Memory and Markets*

Sergei Kovbasyuk [†] Giancarlo Spagnolo [‡]

September 14, 2016

Abstract

We analyze the effects of erasing past records on long-run outcomes in a dynamic market with heterogeneous sellers whose quality changes with time. Buyers leave positive or negative feedback on sellers with an information intermediary. When average seller quality is low, perfect records of past feedback lead to low information production and no trade in the long run. Limited records encourage information production and sustain stationary equilibria with trade when memory of positive records is short and memory of negative ones is long. The stationary equilibrium with the highest social welfare requires the memory of negative records to be limited.

Keywords: Limited records, asymmetric information, rating systems, credit registers, privacy, online reputation mechanisms.

JEL classification: D82, D53, G20, G28, K35, L14, L15.

*We are grateful to Fernando Alvarez, Marco Battaglini, Alessandro Bonatti, Margherita Bottero, Piero Gottardi, Veronica Guerrieri, Hugo Hopenhayn, Johannes Hörner, Patrick Legros, Marco Ottaviani, Facundo Pigguilem, Andrea Pratt, Tomás Rodríguez Barraquer, Maryam Saeedi, Andy Skrzypacz, Steven Tadelis, Daniele Terlizzese and Laura Veldkamp for insightful remarks and suggestions. We also thank participants in seminars at EIEF, Tor Vergata, EUI, Bocconi, the AEA annual meeting 2015, ESSET Gerzensee 2015, CRESSE 2015, Barcelona Graduate Forum 2016, SITE 2016, and AMES 2016 meeting in Kyoto.

†EIEF, skovbasyuk@gmail.com

‡SITE-Stockholm School of Economics, EIEF & CEPR.

Introduction

In many environments, market participants have the possibility to leave feedback on their counterparts that can be observed by other market participants in the future. Airbnb, Amazon, Ebay, Tripadvisor, Uber, Yelp, and other electronic platforms collect and publicize feedback on sellers and buyers. Credit bureaus collect and make information on borrowers' past financial transactions accessible to lenders. A natural question arises in these environments: For how long should records of past feedback be retained and made publicly accessible? And does the answer depend on the type of recorded information? Analogous questions arise in the privacy debates on how long past (e.g. juvenile) offenses should be accessible to potential employers, and on "the right to be forgotten" regarding records present on the Internet and made available by search engines.

In this paper we try to shed light on these fundamental questions by studying how the timespan of past records provided by an information intermediary affects the amount of information and trade prevailing in a market in the long run. We develop and analyze a continuous-time model of a dynamic market populated by long-lived sellers of unknown quality, and short-lived buyers. At any point in time the seller can be of "high" or "low" quality and his quality changes stochastically over time. After transacting, the buyers learn the seller's quality and can leave "positive" or "negative" public feedback, with an information intermediary which makes the record accessible to future cohorts of buyers.[1] We focus on the most interesting case when the role of information intermediary is crucial, i.e. when the average quality of sellers is poor so that with no information on individual sellers there is no trade, and study the effects of different lengths of public memory for positive and negative records, respectively, on equilibrium and welfare in the long run. Distinguishing the memory of positive from that of negative records is a novel feature of our analysis that turns out to be critical to understand the effect of limited records on market outcomes.

First, we show that unlimited memory of past records may have dramatic negative consequences in the long run. Our first result is that perfect memory of past records necessarily leads to (almost) no trade in the long run (Theorem 1). This happens even if the market starts with full information about sellers. The reason is simple. A seller with a negative record does not trade because the buyers' willingness to pay for his product

[1] The distinction between positive and negative records is self-evident in financial markets, where credit records may include "black" (credit remarks, past arrears, defaults and bankruptcies) and "white" information (patterns of repayments, open and closed credit accounts, new loans, debt maturity, guarantees and assets); and in electronic markets (e.g. eBay's positive and negative feedbacks). But as we explain later, the distinction is relevant for most environments.

is below the seller's reservation value/cost. A seller without a negative record may trade and get feedback from the buyer. Since sellers' types change over time, in the long run each good seller may become bad and get a negative record. Because of perfect memory, from that moment such sellers are out of the market forever. As time goes to infinity, each seller is almost surely excluded from trading and the fraction of sellers trading converges to zero. Hence, in this environment, perfect memory of past feedback necessarily leads to market breakdown.

Our second main result is that in the same poor market conditions, a stationary equilibrium with a constant, positive fraction of sellers trading at each moment in time can instead be sustained, if past records are limited in a specific way. With limited records, market breakdown can be avoided by retaining past negative records for a sufficiently long time but deleting positive records quickly or not recording them at all (Theorems 2 and 4). The mechanism behind this second finding is also rather intuitive. In our model, longer memory of negative records allows the bad sellers to be identified and kept out of the pool of sellers with no records for longer, thereby improving the average quality of that pool. This encourages buyers to offer higher prices to sellers with no records and gives the latter a chance to trade and get feedback. A longer memory of positive records, instead, allows more good sellers to separate from the pool of sellers with no records, thereby discouraging trade with sellers with no records and the production of information on their quality. This makes stationary equilibria with trade harder to sustain. The proposed limits to records - short for positives and long for negatives - encourage buyers to "experiment" by buying from sellers with no records, producing new information and leading to better long-run outcomes.[2]

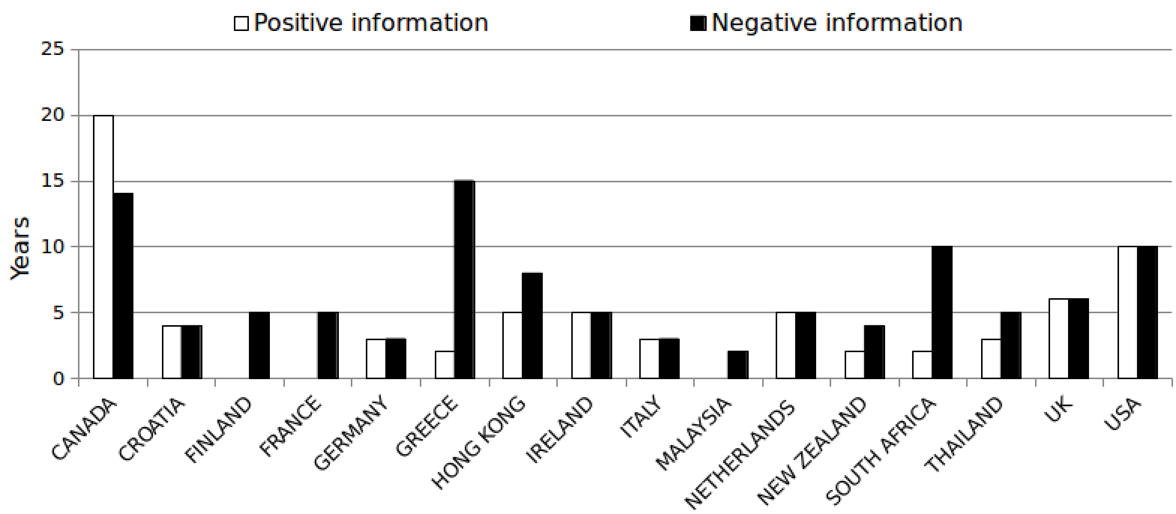
Our third main result concerns about the optimal memory policy that maximizes social welfare in our environment. Provided that trade can be sustained in a stationary equilibrium, social welfare is maximized when negative feedback is deleted from public records after a specific amount of time. In other words, if it does not compromise equilibrium existence, then it is socially beneficial to also limit the memory for negative records (Theorem 3). Keeping negative records for a long time excludes bad sellers from the market and sustains trade in the long run, as our previous results have shown. However, excluding sellers for too long involves a social loss: after enough time the seller's type will likely have changed and it will be socially optimal to trade with the seller in order to learn his

[2] An ancillary result worth noting here is that positive and negative records not only differ in their opposite effect on equilibrium existence, they also differ in terms of the intensity of their effects: negative records simply prevent the seller from trading for a given amount of time, while positive records have a non-linear, self-enforcing effect because they induce further trade and new records right away.

true type. There is a positive social value of learning the seller's type, associated with the information generated by buyers that purchase and leave feedback. However, individual buyers do not internalize this social value of information and do not buy from buyers that have old negative feedback even if this is socially optimal. By deleting old negative feedback it is possible to induce buyers to buy from those sellers: once negative feedback is deleted the seller is pooled with good sellers and is given a fresh start in the market. This result shows that due to the information externality, the market may not "forgive" sellers with negative feedback and a policy intervention requiring the market to "forget" negative feedback can be beneficial.[3]

Although our model is rather abstract, we believe our results have important policy implications. Informational problems are among the main sources of market and government failures. Increasing the amount of information available is often regarded as a natural remedy. Privacy concerns, on the other hand, have been at the center of several recent debates on electronic markets and the Internet, particularly after Erik Snowden's revelations. To some, a privacy regulation that, for example, mandates the removal of data from the public domain is simply unjustified because it opens the door to more "fraud".[4] Many disagree with this view. We believe our results bring novel, theoretically grounded arguments to this important debate.

Figure 1: Retention periods for credit records



[3]Here we use the language of Elul and Gottardi (2015) who how forgetting past information can improve incentives for borrowers in a credit market.

[4]From Posner's blog, 8th May 2005. http://www.becker-posner-blog.com/2005/05/index.html
See also Posner (1983) and Nock (1993) .

While little economic theory was available to guide policy on how long "public memory" should be in different environments and for different types of records, regulation on data retention has been in place for quite some time in many countries. Since regulation could not be based on much rigorous research about its likely effects, it is not surprising that it has remained rather generic.[5] Credit markets exemplify the variation in adopted retention policies that this lack of theoretical guidance and generic regulation has led to. Figure 1 plots the number of years after which positive and negative information about borrowers must be erased by credit bureaus for a handful of countries. Positive information generally contains the pattern of repayments, open and closed credit accounts and new loans, while negative information is about defaults, bankruptcies, delinquencies, arrearls. As one can see from Figure 1, retention limits differ substantially even among similar countries.

Internet platforms collecting feedback on participants also have to decide for how long to leave past records accessible, and how much memory to assign to summary indicators. For example, in 2008 eBay changed its reputational indicator and made it a function of feedback left in the last year only, instead of all past records, while still leaving all past feedback accessible but at the cost of some search effort.

Even though excessively long memory of records may have the negative aggregate/social effects we described, in our environment these records are still valuable to individual buyers, so strong private incentives exist for intermediaries to collect and distribute them. Therefore, our model suggests that privacy regulation limiting data retention of negative and positive records separately may be necessary or desirable. Our analysis also suggests that the current regulation, where present, may not be optimal, at least in markets where average quality is poor. Furthermore, the current trend of credit bureaus increasingly collecting positive records (in addition to the usual negative ones) may end up having harmful long-term consequences in terms of aggregate credit market outcomes. All these positive past records may make it very difficult for borrowers without any record to enter (or re-enter) the market and obtain credit in the first place, even if their repayment likelihood is very high.[6] Similarly, our results suggest that eBay should at least have considered the possibility of assigning a different memory to positive and negative feedback in its new

[5]For instance, the EU Directive on Data Protection (95/46/EC) mandates that "[M]ember States shall provide that personal data must be [...] adequate, relevant and not excessive in relation to the purposes for which they are collected and/or further processed".

[6]A recent New York Times article by Patricia Cohen (Oct. 10, 2014) documents how difficult it is for people with no credit history to obtain a first loan in the highly informed US credit market, leading them to form neighborhood-based rotating saving and credit associations to obtain their first records. This suggests that the long-term effects we highlight may actually manifest themselves rather quickly in certain markets.

reputation indicator, and should consider limiting the history of the feedback it makes accessible to buyers.[7]

For other environments, it may be more difficult to distinguish, ex ante, positive from negative information. For example, frequently switching jobs may be considered negative information by some and positive by others. This, however, does not mean that our results do not apply. Our results suggest, for example, that the spring 2014 ruling of the European Court of Justice which forces Google to cancel the past records of people that ask for it under certain (unclear) conditions is not necessarily benefiting society. The reason is that people know which of their past records are good or bad given their private situation and future plans, and are likely to ask for the removal of negative past information only. The Court's decision represnts a step in the right direction by avoiding an excessively long memory for negative records, but it also allows a very long memory for positive records, which may not be desirable (as we show in this paper).

Criminal records typically do not contain "positive" past records, but focus on negative information only. Police and court databases tend to automatically store only negative past information, and in many countries there are (different) rules that limit for how long these can be accessed. These policies appear closer to what our model suggests could be optimal.

The paper unfolds as follows. The next section reviews the related literature. Section 2 presents our general model. Section 3 studies the case of perfect past records, and Section 4 is devoted to limited records. In Section 5 we analyze welfare. Section 6 develops several extensions of the model and Section 7 briefly concludes.

1 Related literature

The mechanisms unveiled by our model are relevant to markets with hidden information, whether or not there are problems of adverse selection, as they are based on i) the positive informational externality generated by a buyer when he trades with a seller and leaves feedback; and ii) the reduced incentives to experiment with sellers without ratings when many sellers with positive ratings are available, thanks to the long memory for positive feedback. Our paper is thus related to the literature on experimentation in markets, including Bergemann and Välimäki (1996, 2000), Bolton and Harris (1999), Keller and Rady (1999), Keller et al. (2005) and Kremer et al. (2014), among others. The relationship is not immediate because this literature focuses mostly on *strategic* experimentation at the

[7]Moreover, on eBay the feedback is not verifiable, so the possibility and incentives to provide false feedback, such as buyers blackmailing sellers, may make a long memory of feedback not advisable.

agent level linked to oligopolistic competition and/or the intertemporal "encouragement effect" of experimenting today to induce others to do it tomorrow, and mainly analyzes short-term experimentation policies. In contrast to this literature, our paper focuses on competitive effects at the economy level and on long-term stationary market outcomes. In common with this literature, in our model buyers produce a positive informational externality when buying and leaving feedback for other buyers, so that information is an under-provided public good. However, in our setting incentives to experiment are not provided strategically by a profit-maximizing market participant (as all agents in our model are small and competitive and do not internalize any effects of information production), but are rather provided by the information intermediary, which puts sellers of different types into one "unknown" pool and ensures that the composition of this pool is good enough that the buyers want to buy from it thereby producing feedback. Limits to the memory of past feedback are then able to provide incentives to buyers to experiment more by trading with sellers in the pool.

Markets with adverse selection have been studied intensively following the recent financial crisis.[8] From this side, our work is probably most related to papers focusing on the effects of different information rules on the performance of dynamic lemon markets. For example, Hörner and Vieille (2009) study the role of availability of information on previous offers in a dynamic bargaining model between a long-lived seller with private information and a sequence of short-run buyers. They find that when previous offers are observable, bargaining is likely to end up in an impasse, while if past offers are not observable, agreement is always reached. In the same vein, Fuchs et al. (2015) analyze price transparency in a dynamic market where uninformed buyers compete inter- and intra-temporarily for a good sold by an informed seller. They contrast public with private price offers and show that in a two-period model, all equilibria with private offers Pareto-dominate the equilibrium with public offers. Daley and Green (2012) consider a dynamic setting in which a single seller faces a competitive market and news (public signals about the seller's type) arrives over time. Among other things, they show that increasing the news quality may or may not improve efficiency. Perhaps more closely related, Kim (2015) studies a dynamic market under adverse selection where, as in our model, uninformed players' (buyers') inferences on the quality of goods rely on the information they have about informed players' (sellers') past behavior. He examines three different information regimes on past play: no information, buyers observe time on the market, and buyers observe the number of previous matches. He finds that market efficiency depends crucially on what information is

[8] See e.g. Guerrieri et al. (2010), Attar et al. (2011), Philippon and Skreta (2012), Tirole (2012), Kurlat (2013), Guerrieri and Shimer (2014), and Jullien and Park (2014)

available under what market conditions (i.e. the size of any frictions present). Like most of these papers, we study the effects of changing the level of information asymmetry - represented in our setting by the length of memory - given a certain information structure. Like some of these papers, we also study the effects of changing this information structure, i.e. the availability of negative past records, positive records, or both. Yet, unlike our paper, none of these papers studies the information provision by an information intermediary that can have a short or long memory, and may have a different memory for different types of information.

Our work is also related to papers studying the effects of limiting information on past behavior/outcomes on incentives in reputation models. Vercammen (1995) shows that limiting the retention of positive records may improve incentives in a credit market by preventing borrowers with a long history of positive records from "sitting on their laurels". Ekmekci (2011) shows in a more general reputation game that censoring past information prevents long-term learning, allowing reputation to become "permanent", where Cripps et al. (2004) showed it would be impermanent otherwise. Elul and Gottardi (2015) find that allowing bankruptcy to be forgotten with some positive probability may improve credit market outcomes when both moral hazard and adverse selection problems are mild. Most recently, Hörner and Lambert (2015) investigate the optimal rating system in a classic career concern setting, showing when and how past information should be discounted to provide incentives at the cost of worsening the information available to the market.[9]

While these studies are close to ours in terms of research questions, the mechanisms they discuss are very different from the ones we identify here. In our market there are no incentive problems/effects, and although we also find that it may be optimal to limit access to past performance information, the reason behind this is completely different: it is to encourage the unrated side of the market to trade and produce information on the rated side, and thereby prevent a market breakdown induced by lack of information production, not to provide incentives to the rated parties. For the same reason, although closely related in spirit, the literature on repeated games with restricted memory, where there is no hidden information at all, is even further away from what we are doing here.[10]

Our work is also related to the literature on credit bureaus and credit registers, started by the seminal contributions by Jappelli and Pagano (1993, 2002) and Padilla and Pagano (1997). Closest in spirit to our results is Padilla and Pagano (2000), in which the authors

[9]See also Dellarocas (2006), Chatterjee et al. (2011), Bottero and Spagnolo (2012) and Liu and Skrzypacz (2014); Musto (2004) and Bos and Nakamura (2012) provide empirical evidence on the effect of limiting the retention of negative credit records.

[10]Barlo, Carmona, and Sabourian (2009), Mailath and Olzewsky (2010) and Doraszelski and Escobar (2012).

show that too much information provided by the credit bureau may lead to very poor credit market outcomes. Finally, limiting retention of past records is a form of privacy, hence our work is also relevant to the literature on privacy and its regulation surveyed in detail by Acquisti et al. (2015).

2 Environment

Consider an economy populated by sellers, buyers and an information intermediary who interact in continuous time $t \in [0, \infty)$.

Sellers. There is a unit mass of infinitely lived sellers $i \in [0, 1]$ in the economy. At each instant, a seller may be active on the market (i.e. may have a product to sell) or not. Seller i is active whenever there is a jump in a counting process $\{N_t^i\}_{t\geq 0}$ with Poisson intensity $m > 0$. Markets with many (few) transactions per unit of time can be described by a process with a high (low) intensity m. For instance, in the context of Ebay one can think that with a certain probability, a person may decide to sell an old gadget. Similarly, in the context of a credit market, with a certain probability a potential borrower (seller of debt) may need to borrow from (sell debt to) a bank (buyer of debt).

We normalize the value of the product to the seller to one (it can be the value the seller derives from alternative use, the cost of production or, in the case of the credit market, the amount of necessary investment). The product price $P_i(t)$ is determined by the buyers' willingness to pay, which in turn depends on their expectation of the seller's quality. The seller decides whether to sell the product ($s_i = 1$) or not ($s_i = 0$). His instantaneous payoff is $V_i(t, s_i) = s_i[P_i(t) - 1] + 1$. We assume that the seller is impatient (myopic), that is, the seller heavily discounts the future and cares only about his instantaneous payoff.

Product quality. The buyers' valuation of seller i's product (product quality) $\theta_i(t)$ is stochastic: it can be high ($\theta^H > 1$) or low ($\theta^L = 0$). We refer to θ_i as seller i's type; a good seller's product has quality θ^H and a bad seller's product has quality $\theta^L < \theta^H$. Note that we do not specify whether the seller knows his type or not, as our analysis holds in both cases. The quality of each seller may change over time. For instance, there can be innovations in products offered, changes in the seller's management or ownership, or an evolution of buyers' preferences. Seller i's product quality follows an exogenous time-homogeneous Markov process $\theta_i(t), t \in [0, \infty)$, with an initial probability distribution $\pi_i(0) = Pr(\theta_i(0) = \theta^H)$, and a transition rate matrix $\mathbf{Q}$. For convenience, we introduce the following assumption:

Assumption 1. *At $t = 0$ there is a mass $\mu > 0$ of good sellers $\int_0^1 \pi_i(0)di = \mu$, and for* *any $\varphi \in (0,\infty)$ the transition rate matrix takes the form:*

$$\mathbf{Q} = \varphi \begin{bmatrix} -(1-\mu) & 1-\mu \\ \mu & -\mu \end{bmatrix}.$$

Assumption 1 ensures that the fraction of good sellers in the population is constant. Indeed, for $t > 0$ the probability of seller $i \in [0,1]$ being of high type satisfies the equation $\frac{d\pi_i(t)}{dt} = -\varphi(1-\mu)\pi_i(t) + \varphi\mu(1-\pi_i(t))$. The solution is:

$$\pi_i(t) = \pi_i(0)e^{-\varphi t} + \mu(1 - e^{-\varphi t}), \tag{1}$$

which, together with $\int_0^1 \pi_i(0)di = \mu$, implies $\int_0^1 \pi_i(t)di = \mu$ for any $t \in [0,\infty)$.

Assumption 2. *The average quality of sellers in the population is low, $\mu\theta^H < 1$.*

Assumption 2 implies that without any information, the average seller will not trade. This assumption makes the information intermediary and its feedback retention policy crucial for the market.

Buyers. At each moment $t \in [0,\infty)$, many competitive risk-neutral buyers are matched to active sellers. A buyer is never matched to the same seller twice. Alternatively, we could assume that buyers consume only once in their lifetime, or that they are short-lived, so in each instant buyers are different. We do not model competition between buyers explicitly, but follow Holmström (1999) and Mailath and Samuelson (2001) in simply assuming that buyers are ready to buy a product for a price equal to its expected quality. If the price is above the seller's valuation, the seller sells the product to one of the buyers that he chooses randomly. Before the buyer purchases the product from seller i, he does not know its quality and relies on past feedback h_i^t about seller i provided by the information intermediary (described below). The buyers have no other information about the seller except past feedback and the prior; they believe that the seller is high quality with probability $\mu(h_i^t, \pi_i(0))$, and these beliefs are updated using Bayes' rule. To shorten notation, we will often write $\mu(h_i^t)$ for the beliefs omitting the prior $\pi_i(0)$. After the buyer purchases the product from seller i at time t, he learns the quality of the product $\theta_i(t)$ and derives utility $\theta_i(t)$.

Information intermediary. After a buyer has purchased the product from seller i and learned its quality, he can leave his feedback f_i^t on it with the information intermediary. The feedback can be positive $f_i^t = S$ (Satisfied) or negative $f_i^t = D$ (Dissatisfied), or there may be no feedback $f_i^t = N$ (No), with no loss of generality. If the seller does not trade there is no feedback $f_i^t = N$. At each $t \in [0,\infty)$ for each seller $i \in [0,1]$, the information intermediary records feedback $f_i^t \in \{S, D, N\}$ and makes records of past feedback $h_i^t : [0,t) \to \{S, D, N\}$ available to the buyers.

Note that after the purchase the buyer has no incentive to leave feedback. Yet, in reality many buyers do leave feedback. We abstract from the buyer's motivation to leave honest feedback, and simply introduce the following:

Assumption 3. *After purchasing a high quality product the buyer leaves positive feedback S, and after purchasing a low quality product the buyer leaves negative feedback D.*

Having described the model, we can turn to the equilibrium specification.

Market equilibrium. Informally, an equilibrium at each moment in time is characterized by the information the intermediary provides, the buyers' beliefs about sellers and the prices they offer to active sellers, active sellers' decisions to sell, and the feedback that buyers leave about sellers. Formally, for any prior information about sellers $\pi_i(0)$, $i \in [0,1]$, an equilibrium for each $t \in [0,\infty)$ specifies is characterized by public histories h_i^t of past feedback published by the information intermediary for all sellers $i \in [0,1]$; buyers' beliefs about sellers' types $\mu(h_i^t, \pi_i(0))$; realizations of Poisson shocks dN_t^i that determine the sellers active at t; prices $P_j(t) \in R^+$ offered by buyers to each active seller j; the optimal selling decision $s_j^t \in \{0,1\}$ for each active seller j; and feedback $f_i^t \in \{S, D, N\}$ recorded for each seller $i \in [0,1]$ by the information intermediary. Buyers use Bayes' rule to update their beliefs about sellers.

Let us describe the equilibrium. If a Poisson shock hits seller i at time t, he becomes active and gets matched with many competitive buyers. Buyers use history of past feedback h_i^t to form their belief about the seller's quality $\mu(h_i^t)$. Competitive risk-neutral buyers offer a price equal to the expected value of the product $P_i(t) = \mu(h_i^t)\theta^H$.

Having observed the price, each active seller decides whether to sell the product $s_i = 1$ or not $s_i = 0$ in order to maximize his instantaneous payoff $V_i(t, s_i) = s_i[P_i(t) - 1] + 1$. It immediately follows that an active seller sells his product $s_i = 1$ whenever $P_i(t) = \mu(h_i^t)\theta^H \geq 1$. Note that the seller's payoff does not depend on his type θ_i, that is, both types of sellers sell whenever they can get a price of at least one. If the seller decides to sell, the buyer learns perfectly the quality of the product and leaves corresponding feedback $f_i^t = S$ if $\theta_i^t = \theta^H$ and $f_i^t = D$ if $\theta_i^t = \theta^L$. If the seller is not active or if he does not sell, there is no feedback $f_i^t = N$.

As one can see, the equilibrium behavior of all agents can be easily described once one knows the buyers' beliefs $\mu(h_i^t)$ that in turn depend on the information policy of the intermediary. This is the key object of our analysis. We first consider the case of perfect memory of past feedback and then turn to the case when the intermediary deletes past feedback.

3 Perfect past records

According to Assumption 2, the average quality of sellers is low and there can't be trade unless the buyers are able to tell apart at least some good sellers from the bad ones. In such a situation, one may expect that providing information to the buyers and retaining it indefinitely would be beneficial. For instance, one may think that the availability of perfect information about sellers at $t = 0$ and of full history of feedback at any $t > 0$ would facilitate trade. It turns out this is not the case in the long run.

Theorem 1. *If the intermediary provides full history of past feedback for any seller, then the fraction of sellers trading in equilibrium converges to zero with time.*

The proof can be found in the appendix. The fact that the full provision of past information has such a negative effect on trade in the long run is striking, and even more so when contrasted with the potential positive effects on trade in the long run of limited past information retention policies analyzed in the next section.

The logic behind Theorem 1 is very simple. With time, good sellers happen to become bad and get negative feedback. From that moment they are excluded from the market forever because buyers' posterior about their quality never exceeds the unconditional probability of a high-quality seller in population μ and, according to Assumption 2, the unconditional expected quality of a seller is low ($\mu\theta^H < 1$). As time goes to infinity, each seller is almost surely excluded from trading. In spite of being intuitive, in our view the result is surprising because full provision of past information by the information intermediary leads to market break-down in the long run.

Having illustrated the potential long-run drawbacks of full provision of past information, we ask the following question: Can a restriction on information provision by the information intermediary affect trade in the long run? More specifically, can the deletion of past records after a certain period of time facilitate trade in the market? Our answer to the second question is positive and is provided in the next section.

4 Limited records

From now on, we assume that the information intermediary does not provide all past feedback on sellers to the buyers. Instead, the information intermediary deletes negative and positive feedback after $T^- \leq \overline{T}$ and $T^+ \leq \overline{T}$ periods correspondingly. As we mentioned previously, the intermediary does not have to physically delete past information, it can simply choose not to disclose it to the public.

4.1 Relevant records and stationary equilibrium

First, we establish an intermediate result about relevant information that simplifies the analysis considerably. Note that the recorded history of a seller h_i^t can be a rather complex object, as it contains all positive feedback left in the last T^+ periods and all negative feedback left in the last T^- periods. However, the Markov nature of the seller's stochastic quality $\theta_i(t)$ guarantees that just the latest record available at t contains the sufficient information to determine the belief $\mu(h_i^t)$. For instance, if the latest record about seller i was left at $t-\tau$ and was positive $S(\tau)$, then the buyer knows that seller was of high quality at $t-\tau$ but understands that the seller's type might have changed since then. At the same time, any previous record left for the same seller before $t-\tau$ is irrelevant, as it was already obsolete at $t-\tau$ when the latest record was left.

Positive records are deleted T^+ periods after they are left, hence, without loss of generality we can say that the seller has a record $r_i^t = S(\tau)$, $\tau \in [0, T^+]$ at time t if his latest record was positive and was left at $t-\tau$. Denote by $\mathbf{S} = \{S(\tau) : \tau \in [0, T^+]\}$ the set of all possible positive records. Analogously, for negative records we say that the seller has a record $r_i^t = D(\tau^-)$, $\tau^- \in [0, T^-]$ at time t if his latest record was negative and was left at $t-\tau^-$. Also let $\mathbf{D} = \{D(\tau^-) : \tau^- \in [0, T^-]\}$ be the set of all possible negative records. Finally, the seller may not have any record, because his past records were deleted and he received no good feedback in the last T^+ periods and no bad feedback in the last T^- periods, in which case we say that the seller has a record $r_i^t = N$. Denote by $G = \mathbf{S} \cup \mathbf{D} \cup N$ the set of all possible records that a seller might have. The above arguments imply the following:

Lemma 1. *For any seller $i \in [0,1]$ at any time $t \in [0, \infty)$, the latest record $r_i^t \in G$ contains all type-relevant public information about the seller.*

In what follows we use records $r \in G$ instead of histories. This considerably simplifies the analysis: the distribution of sellers' types to records in G contains all relevant information about the economy at any moment of time t and, therefore, pins down the buyers' beliefs about sellers, the prices offered by the buyers and the sellers' selling decisions. This distribution consists of six components: $\mathbf{\Delta}_t = \{\rho_t^N, \rho_t^S(.), \rho_t^D(.), \eta_t^N, \eta_t^S(.), \eta_t^D(.)\}$. Here, ρ_t^N and η_t^N are masses of good and bad sellers with an N record at time t, density functions $\rho_t^S(\tau)$ and $\eta_t^S(\tau)$, $\tau \in [0, T^+]$ determine densities of good and bad sellers with records $S_t(\tau)$ at time t, and density functions $\rho_t^D(\tau^-)$ and $\eta_t^D(\tau^-)$, $\tau^- \in [0, T^-]$ determine densities of good and bad sellers with records $D(\tau^-)$ at time t. Note that Assumption 1 guarantees that $\rho_t^N + \int_0^{T^+} \rho_t^S(\tau) d\tau + \int_0^{T^-} \rho_t^D(\tau^-) d\tau^- = \mu$ and $\eta_t^N + \int_0^{T^+} \eta_t^S(\tau) d\tau + \int_0^{T^-} \eta_t^D(\tau^-) d\tau^- = 1 - \mu$ for any $t \geq 0$.

Stationary equilibrium. Stationary equilibrium is a market equilibrium in which the distribution of sellers' types to records is time invariant,[11] that is, $\boldsymbol{\Delta}_t = \boldsymbol{\Delta}$.

In the subsequent analysis we focus on stationary equilibria. We find conditions under which different stationary equilibria exist and study the properties of these equilibria.[12] First, note that a stationary equilibrium with no trade exists. This observation echoes the negative result of Theorem 1 as trade collapses in the long run, and is very intuitive. If no seller trades, there is no feedback. When there is no feedback, in the long run all sellers look identical to the buyers and are believed to be of a high quality with the same probability μ. Assumption 2 implies that sellers do not sell because the buyers' willingness to pay is below the sellers reservation value $\mu\theta^H < 1$. Hence, there is no trade in equilibrium.

The next section shows that certain record-keeping policies by an information intermediary can support a stationary equilibrium with trade.

4.2 Stationary equilibrium with trade

In this section we show that with limited records, trade can often be sustained in the long run. Here we establish conditions that guarantee the existence of a stationary equilibrium in the economy in which a constant positive mass of sellers trades. We start by describing some general features of the stationary equilibrium with trade.

Active sellers with negative records do not trade. Naturally, sellers with negative records do not sell because the buyers believe their products to be of low quality and offer prices below the reservation value of the sellers. To prove this formally, consider an active seller i at time t who got negative feedback at $t - \tau^-$ and no feedback since then, he has a $D(\tau^-)$ record at time t. He must have been a low type τ^- periods ago $\pi_i(t - \tau^-) = 0$ because he got negative feedback. Since then his type evolved according to Assumption 1, hence using (1), the posterior probability of this seller being of high type now is given by $\pi_i(t) = \mu(1 - e^{-\varphi\tau^-})$. This seller's expected quality is below the seller's reservation value, $\mu(1 - e^{-\varphi\tau^-})\theta^H < 1$, for any τ^-, and the seller does not sell.

Active sellers with an N record sell. Intuitively, if active sellers with an N record were not selling in equilibrium then with time all sellers would get an N record and would stop selling. Indeed, the seller's type changes and any seller can get negative feedback after selling a low-quality product. For trade to exist in the long run, the seller must have an

[11] In principle, our model may have non-stationary equilibria that are outside the scope of this paper.

[12] Note that $\boldsymbol{\Delta}$ specifies the distribution of seller's types to records but not the distribution of sellers to records. In a stationary equilibrium the records of individual sellers constantly change, but this is not important from the aggregate point of view as far as the quality distribution of sellers with a particular record is stationary.

opportunity to trade in the future, which arises when his negative feedback is deleted and he joins the pool of sellers with an N record. Therefore, in a stationary equilibrium with trade, active sellers with an N record must be selling.

Active sellers with positive records sell. Intuitively, in a stationary equilibrium the pool of sellers with an N record consists of two groups: sellers that had a positive record $S(T^+)$ in the past which got deleted with time, and sellers with a negative record $D(T^-)$ which got deleted with time. Therefore, the expected quality of a sellers with an N record is higher than the expected quality of a seller with an $D(T^-)$, record but lower than the expected quality of a seller with an $S(T^+)$ record. Previously, we have shown that in a stationary equilibrium with trade an active seller with an N record sells, therefore his expected quality is higher than the seller's reservation value of the product. This immediately implies that an active seller with an $S(T^+)$ record also prefers to sell. Note that this also applies to a seller with any positive record $S(\tau)$, $\tau \in [0, T^+]$ because the seller's expected quality is decreasing with the time passed since the latest positive feedback τ. Indeed, a seller with an $S(\tau)$ record was high quality τ periods ago and since then his type evolved according to the Markov process described in Assumption 1, he is high quality now with posterior probability $\pi_i(t) = \mu + (1-\mu)e^{-\varphi\tau}$, which is decreasing with τ.

The above arguments pave the path to proving our main result about the existence of a stationary equilibrium with trade, which is established in the next theorem.

Theorem 2. *1. A stationary equilibrium with trade exists if and only if: i) positive feedback is recorded for a sufficiently short interval of time, $T^+ < \overline{T}^+$, ii) negative feedback is recorded for a sufficiently long interval of time, $T^- \geq \underline{T}^-$, and iii) the seller's type is sufficiently persistent:*

$$\frac{\varphi}{m} < \frac{\mu(\theta^H - 1)}{1 - \mu\theta^H}. \tag{2}$$

$\overline{T}^+$ and $\underline{T}^-$ are given by solutions to

$$\frac{1-\mu}{\mu}\frac{\varphi}{\varphi+m}[1-\mu\theta^H] = e^{-m\overline{T}^+}\left[\mu\theta^H + (1-\mu)\theta^H\frac{m}{m+\varphi}e^{-\varphi\overline{T}^+} - 1\right], \tag{3}$$

$$\frac{1-\mu}{\mu}\frac{\varphi}{\varphi+m(1-e^{-\varphi\underline{T}^-})} = \theta^H\left[\mu e^{-mT^+} + (1-\mu)\frac{\varphi + me^{-(m+\varphi)T^+}}{m+\varphi}\right] - e^{-mT^+}. \tag{4}$$

2. There can be at most one stationary equilibrium with trade.

The proof is in the appendix. The rough idea of the proof is as follows. First, condition (2) is very natural because for records of past feedback to have any value, the sellers' types must be persistent enough. Indeed, if the sellers' types change very quickly, past feedback becomes obsolete very quickly and it does not matter how long the information intermediary keeps it for. Second, a stationary equilibrium with trade exists if and only if the average quality of the anonymous (unknown) pool of sellers with N records is high. Otherwise, N records can become a "black hole" for the market: if sellers with an N record do not sell, then there may be no trade in the long run, as all sellers almost surely enter this pool with time and can't exit it. The conditions provided in Theorem 2 guarantee that the expected quality of sellers with an N record is high enough to sustain trade.

At first, the fact that limited memory can lead to better long-run outcomes (stationary equilibrium with trade) than full memory (collapse of trade) is surprising. Yet, this result has a clear and robust rationale behind it. When buying, buyers produce a positive informational externality - the feedback on the seller. With perfect information about past feedback, buyers do not experiment enough - they only buy from sufficiently good sellers. Because of low experimentation and the Markov nature of sellers' types, past feedback becomes obsolete with time. Limited memory creates a pool of sellers with unknown history, and if the quality of this pool is sufficiently high, buyers will be ready to experiment by buying from the sellers in this pool and producing information. Essentially, by forcing good records to be forgotten, one can encourage buyers to experiment and trade with unknown sellers, which encourages information production and sustains trade in the long run.

Example 1. In order to illustrate Theorem 2, consider a simple example. Suppose that (2) holds and positive feedback is not recorded $T^+ = 0$ (no positive memory). It is easy to check that in this case $T^+ < \overline{T}^+$, and condition $T^- \geq \underline{T}^-$ is necessary and sufficient for an equilibrium to exist. Using (4) condition $T^- \geq \underline{T}^-$ can be rewritten as:

$$\mu\theta^H \geq \frac{1-\mu}{1+\frac{m}{\varphi}(1-e^{-\varphi T^-})} + \mu.$$

Clearly, when negative memory is short $T^- = 0$ it is not satisfied, since $\mu\theta^H < 1$. However, for long negative memory $T^- \to \infty$ it becomes $\mu\theta^H \geq \frac{\varphi}{m+\varphi}(1-\mu) + \mu$ and is equivalent to (2), which is satisfied if $\frac{\varphi}{m}$ is small. In other words, if the seller's type changes slowly relative to the intensity of trade in the market, then introducing a long memory for negative feedback can support trade in the long run and prevent the market from collapsing.

It turns out the result illustrated in the above example is more general. From Theorem 2 it becomes clear that the existence condition is least stringent when negative memory is longest $T^- = \overline{T}$ and positive memory is shortest $T^+ = 0$. The intuition is as follows: positive memory depletes the average quality of the pool of unknown sellers with an N

record, hence, short memory for positive records improves the average quality of this pool and relaxes the existence condition for the stationary equilibrium equilibrium with trade. The long memory for negative feedback also helps to keep bad sellers out of the pool of sellers with an N record and to improve its average quality.

Note, that sometimes for an equilibrium to exist, it must be technologically possible to keep past records for a sufficiently long time $\overline{T} \geq \underline{T}^-$. Then, with an appropriate choice of record-keeping policy, one can guarantee that the stationary equilibrium with trade exists.

Example 2. Hidden in Theorem 2 is another difference between positive and negative records, in addition to the main one that they influence equilibrium existence condition in opposite ways. To illustrate this second difference between positive and negative records, consider the special case where the sellers' types are almost permanent, that is, their types change with a very low intensity $\varphi \to 0$. Substituting for $\underline{T}^-$ from (4) into $T^- \geq \underline{T}^-$ and taking the limit when $\varphi \to 0$, we obtain the existence condition for a stationary equilibrium with trade when $\varphi = 0$:

$$(1 + mT^-)e^{-mT^+} \geq \frac{1-\mu}{\mu(\theta^H - 1)}.$$

Clearly, longer retention of negative feedback T^- relaxes the existence condition, while longer retention of positive feedback T^+ has the opposite effect. However, the strength of the two effects is also different. While the positive effect of T^- is linear, the negative effect of T^+ is exponential, that is, potentially much stronger. To see why this is the case, consider a good and a bad seller that have an N record and happen to trade at a given moment. The bad seller gets negative feedback after selling the product and leaves the pool of sellers with an N record. He is effectively excluded from trade for the time the negative feedback is retained T^-, after which the feedback is deleted and he enters back into the pool of sellers with an N record. Now consider the good seller. After selling the product he gets positive feedback and leaves the pool of sellers with an N record. Given that he has a good record, he can potentially trade again and get new positive feedback. The positive feedback is retained for T^+ periods and if the seller trades before the positive feedback is deleted, he gets new positive feedback which will be retained for another T^+ periods and he will be able to trade again. By repeating this argument, one can see that once a good seller leaves the pool of sellers with an N record, he can spend much longer than T^+ periods outside of this pool before entering again. For instance, if both kinds of feedback are retained for the same amount of time $T^+ = T^-$, then because good sellers stay outside of the pool of sellers with an N record for longer than the bad sellers, in a stationary equilibrium the average quality of the sellers in the pool is low. Indeed, if T^+ is long enough, the existence condition would be violated because $(1+mT^+)e^{-mT^+} \to 0$ with

$T^+ \to \infty$. This demonstrates that when seller's types are permanent ($\varphi \to 0$), retaining positive records has a strong negative effect on the average quality of the sellers with an N record that dominates the positive effect of retaining negative records. This provides an additional reason why it is important to treat positive and negative records separately when analyzing models with performance ratings.

5 Welfare analysis

In equilibrium, two types of direct social losses may occur: a) when a bad product is purchased and the buyer suffers, and b) when a good product is not purchased and the potential buyer forgoes consumer surplus. Suppose a stationary equilibrium with trade exists. At each instance in such an equilibrium, only sellers with positive records $S(\tau)$, $\tau \in [0, T^+]$ and sellers with no records N trade their products. The value of the product to the seller is 1. A good product generates utility $\theta^H > 1$ to the buyer, the bad product generates no utility. The mass of good sellers trading during a short time interval dt is $m[\rho(N) + \int_0^{T^+} \rho(S(\tau))d\tau]dt$, the mass of bad sellers trading is $m[\eta(N) + \int_0^{T^+} \eta(S(\tau))d\tau]dt$. The surplus generated over a small interval of time dt can be expressed as follows:

$$Wdt = \left\{ [\rho(N) + \int_0^{T^+} \rho(S(\tau))d\tau](\theta^H - 1) - \eta(N) - \int_0^{T^+} \eta(S(\tau)d\tau \right\} mdt.$$

We take as a measure of welfare the flow of surplus per unit of time W. Note that in a stationary equilibrium welfare does not depend on t.

Lemma 2. *If a stationary equilibrium with trade exists, then welfare in this equilibrium is:*

$$W = m\frac{\mu\theta^H - 1 + m\frac{\mu}{\varphi}(1 - e^{-\varphi T^-})(\theta^H - 1)}{1 + m(1-\mu)T^- + m\frac{\mu}{\varphi}(1 - e^{-\varphi T^-})}. \tag{5}$$

The proof can be found in the Appendix. In the proof we characterize the stationary distributions of sellers of different qualities for different records and then use these distributions to find the mass of good and bad sellers that trade in equilibrium, and to express the welfare.

Theorem 3. *In a stationary equilibrium with trade welfare does not depend on the length of memory for positive records T^+, and is maximized with the length of memory for negative records T_W^- given by a solution to:*

$$1 - \mu\theta^H - (\theta^H - 1)\frac{m\mu}{\varphi} + \mu[\theta^H + m(\theta^H - 1)(T_W^- + \frac{1}{\varphi})]e^{-\varphi T_W^-} = 0. \tag{6}$$

The result is intuitive. If the average quality of sellers in the population is not very high, it is beneficial to exclude sellers with negative records from trade. Indeed, a seller who got negative feedback τ^- periods ago is believed to be of high quality with probability:

$$\pi_i(D(\tau^-)) = \pi_i(-\tau^-)e^{-\varphi\tau^-} + \mu(1 - e^{-\varphi\tau^-}) = \mu(1 - e^{-\varphi\tau^-}). \quad (7)$$

Trade with such a seller is associated with a direct social welfare loss if $\mu(1-e^{-\varphi\tau^-})\theta^H < 1$. Assumption 2 implies that for any $\tau^- < T^- \leq \overline{T}$ this inequality holds, therefore trading with sellers with negative feedback involves a direct social cost. However, there is also an indirect benefit of trading with these sellers which comes from learning their actual types. Indeed, after a trade the buyers leave feedback that allows high and low quality sellers to be separated. This information has a positive social value as it enables future buyers to distinguish the sellers and trade only with those that have sufficiently high expected quality. In essence, there is a positive option value associated with trading with a seller with negative feedback. For a seller who got negative feedback time T_W^- ago, the option value of learning the seller's type compensates for the direct cost of trading with the seller. That is why it is not socially optimal to exclude sellers with negative records from trade forever. When negative records are erased after time T_W^-, sellers with negative records older than T_W^- are able to trade and the social welfare is highest.

The fact that the memory for positive feedback does not affect welfare is also intuitive. In our model a seller with an N record can trade as well as a seller with a positive record $S(\tau)$, $\tau \in [0, T^+]$. Therefore, provided that the stationary equilibrium with trade exists, it does not matter when the positive record is erased as the seller can trade anyway.

6 Robustness and extensions

In this section we discuss the robustness of our findings by relaxing some of the many simplyfying assumptions that make the model tractable, and develop some extensions that are of intrest for particular applications.

6.1 Buyers do not always leave feedback

It is easy to verify that we can relax Assumption 3 that the buyer always leaves feedback after purchasing the product, and assume instead that a buyer leaves feedback only with a certain probability $\lambda \in (0, 1]$. In this case, the probability of trading and getting feedback is $m\lambda$ instead of m, so the results do not change qualitatively. For instance, the existence conditions of Theorem 2 when the feedback is left with probability λ can be obtained by replacing m with $m\lambda$. Other results can be also easily established for $\lambda \in (0, 1]$. This extension of our results is important for online market platforms like Ebay and Amazon, where the probability of a buyer leaving feedback after a transaction can be as low as few percent.

6.2 Uninformed sellers and lemons

Our results hold when sellers are not informed about their type, that is, when there is no asymmetric information between buyers and sellers. Indeed, in the main analysis the strategy of the seller does not depend on his type, and hence it does not matter for the analysis if the seller is informed.

Our results also hold when there is adverse selection. For simplicity, in the main analysis we have assumed that both types of seller have the same product valuation/cost of one. In a standard lemons problem, the low-quality seller has a lower product valuation/cost than the high-quality seller. We can generalize our analysis and assume that the product valuation/cost for a low-quality seller $c_L \in (0, 1)$ is lower than the product valuation/cost for a high-quality seller $c_H = 1$, which is closer to the classic lemons market problem.

In this case, whenever the buyers' willingness to pay is below the product valuation/cost of the high-quality seller $\mu(r_i)\theta^H < 1$, the high-quality seller will not trade. But this in turn implies that the low-quality seller will not be able to trade because no buyer would ever offer a price $P_i \in (0, 1)$ knowing that only the low-quality seller will accept this price. Indeed, if the buyer understands that only low-quality sellers will accept prices below one, his willingness to pay drops to zero, that is, below the low-quality seller product valuation/cost $c_L > 0$. In other words, in this case trade disappears as soon as the buyer's willingness to pay drops below 1, as it was in the previous model with $c_L = 1$. This reasoning implies that our main results regarding market break down with full memory (Theorem 1) and regarding the existence of a stationary equilibrium with trade (Theorem 2) also hold when $0 < c_L < c_H = 1$.

6.3 Quality-insensitive buyers

In the analysis we assumed that no buyer values the low-quality product as much as the seller $\theta^L = 0 < 1$. In other words, the sale of a low-quality product is inefficient. As a result, in our equilibrium sellers with negative feedback are effectively excluded from the market. This is a stark prediction, as one may think that in reality there are always some buyers that will buy even from the low-quality sellers. This may be either because they value low-quality products more than other buyers and the seller, or because they make mistakes. In this section, we analyze the possibility that some buyers are not quality sensitive, - they value the low-quality product as much as the high-quality product and their valuation is equal to the seller's reservation value of one. This way we ensure that any seller may have a chance to trade no matter what his feedback.

We assume that each active seller, when matched with several buyers, may have a quality-insensitive buyer among them with probability $\beta \in (0, 1]$. The quality-insensitive buyer, whenever matched to an active seller i, bids his valuation $b_i = 1$ independently of the seller's posterior probability $\mu(r_i^t)$ of being high type. Note that the buyer does not want to bid less than 1 because the seller's reservation value is also 1 and he only trades when the price is at least 1. It immediately follows that any active seller with expected quality $\mu(r_i^t)\theta^H < 1$ can only sell to a quality-insensitive buyer, which happens with probability β. In other words, sellers with low expected quality trade with a lower intensity $\beta m \leq m$. Apart from not valuing quality, the quality-insensitive buyers are exactly like other buyers: they learn the quality of the product after the purchase and leave feedback with the informational intermediary.[13]

Clearly, in such an environment there will always be trade, as quality-insensitive buyers are ready to purchase from any seller. We are interested in a stationary equilibrium in which sellers with an N record are able to sell to all buyers, not only the information-insensitive ones. We call this equilibrium the "high-trade stationary equilibrium".

Theorem 4. *The high trade stationary equilibrium exists if and only if memory for positive feedback is short enough, memory for negative feedback is long enough, and the sellers' types are persistent enough: $T^+ < T^*$, $T^- \geq \hat{T}$, and*

$$\frac{\varphi}{m} \leq \frac{\theta^H - 1}{1 - \mu\theta^h}. \tag{8}$$

Here T^* solves $A(T^*) = \theta^H(\mu + (1-\mu)\frac{m}{m+\varphi}e^{-\varphi T^*}) = 1$ and $\hat{T}$ solves $C(T^*, \hat{T}) = 0$,

$$C(T^+, T^-) = \frac{Xe^{-mT^+}}{m} + \frac{e^{-m\beta T^-}}{m} - \theta^H\left(X(\frac{\mu e^{-mT^+}}{m} + \frac{(1-\mu)e^{-(m+\varphi)T^+}}{m+\varphi}) + \frac{\mu e^{-m\beta T^-}}{m} - \frac{\mu e^{-(m\beta+\varphi)T^-}}{m+\varphi}\right), \tag{9}$$

$$X = \frac{\mu}{1-\mu}\left(1 + \frac{m(1-\beta)}{m\beta+\varphi}(1 - e^{-(m\beta+\varphi)T^-})\right). \tag{10}$$

The proof is in the Appendix. This result is similar to Theorem 2: the existence condition for the high trade stationary equilibrium is less stringent when memory for positive

[13]If one is not comfortable with quality-insensitive buyers actually learning the true quality of the product, one can instead assume that ordinary buyers sometimes make mistakes: with probability β one of them offers a price of one for a product of expected quality below one.

feedback shortens (T^+ goes down), and when memory for negative feedback lengthens (T^- goes up).

6.4 The 'right to be forgotten' (by Google)

As mentioned in the introduction, on May 13 2014, the European Court of Justice ruled against Google in favor of Costeja (Judgement in case C-131/12), a case brought by a Spanish man, Mario Costeja Gonzalez, who requested the removal of a link to a digitized 1998 article in La Vanguardia newspaper about an auction for his foreclosed home due to debt that he had subsequently repaid. The court ruled in favor of Costeja that search engines are responsible for the content they point to and that individuals have the right - under certain conditions - to ask search engines to remove links with personal information about them. This applies where the information is “excessive in relation to the purposes for which they were processed and in the light of the time that has elapsed.” Suppose that it is in principle possible to erase sellers' past records from search results after some time. In particular, suppose that in our environment a regulation is introduced stating that, if a seller's record is older than T^f, then the seller can ask for that record to be erased from the public information. Suppose the new regulation is binding for negative records, in the sense that $T^f < T^-$. How will the regulation affect our equilibrium?

We showed earlier that in any stationary equilibrium with trade, the sellers without records must be expected to be of sufficiently high quality to induce buyers to trade, while no trade occurs with sellers with negative feedback. Consider a seller's incentive to require his most recent record to be erased when it becomes T^f periods old (records other than the most recent are payoff-irrelevant or are cancelled according to the same logic stated here).

If that record is negative, the seller will indeed request that it is erased, as becoming an N type allows him to start trading again. If that record is positive, the seller has no reason to request its cancellation, as it would not increase the chance of trading. The same reasoning applies to the records older than the most recent one: the seller will ask for negative records that are more than T^f periods old to be removed, but not for the positive ones.

Remark 1. A rule allowing sellers to have their records “forgotten” when they become older that some T^f is either irrelevant (when $T^f \geq T^-$) or will lead to a shorter memory for negative records without affecting the memory for positive records.

With respect to positive records, the right to erase past records has the opposite effect of the policy that makes equilibria with trade easier to sustain, which calls for a short memory of positive records. As for negative records, if an equilibrum with trade is sustainable, then the policy goes in the right direction if T^f is close to T_W^-, as defined in Theorem 3.

7 Conclusion

In this paper we have developed a stylized model of a dynamic market that focuses on the problem of hidden information about the quality of sellers in the market that changes stochastically with time. In the model, as in many real markets, an information intermediary collects past feedback on sellers and publicly reports it in order to ameliorate the information problem. Notably, the model abstracts from incentives problems, and shows that the information problem alone requires careful thinking about the length of "public memory" of past feedback.

Most prominently, our results show that a straightforward transparency argument - "disclosing more information is always better for market efficiency" - is flawed even when not considering incentives. We show that reporting full information on sellers' past feedback can lead to a dramatic decrease in the level of trade in the market, potentially leading the market to collapse when the average quality of sellers is low. This is because buyers are only willing to trade with sellers with positive past records, so sellers that get a bad record do not have a chance to trade again. Since each seller's type changes stochastically over time, any seller can turn bad at some point and can get a bad record. When the average quality of sellers is low, sellers with a bad record don't have a chance to trade and get a new record, so in the long run almost all sellers' latest records will be negative and almost nobody will trade.

Perhaps surprisingly, we find that in the same market with a low average quality of sellers, limiting the memory of the information intermediary can have beneficial effects: it improves information production by the market so that stationary equilibria with trade become sustainable.

We also show that it is crucial to distinguish between positive feedback and negative feedback, as they affect the market equilibrium differently. We find that a stationary equilibrium with trade is possible if the memory of positive records is short and the memory of negative ones is long. The reason for this is that positive and negative past records affect incentives to "experiment" by trading with sellers with no past records in opposite ways. Buyers are always ready to trade with sellers that have positive records, but they may not be willing to trade with sellers that have no record, as many sellers of different quality are pooled in this group. Indeed, buyers are willing to trade with sellers with no past records if their expected quality is high enough. A long memory for negative records keeps bad sellers out of the pool of sellers with no records for a long time, improving the average quality of that pool and thereby encouraging experimentation (i.e. trading with non-rated sellers). The opposite holds for positive records: the longer they are visible, the longer good sellers are kept out of the pool of sellers with no records, the lower the average quality in that pool, and the less buyers are inclined to trade with those non-rated sellers. If positive memory is long and negative memory is short, the buyers do not buy from sellers with no records and trade may collapses in the long run.

Finally, we find that optimal regulation of the information intermediary may require limiting both positive and negative memory, although to a different extent. As has been argued above, long positive memory may prevent buyers from experimenting with sellers that have no records and may lead to a market collapse. The effect of negative memory on welfare is more subtle. Generally, long negative memory helps to sustain trade in the long run, but if negative memory is too long, the level of trade may be suboptimal. Buyers' willingness to pay for a product from a seller with a negative record is always too low for trade to be privately profitable, but each trade produces feedback which is useful for future buyers. In other words, each trade has informational social value attached to it that buyers do not internalize. Since the sellers' type is stochastic, it may be socially optimal to trade with a seller that has "old" negative feedback and learn his type, but such trades do not happen because it is not profitable for the buyers; loosely speaking, the market "does not forgive" sellers with negative feedback on record. This is when limiting negative memory and "forcing the market to forget" can improve the situation. The information intermediary can delete negative records older than a certain socially optimal threshold, thereby encouraging buyers to buy from the sellers for which the social value of learning their quality is high enough.

Some of our recommendations for public memory may not arise spontaneously in markets. Indeed, it is natural to think that all kinds of records, including positive ones, are valuable for buyers, and therefore private parties will have incentives to collect/retain and distribute/sell this information. Limiting the length of public memory for such records may therefore need regulation.

Note that our arguments provide a logic for rules forcing past records to be forgotten even in environments with no incentive problems (which are already known to require records to be forgotten to allow a "fresh start" and incentives to be maintained). A main novelty is that the forces we highlight suggest that the memory of both *positive* and *negative* records may have to be limited, while incentive problems typically require that only the memory of *negative* past records should be limited.

We believe these results provide a novel perspective relevant to a number of important contexts, and one which is particularly useful to understand the possible aggregate effects of the design of information retention rules and rating systems, which are often analyzed from a more micro perspective. Our results could also be useful to evaluate the potential aggregate consequences of the current trend toward the accumulation of extensive (positive and negative) information by electronic platforms and private and public agencies (credit bureaus and registers, criminal and health records databases, etc.), and of privacy rules mandating limits to the retention or disclosure of such information, such as the 2014 European Court of Justice decision on Google and "the right to be forgotten".

As a final remark, we offer a word of caution. We have studied the long-run effects of limited memory of an information intermediary on information and trade in a market. It is very possible that the short-term effects are very different from the long-run ones. For instance, an increase in memory for positive records can improve information and can be beneficial in the short term. However, as we have shown, such an increase can be detrimental for future information production and trade. This suggests that an empirical evaluation of the effects of changes in memory policy can be challenging, as some of the effects may only come into play with time.

Appendix

Proof of Theorem 1. First, at any t competitive buyers offer a price $P_i = \mu(h_i^t)\theta^H$ to each active seller i, the seller i maximizes his payoff $V_i(t, s_i)$ and sells whenever $P_i \geq 1$. Under Assumption 1, for any $t > 0$ without any feedback, the probability of seller $i \in [0, 1]$ being of high type is given by $\pi_i(t) = \pi_i(0)e^{-\varphi t} + \mu(1 - e^{-\varphi t})$.

Clearly, at $t = 0$ sellers with $\pi_i(0) < \frac{1}{\theta^H}$ do not sell because $P_i = \pi_i(0)\theta^H < 1$. These sellers will never trade and will never get any feedback because $\mu(h_i^t) = \pi_i(t) = \pi_i(0)e^{-\varphi t} + \mu(1 - e^{-\varphi t})$ and Assumption (2) guarantees $\pi_i(t)\theta^H < 1$. If at $t = 0$ buyers' prior about all sellers is low $\pi_i(0) < \frac{1}{\theta^H}$ for all $i \in [0, 1]$, then clearly there is no trade ever. Suppose that some sellers at $t = 0$ have high prior probability of being high quality $\pi_i(0) \geq \frac{1}{\theta^H}$. Denote the total mass of these sellers by $x < 1$. Note that as soon as a seller sells a low-quality product, he gets negative feedback and is revealed to be of low type. From that moment the seller is excluded from trading forever, just like those sellers who have low prior probability of being high type at $t = 0$. Each high quality seller from $t = 0$ onward may randomly become low quality, and then sell and get negative feedback. This happens with intensity $(1 - \mu)\varphi m$. At $t = 0$ the total mass of sellers that can in principle trade is $x < 1$, therefore for any $t \geq 0$, the mass of sellers that can trade does not exceed $\overline{\mu}(t) = xe^{-(1-\mu)\varphi mt}$. As $t \to \infty$, the mass $\overline{\mu}(t) \to 0$, therefore the fraction of sellers trading in equilibrium converges to zero with time. QED.

Proof of Theorem 2. For brevity, in what follows we call a *stationary equilibrium with trade* simply an *equilibrium*. In equilibrium active sellers with an N record must be selling, that is, their average quality must be high enough $\mu(N)\theta^H \geq 1$. The rest of the proof finds necessary and sufficient conditions for sellers with an N record to trade.

As argued in the text, in equilibrium the average quality of sellers with an N record is lower than the quality of sellers with an $S(T^+)$ record, that is, $\mu(N) < \pi_i(S(T^+))$. Here $\pi_i(S(T^+)) = \mu + (1-\mu)e^{-\varphi T^+}$ is the posterior probability of a seller with an $S(T^+)$ record being of high type. In order to have $\mu(N)\theta^H \geq 1$, one must have $\pi_i(S(T^+))\theta^H > 1$. Given that $\pi_i(S(T^+))$ decreases with T^+, the memory for positive records must be short enough for an equilibrium to exist:

$$\mu\theta^H + (1 - \mu)\theta^H e^{-\varphi T^+} > 1. \tag{11}$$

Note that we can consider $T^+ \leq T^-$ without loss of generality. Indeed, if $T^+ > T^-$, an equilibrium does not exist. Intuitively, suppose $T^+ > T^-$ and an equilibrium exists. Then a good seller is more likely than a bad seller to have a record other than N than a bad seller, because the good seller is more likely to get an S record, which is remembered longer than an D record. But this implies that the fraction of good sellers with an N

record among all sellers with an N record will be lower than the fraction of good sellers in population $\mu(N) \leq \mu$, and because of Assumption 2, sellers with an N record would not trade. This can't happen in equilibrium, so is a contradiction. From now on we consider $T^+ \leq T^-$.

In a stationary equilibrium the distribution $\mathbf{\Delta}$ of sellers' types to records is constant. Recall that we denote masses of high quality sellers with records N, $S(\tau)$ and $D(\tau^-)$ in a stationary equilibrium by $\rho(N)$, $\rho(S(\tau))$ and $\rho(D(\tau^-))$ correspondingly. Analogously, for low-quality sellers we denote masses $\eta(N)$, $\eta(S(\tau))$ and $\eta(D(\tau^-))$. From Assumption 1 it follows that the total mass of high-quality sellers in the population is constant and equal to μ, hence in a stationary equilibrium we have:

$$\rho(N) + \int_0^{T^+} \rho(S(\tau))d\tau + \int_0^{T^-} \rho(D(\tau^-))d\tau^- = \mu, \tag{12}$$

$$\eta(N) + \int_0^{T^+} \eta(S(\tau))d\tau + \int_0^{T^-} \eta(D(\tau^-))d\tau^- = 1 - \mu. \tag{13}$$

In a stationary equilibrium, at any time t a seller i with $D(\tau^-)$ does not sell. A seller with a $D(\tau^-)$ record was of low quality τ^- time ago, hence (1) allows us to express the posterior probability of this seller being of high type $\mu(1 - e^{-\varphi\tau^-})$, which coincides with the fraction of high-quality sellers among those with a $D(\tau^-)$ record according to the law of large numbers. Denoting by $\eta^D = \eta(D(0))$ the total mass of sellers with a $D(0)$ record, for any $\tau^- \in [0, T^-]$ we get:

$$\begin{aligned} \rho(D(\tau^-)) &= \eta^D \mu(1 - e^{-\varphi\tau^-}), \\ \eta(D(\tau^-)) &= \eta^D(1 - \mu + \mu e^{-\varphi\tau^-}). \end{aligned} \tag{14}$$

Consider a seller j with an $S(\tau)$ record $\tau \in [0, T^+]$. He got positive feedback τ periods ago, that is, he was of a high type then $\pi_j(-\tau) = 1$. He hasn't traded since then, hence, using (1), the posterior probability of this seller being of high type is $\pi_j(S(\tau)) = \pi_j(-\tau)e^{-\varphi\tau} + \mu(1 - e^{-\varphi\tau}) = \mu + (1 - \mu)e^{-\varphi\tau}$. By the law of large numbers, this posterior probability also determines the fraction of high-quality sellers among those with an $S(\tau)$ record. Since $\mu + (1-\mu)e^{-\varphi\tau} \geq \mu$ and $\mu\theta^H \geq 1$, sellers with an $S(\tau)$ record $\tau \in [0, T^+]$ can trade. Denote by $\rho^S = \rho(S(0))$ the total mass of sellers with an$S(0)$ record. Both types of sellers with records $S(\tau)$, $\tau \in [0, T^+]$ trade with the same intensity m and leave the state $S(\tau)$. When a high-quality seller trades he gets positive feedback and his record is updated to $S(\tau = 0)$. When a low-quality seller trades he gets negative feedback and his record is updated to $D(\tau^- = 0)$. Therefore, the total mass of sellers with an $S(\tau)$ record exponentially decays with τ, that is, $\rho(S(\tau))+\eta(S(\tau))=\rho^S e^{-m\tau}$. Using this fact for any $\tau\in[0,T^+]$ we get:

$$\begin{aligned}\rho(S(\tau)) &= \rho^S e^{-m\tau}(\mu+(1-\mu)e^{-\varphi\tau}),\\ \eta(S(\tau)) &= \rho^S e^{-m\tau}(1-\mu)(1-e^{-\varphi\tau}).\end{aligned} \tag{15}$$

Sellers with N and $S(\tau)$, $\tau\in[0,T^+]$ records trade with intensity m. Upon trading their type is revealed: high-quality sellers get an $S(0)$ record and low-quality sellers get a $D(0)$ record. At the same time each instance, all sellers in states $S(0)$ or $D(0)$ exit these states and in a stationary equilibrium we must have:

$$\begin{aligned}\rho^S &= \rho(S(0)) = m\rho(N)+m\int_0^{T^+}\rho(S(\tau))d\tau,\\ \eta^D &= \eta(D(0)) = m\eta(N)+m\int_0^{T^+}\eta(S(\tau))d\tau.\end{aligned} \tag{16}$$

Finally, in the stationary equilibrium masses of high- and low-quality sellers with N records must stay constant. Each instance, $S(T^+)$, $D(T^-)$ records are deleted and a mass $\rho(S(T^+))+\rho(D(T^-))$ of high-quality sellers enters state N, at the same time high-quality sellers with an N record trade and leave this state with intensity m. Analogous reasoning is true for low-quality sellers. Finally, high-quality sellers with intensity $\varphi(1-\mu)$ change their type and become low-quality sellers, while low-quality sellers become high-quality sellers with intensity $\varphi\mu$, and we obtain:

$$\begin{aligned}\dot{\rho}(N) &= \rho(S(T^+))+\rho(D(T^-))-m\rho(N)-\varphi(1-\mu)\rho(N)+\varphi\mu\eta(N)=0,\\ \dot{\eta}(N) &= \eta(S(T^+))+\eta(D(T^-))-m\eta(N)+\varphi(1-\mu)\rho(N)-\varphi\mu\eta(N)=0.\end{aligned} \tag{17}$$

Summing the above two equations we get:

$$m(\rho(N)+\eta(N)) = \rho(S(T^+))+\rho(D(T^-))+\eta(S(T^+))+\eta(D(T^-)). \tag{18}$$

Substitute for $\rho(S(T^+))$, $\rho(D(T^-))$, $\eta(S(T^+))$, $\eta(D(T^-))$ in (18) and we obtain:

$$m(\rho(N)+\eta(N)) = \rho^S e^{-mT^+}+\eta^D = Z. \tag{19}$$

Since $\eta(N)=Z/m-\rho(N)$, we get from the first equation of (17) that:

$$(m+\varphi)\rho(N) = \rho^S(\mu e^{-mT^+}+(1-\mu)e^{-(m+\varphi)T^+})+\eta^D\mu(1-e^{-\varphi T^-})+\varphi\mu Z/m. \tag{20}$$

A seller with an N record can trade only if $\frac{1}{\mu(N)}=\frac{\rho(N)+\eta(N)}{\rho(N)}\le\theta^H$, from (20) we get:

$$\frac{(m+\varphi)[\rho^S e^{-mT^+}+\eta^D]}{m\rho^S[\mu e^{-mT^+}+(1-\mu)e^{-(m+\varphi)T^+}]+m\eta^D\mu[1-e^{-\varphi T^-}]+\mu\varphi[\rho^S e^{-mT^+}+\eta^D]}\le\theta^H. \tag{21}$$

Compute:

$$F(T^+) = \frac{1}{\rho^S}\int_0^{T^+} \rho(S(\tau))d\tau = \frac{\mu}{m}(1 - e^{-mT^+}) + \frac{1-\mu}{m+\varphi}(1 - e^{-(m+\varphi)T^+}), \tag{22}$$

and use it in the first equation of (16) in order to obtain:

$$\rho^S = m\rho(N) + m\rho^S F(T^+). \tag{23}$$

Compute:

$$F^-(T^-) = \frac{1}{\eta^D}\int_0^{T^-} \rho(D(\tau^-))d\tau^- = \mu T^- - \frac{\mu}{\varphi}(1 - e^{-\varphi T^-}). \tag{24}$$

Substitute in (12) to obtain:

$$\rho(N) + \rho^S F(T^+) + \eta^D F^-(T^-) = \rho^S/m + \eta^D F^-(T^-) = \mu. \tag{25}$$

Compute:

$$G(T^+) = \frac{1}{\rho^S}\int_0^{T^+} \eta(S(\tau))d\tau = \frac{1-\mu}{m}(1 - e^{-mT^+}) - \frac{1-\mu}{m+\varphi}(1 - e^{-(m+\varphi)T^+}), \tag{26}$$

and use it in the second equation of (16) in order to obtain:

$$\eta^D = m\eta(N) + m\rho^S G(T^+). \tag{27}$$

Compute:

$$G^-(T^-) = \frac{1}{\eta^D}\int_0^{T^-} \eta(D(\tau^-))d\tau^- = (1-\mu)T^- + \frac{\mu}{\varphi}(1 - e^{-\varphi T^-}). \tag{28}$$

Substitute into (13) and, using (27), obtain:

$$\eta(N) + \rho^S G(T^+) + \eta^D G^-(T^-) = \eta^D/m + \eta^D G^-(T^-) = 1 - \mu. \tag{29}$$

which delivers:

$$\eta^D = m\frac{1-\mu}{1 + mG^-(T^-)}. \tag{30}$$

Using (25) we get:

$$\rho^S = m\frac{\mu + m\mu G^-(T^-) - m(1-\mu)F^-(T^-)}{1 + mG^-(T^-)}. \tag{31}$$

Combining the above equations we get:

$$\frac{\eta^D}{\rho^S} = \frac{1-\mu}{\mu + \mu m T^- - mF^-(T^-)}. \tag{32}$$

Using (23), (27) and (32) we get:

$$\frac{\eta(N)}{\rho(N)} = \frac{\eta^D - m\rho^S G(T^+)}{\rho^S(1 - mF(T^+))} = \frac{\eta^D/\rho^S - mG(T^+)}{1 - mF(T^+)} \tag{33}$$

$$\frac{\rho(N) + \eta(N)}{\rho(N)} = 1 + \frac{\eta(N)}{\rho(N)} = 1 + \frac{\frac{1-\mu}{\mu+\mu m T^- - mF^-(T^-)} - mG(T^+)}{1 - mF(T^+)} \leq \theta^H. \tag{34}$$

Finally, substituting for $F(T^+)$, $F^-(T^-)$, $G(T^+)$ from (22), (24) and (26) we get a necessary and sufficient condition for a stationary equilibrium with trade to exist:

$$1 + \frac{\eta(N)}{\rho(N)} = \frac{\frac{1-\mu}{\mu}\frac{1}{1+\frac{m}{\varphi}(1-e^{-\varphi T^-})} + e^{-mT^+}}{\mu e^{-mT^+} + (1-\mu)\frac{\varphi + me^{-(m+\varphi)T^+}}{m+\varphi}} \leq \theta^H,$$

which can be rewritten as

$$\frac{1-\mu}{\mu}\frac{\varphi}{\varphi + m(1-e^{-\varphi T^-})} \leq \theta^H \left[\mu e^{-mT^+} + (1-\mu)\frac{\varphi + me^{-(m+\varphi)T^+}}{m+\varphi}\right] - e^{-mT^+}. \tag{35}$$

In order to prove that this condition is sufficient, we need to show that it implies the necessary condition (11). Note that the left-hand side of (35) is decreasing in T^-. In order for (35) to hold for some finite T^-, we must have that in the limit when $T^- \to \infty$ the condition holds as a strict inequality, that is:

$$\frac{1-\mu}{\mu}\frac{\varphi}{\varphi+m}[1 - \mu\theta^H] < e^{-mT^+}\left[\mu\theta^H + (1-\mu)\theta^H\frac{m}{m+\varphi}e^{-\varphi T^+} - 1\right]. \tag{36}$$

Note that $1 - \mu\theta^H > 0$ by Assumption 2, hence the condition implicitly requires $\mu\theta^H + (1-\mu)\theta^H\frac{m}{m+\varphi}e^{-\varphi T^+} - 1 > 0$, which in turn implies (11).

Now we need to show that the necessary and sufficient existence condition (35) is equivalent to the requirements of Theorem 2.

Given that the left-hand side is decreasing in T^- we can rewrite (35) as $T^- \geq \underline{T}^-$ provided that there exists $\underline{T}^- < \infty$ which solves (35) as equality. This is possible if and only if for $T^- \to \infty$ (35) holds as a strict inequality, that is, (36) holds. In other words, (36) is a necessary condition, and together with $T^- \geq \overline{T}^-$ these two conditions are sufficient.

Denote by $\zeta(T^+) = e^{-mT^+}\left[\mu\theta^H + (1-\mu)\theta^H\frac{m}{m+\varphi}e^{-\varphi T^+} - 1\right]$ the term on the right-hand side of (35) and of (36), differentiate it with respect to T^+, and obtain $\frac{\partial\zeta(T^+)}{\partial T^+} = -me^{-mT^+}[\mu\theta^H - 1 + (1-\mu)\theta^H e^{-\varphi T^+}] < 0$ because (36) requires $\mu\theta^H + (1-\mu)\theta^H\frac{m}{m+\varphi}e^{-\varphi T^+} -$

$1 > 0$. Given that the right-hand side of (36) is decreasing in T^+, we can rewrite (36) as $T^+ < \overline{T}^+$, provided that there exists $\overline{T}^+ > 0$ such that (36) holds as equality. Such $\overline{T}^+ \geq 0$ exists if and only if (36) holds for $T^+ = 0$, that is, if and only if $\frac{\varphi}{m} < \frac{\mu(\theta^H - 1)}{1 - \mu\theta^H}$, which is equivalent to (2).

To sum up we have established that if (2) holds, then there exists $\overline{T}^+ > 0$ that solves:

$$\frac{1-\mu}{\mu}\frac{\varphi}{\varphi+m}[1-\mu\theta^H] = e^{-m\overline{T}^+}\left[\mu\theta^H + (1-\mu)\theta^H\frac{m}{m+\varphi}e^{-\varphi\overline{T}^+} - 1\right]. \tag{37}$$

If also $T^+ < \overline{T}^+$, then there exists $\underline{T}^- < \infty$ that solves:

$$\frac{1-\mu}{\mu}\frac{\varphi}{\varphi+m(1-e^{-\varphi T^-})} = \theta^H\left[\mu e^{-mT^+} + (1-\mu)\frac{\varphi + me^{-(m+\varphi)T^+}}{m+\varphi}\right] - e^{-mT^+}. \tag{38}$$

If also $T^- \geq \underline{T}^-$, then a stationary equilibrium with trade exists because (35) is satisfied. In other words, conditions (2), $T^+ < \overline{T}^+$ and $T^- \geq \underline{T}^-$ are sufficient. They are also necessary, because if one of them is violated then (35) is not satisfied.

Finally, in a stationary equilibrium with trade the distribution $\boldsymbol{\Delta}$ is uniquely defined, hence at most one stationary equilibrium with trade exists. QED.

Proof of Lemma 2. From (22) and from (26) we get $\int_0^{T^+} \rho(S(\tau))d\tau = \rho^S F(T^+)$ and $\int_0^{T^+} \eta(S(\tau))d\tau = \rho^S G(T^+)$ correspondingly. From (23) we express $\rho(N) = \rho^S(\frac{1}{m} - F(T^+))$. Using (27) and (32) we express $\eta(N) = \rho^S(\frac{1}{m}\frac{1-\mu}{\mu+m\mu T^- - mF^-(T^-)} - G(T^+))$. This allows us to express welfare as:

$$W = \left(\theta_H - 1 - \frac{1-\mu}{\mu + m\mu T^- - mF^-(T^-)}\right)\rho^S. \tag{39}$$

Finally, substituting for ρ^S, $F^-(T^-)$ and $G^-(T^-)$ from (31), (24) and (28) one obtains (5). QED.

Proof of Theorem 3. It is clear that T^+ does not affect welfare in equilibrium. To see how welfare changes with T^-, denote $B(T^-) = 1 + m(1-\mu)T^- + m\frac{\mu}{\varphi}(1 - e^{-\varphi T^-})$ and rewrite welfare as $W = m\left(\theta^H - 1 - \frac{(1-\mu)\theta^H + (\theta^H - 1)m(1-\mu)T^-}{B(T^-)}\right)$. Compute the derivative:

$$\frac{\partial W}{\partial T^-} = -m\frac{(\theta^H - 1)m(1-\mu)}{B(T^-)} + m\frac{((1-\mu)\theta^H + (\theta^H-1)m(1-\mu)T^-)(m(1-\mu) + m\mu e^{-\varphi T^-})}{B(T^-)^2}$$

After manipulations one obtains $\frac{\partial W}{\partial T^-} = \frac{m^2(1-\mu)}{B(T^-)^2}h(T^-)$, where $h(T^-) = 1 - \mu\theta^H - (\theta^H - 1)\frac{m\mu}{\varphi} + \mu[\theta^H + m(\theta^H - 1)(T^- + \frac{1}{\varphi})]e^{-\varphi T^-}$. Note that $h(T^-)$ is decreasing because $\frac{\partial h(T^-)}{\partial T^-} =$

$-\varphi\mu m(\theta^H - 1)T^- e^{-\varphi T^-} < 0$, for $T^- > 0$. Also note that for $T^- = 0$ we have $h(0) = 1 > 0$. On the other hand, for $T^- \to \infty$ we have $h(T^-) \to 1 - \mu\theta^H - (\theta^H - 1)\frac{m\mu}{\varphi} < 0$, because (2) implies $\frac{1-\mu\theta^H}{\mu(\theta^H-1)} < \frac{m}{\varphi}$. It follows that there is a unique $T_W^- > 0$ such that $h(T_W^-) = 0$.

Given that the sign of $\frac{\partial W}{\partial T^-}$ is determined by the sign of $h(T^-)$, welfare $W(T^-)$ is maximized for $T^- = T_W^-$ that satisfies $h(T_W^-) = 0$, that is, it solves (6). Finally, one can easily check that $T_W^- > \underline{T}^-$ if $T^+ = 0$. That is, optimal negative memory can be implemented in equilibrium if positive memory is not too long. QED.

Appendix for Online Publication

Proof of Theorem 4. Each active seller is matched with many buyers, and with probability $\beta \leq 1$ there is a quality-insensitive buyer among them. Suppose seller i is active and has a quality-insensitive buyer among other buyers he is matched with. The quality-insensitive buyer valuation of a product is equal to one independently of the product's quality, therefore he always offers a price of one to the seller. Other buyers offer the price $P_i^t = \mu(r_i^t)\theta^H$. It follows that, in equilibrium, active sellers with $\mu(r_i^t)\theta^H \geq 1$ trade at price $P_i^t = \mu(r_i^t)\theta^H$ with probability one, while active sellers with $\mu(r_i^t)\theta^H < 1$ trade at price $P_i^t = 1$ with probability β and do not trade with probability $1 - \beta$.

In a high-trade stationary equilibrium, active sellers with an N record must be able to sell to all buyers: $\mu(N)\theta^H \geq 1$. In this equilibrium, sellers with $S(\tau)$, $\tau \in [0, T^+]$ records must be able to sell to all buyers, that is $\mu(S(\tau))\theta^H \geq 1$. Suppose otherwise $\mu(S(\tau))\theta^H < 1$ for some $\tau < T^+$, then $\mu(S(T^+))\theta^H < 1$ because $\mu(S(\tau)) = \mu + (1-\mu)e^{-\varphi\tau}$ decreases with τ. It follows that sellers with $S(T^+)$ can't sell to all buyers. Moreover, $\mu(D(T^-)) = \mu(1 - e^{-\varphi T^-}) < \mu(S(T^+)) = \mu + (1-\mu)e^{-\varphi T^+}$ implies that sellers with $D(T^-)$ record also can't sell to all buyers. In a high-trade stationary equilibrium, sellers get an N record only if in the previous instance they had an $S(T^+)$ or $D(T^-)$ record that got deleted, which implies $\mu(N) \leq \mu(S(T^+))$. This means that sellers with an N record can't sell to all buyers: $\mu(N)\theta^H < 1$, i.e. a contradiction. It follows that in a high-trade stationary equilibrium $\mu(S(\tau))\theta^H \geq 1$ must hold for any $\tau \in [0, T^+]$. Given that $\mu(S(\tau))\theta^H$ decreases with τ, we get the first necessary condition for a high-trade equilibrium to exist:

$$\mu(S(\tau))\theta^H = \theta^H(\mu + (1-\mu)e^{-\varphi T^+}) \geq 1. \tag{40}$$

As before, $\rho(r)$ and $\eta(r)$ denote masses of high- and low-quality sellers with record $r \in G$ correspondingly. In a stationary equilibrium these masses must not change with time and equations (12) and (13) hold.

A seller with record $S(\tau)$, $\tau \in [0, T^+]$ trades whenever he gets active. Hence, masses of high- and low-quality sellers with $S(\tau)$, $\tau \in [0, T^+]$ records are given by (15), where, $\rho^S = \rho(S(0))$.

In the previous analysis, sellers with $D(\tau^-)$, $\tau^- \in [0, T^-]$ were not able to sell, but now these sellers can sell to quality-insensitive buyers and only to these buyers. Indeed $\mu(D(\tau^-))\theta^H \leq 1$ for any $\tau^- \in [0, T^-]$, because $\mu(D(\tau^-)) = \mu(1 - e^{-\varphi T^-}) \leq \mu(D(T^-)) = \mu(1 - e^{-\varphi T-})$ which, together with Assumption 2 and $T^- \leq \overline{T}$, implies $\mu(D(\tau^-))\theta^H < 1$ for any $\tau^- \in [0, T^-]$. Therefore, in a stationary equilibrium, at any time t a seller i with a $D(\tau^-)$ record $\tau^- \in [0, T^-]$ may become active with Poisson arrival rate m and try to sell. He can sell only if he meets a quality-insensitive seller, which happens with probability β. Essentially, these sellers trade with intensity $m\beta$ and get a new record: bad sellers get a $D(0)$ record and good sellers get an $S(0)$ record. Denote by $\eta^D = \eta(D(0))$ the mass of sellers with $D(0)$ at any moment in time, from the above argument it follows that the total mass of sellers with a $D(\tau^-)$ record is given by $\Delta^D(\tau^-) = \eta(D(\tau^-)) + \rho(D(\tau^-)) = \eta^D e^{-m\beta\tau^-}$. According to the law of large numbers and (7), the fraction of high-quality sellers among those with a $D(\tau^-)$ record is given by $\mu(1 - e^{-\varphi\tau^-})$. For any $\tau^- \in [0, T^-]$ we get:

$$\begin{aligned} \rho(D(\tau^-)) &= \eta^D e^{-m\beta\tau^-} \mu(1 - e^{-\varphi\tau^-}), \\ \eta(D(\tau^-)) &= \eta^D e^{-m\beta\tau^-} (1 - \mu + \mu e^{-\varphi\tau^-}). \end{aligned} \tag{41}$$

Sellers with N and $S(\tau)$, $\tau \in [0, T^+]$ records trade with intensity m, while sellers with $D(\tau^-)$, $\tau^- \in [0, T^-]$ records trade with intensity βm. Upon trade their type is revealed, high-quality sellers get an $S(0)$ record and low-quality sellers get a $D(0)$ record. At the same time each instance, all sellers in states $S(0)$ or $D(0)$ exit these states and in a stationary equilibrium we must have:

$$\begin{aligned} \rho^S = \rho(S(0)) &= m\rho(N) + m\int_0^{T^+} \rho(S(\tau))d\tau + m\beta\int_0^{T^-} \rho(D(\tau^-))d\tau^-, \\ \eta^D = \eta(D(0)) &= m\eta(N) + m\int_0^{T^+} \eta(S(\tau))d\tau + m\beta\int_0^{T^-} \eta(D(\tau^-))d\tau^-. \end{aligned} \tag{42}$$

In the stationary equilibrium, masses of high- and low-quality sellers with N records must stay constant. Each instance, $S(T^+)$, $D(T^-)$ records are deleted and a mass $Z = \Delta(S(T^+)) + \Delta(D(T^-)) = \rho^S e^{-mT^+} + \eta^D e^{-m\beta T^-}$ of sellers gets an N record, at the same time sellers with an N record trade and leave this state with intensity m:

$$m(\rho(N) + \eta(N)) = Z = \rho^S e^{-mT^+} + \eta^D e^{-m\beta T^-}. \tag{43}$$

For convenience, we express $Z = a_Z\rho^S + b_Z\eta^D$ and $\eta(N) = Z/m - \rho(N)$, here $a_Z = e^{-mT^+}$ and $b_Z = e^{-m\beta T^-}$. Note that (22) and (26) hold, hence we can write $\int\limits_0^{T^+} \rho(S(\tau))d\tau = \rho^S F(T^+)$ and $\int\limits_0^{T^+} \eta(S(\tau))d\tau = \rho^S G(T^+)$. Denote:

$$F'^-(T^-) = \frac{1}{\eta^D}\int\limits_0^{T^-} \rho(D(\tau^-))d\tau^- = \frac{\mu}{m\beta}(1 - e^{-m\beta T^-}) - \frac{\mu}{m\beta + \varphi}(1 - e^{-(m\beta+\varphi)T^-}), \quad (44)$$

$$G'^-(T^-) = \frac{1}{\eta^D}\int\limits_0^{T^-} \eta(D(\tau^-))d\tau^- = \frac{1-\mu}{m\beta}(1 - e^{-m\beta T^-}) + \frac{\mu}{m\beta + \varphi}(1 - e^{-(m\beta+\varphi)T^-}). \quad (45)$$

In a stationary equilibrium, the masses of high- and low-quality sellers with an N record must be constant and (17) holds. Using the first equation from (17) we can express:

$$m\rho(N) + \varphi(1-\mu)\rho(N) - \varphi\mu[Z/m - \rho(N)] = \rho(S(T^+)) + \eta(D(T^-)),$$

rearranging and substituting for $\rho(S(T^+))$, $\eta(D(T^-)$ from (15), (41) we obtain:

$$(m+\varphi)\rho(N) = \rho^S(\mu e^{-mT^+} + (1-\mu)e^{-(m+\varphi)T^+}) + \eta^D \mu e^{-m\beta T^-}(1 - e^{-\varphi T^-}) + \varphi\mu Z/m. \quad (46)$$

To shorten notations, we rewrite the above expression as $\rho(N) = \rho^S a_X + \eta^D b_X + \frac{\varphi\mu}{(\mu+\varphi)m}(a_Z\rho^S + b_Z\eta^D)$, here $a_X = \frac{(\mu e^{-mT^+} + (1-\mu)e^{-(m+\varphi)T^+})}{m+\varphi}$, $b_X = \frac{\mu e^{-m\beta T^-}(1-e^{-\varphi T^-})}{m+\varphi}$. Further, $\rho(N) = a_\rho\mu^S + b_\rho\eta^D$, where $a_\rho = a_X + \frac{\varphi\mu}{(m+\varphi)m}a_Z$, $b_\rho = b_X + \frac{\varphi\mu}{(m+\varphi)m}b_Z$. Given that $\eta(N) = Z/m - \rho(N)$ we can express $\eta(N) = a_\eta\mu^S + b_\eta\eta^D$, where $a_\eta = -a_X + \frac{1}{m}(1 - \frac{\varphi\mu}{m+\varphi})a_Z$, $b_\eta = -b_X + \frac{1}{m}(1 - \frac{\varphi\mu}{m+\varphi})b_Z$. After substitutions we get:

$$\begin{aligned}
a_\rho &= \frac{\mu}{m}e^{-mT^+} + \frac{1-\mu}{m+\varphi}e^{-(m+\varphi)T^+}, \\
b_\rho &= \frac{\mu}{m}e^{-m\beta T^-} - \frac{\mu}{m+\varphi}e^{-(m\beta+\varphi)T^-}, \\
a_\eta &= \frac{1-\mu}{m}e^{-mT^+} - \frac{1-\mu}{m+\varphi}e^{-(m+\varphi)T^+}, \\
b_\eta &= \frac{1-\mu}{m}e^{-m\beta T^-} + \frac{\mu}{m+\varphi}e^{-(m\beta+\varphi)T^-}.
\end{aligned} \quad (47)$$

In a high-trade stationary equilibrium, a seller with an N record must be able to sell to all buyers, that is, we must have $\mu(N) = \frac{\rho(N)}{\rho(N)+\eta(N)} \geq \frac{1}{\theta^H}$, or equivalently $\frac{\eta(N)}{\rho(N)} \leq \theta^H - 1$. Denoting $X = \frac{\rho^S}{\eta^D}$ we can express:

$$\frac{\eta(N)}{\rho(N)} = \frac{a_\eta\rho^S + b_\eta\eta^D}{a_\rho\rho^S + b_\rho\eta^D} = \frac{a_\eta X + b_\eta}{a_\rho X + b_\rho}. \quad (48)$$

The existence condition for a high-trade equilibrium $\frac{\eta(N)}{\rho(N)} \leq \theta^H - 1$ can be rewritten as:

$$(a_\rho + a_\eta)X + b_\rho + b_\eta \leq \theta^H(a_\rho X + b_\rho). \tag{49}$$

which, after substitutions, becomes:

$$\begin{aligned} C(T^+, T^-) = &\frac{Xe^{-mT^+}}{m} + \frac{e^{-m\beta T^-}}{m} - \\ &\theta^H \left(X(\frac{\mu e^{-mT^+}}{m} + \frac{(1-\mu)e^{-(m+\varphi)T^+}}{m+\varphi}) + \frac{\mu e^{-m\beta T^-}}{m} - \frac{\mu e^{-(m\beta+\varphi)T^-}}{m+\varphi} \right) \leq 0. \end{aligned} \tag{50}$$

To complete the proof, an expression for X remains to be found. The second equation in (42) can be rewritten in the following way: $\eta(N) + \rho^S G(T^+) + \eta^D G'^-(T^-) = \eta^D/m + (1-\beta)\eta^D G'^-(T^-)$. Using (13) we get:

$$\eta(N) + \rho^S G(T^+) + \eta^D G'^-(T^-) = \eta^D/m + (1-\beta)\eta^D G'^-(T^-) = 1 - \mu, \tag{51}$$

which implies:

$$\eta^D = \frac{(1-\mu)m}{1 + m(1-\beta)G'^-(T^-)}. \tag{52}$$

Analogously, the first equation in (42) can be rewritten in the following way: $\rho(N) + \rho^S F(T^+) + \eta^D F'^-(T^-) = \rho^S/m + (1-\beta)\eta^D F'^-(T^-)$. Using (12) we get:

$$\rho(N) + \rho^S F(T^+) + \eta^D F'^-(T^-) = \rho^S/m + (1-\beta)\eta^D F'^-(T^-) = \mu, \tag{53}$$

which, together with (52), implies:

$$\rho^S = \mu m - \frac{m(1-\beta)m(1-\mu)F'^-(T^-)}{1 + m(1-\beta)G'^-(T^-)}. \tag{54}$$

Using (52) and (54), we express $\frac{\rho^S}{\eta^D} = \frac{\mu}{1-\mu} + \frac{m(1-\beta)}{1-\mu}(\mu G'^-(T^-) - (1-\mu)F'^-(T^-))$. Substituting for $G'^-(T^-)$ and $F'^-(T^-)$ from (44) and (45) we get $\mu G'^-(T^-) - (1-\mu)F'^-(T^-) = \frac{\mu}{m\beta+\varphi}(1 - e^{-(m\beta+\varphi)T^-})$. Therefore we obtain (10):

$$X = \frac{\rho^S}{\eta^D} = \frac{\mu}{1-\mu}\left(1 + \frac{m(1-\beta)}{m\beta+\varphi}(1 - e^{-(m\beta+\varphi)T^-})\right).$$

Together, equations (40),(50) provide necessary and sufficient conditions for the existence of a high-trade stationary equilibrium. Let us consider the effect of T^- on these conditions. Condition (40) is not affected by T^-. Let us differentiate $C(T^+, T^-)$ given by (50) with respect to T^-, taking into account that X given by (10) also depends on T^-: $\frac{dC}{dT^-} = \frac{\partial C}{\partial T^-} + \frac{\partial C}{\partial X}\frac{\partial X}{\partial T^-}$. Using $\frac{\partial X}{\partial T^-} = \frac{\mu}{1-\mu}m(1-\beta)e^{-(m\beta+\varphi)T^-}$ we obtain:

$$\frac{dC}{dT^-} = -\beta e^{-m\beta T^-} - \theta^H \left(-\beta\mu e^{-m\beta T^-} + \frac{\mu(m\beta+\varphi)}{m+\varphi} e^{-(m\beta+\varphi)T^-}\right) +$$
$$\left(\frac{e^{-mT^+}}{m} - \theta^H \left(\frac{\mu e^{-mT^+}}{m} + \frac{(1-\mu)e^{-(m+\varphi)T^+}}{m+\varphi}\right)\right) \frac{\mu}{1-\mu} m(1-\beta)e^{-(m\beta+\varphi)T^-} =$$
$$-\beta e^{-m\beta T^-}(1-\mu\theta^H)$$
$$\frac{\mu e^{-(m\beta+\varphi)T^-}}{1-\mu} \left((1-\beta)[(e^{-mT^+}(1-\mu\theta^H - \frac{(1-\mu)\theta^H m e^{-\varphi T^+}}{m+\varphi})] - \frac{(1-\mu)\theta^H(m\beta+\varphi)}{m+\varphi}\right) < 0.$$

Indeed, $\mu\theta^H < 1$ implies $-\beta e^{-m\beta T^-}(1-\mu\theta^H)$. Moreover,

$$(1-\beta)[(e^{-mT^+}(1-\mu\theta^H - \frac{(1-\mu)\theta^H m e^{-\varphi T^+}}{m+\varphi})] - \frac{(1-\mu)\theta^H(m\beta+\varphi)}{m+\varphi} =$$
$$(1-\beta)[e^{-mT^+}(1-\mu\theta^H - (1-\mu)\theta^H e^{-\varphi T^+}]+ \quad (55)$$
$$(1-\beta)\varphi\frac{(1-\mu)\theta^H}{m+\varphi}e^{-(m+\varphi)T^+} - (m\beta+\varphi)\frac{(1-\mu)\theta^H}{m+\varphi} < 0,$$

Because $(1-\beta)\varphi e^{-(m+\varphi)T^+} < m\beta+\varphi$, condition (40) implies $1-\mu\theta^H-(1-\mu)\theta^H e^{-\varphi T^+} \leq 0$. We have shown that $C(T^+, T^-)$ decreases with T^-, that is, the existence condition for the high-trade stationary equilibrium is easier to satisfy when negative records are kept for a long time.

Consider now the effect of T^+. Note, that X depends on T^- but not on T^+. Let's differentiate $C(T^+, T^-)$ given by (50) with respect to T^+:

$$\frac{\partial C}{\partial T^+} = -Xe^{-mT^+}\left(1-\theta^H(\mu+(1-\mu)e^{-\varphi T^+})\right). \quad (56)$$

Condition (40) implies $1 \leq \theta^H(\mu+(1-\mu)e^{-\varphi T^+})$ and $\frac{\partial C}{\partial T^+} > 0$ unless $1 = \theta^H(\mu+(1-\mu)e^{-\varphi T^+})$. It follows that as T^+ increases, the existence conditions (40) and (50) are harder to satisfy.

Let us find the smallest T^+, denoted T^*, which rules out the existence of the high-trade equilibrium. From the previous analysis, we know that increasing T^- always relaxes the existence condition (50), therefore if the condition does not hold for $T^- \to \infty$ it will not hold for any finite T^-. As $T^- \to \infty$, $X \to \frac{\mu}{1-\mu}\frac{m+\varphi}{m\beta+\varphi}$ and (50) becomes:

$$C(T^+, \infty) = \frac{Xe^{-mT^+}}{m}\left(1-\mu\theta^H - \frac{(1-\mu)\theta^H m e^{-\varphi T^+}}{m+\varphi})\right) \leq 0. \quad (57)$$

Take T^*, which solves $A(T^*) = \theta^H(\mu+(1-\mu)\frac{m}{m+\varphi}e^{-\varphi T^*}) = 1$. Clearly, for any $T^+ > T^*$, (57) is not satisfied. This in turn implies that for any finite T^- and $T^+ \geq T^*$, the existence condition (50) is violated, while for $T^+ < T^*$ one can find $T^- = \hat{T} < \infty$ such that (50) holds as equality $C(T^*, \hat{T}) = 0$:

$$\begin{aligned} \frac{Xe^{-mT^+} + e^{-m\beta\hat{T}}}{\mu\theta^H} =& X[e^{-mT^+} + \frac{(1-\mu)m}{\mu(m+\varphi)}e^{-(m+\varphi)T^+}] + e^{-m\beta\hat{T}} - \frac{m}{m+\varphi}e^{-(m\beta+\varphi)\hat{T}}, \\ X\frac{1-\mu}{\mu} =& 1 + \frac{m(1-\beta)}{m\beta+\varphi}(1 - e^{(m\beta+\varphi)\hat{T}}). \end{aligned} \tag{58}$$

Given that $C(T^+, T^-)$ increases with T^+ and decreases with T^-, for the high-trade stationary equilibrium to exist we must have $T^+ < T^*$ and $T^- \geq \hat{T}$. Clearly, if $T^* < 0$ the stationary equilibrium does not exists because memory T^+ can't be negative and we must have $T^* \geq 0$. Since $A(T^*)$ decreases with T^*, condition $T^* \geq 0$ is equivalent to $A(0) \geq 1$, which can be written as (8):

$$\theta^H(\mu\varphi + m) \geq \varphi + m.$$

Note that $T^+ \leq T^*$ implies the first necessary condition (40), indeed, $\theta^H(\mu + (1 - \mu)e^{-\varphi T^+}) > A(T^+) \geq 1$ for any $T^+ \leq T^*$. Therefore we have proven that if (8) holds, $T^+ < T^*$ and $T^- \geq \hat{T}$, then the average quality of sellers in the pool with an N record is high enough that all buyers are ready to buy from them and a high-trade stationary equilibrium exists, that is, these conditions are sufficient. These conditions are also necessary, because if any of them is violated there is no stationary distribution of sellers to records such that the average quality of sellers in the pool with an N record is high enough to sustain trade with all buyers, and condition (50) is violated. QED

References

Acquisti, A., C. R. Taylor, and L. Wagman (2015): "The economics of privacy," *Available at SSRN 2580411.*

Attar, A., T. Mariotti, and F. Salanié (2011): "Nonexclusive competition in the market for lemons," *Econometrica*, 79, 1869–1918.

Bergemann, D. and J. Välimäki (1996): "Learning and Strategic Pricing," *Econometrica*, 64, 1125–1149.

——— (2000): "Experimentation in Markets," *The Review of Economic Studies*, 67, 213–234.

Bolton, P. and C. Harris (1999): "Strategic Experimentation," *Econometrica*, 67, 349–374.

Bos, M. and L. Nakamura (2012): "Should Defaults be Forgotten?: Evidence from Legally Mandated Removal," Tech. rep., Citeseer.

Bottero, M. and G. Spagnolo (2012): "Reputation, privacy and limited records: a survey," .

Chatterjee, S., D. Corbae, and J.-V. Rios-Rull (2011): "A theory of credit scoring and competitive pricing of default risk," *Unpublished paper, University of Minnesota.[672]*, 31.

Cripps, M. W., G. J. Mailath, and L. Samuelson (2004): "Imperfect Monitoring and Impermanent Reputations," *Econometrica*, 72, 407–432.

Daley, B. and B. Green (2012): "Waiting for News in the Market for Lemons," *Econometrica*, 80, 1433–1504.

Dellarocas, C. (2006): "Research Note: How Often Should Reputation Mechanisms Update a Trader's Reputation Profile?" *Information Systems Research*, 17, 271–285.

Ekmekci, M. (2011): "Sustainable reputations with rating systems," *Journal of Economic Theory*, 146, 479–503.

Elul, R. and P. Gottardi (2015): "Bankruptcy: Is It Enough to Forgive or Must We Also Forget?" *American Economic Journal: Microeconomics*, 7, 294–338.

Fuchs, W., A. Öry, and A. Skrzypacz (2015): "Transparency and distressed sales under asymmetric information," .

Guerrieri, V. and R. Shimer (2014): "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality," *American Economic Review*, 104, 1875–1908.

Guerrieri, V., R. Shimer, and R. Wright (2010): "Adverse selection in competitive search equilibrium," *Econometrica*, 78, 1823–1862.

Holmström, B. (1999): "Managerial incentive problems: A dynamic perspective," *The Review of Economic Studies*, 66, 169–182.

Hörner, J. and N. Lambert (2015): "Motivational Ratings," Tech. rep., Working Paper, Yale University and Stanford University.

Hörner, J. and N. Vieille (2009): "Public vs. private offers in the market for lemons," *Econometrica*, 77, 29–69.

Jappelli, T. and M. Pagano (1993): "Information sharing in credit markets," *The Journal of Finance*, 48, 1693–1718.

——— (2002): "Information sharing, lending and defaults: Cross-country evidence," *Journal of Banking & Finance*, 26, 2017–2045.

Jullien, B. and I.-U. Park (2014): "New, Like New, or Very Good? Reputation and Credibility," *The Review of Economic Studies*.

Keller, G. and S. Rady (1999): "Optimal Experimentation in a Changing Environment," *The Review of Economic Studies*, 66, 475–507.

Keller, G., S. Rady, and M. Cripps (2005): "Strategic Experimentation with Exponential Bandits," *Econometrica*, 73, 39–68.

Kim, K. (2015): "Information about sellers past behavior in the market for lemons," *University of Iowa, Unpublished Manuscript.*

Kremer, I., Y. Mansour, and M. Perry (2014): "Implementing the Wisdom of the Crowd," *Journal of Political Economy*, 122, 988–1012.

Kurlat, P. (2013): "Lemons markets and the transmission of aggregate shocks," *The American Economic Review*, 103, 1463–1489.

Liu, Q. and A. Skrzypacz (2014): "Limited records and reputation bubbles," *Journal of Economic Theory*, 151, 2–29.

Mailath, G. J. and L. Samuelson (2001): "Who wants a good reputation?" *The Review of Economic Studies*, 68, 415–441.

Musto, D. K. (2004): "What happens when information leaves a market? evidence from postbankruptcy consumers*," *The Journal of Business*, 77, 725–748.

Nock, S. L. (1993): *The costs of privacy: Surveillance and reputation in America*, Transaction Publishers.

Padilla, A. J. and M. Pagano (1997): "Endogenous communication among lenders and entrepreneurial incentives," *Review of Financial Studies*, 10, 205–236.

——— (2000): "Sharing default information as a borrower discipline device," *European Economic Review*, 44, 1951–1980.

Philippon, T. and V. Skreta (2012): "Optimal Interventions in Markets with Adverse Selection," *American Economic Review*, 102, 1–28.

Posner, R. A. (1983): *The economics of justice*, Harvard University Press.

Tirole, J. (2012): "Overcoming adverse selection: How public intervention can restore market functioning," *The American Economic Review*, 29–59.

Vercammen, J. A. (1995): "Credit bureau policy and sustainable reputation effects in credit markets," *Economica*, 461–478.