Multi-product Supply function equilibria

Work in Progress

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OVERVIEW

Multi-product Auctions

- Many auctions deal with heterogenous but closely related goods:
  - Spectrum auctions for different regions
  - Government bonds with different maturities
  - Electricity delivery at different time periods
  - Landing slots at an airport
- Both the auctioneer and bidders have non-separable preferences for goods
  - Goods can be substitutes or complements for consumers
  - Economies of scope in production

How to deal with those multi-product auctions?

- Simultaneously operating simple auctions
  - European power markets (hourly markets for different regions)
  - U.S. spectrum auctions: simultaneous multi-round ascending auctions
- Complex auction where firms submit their preference on bundles or packages
  - PJM-market: Bid-based, security-constrained, economic dispatch with nodal prices
  - U.S. Spectrum auctions: Hierarchical package bidding
  - Block bids in European electricity market

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This paper: Procurement auction

- Two heterogeneous goods
  - goods are divisible
- Demand: competitive
  - goods can be substitutes or complements
  - demand is stochastic
- Supply: duopoly
  - (dis)economies of scope
  - can produce multiple units of each good

This paper: Procurement auction

- Supplier submit a bid function for each good
  - for each price level how much the firm is willing to supply
  - quantity for one good depends not only that good's price, but also of the other
  - *i.e.* each firm chooses $s_1(p_1, p_2)$ and $s_2(p_1, p_2)$
- Market equilibrium Uniform Price Auction
  - combine supply bids & particular realization of demand
  - market clearing $\rightarrow$ equilibrium prices, production quantities
  - Infra-marginal incentives matter

Related Literature

Multiple-good oligopolies

- Product interactions might soften or weaken competition between oligopoly producers
- Bülow, Geanakoplos, and Klemperer (1985), Cabral and Villas-Boas (2005)
- Our contribution: Introduce demand uncertainty and allow for a larger strategy space

Combinatorial auctions

- Ausubel and Cramton (2004), Ausubel, Cramton, and Milgrom (2006), Milgrom (2000), Ausubel (2004)

LITERATURE

Supply function equilibria (SFE)

- Firms compete by a choosing **supply function** $s(p)$
- Seminal paper by Klemperer and Meyer (1989)
  - Demand uncertainty pins down supply functions by differential equation
  - Unbounded support unique symmetric equilibrium
- Many applications i.a. in electricity markets
  - (Hortaçsu & Puller, 2008; Sioshansi & Oren, 2007; Holmberg & Newbery, 2010)
- Our contribution: We look at multiple products

REST OF PRESENTATION

- Derive 2-Dimensional version of SFE-model of Klemperer and Meyer
  - Similar first and second order conditions
- Bundling goods (equivalent to coordinate transformation)
  - Decouple demand, costs and bid functions either locally or globally
  - Apply results of standard 1-Dimensional SFE-model

Overview

Set-up

- Two goods:
  - prices $\mathbf{p} = [p_1, p_2]^\top \in \mathcal{P} \subset \mathbb{R}^2$
  - quantities $\mathbf{q} = [q_1, q_2]^\top \in \mathcal{Q} \subset \mathbb{R}^2$
- Stochastic demand function $\mathbf{d} : \mathcal{P} \times \mathcal{E} \to \mathcal{Q}$
  - $\mathbf{q} = \mathbf{d}(\mathbf{p}, \boldsymbol{\varepsilon})$
  - demand shock $\boldsymbol{\varepsilon} = [\varepsilon_1, \varepsilon_2]^\top$
    joint cumulative distribution function $\Phi(\boldsymbol{\varepsilon})$ on $\mathcal{E}$.
- Profit of supplier $k \in \mathcal{K}$= Revenue minus Costs:

$$\pi_k(\mathbf{q}, \mathbf{p}) = \mathbf{p}^\top \mathbf{q} - c_k(\mathbf{q})$$

Set-up

Bidding and equilibrium

- Firm k bids supply function $\mathbf{s}_k : \mathcal{P} \to \mathcal{Q} : \mathbf{q} = \mathbf{s}_k(\mathbf{p})$ and maximizes expected profit

$$\Pi_k = \int_{\mathcal{E}} \pi_k(\mathbf{p}^{eq}(\varepsilon), \mathbf{s}_k(\mathbf{p}^{eq}(\varepsilon)))\, \mathrm{d}\Phi(\varepsilon) \tag{1}$$

where the market equilibruim price $\mathbf{p}^{eq}(\varepsilon)$ is determined by market clearing

$$\mathbf{s}_k(\mathbf{p}) = \mathbf{d}_k(\mathbf{p}, \varepsilon) \equiv \mathbf{d}(\mathbf{p}, \varepsilon) - \sum_{\mathcal{K} \setminus k} \mathbf{s}_{k'}(\mathbf{p}) \tag{2}$$

Setup: Assumptions

Demand and Cost

- Convex cost ($\partial^2 c_k / \partial \mathbf{q}^2 > 0$)
- Downward sloping demand ($\partial \mathbf{d} / \partial \mathbf{p} < 0$)
- Non-crossing demand ($\partial \mathbf{d} / \partial \varepsilon$ has full rank)

Restrictions on Bidding format

- Upward sloping supply ($\partial \mathbf{s}_k / \partial \mathbf{p} > 0$)
  Implies symmetry $\partial s_{k,i} / \partial p_j = \partial s_{k,j} / \partial p_i$

- Equivalently, firm k submits convex cost function $Z_k(\mathbf{q})$ such that $(\partial Z_k / \partial \mathbf{q})^{-1} = \partial \mathbf{s}_k / \partial \mathbf{p}$.

MARKET COUPLING?

- A firm's **supply of good 1** can depend on the **price of good 2** because:

  1. Economies of scope in production
  2. Demand substitutes or complements
  3. Correlated demand shocks
  4. Strategic considerations

Overview

Introduction

Model

Nash Equilibrium

Analysis

Extensions

First Order Conditions

- 2-dimensional version of Klemperer and Meyer conditions:

$$\mathbf{s}_k(\mathbf{p}) + \frac{\partial \mathbf{d}_k(\mathbf{p}, \varepsilon)}{\partial \mathbf{p}}^\top \left(\mathbf{p} - \frac{\partial c_k(\mathbf{s}_k(\mathbf{p}))}{\partial \mathbf{q}}^\top \right) = 0 \qquad (3)$$

- Non-crossing of demand functions leads to ex-post optimality
  - After realization of demand shock ε firm k has no incentive change its bid
- **Note:** Correlation of demand shocks does not affect coupling of supply functions

First Order Conditions

- First order condition for firm k and good i rewrites as:

$$\frac{p_i - c_{k,i}}{p_i} = \epsilon_{k,i}$$

  with $\epsilon_{k,i}$ the super-elasticity of the residual demand of firm k of good i
- Firm takes into account own price and cross-price elasticity of demand
- Nash equilibrium: Solve set of Partial Differential Equations and check second order conditions

Overview

Bundling of goods

- Consider the procurement of two bundles $i' = 1', 2'$
- Each bundle is divisible and consists of fixed proportions of goods $1, 2$.
- Bundle $1'$ consists of $A_{1'1}$ units of good 1 and $A_{1'2}$ units of good 2, etc...
- **So: $\tilde{\mathbf{q}}$ bundles contain $\mathbf{q} = \boldsymbol{A}\tilde{\mathbf{q}}$ goods**

Cost and Demand for bundles: $\tilde{(\cdot)}$

- Cost $\tilde{c}_k(\tilde{\mathbf{q}}) = c_k(\boldsymbol{A}\tilde{\mathbf{q}})$
- Demand $\tilde{\mathbf{d}}(\tilde{\mathbf{p}}, \varepsilon) = \boldsymbol{B}\,\mathbf{d}(\boldsymbol{B}^\top\tilde{\mathbf{p}}, \varepsilon)$ with $\boldsymbol{B} = \boldsymbol{A}^{-1}$.

BUNDLING OF GOODS: INVARIANCE

Theorem

Strategy profile $\{\tilde{\mathbf{s}}_k(\tilde{\mathbf{p}})\}_{k\in\mathcal{K}}$ *is a SFE for bundles*

$$\Updownarrow$$

Strategy profile $\{\mathbf{s}(\mathbf{p})\}_{k\in\mathcal{K}} = \left\{A\tilde{\mathbf{s}}_k(A^\top\tilde{\mathbf{p}})\right\}_{k\in\mathcal{K}}$ *is a SFE for goods*

- So, Supply Function Equilibria are invariant to bundling.
- We can think of bundling as a coordinate transformation.

Linear problem

With linear demand, additive shocks, quadratic costs and symmetric firms:

$$\mathbf{d}(\mathbf{p}, \boldsymbol{\varepsilon}) = \boldsymbol{D}\mathbf{p} + \boldsymbol{\varepsilon}$$
$$c(\mathbf{q}) = \frac{1}{2}\mathbf{q}^\top \boldsymbol{C}\mathbf{q}$$

with $\boldsymbol{D}$ and $\boldsymbol{C}$ matrices.

Theorem (Diagonalization)

$\exists$ bundling $\boldsymbol{A}$ such that $\tilde{\mathbf{D}} = \boldsymbol{A}\boldsymbol{D}\boldsymbol{A}^\top$ and $\tilde{\mathbf{C}} = \boldsymbol{B}^\top \boldsymbol{C}\boldsymbol{B}$ are diagonal matrices

Linear problem: Diagonalization

- Diagonalization Theorem implies that the demand and cost for bundles are fully decoupled:

$$\tilde{d}_1(\tilde{\mathbf{p}}, \tilde{\boldsymbol{\varepsilon}}) = \tilde{D}_{11}\tilde{p}_1 + \tilde{\varepsilon}_1$$
$$\tilde{d}_2(\tilde{\mathbf{p}}, \tilde{\boldsymbol{\varepsilon}}) = \tilde{D}_{22}\tilde{p}_2 + \varepsilon_2$$
$$\tilde{c}(\tilde{\mathbf{q}}) = \frac{1}{2}\tilde{q}_1\tilde{C}_{11}\tilde{q}_1 + \frac{1}{2}\tilde{q}_2\tilde{C}_{22}\tilde{q}_2$$

Linear problem: Market Decoupling

Theorem (Full Decoupling)

If demand and cost are fully decoupled, then in equilibrium also the supply functions are decoupled:

$$\partial s_i(\mathbf{p})/\partial p_j = 0.$$

- Consequence of requiring $\partial \mathbf{s}/\partial \mathbf{p}$ to be symmetric. Proof is not fully straightforward.
- Hence, there are no strategic considerations to couple supply if demand and costs are separate.

Linear problem: Equilibria

- Bundle markets such that each
- **Market for each bundle can be considered separately.**
- All results of the standard Klemper and Meyer SFE-model hold for each bundle.
- We can derive the SFE for the individual goods by doing the inverse transformation $\boldsymbol{A}^{-1}$.

Corollary

With unbounded shocks, linear demand and quadratic costs, the linear supply function $\mathbf{s} = \mathbf{Sp}$ *is the unique SFE.*

GENERAL PROBLEM

- In general it is not possible to find a linear bundle that decouples markets *globally*
- However, a bundle that decouples markets *locally* always exist
  (Taylor expansion of demand and costs)
- It can be shown that with this bundle also the supply functions are locally decoupled.
- This means that:
  - local properties of the one-dimensional SFE model carry-over (such as around origin).
  - once a single point $\mathbf{p}_0, \mathbf{q}_0$ is known on the supply function, the SFE is uniquely defined and can be integrated numerically

Overview

Extensions

- Generalization
  - All results hold directly for N products and M firms
  - Demand functions that cross / higher dimensional demand shocks ($> N$) = Work-in-progress
    - Re-parameterize demand with N dimensional demand shock such that demand doesn't cross.
  - Private cost-types (as in Vives).
    - Currently only feasible without correlation of cost types
- Adding examples
  - Study effect of substitutes and complements
  - Compare complex bid auctions ($\partial s_{k,i}/\partial p_j \neq 0$) and simple bid auctions ($\partial s_{k,i}/\partial p_j = 0$)