# Privacy and Quality*

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**Abstract**

This paper analyzes the effects of a privacy regulation that caps the level of data disclosure on investment in quality and social welfare. We consider a monopolist who offers a service for free and derives revenues from disclosing users' personal information to third parties. If the market is fully covered, a disclosure cap always leads to lower quality. Consequently, its impact on social welfare depends on the degree of complementarity between quality and information. If the market is partially covered, a cap may induce higher quality and may be socially desirable even when quality and information are strongly complementary.

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# 1 Introduction

Data has often been called the oil of the digital era—a highly valuable resource and a force of change and growth. Unlike oil, however, data can be "extracted, refined, valued, bought and sold in different ways" (The Economist, May 6, 2017). Many of these operations, which are potentially privacy-intrusive, occur without the knowledge or consent of the consumers who supply this resource. This has led to calls for firms to be more transparent about their data practices, and to obtain consumer consent for the collection and use of their data (Federal Trade Commission, 2012; The White House, 2012).[1] Some have expressed doubts as to whether greater transparency will lead to more privacy (Adjerid et al., 2013; Solove, 2013). It is widely established that consumers rarely read and understand the implications of privacy policies (The White House, 2014). Further, even if they do, they may lack meaningful choice.[2] Firms typically make complex take-it-or-leave-it offers that consumers have to accept in order to access their services (Cate et al., 2013; The White House, 2014). Moreover, if a company provides services for which there are no (good) alternatives, it may be able to impose unfair terms of data collection and use on consumers.[3]

An alternative to the market-based approach of notice and consent is direct regulatory intervention. Regulators can impose restrictions on the types of data that firms are allowed to collect—a *collection* regulation—or set limitations on the use of that data —a *use* regulation. Though privacy-enhancing, these regulations inevitably undermine the ability of firms to monetize user data. Goldfarb and Tucker (2011) find, for instance, that a regulation restricting the collection of data for ad targeting lowers the effectiveness of online advertising and consequently advertising revenues. Given that the majority of free Internet services are primarily ad-financed, this reduction in revenues may dampen the incentives of these Internet firms to invest and innovate (Castor, 2010; Thierer, 2010; Athey, 2014). It is thus unclear *a priori* whether a regulation on data practices is socially desirable. This paper aims to shed light on this issue. Specifically, we examine whether

[1]Transparency, defined as the consumer's right to "easily understandable and accessible information about privacy and security practices", is one of the principles put forth in the proposed Consumer Privacy Bill of Rights in the U.S. (The White House, 2012)

[2]Another reason why increasing transparency may not be sufficient—even when consumers are fully informed of and understand the implications of these practices—is the presence of negative privacy externalities. Choi et al. (2018) show that the market equilibrium level of privacy would still be too low relative to the social optimum in this case.

[3]The German competition authority is currently investigating Facebook over a similar matter. A preliminary assessment of this case by the German competition authority is available at `http://www.bundeskartellamt.de/SharedDocs/Meldung/DE/Pressemitteilungen/2017/19_12_2017_Facebook.html`

a trade-off between privacy and quality indeed exists and consider the implications of a regulation restricting data disclosure on social welfare.

We develop a model in which a monopolist offers a service for free to consumers and derives revenues from disclosing personal data of its users to third parties (e.g., for the purpose of targeted advertising). We assume that the firm is able to commit to a disclosure level;[4] further, it may invest in the quality of its service. Given the firm's choice of quality and disclosure levels, each consumer decides whether or not to use the firm's service, and how much information to provide when using it. This information can take the form of purchase and browsing histories, location information, personal information shared on a social media profile, reviews and comments and so on.[5] The consumer incurs a privacy cost when providing information. This cost depends on the strength of her preference for privacy, her level of information provision and the monopolist's level of disclosure.

An important feature of our model is the endogenous choice of quality by the firm. The firm's quality level and the consumer's information provision level jointly determine the gross utility that she derives from using the service. The consumer's gross utility is increasing in both quality and information provision levels; moreover, quality and information are complements from her perspective. In other words, the marginal benefit of information provision, and hence the amount of information provided, is increasing in the quality level of the service. As an example, consider the services provided by social media platforms such as Facebook and Instagram. By innovating and creating new and better sharing tools (i.e., improving the quality of its service), a social media site enhances its users' sharing experience; users share more content as a result. The services provided by product recommendation or matching sites serve as another example. The higher the quality of a firm's recommendation/matching algorithm, the higher a consumer's marginal benefit from revealing her preferences/personal information to the firm.

We first compare the privately and socially optimal choices of quality and disclosure levels. Our analysis shows that the monopolist under-provides quality for a given level of disclosure, and over-discloses information for a given level of quality, relative to the social optimum. We then consider a privacy regulation taking the form of a cap on information disclosure.[6] This could correspond, for example, to a set of restrictions on the purposes

[4]For the sake of exposition, the disclosure level is defined in our model as the share of third parties the monopolist sells personal information to. However, it can also be interpreted as an inverse measure of other self-imposed restrictions on the sharing of data (e.g., regarding the type or share of data disclosed).

[5]One could also consider the level of usage as the amount of information input provided by a consumer. By increasing her level of usage, the consumer provides the firm with more opportunities to gather information about herself (e.g., the firm could track the consumer's activities by means of cookies).

[6]This corresponds to a *use* regulation. A *use* regulation is more relevant in our framework because we implicitly assume that the firm only collects the information that the consumer chooses to provide for the

for which data may be disclosed or the types of third parties data may be shared with. In the U.S., both the Gramm-Leach-Bliley Act and the Fair Credit Reporting Act forbid the sharing of consumer information with non-affiliated third parties. Japan's personal data protection law allows companies to sell consumer data to third parties (without requesting their consent) only if the data is anonymized. Many countries also impose restrictions on the international transfer of data,[7] allowing for such transfers only if the foreign country has an adequate level of data protection.

We examine the social desirability of such a disclosure cap. The disclosure cap affects social welfare via two channels: a direct reduction in the privacy cost associated with information provision, and a strategic change in the quality level chosen by the firm. The direct effect is always positive, while the strategic effect could be either positive or negative, depending on the impact of the cap on the benefit of investing in quality. We first examine the strategic effect of the cap, before determining its overall impact on social welfare.

We show that the effects of the cap on quality and social welfare depend on whether the market is fully or partially covered.[8] When the market is fully covered, the cap only results in a reduction in the firm's ability to monetize user data, which decreases the benefit of investing in quality. This leads to a lower quality level, which reduces social welfare, counteracting the positive direct effect of the cap—a trade-off between privacy and quality exists. A disclosure cap is socially desirable in this scenario if the strategic effect does not outweigh the direct effect, which is the case when quality and information are not strongly complementary. In the partial market coverage scenario, the cap also affects demand. It results in *higher* quality level when it increases substantially the sensitivity of demand to changes in service quality.[9] In this case, there is *no privacy-quality trade-off*, and social welfare is unambiguously higher under the cap. When the sensitivity of demand to changes in service quality does not considerably increase following a decrease in disclosure level, the impact of a disclosure cap on quality further depends on the elasticity of demand with respect to disclosure. The cap reduces quality level when demand is relatively disclosure-

use of its service.

[7]This includes in particular the sale of personal data to foreign third parties.

[8]Full market coverage is the scenario where all consumers in the market choose to use the firm's service. This arises when the opportunity cost of using the firm's service is sufficiently low. Under full market coverage, quality and disclosure levels only affect the amount of information provided by consumers (the intensive margin) but not their participation decision (the extensive margin). Under partial market coverage, both the intensive and extensive margins are affected. As discussed later, the full (resp. partial) market coverage assumption can be reinterpreted as meaning that the demand for the service is unresponsive (resp. responsive) to changes in quality and disclosure levels.

[9]To be more precise, this happens when the elasticity of the marginal effect of quality on demand with respect to disclosure is greater than 1, which is equivalent to the condition that the cdf of the privacy cost parameter is convex.

inelastic and has an ambiguous effect otherwise. The overall impact of the cap on social welfare is ambiguous in both cases.

Finally, we present two extensions to our model. First, we consider a setting in which the third-party data buyers are heterogeneous in the privacy costs that they induce. We find that a disclosure cap is more likely to have a positive impact on quality when buyers are heterogeneous rather than homogeneous. As a second extension, we explore the case where the regulator is a consumer protection agency that cares only about consumer surplus. While the results in this case do not qualitatively differ from those derived under a welfare-maximizing regulator, we show that the consumer protection agency will set a (weakly) lower disclosure cap.

Our analysis suggests that regulators should pay attention to the responsiveness of demand to changes in quality and privacy levels when deciding whether or not to impose a cap on data disclosure. In markets where demand (i.e., the consumers' participation decision) is essentially unresponsive to changes in quality and privacy levels,[10] a cap on disclosure is likely to reduce investment in quality. In this case, regulators will have to weigh the negative effect of the cap on quality against its positive direct effect on consumers' utility. Setting a cap is socially desirable if quality and information are not strong complements from consumers' perspective but may be socially undesirable otherwise. In markets where demand responds significantly to changes in quality and privacy, regulators may not always face a privacy-quality trade-off. In particular, if the disclosure cap raises substantially the sensitivity of demand to changes in quality, firms invest more in quality under the cap.

**Related literature.** Our work contributes to the growing pool of research examining privacy issues surrounding firms' data practices, particularly those made possible by technological progress.[11] In particular, our work forms part of an emerging strand of literature focused on online services for which the collection and disclosure of data directly create value to both the consumers—by enhancing the service—and the firms—by generating disclosure revenues (Casadesus-Masanell and Hervas-Drane, 2015; Bloch and Demange, 2018; Bourreau et al., 2018). Of these works, our paper most closely relates to Casadesus-Masanell and Hervas-Drane (2015), which examines the impact of competition on data disclosure.[12] Both works allow for the level of data collection, which is determined by users, to differ from the level of data disclosure, which is chosen by the firm. By contrast,

[10]In this scenario, consumers may however respond to changes in quality and privacy levels by changing the amount of information they provide to the firm.

[11]See Section 2.3 of Acquisti et al. (2016) for a survey.

[12]The impact of competition on disclosure levels have also been studied by Dimakopoulos and Sudaric (2017).

Bloch and Demange (2018) and Bourreau et al. (2018)—which mainly focus on the impact of taxation on data collection—only examine the case of full disclosure/exploitation. That said, we consider a different revenue model from Casadesus-Masanell and Hervas-Drane (2015); the firm derives revenues solely from disclosure in our model, whereas it can also charge a positive price to consumers in theirs.[13] Further, while our focus is on the impact of (privacy) regulation, theirs is on that of competition. More importantly, our work is distinct from the existing literature in that it endogenizes the firm's investment in quality. To the best of our knowledge, we are the first to examine theoretically the relationship between a firm's data practices—more precisely, its disclosure level—and its incentives to invest in quality. This ultimately enables us to discuss how a privacy regulation (taking the form of a disclosure cap) affects quality investment and social welfare.

The implications drawn from our analysis also complement the findings in the empirical literature that examines the interlinkage between privacy regulations and data-driven innovation. Goldfarb and Tucker (2012) analyze several empirical studies—in the healthcare (see Tucker and Miller 2009, 2011a and 2011b) and the online advertising (see Goldfarb and Tucker, 2011) sectors—and found that privacy regulations may raise the costs and/or lower the benefits associated with data-driven innovation, hence weakening the investment incentives.[14] Our work shows, in addition, that a privacy regulation can affect the level of service innovation (quality) even when data is not a direct input for innovation.

The rest of the paper is structured as follows. In Section 2, we introduce the model setup. In Section 3, we examine the consumers' participation and information provision decisions. We then compare, in Section 4, the privately and socially optimal choices of quality and disclosure levels, before presenting our analysis of a privacy regulation taking the form of a cap on data disclosure in Section 5. In Section 6, we present two extensions to our model: we first consider heterogeneous third parties, and then analyze the scenario in which the regulator maximizes consumer surplus instead of social welfare. We provide some general discussions in Section 7 and conclude in Section 8.

# 2 Baseline Model

Consider a firm that offers a service to a unit mass of consumers for free, and derives revenues from selling its customers' personal information to (a subset of) a unit-mass of

[13] In Section 7.4 we explain why allowing the firm to charge consumers would substantially complicate our analysis.

[14] See also Adjerid et al. (2016) who study empirically the impact of privacy regulation on health information exchanges.

third parties (e.g., advertisers) uniformly distributed over the interval $[0,1]$. The firm can choose the quality level $q \geq 0$ of its service, and a disclosure level $d \in [0,1]$, which defines the extent to which personal information is shared with third parties. More precisely, information is sold to the third parties located in $[0,d]$ and not sold to those in $(d,1]$.[15]

**Consumers' utility.** Consumers benefit from a better service when they provide (more) personal information to the firm, but incur a utility loss when that information is disclosed to third parties. The utility of a consumer who provides an amount of information $x \in [0,1]$ is

$$U(x,\theta,q,d) \equiv V(x,q) - P(\theta,d,x) - \alpha x - K,$$

where $V(x,q)$ is the gross utility the consumer derives from consuming the service, $\theta$ is an idiosyncratic privacy cost parameter distributed over an interval $\left[\underline{\theta},\bar{\theta}\right)$ according to a differentiable density function $f(.)$, $P(\theta,d,x)$ is the privacy cost incurred by the consumer, $\alpha$ is the marginal cost of providing personal information,[16] and $K \geq 0$ is a fixed opportunity cost of using the service.[17] The parameter $\theta$ captures the intensity of a consumer's preference for privacy; the higher the value of $\theta$, the stronger the consumer's preference for privacy, i.e., $P(\theta,d,x)$ is increasing in $\theta$. The gross utility $V(x,q)$ is assumed to be bounded, twice continuously differentiable, increasing and concave in both its arguments.

For tractability reasons, we consider a specific functional form for the privacy cost function $P(\theta,d,x)$. More precisely, we assume that sharing a unit of personal information with a third party of type $u \in [0,1]$ induces a privacy cost $\theta g(u)$ for the consumer, where $g(.)$ is a weakly increasing function over $[0,1]$.[18] The (total) privacy cost incurred by a consumer of type $\theta$ who provides an amount of information $x \in [0,1]$ at a given level of disclosure $d \in [0,1]$ is therefore

$$P(\theta,d,x) = \theta x \int_0^d g(u)du.$$

In the baseline model, we focus on the case where third parties are homogeneous from consumers' perspective: the privacy cost induced by each third party is the same, i.e., $g(u)$ is constant over $[0,1]$.[19] Normalizing $g(u)$ to 1, the total privacy cost becomes $P(\theta,d,x) =$

[15] As discussed later, our model allows for other interpretations of the disclosure level $d$.

[16] This could be for instance the cost of uploading content on a social network or the cost of filling out an electronic form on a website. An alternative interpretation of this parameter is provided in Section 7.1.

[17] For notational convenience, we do not include $K$ in the arguments of the utility function.

[18] This assumption ensures that it is always (weakly) optimal for the firm to sell personal information to a subset of third parties of the form $[0,d]$ where $d \in [0,1]$.

[19] We consider the case of heterogeneous third parties in Section 6.1.

$\theta dx$, which yields the following utility function:

$$U(x, \theta, q, d) = V(x, q) - (\alpha + \theta d)\, x - K. \tag{1}$$

**Value of personal information.** We suppose that all the third parties that are interested in buying customer data have the same willingness to pay $r > 0$ for a unit of personal information; we call $r$ the *value of information*. In addition, we assume that the firm is a monopolist in the market for personal information. These simplifying assumptions have two straightforward implications. First, the firm always sets the unit price for personal information to $r$, independently of its other strategic choices. Second, the price set by the monopolist leaves no surplus to the third parties buyers, which simplifies our welfare analysis.[20]

**Firm's profit and social welfare.** Denote by $\mathbf{x}$ the function mapping each $\theta \in \left[\underline{\theta}, \bar{\theta}\right)$ to the amount of information $x(\theta) \in [0, 1]$ provided by a consumer of type $\theta$.[21] The firm's profit is

$$\Pi(\mathbf{x}, q, d) = rd \int_{\underline{\theta}}^{\bar{\theta}} x(\theta)\, f(\theta)\, d\theta - C(q), \tag{2}$$

where $C(q)$ is the (fixed) cost of producing a service of quality level $q$. Assume that $C(.)$ is twice differentiable, with $C(0) = 0$, $C(q) \underset{q \to +\infty}{\longrightarrow} +\infty$, $C'(0) = 0$, $C'(q) > 0$ for any $q > 0$, and $C''(q) > 0$ for any $q \geq 0$. Social welfare is defined as the sum of the firm's profit and the consumers' utility.

Before proceeding further, it is worth highlighting that the expressions of the firm's profit and the consumer's utility functions (expressions (1) and (2)) allow for an alternative interpretation of our baseline model: we could assume that the firm sells a share $d \in [0, 1]$ of the personal information provided by each consumer to all third parties, instead of assuming that its sells all the personal information provided by consumers to a subset $[0, d]$ of third parties.[22]

**Interdependency between quality and information.** We capture the (local) interdependency between quality and information from consumers' perspective through the

[20] In Section 7.2 we discuss the scenario in which third parties buying personal data pay a price lower than $r$, thus making a positive surplus.

[21] The amount of information $x(\theta)$ can be equal to zero either because the consumer of type $\theta$ decides not to use the service, or because she uses the service but decides to provide no personal information at all.

[22] More generally, a lower disclosure level can be interpreted either as more (self-imposed) restrictions on the type/share of data that can be sold to third parties and/or more restrictions on the set of third parties that the data can be sold to.

following parameter:

$$\gamma(x,q) \equiv -\frac{\frac{\partial^2 V}{\partial x \partial q}}{\frac{\partial^2 V}{\partial x^2}}.$$

The most interesting scenario for the purpose of our analysis is the one in which quality and information are complements from consumers' perspective, i.e., $\gamma(x,q) > 0$.[23] To simplify the analysis, we also suppose that $\gamma(x,q)$ is constant: $\gamma(x,q) = \gamma > 0$ for any $(x,q) \in [0,1] \times [0,+\infty)$.[24]

**Timing.** We consider the following two-stage game:

1. The firm chooses a quality level $q$ and commits to a disclosure level $d$.

2. Consumers observe the levels of quality and disclosure. They decide then whether to patronize the firm and, if they do, how much personal information to provide.

# 3 Consumers' choice

We begin our analysis with the consumers' problem. Having observed the firm's quality and disclosure levels, a consumer has to decide whether or not to patronize the firm and how much information to provide if she patronizes it.

Let us first examine a consumer's optimal level of information provision. Conditional on patronizing the firm, a consumer chooses her level of information provision so as to maximize her utility $U(x,\theta,q,d)$.[25] Denote[26]

$$\tilde{x}(\theta,q,d) \equiv \arg\max_{x\in[0,1]} U(x,\theta,q,d).$$

[23]This is likely to be the relevant case when we consider social media platforms. For instance, Facebook has recently developed and introduced new sharing tools (including Facebook Live video and the "On this day" feature) in an attempt to boost the sharing of original personal content (for full story, visit `https://www.bloomberg.com/news/articles/2016-04-07/facebook-said-to-face-decline-in-people-posting-personal-content`). This provides support for our assumption. We discuss the case where quality and information are substitutes in Section 7.3.

[24]This amounts to restricting our attention to the class of gross utility functions $V(x,q)$ for which there exists a real number $\gamma$, a twice continuously differentiable function $l(.)$, and a continuously differentiable function $h(.)$ such that

$$V(x,q) = l(q) + \int_0^x h(u - \gamma q)du.$$

[25]In essence, the consumer performs a "privacy calculus" (see Culnan and Armstrong, 1999; Dinev and Hart, 2006), weighing the benefits of providing more information (i.e., gross utility from using the service) and the privacy costs of sharing information.

[26]The existence and uniqueness of $\tilde{x}(\theta,q,d)$ follows from the fact that $U(x,\theta,q,d)$ is concave in $x$ over the compact set $[0,1]$.

To ease the exposition, we assume throughout the paper that

$$\alpha > \sup_{q \geq 0} \frac{\partial V}{\partial x} (1, q)$$

and

$$\bar{\theta} < \inf_{q \geq 0} \frac{\partial V}{\partial x} (0, q) .$$

These conditions ensure that, conditional on using the service, the amount of information that a consumer provides to the firm is always interior, i.e., $\tilde{x}(\theta, q, d) \in (0, 1)$. The following lemma shows the effect of quality and disclosure levels on the amount of personal information provided by consumers.

**Lemma 1** *(Comparative statics - Information amount) Conditional on using the service, the amount of information that a consumer provides to the firm is decreasing in the disclosure level and the idiosyncratic privacy cost parameter, and is increasing in the quality level. More precisely,*

$$\frac{\partial \tilde{x}}{\partial d} (\theta, q, d) = \frac{\theta}{\frac{\partial^2 V}{\partial x^2} (\tilde{x}(\theta, q, d), q)} < 0,$$

$$\frac{\partial \tilde{x}}{\partial \theta} (\theta, q, d) = \frac{d}{\frac{\partial^2 V}{\partial x^2} (\tilde{x}(\theta, q, d), q)} < 0,$$

*and*

$$\frac{\partial \tilde{x}}{\partial q} (\theta, q, d) = \gamma > 0.$$

**Proof.** See Appendix A. ■

The above results are intuitive. An increase in the disclosure level or the value of her idiosyncratic privacy cost parameter raises the consumer's marginal privacy cost of information provision; this leads her to provide less information. By contrast, an increase in the quality level raises her marginal gross utility from providing information; hence, she finds it optimal to provide more information.

We now consider the participation decision of a consumer. Denoting

$$\tilde{U}(\theta, q, d) \equiv U(\tilde{x}(\theta, q, d); \theta, q, d),$$

a consumer of type $\theta$ chooses to patronize the firm if and only if[27]

$$\tilde{U}(\theta, q, d) > 0.$$

[27] For technical reasons, we assume that a consumer who is indifferent between patronizing the firm or not decides not to patronize it.

The following lemma characterizes the demand for the service offered by the firm and shows how it is affected by the levels of quality and disclosure.

**Lemma 2** *(Comparative statics - Demand). There exists a threshold* $\tilde{\theta}(q,d) \in \left[\underline{\theta}, \bar{\theta}\right]$ *such that a consumer patronizes the firm if and only if*

$$\theta < \tilde{\theta}(q,d).$$

*The threshold* $\tilde{\theta}(q,d)$ *is weakly increasing in the quality level, and weakly decreasing in the disclosure level. Moreover, the following expressions hold whenever* $\tilde{\theta}(q,d) \in \left(\underline{\theta}, \bar{\theta}\right)$ *:*

$$\frac{\partial \tilde{\theta}}{\partial q}(q,d) = \frac{\frac{\partial V}{\partial q}(\tilde{x}(\tilde{\theta}(q,d), q, d), q}{d\tilde{x}(\tilde{\theta}(q,d), q, d)} > 0$$

*and*

$$\frac{\partial \tilde{\theta}}{\partial d}(q,d) = -\frac{\tilde{\theta}(q,d)}{d} < 0.$$

**Proof.** See Appendix A. ■

The above lemma tells us that the demand for the service, which is given by $F(\tilde{\theta}(q,d))$, is weakly increasing in the firm's quality level but weakly decreasing in its disclosure level.[28]

# 4 Private versus social incentives

We now examine the privately and socially optimal choice of quality and disclosure levels. Let us first consider the private incentives to invest in quality and to disclose personal information. The firm's profit when consumers make their participation and information provision decision optimally is

$$\tilde{\Pi}(q,d) = rd \int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \tilde{x}(\theta, q, d) f(\theta) d\theta - C(q). \tag{3}$$

[28] The above analysis can be extended to the case where the use of data by the firm in the provision of its service (e.g., for service personalization) also imposes a privacy cost on the consumer. More precisely, the consumer's gross utility $V$ will further depend on her type $\theta$ in this case. The presence of this additional privacy cost does not fundamentally alter the analysis.

From (3), it follows that the firm's net marginal benefit from investing in quality is

$$\frac{\partial\tilde{\Pi}}{\partial q} = \underbrace{rd \int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \frac{\partial\tilde{x}}{\partial q} (\theta, q, d) f(\theta) d\theta}_{\text{intensive margin effect}} + \underbrace{rd \frac{\partial\tilde{\theta}}{\partial q} \tilde{x} \left(\tilde{\theta}(q,d), q, d\right) f\left(\tilde{\theta}(q,d)\right)}_{\text{extensive margin effect}} - C'(q). \tag{4}$$

Lemma 1 and Lemma 2 tell us respectively that the sign of the intensive margin effect is positive and that of the extensive margin effect is weakly positive. The intensive margin effect here captures how a change in quality level affects the firm's revenue via its impact on the level of information provision. Due to the complementarity between quality and information, a higher quality level induces consumers to provide more information; this increases the firm's disclosure revenues. The extensive margin effect captures how the change in demand resulting from a change in quality impacts the firm's profit. This effect is weakly positive because the firm's demand is weakly increasing in the quality level.

The net marginal benefit from increasing the disclosure level is

$$\begin{aligned}\frac{\partial\tilde{\Pi}}{\partial d} =& \underbrace{r \int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \tilde{x}(\theta, q, d) f(\theta) d\theta + rd \int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \frac{\partial\tilde{x}}{\partial d} (\theta, q, d) f(\theta) d\theta}_{\text{intensive margin effect}} \\ &+ \underbrace{rd \frac{\partial\tilde{\theta}}{\partial d} \tilde{x} \left(\tilde{\theta}(q,d), q, d\right) f\left(\tilde{\theta}(q,d)\right)}_{\text{extensive margin effect}}.\end{aligned}$$

Lemma 1 implies that the sign of the intensive margin effect is ambiguous, while Lemma 2 shows that the extensive margin effect is weakly negative. The sign of the intensive margin effect is ambiguous because a change in the disclosure level generates two opposing effects: it raises the firm's disclosure revenues per unit of information provided but lowers the level of information provision.

Let us assume that $\tilde{\Pi}(.,.)$ is quasi-concave in each of its arguments. The privately optimal level of quality for a given level of disclosure and the privately optimal level of disclosure for a given level of quality are defined as follows:[29]

$$q^M(d) \equiv \arg\max_{q\in[0,+\infty)} \tilde{\Pi}(q,d);$$

$$d^M(q) \equiv \arg\max_{d\in[0,1]} \tilde{\Pi}(q,d).$$

We futher assume that the privately optimal pair of quality and disclosure levels

$$\left(\tilde{q}^M, \tilde{d}^M\right) \equiv \arg\max_{(q,d)\in[0,+\infty)\times[0,1]} \tilde{\Pi}(q,d)$$

is unique.[30]

Let us now consider the social incentives for quality provision and information disclosure. The social planner's objective function is given by

$$\tilde{W}(q,d) \equiv \tilde{\Pi}(q,d) + \widetilde{CS}(q,d)$$

where

$$\widetilde{CS}(q,d) = \int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \tilde{U}(\theta,q,d)\, f(\theta)\, d\theta.$$

Hence, the net marginal social benefit from investing in quality is

$$\begin{aligned}\frac{\partial \tilde{W}}{\partial q} &= \frac{\partial \tilde{\Pi}}{\partial q} + \int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \frac{\partial \tilde{U}}{\partial q}(\theta,q,d)\, f(\theta)\, d\theta + \frac{\partial \tilde{\theta}}{\partial q}\tilde{U}\left(\tilde{\theta}(q,d),q,d\right) f\left(\tilde{\theta}(q,d)\right) \qquad (5)\\ &= \frac{\partial \tilde{\Pi}}{\partial q} + \underbrace{\int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \frac{\partial V}{\partial q}\left(\tilde{x}(\theta,q,d),q\right) f(\theta)\, d\theta}_{>0} + \underbrace{\frac{\partial \tilde{\theta}}{\partial q}\tilde{U}\left(\tilde{\theta}(q,d),q,d\right) f\left(\tilde{\theta}(q,d)\right)}_{=0 \text{ by definition of } \tilde{\theta}(q,d)}.\end{aligned}$$

The last term is equal to zero both when $\tilde{\theta}(q,d) < \bar{\theta}$ and when $\tilde{\theta}(q,d) = \bar{\theta}$.[31] The net

[29] The existence and uniqueness of $q^M(d)$ follows from the fact that $\tilde{\Pi}(q,d)$ is continuous and quasi-concave in $q$ and $\tilde{\Pi}(q,d) \underset{q\longrightarrow+\infty}{\longrightarrow} -\infty$, while the existence and uniqueness of $d^M(q)$ follows from the fact that $\tilde{\Pi}(q,d)$ is continuous and quasi-concave in $d$ over the compact set $[0,1]$.

[30] The existence of the optimal pair of quality and disclosure levels follows from tha fact that $\tilde{\Pi}(q,d)$ is continuous in $(q,d)$ and $\tilde{\Pi}(q,d)$ goes to $-\infty$ uniformly with respect to $d$ when $q \to -\infty$ (which allows to reduce the maximization over $[0,+\infty) \times [0,1]$ to a maximization over a compact set).

[31] When $\tilde{\theta}(q,d) < \bar{\theta}$, this follows from the fact that $\tilde{U}\left(\tilde{\theta}(q,d),q,d\right) = 0$, and when $\tilde{\theta}(q,d) = \bar{\theta}$ it follows

marginal social benefit from investing in quality is greater than the corresponding private benefit because the social planner also accounts for the increase in the infra-marginal consumers' utility resulting from higher quality.

Likewise, the net marginal social benefit from information disclosure is

$$\begin{aligned}\frac{\partial \tilde{W}}{\partial d} &= \frac{\partial \tilde{\Pi}}{\partial d} + \int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \frac{\partial \tilde{U}}{\partial d}(\theta, q, d) f(\theta) d\theta + \frac{\partial \tilde{\theta}}{\partial d} \tilde{U}\left(\tilde{\theta}(q,d), q, d\right) f\left(\tilde{\theta}(q,d)\right) \\ &= \frac{\partial \tilde{\Pi}}{\partial d} \underbrace{- \int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \theta \tilde{x}(\theta, q, d) f(\theta) d\theta}_{<0} + \underbrace{\frac{\partial \tilde{\theta}}{\partial d} \tilde{U}\left(\tilde{\theta}(q,d), q, d\right) f\left(\tilde{\theta}(q,d)\right)}_{=0}.\end{aligned}$$

It is lower than the corresponding marginal private benefit of disclosure as the social planner takes into consideration the negative (direct) effect of disclosure on the infra-marginal consumers' utility.

We further assume that $\tilde{W}(.,.)$ is also quasi-concave in each of its arguments. The socially optimal quality level for a given level of disclosure and the socially optimal disclosure level for a given level of quality are defined as follows:[32]

$$q^W(d) \equiv \arg\max_{q \in [0,+\infty)} \tilde{W}(q,d);$$

$$d^W(q) \equiv \arg\max_{d \in [0,1]} \tilde{W}(q,d).$$

In the following lemmas, we compare the socially and privately optimal levels of quality for a given level of disclosure and those of disclosure for a given level of quality. In both cases, the results stem from the fact that the monopolist does not internalize the effects of its choices on infra-marginal consumers.

**Lemma 3** *(Under-provision of quality) For a given disclosure level, the monopolist under-provides quality from a social welfare perspective:* $q^M(d) \leq q^W(d)$ *for any* $d \in [0,1]$.

**Proof.** See Appendix A. ■

from the fact that $\frac{\partial \tilde{\theta}}{\partial q} = 0$.

[32] The existence and uniqueness of $q^W(d)$ follows from the fact that $\tilde{W}(q,d)$ is continuous and quasi-concave in $q$ and $\tilde{W}(q,d) \underset{q \longrightarrow +\infty}{\longrightarrow} -\infty$, while the existence and uniqueness of $d^W(q)$ follows from the fact that $\tilde{W}(q,d)$ is continuous and quasi-concave in $d$ over the compact set $[0,1]$.

**Lemma 4** *(Over-disclosure of information) For a given quality level, the monopolist over-discloses information from a social welfare perspective:* $d^M(q) \geq d^W(q)$ *for any* $q \in [0, +\infty)$.

**Proof.** See Appendix A. ■

Under the regularity conditions that we have imposed, Lemmas 3 and 4 can help us determine the direction of the impact of a change in quality (resp. disclosure) level holding disclosure (resp. quality) level fixed on the firm's profit or on social welfare. These results will come in handy for the analysis of the privacy regulation examined in the next section.[33]

# 5 Privacy regulation

We now turn to the core question of our paper: is it desirable to regulate the information disclosure level? The first step to answering this question, as will become apparent, is to understand the impact of such a regulation on the firm's quality choice.

We consider a scenario where the only available regulatory instrument is a cap on the disclosure level and study the decision of a social-welfare-maximizing regulator to implement such a cap. We further assume that the cap is set before the firm decides on its investment in quality.[34]

The timing of the game is as follows:

- First, the regulator decides whether to impose a cap on the disclosure level, and sets the value of that cap $\bar{d}$ if it does so.

- Second, the firm decides on its disclosure and quality levels.

- Third, consumers decide whether to patronize the firm and how much information to provide if they do.

Let us first analyze the firm's behavior in the second stage when the regulator imposes a cap $\bar{d}$ in the first stage. The firm's optimal disclosure level maximizes $\tilde{\Pi}\left(q^M(d), d\right)$ subject to the constraint $d \leq \bar{d}$. If $\bar{d} \geq \tilde{d}^M$, the constraint is not binding. In this case, the firm's decision will be the same as in the unregulated scenario; i.e., the firm will set its disclosure level at $d = \tilde{d}^M$. If $\bar{d} < \tilde{d}^M$, however, the constraint binds. Under the additional assumption that $\tilde{\Pi}\left(q^M(d), d\right)$ is quasi-concave in $d$, which we make in the remainder of the paper, the firm will choose $d = \bar{d}$ whenever $\bar{d} < \tilde{d}^M$.

Let us now consider the regulator's decision in the first stage. Notice first that setting no cap or setting a cap $\bar{d} > \tilde{d}^M$ is the same as setting a cap $\bar{d} = \tilde{d}^M$; in all three scenarios,

[33] The comparison of the socially and privately optimal *pair* of quality and disclosure levels is not necessary for our policy analysis.

[34] In other words, we are analyzing an *ex ante* privacy regulation.

the firm's behavior is unaffected, and so is social welfare. We focus on the (interesting) scenario in which the firm invests in quality in the absence of a disclosure cap (i.e. $\tilde{q}^M > 0$) and the regulator does not find it optimal to impose a disclosure cap that induces no investment in quality at all (i.e. a cap such that $q^M(\bar{d}) = 0$). Then, denoting[35]

$$\underline{d} \equiv \inf \left\{ d \in \left[0, \tilde{d}^M\right) \mid q^M(d) > 0 \right\},$$

we can restrict the analysis to disclosure caps $\bar{d}$ between $\underline{d}$ and $\tilde{d}^M$. More precisely, the regulator's maximization program can be written as

$$\max_{\bar{d} \in \left[\underline{d}, \tilde{d}^M\right]} \hat{W}\left(\bar{d}\right)$$

where

$$\hat{W}\left(\bar{d}\right) \equiv \tilde{W}\left(q^M(\bar{d}), \bar{d}\right) = \tilde{\Pi}\left(q^M(\bar{d}), \bar{d}\right) + \int_{\underline{\theta}}^{\tilde{\theta}\left(q^M(\bar{d}), \bar{d}\right)} \tilde{U}\left(\theta, q^M(\bar{d}), \bar{d}\right) f(\theta)\, d\theta.$$

Further, assume that $\hat{W}(.)$ is quasi-concave over $\left[\underline{d}, \tilde{d}^M\right]$. Under this regularity assumption, the regulator finds it strictly optimal to set a binding cap, i.e. $\bar{d} < \tilde{d}^M$, if and only if $\left.\frac{\partial \hat{W}}{\partial \bar{d}}\right|_{\bar{d}=\tilde{d}^M} < 0$ or, equivalently,

$$-\left.\frac{\partial \hat{W}}{\partial \bar{d}}\right|_{\bar{d}=\tilde{d}^M} > 0.$$

In other words, setting a binding disclosure cap is strictly socially desirable if and only if a marginal decrease in disclosure level starting from the unregulated level leads to an increase in social welfare (taking into account the effect of a reduction in disclosure level on the firm's choice of quality).

[35] Our focus on the case in which $q^M\left(\tilde{d}^M\right) = \tilde{q}^M$ is positive, combined with the fact that $q^M(d)$ is continuous at $d = \tilde{d}^M$, ensures that the considered set is not empty and that its lower bound $\underline{d}$ is well-defined in $\left[0, \tilde{d}^M\right)$.

The marginal effect of reducing the disclosure cap on social welfare is

$$
\begin{aligned}
-\frac{\partial \hat{W}}{\partial \bar{d}} &= -\frac{\partial \tilde{\Pi}}{\partial d}\left(q^M(\bar{d}),\bar{d}\right) - \int_{\underline{\theta}}^{\tilde{\theta}\left(q^M(\bar{d}),\bar{d}\right)} \left[\frac{\partial \tilde{U}}{\partial q}\left(\theta, q^M(\bar{d}),\bar{d}\right)\frac{\partial q^M}{\partial d} + \frac{\partial \tilde{U}}{\partial d}\left(\theta, q^M(\bar{d}),\bar{d}\right)\right] f(\theta)\, d\theta \\
&\quad \underbrace{-\left[\frac{\partial \tilde{\theta}}{\partial q}\frac{\partial q^M}{\partial d} + \frac{\partial \tilde{\theta}}{\partial d}\right]\tilde{U}\left(\tilde{\theta}\left(q^M(\bar{d}),\bar{d}\right), q^M(\bar{d}),\bar{d}\right) f\left(\tilde{\theta}(q^M(\bar{d}),\bar{d})\right)}_{=0} \\
&= -\frac{\partial \tilde{\Pi}}{\partial d}\left(q^M(\bar{d}),\bar{d}\right) - \int_{\underline{\theta}}^{\tilde{\theta}\left(q^M(\bar{d}),\bar{d}\right)} \left[\frac{\partial V}{\partial q}\left(\tilde{x}\left(\theta, q^M(\bar{d}),\bar{d}\right), q^M(\bar{d})\right)\frac{\partial q^M}{\partial d} - \theta\tilde{x}\left(\theta, q^M(\bar{d}),\bar{d}\right)\right] f(\theta)\, d\theta.
\end{aligned}
$$

where the second equality follows from the application of the Envelope Theorem. Evaluating this at $\bar{d} = \tilde{d}^M$, and using the fact that

$$
\frac{\partial \tilde{\Pi}}{\partial d}\left(q^M(\tilde{d}^M),\tilde{d}^M\right) = \frac{\partial \tilde{\Pi}}{\partial d}\left(\tilde{q}^M,\tilde{d}^M\right) = 0,
$$

we obtain

$$
\begin{aligned}
-\left.\frac{\partial \hat{W}}{\partial \bar{d}}\right|_{\bar{d}=\tilde{d}^M} &= \underbrace{\int_{\underline{\theta}}^{\tilde{\theta}\left(\tilde{q}^M,\tilde{d}^M\right)} \theta\tilde{x}\left(\theta,\tilde{q}^M,\tilde{d}^M\right) f(\theta)\, d\theta}_{\text{direct effect}} \\
&\quad \underbrace{-\int_{\underline{\theta}}^{\tilde{\theta}\left(\tilde{q}^M,\tilde{d}^M\right)} \frac{\partial V}{\partial q}\left(\tilde{x}\left(\theta,\tilde{q}^M,\tilde{d}^M\right),\tilde{q}^M\right)\left.\frac{\partial q^M}{\partial d}\right|_{d=\tilde{d}^M} f(\theta)\, d\theta.}_{\text{strategic effect}}
\end{aligned}
$$

This shows that a (marginal) decrease in disclosure level starting from the unregulated level generates two effects: a *direct* effect on the privacy costs incurred by consumers (holding the quality level constant), and a *strategic* effect, which captures how a decrease in the disclosure level (due to the disclosure cap) alters the firm's quality choice. The direct effect is always positive because there is over-disclosure by the firm from a social perspective, while the sign of the strategic effect depends on how the firm's quality choice responds to the reduction in disclosure level. Since the firm under-provides quality from a social perspective, social welfare is increasing in quality level at the unregulated equilibrium. Therefore, the strategic effect is weakly positive if the firm weakly increases its quality level when disclosure level is decreased, i.e., if

$$\left.\frac{\partial q^M}{\partial d}\right|_{d=\tilde{d}^M} \leq 0.$$

When the strategic effect is positive, the overall effect of a (marginal) decrease in disclosure level starting from the unregulated level on social welfare is unambiguously positive, which implies that it is strictly socially desirable to set a binding disclosure cap. However, if

$$\left.\frac{\partial q^M}{\partial d}\right|_{d=\tilde{d}^M} > 0,$$

the strategic effect is negative and the overall effect of a disclosure cap on social welfare becomes *a priori* ambiguous.

Let us now analyze the effect of a disclosure cap on quality, which determines the sign of the strategic effect described above. The following lemma relates the effect of a change in disclosure level on the firm's optimal quality level to the cross-effect of quality and disclosure on the firm's profit.

**Lemma 5** *(Effect of the disclosure level on quality) If $q^M(d) > 0$, then*

$$\frac{\partial q^M}{\partial d} = -\frac{\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}\left(q^M(d), d\right)}{\frac{\partial^2 \tilde{\Pi}}{\partial q^2}\left(q^M(d), d\right)}.$$

**Proof.** See Appendix A. ■

From Lemma 5, we see that the effect of a change in disclosure level on the firm's choice of quality has the same sign as the cross-effect of quality and disclosure on the firm's profit.[36] The intuition behind this result is straightforward. If the marginal benefit of investing in quality is increasing in the level of disclosure, the firm will invest more at higher levels of disclosure.

For our analysis, we distinguish between the scenarios where the market is fully covered and where it is partially covered. A change in quality level only generates an intensive margin effect when the market is fully covered while it also creates an extensive margin effect (arising from a change in demand) when the market is partially covered. Consequently, the cross-effect of quality and disclosure on the firm's profit will depend on whether or not the market is fully covered.

[36] We use the fact that $\frac{\partial^2 \tilde{\Pi}}{\partial q^2}\left(q^M(d), d\right) < 0$, which is given by the second-order condition of the maximization of $\tilde{\Pi}(q, d)$ with respect to $q$.

## 5.1 Full market coverage

Suppose that $\tilde{\theta}\left(q^M(d), d\right) = \bar{\theta}$ for any $d \in [0,1]$.[37] Under this assumption, the market is fully covered whatever the regulator's decision in the first stage of the game.

Consider the firm's optimal choice of quality for a given level of disclosure. The firm's marginal benefit from investing in quality is given by (4), with the term capturing the effect of disclosure on the extensive margin effect equal to zero under full market coverage. Substituting $\frac{\partial \tilde{x}}{\partial q}$ by its expression in Lemma 1, we obtain

$$\frac{\partial \tilde{\Pi}}{\partial q} = rd\gamma - C'(q).$$

Since $C'(0) = 0$, the marginal benefit of investing in quality is positive when evaluated at $q = 0$ for any $d > 0$:

$$\left.\frac{\partial \tilde{\Pi}}{\partial q}\right|_{q=0} = rd\gamma.$$

This implies that the firm's optimal choice of quality level for a given level of disclosure, $q^M(d)$, is positive for all levels of disclosure $d > 0$ (and is zero for $d = 0$).[38] From Lemma 5, we know that the sign of the effect of a change in disclosure level on the firm's optimal quality level is the same as the sign of the cross-effect of quality and disclosure on the firm's profit. Under full market coverage,

$$\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}\left(q^M(d), d\right) = r\gamma > 0.$$

Therefore, the firm's optimal level of quality is increasing in the level of disclosure. More precisely,

$$\frac{\partial q^M}{\partial d} = \frac{r\gamma}{C''(q^M(d))} > 0. \tag{6}$$

This leads us to the following proposition.

**Proposition 1** *(Effect of a disclosure cap on quality under full market coverage) When the market is fully covered, a disclosure cap $\bar{d} \in \left(\underline{d}, \tilde{d}^M\right)$ has a negative impact on quality.*

The intuition behind the above proposition is as follows. The marginal benefit from investing in quality arises solely from the complementarity between quality and information when the market is fully covered. Consumers provide more information when the firm's

[37] A sufficient—but not necessary—condition for this to hold is that $K < V(0,0)$.

[38] This implies that $\underline{d} = 0$ under full market coverage.

service is of better quality; consequently, the firm obtains higher revenues from disclosure. A cap on disclosure level reduces the ability of the firm to monetize the additional information that consumers provide when it offers a higher quality level. In other words, the cap decreases the marginal benefit of investing in quality. The firm's investment in quality is therefore lower if a binding disclosure cap is implemented. A direct implication of Proposition 1 is that there always exists a trade-off between privacy and quality when the market is fully covered.

Let us now turn to the social desirability of a disclosure cap. Recall from the preceding discussion that it is strictly optimal for the regulator to set a disclosure cap if and only if $-\left.\frac{\partial \hat{W}}{\partial \bar{d}}\right|_{\bar{d}=\tilde{d}^M} > 0$. Using (6), the social marginal benefit of decreasing the disclosure level, evaluated at $\bar{d} = \tilde{d}^M$, can be expressed as follows:

$$-\left.\frac{\partial \hat{W}}{\partial \bar{d}}\right|_{\bar{d}=\tilde{d}^M} = \int_{\underline{\theta}}^{\bar{\theta}} \theta \tilde{x}(\theta, \tilde{q}^M, \tilde{d}^M) f(\theta) d\theta - \frac{r\gamma}{C''(\tilde{q}^M)} \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial V}{\partial q}(\tilde{x}(\theta, \tilde{q}^M, \tilde{d}^M), \tilde{q}^M) f(\theta) d\theta.$$

Since $\tilde{q}^M$, $\tilde{d}^M$ and $\tilde{x}(\theta, ., .)$ also depend on $\gamma$, it is unclear how the expression above depends on $\gamma$. However, we can show that this expression is positive for $\gamma$ below a certain threshold. To see why, define

$$\tilde{\gamma} = \sup S \equiv \left\{ \gamma' > 0 | \left.\frac{\partial \hat{W}}{\partial d}\right|_{\bar{d}=\tilde{d}^M} < 0 \text{ for all } \gamma \in (0, \gamma') \right\}.$$

From the fact that $\left.\frac{\partial \hat{W}}{\partial d}\right|_{\bar{d}=\tilde{d}^M}$ is continuous in $\gamma$ and (strictly) negative when $\gamma \to 0$, it follows that the set $S$ is not empty, which ensures that its upper bound $\tilde{\gamma}$ is well-defined in $\mathbb{R}_{++} \cup \{+\infty\}$.[39] Thus, a cap on the disclosure level is strictly socially desirable whenever $\gamma < \tilde{\gamma}$, i.e., the complementarity between quality and information is not too strong. In other words, under this condition, there exists a cap $\bar{d}$ between $\underline{d}$ and $\tilde{d}^M$ that leads to a strict increase in social welfare as compared to the unregulated scenario. However, when quality and information are sufficiently strong complements (i.e., $\gamma > \tilde{\gamma}$), the sign of $-\left.\frac{\partial \hat{W}}{\partial \bar{d}}\right|_{\bar{d}=\tilde{d}^M}$ is ambiguous and, therefore, so is the impact of a disclosure cap on social welfare.[40] In this case, one cannot exclude the possibility that the decrease in social welfare due to the reduction in quality (the strategic effect) may outweigh the increase in welfare resulting from the reduction in privacy costs (the direct effect).

[39] One cannot exclude *a priori* that $\tilde{\gamma}$ takes an infinite value.

[40] If $\tilde{\gamma}(r)$ takes an infinite value, privacy regulation would be socially desirable whatever the level of complementarity between quality and information.

More intuitively, consumers respond to the reduction in quality level induced by a disclosure cap by lowering the amount of information they provide. This reduction in the amount of information provided decreases both the gross utility obtained by the consumers and the disclosure revenues of the firm, thereby lowering social welfare. However, this negative impact on social welfare is dominated by the impact of the disclosure cap on consumers' privacy costs when the degree of complementarity between quality and information provision is low. This explains why setting a disclosure cap is socially desirable in this case.

The following proposition summarizes the above analysis.

**Proposition 2** *(Social desirability of a disclosure cap under full market coverage) When the market is fully covered, a privacy regulation taking the form of a binding disclosure cap is strictly socially desirable if the complementarity between quality and information is not too strong (i.e., $\gamma < \tilde{\gamma}$). Otherwise, such a regulation may not be socially desirable.*

The full market coverage assumption made in this section can be interpreted more broadly as meaning that the demand for the service is unresponsive to changes in quality and disclosure levels. Consumers react to such changes only by reducing or increasing the amount of information they provide to the firm. In such a setting, our results show that a regulation taking the form of a disclosure cap adversely affects quality-improving investments. Because of this trade-off between privacy and quality, a disclosure cap may not be socialy desirable if quality and information are strongly complementary but would still be welfare-enhancing if they are not.

## 5.2 Partial market coverage

We now consider the scenario in which $\tilde{\theta}\left(q^M(d), d\right) < \bar{\theta}$ for any $d \in [0,1]$. In this case, the market is only partially covered whatever the regulator's decision in the first stage.

Denoting $F(.)$ the cumulative distribution function of $\theta$, it follows from expression (4) and Lemmas 1 and 2 that the firm's (net) marginal benefit from investing in quality is

$$\frac{\partial\tilde{\Pi}}{\partial q} = \underbrace{rd\gamma F\left(\tilde{\theta}\left(q,d\right)\right)}_{\text{intensive margin effect}} + \underbrace{r\frac{\partial V}{\partial q}\left(\tilde{x}\left(\tilde{\theta}\left(q,d\right),q,d\right),q\right)f\left(\tilde{\theta}\left(q,d\right)\right)}_{\text{extensive margin effect}} - C'(q) \tag{7}$$

whenever the demand is positive and the market is partially covered, i.e., $\tilde{\theta}\left(q,d\right) \in \left(\underline{\theta},\bar{\theta}\right)$.

Let us consider the effect of the disclosure level on the firm's optimal choice of quality, which is driven by the cross-effect of quality and disclosure on the firm's profit. Differentiating expression (7) with respect to $d$ yields[41]

$$\begin{aligned}\frac{\partial^2\tilde{\Pi}}{\partial q\partial d} &= r\gamma F\left(\tilde{\theta}\left(q,d\right)\right)+rd\gamma\frac{\partial\tilde{\theta}}{\partial d}f\left(\tilde{\theta}\left(q,d\right)\right)+r\frac{\partial V}{\partial q}\frac{\partial\tilde{\theta}}{\partial d}f'\left(\tilde{\theta}\left(q,d\right)\right)\\ &\quad +rf\left(\tilde{\theta}\left(q,d\right)\right)\frac{\partial^2 V}{\partial q\partial x}\left[\frac{\partial\tilde{x}}{\partial\theta}\frac{\partial\tilde{\theta}}{\partial d}+\frac{\partial\tilde{x}}{\partial d}\right].\end{aligned}$$

From the expressions of $\frac{\partial\tilde{x}}{\partial\theta}$, $\frac{\partial\tilde{x}}{\partial d}$ and $\frac{\partial\tilde{\theta}}{\partial d}$ provided by Lemmas 1 and 2, it follows that

$$\frac{\partial\tilde{x}}{\partial\theta}\frac{\partial\tilde{\theta}}{\partial d}+\frac{\partial\tilde{x}}{\partial d}=0,$$

and, therefore,

$$\frac{\partial^2\tilde{\Pi}}{\partial q\partial d}=\underbrace{r\gamma F\left(\tilde{\theta}\left(q,d\right)\right)}_{\equiv A}+\underbrace{rd\gamma\frac{\partial\tilde{\theta}}{\partial d}f\left(\tilde{\theta}\left(q,d\right)\right)}_{\equiv B}+\underbrace{r\frac{\partial V}{\partial q}\frac{\partial\tilde{\theta}}{\partial d}f'\left(\tilde{\theta}\left(q,d\right)\right)}_{\equiv C}. \tag{8}$$

Term $A+B$ shows how the intensive margin effect of investment in quality depends on the disclosure level. More specifically, term $A$ captures the effect of an increase in the disclosure level on the marginal benefit from investing in quality for a *given* demand for the service. This is the only term that appears in our analysis of the full market coverage scenario (where $\tilde{\theta}\left(q,d\right)=\bar{\theta}$), and its sign is positive. Under partial market coverage, the magnitude of the intensive margin effect is also affected by the drop in demand resulting from an increase in the disclosure level. This effect is captured by term $B$ and is negative. Finally, term $C$ shows how the extensive margin effect of investment in quality depends on the disclosure level. Since the extensive margin effect is proportional to the density of consumers at the margin, and an increase in the disclosure level leads to a decrease in demand, term $C$ is positive (resp. negative) if the density function is locally decreasing (resp. increasing). Using Lemmas 1 and 2 again, we can rewrite (8) as

$$\frac{\partial^2\tilde{\Pi}}{\partial q\partial d}=\underbrace{r\gamma F\left(\tilde{\theta}\left(q,d\right)\right)\left[1-\frac{\tilde{\theta}\left(q,d\right)f\left(\tilde{\theta}\left(q,d\right)\right)}{F\left(\tilde{\theta}\left(q,d\right)\right)}\right]}_{A+B}\underbrace{-r\frac{\partial V}{\partial q}\frac{\tilde{\theta}\left(q,d\right)}{d}f'\left(\tilde{\theta}\left(q,d\right)\right)}_{C}. \tag{9}$$

[41]For notational convenience we drop the arguments of $\frac{\partial V}{\partial q}\left(q,\tilde{x}\left(\tilde{\theta}\left(q,d\right),q,d\right)\right)$ and $\frac{\partial^2 V}{\partial q\partial x}\left(q,\tilde{x}\left(\tilde{\theta}\left(q,d\right),q,d\right)\right)$.

This shows that the impact of the disclosure level on the intensive margin effect of investment in quality (captured by $A+B$) depends on the elasticity of the cumulative distribution $F(.)$, while its impact on the extensive margin effect (captured by $C$) depends on the convexity/concavity of $F(.)$. More precisely, we have the following result:

**Lemma 6** *(Impact of the disclosure level on the intensive and extensive margin effects)*

*- If $F(.)$ is relatively inelastic (resp. elastic), i.e., $\frac{\theta f(\theta)}{F(\theta)} < 1$ (resp. $> 1$) for all $\theta \in \left[\underline{\theta}, \bar{\theta}\right)$, the impact of the disclosure level on the intensive margin effect of investment in quality is positive (resp. negative).*

*- If $F(.)$ is convex (resp. concave) over $\left[\underline{\theta}, \bar{\theta}\right)$, the impact of the disclosure level on the extensive margin effect of investment in quality is negative (resp. positive).*

The elasticity of $F(.)$ can be related to the shape of the demand for the service, while the second derivative of $F(.)$ can be related to the shape of the derivative of demand with respect to quality. More precisely, in Appendix B, we show that the elasticity of $F(.)$ is equal to the elasticity of the demand for the service with respect to the disclosure level, holding the amount of information constant. We also show that the convexity/concavity of $F(.)$ can be related to the elasticity of the marginal effect of quality on demand with respect to the disclosure level (again holding the amount of information constant): the latter is greater (resp. less) than 1 if $F(.)$ is convex (resp. concave).

Lemma 6 suggests that we should distinguish between four scenarios. However, there are only three possible scenarios because the elasticity of $F(.)$ is always greater than 1 if $F(.)$ is (globally) convex. To see why, notice that

$$\frac{\theta f\left(\theta\right)}{F\left(\theta\right)} = 1 + \frac{\underline{\theta} f(\theta) - \int_{\underline{\theta}}^{\theta} \left[f(u) - f\left(\theta\right)\right] du}{F\left(\theta\right)},$$

which is greater than 1 if $f(.)$ is increasing. Before stating the main result of this section, let us consider the limiting case where quality and information are independent, i.e., $\gamma = 0$. In this scenario, the intensive margin effect is zero whatever the disclosure level, while the extensive margin effect is increasing (resp. decreasing) in the disclosure level if $F(.)$ is concave (resp. convex). Consequently, an increase in disclosure level leads to an increase (resp. decrease) in the firm's marginal benefit from investing in quality if $F(.)$ is concave (resp. convex).

Using Lemmas 5 and 6 and the observations above, we obtain the following results on the effect of an *increase* in disclosure level on quality. First, if $F(.)$ is (weakly) convex, the effect of an increase in disclosure level on quality is (weakly) negative. Second, if $F(.)$ is concave and relatively inelastic, the effect of disclosure on quality is positive. Third, if $F(.)$ is concave and relatively elastic, the effect is positive if the complementarity between quality and information is not too strong: this effect is (strictly) positive for $\gamma = 0$ and remains positive (by continuity) for sufficiently low values of $\gamma$. These results provide us with the implications of a disclosure cap, which corresponds to a *reduction* in disclosure level, on quality level.

**Proposition 3** *(Effect of a disclosure cap on quality under partial market coverage) Assume that the market is partially covered.*

*- If $F(.)$ is (weakly) convex, the effect of a disclosure cap $\bar{d} \in \left(\underline{d}, \tilde{d}^M\right)$ on quality is (weakly) positive.*

*- If $F(.)$ is concave and relatively inelastic, the effect of a disclosure cap $\bar{d} \in \left(\underline{d}, \tilde{d}^M\right)$ on quality is negative.*

*- If $F(.)$ is concave and relatively elastic, the effect of a disclosure cap $\bar{d} \in \left(\underline{d}, \tilde{d}^M\right)$ on quality is negative if the complementarity between quality and information is not too strong, and is ambiguous otherwise.*

This proposition shows that, unlike in the case where the market is fully covered, a disclosure cap can lead to an increase in quality level when the market is partially covered. This is because, under partial market coverage, the reduction in disclosure level has an additional effect of boosting the level of demand and may also affect its responsiveness to changes in quality level. As a result, the disclosure cap may have a positive impact on the marginal benefit of investing in quality overall, thereby resulting in a higher level of investment. In other words, there may be cases (as presented in the proposition) where there is *no trade-off* between more privacy and higher quality.

The size of the demand-boosting effect of a disclosure cap depends on the elasticity of demand with respect to disclosure level, holding the amount of information constant; this corresponds to the elasticity of $F(.)$. The impact of the cap on the responsiveness of demand to a change in quality level is related to elasticity of the marginal effect of quality on demand with respect to disclosure level, holding the amount of information constant; this is determined by the curvature of $F(.)$. Proposition 3 shows that the effect of a disclosure cap on quality is positive if the elasticity of the marginal effect of quality on demand with respect to the disclosure level is high enough. If it is not, then the effect is either negative or ambiguous, depending on the elasticity of the demand for the service with respect to disclosure level and the level of complementarity between quality and information.

Combining Proposition 3 with the fact that the direct effect of a disclosure cap on social welfare is always (strictly) positive[42] leads us to the following result about the social desirability of a cap on the disclosure level.

**Proposition 4** *(Social desirability of a disclosure cap under partial market coverage) Assume that the market is partially covered.*

*- If $F(.)$ is weakly convex, a privacy regulation taking the form of a binding disclosure cap is strictly socially desirable.*

*- If $F(.)$ is concave, a privacy regulation taking the form of a binding disclosure cap may not be socially desirable.*

Thus, when the market is partially covered, a disclosure cap may be socially desirable even when quality and information are strong complements from consumers' perspective. In particular, this is the case if the reduction in disclosure level increases substantially the responsiveness of demand to a change in quality level (i.e., if $F(.)$ is weakly convex). In this scenario, the regulator can improve both consumer privacy and service quality by setting a disclosure cap.[43]

# 6 Extensions

## 6.1 Heterogeneous third parties

In this extension, we analyze the scenario where third parties are heterogeneous in the privacy cost that they induce for consumers. This heterogeneity could reflect, for instance, differences in data use practices of these third parties. Consider the case of third-party advertisers. Some advertisers may, for instance, choose to target their advertisements using data at a more aggregated level than others (e.g., based on demographic groups instead of individual characteristics), thereby resulting in lower privacy costs. We suppose that the privacy cost incurred by a consumer of type $\theta \in \left[\underline{\theta}, \bar{\theta}\right)$ when a unit of her personal information is sold to a third party of type $u \in [0,1]$ is given by $\theta g(u) = 2\theta u$, which is increasing in the third party's type. The total privacy cost incurred by a consumer of type

[42]This implies in particular that a disclosure cap is always socially desirable if $\tilde{q}^M = 0$.

[43]Note that we do not consider in this paper the scenario in which the market can be fully or partially covered *depending* on the regulator's decision in the first stage. In such a scenario, the regulator's problem is less smooth than in the two cases we considered. Because of that, solving this problem requires to determine the optimal disclosure cap under each of the two regimes (i.e., full market coverage and partial market coverage) and compare the corresponding social welfare values. While this substantially complicates the analysis, it is unclear whether it would provide additional insights into the desirability of privacy regulation.

$\theta$ when an amount $x$ of her personal information is sold to third parties located in $[0, d]$ is given by

$$P(\theta, d, x) = \int_0^d 2\theta u x du = \theta d^2 x.$$

Correspondingly, the consumer's utility function is

$$U(x, \theta, q, d) = V(x, q) - \left(\alpha + \theta d^2\right) x - K.$$

It can be shown easily that, with this utility function,

$$\frac{\partial \tilde{x}}{\partial d} = \frac{2\theta d}{\frac{\partial^2 V}{\partial x^2}}; \quad \frac{\partial \tilde{x}}{\partial \theta} = \frac{d^2}{\frac{\partial^2 V}{\partial x^2}}; \quad \frac{\partial \tilde{x}}{\partial q} = \gamma,$$

and

$$\frac{\partial \tilde{\theta}}{\partial q} = \frac{\frac{\partial V}{\partial q}}{d^2 \tilde{x}}; \quad \frac{\partial \tilde{\theta}}{\partial d} = -2\tilde{\theta}.$$

Under the full market coverage scenario, the impact of a disclosure cap on quality and social welfare are qualitatively the same as in our baseline model: a disclosure cap raises the investment in quality and is socially desirable if quality and information are sufficiently weak complements.

However, under the partial market coverage scenario, the results are qualitatively affected by the assumption that third parties are heterogeneous. More specifically, the firm's marginal benefit from investing in quality becomes

$$\frac{\partial \tilde{\Pi}}{\partial q} = \underbrace{r d \gamma F\left(\tilde{\theta}\left(q, d\right)\right)}_{\text{intensive margin effect}} + \underbrace{\frac{r}{d}\frac{\partial V}{\partial q}\left(\tilde{x}\left(\tilde{\theta}\left(q, d\right), q, d\right), q\right) f\left(\tilde{\theta}\left(q, d\right)\right)}_{\text{extensive margin effect}} - C'(q).$$

Therefore, the cross-effect of quality and disclosure on profit is given by

$$\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d} = \underbrace{r\gamma F\left(\tilde{\theta}\left(q, d\right)\right)}_{=A} + \underbrace{r\gamma \frac{\partial \tilde{\theta}}{\partial d} f\left(\tilde{\theta}\left(q, d\right)\right)}_{=\frac{1}{d}B} + \underbrace{\frac{r}{d}\frac{\partial V}{\partial q}\frac{\partial \tilde{\theta}}{\partial d} f'\left(\tilde{\theta}\left(q, d\right)\right)}_{=\frac{1}{d}C} \underbrace{- \frac{1}{d^2}\frac{\partial V}{\partial q} f\left(\tilde{\theta}\left(q, d\right)\right)}_{\equiv D}$$

whereas it corresponds to $A + B + C$ in our baseline model. Rewriting this as

$$\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d} = \frac{1}{d}\left(A + B + C\right) + \underbrace{\left(1 - \frac{1}{d}\right)}_{\leq 0} A + \underbrace{D}_{\leq 0},$$

we can see that the cross-effect is negative whenever its counterpart in the baseline model is negative. This means that the effect of a disclosure cap on quality is positive in this extension whenever it was positive in the baseline model. Therefore, we get the following result.

**Proposition 5** *(Effect of a disclosure cap with heterogeneous third parties)*

*- Under full market coverage, a disclosure cap $\bar{d} \in \left(\underline{d}, \tilde{d}^M\right)$ has a negative effect on quality. However, setting a binding disclosure cap is socially desirable if quality and information are not strong complements.*

*- Under partial market coverage, the effect of a disclosure cap $\bar{d} \in \left(\underline{d}, \tilde{d}^M\right)$ on quality is weakly positive if $F(.)$ is weakly convex and is ambiguous otherwise. Consequently, setting a binding disclosure cap is socially desirable if $F(.)$ is weakly convex.*

## 6.2 Consumer-surplus-maximizing regulator

In the analysis of privacy regulation presented previously, we considered the decision problem of a regulator who seeks to maximize social welfare, which is given by the sum of the firm's profit and consumer surplus. We now examine the case where the regulator is a consumer protection agency, whose objective is to maximize consumer surplus. Let us focus again on the scenario in which the regulator does not find it optimal to choose a disclosure cap that induces no investment in quality.[44] The regulator seeks to maximize

$$\widehat{CS}(\bar{d}) = \widetilde{CS}(q^M(\bar{d}), \bar{d}) = \int_{\underline{\theta}}^{\tilde{\theta}(q,d)} \tilde{U}(\theta, q^M(\bar{d}), \bar{d}) f(\theta)\, d\theta$$

over $\left[\underline{d}, \tilde{d}^M\right]$. Assuming that $\widehat{CS}(.)$ is quasi-concave over this interval, setting a binding cap on disclosure is strictly desirable from the perspective of the consumer agency if the marginal benefit of disclosure to the consumers is negative when evaluated at $\bar{d} = \tilde{d}^M$; i.e.,

$$\left.\frac{\partial \widehat{CS}}{\partial \bar{d}}\right|_{d=\tilde{d}^M} < 0.$$

Since

$$\left.\frac{\partial \widehat{CS}}{\partial \bar{d}}\right|_{\bar{d}=\tilde{d}^M} = \left.\frac{\partial \hat{W}}{\partial \bar{d}}\right|_{\bar{d}=\tilde{d}^M} - \underbrace{\frac{\partial \tilde{\Pi}}{\partial d}\left(q^M(\tilde{d}^M), \tilde{d}^M\right)}_{=0} = \left.\frac{\partial \hat{W}}{\partial \bar{d}}\right|_{\bar{d}=\tilde{d}^M},$$

[44] If the consumer protection agency finds it optimal to choose a disclosure cap that leads to no investment in quality, it will choose $\bar{d} = 0$ (as this will minimize the privacy costs incurred by consumers).

the condition under which a disclosure cap is desirable from the perspective of the consumer protection agency is the same as that under which it is desirable from the perspective of a social-welfare-maximizing regulator. The intuition behind this result is that a marginal decrease of the disclosure level starting from the privately optimal disclosure level $\tilde{d}^M$ has a second-order effect on the firm's profit while it has a first-order effect on consumer surplus.

Let us now assume that setting a disclosure cap is strictly socially desirable and compare the optimal cap $\bar{d}^W$ for a social-welfare-maximizing regulator and the optimal cap $\bar{d}^C$ for a consumer-surplus-maximizing regulator. Since the disclosure cap is binding and $\tilde{\Pi}\left(q^M(d), d\right)$ is quasi-concave with respect to $d$,

$$\frac{\partial \tilde{\Pi}}{\partial d}\left(q^M(\bar{d}^W), \bar{d}^W\right) > 0.$$

Therefore,

$$\left.\frac{\partial \widehat{CS}}{\partial \bar{d}}\right|_{\bar{d}=\bar{d}^W} < \left.\frac{\partial \hat{W}}{\partial \bar{d}}\right|_{\bar{d}=\bar{d}^W} \leq 0,$$

where the second inequality follows from the fact that $\bar{d}^W > 0$.[45] From the quasi-concavity of $\widehat{CS}(.)$, it then follows that

$$\bar{d}^C \leq \bar{d}^W.$$

The next proposition summarizes the above results.

**Proposition 6** *(Consumer-surplus-maximizing vs socially optimal disclosure cap)*

*- A privacy regulation taking the form of a disclosure cap is strictly desirable for a consumer protection agency if and only if it is strictly socially desirable.*

*- When such a regulation is strictly desirable, the optimal disclosure cap from a consumer protection agency's perspective is weakly lower than the socially optimal disclosure cap.*

# 7 Discussion

## 7.1 Alternative interpretation of the model

In our analysis, we interpreted the consumer's input, $x$, as the amount of information that a consumer provides to the firm. This interpretation is appropriate when considering a matching website, where each user decides on the preferences she reveals, or in the context

[45] Under the assumption that a social-welfare-maximizing regulator does not find it optimal to set a disclosure cap that leads to no investment in quality, it must hold that $\bar{d}^W > 0$.

of services involving user-generated content, where each user decides on the amount of content to share (e.g., on a social media platform). This interpretation may, however, be less suited to other forms of Internet services, such as email and search. In these contexts, consumers do not directly supply information to the firm; instead, information is generated as a result of their use of the firm's service. The relevant consumer choice variable (and hence the appropriate interpretation of $x$) is therefore *usage intensity*, rather than information provision level. Correspondingly, $\alpha$, which formerly captured the marginal cost of providing information, can be interpreted more generally as the marginal opportunity cost of using the firm's service.

## 7.2 Positive surplus for buyers of personal information

We assumed in our baseline model that the firm was able to appropriate all the surplus generated by a transaction with a buyer of personal information. Let us relax this assumption by allowing third parties buying personal information to capture a positive share of that surplus. More precisely, let us assume that the unit price of personal information is $\beta r$, where $\beta \in (0, 1]$.

The firm's profit function in this variant of our model can be derived from the one in our baseline setting by replacing $r$ with $\beta r$. This implies in particular that the effect of a disclosure cap on firm's choice of quality is (qualitatively) the same as in the baseline model. More precisely, Proposition 3 still holds.

That said, when it comes to the effect of a disclosure cap on social welfare —defined as the sum of the firm's profit, the third parties' surplus and consumer surplus—it becomes more complicated to derive unambiguous results in the current setting. To see why, consider first the *direct effect* of a marginal decrease in the disclosure level (starting from the firm's optimal disclosure level) on social welfare. In our baseline model (i.e., $\beta = 1$) this effect is always positive. However, when the surplus of third parties buying personal information is positive (i.e., $\beta < 1$), this need not be true. A decrease in the disclosure level still leads to an increase in consumer surplus (through a decrease in privacy costs), but it also results in a decrease in the surplus of the third-party data buyers. Moreover, when the surplus of third parties is positive, there is an additional indirect effect, besides the strategic effect we identified in our baseline model. This additional effect corresponds to the positive impact of a disclosure cap on the amount of information provided by consumers and, therefore, on the surplus of the third parties buying personal information. When this indirect effect and the strategic effect we identified in our baseline model do not have the same sign (which is the case when a disclosure cap has a negative effect on quality), the sign of the overall indirect effect of a disclosure cap depends on their relative magnitudes.

Importantly, notice that if the regulator maximizes consumer surplus instead of social welfare, allowing buyers of personal information to make a positive surplus does not affect the desirability of a disclosure cap (as long as $r$ is replaced with $\beta r$ in the analysis).

## 7.3 Substitutability between quality and information

For many Internet services, it is natural to think of the firm's quality level and the consumer information provision level as complements (i.e., $\gamma > 0$): the better the quality of a firm's service, the higher the level of usage or information provision by consumers. There may also be cases, however, where the firm's quality level and the consumer's information exhibit substitutability (i.e., $\gamma < 0$). User authentication is one such scenario. Interpreting the firm's quality level as its ability to verify its user identity without the use of personal information provided by the consumer,[46] the higher the firm's quality level, the lower the consumer's utility from providing additional pieces of personal information (phone number, secondary email address, etc.) for authentication purposes.

Note that when quality and information are substitutes, a cap on the level of disclosure is always socially desirable under the full market coverage scenario. There is no trade-off between privacy protection and quality provision in this case because the firm never invests in quality. When the market is partially covered, however, a trade-off between privacy and quality may also exist in the substitutes case.

## 7.4 Positive prices for consumers

Our baseline model has focused on the case in which the website offers the service for free to consumers. While this is a widespread scenario in practice, it may also be the case that the website finds it optimal to charge consumers a positive price (in addition to disclosing their personal data).[47] Incorporating this possibility in our model would complicate the analysis substantially. To see why, notice that in a simpler setting without investment in quality and without an intensive margin (i.e., a setting where consumers would not be able to choose the amount of data they provide), we would expect such a cap to lead to an increase in the price charged to consumers. This is due to the so-called *see-saw effect* in two-sided markets: a cap on data disclosure would reduce the marginal benefit from attracting an additional consumer, which would make it optimal for the website to increase the price on

[46] For example, the firm could make use of the IP address or geographical location to assess if a login attempt is potentially fraudulent.

[47] Note that the parameter $K$ in our baseline model could be interpreted as an *exogenous* price.

the consumer side. However, in our setting, a cap on data disclosure would lead to a joint adjustment of both price and quality. A quick inspection of the relevant cross-derivatives of the website's profit function suggests that the effects of a disclosure cap on both price and quality are quite complex, and are likely to be ambiguous.

### 7.5 Regulating quality instead of disclosure

Regulating the disclosure level is one way of addressing (partially) the market failure identified in Section 4. An alternative way of doing so is to regulate the quality level. In Appendix C, we study the effect of setting a minimum quality requirement on the disclosure level chosen by the firm and analyze the social desirability of such a regulation. We show that the sign of the effect of a minimum quality requirement on the disclosure level is the opposite of the sign of the effect of a disclosure cap on quality: whenever a disclosure cap leads to a lower (resp. higher) quality, a minimum quality requirement leads to a higher (resp. lower) disclosure level. The reason is that both effects are driven by the (same) cross-effect of quality and disclosure on profit. This "duality" between quality and disclosure extends to the welfare effects of a regulation targeting one of them. More precisely, because the direct effect of a marginal increase in quality above the unregulated level on social welfare is positive (and, therefore, has the same sign as the direct effect of a disclosure cap on social welfare), the conditions under which setting a minimum quality requirement is socially desirable turn out to be qualitatively similar to those under which setting a disclosure cap is desirable.

## 8 Conclusion

In this paper, we study how a privacy regulation—specifically, a cap on data disclosure—affects a monopolist's incentives to invest in the quality of its service and social welfare. We find that the impact of a reduction in disclosure level on the monopolist's optimal choice of quality is negative when the market is fully covered, and depends on the effect of disclosure on the sensitivity of demand to quality when the market is partially covered. Under full market coverage, a cap on the disclosure level is socially desirable when the degree of complementarity between quality and information is not too strong. Under partial market coverage, a cap is desirable when the marginal effect of quality on demand is (sufficiently) elastic with respect to disclosure. As extensions, we also analyzed the case where third parties are heterogeneous and the scenario in which the regulator's objective is consumer surplus maximization.

Our results suggest it may be important for regulators to take into account the maturity of a market when considering the setting of a disclosure cap. In mature markets, demand is more likely to be (essentially) unresponsive to changes in quality and disclosure levels. This corresponds to the full market coverage scenario in our analysis. By contrast, in younger markets (e.g., for new services), demand is more likely to be (significantly) responsive to changes in quality and disclosure levels and the partial market coverage scenario is more relevant.

In addition to a disclosure cap, another privacy regulation that can be explored in our framework is the taxation of disclosure revenues. The taxation of digital monopoly platforms have been studied, for instance, by Bloch and Demange (2018) and Bourreau et al. (2018); however, to the best of our knowledge, no paper has considered the impact of taxation on the firm's incentives to invest in quality. A (unit) tax on the monopolist's disclosure revenues would translate to a reduction in the value of information in our model. Since this reduction affects both the optimal quality and disclosure levels of the firm, the impact of a tax is *a priori* unclear.

Finally, our model may also be interpreted more generally than one of privacy and quality. For example, the value of information can be thought of (more broadly) as the value that the firm derives from the exploitation of consumer data.[48] Correspondingly, the level of disclosure could instead be interpreted as the degree of data exploitation and the privacy cost parameter as a more general parameter reflecting the cost of sharing information. One can even take a step further and consider other types of inputs (besides personal information) that consumers may provide. For instance, consumers could provide time or attention rather than personal information. The interpretation of the consumers' cost parameter (which captured the intensity of privacy preferences in the case of information provision) would then change depending on the input that we are considering.

# A Appendix: Proofs

**Proof of Lemma 1.** Differentiating

$$\frac{\partial U}{\partial x}\left(\tilde{x}\left(\theta,q,d\right),\theta,q,d\right)=\frac{\partial V}{\partial x}\left(\tilde{x}\left(\theta,q,d\right),q\right)-\left(\alpha+\theta d\right)=0 \tag{10}$$

[48] Bloch and Demange (2017) provide several interpretations for the degree of data exploitation.

with respect to $d$ yields

$$\frac{\partial^2 V}{\partial x^2}\left(\tilde{x}\left(\theta,q,d\right),q\right)\frac{\partial \tilde{x}}{\partial d}\left(\theta,q,d\right)-\theta=0$$

and, therefore,

$$\frac{\partial \tilde{x}}{\partial d}\left(\theta,q,d\right)=\frac{\theta}{\frac{\partial^2 V}{\partial x^2}\left(\tilde{x}\left(\theta,q,d\right),q\right)}<0$$

Differentiating (10) with respect to $\theta$ and $q$ leads to

$$\frac{\partial \tilde{x}}{\partial \theta}\left(\theta,q,d\right)=\frac{d}{\frac{\partial^2 V}{\partial x^2}\left(\tilde{x}\left(\theta,q,d\right),q\right)}<0$$

$$\frac{\partial \tilde{x}}{\partial q}\left(\theta,q,d\right)=-\frac{\frac{\partial^2 V}{\partial x\partial q}\left(\tilde{x}\left(\theta,q,d\right),q\right)}{\frac{\partial^2 V}{\partial x^2}\left(\tilde{x}\left(\theta,q,d\right),q\right)}=\gamma.$$

**Proof of Lemma 2.** Since $U\left(x,\theta,q,d\right)$ is decreasing in $\theta$ then $\tilde{U}\left(\theta,q,d\right)=\max_{x\in[0,1]}U\left(x,\theta,q,d\right)$ is decreasing in $\theta$ (by the Envelope Theorem). Therefore, there exists $\tilde{\theta}\left(q,d\right)\in\left[\underline{\theta},\bar{\theta}\right]$ such that, for any $\theta\in\left[\underline{\theta},\bar{\theta}\right)$, the following equivalence holds:

$$\tilde{U}\left(\theta,q,d\right)>0\Longleftrightarrow\theta<\tilde{\theta}\left(q,d\right)$$

Moreover, whenever $\tilde{\theta}\left(q,d\right)\in\left(\underline{\theta},\bar{\theta}\right)$, it is defined by

$$\tilde{U}\left(\tilde{\theta}\left(q,d\right),q,d\right)=0.$$

Differentiating the latter with respect to $q$ and $d$, and using the Envelope Theorem, we get that

$$\frac{\partial\tilde{\theta}}{\partial q}=-\frac{\frac{\partial\tilde{U}}{\partial q}}{\frac{\partial\tilde{U}}{\partial\theta}}=-\frac{\frac{\partial U}{\partial q}}{\frac{\partial U}{\partial\theta}}=\frac{\frac{\partial V}{\partial q}}{d\tilde{x}\left(\tilde{\theta}\left(q,d\right),q,d\right)}>0;$$

$$\frac{\partial\tilde{\theta}}{\partial d}=-\frac{\frac{\partial\tilde{U}}{\partial d}}{\frac{\partial\tilde{U}}{\partial\theta}}=-\frac{\frac{\partial U}{\partial d}}{\frac{\partial U}{\partial\theta}}=-\frac{\tilde{\theta}\left(q,d\right)}{d}<0;$$

**Proof of Lemma 3.** The result follows directly from the fact that

$$\frac{\partial\tilde{\Pi}}{\partial q}<\frac{\partial\tilde{W}}{\partial q}$$

and the quasi-concavity of $\tilde{\Pi}$ and $\tilde{W}$ with respect to $q$.

**Proof of Lemma 4.** The result follows directly from the fact that

$$\frac{\partial \tilde{\Pi}}{\partial d} > \frac{\partial \tilde{W}}{\partial d}$$

and the quasi-concavity of $\tilde{\Pi}$ and $\tilde{W}$ with respect to $d$.

**Proof of Lemma 5**. Assume that $q^M(d) > 0$. Then, by continuity, $q^M(d') > 0$ for $d'$ sufficiently close to $d$. Therefore, or $d'$ sufficiently close to $d$, $q^M(d')$ is an interior solution given by the first-order condition

$$\frac{\partial \tilde{\Pi}}{\partial q}\left(q^M(d'), d'\right) = 0$$

Differentiating this with respect to $d'$ and evaluating it at $d' = d$ yields

$$\frac{\partial^2 \tilde{\Pi}}{\partial q^2}\left(q^M(d), d\right)\frac{\partial q^M}{\partial d} + \frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}\left(q^M(d), d\right) = 0$$

which leads to the result.

# B Appendix: Elasticities

Denote

$$\breve{V}(x, q) \equiv V(x, q) - \alpha x.$$

The demand addressed to the firm when the amount of information is chosen optimally by consumers is

$$\tilde{D}(q, d) = F\left(\tilde{\theta}(q, d)\right) = F\left(\min\left(\frac{\breve{V}\left(\tilde{x}\left(\tilde{\theta}(q, d), q, d\right), q\right) - K}{d\tilde{x}\left(\tilde{\theta}(q, d), q, d\right)}, \bar{\theta}\right)\right).$$

Consider the following function:

$$D(x, q, d) = F\left(\min\left(\frac{\breve{V}(x, q) - K}{dx}, \bar{\theta}\right)\right),$$

which can be interpreted as the demand addressed to the firm if all consumers using the service (are required to) provide the same amount of information $x$. Notice that

$$\tilde{D}\left(q,d\right) = D(\tilde{x}\left(\tilde{\theta}\left(q,d\right),q,d\right),q,d).$$

The elasticity of $D(x,q,d)$ with respect to $d$ is (in absolute value)

$$-d\frac{\frac{\partial D}{\partial d}}{D} = \frac{d\frac{1}{d^2}\frac{\check{V}(x,q)-K}{x}f\left(\frac{\check{V}(x,q)-K}{dx}\right)}{F\left(\frac{\check{V}(x,q)-K}{dx}\right)} = \frac{\frac{\check{V}(x,q)-K}{dx}f\left(\frac{\check{V}(x,q)-K}{dx}\right)}{F\left(\frac{\check{V}(x,q)-K}{dx}\right)}$$

whenever $D(x,q,d) \in (0,1)$. In particular, under partial (positive) market coverage,

$$-d\left.\frac{\frac{\partial D}{\partial d}}{D}\right|_{x=\tilde{x}\left(\tilde{\theta}(q,d),q,d\right)} = \frac{\tilde{\theta}\left(q,d\right)f\left(\tilde{\theta}\left(q,d\right)\right)}{F\left(\tilde{\theta}\left(q,d\right)\right)},$$

which shows that the elasticity of demand holding the amount of information constant (at the level of the marginal consumer) is the same as the elasticity of the cumulative distribution function (computed for the marginal type).

Similarly, straightforward algebraic manipulations show that the elasticity of $\frac{\partial D}{\partial q}$ with respect to $d$ is given by:

$$-d\frac{\frac{\partial^2 D}{\partial q \partial d}}{\frac{\partial D}{\partial q}} = 1 + \frac{\frac{\check{V}(x,q)-K}{dx}f'\left(\frac{\check{V}(x,q)-K}{dx}\right)}{f\left(\frac{\check{V}(x,q)-K}{dx}\right)}$$

whenever $D(x,q,d) \in (0,1)$. In particular, under partial (positive) market coverage,

$$-d\left.\frac{\frac{\partial^2 D}{\partial q \partial d}}{\frac{\partial D}{\partial q}}\right|_{x=\tilde{x}\left(\tilde{\theta}(q,d),q,d\right)} = 1 + \frac{\tilde{\theta}\left(q,d\right)f'\left(\tilde{\theta}\left(q,d\right)\right)}{f\left(\tilde{\theta}\left(q,d\right)\right)} = 1 + \underbrace{\frac{\tilde{\theta}\left(q,d\right)F''\left(\tilde{\theta}\left(q,d\right)\right)}{F'\left(\tilde{\theta}\left(q,d\right)\right)}}_{\text{curvature of } F(.)}$$

This implies that the elasticity of $\frac{\partial D}{\partial q}$ with respect to $d$ holding the amount of information constant is related to the curvature of $F(.)$. It is greater (resp. less) than 1 if $f'(.)$ is positive (resp. negative), that is, if $F(.)$ is convex (resp. concave).

# C Appendix: Minimum quality requirement

In this section, we investigate the social desirability of a policy whereby the authority does not regulate the disclosure level but, instead, regulates the quality level (before the disclosure level is set by the firm). Specifically, we study the decision of a social-welfare-maximizing regulator whose only instrument is a minimum quality standard. This requires, in particular, an understanding of the effect of a minimum quality requirement on the disclosure level chosen by the firm.

More precisely, consider the following game:

- First, the regulator decides whether to impose a minimum quality requirement, and sets the value of that requirement $\underline{q}$ if it does so.

- Second, the firm decides on its disclosure and quality levels.

- Third, consumers decide whether to patronize the firm and how much information to provide if they do.

Let us first analyze the firm's behavior for a given regulator's choice. The firm's optimal quality level maximizes $\tilde{\Pi}\left(q, d^M(q)\right)$, which we assume to be quasi-concave in $q$, subject to the constraint $q \geq \underline{q}$. If $\underline{q} \leq \tilde{q}^M$, the constraint is not binding; this means that the firm's decision will be the same as in the unregulated scenario. If $\underline{q} > \tilde{q}^M$, however, the constraint is binding. From the quasi-concavity of $\tilde{\Pi}\left(q, d^M(q)\right)$ with respect to $q$, it then follows that the firm will choose $q = \underline{q}$.

Consider now the regulator's decision in the first stage. Note first that the regulator must take account of the firm's particpation constraint, i.e.,

$$\tilde{\Pi}\left(\underline{q}, d^M\left(\underline{q}\right)\right) \geq 0.$$

It can be easily shown that there exists $\bar{q} > \tilde{q}^M$ such that the participation constraint above holds if and only if $\underline{q} \leq \bar{q}$.[49] Therefore, the regulator seeks to maximize

$$\check{W}\left(\underline{q}\right) \equiv \tilde{W}\left(\underline{q}, d^M\left(\underline{q}\right)\right) = \tilde{\Pi}\left(\underline{q}, d^M\left(\underline{q}\right)\right) + \int_{\underline{\theta}}^{\tilde{\theta}\left(\underline{q}, d^M\left(\underline{q}\right)\right)} \tilde{U}\left(\theta, \underline{q}, d^M\left(\underline{q}\right)\right) f(\theta)\, d\theta$$

with respect to $\underline{q} \in \left[\tilde{q}^M, \bar{q}\right]$. Assuming that $\check{W}(.)$ is quasi-concave over this interval, the regulator finds it strictly optimal to set a binding minimum quality requirement if and only

[49] This follows from the fact that $\tilde{\Pi}\left(q, d^M(q)\right)$ is continuous and decreasing in $q$ over $\left[\tilde{q}^M, +\infty\right)$ (because it is quasi-concave in $q$ and reaches its maximum at $q = \tilde{q}^M$).

if

$$\left.\frac{\partial \check{W}}{\partial \underline{q}}\right|_{\underline{q}=\tilde{q}^M} > 0$$

Moreover, we have

$$\begin{aligned}
\frac{\partial \check{W}}{\partial \underline{q}} &= \frac{\partial \tilde{\Pi}}{\partial q}\left(\underline{q}, d^M\left(\underline{q}\right)\right) + \int\limits_{\underline{\theta}}^{\tilde{\theta}\left(\underline{q}, d^M\left(\underline{q}\right)\right)} \left[\frac{\partial \tilde{U}}{\partial d}\left(\theta, \underline{q}, d^M\left(\underline{q}\right)\right)\frac{\partial d^M}{\partial q} + \frac{\partial \tilde{U}}{\partial q}\left(\theta, \underline{q}, d^M\left(\underline{q}\right)\right)\right] f\left(\theta\right) d\theta \\
&\quad + \left[\frac{\partial \tilde{\theta}}{\partial q} + \frac{\partial \tilde{\theta}}{\partial d}\frac{\partial d^M}{\partial q}\right] \underbrace{\tilde{U}\left(\tilde{\theta}\left(\underline{q}, d^M\left(\underline{q}\right)\right), \underline{q}, d^M\left(\underline{q}\right)\right)}_{=0} f\left(\tilde{\theta}(\underline{q}, d^M\left(\underline{q}\right))\right) \\
&= \frac{\partial \tilde{\Pi}}{\partial q}\left(\underline{q}, d^M\left(\underline{q}\right)\right) + \int\limits_{\underline{\theta}}^{\tilde{\theta}\left(\underline{q}, d^M\left(\underline{q}\right)\right)} \left[-\theta \tilde{x}\left(\theta, \underline{q}, d^M\left(\underline{q}\right)\right) + \frac{\partial V}{\partial q}\left(\tilde{x}\left(\theta, \underline{q}, d^M\left(\underline{q}\right)\right), \underline{q}\right)\right] f\left(\theta\right) d\theta
\end{aligned}$$

where the second equality follows from the application of the Envelope Theorem. Evaluating this at $\underline{q} = \tilde{q}^M$ and using the fact that $\tilde{d}^M = d^M(\tilde{q}^M)$ yields

$$\begin{aligned}
\left.\frac{\partial \check{W}}{\partial \underline{q}}\right|_{\underline{q}=\tilde{q}^M} &= \underbrace{\frac{\partial \tilde{\Pi}}{\partial d}\left(\tilde{q}^M, \tilde{d}^M\right)}_{=0} + \\
&\quad \int\limits_{\underline{\theta}}^{\tilde{\theta}\left(\tilde{q}^M, \tilde{d}^M\right)} \left[-\theta \tilde{x}\left(\theta, \tilde{q}^M, \tilde{d}^M\right) \left.\frac{\partial d^M}{\partial q}\right|_{q=\tilde{q}^M} + \frac{\partial V}{\partial q}\left(\tilde{x}\left(\theta, \tilde{q}^M, \tilde{d}^M\right), \tilde{q}^M\right)\right] f\left(\theta\right) d\theta
\end{aligned}$$

or, equivalently,

$$\begin{aligned}
\left.\frac{\partial \check{W}}{\partial \underline{q}}\right|_{\underline{q}=\tilde{q}^M} &= \underbrace{\int\limits_{\underline{\theta}}^{\tilde{\theta}\left(\tilde{q}^M, \tilde{d}^M\right)} \frac{\partial V}{\partial q}\left(\tilde{x}\left(\theta, \tilde{q}^M, \tilde{d}^M\right), \tilde{q}^M\right) f\left(\theta\right) d\theta}_{\text{direct effect}} \\
&\quad \underbrace{- \int\limits_{\underline{\theta}}^{\tilde{\theta}\left(\tilde{q}^M, \tilde{d}^M\right)} \theta \tilde{x}\left(\theta, \tilde{q}^M, \tilde{d}^M\right) \left.\frac{\partial d^M}{\partial q}\right|_{q=\tilde{q}^M} f\left(\theta\right) d\theta.}_{\text{strategic effect}}
\end{aligned}$$

This shows that a (marginal) increase in the quality level starting from the unregulated level has two effects: a *direct* effect on the value of the service for consumers (keeping the disclosure level constant), and a *strategic* effect capturing how an increase in the quality level alters the firm's choice of disclosure level. The direct effect is always positive, while the sign of the strategic effect depends on whether the firm's optimal disclosure level increases or decreases in response to an increase in quality. This strategic effect is weakly positive if the firm weakly decreases its disclosure level when quality level is increased (starting from $q = \tilde{q}^M$), i.e., if

$$\left.\frac{\partial d^M}{\partial q}\right|_{q=\tilde{q}^M} \leq 0.$$

In this case, the overall effect of a marginal increase in quality (starting from the unregulated level) on social welfare is unambiguously positive, which implies that it is strictly socially desirable to set a minimum quality requirement (under our regularity conditions). However, if

$$\left.\frac{\partial d^M}{\partial q}\right|_{q=\tilde{q}^M} > 0,$$

the strategic effect is negative and, therefore, the overall effect of a minimum quality requirement is *a priori* ambiguous. The following lemma relates the effect of a change in quality level on the firm's optimal disclosure level to the cross-effect of quality and disclosure on the firm's profit.

**Lemma 7** *(Effect of the quality level on disclosure) If* $d^M(q) \in (0,1)$*, then*

$$\frac{\partial d^M}{\partial q} = -\frac{\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}\left(q, d^M(q)\right)}{\frac{\partial^2 \tilde{\Pi}}{\partial d^2}\left(q, d^M(q)\right)}.$$

**Proof.** Similar to the proof of Lemma 5. ■

From Lemma 7, we see that the effect of a change in disclosure level on the firm's choice of quality has the same sign as the cross-effect of quality and disclosure on the firm's profit.[50]

We now study the sign of the effect of a change in quality level on disclosure, which in turn determines the sign of the strategic effect of a minimum quality level.

We first focus on the scenario in which the market is fully covered, and then turn to the scenario in which the market is partially covered.

[50]We use the fact that $\frac{\partial^2 \tilde{\Pi}}{\partial d^2}\left(q, d^M(q)\right) < 0$, which is given by the second-order condition of the maximization of $\tilde{\Pi}(q,d)$ with respect to $d$.

## C.1 Full market coverage

Suppose that $\tilde{\theta}\left(q, d^M(q)\right) = \bar{\theta}$ for any $q \in [0, \bar{q}]$. Under this assumption, the market is fully covered whatever the regulator's decision in the first stage of the game.

Consider the firm's optimal choice of disclosure for a given level of quality. The firm's marginal benefit from increasing disclosure is given by (4), with the term capturing the effect of disclosure on the extensive margin effect equal to zero (due to full market coverage). Under the assumption that the firm's optimal choice of disclosure level, $d^M(q)$, is interior for any quality level $q$ (in the relevant range), which we make in this extension, Lemma 7 implies that

$$\frac{\partial d^M}{\partial q} = -\frac{\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}\left(q, d^M(q)\right)}{\frac{\partial^2 \tilde{\Pi}}{\partial d^2}\left(q, d^M(q)\right)} = -\frac{r\gamma}{\frac{\partial^2 \tilde{\Pi}}{\partial d^2}\left(q, d^M(q)\right)} > 0.$$

Therefore, the firm's optimal level of quality is increasing in the level of disclosure. This leads to the following result.

**Proposition 7** *(Effect of a minimum quality requirement on disclosure under full market coverage) When the market is fully covered, a binding minimum quality requirement* $\underline{q} \in \left(\tilde{q}^M, \bar{q}\right]$ *leads to an increase in the disclosure level chosen by the firm.*

Let us now study the social desirability of a minimum quality requirement. Recall that it is strictly optimal for the regulator to set such a requirement if and only if $\left.\frac{\partial \check{W}}{\partial \underline{q}}\right|_{\underline{q}=\tilde{q}^M} > 0$. The social marginal benefit of increasing the quality level, evaluated at $\underline{q} = \tilde{q}^M$, is

$$\left.\frac{\partial \check{W}}{\partial \underline{q}}\right|_{\underline{q}=\tilde{q}^M} = \int_{\underline{\theta}}^{\tilde{\theta}\left(\tilde{q}^M, \tilde{d}^M\right)} \frac{\partial V}{\partial q}\left(\tilde{x}\left(\theta, \tilde{q}^M, \tilde{d}^M\right), \tilde{q}^M\right) f(\theta)\, d\theta - \int_{\underline{\theta}}^{\tilde{\theta}\left(\tilde{q}^M, \tilde{d}^M\right)} \theta \tilde{x}\left(\theta, \tilde{q}^M, \tilde{d}^M\right) \left.\frac{\partial d^M}{\partial q}\right|_{q=\tilde{q}^M} f(\theta)\, d\theta.$$

Let

$$\check{\gamma} = \sup\left\{\gamma' > 0 \middle| \left.\frac{\partial \check{W}}{\partial q}\right|_{q=\tilde{q}^M} > 0 \text{ for all } \gamma < \gamma'\right\}.$$

Using the fact that $\left.\frac{\partial \check{W}}{\partial \underline{q}}\right|_{\underline{q}=\tilde{q}^M}$ is continuous in $\gamma$ and (strictly) positive when $\gamma \to 0$, it follows that $\check{\gamma}$ is well defined in $\mathbb{R}_{++} \cup \{+\infty\}$. By definition of $\check{\gamma}$, we obtain that setting a minimum quality requirement is socially desirable when $\gamma < \check{\gamma}$. Thus, a minimum quality requirement is socially desirable whenever the complementarity between quality and information is not too strong. When quality and information are sufficiently strong complements (i.e., $\gamma > \check{\gamma}$), the sign of $\left.\frac{\partial \check{W}}{\partial q}\right|_{\underline{q}=\tilde{q}^M}$ is ambiguous and, therefore, so is the impact of a minimum quality requirement on social welfare. The following proposition summarizes the above analysis.

**Proposition 8** *(Social desirability of a minimum quality requirement under full market coverage) When the market is fully covered, a regulation taking the form of a minimum quality requirement is socially desirable if the complementarity between quality and information is not too strong (i.e., $\gamma < \check{\gamma}$). Otherwise, such a regulation may be not be socially desirable.*

## C.2 Partial market coverage

We now assume that $\tilde{\theta}\left(q, d^M(q)\right) < \bar{\theta}$ for any $q \in [0, \bar{q}]$. Under this assumption, the market is only partially covered whatever the regulator's decision in the first stage of the game. The sign of the effect of a higher quality on the firm's optimal disclosure level is given by the sign of the cross-effect $\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}$, which has already been studied in the analysis of the disclosure regulation. Therefore, we get the following result which is the counterpart of Proposition 3 when the authority regulates quality instead of privacy.

**Proposition 9** *(Effect of a minimum quality requirement on disclosure under partial market coverage) Assume that the market is partially covered.*

*- If $F(.)$ is (weakly) convex, a minimum quality requirement $\underline{q} \in \left(\tilde{q}^M, \bar{q}\right]$ leads to a (weakly) lower disclosure level.*

*- If $F(.)$ is concave and relatively inelastic, a minimum quality requirement $\underline{q} \in \left(\tilde{q}^M, \bar{q}\right]$ leads to a higher disclosure level.*

*- If $F(.)$ is concave and relatively elastic, the effect of a minimum quality requirement $\underline{q} \in \left(\tilde{q}^M, \bar{q}\right]$ is negative if quality and information are weak complements, and is ambiguous otherwise.*

Combining Proposition 9 with the fact that the direct effect of a minimum quality requirement on social welfare is always positive, we arrive at the following result.

**Proposition 10** *(Social desirability of a minimum quality requirement under partial market coverage) Assume that the market is partially covered.*

*- If the distribution of the idiosyncratic privacy cost exhibits a weakly increasing density function (i.e., $F(.)$ is weakly convex), a regulation taking the form of a binding minimum quality requirement is strictly socially desirable.*

*- If the distribution of the idiosyncratic privacy cost exhibits a decreasing density function (i.e., $F(.)$ is concave), a regulation taking the form of a binding minimum quality requirement may not be socially desirable.*

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