Vertical integration in the e-commerce sector

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1 Introduction

We study the implications of *vertical* integration in the e-commerce sector. Specifically, we consider the possibility that a (major) retailer and/or a platform buys one or several of the parcel delivery operators, or sets up its own delivery network.

Horizontal mergers are typically considered as "suspicious" and potentially anti-competitive. In the e-commerce sector, this includes the emergence of platforms which may bring about significant market power both in the retail and indirectly in the upstream parcel delivery market; see e.g., Borsenberger *et al.* (2016).

The economic literature on *vertical* mergers yields more mixed results. It generates a number of potential benefits. These include the reduction of transaction costs, technological economies, and probably most significantly, the elimination of successive monopolies or oligopolies and thus of the double marginalization these entail.[1] However, on the downside, it also involves the danger of "foreclosure". There is an extensive literature on this concept and its scope is quite large; see for instance Rey and Tirole (2007) or Motta (2004, Ch.6). Probably the most extreme example is when the merger deprives the competing firms from an essential input and thus effectively excludes them from the market. But the concept also covers a wider range of anti-competitive practices made possible by a vertical merger, including various types of vertical restraints (tying exclusive territories, etc.), the extension of market power in one market segment (upstream or downstream) to a different market segment, the possibility to raise competitor's cost etc.

In the postal sector this issue is particularly relevant. Some big retailers/platforms already have significant market power in their relevant markets, which gives them monopsony power towards parcel delivery operators.

In a first step, Sections 2–4, we make the assumption that a vertical merger with a major retailer buying a delivery operator and/or setting up its own delivery network will in the long run result in an integrated monopoly. We revisit this assumption later and show how the integrated monopoly may come about when the number of

[1] See Viscusi, Vernon and Harrington (1998) or Motta (2004) for a detailed overview of the various effects of vertical integration.

active firms is endogenous. We compare the integrated monopoly to a competitive scenario with independent retailers and delivery operators. This comparison involves a tradeoff between competition which tends to decrease prices and double marginalization which will have the opposite effect. Consequently, we cannot expect a general and unambiguous result. We show that with linear demands the integrated monopoly sets a higher price and achieves a lower total surplus than the independent oligopoly provided that there are at least 3 retailers and delivery operators. With a constant elasticity of demand on the other hand surplus is larger even for an independent duopoly. In this first step we evaluate welfare gross of fixed costs. This implies that a larger surplus may not be sufficient to yield a larger welfare.

In Section 6 we do account for fixed costs and their impact on welfare and on the number of active firms in a setting, where the number of firms is endogenous and determined by the opportunity to earn positive profits *net* of fixed costs. This issue is too complex to deal with analytically and we resort to numerical illustrations.[2] However, to set the grounds for this we first need to define and study the equilibrium with a single integrated firm and several independent retailers and/or delivery operators; this is done in Section 5.

The numerical results then yield a number of interesting insights. First, while the integration of a single retailer-delivery operator pair may initially be welfare improving, the resulting market structure may not be sustainable when the induced decrease in the competitors profits leads to their exit. Depending on the fixed costs this may well result in an integrated monopoly as only sustainable configuration (and as initially assumed). This requires fixed costs to be sufficiently large, which in turn pleads for a small number of firms. Interestingly, it turns out that there exist a range of fixed costs for which the integrated monopoly emerges (following a single integration) and is welfare inferior to the initial independent equilibrium *even when the reduction in the number of fixed costs is taken into account.*

The second interesting lesson that emerges is that multiple integration is typically

[2] For the cases of linear and of constant elasticity demand, analytical solutions can be obtained (and some expressions are provided in the Appendix). However the expressions are not very telling so that examples are useful to illustrate the cases that can arise.

welfare superior (for a given total number of firms) to the integration of a single retailer-delivery operator.

The settings discussed so far neglect one crucial characteristic of the parcel delivery sector, namely that delivery costs differ across customers. In Section 7 we consider an extension in which we distinguish between two types of customers according to their location: urban or rural. Delivery costs are larger for rural than for urban customers. We assume that delivery operators (when independent) charge a uniform delivery rate and retailers a uniform price. A vertically integrated firm on the other hand is likely to deliver only in urban areas and take advantage of an independent delivery operator's uniform pricing for customers in high cost areas. We reexamine the implications of vertical integration in this context, while considering the simplest possible initial situation, namely an independent duopoly (two retailers and two delivery operators). We show through analytical and numerical examples that urban integration is more likely to have an adverse effect on welfare than full integration. A crucial factor in the comparison turns out to be the proportion of rural customers (which must be sufficiently large), but at least for the considered demand functions the result obtains for proportions which are consistent with stylized empirical facts.

When examining this issue we assume in a first step that the integrated firm finds it beneficial not to deliver in rural areas. While this is intuitive it is not *a priori* obvious because the operators' delivery rate will include a markup above marginal cost. In a second step, we show through some numerical examples that this is not an empty assumption.

2 Independent retailers and delivery operators

There are (potentially) I upstream delivery operators $i = 1, ..., I$. Each delivers y_i parcels at a constant marginal cost k_i and fixed cost F_i. There are J downstream retailers $j = 1, ..., J$ who sell a homogenous product x_j at a variable cost $c_j(x_j)$ and pay a per unit delivery rate of t. Retailers also face a fixed cost $G_j \geq 0$. The demand for the final good is represented by its demand function $X(p)$ or equivalently, the inverse demand function $p(X)$ where X is the quantity and p its consumer price.

The timing of the game is as follow:

1. Stage 1: The delivery operator i sets a quantity of parcels y_i taking as given the quantity chosen by their competitor (but anticipating the inverse input demand function induced by the second stage equilibrium).
2. Stage 2: The retailer j sets a quantity of the final good x_j taking as given the quantity chosen by its competitor.
3. Stage 3: Demand is realized at a price $p(X)$.

We study the subgame perfect (Cournot-)Nash equilibrium and, as usual solve the model by backward induction. We derive general price formula and illustrate them using analytical and numerical examples. All of these assume that marginal cost is constant, $c_j(x) = c_j x$, and that demand is either linear $p(X) = a - bX$, or that demand elasticity ε defined by $|X'(p)p/X(p)|$ is constant.

2.1 Stage 2

Each retailer j chooses x_j that solves $p(X)$ such that:

$$\max_{x_j} \quad px_j - c(x_j) - tx_j - G_j$$
$$\text{s.t.} \quad p = p\left(\sum_j x_j\right)$$

The FOCs for each retailer $j = 1, ..., J$, are given by

$$p(X) + p'(X)\, x_j - c_j'(x_j) - t = 0, \tag{1}$$

which implies

$$\frac{p(X) - c_j'(x_j) - t}{p(X)} = -\frac{p'(X)\, x_j}{p(X)} \tag{2}$$

This system of J simultaneous equations defines the (second stage) Nash equilibrium quantities $x_j(t)$ and the total output

$$X(t) = \sum_j x_j(t),$$

and we can define an inverse demand function for the upstream market as

$$t\left(X\right)=t\left(\sum_{j}x_{j}\right). \tag{3}$$

Let us illustrate this procedure through the two examples mentioned above.

2.1.1 Example 1: linear demand

In this case, equation (1) is given by

$$a-b\sum_{k=1}^{J}x_{k}-bx_{j}-c_{j}-t=0,\ j=1,...,J.$$

Summing over all j yields

$$J\left(a-t\right)-bJX-bX-\sum_{k}c_{k}=0,$$

so that

$$X\left(t\right)=\frac{J}{J+1}\frac{\left(a-t-\bar{c}\right)}{b},$$

where

$$\bar{c}=\frac{1}{J}\sum_{k}c_{k},$$

is the average marginal cost of the retailers, excluding delivery. Inverting this function we obtain

$$t\left(X\right)=a-\bar{c}-\frac{J+1}{J}bX \tag{4}$$

2.1.2 Example 2: constant elasticity demand

Summing (1) over j yields

$$Jt=Jp\left(X\right)+p'\left(X\right)X-\sum_{j=1}^{J}c_{j}$$

so that

$$\frac{p(X)-\bar{c}-t}{p(X)}=\frac{1}{J\varepsilon},$$

which is the classical expression, best known in the monopoly case with $J = 1$. This equation holds for any demand function, but it yields a closed form solution only when ε is constant. Solving for t yields

$$t(X) = p(X)\left(1 - \frac{1}{J\varepsilon}\right) - \bar{c}. \tag{5}$$

2.2 Stage 1

Each delivery operator chooses y_i to solve

$$\begin{aligned} \max_{y_i} \quad & ty_i - k_i y_i - F_i, \\ \text{s.t.} \quad & t = t(X),\ X = Y = \sum_i y_i. \end{aligned} \tag{6}$$

This is exactly like a traditional Cournot oligopoly with inverse demand $t(X)$. Subgame perfection requires that the level of t induces a second stage equilibrium with aggregate output $X = Y = \sum_i y_i$. The FOC associated with delivery operator i's problem is given by

$$y_i \frac{\partial t(Y)}{\partial y_i} + t(y_i) - k_i = 0;\ i = 1, ..., I. \tag{7}$$

To obtain the equilibrium of the full game, one has to substitute $t(\cdot)$ from (3) and solve this system of I equations. This gives us the y_i's from which we can obtain t and thus also the equilibrium outputs of the retailers x_j. The fixed cost play no direct role in this problem as they are a constant in the profit maximization problem. However, the equilibrium is sustainable only if all delivery operators realize a positive profit in equilibrium. We assume for the time being that this is the case.

To illustrate these conditions and to show how they can be used to determine the equilibrium of the full game, we return to our two examples.

2.2.1 Example 1

Substituting (4) into (7) yields the following equations for $i = 1, \ldots, I$

$$-\frac{J+1}{J} b y_i + a - \bar{c} - \frac{J+1}{J} bY - k_i = 0$$

which, after simplification can be written as

$$-y_i - Y + \frac{J}{J+1}\frac{(a - \bar{c} - k_i)}{b} = 0.$$

Summing over I, using $X = Y$ and rearranging yields

$$X = \frac{I}{I+1}\frac{J}{J+1}\frac{\left(a - \bar{c} - \bar{k}\right)}{b}, \tag{8}$$

where

$$\bar{k} = \frac{1}{I}\sum_{i=1}^{I} k_i$$

denotes the average of the delivery operator's marginal delivery costs.

2.2.2 Example 2

We now substitute $t(\cdot)$ from (5) into (7) to obtain

$$y_i p'(Y)\left(1 - \frac{1}{J\varepsilon}\right) + p(Y)\left(1 - \frac{1}{J\varepsilon}\right) - \bar{c} - k_i = 0$$

Summing over i, using $X = Y$ and rearranging successively yields

$$Yp'(Y)\left(1 - \frac{1}{J\varepsilon}\right) + Ip(Y)\left(1 - \frac{1}{J\varepsilon}\right) - I\bar{c} - \sum_{i=1\ldots I} k_i = 0$$

$$p(X)\left(1 - \frac{1}{J\varepsilon}\right)\left(1 - \frac{1}{I\varepsilon}\right) - \bar{c} - \bar{k} = 0,$$

so that

$$p(X) = \frac{\bar{c} + \bar{k}}{\left(1 - \frac{1}{J\varepsilon}\right)\left(1 - \frac{1}{I\varepsilon}\right)} \tag{9}$$

3 N integrated firms

We now suppose that there are N integrated firms denoted by subscript $n = 1, ..., N$. An integrated firm maximizes total profits obtained from its up- and downstream activities. This implies that the two stages collapse into a single stage, where firm n chooses x_n that solves

$$\max_{X_n} p(X)\, x_n - c_n(x_n) - k_n x_n - F_n - G_n$$

The FOC is

$$p'(X)\,x_n + p(X) - c'_n(x_n) - k_n = 0 \tag{10}$$

We once again present the solution for the two examples

3.1 Example 1

The FOCs are then given by

$$a - bX - bx_n - c_n - k_n = 0.$$

Summing over N and solving for X yields

$$X = \frac{N}{N+1}\frac{a - \bar{c} - \bar{k}}{b}, \tag{11}$$

where

$$\bar{c} = \frac{\sum_{n=1}^{N} c_n}{N} \qquad \bar{k} = \frac{\sum_{n=1}^{N} k_n}{N}$$

denote the average of retailers' and delivery operators' costs.

3.2 Example 2

Summing condition (10) over N and solving for p shows that

$$p(X) = \frac{\bar{c} + \bar{k}}{\left(1 - \frac{1}{N\varepsilon}\right)}. \tag{12}$$

which, once again, represents a closed form solution when ε is constant.

4 Independent vs integrated operators

We now compare the independent and integrated equilibria for our two examples. We assume for the time being that $\bar{c}$ and $\bar{k}$ are the same under the two scenarios. To compare total surplus we can then either compare X or p, keeping in mind that the best solution is the one which gives the larger output and the lower price. For the time being, we restrict our attention to surplus, which does not account for fixed costs. These will be reintroduced and included in the welfare analysis in Section 6.

4.1 Example 1

In this setting it is easier to compare equilibrium aggregate output levels. Using (8) and (11) shows that the equilibrium with independent operators yields a larger output than the integrated solution if and only if

$$\frac{I}{I+1}\frac{J}{J+1} > \frac{N}{N+1}.$$

With $N = 1$, this condition is violated for $J = 2, I = 2$, $4/9 < 1/2$. Consequently the integrated monopoly yields a better solution than two independent retailers and delivery operators. In other words, with two firms at each level, competition is not strong enough to compensate for the double marginalization that occurs when delivery operators are independent. Furthermore, when $J = 3$ and $I = 2$ or $J = 2$ and $I = 3$ the two solutions are equivalent. To obtain a better solution than under the integrated monopoly it takes at least 3 retailers and 3 delivery operators.[3]

4.2 Example 2

Turning to the constant demand elasticity case, we use (9) and (11) to show that an integrated monopoly yields a higher price and is welfare inferior if and only if

$$\left(1-\frac{1}{N\varepsilon}\right) < \left(1-\frac{1}{J\varepsilon}\right)\left(1-\frac{1}{I\varepsilon}\right) \tag{13}$$

Suppose again that $N = 1$, $J = 2, I = 2$, so that (13) reduces to

$$\begin{aligned}
&\left(1-\frac{1}{\varepsilon}\right) < \left(1-\frac{1}{2\varepsilon}\right)\left(1-\frac{1}{2\varepsilon}\right) \\
&\frac{1}{\varepsilon}(\varepsilon-1) < \frac{1}{4\varepsilon^2}(2\varepsilon-1)^2 \\
&4\varepsilon(\varepsilon-1) < (2\varepsilon-1)^2 = 4\varepsilon(\varepsilon-1)+1
\end{aligned}$$

a condition which is *always* satisfied.[4] Consequently, the competition under vertical separation dominates as long as there are at least two retailers and two delivery operators.

[3] Or 2 retailers with 4 delivery operators, etc.

[4] However, $J = 2$ and $I = 1$ or $J = 1$ and $I = 1$ is not enough. Condition (13) then requires

$$\left(1-\frac{1}{\varepsilon}\right) < \left(1-\frac{1}{2\varepsilon}\right)\left(1-\frac{1}{\varepsilon}\right),$$

which is *never* satisfied.

It thus turns out that constant elasticity demand leads to a more intense competition. Its downward pressure on the price outweighs the cost of double marginalization even for a duopoly.

5 A single integrated firm competing with non integrated retailers and delivery operators

So far we have assumed that the integration of one of the retailers and delivery operators results in a monopoly. To show how this can come about we shall now consider a setting where the number of actors is endogenous and determined as the maximum number of retailers and delivery operators who can realize positive equilibrium profits. In other words, their profits gross of fixed costs must exceed their fixed costs. The no integration equilibrium with I independent delivery operators and J retailers has been studied in Section 2. The equilibrium profits determine the range of fixed costs for which this equilibrium is sustainable. Alternatively one can set given levels of fixed costs and determine I and J endogenously. Either way the relevant equilibrium to consider is that determined in Section 2.

To study the equilibrium number of delivery operators and retailers when one pair is vertically integrated, we have to study the equilibrium with $J-1$ independent retailers, $I-1$ independent delivery operators and one integrated retailer cum delivery operator.

To avoid tedious repetitions, we concentrate on the proper specification of the game and the general conditions. Their counterparts for the two considered examples are given in Appendix A. They are used to solve the numerical illustrations presented in the next section.

5.1 Stage 2

Retailers $2,\ldots,J$ solve

$$\max_{x_j} \quad p(X)x_j - c(x_j) - tx_j - G_j$$

while the integrated retailer 1 solves

$$\max_{x_1} p(X)\, x_1 - c_1(x_1) - k_1 x_1 - F_1 - G_1$$

The FOC's are given by

$$p(X) + p'(X)x_j - c_j - t = 0,\ j = 2, ..., J \tag{14}$$

$$p(X) + p'(X)x_1 - c_1 - k_1 = 0 \tag{15}$$

5.2 Stage 1

Delivery operators $2, \ldots, I$ solve

$$\begin{aligned} \max_{y_i} \quad & ty_i - k_i y_i - F_i, \\ \text{s.t.} \quad & t = t(X_{-1}),\ X_{-1} = Y_{-1}. \end{aligned}$$

The first order condition yields:

$$t'(X_{-1})x_i + t(X_{-1}) - k_i = 0,\ i = 2, ...I \tag{16}$$

6 Numerical examples

These numerical examples bring together the specifications considered in Sections 2, 3 and 5. Most importantly we use the equilibrium profits they yield to study which market structure is sustainable when the number of delivery operators and retailers is endogenously determined. This shows that for suitable levels of fixed costs, the integration of a single retailer-delivery operator pair indeed results in an integrated monopoly.

For each scenario we report only the most relevant properties of the equilibrium, including, total output as well as profits and total surplus, both of these being defined gross of possible fixed costs. However, we do examine the role played by fixed costs, both for entry and exit and for welfare comparisons.

All scenarios considered in this section assume $k = 0.05$ and $c = 0.1$.

6.1 Examples starting with $I = J = 2$.

Assume that the inverse demand function is given by $p(X) = X^{-1/\varepsilon}$, so that demand elasticity is constant and equal to ε. We start from the independent equilibrium with 2 delivery operators and 2 retailers.

6.1.1 Scenario 1: $\varepsilon = 2$.

With $I = J = 2$ the independent equilibrium (Section 2) yields a total output of 14.06 and a total surplus (TS) of 5.39. When retailer 1 and delivery operator 1 integrate (Section 3) total output is 18.04 and TS increases to 5.78. Thus in a first step, integration has a positive impact on welfare. However, when $G_2 > 0.15$ or $F_2 > 0.26$ (if one of the two independent actors disappears there is no room for the other actor to exist since there is no available market for them), the integrated monopoly is the only sustainable equilibrium where total output is 11.11 and a TS of 5. The independent 2*2 equilibrium is thus the only sustainable equilibrium and better than the integrated monopoly if the avoided fixed costs are not too large: $F_2 + G_2 < 5.39 - 5 = 0.39$ while we have either $G_2 > 0.15$ or $F_2 > 0.26$.[5]

Scenario	2*2	1i, 1r, 1o	1i
Total surplus	5.39	5.78	5
Total output	14.06	18.04	11.11
Profit integrated	–	1.11	1.11
Profit retailer(s)	0.46	0.15	–
Profit delivery operator(s)	0.35	0.26	–

6.1.2 Scenario 2 $\varepsilon = 0.9$.

We now consider a smaller level of elasticity, namely $\varepsilon = 0.9$. This yields the following results:

Scenario	2*2	1i, 1r, 1o
Total surplus	−9.92	−10.13
Total output	1.31	1.01
Profit integrated	–	0.65
Profit retailer(s)	0.26	0.05
Profit delivery operator(s)	0.12	0.13

In this case, integration reduces welfare even for a given number of retailers and delivery operators. The new equilibrium may or may not be sustainable depending on the fixed costs but for this level of demand elasticity no interior solution exists for the integrated monopoly case.

[5] We use the following notation to identify the scenarios. 2*2 or 3*3 etc. refers to a market with 2 or 3 independent delivery operators and retailers; 1i, 1r, 1o, for instance, means that there is one integrated firm, one independent retailer and one independent operator. The other labels follow the same logic and should be self-explanatory.

6.2 Examples starting with $I = J = 3$

Now for each scenario, we study whether integration leads to the exit of firms (retailers or delivery operators) for some configurations of fixed costs and study whether the equilibrium with exit leads to a lower or higher social welfare for this configuration of fixed costs. In the process we also study scenarios with multiple integrated firms.

6.2.1 Linear demand

Assume $p(X) = a - bX$, $a = 20, b = 1$ (low elasticity of demand). Starting with the scenarios where at most one retailer-delivery operator pair integrates we obtain

Scenario	3*3	1i, 2r, 2o	1i, 1r, 2o	1i, 2r, 1o	1i, 1r, 1o	1i
Total surplus	559	575	567	569	562	547
Total output	11.16	13.23	12.13	12.40	11.57	9.92
Profit integrated	–	43.78	59.58	55.40	68.40	98.50
Profit retailer(s)	13.85	10.94	19.45	6.15	10.94	–
Profit delivery operator(s)	18.46	10.94	7.29	24.62	16.41	–

We obtain 1i (one integrated firm) as a free entry equilibrium when $G_2 > 10.94$ and $F_2 > 16.41$. The 1i equilibrium implies a loss in total surplus of $559 - 547 = 12$ compared to the 3*3 setting but this is not enough to justify the extra fixed costs incurred in the 3*3 case. In this case integration of a single retailer-delivery operator pair appears at first beneficial. However, the following table shows that for any given total number of firms multiple integration always welfare dominates that of a single retailer-delivery operator pair. Specifically, 3i dominates (2i, 1r, 1o) which in turn dominates (1i, 2r, 2o). Similarly, 2i yields a higher level of welfare than (1i, 1r, 1o). This is not surprising: with multiple integration double marginalization is eliminated while the number of competing retailers remains constant.

Scenario	3i	2i, 1r, 1o	2i
Total surplus	584	580	575
Total output	14.88	14.06	13.23
Profit integrated	24.62	33.51	43.78
Profit retailer(s)		6.15	
Profit delivery operator(s)		8.20	

To test the robustness of these results we have considered a number of alternative scenarios with different parameter values of a and b and they all give exactly the same pattern of results. To avoid repetitions we do not report them and instead now turn to a different specification of demand.

6.2.2 Constant elasticity demand

Scenario 1: $\varepsilon = 2$ Considering the same scenarios as in the linear case we obtain the following results.

Scenario	3*3	1i, 2r, 2o	1i, 1r, 2o	1i, 2r, 1o	1i, 1r, 1o	1i
Total output	21.43	25	20.81	20.95	18.04	11.11
Total surplus	6.04	6.25	6	6.01	5.78	5
Profit integrated	–	0.625	0.90	0.89	1.11	1.66
Profit retailer(s)	0.257	0.156	0.31	0.07	0.159	–
Profit delivery operator(s)	0.214	0.156	0.11	0.37	0.26	–

Suppose we start from 3*3 and that in a first step a single retailer-delivery operator pair integrates. Suppose that $G_j > G_{\min} = 0.159$ and $F_i > F_{\min} = 0.156$. While this integration yields initially a welfare gain, the resulting equilibrium is not sustainable and with endogenous entry and exit we'll end up with 1i. This implies a gross social welfare loss of $6.04 - 5 = 1.04$. The social welfare gain stemming from decreased fixed costs is at least $2 * G_{\min} + 2 * F_{\min} = 0.63$. Moving from 3*3 to 1i thus involves a welfare loss if $1.04 > 2 * G_j + 2 * F_i > 0.63$. Notice that when 3*3 is sustainable (along with $G_j > G_{\min} = 0.159$ and $F_i > F_{\min} = 0.156$) then the move to the integrated monopoly involves a welfare loss even when the savings in fixed costs are accounted for.

The following table shows the results obtained under multiple integration. Like in the linear case it shows that for any given number of firms multiple integration is welfare superior.

Scenario	3i	2i, 1r, 1o	2i
Total output	30.86	27.93	25
Total surplus	6.48	6.38	6.25
Profit integrated	0.30	0.45	0.625
Profit retailer(s)		0.077	–
Profit delivery operator(s)		0.11	–

Scenario 2: $\varepsilon = 1.1$

scenario	3*3	1i, 2r, 2o	1i, 1r, 2o	1i, 2r, 1o	1i, 1r, 1o	1i
Total output	3.72	4.23	3.27	3.07	2.42	0.63
Total surplus	10.84	10.91	10.76	10.72	10.56	9.46
Profit integrated	–	0.259	0.39	0.43	0.539	0.86
Profit retailer(s)	0.114	0.064	0.14	0.030	0.066	–
Profit delivery operator(s)	0.079	0.064	0.05	0.167	0.122	–

As in the previous case we start from 3*3 and consider integration of a single firm. When $G_j > G_{\min} = 0.066$ and $F_i = F_{\min} > 0.064$, we end up with an integrated monopoly 1i for which total surplus is 9.46. The gross welfare loss brought about by integration is thus 1.38. The welfare gain due to saved fixed costs is at least equal to $2 * 0.066 + 2 * 0.064 = 0.26$. Integration of a single firm and the subsequent changes in market structure thus lead to a welfare loss if the following three conditions hold: *(i)* $1.38 > 2 * G_j + 2 * F_i$, *(ii)* $G_j > G_{\min} = 0.066$, and *(iii)* $F_i = F_{\min} > 0.064$. The first condition is necessarily satisfied if 3*3 is sustainable.

Considering the possibility of multiple integration yields the results shown in the following table. The pattern of results is exactly the same as in the previous scenario.

Scenario	3i	2i, 1r, 1o	2i
Total output	5.53	4.88	4.23
Total welfare	11.03	10.98	10.91
Profit integrated	0.12	0.18	0.26
Profit retailer(s)		0.03	
Profit delivery operator(s)		0.04	

Comparing the two scenarios suggests that the range of fixed costs for which the integrated monopoly obtains and yields to a welfare reduction (even when the fixed cost is accounted for), is larger the smaller is the demand elasticity. This is also not a surprise; a monopoly will have more market power, the lower is the demand elasticity.

The qualitative results obtained in these two scenarios carry over to other scenarios with different parameter values and particularly different demand elasticities.

7 Extension: two delivery areas

We now distinguish between two types of customers according to their location: urban or rural. Delivery costs are larger for rural than for urban customers. Delivery operators (when independent) charge a uniform delivery rate and retailers a uniform price. A vertically integrated firm on the other hand delivers only in urban areas. Urban and rural customers have identical demand functions. Let α^U and $\alpha^R = 1 - \alpha^U$ denote the share of urban and rural customers respectively. Total demand is then given by $X(p) = \alpha^U X(p) + \alpha^R X(p)$. Rural and urban deliveries involve specific fixed costs denoted by F_i^U and F_i^R. Marginal delivery costs of delivery operator i, are denoted k_i^U and k_i^R.

7.1 No integration

There are two retailers $j = 1, 2$ and two delivery operators $i = 1, 2$ playing the Cournot game specified in Section 2. We proceed again by backward induction but restrict ourselves to recalling the main results.

7.1.1 Stage 2

Retailer j chooses x_j to solve

$$\begin{aligned} \max_{x_j} \quad & p(X)\, x_j - c_j x_j - t x_j - G_j \\ \text{s.t.} \quad & X = x_1 + x_2. \end{aligned}$$

This is exactly the same problem as in subsection 2.1, which yields a total equilibrium output $X(t)$ and thus an inverse demand function $t(X)$.

7.1.2 Stage 1

Delivery operators choose y_i to solve

$$\begin{aligned} \max_{y_i} \quad & t y_i - \left(\alpha^R k_i^R + \alpha^U k_i^U\right) y_i - \left(F_i^U + F_i^R\right) \\ \text{s.t.} \quad & t = t(X),\; X = Y = \sum_i y_i. \end{aligned}$$

From the results derived in Section 2, and in particular (4) and (8) derived for linear demands we obtain

$$t(X) = \frac{a - \bar{c} + 2\bar{k}}{3}, \tag{17}$$

$$X = \frac{4\left(a - c - \bar{k}\right)}{9b}, \tag{18}$$

where $\bar{k}$ is redefined as

$$\bar{k} = \sum_{i=1,2} \frac{\alpha^R k_i^R + \left(1 - \alpha^R\right) k_i^U}{2}.$$

7.2 With integration

We compare this scenario with the case where retailer 1 and delivery operator 1 are integrated. Integrated delivery operator 1 delivers only urban parcels, while delivery operator 2 continues to be independent and delivers to both areas at a uniform rate. Retailer 2 ships all its parcels via operator 2, while retailer 1 uses its own delivery operator for urban parcels and delivery operator 2 for the rural ones.

7.2.1 Stage 2

Integrated retailer1 and independent retailer 2 compete. The problem of firm 1 is:

$$\max_{x_1} p(X) x_1 - cx_1 - \alpha^U k_1^U x_1 - \alpha^R t x_1 - G_1 - F_1^U$$

and the problem of independent retailer 2 is:

$$\max_{x_2} p(X) x_2 - cx_2 - tx_2 - G_2$$

where $X = x_1 + x_2$.

The FOCs are given by

$$p'(X) x_1 + p(X) - c - \alpha^U k_1^U - \alpha^R t = 0$$

$$p'(X) x_2 + p(X) - c - t = 0$$

which for the case of linear demands can be rewritten as

$$a - 2bx_1 - bx_2 - c - \alpha^U k_1^U - \alpha^R t = 0 \tag{19}$$

$$a - 2bx_2 - bx_1 - c - t = 0 \tag{20}$$

This yields the equilibrium levels of x_i as function of t

$$x_1(t) = \frac{a - c - 2\alpha^U k_1^U + t\left(1 - 2\alpha^R\right)}{3b},$$
$$x_2(t) = \frac{a - c + \alpha^U k_1^U - \left(2 - \alpha^R\right)t}{3b},$$

and similarly for the equilibrium aggregate output

$$X(t) = x_1(t) + x_2(t) = \frac{2(a-c) - \alpha^U k_1^U - t\left(1 + \alpha^R\right)}{3b} \tag{21}$$

Note that

$$\frac{\partial x_2}{\partial t} = -\frac{\left(2 - \alpha^R\right)}{3b} \tag{22}$$
$$\frac{\partial (x_1 + x_2)}{\partial t} = -\frac{\left(1 + \alpha^R\right)}{3b} \tag{23}$$

Defining $X_{-1}(t) = \alpha^R x_1(t) + x_2(t)$, one has

$$X_{-1} = \frac{\left(1 + \alpha^R\right)(a-c) - \alpha^U k_1^U \left(2\alpha^R - 1\right) - 2t\left(\alpha^{R^2} - \alpha^R + 1\right)}{3b}.$$

Solving for t and rearranging yields

$$t(X_{-1}) = \frac{\left(1 + \alpha^R\right)(a-c) - \alpha^U k_1^U \left(2\alpha^R - 1\right) - 3bX_{-1}}{2\left(\alpha^{R^2} - \alpha^R + 1\right)} \tag{24}$$

and

$$t'(X_{-1}) = -\frac{3b}{2\left(\alpha^{R^2} - \alpha^R + 1\right)}. \tag{25}$$

7.2.2 Stage 1

Delivery operator 2 is the sole player at this stage and chooses y_2^R and y_2^U to solve

$$\max_{y_2^R, y_2^U} \quad t\left(y_2^R + y_2^U\right) - k_2^R y_2^R - k_2^U y_2^U - \left(F_2^U + F_2^R\right) \qquad \text{s.t.} \qquad y_2^R + y_2^U = X_{-1}$$
$$t \equiv t(X_{-1})$$
$$y_2^R = \alpha^R \left(x_1(t) + x_2(t)\right)$$
$$y_2^U = \alpha^U x_2(t).$$

Substituting the constraints into the objective function the problem of delivery operator 2 can thus be reformulated as

$$\max_{X_{-1}} t\left(X_{-1}\right) X_{-1} - \alpha^{U} k_2^{U} x_2\left(t\left(X_{-1}\right)\right) - \alpha^{R} k_2^{R}\left(x_1\left(t\left(X_{-1}\right)\right) + x_2\left(t\left(X_{-1}\right)\right)\right) - \left(F_2^{U} + F_2^{R}\right),$$

and the FOC is given by

$$t'\left(X_{-1}\right)\left(X_{-1} - \alpha^{U} k_2^{U} \frac{\partial x_2}{\partial t} - \alpha^{R} k_2^{R}\left(\frac{\partial x_1}{\partial t} + \frac{\partial x_2}{\partial t}\right)\right) + t\left(X_{-1}\right) = 0.$$

With the linear demand functions we have

$$\frac{\left(1+\alpha^{R}\right)(a-c) - \alpha^{U} k_1^{U}\left(2\alpha^{R}-1\right) - 3bX_{-1}}{2\left(\alpha^{R^2} - \alpha^{R} + 1\right)}$$
$$-\frac{3b}{2\left(\alpha^{R^2} - \alpha^{R} + 1\right)}\left(X_{-1} + \alpha^{U} k_2^{U} \frac{\left(2-\alpha^{R}\right)}{3b} + \alpha^{R} k_2^{R} \frac{\left(1+\alpha^{R}\right)}{3b}\right) = 0.$$

Solving for X_{-1}, using expression (24) yields

$$t^{I} = \frac{\left(1+\alpha^{R}\right)(a-c) - \alpha^{U} k_1^{U}\left(2\alpha^{R}-1\right) + \left(1+\alpha^{R}\right)\alpha^{R} k_2^{R} + \alpha^{U} k_2^{U}\left(2-\alpha^{R}\right)}{4\left(\alpha^{R^2} - \alpha^{R} + 1\right)}, \quad (26)$$

$$X^{I} = \frac{(a-c)\left(7\alpha^{R^2} - 10\alpha^{R} + 7\right) + k_2^{U}\left(-2 + \alpha^{R} - \alpha^{R^2}\left(2 - \alpha_{R}\right)\right)}{12b\left(\alpha^{R^2} - \alpha^{R} + 1\right)}.$$
$$+ \frac{k_1^{U}\left[\alpha^{R}\left(2\alpha^{R^2} - 7\alpha^{R} + 10\right) - 5\right] - \alpha^{R} k_2^{R}\left(1+\alpha^{R}\right)^2}{12b\left(\alpha^{R^2} - \alpha^{R} + 1\right)} \quad (27)$$

To assess the impact of integration these expressions have to be compared to their counterparts in the nonintegrated case, (17) and (18)

When *marginal* delivery costs are the same in both areas so that $k = k_i^{R} = k_i^{U}$, expressions (18) and (27) reduce to

$$X^{I} = \frac{(a-c-k)\left(7\alpha^{R^2} - 10\alpha^{R} + 7\right)}{12b\left(\alpha^{R^2} - \alpha^{R} + 1\right)},$$
$$X = \frac{4(a-c-k)}{9b},$$

so that

$$\text{sign}[X^{I} - X] = \text{sign}\left[\left(5\alpha^{R^2} - 14\alpha^{R} + 5\right)(a-c-k)\right]$$

and

$$X^I < X \text{ iff } \alpha^R > 0.42.$$

In words, the integrated scenario yields a lower level of output and thus a larger price and a lower welfare than the setting with independent actors as long as the share of rural parcels is larger than 42%.

When costs differ and are given by $k^R = k + \Delta_k$ and $k^U = k$, the two relevant expressions are given by

$$X^I = \frac{(a-c-k)\left(7\alpha^{R^2} - 10\alpha^R + 7\right) - \alpha^R\left(1+\alpha^R\right)^2\Delta_k}{12b\left(\alpha^{R^2} - \alpha^R + 1\right)},$$
$$X = \frac{4\left(a-c-k-\alpha^R\Delta_k\right)}{9b}.$$

And we have

$$\begin{aligned}&\text{sign}[X^I - X] \\ &= \text{sign}\left[\left(5\alpha^{R^2} - 14\alpha^R + 5\right)(a-c-k) + \alpha^R\Delta_k\left(13\alpha^{R^2} - 22\alpha^R + 13\right)\right]\end{aligned}$$

Since $\left(13\alpha^{R^2} - 22\alpha^R + 13\right) > 0$, this means that the difference between X^I and X increases with the cost difference, so that the critical level of α^R above which integration reduces welfare is increasing.

Observe that integration has two conflicting effects. First, under plausible conditions it increases the competitor's cost, that is t. To see this use (17) and (26) to show that when costs are equal across delivery areas, $k = k_i^R = k_i^U$, we have

$$\text{sign}[t^I - t] = \text{sign}\left[-\frac{\left[4\alpha^{R^2} - 7\alpha^R + 1\right][a-c-k]}{12\left(\alpha^{R^2} - \alpha^R + 1\right)}\right]$$

so that

$$t^I - t > 0 \quad \text{iff} \quad \left[4\alpha^{R^2} - 7\alpha^R + 1\right] < 0,$$

a condition which is satisfied $\alpha^R > (7-\sqrt{33})/8 \approx 0.15$, that is when the rural area represents more than 15% of deliveries, which we can safely assume. When costs differ so that $k^R = k + \Delta_k$ and $k^U = k$,

$$t^I = \frac{\left(1+\alpha^R\right)(a-c) + k\left(4\alpha^{R^2} - 5\alpha^R + 3\right) + \alpha^R\left(1+\alpha^R\right)\Delta_k}{4\left(\alpha^{R^2} - \alpha^R + 1\right)}$$

so that

$$\begin{aligned} &\text{sign}[t^I - t] \\ &= \text{sign}\left[-\left[4\alpha^{R^2} - 7\alpha^R + 1\right]\left[(a-c) - k\right] + \alpha^R \Delta_k \left[8\alpha^{R^2} - 11\alpha^R + 5\right]\right]. \end{aligned}$$

Since $\left[8\alpha^{R^2} - 11\alpha^R + 5\right] > 0$ then $\alpha^R > (7 - \sqrt{33})/8 \approx 0.15$ remains a sufficient sufficient condition when $\Delta_k > 0$. To sum up, we can assume that irrespective of the cost structure integration increases t. This in turn will have a negative effect on welfare at it will reduce the independent retailer's output. This effect is boosted by the impact t has on the rural delivery cost of the integrated firm. However, integration also reduces the urban delivery cost of the integrated firm which in turn is welfare improving. To be more precise, integration eliminates the double marginalization on the urban segment and this effect is reinforced as k^U increases (Δ_k decreases). Consequently, it is not surprising that the welfare impact of integration depends on the share of rural parcels. Specifically when it is sufficiently large, one can expect the first effect to dominate.

Finally, observe that we have assumed for simplicity that *marginal* delivery costs are larger in the rural area. Alternatively, one could assume that *average* rural costs are larger; this may be due to smaller volumes which imply that the fixed cost per parcel is larger. From that perspective it may actually be the case that *marginal* rural cost are smaller (due to excess capacities). This would not change our analysis except that Δ_k could now be negative which would effectively reinforce our conclusions.

7.3 Numerical illustrations

The following examples illustrate these results. They show cases where "full" integration of a single firm increases welfare (given the number of firms) but where urban integration decreases output and surplus.

Furthermore the examples allow us to compare profits of the integrated firm across the different scenarios. So far we have assume that it is optimal for the integrated firm to integrate urban delivery only. While this is in line with intuition, it is not *a priori* obvious because the rural delivery rate faced by the integrated firm is subject to a markup (it is above the firm's marginal cost). The examples illustrate situations where this is indeed true: the integrated retailer's profits are larger with urban-only integration.[6]

7.3.1 Scenario 1: $k_U = 0.05,\ k_R = 0.1,\ \alpha^R = 0.25,\ c = 0.1,\ p(X) = X^{-1/\varepsilon}$, $\varepsilon = 2$.

The example is based on the demand function with elasticity of 2 already used above.

Scenario	2*2	Integration (1i+1r+1o)	Urban Integration
Total output	11.98	15.37	10.74
Uniform delivery rate t	0.113	0.116	0.156
Total surplus	4.97	5.34	4.89
Profit integrated	–	1.03	1.07
Profit retailer(s)	0.43	0.14	0.05
Profit delivery operator(s)	0.32	0.24	0.47

7.3.2 Scenario 2: $k_U = 0.05$, $k_R = 0.1$, $\alpha^R = 0.25$, $c = 0.1$, $p(X) = X^{-1/\varepsilon}$, $\varepsilon = 1.11$.

This examples revisits the case where the demand elasticity is smaller.

Scenario	2*2	Integration (1i+1r+1o)	Urban Integration
Total output	1.99	2.22	0.65
Uniform delivery rate t	0.19	0.26	0.80
Total surplus	10.39	10.47	9.48
Profit integrated	–	0.53	0.55
Profit retailer(s)	0.24	0.06	0.03
Profit delivery operator(s)	0.13	0.12	0.27

8 Summary and conclusion

We have studies vertical integration of a retailer and an operator in the e-commerce sector. Our main results can be summarized as follows.

First, the comparison between independent oligopoly and integrated monopoly involves a tradeoff between competition and double marginalization which will have the opposite effect. No general result unambiguous result can be obtains. However, we have

[6] These are of course just illustrations. However, try as we might, we did not manage to find a counter-example.

shown that with linear demand we need at least 3 firms (upstream and downstream) for the independent oligopoly to yield larger surplus. With constant elasticity demand, on the other hand, this is always true.

Second, we have considered a setting wherein the number of firms is endogenous and determined such that gross profits cover fixed costs. We have shown that while the integration of a single retailer-delivery operator pair may initially be welfare improving, the resulting market structure may not be sustainable. Furthermore, there exist a range of fixed costs for which the integrated monopoly emerges (following a single integration) and is welfare inferior to the initial independent equilibrium *even when the reduction in the number of fixed costs is taken into account.* Within this setting we have also show that multiple integration is typically welfare superior (for a given total number of firms) to the integration of a single retailer-delivery operator.

Third and last, we have considered an extension incorporating an important feature of the delivery sector, namely that customers differ according to their location, urban or rural, involving different delivery costs. We have shown that urban integration is more likely to have an adverse effect on welfare than full integration. Finally, we have provided examples where the integrated firm finds it indeed beneficial not to deliver in rural areas, even though the operators' delivery rate will include a markup above marginal cost.

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Appendix

A First-order conditions and equilibria in Section 5 with linear and constant elasticity demands

A.1 Linear demand

With the linear demand function (example 1) specified in Section 2, the relevant expressions for the two stages are as follows.

A.1.1 Stage 2

The FOCs are given by

$$a - bX - bx_j - c_j - t = 0,\ j = 2, ..., J. \tag{A1}$$

$$a - bX - bx_1 - c_1 - k_1 = 0 \tag{A2}$$

From $(A2)$, and using $X = X_{-1} + x_1$, one has

$$x_1 = \frac{a - bX_{-1} - c_1 - k_1}{2b}.$$

Moreover, summing $(A1)$ over $j = 2, ..., J$ yields

$$(J-1)\,a - b\,(J-1)\,X - bX_{-1} - \sum_{j=2}^{J} c_j - (J-1)\,t = 0,$$

which after some computations yields:

$$t\,(X_{-1}) = \frac{1}{2}a - \frac{b}{2}X_{-1}\frac{(J+1)}{(J-1)} - \frac{\bar{c}}{(J-1)} + \frac{c_1}{2}\frac{(J+1)}{(J-1)} + \frac{k_1}{2} \tag{A3}$$

With $c_1 = \bar{c}$ and $k_1 = \bar{k}$ this reduces to

$$t\,(X_{-1}) = \frac{1}{2}a - \frac{b}{2}X_{-1}\frac{(J+1)}{(J-1)} - \frac{\bar{c}}{2} + \frac{\bar{k}}{2}$$

A.1.2 Stage 1

Summing (16) over $i = 2, ...I$ yields:

$$t'\,(X_{-1})\,X_{-1} + (I-1)\,t\,(X_{-1}) - \bar{k} + k_1 = 0 \tag{A4}$$

where $t\left(X_{-1}\right)$ is given by $(A3)$ and $t'\left(X_{-1}\right) = -\left(b/2\right)\left(J+1\right)/\left(J-1\right)$. Substituting into $(A4)$ and rearranging yields

$$X_{-1} = \frac{(J-1)}{(J+1)}\frac{(I-1)}{I}\frac{a}{b} - \frac{2}{b}\frac{(I-1)}{I(J+1)}\bar{c} - \frac{2}{b}\frac{(J-1)}{I(J+1)}\bar{k} + \frac{1}{b}\frac{(I-1)}{I}c_1 + \frac{1}{b}\frac{(I+1)}{I}\frac{(J-1)}{(J+1)}k_1.$$

Since

$$x_1 = \frac{a - bX_{-1} - c_1 - k_1}{2b}$$

$$X = X_{-1} + \frac{a - bX_{-1} - c_1 - k_1}{2b}$$

which can be rearranged as

$$X = \frac{2IJ - J + 1}{I(J+1)}\frac{a}{2b} - \frac{1}{b}\frac{(I-1)}{I(J+1)}\bar{c} - \frac{1}{b}\frac{(J-1)}{I(J+1)}\bar{k} - \frac{1}{2b}\frac{1}{I}c_1 + \frac{1}{2b}\frac{(J-2I-1)}{I(J+1)}k_1.$$

With $c_1 = \bar{c}$ and $k_1 = \bar{k}$, this reduces to

$$X^I = \frac{2IJ - J + 1}{I(J+1)}\frac{a}{2b} - \frac{1}{b}\frac{2I + J - 1}{2I(J+1)}\left(\bar{c} + \bar{k}\right).$$

A.2 Constant elasticity demand

A.2.1 Stage 2

Summing (14) over $j = 2...J$ and adding it to (15) yields

$$Jp(X)\left(1 - \frac{1}{J\varepsilon}\right) - J\bar{c} - (J-1)t - k_1 = 0,$$
$$p(X) + p'(X)x_1 - c_1 - k_1 = 0$$

Denoting $X_{-1} = X - x_1$, we thus have the following system of two equations with two unknowns X_{-1} and x_1:

$$Jp\left(X_{-1} + x_1\right)\left(1 - \frac{1}{J\varepsilon}\right) - J\bar{c} - (J-1)t - k_1 = 0 \quad \text{(A5)}$$
$$p\left(X_{-1} + x_1\right) + p'\left(X_{-1} + x_1\right)x_1 - c_1 - k_1 = 0 \quad \text{(A6)}$$

The second equation defines $x_1 \equiv x_1\left(X_{-1}\right)$ with

$$\frac{dx_1}{dX_{-1}} = -\frac{p' + x_1 p''}{2p' + x_1 p''}$$

where $2p' + x_1 p'' < 0$ (SOC) so that

$$t\left(X_{-1}\right) = \frac{J}{J-1}\left(1 - \frac{1}{J\varepsilon}\right) p - \frac{J}{J-1}\bar{c} - \frac{1}{J-1}k_1 \tag{A7}$$

with

$$\begin{aligned} \frac{dt}{dX_{-1}} &= \frac{J}{J-1}\left(1 - \frac{1}{J\varepsilon}\right)\left(1 + \frac{dx_1}{dX_{-1}}\right) p' \\ &= \frac{J}{J-1}\frac{p'}{2p' + x_1 p''}\left(1 - \frac{1}{J\varepsilon}\right) p' \\ &= \frac{J}{J-1}\frac{1}{2 + \frac{x_1 p''}{p'}}\left(1 - \frac{1}{J\varepsilon}\right) p' < 0 \text{ because of SOC} \end{aligned} \tag{A8}$$

Note that

$$\frac{Xp'}{p} = -\frac{1}{\varepsilon}$$

so that

$$Xp' + \frac{p}{\varepsilon} = 0$$

Totally differentiating this w.r.t. X yields:

$$\frac{Xp''}{p'} = -\left(1 + \frac{1}{\varepsilon}\right) \tag{A9}$$

Substituting $(A9)$ into $(A8)$ we obtain

$$\frac{dt}{dX_{-1}} = \frac{J}{J-1}\left(1 - \frac{1}{J\varepsilon}\right)\frac{1}{2 - \frac{x_1}{X}\left(1 + \frac{1}{\varepsilon}\right)} p' \tag{A10}$$

A.2.2 Stage 1

Substituting $(A7)$ and $(A10)$ into (16) we obtain

$$\frac{J}{J-1}\left(1 - \frac{1}{J\varepsilon}\right)\frac{1}{2 - \frac{x_1}{X}\left(1 + \frac{1}{\varepsilon}\right)} p' x_i + \frac{J}{J-1} p\left(1 - \frac{1}{J\varepsilon}\right) - \frac{J}{J-1}\bar{c} - \frac{1}{J-1}k_1 - k_i = 0,\ i = 2, ...I.$$

Summing over $i = 2...I$ yields

$$\frac{J}{J-1}\left(1 - \frac{1}{J\varepsilon}\right) p'(X)\frac{1}{2 - \frac{x_1}{X}\left(1 + \frac{1}{\varepsilon}\right)}\sum_{i=2}^{I} x_i + \frac{(I-1)J}{J-1}\left(1 - \frac{1}{J\varepsilon}\right) p(X) \tag{A11}$$

$$-\frac{(I-1)J}{J-1}\bar{c} - \frac{I-1}{J-1}k_1 - \sum_{i=2}^{I} k_i = 0 \tag{A12}$$

Note that

$$\sum_{i=2}^{I} x_i = X - x_1.$$

Using (A6) this implies

$$\sum_{i=2}^{I} x_i = X - \frac{c_1 + k_1}{p'(X)} + \frac{p(X)}{p'(X)},$$

so that ($A11$) can be rewritten as

$$\frac{J}{J-1}\left(1-\frac{1}{J\varepsilon}\right)p'(X)\frac{1}{2-\frac{x_1}{X}\left(1+\frac{1}{\varepsilon}\right)}(X-x_1)+\frac{(I-1)J}{J-1}\left(1-\frac{1}{J\varepsilon}\right)p(X) \quad \text{(A13)}$$

$$-\frac{(I-1)J}{J-1}\bar{c}-\frac{I-1}{J-1}k_1-\sum_{i=2}^{I}k_i=0. \quad \text{(A14)}$$

Solving for p we obtain

$$p(X)=\frac{\bar{c}+\frac{J-1}{J}\frac{I}{(I-1)}\overline{k}-\frac{J-I}{(I-1)J}k_1}{\left(1-\frac{1}{J\varepsilon}\right)\left[1-\frac{1}{(I-1)\varepsilon}\left(\frac{1-\frac{x_1}{X}}{2-\frac{x_1}{X}\left(1+\frac{1}{\varepsilon}\right)}\right)\right]}$$

which for $k_1 = \bar{k}$ reduces to

$$p^I(X)=\frac{\bar{c}+\overline{k}}{\left(1-\frac{1}{J\varepsilon}\right)\left[1-\frac{1}{(I-1)\varepsilon}\left(\frac{1-\frac{x_1}{X}}{2-\frac{x_1}{X}\left(1+\frac{1}{\varepsilon}\right)}\right)\right]}.$$