future ecosystem service, i.e., it underestimates the risk-adjusted discount rate. This effect is increasing in maturity, which means that it tends to make the term structure of the risk-adjusted discount rate *increasing*. The existence of this correlation between the conditional discount rate and the conditional future benefit explains the second term in the right side of equation (30). This is the correlation effect.

Equations (28) and (30) imply that, under a stationary and symmetric distribution for β_t, the term structure is increasing if and only if the correlation effect given by $\pi\left(\mu_c - \mu_x + bE\beta_{0_+}\right)Var(\beta)$ dominates the Weitzman effect given by $-0.5\pi^2 Var(\beta)$. We have shown earlier that this condition is likely to hold in most real world applications.

5.2. The growth rate of the expected monetary value of ecosystem services

How can it be possible to have a decreasing ecological discount rate ρ_t – which magnifies the role of long-term benefits -- and an increasing risk-adjusted monetary discount rate r_t – which does the opposite -- at the same time? The answer to this question can be found in the fact that r_t discounts a flow of expected monetary values F_t that are increasing with maturity. To see this, it is useful to define the growth rate f_t of the expected monetary benefit as

$$f_t = t^{-1} \ln \frac{EF_t}{F_0}, \tag{31}$$

so that equation (20) can be rewritten as follows :

$$P = F_0 e^{-(r_t - f_t)t} \Delta_t. \tag{32}$$

By analogy with equation (4), we obtain that

$$\rho_t = r_t - f_t. \tag{33}$$

The ecological discount rate ρ_t is equal to the difference between the risk-adjusted monetary discount rate r_t and the growth rate of the expected monetary benefit f_t. Thus, a decreasing ρ_t is compatible with an increasing r_t if f_t is sufficiently increasing.

From equation (21), the growth rate of F_t is given by $(g_{ct} - g_{xt})\beta_t$, whose conditional expectation is $(\mu_c - \mu_x)\beta_t$ and conditional variance is $b\beta_t^2$. Thus, conditional to β_t, the growth rate of expected monetary value is equal to $(\mu_c - \mu_x)\beta_t + 0.5b\beta_t^2$. As in Pastor and Veronesi (2003),[5] the growth rate of monetary benefits is uncertain. Using the same methodology as in the proof of Proposition 1, we obtain that the growth rate of the expected monetary benefit is equal to

$$f_t = t^{-1}\chi\left(t, (\mu_c - \mu_x)\beta_t + 0.5b\beta_t^2\right). \tag{34}$$

Moreover, we obtain the following results. First, the growth rate f_t of expected monetary benefits is increased by any mean preserving spread of β_t. A similar result is obtained by Pastor and Veronesi (2003). Second, the term structure of f_t is increasing when the uncertainty affecting β_t

[5] Our approach is different from Pastor and Veronesi (2003), who assume risk neutrality or, alternatively, no correlation between the growth rates of dividends and of aggregate consumption. In our model, these two variables are statistically linked, as shown by equation (21) for example.

is increasing with maturity. For example, if the distribution of β_t is independent of t in the neighborhood of $t = 0_+$, then equation (34) implies that

$$\lim_{t\to 0_+} \frac{\partial f_t}{\partial t} = -0.5Var\left(\left(\mu_c - \mu_x\right)\beta_{0_+} + 0.5b\beta_{0_+}^2\right). \tag{35}$$

If the support of the distribution of β_t is bounded when t tends to infinity, with $\lim_{t\to\infty} \text{supp}\, \beta_t = \left[\beta_{\min}, \beta_{\max}\right]$, then the growth rate of the expected monetary value asymptotically tend to

$$\lim_{t\to\infty} f_t = \begin{cases} \left(\mu_c - \mu_x\right)\beta_{\min} + 0.5b\beta_{\min}^2 & if \quad \mu_c - \mu_x + b\beta^* \le 0 \\ \left(\mu_c - \mu_x\right)\beta_{\max} + 0.5b\beta_{\max}^2 & if \quad \mu_c - \mu_x + b\beta^* > 0. \end{cases} \tag{36}$$

To sum up this financial approach to the value of natural capital, we have shown in this section that the uncertainty affecting the substitutability of ecosystem services is transmitted to the growth rate of the monetary value of these services. As shown by Pastor and Veronesi (2003), this tends to raise the growth rate of expected cash flow, and this effect is magnified by the long-term nature of these services. But the uncertainty surrounding the substitutability of ecosystem services also raises the risk-adjusted monetary rate at which this cash flow should be discounted. This effect is also magnified by the time horizon. However, we know from Propositions 2 and 3 that the net effect of the uncertainty affecting β is to reduce the ecological discount rate increasingly with the maturity, thereby raising the economic value of natural capital.

6. The Gaussian case

We now examine the special case in which the beliefs about β_t can be represented by a normal distribution. Our results in this section are derived from Lemma 2, whose proof is relegated to the Appendix.

Lemma 2: *Suppose that random variable* z *is normally distributed with mean* μ_z *and standard deviation* σ_z. *Consider any pair* $(a,b) \in \mathbb{R}^2$ *such that* $b < \sigma_z^{-2}$. *Then, we have that*

$$E\exp(az+0.5bz^2)=\left(1-b\sigma_z^2\right)^{-1/2}\exp\left(\frac{a\mu_z+0.5a^2\sigma_z^2+0.5b\mu_z^2}{1-b\sigma_z^2}\right). \tag{37}$$

The following proposition is a direct consequence of Lemma 2 applied to equations (12), (24) and (34).

Proposition 4 : *Suppose that random variable β_t is normally distributed with mean μ_{β_t} and standard deviation σ_{β_t}. Then, as long as $b\sigma_{\beta_t}^2 t<1$,*

$$\rho_t=\rho_f+\frac{1}{2t}\ln\left(1-b\sigma_{\beta_t}^2 t\right)-\frac{a\mu_{\beta_t}+0.5b\mu_{\beta_t}^2+0.5a^2\sigma_{\beta_t}^2 t}{1-b\sigma_{\beta_t}^2 t} \tag{38}$$

$$r_t=\rho_f+\frac{\mu_{\beta_t}+\left(\left(\mu_c-\mu_x\right)-0.5\pi\right)\sigma_{\beta_t}^2 t}{1-b\sigma_{\beta_t}^2 t}\pi \tag{39}$$

$$f_t=-\frac{1}{2t}\ln\left(1-b\sigma_{\beta_t}^2 t\right)+\frac{\left(\mu_c-\mu_x\right)\mu_{\beta_t}+0.5b\mu_{\beta_t}^2+0.5\left(\mu_c-\mu_x\right)^2\sigma_{\beta_t}^2 t}{1-b\sigma_{\beta_t}^2 t}. \tag{40}$$

Notice that these rates exist only for maturities t such that $b\sigma_{\beta_t}^2 t$ is smaller than unity. The ecological discount rate unbounded below for all other maturities. This means that any natural capital that delivers a positive service in time horizons t such that $b\sigma_{\beta_t}^2 t>1$ has an unbounded value. In Corollary 1 we illustrate this result in the special case in which β_t evolves stochastically from the current β_0 by following an arithmetic Brownian motion. This is a direct application of Proposition 4 with $\mu_{\beta_t}=\beta_0+\mu_\beta t$ and $\sigma_{\beta_t}^2=t\sigma_\beta^2$.

Corollary 1: *Suppose that β_t follows an arithmetic Brownian motion with drift μ_β and volatility σ_β. This implies that the term structure of ecological discount rates exists for all maturities $t<T=(b\sigma_\beta^2)^{-1/2}$, with*

$$\rho_t=\rho_f+\frac{1}{2t}\ln\left(1-b\sigma_\beta^2 t^2\right)-\frac{a\left(\beta_0+\mu_\beta t\right)+0.5b\left(\beta_0+\mu_\beta t\right)^2+0.5a^2\sigma_\beta^2 t^2}{1-b\sigma_\beta^2 t^2} \tag{41}$$

$$r_t = \rho_f + \frac{\left(\beta_0 + \mu_\beta t\right) + \left(\left(\mu_c - \mu_x\right) - 0.5\pi\right)\sigma_\beta^2 t^2}{1 - b\sigma_\beta^2 t^2}\pi \tag{42}$$

$$f_t = -\frac{1}{2t}\ln\left(1 - b\sigma_\beta^2 t^2\right) + \frac{\left(\mu_c - \mu_x\right)\left(\beta_0 + \mu_\beta t\right) + 0.5b\left(\beta_0 + \mu_\beta t\right)^2 + 0.5\left(\mu_c - \mu_x\right)^2 \sigma_\beta^2 t^2}{1 - b\sigma_\beta^2 t^2}. \tag{43}$$

When the inverse of the elasticity of substitution follows a Brownian motion, the term structure of ecological discount rates is decreasing and tends to $-\infty$ when the maturity tends to $T = (b\sigma_\beta^2)^{-1/2}$. The risk-adjusted monetary discount rates and the growth rates of the expected monetary value also divergence at that maturity. In order to estimate the order of magnitude of this bliss maturity *T*, let us consider the case of a natural capital that delivers a sure flow of services, so that $\sigma_x = 0$ and $b = \sigma_c^2$. This implies that $T = 1/\sigma_c\sigma_\beta$. The volatility of the growth rate of aggregate consumption over the last century in the western world has been between 2% and 4% per year. If we assume that the volatility of the growth rate of β is also between 2% and 4%, we find that the bliss maturity *T* is somewhere between 625 and 2500 years. If the natural capital delivers a positive service above this bliss maturity *T*, it has an unbounded economic value.

7. Conclusion

The uncertainty affecting the substitutability of ecosystem services in the future is an important source of complexity to estimate the economic value of natural capital. We have shown in this paper that taking account of this uncertainty may indeed have a crucial importance, in particular if this natural capital is expected to deliver services in the distant future. This uncertainty makes the investment in natural capital risky, because the flow of monetary benefits generated by it becomes uncertain even when its ecological benefit is certain. Under the standard asset pricing methodology, the value of natural capital would be obtained by first measuring the flow of monetary benefits of the ecosystem services. One would then compute the present value of the flow of expected monetary benefits by using a discount rate that is adjusted for the riskiness of these benefits. In the special case in which the flow of ecosystem services is independent of economic growth, the CCAPM beta of natural capital is the inverse of the elasticity of substitution. This implies that the risk-adjusted monetary discount rate is also uncertain. We have shown that this uncertainty tends to raise this discount rate, and to make it increasing with maturity, contrary to what is suggested by Weitzman (1998, 2001).

This result is misleading because it does not provide a complete picture of the effect of the uncertain substitutability. This uncertainty also affects the growth rate of monetary benefits. As shown by Pastor and Veronesi (2003) in a different context, the uncertainty surrounding the growth rate of benefits raises the growth rate of expected benefits. The main result of this paper is that this positive effect always dominates the negative discounting effect, so that the economic value of natural capital is always increased by the risk affecting the inverse of the elasticity of substitution.

Finally, we have shown that the impact of this uncertainty on the economic value of ecosystem services is always increasing with maturity. In the Gaussian case, any marginal increment of ecosystem service has an infinite economic value if it is delivered in a time horizon that is larger than some "bliss maturity" that is estimated to be between 625 and 2500 years. This is an extreme version of the more general message of this paper, which is that the long term impacts of natural capital are important for the determination of its economic value.

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Appendix : Proof of Lemma 2:

We have that

$$E\exp(az+0.5bz^2)=\frac{1}{\sigma_z\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left(az+0.5bz^2-\frac{(z-\mu_z)^2}{2\sigma_z^2}\right)dz. \tag{44}$$

After rearranging terms in the integrant, this is equivalent to

$$E\exp(az+0.5bz^2)=\frac{\exp\left(-\frac{\mu_z^2}{2\sigma_z^2}-y\right)}{\sigma_z/\hat{\sigma}}\left[\frac{1}{\hat{\sigma}\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left(-\frac{\left(z-\hat{\mu}\right)^2}{2\hat{\sigma}^2}\right)dz\right], \tag{45}$$

with

$$y=\frac{\left(a+(\mu_z/\sigma_z^2)\right)^2}{2b-(2/\sigma_z^2)},$$

$$\hat{\mu}=-\frac{a+(\mu_z/\sigma_z^2)}{b-(1/\sigma_z^2)},$$

and

$$-\frac{1}{\hat{\sigma}^2}=b-\frac{1}{\sigma_z^2}.$$

Notice that $\hat{\sigma}$ exists only if we assume that $b<1/\sigma_z^2$. Notice also that the bracketed term in equation (45) is the integral of the density function of the normal distribution with mean $\hat{\mu}$ and variance $\hat{\sigma}^2$. This must be equal to unity. This equation can thus be rewritten as

$$\begin{aligned}E\exp(az+0.5bz^2)&=\frac{\hat{\sigma}}{\sigma_z}\exp\left(-\frac{\mu_z^2}{2\sigma_z^2}-\frac{\left(a+(\mu_z/\sigma_z^2)\right)^2}{2b-(2/\sigma_z^2)}\right)\\&=\left(1-b\sigma_z^2\right)^{-1/2}\exp\left(\frac{a\mu_z+0.5a^2\sigma_z^2+0.5b\mu_z^2}{1-b\sigma_z^2}\right).\end{aligned} \tag{46}$$

This concludes the proof of Lemma 2. □