Entry into complementary good markets with network effects*

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Abstract

We study whether complementarities can help a firm enter a market with strong network effects and incumbency advantages. We find that bundling the network good with a complementary good, or using the network good as a loss leader (i.e., pricing below marginal cost) can facilitate entry, but that these strategies involve costs that may render them undesirable for the entrant. We also find that the entrant always prefers to make the complementarity general (so that the incumbent benefits from it as well) over having a firm-specific complementarity and using a loss leading strategy. Pricing and product design strategies are interdependent: bundling (unbundled pricing) should be used if and only if the complementarity is specific (general). Finally, we find that bundling may be socially optimal because it allows entrants to challenge incumbents in markets with network effects, thereby expanding the complementary benefits enjoyed by consumers. This finding contrasts with the standard view of regulators, who see bundling as a way to foreclose entry and prevent competition, as in the recent case of the European Commission vs. Google.

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1 Introduction

Antitrust authorities and industrial organization scholars view network effects as one of the most difficult entry barriers to overcome.[1] Google's attempts to become a prominent actor in the social networking, instant messaging, voice call and online storage industries provide excellent examples. Google's entry into the online storage market, for instance, has been hampered by Dropbox's incumbency advantage, which is based on direct network effects: users in its installed base care about the number of users with whom they can share files. Counteracting this disadvantage, Google provides a range of products complementary to online storage, such as picture-editing software (Google Photos) and online office applications (Google Docs), that might give it an edge when competing against Dropbox.

Likewise, Amazon's entry into the online video streaming market has been difficult because of Netflix's incumbency advantage, which is based on indirect network effects (the more users, the more movies Amazon or Netflix can provide, which in turn benefits users). In addition to online video, Amazon gives its Prime subscribers free two-day shipping. The two products are complementary because one-stop shopping allows consumers to save time when they register to consume both products from the same firm. Interestingly, Amazon provides both services as a bundle, but it could have chosen to sell both products separately.

In this paper, we study whether complementarities can help a firm enter a market with strong network effects and incumbency advantages, and determine the welfare effect of the entrant's pricing and product design policies. More precisely, we consider a model with two firms, two markets, and a continuum of consumers who demand at most one unit in each market. One of the firms (entrant) is already active as a monopolist in one of the markets, and is contemplating entry into the market dominated by the other firm (incumbent). The incumbent's market is characterized by strong network effects that make it winner-take-all, and the incumbent benefits from optimistic expectations by consumers, meaning that consumers always coordinate on consuming the incumbent's network good when other coordination possibilities are feasible. Depending on which network good they buy, consumers may enjoy a complementarity benefit when

[1] For example, in their comprehensive study on EU competition law, Jones and Sufrin (2014, p. 90) adhere to the idea that network effects "can create a truly formidable entry obstacle, sufficient to permit prices to persist above competitive level[s] for a substantial period of time without attracting entry." Likewise, McAfee (2005, p. 15) states that network effects are a "major source of barriers to entry," and Besanko *et al.* (2013, p. 381) argue that "in markets with network effects, the first firm that establishes a large installed base of customers has a decided advantage."

they consume the goods in the two markets. Such complementarity is *specific* if consumers enjoy the extra utility only when buying both products from the same firm, and is *general* if the extra utility is independent of which network good they buy.

Specific complementarities arise when consumers have access to additional functions if they use two products provided by the same firm. For example, a user can store pictures online with Google Drive and edit them with Google Photos without downloading them to a computer. If she stores her pictures in Dropbox instead, she would have to download them to a computer before using Google Photos. Specific complementarities may also arise because of transaction-cost savings from one-stop shopping, as in the case of Amazon discussed above. In turn, the model with a general complementarity represents situations in which the provider of the complementary good shares complementarity benefits with competitors. For example, Google could extend its Google Photos's functionality to Dropbox users, so that they could edit pictures directly online.

Our main results in this setting are as follows. We begin by studying settings with a specific complementarity and show that loss leading can be an effective entry mechanism in such cases. The attractiveness of this strategy for the entrant lies on using the network good as a loss leader to enhance demand for the other product it sells. The drawback is that the entrant ends up selling the network good below cost, but such entry costs fall as the specific complementarity benefits rise. For this reason, the specific complementarity should be large enough for the entrant to be willing to engage in loss leading.

We further show when the complementarities are specific that bundling is a powerful but possibly undesirable entry weapon for the entrant. On the one hand, bundling facilitates entry because it commits to more aggressive pricing and induces consumers to coordinate on using the entrant's network good in order to enjoy its complementarity benefits. On the other, bundling prevents the entrant from adapting its pricing to the specifics of each product it sells, which may greatly harm its profitability and induce it not to enter. We find however that bundling is preferred by the entrant over selling the network good at a loss whenever the complementarity is specific.

We also study product design strategies by the entrant, examining whether it would benefit from making the complementarity general, so that it can be enjoyed by consumers even if they buy the incumbent's network good. For a given pricing strategy (*i.e.*, bundled or unbundled pricing), a general complementarity makes entry harder because the entrant gives up its complementarity advantage, but it has the benefit of enhancing the demand for the complementary good even if the entrant prefers to stay out of the network goods market. Overall, we find that a general complementarity is undesirable if the entrant chooses to bundle its two products because it raises the implicit cost of entry using a bundling strategy. However, a general complementarity makes sense in the case of unbundled pricing because it avoids having to engage in (costly) loss leading. Therefore, the only reason why loss leading may be optimal as an entry strategy is because bundling is forbidden and a general complementarity is undesirable owing to outside factors (*e.g.*, informational or technology spillovers).

From a public policy standpoint, our results regarding entry into winner-take-all markets have relevant consequences for practices that are often considered anticompetitive. When the entrant is the only one to exploit the complementarities in another market, the existence of such specific complementarities implies that there is a socially inefficient dominance of the network goods market by the incumbent. Entry is socially desirable and should be encouraged so that consumers can benefit from the complementarity benefits. Indeed, there will be socially insufficient entry even if the entrant is allowed to engage in loss leading or to use bundling because of the entry costs associated with such strategies. The social desirability of bundling as an entry weapon contrasts with the standard view of regulators, who view bundling as a way to foreclose entry and prevent competition.

Another aspect that needs careful consideration when it comes to policy prescriptions is the social desirability of forcing the entrant to make the complementarity general rather than specific, for example, by forcing the entrant to develop open standards or to guarantee interoperability. Such a move has a positive effect on welfare because it guarantees that consumers will enjoy the complementarity benefits. However, if bundling is also used by the entrant, this benefit may be more than offset by the greater production cost borne by the entrant if the incumbent keeps dominance of the network goods market. In addition, the private desirability of bundling diminishes when a complementarity is general rather than specific, which inefficiently decreases coverage of the complementary goods market in these cases. As a result of all these forces, the socially efficient outcome simply involves a comparison of unbundled pricing with a general complementarity and bundled pricing with a specific complementarity.

Our paper contributes to the literature dealing with durable market dominance in network industries. This literature has long recognized that network effects constitute significant barriers to entry, which may be difficult to overcome by potential entrants. As Farrell and Klemperer (2007, p. 1972) explain, "consumers' expectations may naturally focus on established firms, so entry with network effects [is] hard." The focus has been mainly on whether an entrant can outcompete an established firm enjoying network effects thanks to technological improvements, as in Farrell and Saloner's (1986) pioneering analysis. To the best of our knowledge, the only paper explicitly studying entry strategies in markets with network effects is Katz and Shapiro (1992), but they focus on the optimal timing of entry. Our analysis on optimal entry strategies relies on the relevant but so far neglected roles that product complementarity and bundling may play on the replacement of incumbents in network industries.

Our paper also contributes to the literature on the strategic and welfare effects of bundling. Bundling was initially treated as a price discrimination scheme that allows for better surplus extraction from heterogeneous consumers, as pointed out by Adams and Yellen (1976), Schmalensee (1984), and McAfee et al. (1989). The modern treatment of bundling has focused however on its market foreclosure aspects. More precisely, the literature has analyzed whether a multi-product monopolist can use bundling to foreclose access of a single-product rival to one of the markets it serves (see Whinston, 1990, Carlton and Waldman, 2002, and Nalebuff, 2004). We are the first to examine the private and public implications of bundling when one of the bundled goods is a network good, a practice that is worthwhile studying in detail given the increasing importance of network goods in the real world. More precisely, the novelty of our paper is that we consider the practice of bundling complementary products not as a way to shelter existing dominant positions, but as a strategy to undermine dominant positions sustained by strong network effects. More importantly, we find that bundling may be a privately costly but socially desirable entry strategy.

Our paper is also related to the literature investigating compatibility and bundling decisions when consumers demand systems made of complementary components. In our model, products sold in the two markets can be seen as compatible when the complementarity is general, and as incompatible when the complementarity is firm-specific. Matutes and Regibeau (1988, 1992) and Economides (1989) study systems with multiple components and show that firms choose to sell compatible components when all firms in the market offer all components. Denicolò (2000) shows that this conclusion changes if generalist firms that offer all components compete against specialist firms that offer only one component, in which case firms may choose to sell incompatible products. The reason is that, when components are incompatible, the generalist internalizes cross-price effects, so it has an incentive to lower the prices of its components. Specialist firms respond by increasing the price of their components, so that the generalist may benefit from incompatibility if the price rises enough.

As in Denicolò (2000), we show that firms may choose incompatibility, but the reason differs: incompatibility implies that the generalist (the entrant in our paper) has an advantage over the specialist (the incumbent), which may allow it to capture the incumbent's market when strong network effects lead to winner-take-all outcomes. An additional difference with respect to all these papers in the systems compatibility literature is that they assume that all components are essential, and therefore assume that pure bundling is equivalent to incompatibility by definition. In our paper, consumers may refrain from buying the complementary good provided by the entrant, so we can study the desirability of using an unbundled pricing strategy together with incompatibility. However, we show that the entrant uses bundling to facilitate entry, whereas product compatibility makes entry harder, in which case product compatibility and bundling will tend not to be used together. This result gives support to the assumption made by these earlier papers.

2 The baseline model

We consider a game played by two firms labeled 1 and 2 and a continuum of consumers with unit mass. Consumers demand products in two markets labeled A and B. Firm 1 sells product a in market A and product b_1 in market B, whereas firm 2 sells product b_2 in market B. There are no fixed costs of operation for any of the firms. Also, the marginal costs of production of good a are normalized to zero (which means that the price of such a good should be interpreted as a markup), whereas there is a marginal cost $c \geq 0$ of producing b_i, $i \in \{1, 2\}$.

Consumers are willing to consume at most one unit of the product sold in each of the two markets, and they are identical except for their valuations of product a. In particular, the valuation v of a given consumer is an independent draw from a random variable uniformly distributed between 0 and 1. Each consumer privately observes her valuation before buying any good. We also assume that any consumer gets a gross utility u from consuming any of the goods sold in market B, where u is publicly known.

The goods sold in market B exhibit direct network effects. In particular, if $n_i^e \in [0, 1]$ consumers are expected to consume b_i, the expected valuation of a consumer contemplating to purchase such a product increases by $\alpha\, n_i^e$, where $\alpha \geq 0$ is a known parameter that represents the intensity of network effects.[2]

We initially assume that a consumer who consumes both a and b_1 increases her utility by some known fixed amount $\beta \in [0, 1)$, but this increase in utility is not available for

[2]We are assuming that networks are incompatible, so the value of product b_i for consumers depends upon the number of users with whom they can interact by using such product (Farrell and Saloner, 1985; Katz and Shapiro, 1985; Economides, 1996).

a consumer who consumes a and b_2.[3] This corresponds to the cases in which there is a *firm-specific complementarity* between the products sold by firm 1. In Section 5, we will study the case of a *general complementarity.* That is, we will assume that firm 1 extends the complementarity between products a and b_1 to firm 2, so that consumers of products a and b_2 increase their utility by β as well.

Our baseline model examines unbundled pricing (see Section 4 for the analysis of bundling), so let p_a denote the price of good a, and let p_i denote the price of good b_i, $i \in \{1, 2\}$. The utility derived by a consumer who consumes good b_i but not a is

$$U_{b_i} = u + \alpha\, n_i^e - p_i, \tag{1}$$

whereas the total utility generated by consuming a and b_1 is

$$U_{ab_1}(v) = v - p_a + \beta + u + \alpha n_1^e - p_1, \tag{2}$$

and the total utility generated by consuming a and b_2 is

$$U_{ab_2}(v) = v - p_a + u + \alpha n_2^e - p_2. \tag{3}$$

We assume u is large enough so that market B is fully covered. We also assume that the degree of network effects is large, *i.e.* $\alpha \geq 1/2$. We are therefore examining markets in which network effects are intense. We will show that this assumption leads to a winner-take-all outcome in market B. We finally assume that consumers can freely dispose of product b_1 if they are enticed or forced to purchase b_1 when they are instead interested in consuming b_2; in such a case, consumers enjoy neither the complementarity benefit β nor the network benefits of good b_1.

Unless otherwise stated, we consider a two-period game. In the first period, firms set prices. In the second period, having observed all prices, consumers form rational expectations and simultaneously decide which goods to consume. Our solution concept is subgame perfect Nash equilibrium under a refinement that we introduce in the next section.

[3] The case of $\beta \geq 1$ yields no additional insights, but it unnecessarily lengthens proofs and propositions. Results are available upon request.

3 Solution of the baseline model (unbundled pricing with a specific complementarity)

In solving for a subgame perfect Nash equilibrium, we work backwards and begin by considering the second period, in which consumers simultaneously decide which products to purchase after observing prices. We will solve for the set of Nash equilibria of this second-period adoption game for any set of prices, which will enable us to characterize firms' demands. Given a set of prices, it will usually be the case that there exist several Nash equilibria for the second-period adoption game played by consumers. Therefore, demands will typically be correspondences rather than functions, which is a standard feature of settings involving network effects.[4]

Given that we are interested in contemplating firm 1 as an entrant, we will always select the second-period Nash equilibrium with the largest market share for firm 2. Thus, when pricing is such that consumers can coordinate in several ways, all consumers believe that the prevailing equilibrium is the one favoring firm 2. This refinement is meant to capture historical inertia favoring the incumbent's product (*e.g.*, existence of an installed base of users), and its main implication is to handicap entry by firm 1. Such pessimistic beliefs for the entrant are a standard manner to formalize path dependence favoring incumbents in a static model with network effects (see for example the influential paper by Caillaud and Jullien, 2003). The promising developments by Biglaiser and Crémer (2016) and Halaburda et al. (2016) provide truly dynamic models with a similar flavor.

It is worth noting that firms in our setting are assumed to be symmetric except for two aspects. On the one hand, firm 1 is active in the market of a complementary good. On the other hand, firm 2 benefits from optimistic expectations by consumers. As a consequence, the higher the intensity of network effects, captured by parameter α, the more difficult is for firm 1 to enter.[5]

Given p_a, p_1 and p_2, let $n_i^e \in [0, 1]$ denote the number of consumers who are expected to purchase b_i in market B. Letting $n_i \in [0, 1]$ denote the number of consumers who actually purchase b_i in market B, the fact that each consumer is negligible and forms rational expectations implies that, given prices, $n_i^e = n_i$ for all $i \in \{1, 2\}$.Since market B is assumed to be covered, it follows that $n_1 = 1 - n_2$, so we simply need to find out how

[4]Grilo *et al.* (2001) and Griva and Vettas (2011) also present models in which network effects lead to multiple coordination possibilities by consumers. In contrast with our paper, these papers study price competition between single-product firms selling differentiated products.

[5]If such inertia favored entrants, then network benefits would favor entry rather than hinder it.

much demand is generated by firm 2 in market B given the prices charged by both firms.

Any pricing strategy for firm 1 that involves $p_a > 1 + \beta$ is (weakly) dominated by a strategy with $p_a = 1 + \beta$. Indeed, given our assumption on v, no consumer would buy a and b_1 when $p_a > 1 + \beta$, as buying b_1 in isolation would entail a higher level of utility. Similarly, firm 1 cannot be playing any pricing strategy that involves $p_a < 0$ because it is (weakly) dominated by a strategy with $p_a = 0$. Thus, in what follows we consider strategies such that $p_a \in [0, 1 + \beta]$. It is also worth noting that, because consumers can freely dispose of product b_1, it must hold that firm 1 finds it optimal to charge $p_1 \geq 0$.

Appendix A contains the construction of firm 2's demand correspondence for all admissible values of p_2, p_1 and p_a. Such a correspondence is graphically represented in the left panels of Figure 1, in which we distinguish three cases, depending on the value of p_a. Recall that there is no need to plot firm 1's demand correspondence because its demand equals $1 - n_2$.

Given that in case of multiplicity we select the second-period Nash equilibrium most favorable to firm 2, the right panels of Figure 1 represent firm 2's demand function. For example, when $p_1 + p_a - \alpha - \beta < p_2 < p_1 + \alpha - \beta$ in Figure 1's left top panel, our refinement implies that firm 2 must necessarily be capturing the whole network market, as one can see on the correspondent right top panel. Indeed, only when $p_2 > p_1 + \alpha - \beta$ does firm 2 fail to make some sales in this market, all of which are made by firm 1.

In Appendix B, we prove the following result.

Proposition 1 (Unbundled pricing with a specific complementarity). *An equilibrium exists and is unique. The equilibrium is such that:*

(a) If $\alpha < \min\{\beta(6 + \beta)/4, c + \beta\}$, then firm 1 captures the network goods market B ($n_2^{US} = 0$), $p_a^{US} = (1 + \beta)/2$, $p_1^{US} = c + \beta - \alpha$, $p_2^{US} = c$, $\pi_1^{US} = \beta - \alpha + (1 + \beta)^2/4 > 0$, and $\pi_2^{US} = 0$. Social welfare equals $w^{US} = u + \alpha - c + 3(1 + \beta)^2/8$, and firm 1 uses good b_1 as a loss leader in equilibrium ($p_1^{US} < c$) if and only if $\beta < \alpha$.

(b) If $\alpha \geq \min\{\beta(6 + \beta)/4, c + \beta\}$, then firm 2 maintains its dominance of the network goods market B ($n_2^{US} = 1$), $p_a^{US} = 1/2$ and $\pi_1^{US} = 1/4$. Firm 1 uses good b_1 as a loss leader, but has no sales of this good in equilibrium. In addition:

(b1) If $c < \beta(2 + \beta)/4$, then $p_1^{US} = 0$, $p_2^{US} = \alpha - \beta$, and $\pi_2^{US} = \alpha - \beta - c \geq 0$. Social welfare equals $w^{US} = u + \alpha - c + 3/8$.

(b2) If $c \geq \beta(2 + \beta)/4$, then $p_1^{US} = c - \beta(2 + \beta)/4 \geq 0$, $p_2^{US} = c + \alpha - \beta(6 + \beta)/4$, and $\pi_2^{US} = \alpha - \beta(6 + \beta)/4 > 0$. Social welfare equals $w^{US} = u + \alpha - c + 3/8$.

Superscript "US" indicates that we are considering the equilibrium values for the case with unbundled pricing and a specific complementarity.

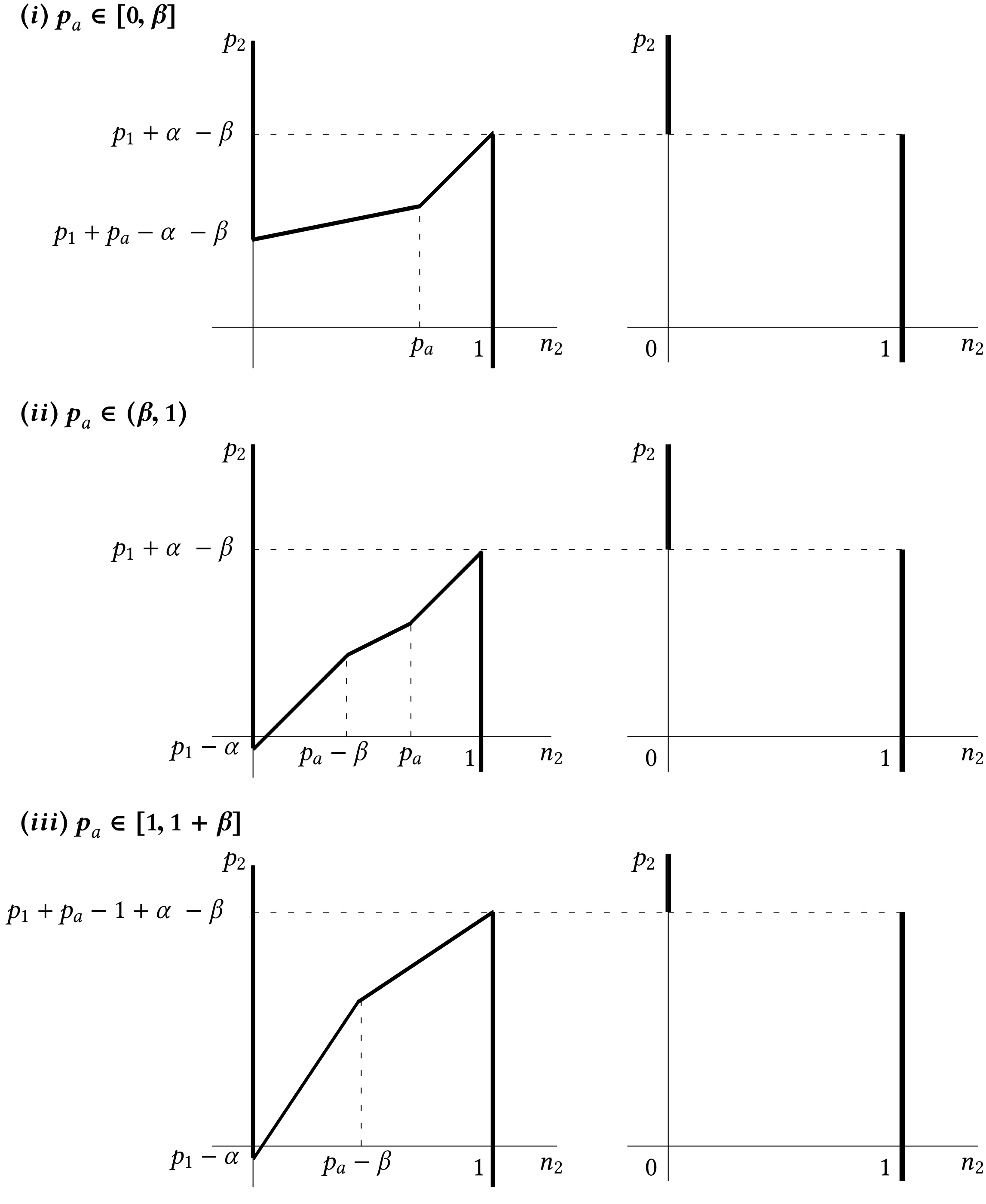


Figure 1: Firm 2's demand correspondence (left panels) and demand function with the equilibrium refinement (right panels) for different values of p_a

Proposition 1 shows that firm 1's equilibrium profit (weakly) increases with β and (weakly) decreases with α, whereas the converse holds for firm 2. This result is a direct consequence of firm 1 exclusively enjoying the complementarity benefit and of firm 2 exclusively enjoying the incumbency advantage with regards to network effects. Moreover, social welfare is maximized when firm 1 captures the network goods market B. This can be explained by the fact that not only do more consumers consume good a, but also that those consumers purchasing both goods benefit from the complementarity benefit.

The top panels of Figure 2 represent the subset of parameters for each type of equilibrium in Proposition 1.[6] As it can be readily seen in the figure, for given values of α and c, there exists a threshold level of β such that the equilibrium has $n_2^{US} = 1$ for values of β below the threshold, and $n_2^{US} = 0$ for values of β above the threshold. This threshold increases with α, which is natural given firm 2's incumbency advantage, and shifts inwards as c grows, which is also natural because c affects firm 2 more than firm 1, given that the latter is active in market A as well as in market B.

Part (a) of Proposition 1 shows that, given α and c, firm 1 enters the network goods market if β is large enough, an intuitive result. Perhaps more surprising is the mechanism at play, since the complementarity benefit β plays a dual role: (i) it enables firm 1 to price firm 2 out of market B; and (ii) it also enables firm 1 to raise its price in market A because of the complementarity between a and b_1. Indeed, the latter role interacts with the former: the strong position of firm 1 in market A induces it to compete more fiercely in market B so as to exploit the complementarity benefit, thereby using b_1 as a loss leader to expand good a's market. In fact, $p_1^{US} - c = \beta - \alpha$ is negative when β is slightly larger than the threshold value that delineates whether firm 1 conquers market B. As the complementarity benefit β grows, firm 1 can increase b_1's markup, and, when it exceeds α, b_1's markup becomes positive: its advantage over firm 2 is so strong that it does not need to incur a cost to make its monopolistic position in market A more valuable. The shaded areas in the bottom panels of Figure 2 represent the parametric values for which firm 1 uses b_1 as a loss leader and captures the network goods market (given c, these are the values of β and α such that $\beta \leq \alpha < \min\{\beta(6+\beta)/4, c+\beta\}$).

Part (b) of Proposition 1 shows that, given α and c, firm 2 maintains its dominance

[6] Based on our assumption on α, in the vertical axis of Figure 2 we consider $\alpha \geq 1/2$. As a consequence, $\beta(6+\beta)/4 > 1/2$ when $\beta > \sqrt{11} - 3 \simeq 0.33$. Note that $\beta(6+\beta)/4 > c + \beta$ when $\beta > \sqrt{4c+1} - 1$ (or $c < \beta(\beta+2)/4$, as reported in Proposition 1), and vice versa. As $\alpha \geq 1/2$ and $\beta \in [0, 1)$ by assumption, it holds that $\sqrt{4c+1} - 1 \geq 1$ if $c \geq 3/4$ and $\sqrt{4c+1} - 1 > \sqrt{11} - 3$ if $c \geq 7/2 - \sqrt{11} \simeq 0.18$. It follows that $c + \beta < \beta(6+\beta)/4$ when $c \in (0, 0.18]$, whereas $c + \beta \geq \beta(6+\beta)/4$ when $c \geq 3/4$. These cases are respectively represented in the left and right panels of Figure 2. When $c \in (0.18, 3/4)$, the two threshold values of α intersect in $\beta = \sqrt{4c+1} - 1$, as it can be seen in the central panels of Figure 2.

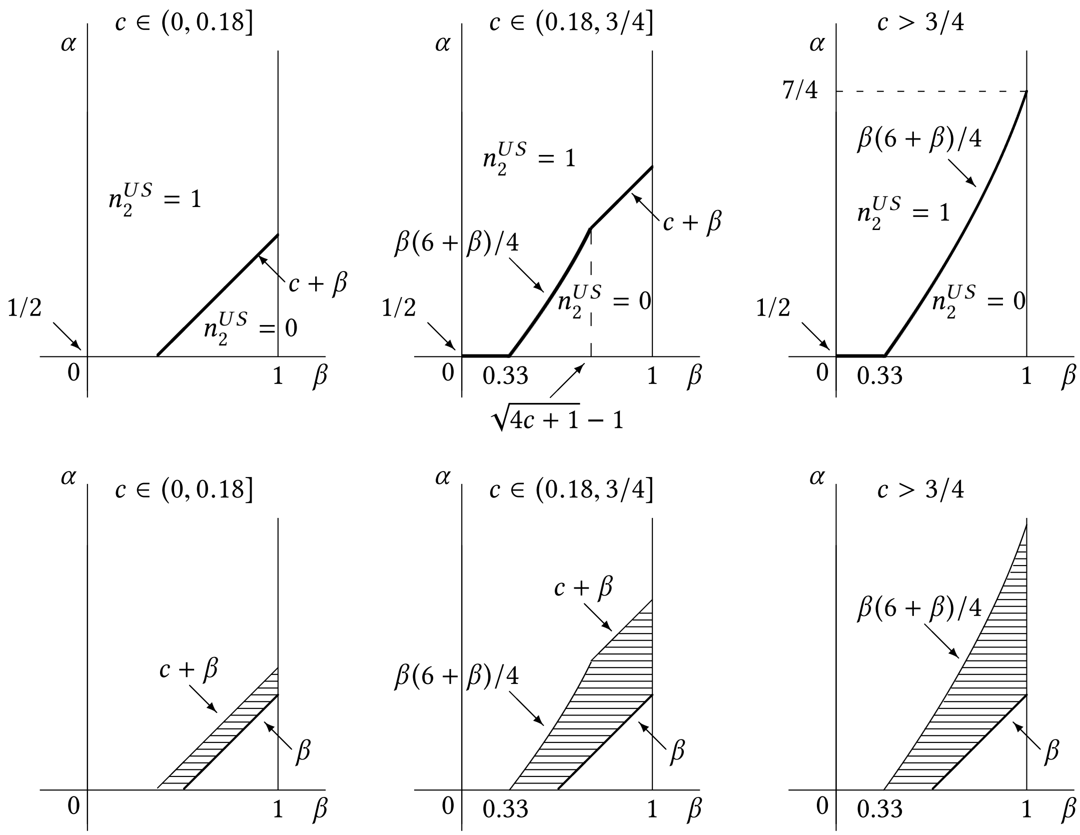


Figure 2: Equilibrium market shares (top panels) and successful entry with loss leading (shaded area in bottom panels)

of market B when β is sufficiently small. In this case, firm 1 is induced to price b_1 at its perceived marginal cost, which is its actual marginal cost c in market B minus the profit increase in market A associated with conquering market B, namely $(1+\beta)^2/4 - 1/4 = \beta(2+\beta)/4$. This last component captures firm 1's willingness to engage in loss leading in market B. On the one hand, when such component is more prominent than c (i.e., $c < \beta(2+\beta)/4$, case (b1)), the fact that p_1^{US} cannot be negative because of free disposal implies that $p_1^{US} = 0$. As a result, firm 2's price always falls with β because those consumers purchasing firm 1's good in market A also weigh whether to purchase the other good sold by firm 1 or to acquire that of firm 2. Because this comparison depends on β, and greater values of β makes firm 2's good relatively less appealing, firm 2 is led to decrease its price as β grows. On the other hand, when $c \geq \beta(2+\beta)/4$ (case (b2)), it holds that $p_1^{US} = c - \beta(2+\beta)/4 \geq 0$, so firm 2's price falls with β for the same reason as in case (b1) as well as because firm 1's incentive to engage in loss leading in market B becomes more intense as β grows. Even though such a loss leading strategy is ineffective, it constraints the equilibrium price of firm 2, which decreases with the degree of complementarity. Note that in case (b) firm 1 could use a more aggressive loss leading approach to conquer market B, but it refrains from doing so because the cost would be too high in comparison with the market expansion benefits that it brings.

A final aspect worth highlighting regarding Proposition 1 is that it shows that firm 1 has no incentives to use good a as a loss leader. The point is that lowering p_a does not expand the demand for good b_1, as can be easily seen in Figure 1.

4 Bundling with a specific complementarity

In the previous section firm 1 charged separate prices for a and b_1. Another option is to consider the adoption of a bundling strategy. In subsection 4.1, we shall examine the consequences of having firm 1 charge a single price for the bundle of the two products it sells (pure bundling). In subsection 4.2, we shall examine the implications of having firm 1 charge a price in each of the markets in which it operates together with a discount offered to those consumers who purchase both of its products (mixed bundling). We will show that any equilibrium arising under mixed bundling is characterized by the same profit or less for firm 1 than the unique equilibrium of a game in which it chooses between unbundled pricing and pure bundling. Therefore, from firm 1's standpoint, there is no loss of generality in restricting our analysis to the decision between unbundled pricing and pure bundling. Regardless of whether bundling is pure or mixed, recall that consumers can freely dispose of product b_1 should they be interested in consuming b_2.[7] Also, we continue to give an incumbency advantage to firm 2 in the second period of this variant of the baseline game.

4.1 Pure bundling with a specific complementarity

Let p be the price of a bundle composed by one unit of a and one unit of b_1. Because product b_1 can be disposed of at no cost, a consumer may buy the bundle of firm 1 together with good b_2, and choose to use products a and b_2, thus forgoing the complementarity

[7] In the baseline model, free disposal has the effect of ruling out prices for b_1 below 0, since consumers could otherwise pocket firm 1's money, throw away b_1, and then purchase b_2. With pure bundling, free disposal limits firm 1's ability to enter the network market. Without free disposal, a consumer with a high valuation for a would be forced to acquire and *use* b_1, given that firm 1 offers no other possibility. It would be therefore easier for firm 1 to gain market shares in the network market. On the contrary, with free disposal such an advantage is reduced because the consumer who buys the bundle may choose to dispose of good b_1 and use good b_2 instead. In our analysis, we therefore consider the most difficult scenario for firm 1 to enter.

and network benefits of good b_1. The utility generated by consuming a and b_1 is

$$U_{ab_1}(v) = v - p + u + \alpha n_1^e + \beta, \tag{4}$$

whereas the utility generated by consuming a and b_2 is

$$U_{ab_2}(v) = v - p + u + \alpha n_2^e - p_2. \tag{5}$$

and the utility generated by consuming only b_2 is

$$U_{b_2} = u + \alpha n_2^e - p_2. \tag{6}$$

Appendix C shows how to find out firm 2's demand correspondence, together with its graphical representation. Figure 3 represents the demand function that results from using the equilibrium refinement described in Section 3.

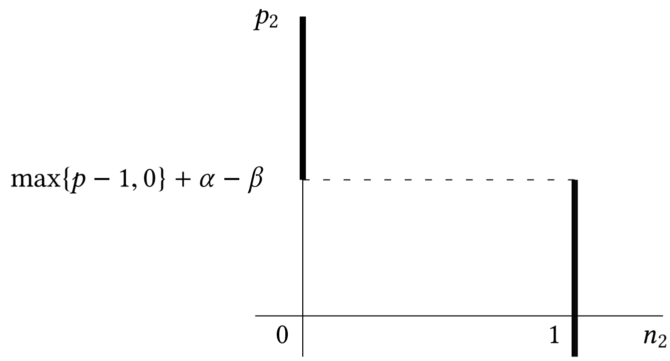


Figure 3: Demand function with pure bundling and a specific complementarity

The most noteworthy feature of the demand function illustrated in Figure 3 is that firm 2's sales do not vary with p if $p \leq 1$. On the one hand, if $p > 1$, it holds that $U_{ab_2}(v) < U_{b_2}$ for all v, so consumers do not consider buying b_2 together with firm 1's bundled goods. Thus, the relevant utility comparison for a consumer with valuation v for good a is U_{b_2} versus $U_{ab_1}(v)$, a comparison that depends on p. Consequently, firm 2's demand depends on p for $p > 1$. If $p \leq 1$, on the other hand, consumption of b_2 alone does not dominate consumption of a and b_2 for all v, and the comparison between $U_{ab_1}(v)$ and $U_{ab_2}(v)$ does not depend on p. Because of firm 2's incumbency advantage, consumers choose to coordinate on $n_2 = 0$ only if $p_2 > \alpha - \beta$, and they choose to coordinate on $n_2 = 1$ otherwise. Thus, firm 2's demand does not depend on p if $p \leq 1$.

It is straightforward to characterize equilibria with the aid of Figure 3. When $\alpha \geq c+\beta$, firm 2 can always guarantee conquering market B by charging price $p_2 = \alpha - \beta \geq c$. Provided $p < 1$, firm 2 has no incentive to deviate, whereas firm 1 best responds to such pricing by focusing on market A and charging the monopoly price in this market given marginal cost c, so $p = (1 + c)/2 < 1$ as long as $c < 1$. Therefore, a unique equilibrium exists when $\alpha \geq c + \beta$ and $c < 1$, and it is such that $n_2 = 1$, with profits for firms 1 and 2 equal to $\pi_1 = (1 - c)^2/4$ and $\pi_2 = \alpha - \beta - c$.

It remains to study whether we can have an equilibrium with $n_2 = 1$ and $p \geq 1$. In such a case, firm 2 should be charging price $p_2 = \alpha - \beta + p - 1$. Firm 1 should have no incentive to slightly reduce p and get all the demand. Thus, it should hold that $p - \epsilon - c < 0$ for any arbitrarily small $\epsilon > 0$, so we should have $p = c$ and $p_2 = \alpha - \beta + c - 1$, with $1 \leq p = c$ and $\alpha \geq 1 + \beta$ in order for to have an equilibrium such that $n_2 = 1$ and $p \geq 1$.

By an analogous argument, firm 2 cannot profitably capture market B when $\alpha < \min\{1, c\} + \beta$. Indeed, firm 2 must be optimally charging $p_2 = c$ if an equilibrium exists. Given such pricing, firm 1 must be charging some $p \leq 1 + \beta - \alpha + c$ so that it serves all consumers (in both markets). It follows that $p = 1 + \beta - \alpha + c \geq 1$ in order for firm 1 to maximize profit. Because $\alpha < \min\{1, c\} + \beta$, we have that the unique equilibrium is such that $n_2 = 0$, with profits for firms 1 and 2 equal to $\pi_1 = 1 + \beta - \alpha \geq 0$ and $\pi_2 = 0$.

The following proposition summarizes these findings.

Proposition 2 (Pure bundling with a specific complementarity). *An equilibrium exists and is unique. The equilibrium is such that:*

(a) If $\alpha < \min\{1, c\} + \beta$, then firm 1 captures the network goods market B ($n_2^{PS} = 0$), $p^{PS} = 1 + \beta - \alpha + c$, $p_2^{PS} = c$, $\pi_1^{PS} = 1 + \beta - \alpha$, and $\pi_2^{PS} = 0$. In equilibrium, no consumer who buys the bundle disposes of good b_1.

(b) If $\alpha \geq \min\{1, c\}+\beta$, then firm 2 maintains its dominance of the network goods market B ($n_2^{PS} = 1$), $p^{PS} = \max\{c, (1+c)/2\}$, $p_2^{PS} = \alpha - \beta + c - \min\{1, c\}$, $\pi_1^{PS} = (\max\{0, (1-c)/2\})^2$, and $\pi_2^{PS} = \alpha - \beta - \min\{1, c\}$. In equilibrium, all consumers who buy the bundle dispose of good b_1.

Superscript "*PS*" indicates that we are considering the equilibrium values for the case with pure bundling and a specific complementarity.

All consumers purchase some good in market B. When $\alpha < \min\{1, c\} + \beta$, the fact that good b_2 is (rationally) expected to yield only weak network benefits implies that all consumers want to acquire good b_1 even if b_2 is sold at its marginal cost. Because acquiring good b_1 requires purchasing a as well, pure bundling implies that firm 1 is able to effectively capture all demand when the complementarity benefit β is large enough to offset firm 2's incumbency advantage, captured by α. As in Proposition 1, the greater α is, the greater the complementarity benefit needs to be in order for firm 1 to conquer the network market. However, given that $\min\{\beta(6+\beta)/4, c+\beta\} \leq \min\{1, c\} + \beta$, it follows from Propositions 1 and 2 that pure bundling makes it easier for firm 1 to successfully attack the network market.

The dotted area in Figure 4 represent the parametric region for which firm 1 fails to enter with unbundled pricing whereas it successfully captures the network goods market with bundling (*i.e.*, $n_2^{US} = 1$ and $n_2^{PS} = 0$). Such a region appears when $c \geq \beta(2+\beta)/4$ and it enlarges with c.[8] The following explains why greater c makes it harder for firm 2 to successfully defend market B when firm 1 uses pure bundling over unbundled pricing. When firm 2 keeps market B under unbundled pricing, its markup/profit does not vary with c if $c \geq \beta(2+\beta)/4$, as we know from Proposition 1. This is because consumers choose between firms based on differences in prices, thus it is the cost differential that matters when firms compete for consumers in market B; increasing c has no effect on how easy or how hard it is for firm 1 to successfully conquer the network goods market. However, when firm 2 keeps market B under pure bundling, its markup/profit is (weakly) decreasing in c because, as discussed earlier (see Figure 3), its appeal to consumers does not depend on the bundle price (and hence on firm 1's marginal cost); increasing c in these cases makes it easier for firm 1 to successfully conquer market B.

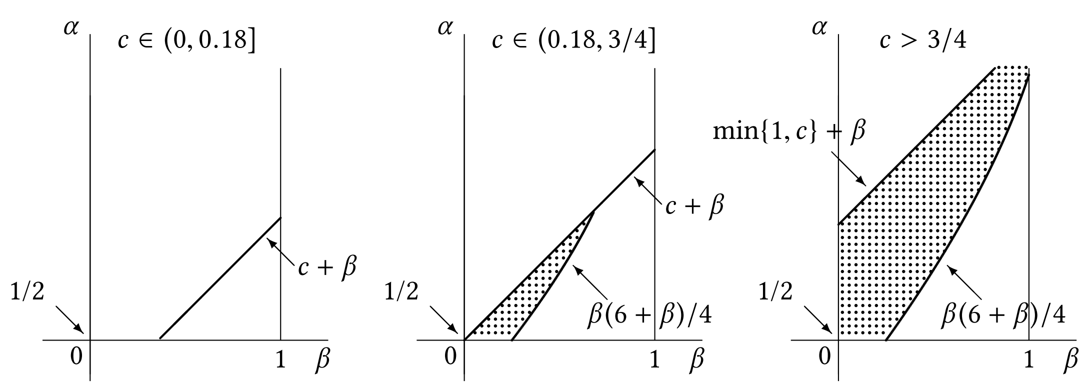


Figure 4: Region of parameters for which pure bundling enables firm 1 to enter with a specific complementarity whereas unbundled pricing does not (dotted area)

Entry is easier because pure bundling allows firm 1 to extend its market power in market A onto market B thanks to the complementarity that exists between a and b_1.

[8] Recall $\alpha > 1/2$ and $\beta \in (0,1)$. Thus, $\min\{\beta(6+\beta)/4, c+\beta\} = \min\{1, c\} + \beta$ when $c < \beta(2+\beta)/4 < 3/4$.

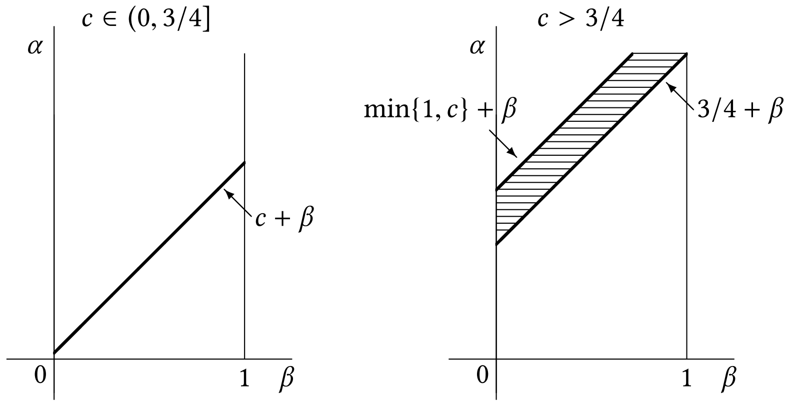


Figure 5: Region of parameters for which firm 1 prefers unbundled pricing without entry over bundled pricing with entry, when complementarity is specific (dashed area)

However, this advantage comes at a cost, because firm 1 loses its ability to adapt its pricing to the specifics of market A.[9] Only when the complementarity gain β is sufficiently high does such a disadvantage become relatively less relevant, as consumers buy the two products sold by firm 1 even if their stand-alone valuation for product a is very small. It follows that firm 1 prefers pure bundling over unbundled pricing if and only if β is large enough. Indeed, because $(\max\{0, (1-c)/2\})^2 \leq 1/4$, $1 + \beta - \alpha > \beta - \alpha + (1+\beta)^2/4$, and $1 + \beta - \alpha \geq 1/4$ if and only if $\alpha \leq 3/4 + \beta$, the following result follows from comparing Propositions 1 and 2.

Proposition 3 (Private desirability of pure bundling with a specific complementarity). *Pure bundling is preferred by firm 1 over unbundled pricing if and only if* $\alpha \leq \min\{3/4, c\} + \beta$.

Even though pure bundling makes it easier to enter the network goods market, firm 1 may adopt unbundled pricing instead, precisely because of the cost required in order to successfully enter. Indeed, Propositions 2 and 3 show that there exists an interval region where firm 1 prefers unbundled pricing without entry rather than pure bundling with entry. This happens when $\min\{3/4, c\} + \beta < \alpha \leq \min\{1, c\} + \beta$, which is possible only if $c > 3/4$. The dashed area in Figure 5 illustrates this parametric region.

Having examined the incentives for firm 1 to use pure bundling, a natural question to investigate is under what conditions does pure bundling result in greater social welfare

[9] In this sense, pure bundling can be interpreted as selling b_1 at price p and giving away a to any consumer who buys b_1, regardless of her realized valuation for good a.

than unbundled pricing. When firm 1 cannot conquer market B in Proposition 2, it holds that pure bundling is socially inefficient because firm 1 bears a production cost c for each unit of good a consumed (recall that b_1 is thrown away by consumers in such a case), so bundling artificially creates production costs for good a relative to no bundling.[10] When pure bundling does allow firm 1 to conquer market B (*i.e.*, when $\alpha < \min\{1, c\} + \beta$), social welfare equals

$$w^{PS} = \int_0^1 (v - c + u + \alpha + \beta)dv = u + \alpha - c + \frac{1}{2} + \beta,$$

which is greater than that generated when firm 1 does not bundle a and b_1. Pursuit of entry in market B forces firm 1 to charge a low price for the bundle, such that it cannot exclude any consumer in market A. Firm 1 effectively kills its power to control pricing in market A as a stand-alone monopolist would, which enhances social welfare.

Proposition 4 (Social desirability of pure bundling with a specific complementarity)**.** *Pure bundling results in greater social welfare than unbundled pricing if and only if* $\alpha < \min\{1, c\} + \beta$.

By comparing the private incentives for firm 1 to adopt pure bundling (Proposition 3), and the welfare impact of such a decision (Proposition 4), we obtain the following result.

Corollary 1 (Private vs. social desirability of pure bundling with a specific complementarity)**.** *If* $\alpha \leq \min\{3/4, c\} + \beta$*, firm 1 opts for pure bundling, which is socially efficient. If* $\alpha > \min\{1, c\} + \beta$*, firm 1 prefers unbundled pricing to pure bundling, which is also socially efficient. However, if* $\alpha \in (\min\{3/4, c\} + \beta, \min\{1, c\} + \beta]$*, firm 1 prefers unbundled pricing to pure bundling, whereas social welfare is higher under pure bundling. This last result necessarily requires that* $c > 3/4$.

The area in which private and social incentives are not aligned coincides with the area in which firm 1 prefers unbundling pricing without entry to pure bundling with entry. This area is again the dashed one represented in Figure 5. In terms of total welfare, unbundled pricing increases firm 1's profit (relative to bundling), but such a positive effect cannot compensate for the lower consumption of good a and the lower utility attained by consumers when buying such a good, given that they cannot enjoy the complementarity benefit β.

[10] Additional calculations are available upon request. The precise expression of social welfare for the case in which firm 1 cannot enter into market B ($\alpha \geq \min\{1, c\} + \beta$) is available in Appendix H.

4.2 Mixed bundling with a specific complementarity

With mixed bundling, firm 1 offers consumers three choices: they can buy product a alone and pay a price p_a, they can buy product b_1 and pay a price p_1, or they can buy the bundle formed by products a and b_1 at a discount price of $p_a + p_1 - d$, where d is an endogenously chosen discount such that $d \geq 0$. Because of free disposal, consumers may buy a, b_1 and b_2 with the aim of using a and b_2, in order to obtain an effective discount equal to $d - p_1$ (provided $d > p_1$, of course). In that case, however, consumers forgo the complementarity benefit $\beta \geq 0$ and the network benefits provided by good b_1.

The utility derived by consuming good b_i is again given by:

$$U_{b_i} = u + \alpha n_i^e - p_i. \tag{7}$$

With mixed bundling, the utility generated by consuming a and b_1 is instead given by:

$$U_{ab_1}(v) = v - p_a + \beta + d + u + \alpha n_1^e - p_1, \tag{8}$$

whereas the utility generated by consuming a and b_2 is

$$U_{ab_2}(v) = v - p_a + u + \alpha n_2^e - p_2. \tag{9}$$

Finally, the utility generated by purchasing all goods and disposing of good b_1 is

$$U_{ab_1b_2}(v) = v - p_a + d + u + \alpha n_2^e - p_1 - p_2. \tag{10}$$

As in the previous sections, we assume u is large enough, so that market B is fully covered. In Appendix D, we show how to obtain both the demand correspondence and the demand function that result from firm 2's incumbency advantage. In Appendix E, we conduct the equilibrium analysis, and arrive at the following result.

Proposition 5 (Mixed bundling with a specific complementarity)**.** *An equilibrium exists. Any equilibrium with mixed bundling yields the same profit or less for firm 1 than the (unique) equilibrium of a game in which firm 1 chooses between unbundled pricing and pure bundling before competing with firm 2.*

In principle, mixed bundling offers firm 1 a much wider choice of pricing strategies. For example, unbundled pricing arises if $d = 0$, whereas pure bundling arises for instance if $p_a = 1$ and $d > 0$ (the only reason why good a would be purchased is because b_1 is purchased as well). Despite the rich pricing afforded by mixed bundling, Proposition 5 reveals that, if firm 1 can choose between unbundled pricing and pure bundling, adding the possibility of choosing mixed bundling before competing with firm 2 does not bring extra profit to firm 1. Thus, there is no loss of generality in restricting our analysis to firm 1's decision between unbundled pricing and pure bundling.

5 General complementarity

The previous two sections highlighted the crucial role played by specific complementarities in order to successfully penetrate a network goods market in which an incumbent enjoys favorable expectations. The purpose of this section is to study whether the entrant may have an incentive to share its complementarity with the incumbent, so that consumers who purchase any of the goods in market B increase their utility by $\beta \geq 0$ if they purchase good a as well. We successively examine the cases of unbundled pricing and pure bundling, both with a general complementarity.

5.1 Unbundled pricing with a general complementarity

In the case of unbundled pricing, given a price p_i for product b_i and that n_i^e consumers are expected to purchase such a product, the utility derived by any consumer who consumes such a good is

$$U_{b_i} = u + \alpha n_i^e - p_i, \tag{11}$$

whereas the total utility generated by consuming a and b_i is

$$U_{ab_i}(v) = v - p_a + \beta + u + \alpha n_i^e - p_i. \tag{12}$$

Appendix F shows both how to construct and how to represent firm 2's demand correspondence. Figure 6 displays firm 2's demand function once the second-period equilibrium refinement is used.

Let us first try to sustain $n_2 = 1$ in equilibrium. Clearly, firm 1 must be charging the monopoly price in market A given that customers' utilities increase by β, so

$$p_a = \underset{0 \leq \widehat{p}_a \leq 1+\beta}{\arg\max} \left\{ \widehat{p}_a \ \min\{1, 1 + \beta - \widehat{p}_a\} \right\} = (1 + \beta)/2, \tag{13}$$

and it must be earning

$$\pi_1 = (1 + \beta)^2/4. \tag{14}$$

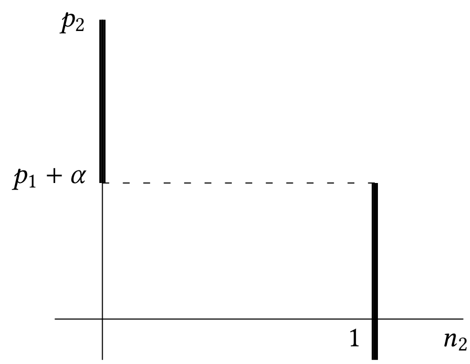


Figure 6: Firm 2's demand function in the unbundled pricing game with a general complementarity

Also, firm 2 must be charging the highest possible price such that its demand equals 1. If firm 1 is charging p_1 in market B, then firm 2 must be charging $p_2 = p_1 + \alpha$ (see Figure 6). Firm 1's optimal deviation is to charge $\widehat{p_1} = p_1 - \epsilon$ so as to attract all consumers in market B, keeping at the same time the price charged in market A as fixed. In order for such a deviation not to be profitable, we must have $p_1 = c$, which implies that $p_2 = c + \alpha$. A similar argument can be used to show that there cannot be an equilibrium in which $n_2 = 0$. Taking into account that the social welfare coincides with the one generated when firm 1 conquers the network market in Proposition 1, we have the following result.

Proposition 6 (Unbundled pricing with a general complementarity). *An equilibrium exists and is unique. Firm 2 maintains its dominance in the network goods market B ($n_2^{UG} = 1$), $p_a^{UG} = (1 + \beta)/2$, $p_1^{UG} = c$ and $p_2^{UG} = c + \alpha$. Firm 1 gains $\pi_1^{UG} = (1 + \beta)^2/4$, and firm 2 gains $\pi_2^{UG} = \alpha$. Social welfare equals $w^{UG} = u + \alpha - c + 3(1 + \beta)^2/8$.*

Superscript "*UG*" indicates that we are considering the equilibrium values for the case with unbundled pricing and a general complementarity.

Relative to Proposition 1, competition in market B is relaxed and firm 1 can always incorporate the complementarity benefit into its market A pricing, even though it cannot hope to conquer market B because of firm 2's incumbency advantage. In light of Propositions 1 and 6, we are now in a position to find out whether firm 1 prefers a specific or a general complementarity for the case of unbundled pricing.

Suppose first that $\alpha \geq \min\{\beta(6+\beta)/4, \beta+c\}$ so that both $n_2^{UG} = 1$ and $n_2^{US} = 1$. In this case, firm 1 cannot conquer market B regardless of its choice between specific and general complementarity, but it can capture part of the surplus created by the complementarity if it chooses to share it with firm 2. Thus, firm 1 always finds it optimal to share the complementarity, *i.e.* $\pi_1^{UG} > \pi_1^{US}$. Now consider the case in which $\alpha < \min\{\beta(6+\beta)/4, \beta+c\}$, so that $n_2^{UG} = 0$ under a specific complementarity. In this case, firm 1 can conquer market B by adopting a specific complementarity, but in so doing it will lose out when it uses product b_1 as a loss leader (see bottom panels of Figure 2). Indeed, it is immediate to notice that $\pi_1^{UG} > \pi_1^{US}$ when $\alpha > \beta$. Therefore, firm 1 will find it optimal to keep the complementarity to itself if and only if $\alpha < \beta$. In terms of social welfare, general complementarity is (weakly) preferred, being strictly higher than specific complementarity when $\alpha \geq \min\{\beta(6+\beta)/4, \beta+c\}$. When $\alpha < \min\{\beta(6+\beta)/4, c+\beta\}$, both alternatives provide the same level of total welfare. We have then proven the following result.

Proposition 7 (Specific vs. general complementarity with unbundled pricing)**.** *In the case of unbundled pricing, firm 1 prefers to maintain a specific complementarity over sharing it with its rival if and only if* $\alpha < \beta$*. This choice also ensures the highest level of total welfare.*

Proposition 7 shows that firm 1 may prefer to share its complementarity with its rival, even though this means it will not be able to capture market B. In particular, loss leading allows firm 1 to enter market B with a specific complementarity, but this strategy is costly in terms of profits, and therefore firm 1 will prefer to renounce to market B instead of engaging in loss leading. This also explains why a general complementarity is preferred in terms of total welfare when $\alpha \geq \min\{\beta(6+\beta)/4, c+\beta\}$, as in such parametric region firm 1 would not enter if the complementarity were specific.

5.2 Compatibility vs. complementarity

A natural question to ask is what happens if the complementarity we have just analyzed arises because customers have access to any network regardless of which good in market B they consume. In case of such compatibility between the networks of firms 1 and 2, someone who buys either b_1 or b_2 expects to increase her utility by $\alpha(n_1^e + n_2^e) = \alpha$ and purchasing good a in addition further increases her utility by β. This situation corresponds to suppressing the network effect (*i.e.*, setting $\alpha = 0$) in the above analysis.[11] Therefore, compatibility harms firm 2 without bringing an extra profit to firm 1. If firm 2 could refuse it, and doing so also implied forgoing enjoying the complementarity benefit β, it would hold that firm 2 would not hesitate in remaining incompatible and thus keep its incumbency advantage. Even though this kind of compatibility offer by firm 1 would

[11]Note that the assumption that $\alpha \geq 1/2$ was actually not used in any part of the analysis we performed, so one can set $\alpha = 0$ directly.

be a poison pill for firm 2, Propositions 1 and 6 imply that firm 2 would be happy to instead accept an offer to enjoy the complementarity benefit β.

5.3 Pure bundling with a general complementarity

In this subsection, we consider the case in which firm 1 bundles goods a and b_1 under the assumption that the complementarity is general. Letting p denote the bundle price, the utility generated by consuming a and b_1 is

$$U_{ab_1}(v) = v - p + u + \alpha n_1^e + \beta, \tag{15}$$

whereas the utility generated by consuming a and b_2 is

$$U_{ab_2}(v) = v - p + \beta + u + \alpha n_2^e - p_2. \tag{16}$$

Finally, the utility generated by consuming only b_2 is

$$U_{b_2} = u + \alpha n_2^e - p_2. \tag{17}$$

Appendix G shows how to obtain firm 2's demand correspondence, and Figure 7 shows the demand function that results from using the equilibrium refinement described in Section 3.

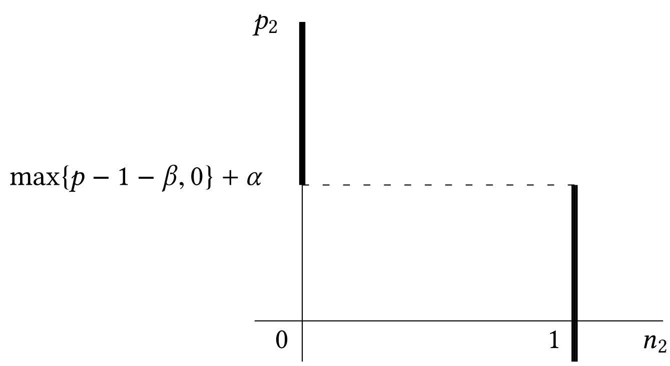


Figure 7: Demand function with bundling and general complementarity

It is straightforward to characterize equilibria with the aid of Figure 7. When $\alpha \geq c$, firm 2 can always guarantee conquering market B by charging price $p_2 = \alpha \geq c$. Provided $p \leq 1 + \beta$, firm 2 has no incentive to deviate, whereas firm 1 best responds to such pricing by focusing on market A and charging the monopoly price in such a market given marginal cost c, so $p = (1 + \beta + c)/2 \leq 1 + \beta$ as long as $c \leq 1 + \beta$. Therefore, a unique equilibrium exists when $\alpha \geq c$ and $c \leq 1 + \beta$, and it is such that $n_2 = 1$, with profits for firms 1 and 2 equal to $\pi_1 = (1 + \beta - c)^2/4$ and $\pi_2 = \alpha - c$. Note that firm 1 makes no sales if it charges a price above $1 + \beta$. Hence, in order for an equilibrium with $n_2 = 1$ and $p \geq 1 + \beta$ to exist, we need that $p = c \geq 1 + \beta$, so $p_2 = \alpha + c - 1 - \beta$, with profits for firms 1 and 2 equal to $\pi_1 = 0$ and $\pi_2 = \alpha - 1 - \beta$. In order for firm 2 not to have an incentive to deviate, we also need $\alpha \geq 1 + \beta$.

As for an equilibrium in which firm 2 does not capture market B, it is clear from Figure 7 that any $p < 1 + \beta$ is dominated by $p = 1 + \beta$, so let us seek for an equilibrium with $p \geq 1 + \beta$. In order for firm 2 not have an incentive to deviate, we need $p - 1 - \beta + \alpha \leq c$. Because firm 1 sells the bundle to all consumers, it earns $p - c$, so $p - c \leq 1 + \beta - \alpha$ implies that $\alpha \leq 1 + \beta$ must be met in order for firm 1 not to earn a negative margin. Because firm 1 maximizes profit by setting $p = 1 + \beta - \alpha + c$ in such a case, $p \geq 1 + \beta$ implies that $\alpha \leq c$ is also required for an equilibrium with $n_2 = 0$ to exist. In summary, it holds when $\alpha \leq c$ and $\alpha \leq 1 + \beta$ that $p = 1 + \beta - \alpha + c$ and $p_2 = c$ is the unique equilibrium such that $n_2 = 0$, with $\pi_1 = 1 + \beta - \alpha$ and $\pi_2 = 0$. In such a case, welfare is the same as with bundling with a specific complementarity. The following proposition summarizes these findings.

Proposition 8 (Pure bundling with a general complementarity)**.** *An equilibrium exists and is unique. The equilibrium is such that:*

(a) If $\alpha < \min\{c, 1 + \beta\}$, then firm 1 captures the network goods market B ($n_2^{PG} = 0$), $p^{PG} = 1 + \beta - \alpha + c$, $p_2^{PG} = c$, $\pi_1^{PG} = 1 + \beta - \alpha$, and $\pi_2^{PG} = 0$. In equilibrium, no consumer who buys the bundle disposes of good b_1.

(b) If $\alpha \geq \min\{c, 1 + \beta\}$, then firm 2 maintains its dominance of the network goods market B ($n_2^{PG} = 1$), $p^{PG} = \max\{c, (1 + \beta + c)/2\}$, $p_2^{PG} = \alpha + c - \min\{1 + \beta, c\}$, $\pi_1^{PG} = (\max\{0, (1 + \beta - c)/2\})^2$, $\pi_2^{PG} = \alpha - \min\{1 + \beta, c\}$. In equilibrium, all consumers who buy the bundle dispose of good b_1.

Superscript "PG" indicates that we are considering the equilibrium values for the case with pure bundling and a general complementarity.

In contrast with the case of unbundled pricing, in which entry was not possible with a general complementarity, bundling allows firm 1 to capture the network goods market even if it shares its complementarity. With a general complementarity, if firm 1 opts for unbundled pricing it completely renounces to its advantage when competing with firm 2, hence it cannot hope to conquer market B. With bundling, firm 1 can still capture the

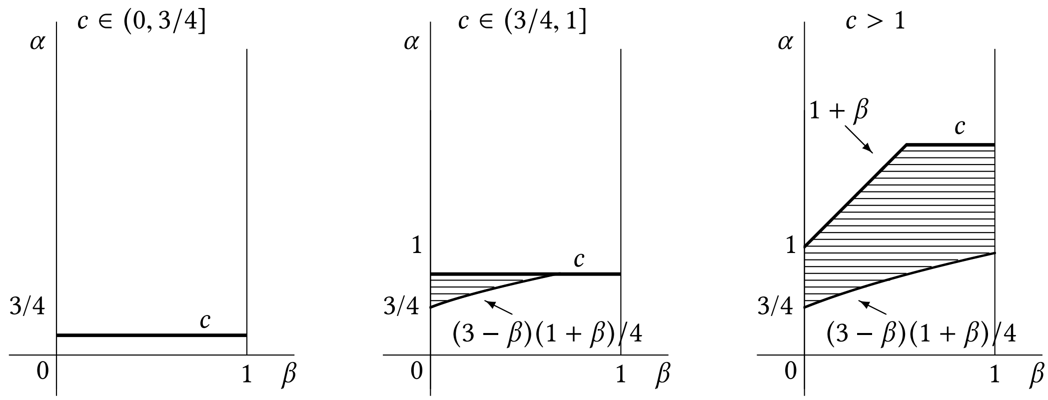


Figure 8: Region of parameters for which firm 1 prefers unbundled pricing without entry to bundled pricing with entry, when complementarity is general (dashed area)

network goods market, because those consumers interested in product a obtain product b_1 as part of the bundle, which gives firm 1 a competitive edge against firm 2.

Similarly to the case of a specific complementarity, pure bundling facilitates entry in comparison with unbundled pricing. In the current case, this occurs in the parametric region in which $\alpha < \min\{c, 1+\beta\}$, where $n_2^{UG} = 1$ but $n_2^{PG} = 0$. Also in this case, however, when firm 1 enters with pure bundling it loses its ability to adapt its pricing to the specifics of market A. Comparing firm 1's profit with unbundled pricing (Proposition 6) and pure bundling (Proposition 8), it always holds that $(1+\beta)^2/4 > (\max\{0, (1+\beta-c)/2\})^2$, whereas $1+\beta-\alpha > (1+\beta)^2/4$ if and only if $\alpha < (3-\beta)(1+\beta)/4$. Notice that $(3-\beta)(1+\beta)/4 \in [3/4, 1)$ when $\beta \in [0, 1)$. The following result follows.

Proposition 9 (Private desirability of pure bundling with a general complementarity)**.** *Pure bundling is preferred by firm 1 over unbundled pricing if and only if* $\alpha < \min\{c, (3-\beta)(1+\beta)/4\}$.

By combining the results of Propositions 8 and 9 we observe that firm 1 may sacrifice entry in order to obtain a higher profit with unbundled pricing. This occurs when $(3-\beta)(1+\beta)/4 < \alpha < \min\{c, 1+\beta\}$, which is satisfied only when $c > 3/4$. Figure 8 parallels Figure 5 as the dashed area represents the parametric region where firm 1 prefers unbundled pricing without entry rather than pure bundling with entry in market B.[12]

The similarities with the case of specific complementarity extend to social welfare as well. Indeed, when firm 1 cannot conquer market B in Proposition 8, pure bundling is

[12]In the right panel of Figure 8, we implicitly assume that $c \in (1, 1+\beta)$.

socially inefficient as it implies for firm 1 an additional cost c for the production of a good that is not consumed.[13] When pure bundling enables firm 1 to enter into the network goods market (*i.e.*, when $\alpha < \min\{c, 1+\beta\}$), welfare is the same as with bundling with a specific complementarity:

$$w^{PG} = \int_0^1 (v - c + u + \alpha + \beta)dv = u + \alpha - c + \frac{1}{2} + \beta.$$

It is immediate to verify that $w^{PG} > w^{UG}$ in such a parametric region.

Proposition 10 (Social desirability of pure bundling with a general complementarity)**.** *With a general complementarity, pure bundling results in greater welfare than unbundled pricing if and only if* $\alpha < \min\{c, 1+\beta\}$.

Finally, by comparing the private and the social desirability of pure bundling with a general complementarity, we find that in the dashed area of Figure 8 firm 1 prefers unbundling without entry to pure bundling with entry, whereas social welfare would be higher with pure bundling. When $(3-\beta)(1+\beta)/4 < \alpha < \min\{c, 1+\beta\}$, in fact, entry in market B, although not profitable for firm 1, would enhance social welfare by expanding the market for good a.

6 Optimal product design and pricing strategies

The analysis of the previous sections has revealed three main results. First, firm 1 can enter the network goods market more easily when it uses pure bundling, but there exist situations where unbundled pricing without entry ensures a higher profit. Second, when entry is not an equilibrium, sharing the complementarity with the rival may increase firm 1's profits with respect to enjoying a specific complementarity. Third, when entry in the network market is an equilibrium, it is also beneficial in terms of social welfare.

However, the private and social comparisons carried out so far are not exhaustive, as they rely on the assumption that one of the strategic decision variables of firm 1 is given. The results of Propositions 3 and 4, for example, are based on the assumption that firm 1 is committed to having a specific complementarity. The results of Propositions 8 and 10, on the contrary, depend on the assumption that firm 1 adopts a general complementarity.In this section, we study firm 1's optimal decision regarding its product design (whether

[13] The precise expression of social welfare for the case $\alpha \geq \min\{c, 1+\beta\}$ can be found in Appendix H. Additional calculations are available upon request.

the complementarity is commonly enjoyed or not) and pricing (bundling or no bundling) strategies.

We now consider a three-period game. In the first period, firm 1 chooses whether or not to share its complementarity with firm 2, and whether to adopt pure bundling or unbundled pricing. In the second period, firms choose prices. In the third period, consumers make consumption decisions. The second and third periods correspond to the various games we have examined earlier, so the current three-period game simply adds an initial stage to such games. It follows that firm 1 faces four relevant options in the first period: (i) unbundled pricing with a specific complementarity; (ii) pure bundling with a specific complementarity; (iii) unbundled pricing with a general complementarity; and (iv) pure bundling with a general complementarity.

Before analyzing the private and social desirability of the different options, we study which of them allows for an easier entry of firm 1 into the network goods market. By considering the results of Subsections 4.1 and 5.3, we know that pure bundling expands the region of parameters for which firm 1 enters market B with respect to the case of unbundled pricing, both if the complementarity is specific or general. We therefore compare a specific with a general complementarity in the case of pure bundling. Recall that $n_2^{PS} = 0$ when $\alpha < \min\{1, c\} + \beta$, and that $n_2^{PG} = 0$ when $\alpha < \min\{c, 1 + \beta\}$. Given that $\min\{c, 1 + \beta\} \leq \min\{1, c\} + \beta$, the following result holds.

Proposition 11 (Entry into the network goods market)**.** *Choosing pure bundling with a specific complementarity maximizes the region of parameters for which firm 1 enters the network goods market in the equilibrium of the continuation game.*

Proposition 11 implies that pure bundling with a specific complementarity is weakly better than the other product design and pricing strategies in terms of facilitating access to the network goods market to firm 1. However, as the previous analysis shows, entry may not always be profitable for the entrant. We next study the optimal product design and pricing strategy, by solving the three-period game described above.

We start by noting that if the firm wants to use pure bundling, then choosing a specific complementarity is weakly optimal, because it leads to a larger region of parameters for which entry is possible, and thus leads to weakly larger profits than choosing a general complementarity. Moreover, if there is a small cost of modifying firm 1's products to make the complementarity general (no matter how small), then a specific complementarity strictly dominates a general complementarity under pure bundling. Therefore, if for some parameters pure bundling with a specific complementarity leads to the same profits as pure bundling with a general complementarity, we select the first strategy as firm 1's optimal strategy. In Appendix H we prove the following proposition.

Proposition 12 (Privately vs. socially optimal product design and pricing strategies)**.** *Firm 1 prefers pure bundling with a specific complementarity if* $\alpha \leq \min\{(3-\beta)(1+\beta)/4, c+\beta\}$*, and prefers unbundled pricing with a general complementarity otherwise. Pure bundling with a general complementarity is a weakly dominated strategy, and unbundled pricing with a specific complementarity is a strictly dominated strategy, both from a private and social point of view. Firm 1's optimal strategy is socially optimal unless* $\min\{(3-\beta)(1+\beta)/4, c+\beta\} < \alpha < \min\{1, c\} + \beta$*, in which case firm 1 prefers unbundled pricing with a general complementarity, but pure bundling with a specific complementarity yields higher social welfare.*

The above proposition confirms the results that we found throughout our analysis. Pure bundling not only represents the best pricing strategy in order to enter in the network goods market, but combined with a specific complementarity it provides the highest profit when β is sufficiently high for fixed α and c. Society at large benefits from entry in market B, as more consumers are induced to buy product a, thereby enjoying the complementarity benefit.

From firm 1's viewpoint, the drawback of pure bundling is that it does not allow it to adapt its pricing to the specifics of market A. Hence, for higher values of α, firm 1 prefers unbundled pricing with a general complementarity, as with this strategy it can raise the price for good a, thereby increasing its profit. Entry in the network goods market is not viable in such a case. If $\alpha \geq \min\{1, c\} + \beta$, firm 1's choice is socially optimal.

However, when $(3-\beta)(1+\beta)/4 < \alpha < \min\{1, c\} + \beta$, firm 1 prefers unbundling with a general complementarity, whereas social welfare is maximized with pure bundling and a specific complementarity. This region is represented by the dashed areas of Figure 9. As in Figures 5 and 8, firm 1 could enter by using a bundling strategy, but it prefers not to do so. Entry in market B, although not profitable for firm 1, would have expanded the consumption of good a at an acceptable cost for society, thus explaining the social desirability of pure bundling.

Proposition 12 presents one of the central findings of our paper. Bundling is typically seen as a strategy adopted by incumbents to protect their market from entry by rivals. Regulatory interventions against bundling practices are therefore often advocated, as in the recent case of the European Commission against Google. In contrast, our paper shows that bundling can be used to enter a market where an incumbent holds a strong

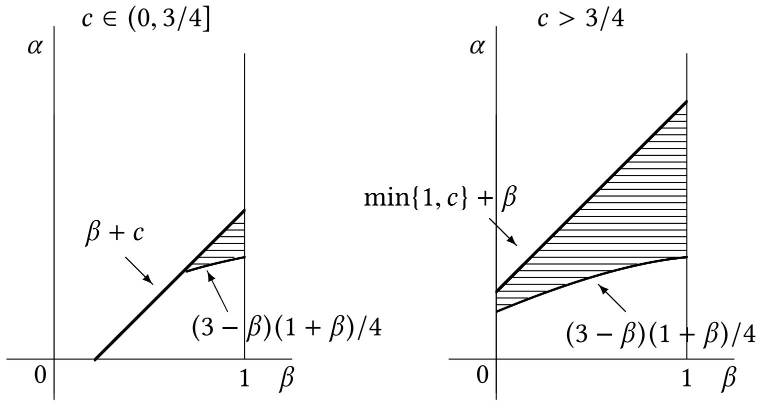


Figure 9: Region of parameters for which firm 1's optimal product design and pricing strategies are not socially optimal (dashed area)

dominant position sheltered by intense network effects, which is a socially efficient entry strategy if there exist significant complementarities between the bundled products. This is likely the case for online social networks (Facebook, Twitter, Google Plus), cloud storage and software (Dropbox, Google Drive, Apple's iCloud), and customer relationship management software (Salesforce). For example, several iPhone and iPad apps and services rely on data storage on Apple's iCloud (for example, iPhone and iPad backups are stored on iCloud, and a user can also upload each photo she takes automatically to iCloud). These products are in turn highly complementary with Apple software such as Keynote, Numbers and Pages (for example, a user may create a document in Keynote in a mac computer, save it on the iCloud, share it with other users, and access and modify it on her iPhone and iPad).

7 Conclusion

We have studied how an incumbent resting on strong network effects may be replaced by a firm already present in the market for a complementary good. We have considered a variety of entry strategies that can be employed by the entrant to displace the incumbent and perhaps enhance efficiency in so doing. These entry strategies include pure and mixed bundling, using the network good as a loss leader to increase revenues from the entrant's complementary product, and sharing the complementarity benefits with the incumbent.

One of the contributions of our paper is to disentangle the roles of both complementarity and bundling in facilitating entry and enhancing profitability, showing that there may be a conflict between entry and profitability when complementarities are specific. Even though competition authorities often advocate banning of strategies involving bundling because they argue it stifles entry, we find that bundling facilitates entry, and it should therefore be encouraged in network markets, at least when complementarities are specific. Loss leading is also socially desirable in these settings, even if it is sometimes viewed as an exploitative practice with exclusionary implications (Chen and Rey, 2012). Finally, we have shown that, even though entry may be possible through bundling, it may be deliberately avoided by the entrant, which may prefer unbundled pricing with a general complementarity. This would be socially inefficient because welfare would be higher if the entrant used bundling and a specific complementarity.

Our paper may hopefully serve as a building block for future analyses that may enhance our understanding of lateral entry into network goods markets. There are cases to which it is relatively simple to extend our insights. For example, the entrant and the incumbent may produce the network good at different costs, or the entrant need not be a monopolist in the complementary good market. In this cases, it is straightforward to show that the mechanisms studied in this paper will also be at work. Extending our results to other settings, however, is not as straightforward. First, one may study what happens if the incumbent has the possibility to respond by becoming active in the market for complementary goods. Previous works (*e.g.*, Bakos and Brynjolfsson, 2000, or Nalebuff, 2004, *inter alii*) noted that bundle-to-bundle competition can be very intense, so it would be interesting to examine whether this would still hold if bundles include a network good. Second, one may analyze a setting when consumers are not forced to single-home, but rather are allowed to multi-home in the network goods market (as in Choi, 2010). In such cases, the entrant may not compete so fiercely for the incumbent's installed base, so entry might be more difficult. We believe that these two issues present interesting directions for future research.

Appendix A

Given that we can restrict attention to $p_a \in [0, 1+\beta]$, we distinguish the following cases that may arise based on the firms' pricing, analyzing the implications of each of them in constructing the firms' demand functions/correspondences:

(a) $U_{b_1} > U_{b_2}$.

(b) $U_{b_1} = U_{b_2}$.

(c) $U_{b_2} > U_{b_1}$.

Regarding case (a), $U_{b_1} > U_{b_2}$ implies that $U_{ab_1}(v) > U_{ab_2}(v)$ for all v (since $\beta \geq 0$), which in turn implies that $n_2 = 0$ must hold. Therefore, $U_{b_1} > U_{b_2}$ if and only if

$$u + \alpha - p_1 > u - p_2.$$

This shows that $n_2 = 0$ can arise as firm 2's quantity demanded whenever $p_2 > p_1 - \alpha$ is met.

As for case (b), $U_{b_1} = U_{b_2}$ implies that $U_{ab_1}(v) > U_{ab_2}(v)$ for all v whenever $\beta > 0$ (we treat $\beta = 0$ separately further below). Those consumers with valuation greater than $p_a - \beta$ buy both a and b_1, whereas those consumers with valuation smaller than $p_a - \beta$ do not buy a and are indifferent between b_1 and b_2. If $p_a \leq \beta$, then $n_2 = 0$ must hold. If instead $p_a > \beta$, the measure of consumers who buy only b_2 cannot exceed the total number of consumers who do not buy a and b_1, which is equal to $p_a - \beta$. Hence, $0 \leq n_2 \leq p_a - \beta$ must hold, with $U_{b_1} = U_{b_2}$ yielding that it must also hold that

$$p_2 = p_1 + (2n_2 - 1)\alpha.$$

Note that $n_2 = 0$ is also consistent with $p_a > \beta$. Then, we have shown that $n_2 = 0$ can arise as firm 2's quantity demanded if $p_2 = p_1 - \alpha$, and

$$n_2 = \frac{p_2 - p_1 + \alpha}{2\alpha} \in (0, p_a - \beta]$$

can arise as firm 2's quantity demanded if $p_a > \beta$ and $p_1 - \alpha < p_2 \leq p_1 - \alpha + 2\alpha(p_a - \beta)$.

When $\beta = 0$ in case (b), note that $U_{b_1} = U_{b_2}$ implies that $U_{ab_1}(v) = U_{ab_2}(v)$ for all v. Those consumers with valuation greater than p_a purchase a and are indifferent between buying b_1 and b_2 as well, whereas those consumers with valuation smaller than p_a do not purchase a and are indifferent between b_1 and b_2. Clearly, as $0 \le n_2 \le 1$, when $U_{b_1} = U_{b_2}$ it must also hold that $p_2 = p_1 + (2n_2 - 1)\alpha$, so

$$n_2 = \frac{p_2 - p_1 + \alpha}{2\alpha} \in (0, 1]$$

can arise as firm 2's quantity demanded if $p_1 - \alpha \le p_2 \le p_1 + \alpha$ whenever $\beta = 0$.

Finally, with regards to case (c) $U_{b_2} > U_{b_1}$ implies that $p_2 < p_1 + (2n_2 - 1)\alpha$ must be satisfied. Depending on the comparison between $U_{ab_2}(v)$ and $U_{ab_1}(v)$, we have three subcases:

(c1) $U_{ab_2}(v) > U_{ab_1}(v)$ for all v.

(c2) $U_{ab_2}(v) = U_{ab_1}(v)$ for all v.

(c3) $U_{ab_2}(v) < U_{ab_1}(v)$ for all v.

In subcase (c1), $U_{b_2} > U_{b_1}$ and $U_{ab_2}(v) > U_{ab_1}(v)$ for all v; no consumer wants to buy b_1, and therefore $n_2 = 1$. Inequality $U_{ab_2}(v) > U_{ab_1}(v)$ implies $p_2 < p_1 + (2n_2 - 1)\alpha - \beta$. Note that this condition is consistent with the one for $U_{b_2} > U_{b_1}$, which is $p_2 < p_1 + (2n_2 - 1)\alpha$. Given that $n_2 = 1$ must hold, this subcase arises if $p_2 < p_1 + \alpha - \beta$.

In subcase (c2), $U_{ab_2}(v) = U_{ab_1}(v)$ implies $p_2 = p_1 + (2n_2 - 1)\alpha - \beta$. Recalling that $U_{b_1} < U_{b_2}$, $U_{b_2} > U_{ab_2}(v) = U_{ab_1}(v)$ would imply that any consumer with valuation smaller than $\min\{1, p_a\}$ should purchase b_2, whereas any other consumer should purchase a together with either b_1 or b_2. Therefore, the measure of consumers who buy only b_2 must be greater than or equal to $\min\{1, p_a\}$, and any

$$n_2 = \frac{p_2 - p_1 + \alpha + \beta}{2\alpha} \in [\min\{1, p_a\}, 1]$$

can arise as firm 2's quantity demanded if $p_2 = p_1 + (2n_2 - 1)\alpha - \beta$ holds. Note that $n_2 = 1$ is consistent with fulfilled expectations if $p_2 = p_1 + \alpha - \beta$, and $n_2 \in [p_a, 1)$ is consistent if $p_1 + 2\alpha p_a - \alpha - \beta \le p_2 < p_1 + \alpha - \beta$, and $0 \le p_a < 1$.

In (c3), $U_{ab_2}(v) < U_{ab_1}(v)$ implies that $p_1 + (2n_2 - 1)\alpha - \beta < p_2 < p_1 + (2n_2 - 1)\alpha$. Let

$$\widehat{v} \equiv p_a - \beta + (2n_2 - 1)\alpha + p_1 - p_2$$

denote the unique value of v such that $U_{b_2} = U_{ab_1}(v)$ holds. Note that $\widehat{v} \in (p_a - \beta, p_a)$, and since $p_a - \beta$ may be negative, we need to consider both the case in which $\widehat{v}$ is below 0 and the case in which it is above 0. Similarly, as p_a may exceed 1, we need to consider both the case in which $\widehat{v}$ is above 1 and the case in which it is below 1.

Firstly, suppose that $\widehat{v} \leq 0$ (which necessarily requires $p_a < \beta$), so that $p_a - \beta + (2n_2 - 1)\alpha + p_1 - p_2 \leq 0$. Satisfaction of rational expectations by consumers (as required by Nash equilibrium behavior) implies that we must have $n_2 = 0$, so the condition becomes $p_1 + p_a - \alpha - \beta \leq p_2$, which must hold at the same time as $p_1 - \alpha - \beta < p_2 < p_1 - \alpha$, so we simply need that $p_1 + p_a - \alpha - \beta \leq p_2 < p_1 - \alpha$.

Secondly, suppose that $\widehat{v} \geq 1$ (which necessarily requires that $p_a \geq 1$, since $U_{b_2} > U_{ab_2}(v)$ for all $v \in [0, 1]$), so that $p_2 \leq p_1 + (2n_2 - 1)\alpha + p_a - \beta - 1$. In order for consumers expectations to be fulfilled, it must hold that $n_2 = 1$, so the condition becomes $p_2 \leq p_1 + \alpha + p_a - \beta - 1$. Recall we also need $p_1 + (2n_2 - 1)\alpha - \beta < p_2 < p_1 + (2n_2 - 1)\alpha$, but given that we only consider cases such that $p_a \leq 1 + \beta$, the final restriction is $p_1 + \alpha - \beta < p_2 \leq p_1 + \alpha - (1 + \beta - p_a)$.

Lastly, suppose now that $\widehat{v} \in (0, 1)$: $p_1 + p_a + (2n_2 - 1)\alpha - \beta - 1 < p_2 < p_1 + p_a + (2n_2 - 1)\alpha - \beta$. Then n_2 must equal the measure of consumers whose valuation is smaller than $\widehat{v}$ (since $U_{b_2} > U_{ab_1}(v) > U_{ab_2}(v)$ and $U_{b_2} > U_{b_1}$ for such consumers). That is, $n_2 = \Pr(v \leq \widehat{v})$ must hold in order for consumers expectations to be fulfilled (as required by Nash equilibrium behavior), so the fact that $\Pr(v \leq \widehat{v}) = \widehat{v}$ implies that we must have $n_2 = (2n_2 - 1)\alpha + p_a - \beta + p_1 - p_2$, which yields

$$n_2 = \frac{p_2 - p_1 - p_a + \alpha + \beta}{2\alpha - 1}.$$

The constraints that $p_1 + p_a - 1 + (2n_2 - 1)\alpha - \beta < p_2 < p_1 + p_a + (2n_2 - 1)\alpha - \beta$ and $p_1 + (2n_2 - 1)\alpha - \beta < p_2 < p_1 + \alpha(2n_2 - 1)$ imply that $\max\{0, p_a - \beta\} < n_2 < \min\{1, p_a\}$. Because we assumed that $\alpha > 1/2$, we must have $p_1 + p_a - \alpha - \beta + (2\alpha - 1)\max\{p_a - \beta, 0\} < p_2 < p_1 + p_a - \alpha - \beta + (2\alpha - 1)\min\{p_a, 1\}$.

To summarize, we have shown that firm 2's demand correspondence is as follows:

- $n_2 = 0$ if $p_2 \geq p_1 - \alpha - \max\{\beta - p_a, 0\}$.
- $n_2 = 1$ if $p_2 \leq p_1 + \alpha - \beta + \max\{p_a - 1, 0\}$.
- $n_2 = \dfrac{p_2 - p_1 + \alpha}{2\alpha}$ if $p_1 - \alpha < p_2 \leq p_1 - \alpha + 2\alpha(p_a - \beta)$ and $p_a \in (\beta, 1 + \beta)$.
- $n_2 = \dfrac{p_2 - p_1 + \alpha + \beta}{2\alpha}$ if $p_1 + 2\alpha p_a - \alpha - \beta \leq p_2 < p_1 + \alpha - \beta$ and $p_a \in [0, 1)$.
- $n_2 = \dfrac{p_2 - p_1 - p_a + \alpha + \beta}{2\alpha - 1}$ if $p_1 + p_a - \alpha - \beta + (2\alpha - 1)\max(p_a - \beta, 0) < p_2 \leq p_1 + p_a - \alpha - \beta + (2\alpha - 1)\min\{p_a, 1\}$.

Appendix B

Denote equilibrium values using the "US" superscript. Let us try to sustain $n_2^{US} = 1$ in equilibrium. Clearly, firm 1 must be charging the monopoly price in market A, so $p_a^{US} = 1/2$, and it must be earning $\pi_1^{US} = 1/4$. Also, firm 2 must be charging the highest possible price such that its demand equals 1. If firm 1 is charging p_1^{US} in market B, then firm 2 must be charging $p_2^{US} = p_1^{US} + \alpha - \beta$ (see top and middle panels on the right of Figure 1). Suppose first that $p_1^{US} > 0$. If firm 1 deviates by charging $\widehat{p}_a < 1$, then the best it can do is to charge $\widehat{p}_1 = p_1^{US} - \epsilon$ for $\epsilon > 0$ small enough so as to attract all consumers in market B and attract those consumers in market A whose valuation exceeds $\widehat{p}_a - \beta$. Doing so yields

$$\begin{aligned}\pi_1(\widehat{p}_1, \widehat{p}_a) &= \widehat{p}_1 - c + \widehat{p}_a[\min\{1, 1 + \beta - \widehat{p}_a\}] \\ &= \begin{cases} \widehat{p}_1 - c + \widehat{p}_a & \text{if } \widehat{p}_a \leq \beta \\ \widehat{p}_1 - c + \widehat{p}_a(1 + \beta - \widehat{p}_a) & \text{if } \widehat{p}_a \in (\beta, 1) \end{cases}.\end{aligned}$$

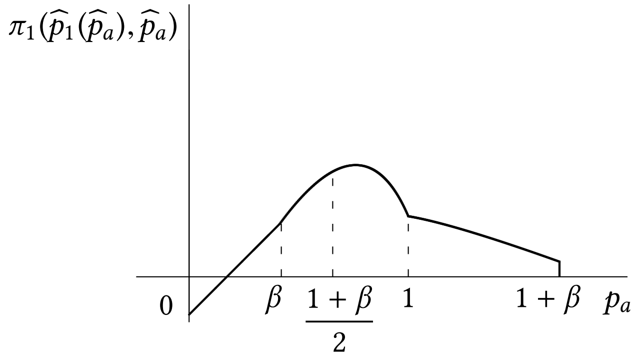
Consider now what happens if firm 1 chooses $\widehat{p}_a \geq 1$. Clearly the best that firm 1 could do is to charge $\widehat{p}_1(\widehat{p}_a) = p_1^{US} - (\widehat{p}_a - 1) - \epsilon$ so as to attract all consumers in market B (see the bottom right panel of Figure 1) and attract those consumers in market A whose valuation exceeds $\widehat{p}_a - \beta$. This can be accomplished if $\widehat{p}_a < p_1^{US} + 1$, since the price charged for b_1 would not be negative; however, if $\widehat{p}_a \geq p_1^{US} + 1$, then it is impossible for firm 1 to get some demand in market B.[14] Taking into account that the non-negativity of good b_1's price limits firm 1's deviations so that $\widehat{p}_a < p_1^{US} + 1$, firm 1 would earn

$$\pi_1(\widehat{p}_1(\widehat{p}_a), \widehat{p}_a) = p_1^{US} - c + 1 - \widehat{p}_a + \widehat{p}_a[\min\{1, 1 + \beta - \widehat{p}_a\}]$$

when charging $\widehat{p}_a \in [1, p_1^{US} + 1)$. Because $\beta < 1$, firm 1's payoff to deviating as a function of $\widehat{p}_a$ is represented in Figure A.

[14] If $\widehat{p}_a \geq p_1^{US} + 1$, we should have $\widehat{p}_1 = 0$ and $\widehat{p}_a = 1 + \beta - \alpha + p_2^{US} - \epsilon = 1 + p_1^{US} - \epsilon$ in order for firm 1 to attract market B's demand, but $\widehat{p}_a = 1 + p_1^{US} - \epsilon < 1 + p_1^{US}$ contradicts $\widehat{p}_a \geq p_1^{US} + 1$.

Figure A



It follows that the optimal deviation for firm 1 involves $\widehat{p}_1^{US} = p_1^{US} - \epsilon > 0$ and $\widehat{p}_a^{US} = (1+\beta)/2$ so as to earn $\pi_1(\widehat{p}_1^{US}, \widehat{p}_a^{US}) = p_1^{US} - c - \epsilon + (1+\beta)^2/4$. Recalling that we have made the working assumption that $p_1^{US} > 0$, firm 1 does not deviate if $1/4 \geq p_1^{US} - c + (1+\beta)^2/4$, that is, $p_1^{US} \leq c - \beta(2+\beta)/4$, with $c > \beta(2+\beta)/4$ in order for $p_1^{US} > 0$. In an equilibrium not sustained by weakly dominated strategies,[15] we must have

$$p_1^{US} = c - \beta(2+\beta)/4 > 0;$$

in such a case,

$$p_2^{US} = \alpha - \beta + c - \beta(2+\beta)/4 = c + \alpha - \beta(6+\beta)/4,$$

and therefore we need $\alpha \geq \beta(6+\beta)/4$ so that firm 2 has no incentive to perform a unilateral deviation. We conclude the analysis of $n_2 = 1$ by considering the case in which $p_1^{US} = 0$. Clearly, we must have $p_2^{US} = \alpha - \beta$ in such situations. Firm 1 cannot undertake any profitable deviation, and ruling out deviations by firm 2 simply requires that $c \leq \alpha - \beta$, or $\alpha \geq \beta + c$. In any equilibrium in which $n_2 = 1$, it holds that the social

[15]It is standard not to allow for weakly dominated strategies in a Bertrand pricing game with asymmetric firms. To give a simple example, consider a Bertrand duopoly where firms bear a marginal cost of production equal to 1 and compete in prices to attract a single unit-demand consumer who values the product of one of the firms at 5 and the product of the other firm at 4. When the smallest monetary unit is $\epsilon > 0$ (where ϵ is not zero but is "small"), that the former firm prices at $1.5 - \epsilon$ and the latter prices at $0.5 < 1$ is certainly a Nash equilibrium, but it is sustained by a weakly dominated strategy. The only Nash equilibrium not sustained by weakly dominated strategies converges as $\epsilon \to 0$ to the well-known equilibrium in which the vertically superior firm charges 2 and the rival sells at marginal cost. Our focus will always be on the kind of equilibrium equivalent to this type in our setting. See Griva and Vettas (2011) for a similar restriction in a setting with direct network effects.

welfare generated equals

$$w^{US} = u + \alpha - c + \int_{\frac{1}{2}}^{1} (v - \frac{1}{2}) dv + \frac{1}{4} = u + \alpha - c + 3/8.$$

Let us now try to sustain $n_2 = 0$ in equilibrium with $p_a^{US} < 1$. In such a case, it must hold that $p_1^{US} = p_2^{US} + \beta - \alpha$. Also, firm 2 should have no incentive to charge $p_2^{US} - \epsilon$, so it must hold that $p_2^{US} \leq c$, which implies that $p_2^{US} = c$ in an equilibrium not sustained by weakly dominated strategies. Therefore,

$$p_1^{US} = c + \beta - \alpha,$$

whereas the price that firm 1 charges in market A must be

$$p_a^{US} = \arg\max_{p_a} \left\{ p_1^{US} - c + p_a[\min\{1, 1 + \beta - p_a\}] \right\}.$$

Because $\beta < 1$, then $p_a^{US} = (1 + \beta)/2$, so $p_a^{US} < 1$ does hold, as initially claimed. Firm 1 then earns the following equilibrium profit:

$$\pi_1^{US} = \beta - \alpha + (1 + \beta)^2 / 4.$$

Besides $p_1^{US} \geq 0$ (*i.e.*, $c \geq \alpha - \beta$), firm 1 should have no incentive to focus on monopolizing market A only, in which case it would earn 1/4. It follows that $\beta - \alpha + (1 + \beta)^2 / 4 \geq 1/4$ requires that $\alpha \leq \beta (6 + \beta) / 4$. Firm 1 should have no incentive either to charge $\widehat{p}_a \geq 1$ and set $\widehat{p}_1(\widehat{p}_a)$ in such a way that $\widehat{p}_1(\widehat{p}_a) = p_2^{US} + \beta - \alpha - (\widehat{p}_a - 1)$, provided $\widehat{p}_1(\widehat{p}_a) > 0$, since otherwise the deviation cannot be profitable. Doing so yields

$$\begin{aligned}
\pi_1(\widehat{p}_1(\widehat{p}_a), \widehat{p}_a) &= \widehat{p}_1(\widehat{p}_a) - c + \widehat{p}_a[\min\{1, 1 + \beta - \widehat{p}_a\}] \\
&= p_2^{US} - c + \beta - \alpha + 1 - \widehat{p}_a + \widehat{p}_a[\min\{1, 1 + \beta - \widehat{p}_a\}] \\
&= p_2^{US} - c + 1 + \beta - \alpha + \widehat{p}_a(\beta - \widehat{p}_a),
\end{aligned}$$

since $\widehat{p}_a \geq 1 > \beta$. The payoff to deviating is maximized at $\widehat{p}_a^{US} = 1$ (note that $\pi_1(\widehat{p}_1(\widehat{p}_a), \widehat{p}_a)$ decreases with $\widehat{p}_a$ whenever $\widehat{p}_a \geq 1$), so we need that

$$\pi_1^{US} = \beta - \alpha + (1 + \beta)^2 / 4 \geq 1 + \beta - \alpha + \beta - 1,$$

which always holds. The social welfare generated in this type of equilibrium is

$$\begin{aligned} w^{US} &= u + \alpha - c + \int_{\frac{1+\beta}{2}-\beta}^{1} \left[v - \left(\frac{1+\beta}{2} - \beta \right) \right] dv + \frac{(1+\beta)^2}{4} \\ &= u + \alpha - c + 3(1+\beta)^2/8. \end{aligned}$$

Finally, let us try to sustain an equilibrium in which $n_2 = 0$ and $p_a^{US} \geq 1$. In such a case, it must hold that firm 1 sells good b_1 at price $p_1^{US}(p_a^{US}) = p_2^{US} + \beta - \alpha - (p_a^{US} - 1) \geq 0$. Obviously, $p_2^{US} = c$ in an equilibrium not sustained by weakly dominated strategies, and

$$p_a^{US} \in \underset{\widehat{p}_a \leq c+1+\beta-\alpha}{\arg\max} \left\{ p_1^{US}(\widehat{p}_a) - c + \widehat{p}_a[\min\{1, 1+\beta - \widehat{p}_a\}] \right\}.$$

Because $\beta < 1$, we should have $p_a^{US} = \min\{\beta/2, c+1+\beta-\alpha\} < 1$, which contradicts $p_a^{US} \geq 1$. It follows that there cannot exist an equilibrium with $n_2 = 0$ and $p_a^{US} \geq 1$.

Appendix C

In order to find out firm 2's demand correspondence, note that charging $p < 0$ is weakly dominated, so we focus on $p \geq 0$ in what follows. We cover all the possible cases that can arise:

- Suppose $U_{ab_1}(v) > U_{ab_2}(v)$ and $U_{ab_1}(v) > U_{b_2}$ for all $v \in [0,1]$. Then $n_2 = 0$ whenever $p_2 > -\alpha - \beta$ and $p_2 > -(v - p) - \alpha - \beta$ for $v = 0$, that is, whenever $p_2 > p - \alpha - \beta$. Because $p \geq 0$, it holds that $n_2 = 0$ whenever $p_2 - p > -\alpha - \beta$.

- Suppose $U_{b_2} \geq U_{ab_1}(v)$ and $U_{b_2} > U_{ab_2}(v)$ for all $v \in [0,1]$. Then $n_2 = 1$ and $p > v$ for all $v \in [0,1]$ as well as $\alpha - \beta - p_2 \geq v - p$ for all $v \in [0,1]$. Hence, $n_2 = 1$ can arise whenever $p \geq 1$ and $p_2 - p \leq \alpha - \beta - 1$. Firm 1 makes no sale of good a.

- Suppose $U_{ab_2}(v) > U_{ab_1}(v)$ for all $v \in [0,1]$. Then it must hold that $n_2 = 1$ and $p_2 < \alpha - \beta$. Those consumers with $v > p$ purchase both a and b_2, and those with $v \leq p$ purchase b_2 only. For $p \leq 1$, firm 1 makes sales of good a equal to $1 - p \in [0,1]$.

- Suppose $U_{ab_1}(v) = U_{ab_2}(v)$ for all $v \in [0,1]$. Then we must have $\alpha(1-n) + \beta = \alpha n_2 - p_2$. If there exists $\widehat{v} \in (0,1)$ such that $U_{ab_2}(v) > U_{b_2}$ for $v > \widehat{v}$ and $U_{ab_2}(v) < U_{b_2}$

for $v < \widehat{v}$, then it must be the case that $p \in (0,1)$ so that $\widehat{v} = p$. Then

$$n_2 = \frac{p_2 + \alpha + \beta}{2\alpha} \in [0,1]$$

for $-\alpha - \beta \leq p_2 \leq \alpha - \beta$. We should also have that $p_2 \geq 2\alpha p - \alpha - \beta$ in order for $\widehat{v} \leq n_2$ (so that consumer expectations are fulfilled), so $p \in (0,1)$ implies that it should hold that $2\alpha p - \alpha - \beta \leq p_2 \leq \alpha - \beta$. Firm 1 makes sales of good a equal to $1 - p$. If instead $\widehat{v} = 1$, which means that $p \geq 1$, then firm 1 makes no sale of good a and on top of that $n_2 = 1$. If instead $\widehat{v} = 0$, which means that $p = 0$, then firm 1 makes sales of good a equal to 1.

- Suppose that $U_{ab_1}(v) > U_{ab_2}(v)$ for all $v \in [0,1]$ and that there exists $\widehat{v} \in (0,1)$ such that $U_{ab_1}(v) > U_{b_2}$ for $v > \widehat{v}$ and $U_{ab_1}(v) < U_{b_2}$ for $v < \widehat{v}$. In this case $\widehat{v} - p + \alpha(1-n) + \beta = \alpha n_2 - p_2$, with $\widehat{v} = n_2$ because of rational expectations, so $\alpha - p + p_2 + \beta = n_2(2\alpha - 1)$. Therefore, $\alpha > 1/2$ implies that

$$n_2 = \frac{p_2 - p + \alpha + \beta}{2\alpha - 1} \in (0,1)$$

for $p - \alpha - \beta < p_2 < p + \alpha - \beta - 1$. We also need $\alpha n_2 - p_2 < \alpha(1-n) + \beta$, that is, $p_2 < 2\alpha p - \alpha - \beta$. Therefore, the condition that must be satisfied is $p - \alpha - \beta < p_2 < \min\{2\alpha p - \alpha - \beta, p + \alpha - \beta - 1\}$. Firm 1 makes sales of a equal to $1 - n$.

To sum up, we have proven that firm 2's demand correspondence is as follows:

- $n_2 = 0$ for $p_2 > p - \alpha - \beta$. In this case, firm 2 makes no sales of good b_2 and firm 1's sales of the bundled good equal 1.

- $n_2 = 1$ for $p < 1$ and $p_2 \leq \alpha - \beta$. In this case, firm 2's sales of good b_2 equal 1 and firm 1's sales of the bundled good equal $1 - p$.

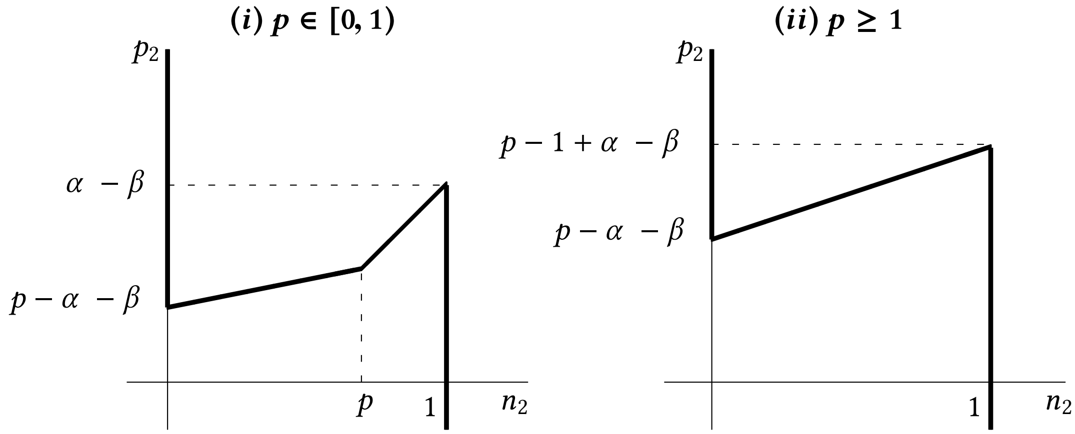
- $n_2 = 1$ for $p \geq 1$ and $p_2 \leq \alpha - \beta + p - 1$. In this case, firm 2's sales of good b_2 equal 1 and firm 1's sales of the bundled good equal 0.

- $n_2 = \frac{p_2 + \alpha + \beta}{2\alpha} \in [0,1]$ for $2\alpha p - \alpha - \beta \leq p_2 \leq \alpha - \beta$, which requires that $p \in (0,1)$. In this case, firm 2 makes n_2 sales of good b_2, and firm 1's sales of the bundled good equal $1 - p$.

- $n_2 = \frac{p_2 - p + \alpha + \beta}{2\alpha - 1} \in [0,1]$ for $p - \alpha - \beta < p_2 < 2\alpha p - \alpha - \beta$ if $p \in (0,1)$ and for $p - \alpha - \beta < p_2 < p + \alpha - \beta - 1$ if $p \geq 1$. In this case, firm 2 makes n_2 sales of good b_2, and firm 1's sales of the bundled good equal $1 - n_2$.

Firm 2's demand correspondence is represented in Figure B, in which we distinguish two cases, depending on the value of p.

Figure B



Appendix D

To find out firm 2's demand correspondence, it is worth noting first that, regardless of v, $U_{ab_2}(v) \geq U_{ab_1b_2}(v)$ for all v if and only if $d \leq p_1$, so we distinguish two situations depending on whether $p_1 - d$ is positive or not.

(i) Consider first the situations in which $d \leq p_1$. Clearly, any pricing strategy for firm 1 that involves $p_a > 1+d+\beta$ is (weakly) dominated by a strategy with $p_a = 1+d+\beta$. Thus, in what follows we consider strategies such that $p_a \leq 1 + d + \beta$. Similarly, firm 1 cannot be playing any pricing strategy that involves $p_a < 0$ because it is (weakly) dominated by a strategy with $p_a = 0$.

Given that we can restrict attention to $p_a \in [0, 1+d+\beta]$, we distinguish the following cases that may arise based on the firms' prices, analyzing the implications of each in constructing the firms' demand functions/correspondences:

(a) $U_{b_1} > U_{b_2}$.

(b) $U_{b_1} = U_{b_2}$.

(c) $U_{b_2} > U_{b_1}$.

Regarding case (a), $U_{b_1} > U_{b_2}$ implies that $U_{ab_1}(v) > U_{ab_2}(v)$ for all v (since $d+\beta \geq 0$), which in turn implies that $n_2 = 0$. Therefore, we have that $U_{b_1} > U_{b_2}$ if and only if

$$u + \alpha - p_1 > u - p_2.$$

This shows that $n_2 = 0$ can arise as firm 2's quantity demanded whenever $p_2 > p_1 - \alpha$ is met.

As for case (b), $U_{b_1} = U_{b_2}$ implies that $U_{ab_1}(v) > U_{ab_2}(v)$ for all v whenever $d + \beta > 0$ (we treat the case $d = \beta = 0$ separately further below). Those consumers with valuation greater than $p_a - d - \beta$ purchase both a and b_1, whereas those consumers with valuation smaller than $p_a - d - \beta$ do not purchase a and are indifferent between b_1 and b_2. If $p_a \leq d + \beta$, then $n_2 = 0$ must hold. If instead $p_a \in (d + \beta, 1 + d + \beta]$, then the measure of consumers who purchase b_2 only cannot exceed the total number of consumers who do not purchase a and b_1, which is equal to $p_a - d - \beta$. Hence, $0 \leq n_2 \leq p_a - d - \beta$ must hold, with $U_{b_1} = U_{b_2}$ yielding that it must also hold that

$$p_2 = p_1 + (2n_2 - 1)\alpha.$$

Note that $n_2 = 0$ is also consistent with $p_a > d + \beta$. Then, we have shown that $n_2 = 0$ can arise as firm 2's quantity demanded if $p_2 = p_1 - \alpha$, and

$$n_2 = \frac{p_2 - p_1 + \alpha}{2\alpha} \in (0, p_a - d - \beta]$$

can arise as firm 2's quantity demanded if $p_a \in (d+\beta, 1+d+\beta]$ and $p_2 = p_1 + (2n_2 - 1)\alpha$, in which case the restrictions on n_2 imply that $p_1 - \alpha < p_2 \leq p_1 - \alpha + 2\alpha(p_a - d - \beta)$.

When $d = \beta = 0$ in case (b), note that $U_{b_1} = U_{b_2}$ implies that $U_{ab_1}(v) = U_{ab_2}(v)$ for all v. Those consumers with valuation greater than p_a purchase a and are indifferent between buying b_1 and b_2 as well, whereas those consumers with valuation smaller than p_a do not purchase a and are indifferent between b_1 and b_2. Clearly, $0 \leq n_2 \leq 1$ must hold, with $U_{b_1} = U_{b_2}$ yielding that it must also hold that

$$p_2 = p_1 + (2n_2 - 1)\alpha,$$

and then

$$n_2 = \frac{p_2 - p_1 + \alpha}{2\alpha}$$

can arise as firm 2's quantity demanded if $-\alpha \leq p_2 \leq \alpha$ whenever $d = \beta = 0$.

Finally, with regards to case (c), $U_{b_2} > U_{b_1}$ implies that $p_2 < p_1 + (2n_2 - 1)\alpha$ must be satisfied. Depending on the comparison between $U_{ab_2}(v)$ and $U_{ab_1}(v)$, we have three subcases:

(c1) $U_{ab_2}(v) > U_{ab_1}(v)$ for all v.

(c2) $U_{ab_2}(v) = U_{ab_1}(v)$ for all v.

(c3) $U_{ab_2}(v) < U_{ab_1}(v)$ for all v.

In case (c1), $U_{b_2} > U_{b_1}$ and $U_{ab_2}(v) > U_{ab_1}(v)$ for all v, which means that no consumer wants to buy b_1, and therefore $n_2 = 1$. Inequality $U_{ab_2}(v) > U_{ab_1}(v)$ implies $p_2 < p_1 - d + (2n_2 - 1)\alpha - \beta$. Note that this condition is consistent with the condition for $U_{b_2} > U_{b_1}$, which is $p_2 < p_1 + (2n_2 - 1)\alpha$. Given that $n_2 = 1$, this case holds if $p_2 < p_1 - d + \alpha - \beta$.

In case (c2), $U_{ab_2}(v) = U_{ab_1}(v)$ implies $p_2 = p_1 + (2n_2 - 1)\alpha - d - \beta$. Then $U_{b_2} > U_{ab_2}(v) = U_{ab_1}(v)$ for consumers whose valuation is weakly smaller than p_a and $U_{ab_2}(v) = U_{ab_1}(v) \geq U_{b_2}$ for consumers whose valuation exceeds p_a. As a result, any consumer with valuation smaller than p_a must purchase b_2, and any consumer with valuation greater than p_a must purchase a and either b_1 or b_2 (whichever of them). The measure of consumers who purchase only b_2 must be greater than or equal to $\min\{1, p_a\}$, so any $n_2 \in [\min\{1, p_a\}, 1]$ can arise as firm 2's quantity demanded if $p_2 = p_1 - d + (2n_2 - 1)\alpha - \beta$ holds. Note that $n_2 = 1$ is consistent with fulfilled expectations if $p_2 = p_1 - d + \alpha - \beta$, and

$$n_2 = \frac{p_2 - p_1 + d + \alpha + \beta}{2\alpha} \in [p_a, 1)$$

is consistent if $p_1 - d + 2\alpha p_a - \alpha - \beta < p_2 < p_1 - d + \alpha - \beta$, and $0 \leq p_a < 1$.

In case (c3), $U_{ab_2}(v) < U_{ab_1}(v)$ implies $p_1 - d + (2n_2 - 1)\alpha - \beta < p_2 < (2n_2 - 1)\alpha$. Let

$$\widehat{v} \equiv p_a - d - \beta + (2n - 1)\alpha + p_1 - p_2$$

denote the unique value of v such that $U_{b_2} = U_{ab_1}(v)$ holds. Note that $\widehat{v} \in (p_a - d - \beta, p_a)$, and since $p_a - d - \beta$ may be negative, we need to consider cases in which $\widehat{v}$ is above 0 and below 0. Similarly, p_a may exceed 1, so we need to consider cases in which $\widehat{v}$ is above 1 and below 1.

Firstly, suppose that $\widehat{v} \leq 0$ (which necessarily requires $p_a < d + \beta$), so that $p_a - d - \beta + (2n - 1)\alpha + p_1 - p_2 \leq 0$. Then, we must have $n_2 = 0$, and the condition becomes $p_1 + p_a - d - \alpha - \beta \leq p_2$, which must hold at the same time as $p_1 - d - \alpha - \beta < p_2 < p_1 - \alpha$. Hence, we simply need that $p_1 + p_a - d - \alpha - \beta \leq p_2 < p_1 - \alpha$.

Secondly, suppose that $\widehat{v} \geq 1$ (which necessarily requires that $p_a > 1$), so that $p_2 \leq p_1 + (2n-1)\alpha + p_a - d - \beta - 1$. In order for consumers expectations to be fulfilled, it must hold that $n_2 = 1$, so the condition becomes $p_2 \leq p_1 + \alpha + p_a - d - \beta - 1$. Recall that we also need $p_1 + (2n_2 - 1)\alpha - d - \beta < p_2 < p_1 + (2n_2 - 1)\alpha$, but given that we only consider cases such that $p_a \leq 1 + d + \beta$, the final restriction is $p_1 - d + \alpha - \beta < p_2 \leq p_1 + p_a - d - 1 + \alpha - \beta$.

Lastly, suppose now that $\widehat{v} \in (0,1)$, that is, $p_1 + p_a - d - 1 + (2n_2 - 1)\alpha - \beta < p_2 < p_1 + p_a - d + (2n_2 - 1)\alpha - \beta$. Then n_2 must equal the measure of consumers whose valuation is smaller than $\widehat{v}$ (since $U_{b_2} > U_{ab_1}(v) > U_{ab_2}(v)$ and $U_{b_2} > U_{b_1}$ for such consumers). That is, $n_2 = \Pr(v \leq \widehat{v})$ must hold in order for consumers expectations to be fulfilled (as required by Nash equilibrium behavior), so the fact that $\Pr(v \leq \widehat{v}) = \widehat{v}$ implies that we must have $n_2 = (2n_2 - 1)\alpha + p_a - d - \beta + p_1 - p_2$, which yields

$$n_2 = \frac{p_2 - p_1 - p_a + d + \alpha + \beta}{2\alpha - 1}.$$

The constraints that $p_1 + p_a - d - 1 + (2n_2 - 1)\alpha - \beta < p_2 < p_1 + p_a - d + (2n_2 - 1)\alpha - \beta$ and $p_1 - d + (2n_2 - 1)\alpha - \beta < p_2 < p_1 + \alpha(2n_2 - 1)$ imply that $\max(0, p_a - d - \beta) < n_2 < \min(1, p_a)$. We must have $p_1 + p_a - d - \alpha - \beta - (1 - 2\alpha)\max\{p_a - d - \beta, 0\} < p_2 < p_1 + p_a - d - \alpha - \beta - (1 - 2\alpha)\min\{p_a, 1\}$ when $\alpha > 1/2$.

To summarize, $d \leq p_1$ implies the following:

- $n_2 = 0$ if $p_2 \geq p_1 - \alpha - \max\{d + \beta - p_a, 0\}$. In this case, firm 1 makes sales of good a equal to $1 - (p_a - d - \beta)$, of good b_1 equal to 1, and firm 2 makes no sales.

- $n_2 = 1$ if $p_2 \leq p_1 - d + \alpha - \beta + \max\{p_a - 1, 0\}$. In this case, firm 1 makes sales of good a equal to $1 - p_a$, of good b_1 equal to 0, and firm 2 makes sales of good b_2 equal to 1.

- $n_2 = \dfrac{p_2 - p_1 + \alpha}{2\alpha}$ if $p_1 - \alpha < p_2 \leq p_1 - \alpha + 2\alpha(p_a - d - \beta)$ and $p_a \in (d + \beta, 1 + d + \beta)$. In this case, firm 1 makes sales of good a equal to $1 - (p_a - d - \beta)$, of good b_1 equal to $1 - n_2$, and firm 2 makes sales of good b_2 equal to n_2.

- $n_2 = \dfrac{p_2 - p_1 + d + \alpha + \beta}{2\alpha}$ if $p_1 + 2\alpha p_a - d - \alpha - \beta \leq p_2 < p_1 - d + \alpha - \beta$ and $p_a \in [0, 1)$. In this case, firm 1 makes sales of good a equal to $1 - p_a$, of good b_1 equal to $1 - n_2$, and firm 2 makes sales of good b_2 equal to n_2.

- $n_2 = \dfrac{p_2 - p_1 - p_a + d + \alpha + \beta}{2\alpha - 1}$ if $p_1 + p_a - d - \alpha - \beta + (2\alpha - 1)\max\{p_a - d - \beta, 0\} < p_2 \leq p_1 + p_a - d - \alpha - \beta + (2\alpha - 1)\min\{p_a, 1\}$. In this case, firm 1 makes sales of good a equal to $1 - n_2$, of good b_1 equal to $1 - n_2$, and firm 2 makes sales of good b_2 equal to n_2.

The graphical representation of firm 2's demand correspondence for both the present and the following case are relatively easy to draw and are not reported as this section is already very long. They are available upon request.

(ii) Consider now the situations in which $d > p_1$. Again, any pricing strategy for firm 1 that involves $p_a > 1+d+\beta$ is (weakly) dominated by a strategy with $p_a = 1+d+\beta$. Thus, in what follows we consider strategies such that $p_a \leq 1 + d + \beta$. Similarly, firm 1 cannot be playing any pricing strategy that involves $p_a < 0$ because it is (weakly) dominated by a strategy with $p_a = 0$. Finally, note when $d > p_1$ that we cannot have $p_1 < 0$, because in such a case a consumer buying b_1 would be subsidized by firm 1 without firm 1 benefiting from it. Moreover, we need to assume that $p_a + p_1 - d \geq 0$, *i.e.* the sum of prices charged by firm 1 cannot be negative.

Given that we can restrict attention to $p_a \in [0, 1+d+\beta]$, we distinguish the following cases that may arise based on the firms' pricing, analyzing the implications of each in constructing the firms' demand functions/correspondences:

(a) $U_{b_1} > U_{b_2}$.

(b) $U_{b_1} = U_{b_2}$.

(c) $U_{b_2} > U_{b_1}$.

Regarding case (a), $U_{b_1} > U_{b_2}$ implies that $U_{ab_1}(v) > U_{ab_1b_2}(v)$ for all v (since $p_1 + \beta \geq 0$), which in turn implies that $n_2 = 0$. Therefore, we have that $U_{b_1} > U_{b_2}$ if and only if

$$u + \alpha - p_1 > u - p_2.$$

This shows that $n_2 = 0$ can arise as firm 2's quantity demanded whenever $p_2 > p_1 - \alpha$ is met.

As for case (b), $U_{b_1} = U_{b_2}$ implies that $U_{ab_1}(v) > U_{ab_1b_2}(v)$ for all v whenever $p_1 > 0$ (we treat the case $p_1 = \beta = 0$ separately further below). Those consumers with valuation greater than $p_a - d - \beta$ purchase both a and b_1, whereas those consumers with valuation smaller than $p_a - d - \beta$ do not purchase a and are indifferent between b_1 and b_2. If $p_a \leq d + \beta$, then $n_2 = 0$ must hold. If instead $p_a \in (d + \beta, 1 + d + \beta]$, then the measure of consumers who purchase b_2 only cannot exceed the total number of consumers who do not purchase a and b_1, which is equal to $p_a - d - \beta$. Hence, $0 \le n_2 \le p_a - d - \beta$ must hold, with $U_{b_1} = U_{b_2}$ yielding that it must also hold that

$$p_2 - p_1 = (2n_2 - 1)\alpha.$$

Note that $n_2 = 0$ is also consistent with $p_a > d + \beta$. Then, we have shown that $n_2 = 0$ can arise as firm 2's quantity demanded if $p_2 = p_1 - \alpha$, and

$$n_2 = \frac{p_2 - p_1 + \alpha}{2\alpha} \in (0, p_a - d - \beta]$$

can arise as firm 2's quantity demanded if $p_a > d + \beta$ and $p_2 = p_1 + (2n_2 - 1)\alpha$, in which case the restrictions on n_2 imply that $p_1 - \alpha < p_2 \le p_1 - \alpha + 2\alpha(p_a - d - \beta)$.

When $p_1 = \beta = 0$ in case (b), note that $U_{b_1} = U_{b_2}$ implies that $U_{ab_1}(v) = U_{ab_1b_2}(v)$ for all v. Those consumers with valuation greater than $p_a - d$ purchase a and are indifferent between buying b_1 and b_2 as well, whereas those consumers with valuation smaller than $p_a - d - \beta$ do not purchase a and are indifferent between b_1 and b_2. Clearly, $0 \le n_2 \le 1$ must hold, with $U_{b_1} = U_{b_2}$ yielding that it must also hold that

$$p_2 = (2n_2 - 1)\alpha,$$

so

$$n_2 = \frac{p_2 + \alpha}{2\alpha}$$

can arise as firm 2's quantity demanded if $-\alpha \le p_2 \le \alpha$ whenever $p_1 = \beta = 0$.

Finally, with regards to case (c), $U_{b_2} > U_{b_1}$ implies that $p_2 < p_1 + (2n_2 - 1)\alpha$ must be satisfied. Depending on the comparison between $U_{ab_2}(v)$ and $U_{ab_1}(v)$, we have three subcases:

(c1) $U_{ab_1b_2}(v) > U_{ab_1}(v)$ for all v.

(c2) $U_{ab_1b_2}(v) = U_{ab_1}(v)$ for all v.

(c3) $U_{ab_1b_2}(v) < U_{ab_1}(v)$ for all v.

Note that, when $p_1 = \beta = 0$, only case (c1) is possible, whereas all three cases are possible when $p_1 + \beta > 0$.

In case (c1), $U_{b_2} > U_{b_1}$ and $U_{ab_1b_2}(v) > U_{ab_1}(v)$ for all v, which means that no consumer wants to consume b_1 even if they buy it, and therefore $n_2 = 1$. Inequality $U_{ab_1b_2}(v) > U_{ab_1}(v)$ implies $p_2 < (2n_2 - 1)\alpha - \beta$. Note that this condition is consistent with the condition for $U_{b_2} > U_{b_1}$, which is $p_2 < p_1 + (2n_2 - 1)\alpha$. Given that $n_2 = 1$, this case holds if $p_2 < \alpha - \beta$. Note that those consumers whose valuation exceeds $p_a + p_1 - d$ purchase all goods, whereas those consumers whose valuation is below $p_a + p_1 - d$ simply purchase b_2.

In case (c2), $U_{ab_1b_2}(v) = U_{ab_1}(v)$ implies $p_2 = (2n_2 - 1)\alpha - \beta$. Take into account that $U_{b_2} > U_{b_1}$, so any consumer with valuation smaller than $p_a + p_1 - d$ must purchase b_2 only, whereas any consumer with valuation greater than $p_a + p_1 - d$ must purchase firm 1's bundle (perhaps together with b_2). Therefore, the measure of consumers who purchase only b_2 must be greater than or equal to $\min\{1, p_a + p_1 - d\}$, so any $n_2 \in [\min\{1, p_a + p_1 - d\}, 1]$ can arise as firm 2's quantity demanded if $p_2 = (2n_2 - 1)\alpha - \beta$ holds. Note that $n_2 = 1$ is consistent with fulfilled expectations if $p_2 = \alpha - \beta$, and

$$n_2 = \frac{p_2 + \alpha + \beta}{2\alpha} \in [p_a + p_1 - d, 1)$$

is consistent if $-\alpha - \beta + 2\alpha(p_a + p_1 - d) < p_2 < \alpha - \beta$, and $0 \leq p_a + p_1 - d < 1$, as we previously specified.

In case (c3), $U_{ab_1b_2}(v) < U_{ab_1}(v)$ implies $(2n_2 - 1)\alpha - \beta < p_2 < (2n_2 - 1)\alpha + p_1$. Let

$$\widehat{v} \equiv p_a - d - \beta + (2n - 1)\alpha + p_1 - p_2$$

denote the unique value of v such that $U_{b_2} = U_{ab_1}(v)$ holds. Note that $\widehat{v} \in (p_a - d - \beta, p_a + p_1 - d)$, and since $p_a - d - \beta$ may be negative, we need to consider cases in which $\widehat{v}$ is above 0 and below 0. Similarly, $p_a + p_1 - d$ may exceed 1, so we need to consider cases in which $\widehat{v}$ is above 1 and below 1.

Firstly, suppose that $\widehat{v} \leq 0$ (which necessarily requires $p_a < d + \beta$), so that $p_a - d - \beta + (2n - 1)\alpha + p_1 - p_2 \leq 0$. Then, we must have $n_2 = 0$, so the condition becomes $p_1 + p_a - d - \beta - \alpha \leq p_2$, which must hold at the same time as $-\alpha - \beta < p_2 < p_1 - \alpha$, so we simply need that $p_1 + p_a - d - \beta - \alpha \leq p_2 < p_1 - \alpha$.

Secondly, suppose that $\widehat{v} \geq 1$ (which necessarily requires that $p_a + p_1 - d > 1$), so that $p_2 \leq p_1 + p_a - d - 1 + (2n - 1)\alpha - \beta$. In order for consumers expectations to be fulfilled, it must hold that $n_2 = 1$, so the condition becomes $p_2 \leq p_1 + p_a - d - 1 + \alpha - \beta$. Recall we also need $(2n_2 - 1)\alpha - \beta < p_2 < p_1 + (2n_2 - 1)\alpha$, but given that we only consider cases such that $p_a \leq 1 + d + \beta$, the final restriction is $\alpha - \beta < p_2 \leq p_1 + p_a - d - 1 + \alpha - \beta$. Note that $U_{b_2} > U_{ab_1b_2}(v)$ for all $v \in [0, 1]$ implies that $p_a + p_1 - d - \beta > 1$, so firm 1 does no sell any unit of good a.

Lastly, suppose now that $\widehat{v} \in (0, 1)$, that is, $p_1 + p_a - d - 1 + (2n - 1)\alpha - \beta < p_2 <$

$p_1+p_a-d+(2n-1)\alpha-\beta$. Then n_2 must equal the measure of consumers whose valuation is smaller than $\widehat{v}$ (since $U_{b_2} > U_{ab_1}(v) > U_{ab_1b_2}(v)$ and $U_{b_2} > U_{b_1}$ for such consumers). That is, $n_2 = \Pr(v \leq \widehat{v})$ must hold in order for consumers expectations to be fulfilled (as required by Nash equilibrium behavior), so the fact that $\Pr(v \leq \widehat{v}) = \widehat{v}$ implies that we must have $n = (2n-1)\alpha + p_a - d - \beta + p_1 - p_2$, which yields

$$n_2 = \frac{p_2 - p_1 - p_a + d + \alpha + \beta}{2\alpha - 1}.$$

The constraints that $p_1 + p_a - d - 1 + (2n-1)\alpha - \beta < p_2 < p_1 + p_a - d + (2n-1)\alpha - \beta$ and $(2n-1)\alpha - \beta < p_2 < p_1 + \alpha(2n-1)$ imply that $\max(0, p_a - d - \beta) < n_2 < \min(1, p_a + p_1 - d)$. We must have $p_1 + p_a - d - \beta - \alpha + (2\alpha - 1)\max\{p_a - d - \beta, 0\} < p_2 < p_1 + p_a - d - 1 - \beta - \alpha + (2\alpha - 1)\min\{p_a + p_1 - d - 1, 0\}$ when $\alpha > 1/2$.

We can now summarize the situations in which $d > p_1$:

- $n_2 = 0$ if $p_2 \geq p_1 - \alpha - \max\{d + \beta - p_a, 0\}$. In this case, firm 1 makes sales of good a equal to $1 - (p_a - d - \beta)$, of good b_1 equal to 1, and firm 2 makes no sales.
- $n_2 = 1$ if $p_2 \leq \alpha - \beta + \max\{p_1 + p_a - d - 1, 0\}$. In this case, firm 1 makes sales of good a equal to $1 - (p_a + p_1 - d)$, of good b_1 equal to $1 - (p_a + p_1 - d)$, and firm 2 makes sales of good b_2 equal to 1.
- $n_2 = \dfrac{p_2 - p_1 + \alpha}{2\alpha}$ if $p_1 - \alpha < p_2 \leq p_1 - \alpha + 2\alpha(p_a - d - \beta)$ and $p_a \in (d + \beta, 1 + d + \beta)$. In this case, firm 1 makes sales of good a equal to $1 - (p_a - d - \beta)$, of good b_1 equal to $1 - n_2$, and firm 2 makes sales of good b_2 equal to n_2.
- $n_2 = \dfrac{p_2 + \alpha + \beta}{2\alpha}$ if $2\alpha(p_a + p_1 - d) - \alpha - \beta \leq p_2 < \alpha - \beta$, and $0 \leq p_1 + p_a - d < 1$. In this case, firm 1 makes sales of good a equal to $1 - (p_a + p_1 - d)$, of good b_1 equal to $1 - n_2$, and firm 2 makes sales of good b_2 equal to n_2.
- $n_2 = \dfrac{p_2 - p_1 - p_a + d + \alpha + \beta}{2\alpha - 1}$ if $p_1 + p_a - d - \alpha - \beta + (2\alpha - 1)\max\{p_a - d - \beta, 0\} < p_2 < p_1 + p_a - d - \beta - \alpha - 1 + (2\alpha - 1)\min\{p_a + p_1 - d - 1, 0\}$. In this case, firm 1 makes sales of good a equal to $1 - n_2$, of good b_1 equal to $1 - n_2$, and firm 2 makes sales of good b_2 equal to n_2.

Appendix E

Denote equilibrium values using the "MS" superscript. Proposition 5 follows from Proposition 3 and the following lemma.

Lemma 1 (Mixed bundling with a specific complementarity). *An equilibrium exists. Equilibria are such that:*

(a) When $\alpha < \beta + c$ *and* $\alpha \leq 3/4 + \beta$, $p_2^{MS} = c$ *and any* $(p_1^{MS}, p_a^{MS}, d^{MS})$ *such that* $p_1^{MS} + p_a^{MS} - d^{MS} = 1 + \beta - \alpha$, *with* $1 \leq p_a^{MS} \leq d^{MS} + \beta$ *and* $d^{MS} \geq 1 - \beta$, *is an equilibrium; in any of such equilibria,* $n_2^{MS} = 0$, $\pi_1^{MS} = 1 + \beta - \alpha > 0$ *and* $\pi_2^{MS} = 0$.

(b1) When $\alpha \geq \beta + c$ *and* $c \leq (2 + \beta^2)/4$, $p_2^{MS} = \alpha - \beta$ *and any* $(p_1^{MS}, p_a^{MS}, d^{MS})$ *such that* $p_a^{MS} = 1/2$, *with* $p_1^{MS} - d^{MS} = 0$, *is an equilibrium; in any of such equilibria,* $n_2^{MS} = 1$, *no consumer buys good* b_1, $\pi_1^{MS} = 1/4$ *and* $\pi_2^{MS} = \alpha - \beta - c$.

(b2) When $c > (2 + \beta^2)/4$ *and* $\alpha \geq \beta + (2 + \beta^2)/4$, $p_2^{MS} = c + \alpha - [\beta + (2 + \beta^2)/4]$ *and any* $(p_1^{MS}, p_a^{MS}, d^{MS})$ *such that* $p_a^{MS} = 1/2$ *with* $p_1^{MS} - d^{MS} = c - (2 + \beta^2)/4$ *is an equilibrium; in any of such equilibria,* $n_2^{MS} = 1$, *no consumer buys good* b_1, $\pi_1^{MS} = 1/4$ *and* $\pi_2^{MS} = \alpha - [\beta + (2 + \beta^2)/4]$.

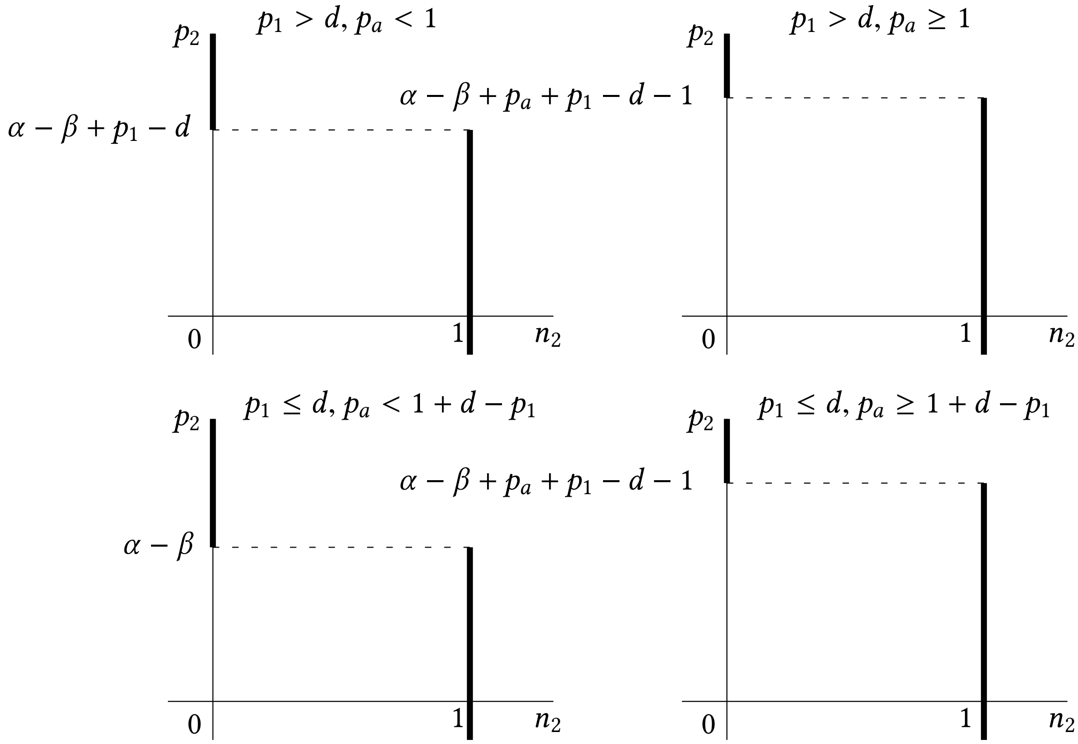
Proof. In our equilibrium analysis, we will make use of Figure C, which represents firm 2's demand function based on our previous analysis in Appendix D. Let us first try to sustain $n_2 = 1$ in equilibrium with no sales of good b_1. Clearly, in an equilibrium with no sales of good b_1, it must hold that $p_a^{MS} = 1/2$ and $p_2^{MS} = \alpha - \beta + \max\{0, p_1^{MS} - d^{MS}\} \geq c$ (see left panels in Figure C).

Case (i): Suppose first that $p_1^{MS} - d^{MS} > 0$. If firm 1 deviates by charging $p_a < 1$ and $p_1 > d$, then the best it can do is to charge $p_1 = p_2^{MS} + \beta - \alpha + d - \epsilon = p_1^{MS} + d - d^{MS} - \epsilon > d$ (see Figure C, top panel on the left) so as to earn

$$p_1^{MS} + d - d^{MS} + (p_a - d)[\min\{1, 1 + d + \beta - p_a\}] =$$
$$= \begin{cases} p_1^{MS} - d^{MS} + p_a & \text{if } p_a \leq d + \beta \\ p_1^{MS} + d - d^{MS} + (p_a - d)(1 + d + \beta - p_a) & \text{if } p_a > d + \beta \end{cases}$$

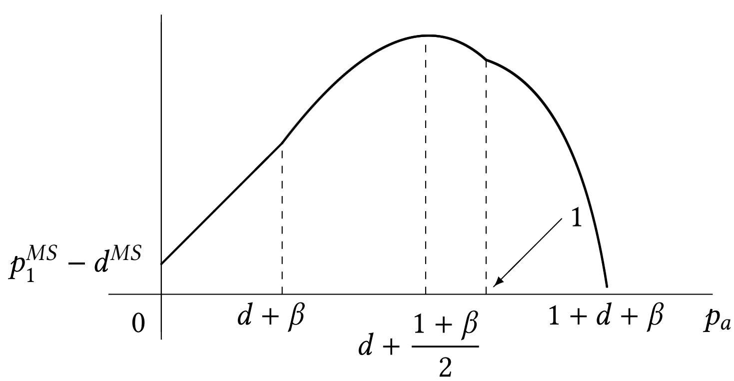
If instead firm 1 chooses $p_a \geq 1$ and $p_1 > d$ when deviating, clearly the best it can do is to charge $p_1 = p_2^{MS} + \beta - \alpha + d + 1 - p_a - \epsilon = p_1^{MS} + d - d^{MS} - (p_a - 1) - \epsilon$ (see Figure C, top panel on the right) so as to earn $p_1^{MS} + d - d^{MS} - c + 1 - p_a + (p_a - d)[\min\{1, 1 + d + \beta - p_a\}]$. Note that $p_1 > d$ if and only if $p_a < 1 + p_1^{MS} - d^{MS}$, which implies that, as soon as $p_a > 1 + p_1^{MS} - d^{MS}$ (and so $p_1 < d$), it holds that $p_1 + p_a - d = p_1^{MS} - d^{MS} + 1 - \epsilon > 1$ (see Figure C, bottom panel on the right), so that firm 1 continues to earn $p_1^{MS} + d - d^{MS} - c + 1 - p_a + (p_a - d)[\min\{1, 1 + d + \beta - p_a\}]$.

Figure C



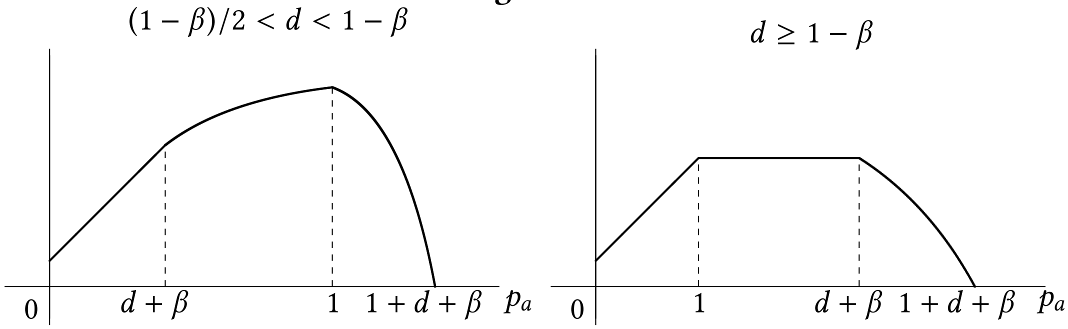
Recalling that $\beta < 1$, if $d \leq (1-\beta)/2$, then the payoff to deviating as a function of p_a is represented in Figure D:

Figure D



As a consequence, when $d \leq (1-\beta)/2$, the optimal deviation (taking d as exogenous) is given by $\widehat{p_1} = d + p_1^{MS} - d^{MS} - \epsilon > d$ and $\widehat{p_a} = d + (1+\beta)/2$ so as to earn $p_1^{MS} - d^{MS} + d - c + (1+\beta)^2/4$. The following figure represents the payoff to deviating as a function of p_a when $(1-\beta)/2 < d < 1 - \beta$ and $d \geq 1 - \beta$:

Figure E



Hence, when $(1-\beta)/2 < d$, the optimal deviation (taking d as exogenous) is given by $\widehat{p_1} = d + p_1^{MS} - d^{MS} - \epsilon > d$ and $\widehat{p_a} = 1$ so as to earn

$$p_1^{MS} - d^{MS} + d - c + (1-d)\min\{1, d+\beta\}.$$

Because $(1+\beta)^2/4 > (1-d)\min\{1, d+\beta\}$ for $d > (1-\beta)/2$, when maximizing firm 1's deviation profit with respect to d, we find that the optimal deviation involves $\widehat{d} = (1-\beta)/2$, and hence firm 1 can earn $p_1^{MS} - d^{MS} + (1-\beta)/2 - c + (1+\beta)^2/4$. Such a deviation is unprofitable if and only if $1/4 \geq p_1^{MS} - d^{MS} + (1-\beta)/2 - c + (1+\beta)^2/4$, that is, $c - (2+\beta^2)/4 \geq p_1^{MS} - d^{MS}$.

Case (ii): Suppose now that $p_1^{MS} - d^{MS} \leq 0$. Clearly, the only reason for firm 1 to deviate is to attract consumers in market B. Because $p_2^{MS} = \alpha - \beta$, it is clearly impossible for firm 1 to find $p_1 > d$ such that it ends up attracting demand in market B (see top panels of Figure C). Similarly, the fact that $p_2^{MS} = \alpha - \beta$ implies that it is impossible to find $p_1 \leq d$ and $p_a \geq 1 + d - p_1$ such that it ends up attracting demand in market B (see right bottom panel of Figure C). As a result, firm 1 has no profitable deviation available, so existence of an equilibrium with $n_2 = 1$ and no sales of good b_1 would simply require $\alpha \geq \beta + c$.

Note that ruling out weakly dominated strategies implies that $p_1^{MS} - d^{MS}$ must be as high as admissible based on our analysis of cases (i) and (ii). If $c \leq (2+\beta^2)/4$, then the conditions required for case (i) (namely, $p_1^{MS} - d^{MS} > 0$ and $p_1^{MS} - d^{MS} \leq c - (2+\beta^2)/4$) cannot possibly hold, so case (ii) implies that an equilibrium with $n_2 = 1$ and no sales of good b_1 exists if $\alpha \geq \beta + c$. If instead $c > (2+\beta^2)/4$, it must hold that $p_1^{MS} - d^{MS} = c - (2+\beta^2)/4 > 0$, so case (i) implies that that an equilibrium with $n_2^{MS} = 1$ and no sales of good b_1 exists if $\alpha \geq \beta + (\beta^2+2)/4$ (so that firm 2 has no incentive to deviate because $p_2^{MS} = \alpha - \beta + p_1^{MS} - d^{MS} \geq c$).

We have therefore shown that $c \leq (2+\beta^2)/4$ and $\alpha \geq \beta + c$ imply that the following equilibrium with $n_2^{MS} = 1$ and no sales of good b_1 exists: $p_a^{MS} = 1/2$, $p_1^{MS} - d^{MS} = 0$, and $p_2^{MS} = \alpha - \beta \geq c$, with $\pi_1^{MS} = 1/4$ and $\pi_2^{MS} = \alpha - \beta - c$. In turn, $c > (2+\beta^2)/4$ and $\alpha \geq \beta + (\beta^2+2)/4$ imply that the following equilibrium with $n_2^{MS} = 1$ and no sales of good b_1 exists: $p_a^{MS} = 1/2$, $p_1^{MS} - d^{MS} = c - (2+\beta^2)/4 > 0$, and $p_2^{MS} = c + \alpha - [\beta + (2+\beta^2)/4] \geq c$, with $\pi_1^{MS} = 1/4$ and $\pi_2^{MS} = \alpha - [\beta + (2+\beta^2)/4]$.

Let us now try to sustain $n_2 = 1$ in equilibrium with positive sales of good b_1 (but no usage of such a good). In such a case, we must have $p_a^{MS} + p_1^{MS} - d^{MS} = \min\{1, (1+c)/2\} < 1$ and $p_2^{MS} = \alpha - \beta$, with $p_1^{MS} - d^{MS} \leq 0$ (so $p_a^{MS} \geq (1-c)/2$). Clearly, we need $c < \alpha - \beta$ and $\alpha > \beta$. We also need that firm 1 has no profitable unilateral deviation. Note that it is impossible for firm 1 to profitably deviate by attempting to conquer market B through some $p_1 > d$ (perhaps changing p_a at the same time): otherwise, such a deviation should be accompanied by price $p_1 = p_2^{MS} + d + \beta - \alpha - \max\{0, p_a - 1\} - \epsilon = d - \max\{0, p_a - 1\} - \epsilon$ charged for b_1, which contradicts $p_1 > d$. So consider deviations such that $p_1 \leq d$ from now on. In such a case, firm 1 cannot find any price such that firm 2 loses its demand, since $p_2^{MS} = \alpha - \beta$ guarantees that firm 2 sells to all consumers in market B regardless of whether firm 1 charges prices such that $p_1 + p_a - d$ exceeds 1 or not (see Figure C, bottom panels). However, firm 1 can always neglect market B and focus on monopolizing market A so as to earn 1/4. Because $1/4 > (\max\{0, (1-c)/2\})^2$, there cannot exist an equilibrium with $n = 1$ and positive sales of good b_1 (but no usage of such a good).

We conclude by trying to sustain $n_2 = 0$ in equilibrium, noting that it must clearly hold that $p_2^{MS} = c$ in such an equilibrium (if it exists). Consider first the cases in which $c \leq \alpha - \beta$. For fixed d and p_a, suppose that firm 1 chooses to price according to $p_1 = p_2^{MS} + d + \beta - \alpha - \max\{0, p_a - 1\} - \epsilon$ (see top panels in Figure C, as well as the bottom panel on the right). Because $c \leq \alpha - \beta$, $p_1 - d = c + \beta - \alpha - \max\{0, p_a - 1\} - \epsilon < 0$, so we would need $p_a \geq 1 + d - p_1$ (see Figure C's bottom panel on the right). Because $p_1 = c + d + \beta - \alpha - \max\{0, p_a - 1\} - \epsilon$ and $c \leq \alpha - \beta$, it cannot possibly hold that $p_a \geq 1 + d - p_1$, so there cannot exist an equilibrium with $n = 0$ whenever $c \leq \alpha - \beta$.[16]

In sustaining $n_2 = 0$ as an equilibrium outcome, we consider henceforth the cases in which $c > \alpha - \beta$. Clearly, there cannot exist any incentive by firm 2 to unilaterally deviate from $p_2^{MS} = c$ because $c > \alpha - \beta$ and attracting all demand would require lowering the price.

[16]Another way to derive this result is to observe in Figure C that, regardless of firm 1's pricing, firm 2 can always charge $p_2 = \alpha - \beta$ and ensure some profit unless $\alpha - \beta < c$.

Next, note that $p_1 - d \leq -\max\{0, p_a - 1\}$ ensures that firm 1 gets all the demand in market B (see Figure C's bottom panel on the left), so that firm 1 earns

$$\pi_1(p_1, p_a, d) = p_1 - c + (p_a - d)[\min\{1, 1 + d + \beta - p_a\}].$$

Maximizing $\pi_1(p_1, p_a, d)$ with respect to p_1, p_a and d subject to the constraints that $p_1 \leq d$ if $p_a < 1$ or $p_1 - d \leq 1 - p_a$ if $p_a \geq 1$ yields that firm 1 earns a profit equal to $1 - c$. Proving this is quite elaborate, so we proceed in steps (roughly speaking, the idea is to successively maximize with respect to p_1, p_a and d in order to take carefully into account important nondifferentiability points of the profit function). Suppose first that $p_a < 1$. Keeping p_a and d fixed, $p_1 = d$ maximizes $\pi_1(p_1, p_a, d)$ with respect to p_1, so firm 1 must maximize $d - c + (p_a - d)[\min\{1, 1 + d + \beta - p_a\}]$ with respect to d and p_a. Keeping d fixed, it holds because $\beta < 1$ that firm 1's profit is maximized by

$$p_a^{MS} = \begin{cases} d + (1+\beta)/2 & \text{if } d \leq (1-\beta)/2 \\ 1 & \text{if } d > (1-\beta)/2 \end{cases}.$$

It follows that firm 1's profit equals $d + (1+\beta)^2/4$ if $d \leq (1-\beta)/2$ and $d + (1-d)[\min\{1, d + \beta\}]$ if $d \geq (1-\beta)/2$, so any $d^{MS} \geq 1 - \beta$ maximizes firm 1's profit, which is equal to $1 - c$.

Thus far, we have shown that any $d^{MS} \geq 1 - \beta$ maximizes firm 1's profit when $p_a < 1$, and the maximal profit equals $1 - c$. We now turn to the cases in which $p_a \geq 1$, so that $p_1 = 1 + d - p_a$ maximizes $\Pi_1(p_1, p_a, d)$ with respect to p_1. Then firm 1 must maximize $1 + d - p_a - c + (p_a - d)[\min\{1, 1 + d + \beta - p_a\}]$ with respect to d and p_a. Keeping d fixed, it holds because $\beta < 1$ that firm 1's profit is maximized by $p_a^{MS} = 1$ if $d \leq 1 - \beta$ and any $p_a^{MS} \in [1, d + \beta]$ if $d > 1 - \beta$. It follows that firm 1's profit as a function of d equals $d + (1 - d)\min\{1, d + \beta\}$, which is maximized for any $d^{MS} \geq 1 - \beta$. This shows that any $d^{MS} \geq 1 - \beta$ maximizes firm 1's profit when $p_a \geq 1$, and the maximal profit equals $1 - c$.

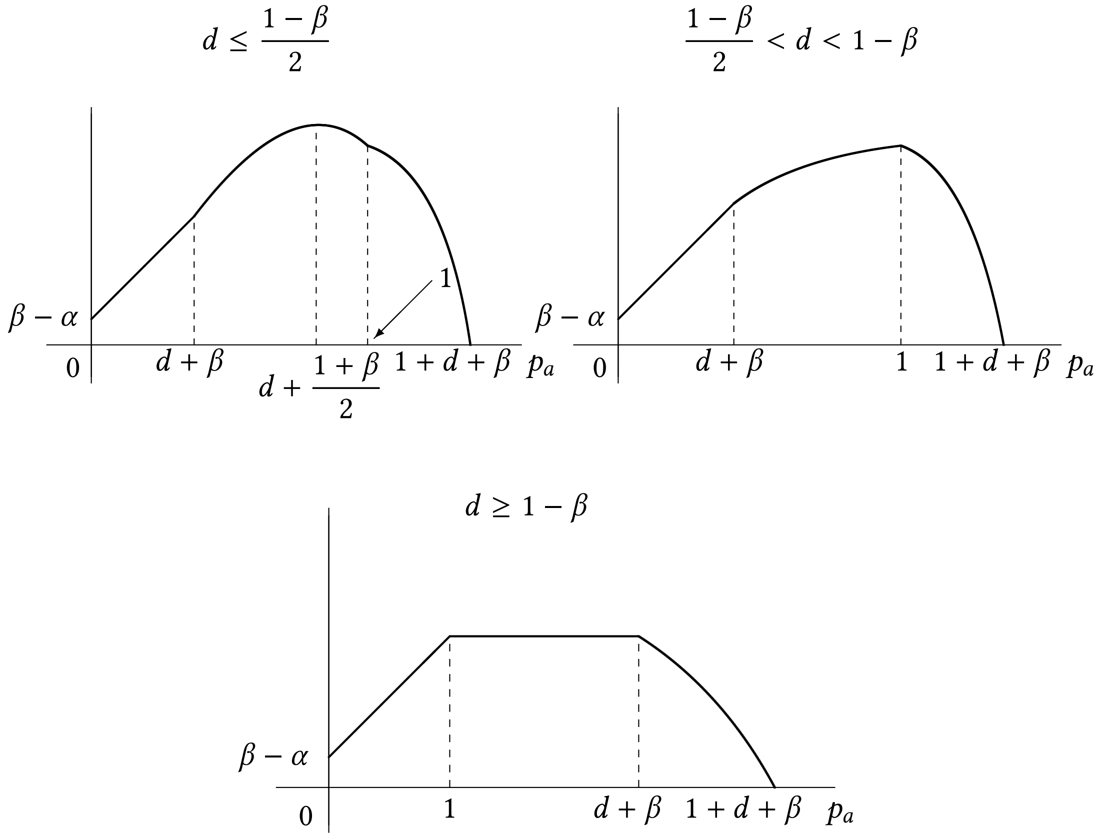
Having shown that firm 1 can earn $1 - c$ by (optimally) following a pricing strategy such that $p_1 - d \leq -\max\{0, p_a - 1\}$, we now show that it can earn more than $1 - c$ by (optimally) following a pricing strategy such that $p_1 - d > -\max\{0, p_a - 1\}$. To this end, suppose now that firm 1 follows a pricing strategy such that $p_1 - d > -\max\{0, p_a - 1\}$ (see all the panels in Figure C except the bottom left one). Given p_a and d, firm 1 must be choosing p_1 so that $p_1 = p_2^{MS} + \beta - \alpha + d - \max\{0, p_a - 1\} - \epsilon$ holds, thus earning

$$\pi_1(p_a, d) = p_2^{MS} + d + \beta - \alpha - \max\{0, p_a - 1\} - c + (p_a - d)[\min\{1, 1 + d + \beta - p_a\}].$$

Note that $p_1 - d > -\max\{0, p_a - 1\}$ is directly satisfied when $p_1 = p_2^{MS} + d + \beta - \alpha -$

$\max\{0, p_a - 1\} - \epsilon$ (for $\epsilon > 0$ small enough) because $c > \alpha - \beta$. Firm 1 then maximizes $\pi_1(p_a, d)$ with respect to p_a and d. To do so, we will first maximize $\pi_1(p_a, d)$ with respect to p_a keeping d fixed, find out the optimal value/s of p_a as a function of d, plug them into the profit function, and then maximize the resulting objective function with respect to d. Note that $\Pi_1(p_a, d)$ as a function of p_a is as represented in Figure F:

Figure F



Therefore, $\beta < 1$ implies that $\pi_1(p_a, d)$ is maximized with respect to p_a as follows: $p_a^{MS} = d + (1+\beta)/2$ if $d \leq (1-\beta)/2$, $p_a^{MS} = 1$ if $(1-\beta)/2 \leq d \leq 1-\beta$ and $p_a^{MS} \in [1, d+\beta]$ if $d \geq 1-\beta$. Firm 1's profit as a function of d is as follows:

$$\widehat{\pi}_1(d) = \begin{cases} \beta - \alpha + d + (1+\beta)^2/4 & \text{if } d \leq (1-\beta)/2 \\ \beta - \alpha + d + (1-d)(d+\beta) & \text{if } (1-\beta)/2 \leq d \leq 1-\beta \\ \beta - \alpha + 1 & \text{if } d \geq 1-\beta \end{cases} .$$

This function is clearly maximized for any $d^{MS} \geq 1-\beta$, since it is increasing. Therefore, any triplet $(p_1^{MS}, p_a^{MS}, d^{MS})$ such that $p_1^{MS} + p_a^{MS} - d^{MS} = \beta - \alpha + 1$, $1 \leq p_a^{MS} \leq d^{MS} + \beta$ and $d^{MS} \geq 1 - \beta$ maximizes firm 1's profit, so that firm 1 earns $\beta - \alpha + 1$. Clearly, $c > \alpha - \beta$

implies that this profit exceeds the one that firm 1 could earn by setting prices optimally under the constraint that $p_1 - d \leq -\max\{0, p_a - 1\}$ (such a profit was equal to $1 - c$). Having shown that firm 1 has no incentive to deviate from this pricing if it is to make sales of good b_1, we need to rule out incentives of such a firm to disregard market B and simply focus on monopolizing market A: in order for $\beta - \alpha + 1 \geq 1/4$, it is necessary that $\alpha \leq \beta + 3/4$ holds. ■

Part (a) of Lemma 1 shows that firm 1 conquers market B by inducing pure bundling of the two goods it sells through the three prices it charges, since $p_a^{MS} \geq 1$. Even though the pricing when no bundling is pursued can be replicated under mixed bundling (by setting the discount equal to 0), firm 1 cannot commit to not offering a discount, and it always has an incentive to use such a lever when competing for market B. So having more degrees of freedom under mixed bundling turns firm 1 a softer competitor for market B and allows firm 2 to defend such a market more easily. Lemma 1 follows.

Appendix F

In order to construct firm 2's demand correspondence, we consider the following cases:

(a) $U_{b_1} > U_{b_2}$.

(b) $U_{b_1} = U_{b_2}$.

(c) $U_{b_2} > U_{b_1}$.

Regarding case (a), $U_{b_1} > U_{b_2}$ implies that $U_{ab_1}(v) > U_{ab_2}(v)$ for all v, which in turn implies that $n_2 = 0$. Therefore, we have that $U_{b_1} > U_{b_2}$ if and only if

$$u + \alpha - p_1 > u - p_2.$$

This shows that $n_2 = 0$ can arise as firm 2's quantity demanded whenever $p_2 > p_1 - \alpha$ is met.

As for case (b), $U_{b_1} = U_{b_2}$ implies that $U_{ab_1}(v) = U_{ab_2}(v)$ for all v. Those consumers with valuation greater than $p_a - \beta$ purchase a together with either b_1 or b_2, whereas those consumers with valuation smaller than $p_a - \beta$ do not purchase a and are indifferent between b_1 and b_2. Since $U_{b_1} = U_{b_2}$ yields that it must hold that

$$p_2 = p_1 + (2n - 1)\alpha,$$

with $0 \leq n_2 \leq 1$, we have then shown that $n_2 = 0$ can arise as firm 2's quantity demanded if $p_2 = p_1 - \alpha$, $n_2 = 1$ can arise as firm 2's quantity demanded if $p_2 = p_1 + \alpha$, and

$$n_2 = \frac{p_2 - p_1 + \alpha}{2\alpha}$$

if $p_1 - \alpha < p_2 < p_1 + \alpha$.

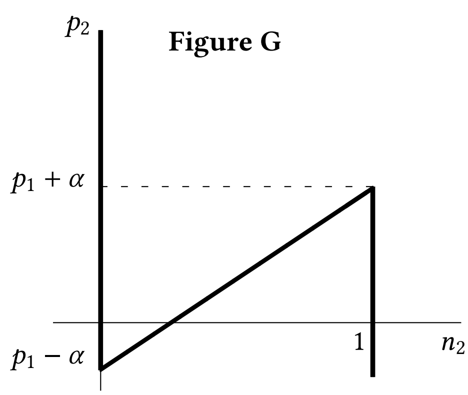
Finally, with regards to case (c), $U_{b_2} > U_{b_1}$ implies that $U_{ab_2}(v) > U_{ab_1}(v)$ for all v, so no consumer wants to buy b_1, and therefore $n_2 = 1$. Also, $U_{b_2} > U_{b_1}$ yields that $p_2 < p_1 + (2n_2 - 1)\alpha$ must be satisfied. Given that $n_2 = 1$, this case holds if $p_2 < p_1 + \alpha$.

To sum up, we have shown that firm 2's demand correspondence is as follows:

- $n_2 = 0$ for $p_2 \geq p_1 - \alpha$.
- $n_2 = 1$ for $p_2 \leq p_1 + \alpha$.
- $n_2 = \dfrac{p_2 - p_1 + \alpha}{2\alpha}$ for $p_1 - \alpha < p_2 < p_1 + \alpha$.

Figure G represents firm 2's demand correspondence.

Figure G



Appendix G

In order to find out firm 2's demand correspondence, note that charging $p < 0$ is weakly dominated, so we focus on $p \geq 0$ in what follows. We cover all the possible cases that can arise:

- Suppose $U_{ab_1}(v) > U_{ab_2}(v)$ and $U_{ab_1}(v) > U_{b_2}$ for all $v \in [0, 1]$. Then $n_2 = 0$ whenever $p_2 > -\alpha$ and $p_2 > -(v-p)-\alpha-\beta$ for $v = 0$, that is, whenever $p_2 > p-\alpha-\beta$.

So it holds that $n_2 = 0$ whenever $p \geq \beta$ and $p_2 - p > -\alpha - \beta$ or whenever $p < \beta$ and $p_2 > -\alpha$.

- Suppose $U_{b_2} \geq U_{ab_1}(v)$ and $U_{b_2} > U_{ab_2}(v)$ for all $v \in [0,1]$. Then $n_2 = 1$ and $p > v+\beta$ for all $v \in [0,1]$ as well as $\alpha - \beta - p_2 \geq v - p$ for all $v \in [0,1]$. Hence, $n_2 = 1$ can arise whenever $p \geq 1 + \beta$ and $p_2 - p \leq \alpha - \beta - 1$. Firm 1 makes no sale of good a.

- Suppose $U_{ab_2}(v) > U_{ab_1}(v)$ for all $v \in [0,1]$. Then it must hold that $n_2 = 1$ and $p_2 < \alpha$. Those consumers with $v > p - \beta$ purchase both a and b_2, and those with $v \leq p - \beta$ purchase b_2 only. For $\beta < p \leq 1 + \beta$, firm 1 makes sales of good a equal to $1 + \beta - p \in [0,1]$, whereas for $p \leq \beta$, firm 1 makes sales of good a equal to 1.

- Suppose $U_{ab_1}(v) = U_{ab_2}(v)$ for all $v \in [0,1]$. Then we must have $\alpha(1-n_2) = \alpha n_2 - p_2$. If there exists $\widehat{v} \in (0,1)$ such that $U_{ab_2}(v) > U_{b_2}$ for $v > \widehat{v}$ and $U_{ab_2}(v) < U_{b_2}$ for $v < \widehat{v}$, then it must be the case that $p \in (\beta, 1+\beta)$ so that $\widehat{v} = p - \beta$. Then

$$n_2 = \frac{p_2 + \alpha}{2\alpha} \in [0,1]$$

  for $-\alpha \leq p_2 \leq \alpha$. We should also have that $p_2 \geq 2\alpha p - \alpha - 2\alpha\beta$ in order for $\widehat{v} \leq n_2$ (so that consumer expectations are fulfilled), so $p \in (\beta, 1+\beta)$ implies that it should hold that $2\alpha p - \alpha - 2\alpha\beta \leq p_2 \leq \alpha$. Firm 1 makes sales of good a equal to $1 + \beta - p$. If instead $\widehat{v} = 1$, which means that $p \geq 1 + \beta$, then firm 1 makes no sale of good a and on top of that $n_2 = 1$. If instead $\widehat{v} = 0$, which means that $p \leq \beta$, then firm 1 makes sales of good a equal to 1.

- Suppose that $U_{ab_1}(v) > U_{ab_2}(v)$ for all $v \in [0,1]$ and that there exists $\widehat{v} \in (0,1)$ such that $U_{ab_1}(v) > U_{b_2}$ for $v > \widehat{v}$ and $U_{ab_1}(v) < U_{b_2}$ for $v < \widehat{v}$. In this case $\widehat{v} - p + \alpha(1-n) + \beta = \alpha n_2 - p_2$, with $\widehat{v} = n_2$ because of rational expectations, so $\alpha - p + p_2 + \beta = n_2(2\alpha - 1)$. Therefore, $\alpha > 1/2$ implies that

$$n_2 = \frac{p_2 - p + \alpha + \beta}{2\alpha - 1} \in (0,1)$$

  for $p - \alpha - \beta < p_2 < p + \alpha - \beta - 1$. We also need $\alpha n_2 - p_2 < \alpha(1-n_2)$, that is, $p_2 < 2\alpha p - \alpha$. Therefore, the condition that must be satisfied is $p - \alpha - \beta < p_2 < \min\{2\alpha p - \alpha, p + \alpha - \beta - 1\}$. Firm 1 makes sales of a equal to $1 - n_2$.

To sum up, we have proven that firm 2's demand correspondence is as follows:

- $n_2 = 0$ for $p_2 > -\alpha + \max\{p - \beta, 0\}$. In this case, firm 2 makes no sales of good b_2 and firm 1's sales of the bundled good equal 1.

- $n_2 = 1$ for $p \geq 1 + \beta$ and $p_2 \leq \alpha - \beta + p - 1$. In this case, firm 2's sales of good b_2 equal 1 and firm 1's sales of the bundled good equal 0.

- $n_2 = 1$ for $p < 1 + \beta$ and $p_2 \leq \alpha$. In this case, firm 2's sales of good b_2 equal 1 and firm 1's sales of the bundled good equal $1 + \beta - p$ if $p > \beta$, and equal 1 if $p \leq \beta$.

- $n_2 = \frac{p_2 + \alpha}{2\alpha} \in [0, 1]$ for $2\alpha p - \alpha - 2\alpha\beta \leq p_2 \leq \alpha$, which requires that $p \in (\beta, 1 + \beta)$. In this case, firm 2 makes n sales of good b_2, and firm 1's sales of the bundled good equal $1 + \beta - p$.

- $n_2 = \frac{p_2 - p + \alpha + \beta}{2\alpha - 1} \in [0, 1]$ for $p - \alpha - \beta < p_2 < \min\{2\alpha p - \alpha, p + \alpha - \beta - 1\}$. Firm 2 makes n sales of good b_2, and firm 1's sales of the bundled good equal $1 - n_2$.

Appendix H

Consider first the optimal decision of firm 1. Based on Proposition 3, we know that $\pi_1^{PS} \geq \pi_1^{US}$ when $\alpha \leq \min\{3/4, c\} + \beta$, whereas from Proposition 7 we know that $\pi_1^{UG} \geq \pi_1^{US}$ when $\alpha \geq \beta$. It is therefore immediate to conclude that unbundled pricing with a specific complementarity can never be optimal, given that $\beta < \min\{3/4, c\} + \beta$. We use a similar argument to eliminate pure bundling with a general complementarity, even though it requires more calculations. Indeed, we only know from Proposition 9 that $\pi_1^{UG} > \pi_1^{PG}$ when $\alpha > \min\{c, (3 - \beta)(1 + \beta)/4\}$, but no other comparisons are provided for π_1^{PG} in the remaining parametric region of α, so we focus on comparing π_1^{PG} with π_1^{PS}. As $\pi_1^{PS} = \pi_1^{PG}$ when $\alpha \leq \min\{c, 1 + \beta\}$, the assumption that there is some (arbitrarily small) cost of sharing the complementarity with firm 2 implies that we take π_1^{PS} to be greater than π_1^{PG} when $\alpha \leq \min\{c, 1 + \beta\}$. If $\alpha > \min\{c, 1 + \beta\}$, $\pi_1^{PS} > \pi_1^{PG}$ when $c \geq 1 + \beta$, and therefore pure bundling with a general complementarity can never be optimal. When $c < 1 + \beta$, $\pi_1^{PS} > \pi_1^{PG}$ if and only if $\alpha < \min\{1 + \beta - (1 - c + \beta)^2/4, \min\{1, c\} + \beta\}$. Yet, pure bundling with a general complementarity can be discarded also in this case because: (i) $\min\{c, (3 - \beta)(1 + \beta)/4\} < \min\{1 + \beta - (1 - c + \beta)^2/4, \min\{1, c\} + \beta\}$ when $c < 1 + \beta$; and (ii) $\pi_1^{UG} > \pi_1^{PG}$ in the parameter region in which $\pi_1^{PG} > \pi_1^{PS}$, namely, $\alpha > \min\{1 + \beta - (1 - c + \beta)^2/4, \min\{1, c\} + \beta\}$. As a consequence, pure bundling with a general complementarity cannot represent the optimal decision for firm 1, which is therefore left with the decision between unbundled pricing with a general complementarity and pure bundling with a specific complementarity. The first part of Proposition 12 directly follows from comparing π_1^{UG} with π_1^{PS}.

Now let us consider social welfare. Proposition 4 explicitly states that $w^{PS} \geq w^{US}$ if and only if $\alpha \leq \min\{1, c\} + \beta$. Proposition 7 is based on the fact that $w^{UG} \geq w^{US}$ when $\alpha \geq \min\{\beta(6+\beta)/4, \beta + c\}$; when $\alpha < \min\{\beta(6+\beta)/4, c+\beta\}$, both alternatives provide the same level of total welfare, so we conclude that unbundled pricing with a specific complementarity is preferred over unbundled pricing with a general complementarity if there is some (small) cost of sharing the complementarity. However, unbundled pricing with a specific complementarity can never be optimal for society at large, given that $\min\{\beta(6+\beta)/4, \beta+c\} \leq \min\{1, c\} + \beta$. Hence, $w^{PS} > w^{US}$ if $\alpha < \min\{\beta(6+\beta)/4, \beta+c\}$, the only region where $w^{US} > w^{UG}$. We use a similar argument to show that pure bundling with a general complementarity may not represent either the best outcome in terms of social welfare. Firstly, Proposition 10 indicates that $w^{UG} > w^{PG}$ when $\alpha > \min\{c, 1+\beta\}$. Secondly, we compare w^{PS} and w^{PG}, whose complete expressions are as follows:

$$w^{PS} = \begin{cases} u + \alpha - c + \frac{1}{2} + \beta \text{ iff } \alpha < \min\{1, c\} + \beta; \\ u + \alpha + [3 - c(14 - 3c]/8 \text{ iff } \alpha \geq c + \beta, \text{ for } c < 1; \\ u + \alpha - c \text{ iff } \alpha \geq 1 + \beta, \text{ for } c \geq 1. \end{cases}$$

$$w^{PG} = \begin{cases} u + \alpha - c + \frac{1}{2} + \beta \text{ iff } \alpha < \min\{c, 1 + \beta\}; \\ u + \alpha - c + [3(1 + \beta - c)^2]/8 \text{ iff } \alpha \geq c, \text{ for } c < 1 + \beta; \\ u + \alpha - c \text{ iff } \alpha \geq 1 + \beta, \text{ for } c \geq 1 + \beta. \end{cases}$$

If there is some (arbitrarily small) cost of sharing the complementarity, we immediately obtain that $w^{PS} > w^{PG}$ when: (i) it holds that $c \geq 1 + \beta$; and (ii) it holds that $\alpha < \min\{c, 1+\beta\}$, since $\min\{c, 1+\beta\} = c < \min\{1, c\} + \beta$ when $c < 1 + \beta$. Moreover, we find that $w^{PS} > w^{PG}$ also for $\alpha \in [c, \min\{1, c\} + \beta)$, meaning that $w^{PS} > w^{PG}$ when $\alpha < \min\{1, c\} + \beta$. Finally, precisely because $\min\{c, 1+\beta\} = c < \min\{1, c\} + \beta$ when $c < 1 + \beta$, it holds that $w^{UG} > w^{PG}$ in the parametric region where $w^{PG} \geq w^{PS}$, *i.e.* for $\alpha \geq \min\{1, c\} + \beta$. It follows that pure bundling with a general complementarity cannot be socially optimal. As a consequence, likewise the private decision of firm 1, the two remaining alternatives are unbundled pricing with general complementarity and pure bundling with specific complementarity. The second part of Proposition 12 can be easily obtained by comparing w^{UG} with w^{PS}.

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