Cash-in-the-market pricing or cash hoarding: how banks choose liquidity*

Jung-Hyun Ahn† Vincent Bignon‡ Régis Breton§ Antoine Martin¶

February 2017

Abstract

We develop a model in which financial intermediaries hold near-cash assets to protect themselves from shocks. Depending on parameter values, banks may choose to hold too much or too little cash on aggregate compared to the socially optimal amount. The model, therefore, provides a unified framework for thinking, on the one hand, about policy measures that can reduce hoarding of cash by banks and, on the other hand, about liquidity requirements of the type imposed by the new Basel III regulation. Key to our results is a market in which banks can obtain cash by selling or repo-ing their marketable securities. The quantity of cash obtained on this market is determined endogenously by the market value of the marketable securities and is subject to cash-in-the-market pricing. When uncertainty about the value of marketable securities is low, banks hold too little cash and liquidity regulation can achieve a better allocation. When uncertainly about the value of marketable securities is sufficiently high, banks hoard cash leading to a market freeze.

Keywords: money market, liquidity regulation, non conventional monetary policy, cash-in-the-market-pricing

JEL: E58, G28, G21

*We are grateful to Cyril Monnet, Bruno Parigi, Eva Schliephake, Alexandros Vardoulakis and Guillaume Vuillemey for helpful comments. We also thank seminar participants of the Oxford Financial Intermediation Theory (OxFIT) Conference, of the TSE-BdF conference on models of the money market, of the SAET conference in Cambridge, and at the universities of Bern, Paris-Dauphine, Padova and St. Gallen. Antoine Martin acknowledges the support of the Fondation Banque de France in the inception of this project. The views expressed herein are those of the authors and do not necessarily represent the views of the Banque de France, the Eurosystem, the Federal Reserve Bank of New York, or the Federal Reserve System.

†NEOMA Business School, jung-hyun.ahn@neoma-bs.fr

‡Banque de France, vincent.bignon@banque-france.fr

§Banque de France, regis.breton@banque-france.fr

¶Federal Reserve Bank of New York and University of Bern, antoine.martin@ny.frb.org

1. Introduction

This paper studies whether the amount of liquidity (near-cash assets) chosen by financial intermediaries to protect themselves from shocks is optimal from the perspective of social welfare. Liquidity has been an important policy issue since the 2007-2009 financial crisis. The new Basel III regulation introduced liquidity requirements, in the form of the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR). This type of regulation stems from the concern that financial intermediaries may not hold enough liquid assets. On the other hand, concerns have been expressed that some financial intermediaries may be hoarding liquid assets such as cash, leading to market freezes, particularly in Europe during the Subprime and Euro crisis (ECB, 2014).[1] In light of these concerns, it seems important to understand under which circumstances the private decisions of decentralized financial intermediaries may lead to too much or too little liquidity in the financial system. Our model provides a unified framework to think about these issues and the policy interventions that could lead to better outcomes.

We develop a simple model of banks that are subject to a liquidity shock, in the spirit of Diamond and Dybvig (1983) and Holmstrom and Tirole (1998). Our banks are endowed with a non-tradable project, which we can think of as a commercial loan, that may require additional cash to successfully come to maturity. Banks also need to decide how much near-cash assets and marketable securities they want to hold. We think of near-cash assets as level 1 high quality liquid assets, such as central bank reserves and sovereign bonds. Examples of marketable securities in our model would be corporate bonds or equities. Marketable securities have a higher expected return than cash but their return is uncertain. In addition, these securities cannot be used directly to meet the liquidity shock, but they can be exchanged against cash in a market. The market may be interpreted either as a secured money market in which banks lend temporarily their securities against cash (e.g. in a repo transaction) or as a market in which the securities are sold.[2] Banks holding securities that turn out to have a low return may not be able to raise sufficient cash when they are hit by a liquidity shock.

We assume that the non-marketable project cannot be pledged or sold to other banks. That project may not be good collateral because its value could depend on the specific skills

[1] For example, the Wall Street Journal reported "Europe's biggest banks are continuing to stash more money at central banks, a move that reflects their lingering fears about the financial system despite signs of improvement." (WSJ "Large European Banks Stash Cash" Nov. 13 2012). For academic work on the topic see Acharya and Merrouche (2012) for the UK, Heider et al. (2015) for Euro zone, de Haan and van den End (2013) for the Netherlands. See also Ashcraft et al. (2011) for empirical evidence of precautionary holding of reserves by U.S. banks in 2007-2008.

[2] As is usual in the literature (for example Allen et al. (2009)), there is no substantial difference between a secured lending market and a market for outright purchases in our model. Indeed, since the return on marketable securities is publicly known when the market opens in our model, there is no difference between selling marketable securities and using them as collateral in secured loans.

of the banker so that its value is very low when the bank fails, which is precisely when the collateral is needed (Diamond and Rajan, 2001).[3] In contrast, marketable securities are good collateral because their value does not depend on the survival of the bank that pledged them.

In choosing the optimal amount of cash, a planner weighs the benefit of an additional unit of cash, which can be used to continue non-marketable projects, against the cost, which is the foregone return on tradable assets. At the optimum, the planner chooses to hold exactly enough cash, on aggregate, to continue all non-marketable projects, because we assume that the return on those projects is sufficiently high.

We show that banks may choose to hold too much or too little cash on aggregate, in equilibrium, depending on parameter values, compared to the socially optimal amount. When uncertainly about the value of the marketable asset is low, banks hold too little cash because they do not internalize the effect their cash-holding decision on the equilibrium price of securities. Low cash holding implies that the value of securities is low because of cash-in-the-market pricing. This, in turns, reduces the extent to which banks hit by a liquidity shock can continue their non-marketable project.

If the uncertainty about the value of the marketable securities is sufficiently high, banks choose to hoard cash; that is, banks hold enough cash to continue their long-term project without needing to access the market. This leads to a market "freeze". Nevertheless, we show that this allocation is constrained optimal.

Our work relates to the body of research analyzing liquidity choices by financial institutions, and the link with asset prices. Following Allen and Gale (2004, 2005), a number of papers have analyzed how *cash-in-the-market-pricing* affects banks' liquidity decisions and the possibility of central bank or public intervention. In particular, our paper is related to Allen et al. (2009), Freixas et al. (2011) and Gale and Yorulmazer (2013). An important difference between these papers and ours is that there can be either too much or too little cash in equilibrium in our model. Acharya et al. (2010) shows how the expectation of fire sales can lead to liquidity choices that are excessive from a social viewpoint. The strategic motive for cash hoarding is also analyzed in Acharya et al. (2012) and Gale and Yorulmazer (2013). Other papers show that liquidity hoarding may be a response to an exogenous increase in counterparty risk (Heider et al., 2015). The interplay between endogenous liquidity choices and private information is analyzed in Bolton et al. (2011) and Malherbe (2014). Bolton et al. (2011) focus on a tradeoff between outside and inside liquidity and show that the timing of trade can lead to an equilibrium with too little outside liquidity and too little cash-in-the-market-pricing. Malherbe (2014) shows how in the absence of cash-in-the-

[3]While we do not explicitly model these frictions, a number of informational friction could help explain why the non-marketable project cannot be sold or its return pledged. See for example Gale and Yorulmazer (2013).

market-pricing initial liquidity choices impose a negative externality on future liquidity by exacerbating adverse selection problems. We consider a setup where information on traded assets is symmetric, and relate socially excessive liquidity to the dispersion of traded assets.

The remainder of the paper proceeds as follows: In section 2 we describe the model. Section 3 characterizes the allocations chosen by the planner, under different constraints. We describe banks' behavior in section 4. The definition and characterization of equilibria is done in section 5. Section 6 considers some policy implications of the model. Section 7 concludes.

2. Environment

The economy is populated by a continuum of identical agents called banks. There are 3 dates indexed by $t \in \{0, 1, 2\}$. At $t = 0$, each bank is endowed with a non-marketable investment project, which we can think of as an industrial loan or a commercial real-estate deal, for example, as well as one unit of cash.[4] Banks can keep the cash or invest in marketable "securities," such as bonds, equities, or asset-backed securities.[5] At $t = 1$, a financial market opens in which each bank can rebalance its portfolio of cash and securities. At $t = 2$, the investment project and the securities mature, and consumption takes place.

2.1. *Cash and securities*

Cash has a net return of 0 and can be costlessly stored from one period to the next. Tradable securities generate a stochastic return at date 2. The actual return depends on the type θ of the security, which is publicly revealed at date 1. That return is H with probability of μ and L otherwise. Tradable securities generates a return R_θ at date 2 with $R_H > R_L$. The expected gross return, denoted $\bar{R}$, is assumed to be greater than 1.

$$\bar{R} \equiv \mathbb{E}\left(R_\theta\right) = \mu R_H + (1 - \mu) R_L > 1 \tag{A1}$$

Let $s \in [0, 1]$ denote the share of the cash in the bank's portfolio at $t = 0$.

2.2. *Project*

Each bank holds a project that yields y at $t = 2$. At $t = 1$, with probability λ, the project is hit by a liquidity shock that requires the injection of an extra amount of cash.[6] To simplify notation, we normalize that amount to 1. In Appendix A, we show that our results

[4] We think of "cash" as including near-cash assets, such as level 1 HQLA.

[5] Empirically we think of this asset as any type of security with an International Security Identification Number (ISIN) or, in the U.S. context, a CUSIP, allowing its trade on an organized market. We do not modeled the process leading to the creation of this type of security.

[6] We assume a law of large number holds so that the share of projects affected by the liquidity shock is also λ.

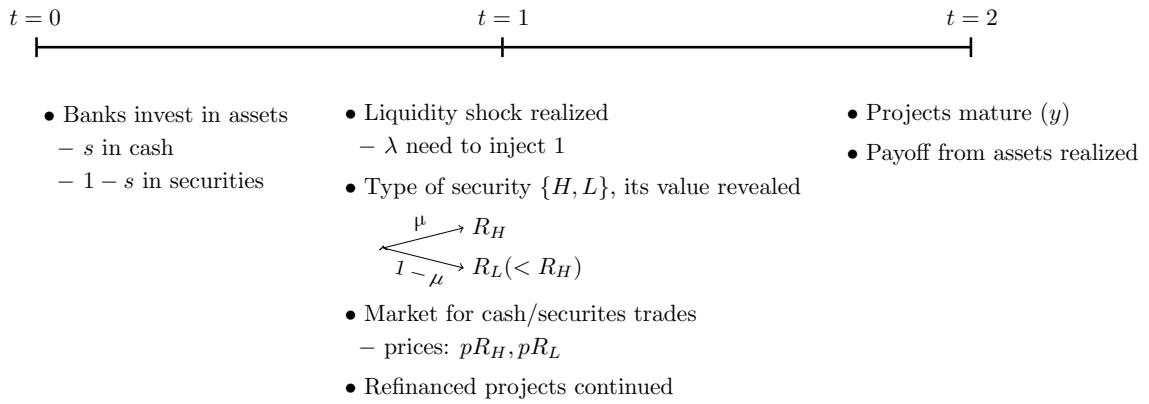


Figure 1: Timing

generalize to the case where the amount of cash to be injected is less than 1. If no cash is injected, the project is liquidated at $t = 1$ and yields nothing. The project is divisible: If a bank does not have enough cash at $t = 1$ to continue the project in its entirety, it can inject $i < 1$ of cash and the project returns $i \cdot y$ at date $t = 2$.

We assume that :

$$y > \bar{R} \tag{A2}$$

This condition implies that, from the perspective of date 0, the return y on the extra amount of cash that needs to be injected to continue to project in case of a liquidity shock is greater than $\bar{R}$, the expected return on securities.

2.3. *Timing*

- $t = 0$: Each bank chooses s, the share of cash it wants to hold, with $1 - s$ representing the share of securities the bank holds.
- $t = 1$: Each bank learns the value of the securities it holds and whether it suffer liquidity shock. The market for securities opens. Bank with a liquidity shock decide whether to inject additional cash to continue their project.
- $t = 2$: The return on long term project and the securities are realized. Banks profit are realized.

3. The planner's problem: Optimal allocation between cash and securities

In this section we determine the date-0 optimal allocation from the point of view of a social planner. Since all agents are risk neutral, we define welfare as total expected output. The social planner aims to maximize social welfare by choosing the cash level s that

individual banks hold at $t = 0$ and by redistributing cash among banks at $t = 1$, so that the banks subject to a liquidity shock have the cash they need to continue their project.

The total output of economy is given by

$$W(s) = \max 1, [1 - \lambda + \lambda(s + c)]y + [(1 - \lambda)s - \lambda c] + (1 - s)\bar{R} \quad (1)$$

Subject to the feasibility constraint

$$(1 - \lambda)s \geq \lambda c,$$

where c is the amount of cash transferred to a bank with a liquidity shock. We define $C \equiv \lambda c$ as the aggregate amount of cash transferred. The first term in (1) is the return from projects that mature. A proportion $1 - \lambda$ of banks are not hit by the liquidity shock and their project will mature yielding y. In addition, a proportion λ of banks are affected by the liquidity shock. Each of these banks holds an amount s of cash and obtains c from the social planner. In aggregate, a proportion $1 - \lambda + \lambda(s + c) \leq 1$ of project, each yielding y, will be continued. The second term represents leftover cash, which cannot be negative, and the third term captures the return of tradable securities in the economy.

Since the expected return of the project, y, is greater than the expected return on securities, $\bar{R}$, which is itself greater than the return on cash, 1, inspection of the objective function reveals that any solution must have $1-\lambda+\lambda(s+c) = 1$ and $(1-\lambda)s = \lambda c$. The first equality state that there should be enough cash in the economy to continue all projects. The second equality states that there should be no leftover cash. Together, these two conditions imply $s^{FB} = \lambda$.

Next we consider the problem of the social planner who maximizes welfare in the economy under the constraint that a bank cannot receive more cash than the value of the securities it holds. For example, banks may be able to hide the cash they hold and the planner can only transfer cash if banks are compensated for the cash they give up. Studying the allocation of the constrained planner is useful to distinguish the inefficiency related to the redistribution of cash in the interbank market from the inefficiency related to the choice of cash held by banks at date 0.

The constrained-optimal, or second-best, level of cash is denoted s^{SB}. A share μ of cash-poor banks hold H-security, which have a value greater than 1 by assumption, so they can receive the amount $1 - s$ of cash against (some of) their securities. However, a share $1-\mu$ of bank with liquidity shock hold L-security, which can have value lower than 1. Let c_θ denote the transfer to banks holding θ-security, $\theta \in \{L, H\}$. The constraint that the social planner faces for banks holding L-security is therefore

$$c_L \leq (1 - s)R_L.$$

Note that the planner would never transfer more than $1-s$, so the above constraint binds only if $R_L < 1$. The aggregate amount of redistributed cash is given by

$$C = \begin{cases} (1-\lambda)\, s & \text{if} \quad R_L \geq 1, \\ \min\left\{(1-\lambda)\, s,\, \lambda\,(1-s)\,(\mu + (1-\mu)\, R_L)\right\} & \text{if} \quad R_L < 1. \end{cases} \tag{2}$$

To find the marginal contribution of cash to output, $\frac{\partial W}{\partial s}$, we can plug (2) into (1) and differentiate with respect to s. Let $\hat{s}$ denote the level of s such that there is no cash left over after transfers if $R_L < 1$. It is the value of s that solves $\lambda\,(1-s)\,(\mu + (1-\mu)\, R_L) = (1-\lambda)\, s$, and is given by

$$\hat{s} \equiv \frac{\lambda\,(\mu + (1-\mu)\, R_L)}{\lambda\,(\mu + (1-\mu)\, R_L) + 1 - \lambda} < s^{FB}. \tag{3}$$

It is easy to verify that $\frac{\partial W}{\partial s} > 0$ if $s < \hat{s}$. Indeed, when $s < \hat{s}$ an additional unit of cash can be used to continue a larger share of the project without resulting in any leftover cash after transfers. When $s \geq \hat{s}$, we have

$$\frac{\partial W}{\partial s} = (1 - \lambda\,(1-\mu)) + \lambda\,(1-\mu)\,(y - R_L\,(y-1)) - \bar{R}$$

The sign of this expression depends on R_L, but not on s. We can derive $\hat{R}$ is the value of R_L such that $\frac{\partial W}{\partial s} = 0$.

$$\hat{R} \equiv 1 - \frac{\bar{R} - 1}{(1-\mu)\,\lambda\,(y-1)}. \tag{4}$$

If $R_L > \hat{R}$, then $\partial W / \partial s < 0$, so the planner wants to minimize the amount of cash held, conditional on continuing as much of the non-marketable projects as possible. In contrast, if $R_L < \hat{R}$, then $\partial W / \partial s > 0$, so the planner wants to maximize the amount of cash.

These results are summarized in the following proposition 1:

Proposition 1. *1. When $R_L \geq 1$, the additional constraint faced by the planner does not bind so that $s^{SB} = s^{FB} = \lambda$.*

2. When $\hat{R} < R_L < 1$, the constrained optimal level of cash s^{SB} is $\hat{s}$ ($< s^{FB}$).

3. When $R_L = \hat{R}$, the constrained optimal level of cash s^{SB} can be any value between $[\hat{s}, 1]$.

4. When $R_L < \hat{R}$, the constrained optimal level of cash s^{SB} is 1 ($> s^{FB}$).

The constrained optimal level of cash depends on R_L, the low return for the securities. When R_L is higher than 1, the planner can achieve the first-best because the constraint does not bind. When R_L is smaller than 1, the planner can no longer transfer the optimal amount of cash. The planner chooses to reduce s for two reasons: First, a reduction of s

is necessary to make sure that there is no unused cash after transfers. Second, reducing s allows banks to hold more securities, which can be used to acquire more cash when the return on the securities is low. For these reasons, s^{SB} decreases as R_L decreases until R_L becomes smaller than $\hat{R}$ at which point it jumps to 1. The intuition for this result is that when R_L is so low, the interbank market is very ineffective at redistributing cash among banks. Hence, the planner prefers that banks hold sufficient cash to continue the project without needing a transfer that is constrained by the value of the securities.

4. Equilibrium

In this section, we derive the equilibrium allocation. At date $t = 0$, banks decide how much cash and securities to hold, taking into account the expected market price of securities in terms of cash at $t = 1$. Working backward, we first derive the market price of securities at $t = 1$, for a given choice of s, and then we consider the optimal choice of s at $t = 0$.

4.1. *Market for securities at* $t = 1$

At date 1, a market opens in which banks can trade cash for securities. Banks that receive a liquidity shock and need to acquire additional cash to continue their project will seek to sell some or all of their securities holding in this market. Banks that did not receive a liquidity shock, as well as banks that are unwilling to continue their project, may want to purchase securities with their cash. Since there exists two types of securities H, L, we first characterize a no arbitrage condition for the price of these securities.

Because there is no informational asymmetry, the no-arbitrage condition requires that the return from purchasing either security be same. Let $\frac{1}{p}$ denote the market return on securities. This notation is convenient because the price of security θ can be written as pR_θ. The no-arbitrage condition can thus be expressed as

$$\frac{R_H}{pR_H} = \frac{R_L}{pR_L} = \frac{1}{p}. \tag{5}$$

At $t = 1$, banks hit by the liquidity shock decide the share of their nonmarketable project, $i \in [0, 1]$, to continue, with full continuation of the project when $i = 1$. This decision depends on the comparison between the return of injecting cash i in the project, $i \cdot y$, and its opportunity cost, which is the return on purchasing securities, $\frac{i}{p}$. The bank will choose to continue its project if

$$y \geq \frac{1}{p}. \tag{6}$$

4.1.1. *Trading strategies and supply and demand of cash*

Banks may be willing to supply cash to the market, and purchase securities, for two reasons. First, banks that are not subject to the liquidity shock will buy securities if their

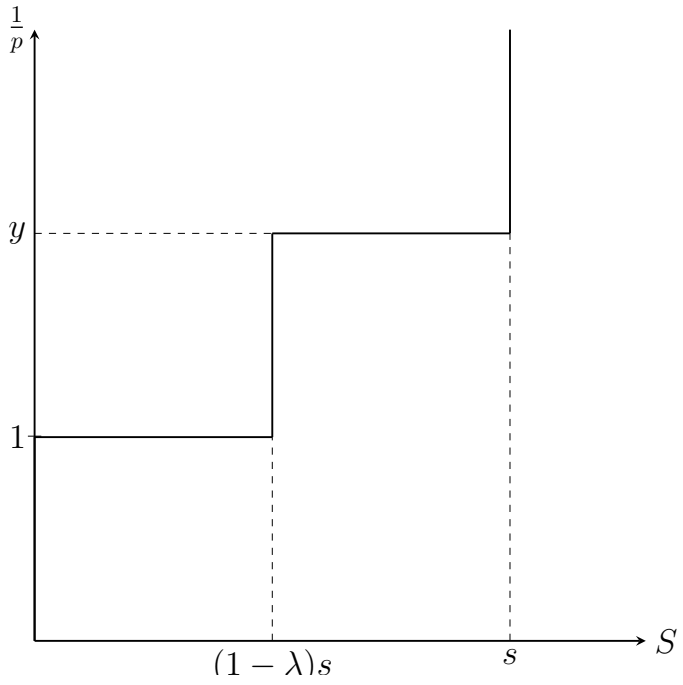


Figure 2: Supply of cash

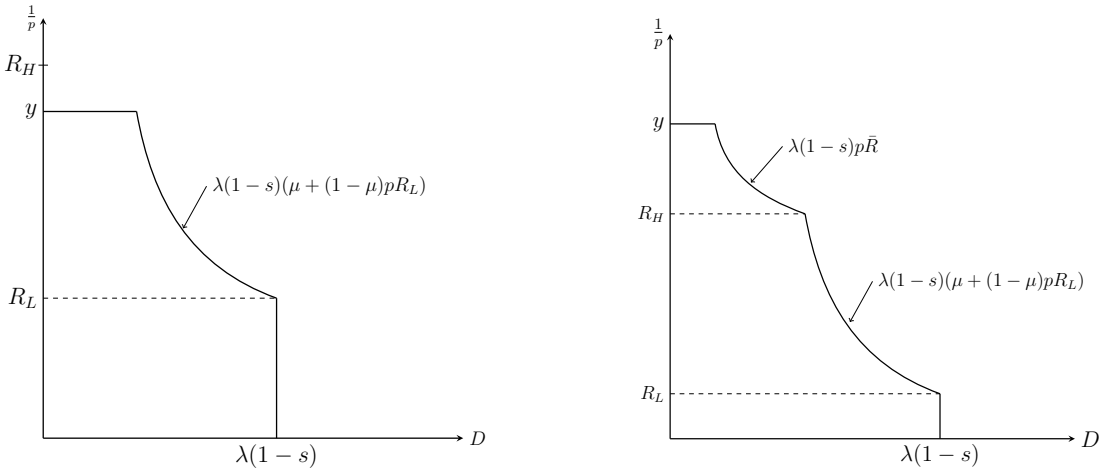
return is at least as high as the return on cash, $\frac{1}{p} \geq 1$. Second, banks hit by the liquidity shock supply cash and purchase securities if the market return on cash is higher than their internal return on cash, $\frac{1}{p} \geq y$. The aggregate supply function of cash, illustrated in Figure 2, is given by:

$$S = \begin{cases} 0 & \text{if} \quad \frac{1}{p} < 1, \\ [0, (1-\lambda)s] & \text{if} \quad \frac{1}{p} = 1, \\ (1-\lambda)s & \text{if} \quad 1 < \frac{1}{p} < y, \\ [(1-\lambda)s, s] & \text{if} \quad \frac{1}{p} = y, \\ s & \text{if} \quad \frac{1}{p} > y. \end{cases} \tag{7}$$

Next, we can derive the demand for cash. Banks that receive a liquidity shock demand cash and supply securities to the market, provided they have an incentive to continue their project. This condition corresponds to inequality (6).

Assuming inequality (6) holds one of two things can happen: If $s + (1-s)\, pR_\theta \geq 1$ (equivalently $pR_\theta \geq 1$), then the bank can acquire $1-s$, the amount of cash that it needs to continue its project. Otherwise, the bank acquires as much as it can by selling all the securities at its disposal, $(1-s)\, pR_\theta$. If inequality (6) does not hold, banks will not demand any cash.

The individual demand function for cash by a bank holding θ-type security and hit by a



(a) $R_L < y \leq R_H$ (b) $R_L < R_H < y$

Figure 3: Demand of cash

liquidity shock, denoted d_θ, is given by:

$$d_\theta = \begin{cases} \min\{1-s, (1-s)\,pR_\theta\} & \frac{1}{p} < y, \\ [0,\, \min\{1-s, (1-s)\,pR_\theta\}] \quad \text{if} & \frac{1}{p} = y, \\ 0 & \frac{1}{p} > y. \end{cases}$$

The aggregate demand function on cash, D, illustrated in Figure 3, is given by the sum of the individual demands:

$$D = \begin{cases} A & \frac{1}{p} < y, \\ [0,\, A] \quad \text{if} & \frac{1}{p} = y, \\ 0 & \frac{1}{p} > y. \end{cases} \tag{8}$$

where

$$A = \lambda\left[\mu \min\{1-s, (1-s)\,pR_H\} + (1-\mu)\min\{1-s, (1-s)\,pR_L\}\right].$$

4.1.2.*Equilibrium price of security at* $t = 1$

This section characterizes the market equilibrium price of securities, given a cash level s, chosen at $t = 0$. The price of θ-type securities is denoted by $p^* R_\theta$. First, we prove the following lemma:

Lemma 1. *Given the cash holding level* s *chosen at* $t = 0$*, the equilibrium market return on cash* $\frac{1}{p^*}$ *always satisfies* $1 \leq \frac{1}{p^*} \leq y$.

Proof. If the market return on cash $\frac{1}{p}$ is lower than 1, then holding cash would have a higher return than purchasing securities, implying that the supply of cash in the market would be

0. Therefore, $\frac{1}{p} < 1$ cannot be an equilibrium. On the other hand, if market return on cash is higher than y, then all cash holders would prefer to buy securities rather than continue their project, implying that there would be no demand for cash in the market. Therefore, $\frac{1}{p} > y$ cannot be an equilibrium.□

Note that since $y \geq \frac{1}{p}$, banks hit by a liquidity shock always prefer to use their cash to continue their project rather than to acquire securities in the market. This implies that the supply of cash in the market is given by banks that are not affected by the liquidity shock,

$$S = (1-\lambda)s. \tag{9}$$

When $1 < \frac{1}{p} < y$, the demand for cash can be written as:

$$D(s) = \begin{cases} \lambda(1-s) & \text{if} \quad 1 \leq pR_L, \\ \lambda(1-s)\left[\mu + (1-\mu)pR_L\right] & \text{if} \quad pR_L < 1 \leq pR_H, \\ \lambda(1-s)\bar{R} & \text{if} \quad 1 > pR_H. \end{cases}$$

Intuitively, if $pR_L \geq 1$, then banks affected by a liquidity shock can sell enough assets to get the cash they need to continue their project regardless of the return on the securities they hold. So a fraction λ of banks demand an amount $(1-s)$ of cash. If $pR_H \geq 1 > pR_L$, then banks affected by a liquidity shock can get as much cash as they would like if they hold securities with return R_H, but are constrained if they hold securities with return R_L. Finally, if $1 > pR_H$, all banks affected by a liquidity shock are constrained.

There exists no equilibrium either when $pR_L > 1$ or when $pR_H < 1$ (see appendix). To see why $pR_L \leq 1$, note that if $pR_L > 1$, then a bank would be able to continue its non-marketable project even when it holds securities with a low return. Since securities have a higher return than cash, banks would increase their holding of securities to earn the extra yield. However, banks do not take into account the fact that when they hold more securities the aggregate amount of cash in the market decreases, leading to cash-in-the-market pricing. In equilibrium, the benefit of a marginal unit of cash, which comes from the ability to continue the non-marketable project when a liquidity shock hits, must be equal to the cost from the foregone income on the marketable securities.

When $pR_L \leq 1 \leq pR_H$, the demand for cash is given by

$$D(p,s) = \lambda(1-s)\left[\mu + (1-\mu)pR_L\right]. \tag{10}$$

Given the expression for supply (9) and demand (10), we can prove proposition 2, which characterizes the equilibrium prices of securities, taking as given the cash level s.

Proposition 2. *Under $pR_L \leq 1 \leq pR_H$, given the level of cash holding s, the equilibrium $p^*(s)$ is determined as follows:*

1. *If $S > D(p=1)$, then $p^*(s) = 1$. The price of security is its fundamental value.*

2. *If* $S \leq D(p=1)$*, then* $p^*(s)$ *is determined by* $S = D$ *yielding*

$$p^*(s) = \frac{1}{R_L}\left[\frac{(1-\lambda)s}{\lambda(1-s)} - \mu\right]\frac{1}{1-\mu}.$$

The price of security is below its fundamental value.

3. *If* $s = \lambda$ *and* $R_L \geq 1$*,* $p^*(s)$ *can be any value in* $\left[\frac{1}{R_L}, 1\right]$*, i.e. any value between* $p^*(s)$ *determined by* $S = D$ *and* 1.

4.2. *Individual Portfolio Choice at* $t = 0$

Each bank decides how much cash to hold at $t = 0$ to maximize its expected profit. Given θ, the marginal return on cash depends on whether or not $s + (1-s)pR_\theta \geq 1$. Since $pR_L \leq 1 \leq pR_H$, it depends on which type of security the bank holds. When the bank holds H-type, it is not liquidity constrained, so the marginal return on cash is simply

$$\frac{1}{p} - R_H;$$

the gross return on one additional unit of cash minus the opportunity cost of that additional unit.

If the bank holds L-type, then we have two cases upon the price of L-type security pR_L. When $pR_L = 1$, the bank is not liquidity constrained, so the marginal return on cash is similar to above and can be written as

$$\frac{1}{p} - R_L.$$

When $pR_L < 1$, the bank is liquidity constrained. In this case, the marginal return is given by

$$\left[\lambda y + (1-\lambda)\frac{1}{p}\right] - \left[\lambda p R_L y + (1-\lambda)R_L\right].$$

The first term in brackets corresponds to the bank's marginal return on cash: if the bank is affected by a liquidity shock, which happens with probability λ, then the marginal return on cash is equal to the marginal return on the project, y, while if the bank is not affected by the liquidity shock, then the cash can be used to purchase securities, with a marginal return of $1/p$. An additional unit of cash also carries an opportunity cost, corresponding to the return of the securities that could have been purchased instead. This is captured by the second term in brackets: if the bank is affected by a liquidity shock, it would sell the additional securities at price pR_L to acquire cash, while if the bank is not affected by the liquidity shock the additional securities would return R_L.

Using these expressions, we can derive the expected marginal (net) return on cash conditional on the price of securities p, which is given by

$$\frac{\partial \pi}{\partial s} = \begin{cases} \frac{1}{p} - \bar{R} & \text{if} \quad pR_L = 1 \\ \left(\frac{1}{p} - \bar{R}\right) + (1-\mu)\lambda\left(y - \frac{1}{p}\right)(1 - pR_L) & \text{if} \quad pR_L < 1 \end{cases}$$

When $pR_L < 1$, the expected marginal return on cash is the sum of two components. The first term, $\left(\frac{1}{p} - \bar{R}\right)$, represents the pecuniary or "fundamental" net return on cash, which can be negative. The second term represents the additional value of cash that comes from its use in continuing the long-term project when a liquidity shock hits. The relative size of these two terms will determine whether there is cash hoarding or cash-in-the-market pricing in equilibrium.

The equilibrium cash holding level s^* given p must maximize the expected return on cash. From this together with the proposition 2, we can characterize market equilibria, represented by the pair of the price p^* and the level s^* of individual cash holding.

Proposition 3. *1. When $R_L = R_H = \bar{R}$, there exist an equilibrium with $s^* = \lambda$ and $p^* = \frac{1}{\bar{R}}$. (optimal)*

2. When $\hat{R} < R_L < \bar{R}$, there exist an equilibrium with $s^* (< s^{SB}) < \lambda$ and $p^* < 1$. (cash-in-the market pricing) s^* and p^* are determined by

$$s^* = \frac{\lambda(\mu + (1-\mu) pR_L)}{\lambda(\mu + (1-\mu) pR_L) + (1-\lambda)} \tag{11}$$

$$\frac{1}{p^*} = \bar{R} - (1-\mu)\lambda\left(y - \frac{1}{p^*}\right)(1 - p^* R_L) \tag{12}$$

3. When $R_L < \hat{R}$, there exist an equilibrium with $s^* = 1$ and $p^* = 1$. (cash hoarding, with the price being the fundamental value)

The proof is provided in the appendix.

Figure 4 illustrates the equilibrium price and cash level upon the value of security with low quality R_L.

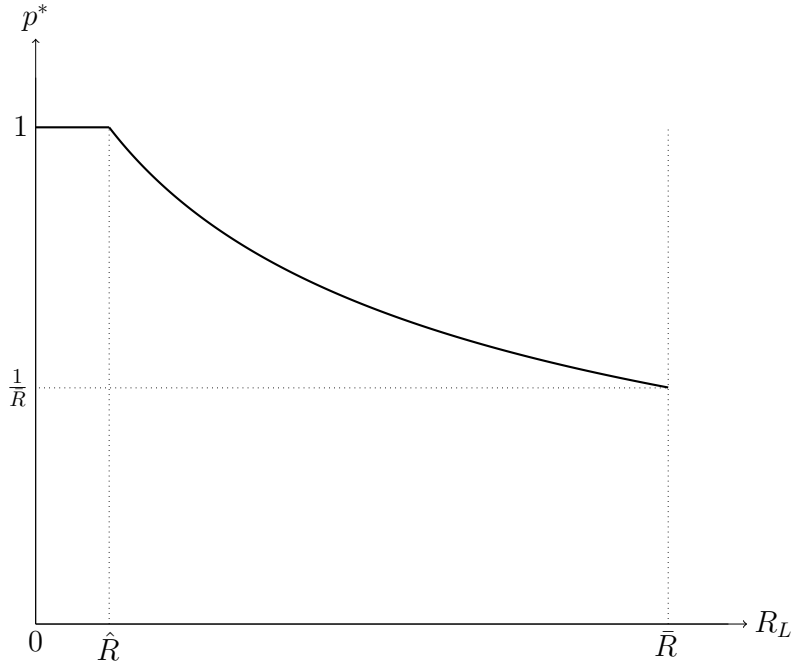
5. Discussion

5.1. *Comparison with optimal allocation*

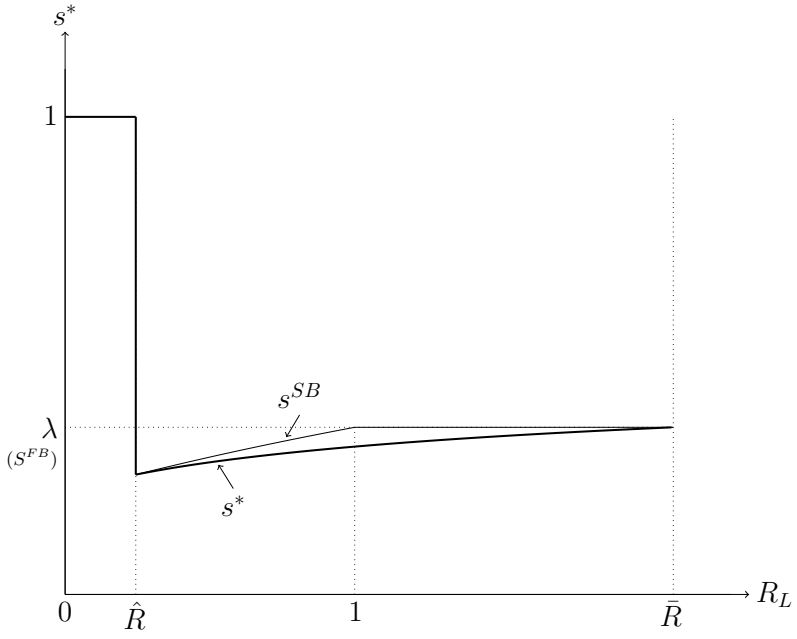
An equilibrium with cash hoarding can only exist when R_L is lower than $\hat{R}$.[7] When it exists, this equilibrium corresponds to a situation where the return on cash that comes from its use to continue the non-marketable project exceeds, in equilibrium, the return from holding securities. In such a situation, the interbank market functions very poorly as a way to redistribute cash between banks. As a consequence, the interbank market freezes; all banks hold as much cash as they need and no trade takes place.

Since there is cash hoarding in this equilibrium, the price of securities always corresponds to their fundamental value. The level of cash is constrained optimal.

[7] Such an equilibrium may not exist, depending on the value of other parameters and if $\hat{R}$ is lower than 0.



(a) Equilibrium p^*



(b) Equilibrium cash holding level s^*

Figure 4: Equilibrium price and cash holding level

When $\hat{R} < R_L < \bar{R}$, the equilibrium level of cash holding by banks is lower than the optimal amount, $s^* < \lambda$, and even lower than the constrained optimal amount, $s^* < s^{SB}$. In this case, p^* and s^* are determined by (11) and (12). Because banks are holding so little cash, equilibrium p^* is always lower than 1. In other words, there is cash-in-the market pricing. As already discussed, in this equilibrium banks do not internalize the effect their cash-holding decision has on the equilibrium price of securities. Banks hold too little cash, which implies that the value of securities is low because of cash-in-the-market pricing. This, in turns, reduces the extent to which banks hit by a liquidity shock can continue their project.

The threshold $\hat{R}$ represents the value of R_L below which individual banks shift their behavior from holding too little cash to holding too much cash, compared to the socially optimal level. Equation (4), shows that this threshold increases as the proportion of low quality securities, $1 - \mu$, and the proportion of projects hit by liquidity shock, λ, increases. If both of these values are high, then the value of R_L below which banks hold too much cash becomes relatively high. Indeed, if μ tends to zero and λ tends to 1, then $\hat{R}$ tends to 1.

In words, if the share of securities with a low return and the share of non-marketable projects that are hit by a liquidity shock are high the interbank market may not be very effective at redistributing cash among banks. The market is likely to freeze as banks attempt to self-insure.

5.2. *Comparative statics*

5.2.1. *Dispersion of return on securities*

Using (4), we can figure out the graphical representation of proposition 3 in the $\left(R_L, \bar{R}\right)$ space, which is illustrated in Figure 5.

For a given level of $\bar{R}$, a change in R_L corresponds to a change in the dispersion of the return on securities. When there is no dispersion, $R_L = \bar{R}$, the first best can be achieved in equilibrium. As dispersion increases (R_L decreases), we have first cash-in-the-market princing. Then, as dispersion increases further there can be cash hoarding in equilibrium. The corresponding behavior of cash holdings is shown in figure 4 (b). Interestingly, we have a non trivial/non monotonous impact of the level of dispersion, with a significant change in the cash holding as dispersion increases.

Note that the first best level of cash, $s^{FB} = \lambda$, is unaffected by the expected return on securities $\bar{R}$, which is assumed to be greater than the return on cash. The intuition is that if nothing prevents the efficient redistribution of cash at date 1, then it is efficient to hold just enough cash to continue all long-term projects, since they have a high return, but no more since the return on cash is lower than the return on securities.

Proposition 4. *There exist non-monotonous relationship between cash holding level of banks*

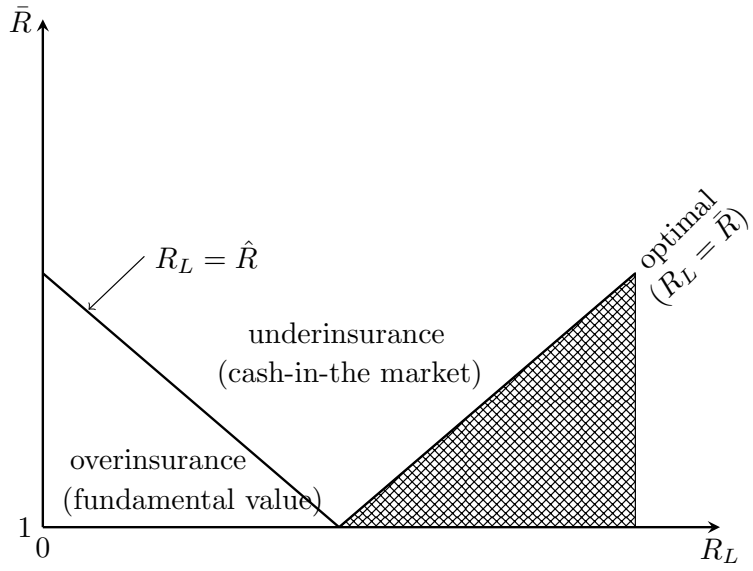


Figure 5: Type of equilibria

and dispersion of return on securities. Cash holding level of banks decreases in the regime with low and moderate dispersion ($R_L < \bar{R}$) and jumps to 1 *at $R_L = \bar{R}$, then it remains unchanged as dispersion increases in the regime with high dispersion ($R_L > \bar{R}$).*[8]

5.2.2.*Expected return on securities ($\bar{R}$)*

Now, we analyze the impact that changes in expected return on securities ($\bar{R}$) have on cash holding behavior of bank. In order to control the impact driven by changes in dispersion of return on securities and to focus on pure impact of change in $\bar{R}$, we conduct a comparative analysis given that dispersion remains unchanged. Let σ denote coefficient of dispersion defined as standard deviation of returns on securities divided by their expeected return. σ is therefore given by

$$\sigma^2 \equiv \frac{\mu \left(R_H - \bar{R}\right)^2 + (1-\mu)\left(R_L - \bar{R}\right)^2}{\bar{R}^2}$$

Using $\bar{R} \equiv \mu R_H + (1-\mu) R_L$, σ can be simplified as

$$\sigma \equiv \sqrt{2\,(1-\mu)}\left(1 - \frac{R_L}{\bar{R}}\right) < \sqrt{2\,(1-\mu)} \tag{13}$$

[8]We show more clear non-monotonous relationship in a general set-up with $\bar{x} < 1$ presented in the appendix A. In this extension, the relationship between cash holding level and dispersion of return on securities are negative when dipersion is low and moderate ($R_L < \bar{R}$) whereas this relationship becomes positive when dispersion is high ($R_L > \bar{R}$).

Using proposition 3, we obtain the following proposition for $\bar{R} \in (1,\, y)$, which is relevant range of $\bar{R}$:

Proposition 5. *1. When $\sigma = 0$, in other words $\bar{R} = R_H = R_L$, equilibrium level of cash holding is $s^* = s^{FB} = \lambda$ independent of level of $\bar{R}$.*

2. When σ is too high such that $\sigma \geq \hat{\sigma}$, equilibrium level of cash holding is $s^* = 1$ (cash hoarding) independent of level of $\bar{R}$.

3. When $0 < \sigma < \hat{\sigma}$, equilibrium level of cash holding is given by the following:

(a) If $\bar{R} > \bar{R}_0$,

$$s^* = \frac{\lambda\left(\mu + (1-\mu)\, kp^*\bar{R}\right)}{\lambda\left(\mu + (1-\mu)\, kp^*\bar{R}\right) + (1-\lambda)} \tag{14}$$

$$\frac{1}{p^*} = \bar{R} - (1-\mu)\,\lambda\left(y - \frac{1}{p^*}\right)\left(1 - kp^*\bar{R}\right) \tag{15}$$

where

$$k \equiv 1 - \frac{\sigma}{\sqrt{2\,(1-\mu)}} \tag{16}$$

(b) If $\bar{R} < \bar{R}_0$, equilibrium level of cash holding and price is respectively$s^* = 1$ and $p^* = 1$.

$\hat{\sigma}$ and $\bar{R}_0$ are given by

$$\hat{\sigma} = \frac{\sqrt{2\,(1-\mu)}\,(y-1)\,((1-\mu)\,\lambda + 1)}{y\,(1-\mu)\,\lambda}$$

$$\bar{R}_0 = \frac{\sqrt{2\,(1-\mu)}\,[(1-\mu)\,\lambda\,(y-1)+1]}{\sqrt{2\,(1-\mu)}\,[(1-\mu)\,\lambda\,(y-1)+1] - \sigma\,(1-\mu)\,\lambda\,(y-1)}$$

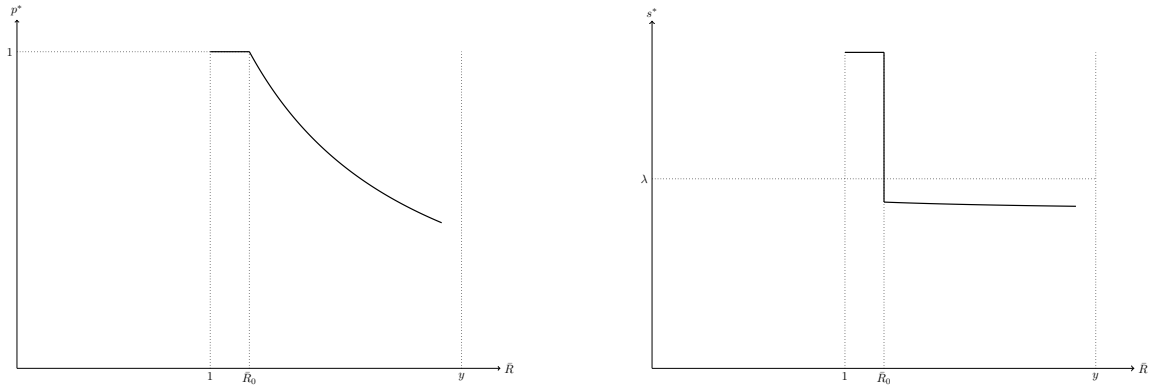
Proposition 6 follows.

Proposition 6. *Equilibrium level s^* of cash holding is (weakly) decreasing to $\bar{R}$. More precisely, as $\bar{R}$ decreases, s^* increases till $\bar{R}_0$ and then jumps to 1 and remains unchanged.*

Proof is relegated to the appendix. Figure 6 illustrates proposition 6. It is noteworthy that unlike relationship between cash holding level of banks and dispersion of return on securities, there exist a monotonous relationship between cash holding level and expected return on securities.

5.2.3.*Probability of liquidity shock*

Now, we analyze how the change in probability of liquidity shock affects the individual bank's decision on liquidity holding. Accordingly, we investigate on the effect of change in probability λ of liquidity shock on the equilibrium level of cash holding (s^*). Proposition 7 follows.



(a) Equilibrium p^* (b) Equilibrium s^*

Figure 6: Return on securities vs. p^* and s^*

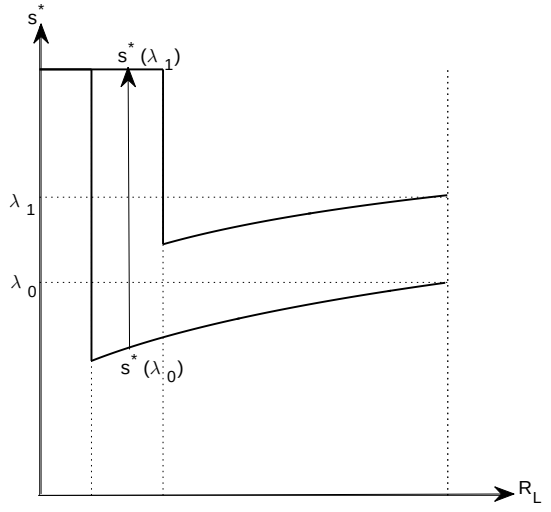


Figure 7: The effect of increase in probability of liquidity shock

Proposition 7. *Given $R_L > \hat{R}$, an increase in λ, from λ_0 to $\lambda_1 > \lambda_0$ results in*

1. *an increase the amount of cash s held by banks;*
2. *an increase in the threshold below which the equilibrium shift from cash-in-the-market pricing to cash hoarding.*

Figure 7 illustrates the above proposition.

6. Intervention policy

6.1. *cash-in-the-market pricing and policy intervention*

6.1.1. *Liquidity regulation*

When the economy faces cash-in-the-market pricing, a liquidity requirement, such as the LCR, can improve welfare. For the sake of illustration, consider the case with $\hat{R} < R_L < R_H$, where the market equilibrium exhibits cash-in-the-market pricing ($s^* < \lambda$). In equilibrium, welfare is given by:

$$W(s) = (1 - \lambda + \lambda s + C^m) y + [(1 - \lambda) s - C^m] + (1 - s) \bar{R},$$

where C^m is the aggregate amount of cash that banks faced with a liquidity shock can obtain in the market. It is easy to verify that $C^m = (1 - \lambda) s$. Plugging this expression into $W(s)$ yields total welfare at the market equilibrium cash level s^*

$$W(s^*) = (1 - \lambda + s^*) y + (1 - s^*) \bar{R}$$

The marginal contribution of cash to welfare, $\frac{\partial W}{\partial s}$, is $y - \bar{R}$, which is always positive. Therefore, requiring banks to hold more cash than s^* improves welfare.

Not surprisingly, the solution for the welfare maximizing level of cash that banks should be required to hold is same as the level of cash chosen by the constrained social planner. This is summarized in the next proposition.

Proposition 8.
1. *When the economy displays cash-in-the-market pricing, liquidity requirements, such as the LCR can improve welfare.*
2. *In such a situation, liquidity requirement can restore first-best when $1 \leq R_L < R_H$ and can restore contained optimal level of welfare (second-best) when $\hat{R} < R_L < 1$.*

By forcing banks to hold more liquidity, a requirement like the LCR increases the value of securities and permits more long-term projects to be continued, resulting in higher welfare.

An interesting implication of this result is that the appropriate level of the LCR should depend in part on the dispersion of the return of marketable securities. More precisely, the

constrained optimal level of cash becomes lower as the dispersion of the return on securities is greater. Since the dispersion of returns is likely to be greater during periods of crisis, our model suggests that the liquidity requirement should be lower in times of stress than in good time.

6.1.2.*Announcement of asset purchase by the central bank*

Another policy intervention that can improve welfare when the economy faces cash-in-the-market pricing is for the central bank to announce at $t = 0$ that it will purchase the securities at their fundamental value at $t = 1$, which implies that $p = 1$.

To analyze the effect of asset purchase by the central bank, we slightly modify our basic model. We suppose that at $t = 0$ the central bank levies a tax $\tau < 1$ on banks in the form of cash. Hence, the amount of cash held by banks is $s \in [0, 1 - \tau]$ and the amount of securities is $1 - s - \tau$. At $t = 1$, banks can obtain the cash by trading their securities either with other cash-rich banks at market price or with the central bank at the price $p = 1$. The central bank can purchase securities until it runs out of cash. The central bank chooses τ to maximize social welfare. The following proposition describes an equilibrium:

Proposition 9. *1. When the economy displays cash-in-the-market pricing, the central bank can improve welfare by levying a tax $\tau > 0$ at $t = 0$ and using the proceeds to purchase securities at their fundamental value ($p = 1$) at $t = 1$.*

(a) When $1 \leq R_L < \bar{R}$, such intervention with $\tau = \lambda$ can restore the first-best allocation.

(b) When $\hat{R} < R_L < 1$, such intervention with $\tau = s^{SB}$ can restore the constrained optimal allocation.

2. In both cases, banks choose to hold no cash $s^* = 0$ in equilibrium. The market for securities is inactive and all trading occurs with the central bank.

Proof. Here, we will demonstrate that when $\hat{R} < R_L < 1$, intervention with $\tau = s^{SB}$ can restore the constrained optimal allocation. The other case can be proved in a similar way. When $\hat{R} < R_L < 1$, we know that the constrained optimal allocation is attained when the total cash in the economy is s^{SB} and if all of the cash redistributed to banks that need it. Thus, it is sufficient to demonstrate that banks have no incentive to deviate from $s^* = 0$ and that the cash held by the central bank will be transferred to the banks with liquidity shock. First, note that this policy is feasible, since $\tau = s^{SB}$ and the central bank sets $p^{CB} = 1$.

If the market equilibrium price is $p^* < 1$, then all banks that need cash sell their securities to the central bank. Since s^{SB} is the maximum amount of cash that is demanded when the securities are traded at their fundamental value, all demand for cash can be satisfied by

the central bank and there is no trade in the market. Therefore, $p^* < 1$ cannot be an equilibrium.

If the market equilibrium price is $p^* = 1$, then banks with liquidity shock are indifferent between selling their securities to the central bank and selling them in the market. However, when $\hat{R} < R_L$ and $p^* = 1$, the marginal return on cash expected at $t = 0$ is always negative. As a consequence, banks will not hold any cash. Therefore, $s^* = 0$ is an equilibrium given $\tau = s^{SB}$ and $p^{CB} = 1$. In such an equilibrium, all the cash s^{SB} is transferred to banks with liquidity shock since it is equal to demand for cash by these banks. □

The central bank's intervention described in the proposition above resembles the Quantitative Easing (QE) polices that several central bank conducted during the recent crisis along some dimensions. These policies can increase the value of securities by "flooding the market" with reserves. This increase brings the value of the securities back to their fundamental value. Flooding the market with reserves typically depress activity in the interbank market and this happens in the model too. With the cash injections from the central bank, there is no need for banks to trade with each other.

6.2. *Cash hoarding and policy intervention.*

When $R_L < \hat{R}$, there is cash hoarding. The decentralized individual choice of cash holding is clearly higher than the optimal amount as defined by the first-best. However, our result shows that the market equilibrium is constrained optimal. Indeed, in such situation the value of cash to continue the long-term project is so high that it more than compensate for the fact that cash has a lower expected "fundamental" return than securities. As a result, the central bank cannot improve welfare except, unless it is willing to purchase securities at a value that is greater than their fundamental value, which would lead to losses.

6.2.1.*Mutual insurance*

In this type of situation, an insurance scheme among banks could restore the first best. If banks can commit to share the available cash, then they face the same problem as an unconstrained planner. This type of insurance mechanisms can be costly to implement and difficult to enforce, so that they might not be set up ex ante. This might be the case for example, if the economy is rarely expected to be in a situation where banks would want to hold so much cash.

That said, the type of clearing house arrangements that arose during banking panics, described in Gorton (1985), have characteristics that are similar to an insurance mechanism that would restore the first best in our model. In response to a panic, clearinghouse member banks would join together to form a new entity overseen by the Clearinghouse Committee.

As Swanson (1908) put it, describing the crisis of 1860, there was "the virtual fusion of the fifty banks of New York into one central bank" (p. 221).[9]

6.2.2. *Central bank's lending against non-marketable project*

It is also possible for a central bank to achieve the unconstrained optimal allocation if it can lend against the return of the non-marketable long-term project. While this result requires assuming that the central bank can choose actions that are not permissible to market participants, it is consistent with empirical evidence.

In practice, some central banks do lend against such assets. In Europe, assets which do not have an International Securities Identification Number (ISIN) are typically very illiquid and cannot be traded in a market on short notice, if at all. Such assets would correspond well to the non-marketable asset of our theory. Nevertheless, the ECB regularly provides liquidity to banks against this type of collateral–which includes credit claims and non-marketable retail mortgage-backed debt instruments (RMBDs)–as long as it complies with some eligibility criteria.[10] The use of non-marketable assets pledged at the Eurosystem open market operations as percentage of the total collateral pledged increased steadily with the Subprime and Euro crisis, jumping from about 10% in 2007 to more than 25% in 2012.[11]

We do not explicitly model the frictions that prevents market participants from purchasing, or lending against, non-marketable assets. One reason might be that revealing the information necessary to evaluate the quality of such assets could provide a competitive advantage to the bank's competitors. Providing this information to the central bank, in contrast, is not an issue.

In order to analyze the effect of lending against project by central bank, we make similar assumption as in the previous subsection. The central bank levies a tax with τ at $t = 0$ before banks allocate their resource between cash and securities. At $t = 1$, banks can obtain the cash either by trading their securities on the market or by borrowing from the central bank pledging their project (non-tradable security). We assume that 1) there is no informational asymmetry between banks and the central bank and therefore the quality of project is known to the central bank; and 2) that the central bank lends the amount of debt that banks want under the constraint of its resource τ with net interest rate r and with haircut 0. The central bank chooses the level of tax τ that maximizes the social welfare W. The following proposition describes an equilibrium:

[9] See Gorton (2010), Chapter 2.

[10] These criteria include, notably, that the asset must be denominated in Euro, the debtor must be located in the Euro area, and the debtor's default probability, as defined by the Basel accord, must be lower than 0.4%. Details on non-marketable accepted in guarantee to ECB loans to credit institutions can be found in Sauerzopf (2007) and in Tamura and Tabakis (2013).

[11] See ECB (2013), p. 81.

Proposition 10. *Given that $r = 0$, we can have an equilibrium achieving the optimal allocation in which the central bank levies tax with the amount $\tau^* = \lambda$ and banks choose their cash holding level $s^* = 0$ and holds only securities $1 - \lambda$. The market will collapse and the central bank substitutes market but optimal allocation is achieved.*

Proof. First, we characterize the decentralized equilibrium given $\tau = \lambda$. Then we demonstrate that the social welfare given by $\tau^* = \lambda$ and s^* is first-best. Since the central bank levies $\tau = \lambda$, banks know that the central bank has enough resources to lend to all banks with a liquidity shock. Hence, banks would only hold cash to purchase securities. This implies that the (net) expected return on additional cash for banks is always $\frac{1}{p} - \bar{R}$. Since banks do not want to sell securities below their fusubsectionndamental value, the only price at which the market could clear is $p = 1$. However, at that price $\frac{1}{p} - \bar{R} < 0$, and banks do not want to hold any cash. As a result, given $\tau = \lambda$, market equilibrium is $s^* = 0$. The total output in the economy in this equilibrium is given by

$$W \quad = \quad y + (1 - \lambda)\, \bar{R},$$

which is the first-best level of welfare. The central bank has no incentive to deviate from $\tau = \lambda$.
□

7. Conclusion

Our paper proposes a simple theoretical framework to analyze the liquidity choice of banks. Banks require liquidity to meet unexpected shocks and can obtain this liquidity from other banks by selling marketable securities in a market. Interestingly, our theory suggests that banks may hold either too much cash or too little cash, depending on parameter values.

The banks' choice if cash holding deviates from the social optimum when the return on marketable securities is sufficiently variable. In this case, a bank needing liquidity but holding marketable securities with a low return will not be able to obtain as much cash as it would like. When the expected return of securities is sufficiently high, then banks will always hold too little cash, compared to the social optimum. The intuition for this result is that banks do not internalize the effect of their cash holding on the market price of securities. Because of cash-in-the-market pricing, the fact that banks hold too little cash, lowers the price of securities. This, in turn, means that banks cannot continue the long-term project to the extent they would like. A liquidity constraint, such as the LCR, that forces banks to hold enough liquidity can restore the constrained optimum in such situations.

The return on marketable securities is sufficiently variable and the expected return is not too high, then banks will choose to hold too much cash. In this situation, the value of cash for the purpose of continuing the long-term project more than compensate for the

fact that cash has a lower "fundamental" return than securities. This can be thought of as a state of crisis, where the market for securities shuts down and banks hoard cash. Central bank intervention can restore the first best in this situation; in particular if the central bank can lend against the value of the non-marketable project.

Appendix

A. Extension: The general case with liquidity shock $x \leq 1$

Until now, we have assumed that bank needs the amount of liquidity 1 to continue its project when it is affected by liquidity shock. In this appendix, we relax this assumption by considering that the amount of liquidity needed to continue project is $x \leq 1$. In this general case, the assumption (A2) is replaced by

$$\frac{y}{x} > \bar{R} \tag{A2'}$$

First, we determine the date-0 optimal allocation from the point of view of a social planner. The total output of economy becomes

$$W_x(s) = \min\left\{1, \left[1-\lambda+\lambda(\frac{s+c}{x})\right]\right\} y + [(1-\lambda)s - \lambda c] + (1-s)\bar{R} \tag{17}$$

The consideration similar to the section 3 yields that the optimal level of cash for economy is $s_x^{FB} = \lambda x$ when the social planner has no constraint in redistribution of cash. The constrained-optimal, or second-best, level of cash is denoted by s_x^{SB}. This is given by the following proposition 11:

Proposition 11. *1. When $R_L \geq \frac{(1-\lambda)x}{1-\lambda x}$, the planner is not constrained to redistribution of cash so that $s_x^{SB} = s_x^{FB} = \lambda x$.*

2. When $\hat{R}_x < R_L < \frac{(1-\lambda)x}{1-\lambda x}$, the constrained optimal level of cash s_x^{SB} is $\hat{s}_x$ ($< s_x^{FB}$) where

$$\hat{s}_x \equiv \frac{\lambda(\mu x + (1-\mu)R_L)}{\lambda(\mu + (1-\mu)R_L) + 1 - \lambda}, \tag{18}$$

and

$$\hat{R}_x \equiv 1 - \frac{\bar{R}-1}{\lambda(1-\mu)\left(\frac{y}{x}-1\right)}. \tag{19}$$

3. When $R_L = \hat{R}_x$, the constrained optimal level of cash s_x^{SB} can be any value between $\left[\hat{s}_x, \frac{x-R_L}{1-R_L}\right]$.

4. When $R_L < \hat{R}_x$, the constrained optimal level of cash s_x^{SB} is $\frac{x-R_L}{1-R_L}$ ($> s_x^{FB}$).

The proof is provided in the appendix E.

The constrained optimal level of cash depends on R_L, the low return for the securities. When R_L is higher than $\frac{(1-\lambda)x}{1-\lambda x}$, the planner can achieve the first-best because the constraint does not bind. When R_L is smaller than $\frac{(1-\lambda)x}{1-\lambda x}$, the planner can no longer transfer the optimal amount of cash. The planner chooses to reduce s for two reasons: First, a reduction of s is necessary to make sure that there is no unused cash after transfers. Second, reducing s allows banks to hold more securities, which can be used to acquire more cash when the return on the securities is low. For these reasons, s^{SB} decreases as R_L decreases until R_L becomes smaller than $\hat{R}$ at which point it jumps to $\frac{x-R_L}{1-R_L}$. The intuition for this result is that R_L becomes so low that the planner prefers that banks hold sufficient cash to continue the project in full scale even when banks hold L-type security. In other words, s^{SB} becomes such that $s + (1-s) R_L = x$. We can easily verify that our main model is a particular case with $x = 1$.

Now, we turn to the decentralized decision on cash holding by individual banks. Consider first aggregate demand and supply of cash in the market. The equilibrium prices and cash holding level satisfy the following lemma 2:

Lemma 2. *1. Given the cash holding level s chosen at $t = 0$, the equilibrium market return on cash $\frac{1}{p^*}$ always satisfies $1 \leq \frac{1}{p^*} \leq \frac{y}{x}$.*

2. Given the market price of securities p, the equilibrium cash holding level s^* always satisfies $s^* + (1-s^*) pR_H \geq x$.

Proof. The proof is similar to that of lemma 1 and appendix B.2.□

Given that $1 \leq \frac{1}{p} \leq \frac{y}{x}$, banks hit by a liquidity shock always prefer to use their cash to continue their project whereas banks that are not affected by liquidity shock always supply their cash in the market. The aggregate supply of cash in the market is

$$S = (1-\lambda) s$$

The demand for cash depends on whether banks hit by liquidity shock is constrained by the value of their securities or not. Under $s^* + (1-s^*) pR_H \geq x$, banks with the H-security is never constrained. The aggregate demand depends on the value of L-security. The demand for cash can be written by:

$$D(s) = \begin{cases} \lambda (x-s) & \text{if} \quad x \leq s + (1-s) pR_L \\ \lambda [\mu (x-s) + (1-\mu)(1-s) pR_L] & \text{if} \quad x > s + (1-s) pR_L \end{cases} \tag{20}$$

Now we consider individual decision on cash holding level by banks. They decide their cash holding level at $t = 0$ maximizing their expected profits. Their decision depends on the

expected marginal (net) return on cash conditional on the price of securities p, which is given by

$$\frac{\partial \pi}{\partial s} = \begin{cases} \frac{1}{p} - \bar{R} & \text{if} \quad s + (1-s)\, pR_L \quad \geq \quad x \\ \left(\frac{1}{p} - \bar{R}\right) + (1-\mu)\,\lambda \left(\frac{y}{x} - \frac{1}{p}\right)(1 - pR_L) & \text{if} \quad s + (1-s)\, pR_L \quad < \quad x \end{cases}$$

The equilibrium cash holding level s^* given p must maximize the expected return on cash, therefore must satisfy the following conditions:

- If $s + (1-s)\, pR_L \neq x$,
$$\frac{\partial \pi}{\partial s} = 0 \tag{21}$$
- If $s + (1-s)\, pR_L = x$
$$\begin{cases} \frac{\partial \pi}{\partial s}\,(s < s^*) & \geq & 0 \\ \frac{\partial \pi}{\partial s}\,(s > s^*) & \leq & 0 \end{cases} \tag{22}$$

The conditions in (22) are equivalent to the condition (21) when π is differentiable for all s. In our case, π is piecewise linear in s but the slope changes at s such that$s + (1-s)\, pR_L = x$. Therefore, $\frac{\partial \pi}{\partial s} = 0$ is not sufficient. From this together with the proposition 2, we can characterize market equilibria, represented by the pair of the price p^* and the level s^* of individual cash holding.

Proposition 12. *1. When $\tilde{R}_x < R_L \leq \bar{R}$, there exist an equilibrium with $s^* = \lambda x$ (optimal) where*

$$\tilde{R}_x = \frac{(1-\lambda)\, x}{(1-\lambda x) - (1-\mu)\,\lambda\,(1-x)} \left[\bar{R} - \frac{(1-\mu)\,\lambda\,(1-x)\, y}{(1-\lambda x)\, x}\right] \tag{23}$$

Equilibrium price is given by

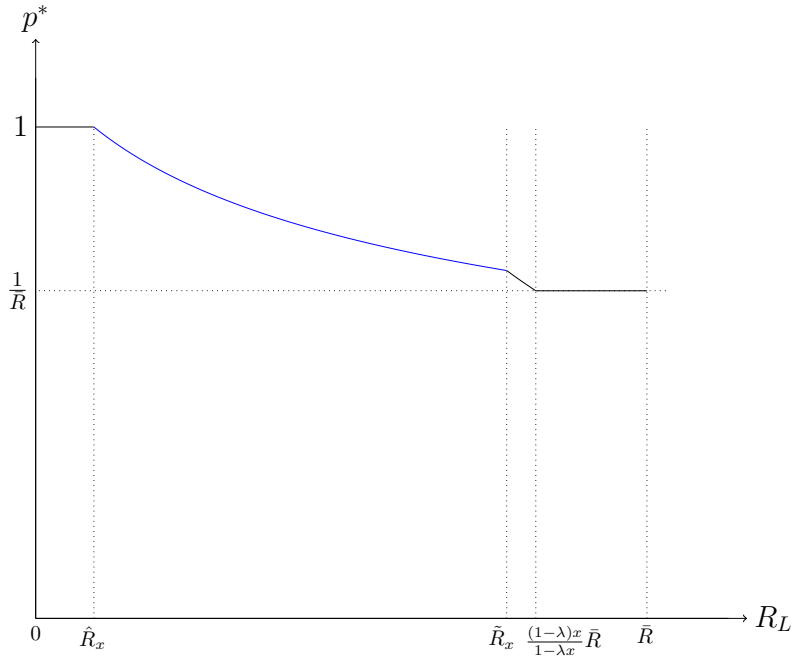
(a) $p^* = \frac{1}{\bar{R}}$ when $\frac{x-\lambda x}{1-\lambda x}\bar{R} \leq R_L \leq \bar{R}$;
(b) $p^* = \frac{(1-\lambda)x}{1-\lambda x}\frac{1}{R_L}$ when $\tilde{R}_x < R_L < \frac{x-\lambda x}{1-\lambda x}\bar{R}$.

2. When $\hat{R}_x < R_L < \tilde{R}_x$, there exist an equilibrium with $s^* (< s_x^{SB}) < \lambda x$ and $p^* < 1$. (cash-in-the market pricing) s^* and p^* are determined by The case with small liquidity shock

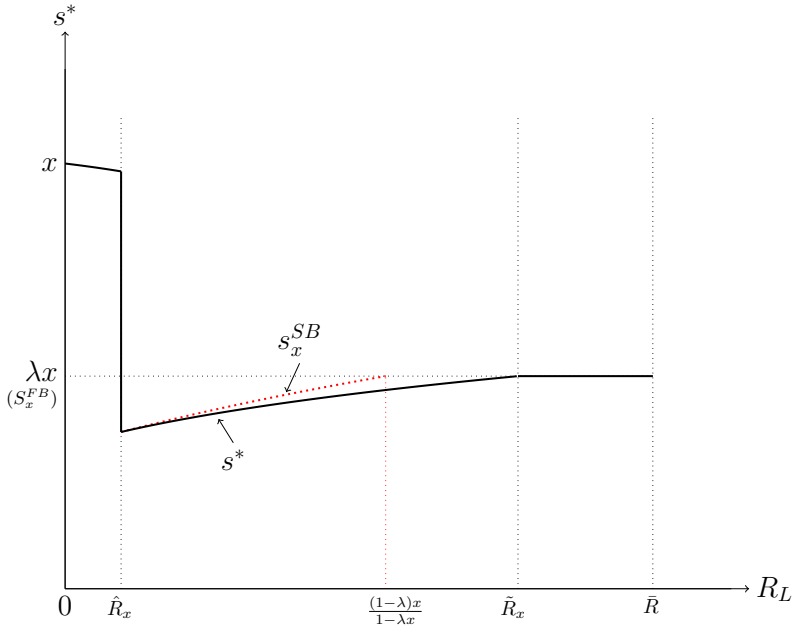
$$s^* = \frac{\lambda\,(\mu x + (1-\mu)\, p^* R_L)}{\lambda\,(\mu + (1-\mu)\, p^* R_L) + (1-\lambda)} \tag{24}$$

$$\frac{1}{p^*} = \bar{R} - (1-\mu)\,\lambda \left(\frac{y}{x} - \frac{1}{p^*}\right)(1 - p^* R_L) \tag{25}$$

3. When $R_L < \hat{R}_x$, there exist an equilibrium with $s^* = \frac{x-R_L}{1-R_L}$ and $p^* = 1$. (cash hoarding, with the price being the fundamental value)



(a) Equilibrium p^*



(b) Equilibrium cash holding level s^*

Figure 8: Equilibrium price and cash holding level with $x < 1$

The proof is provided in the appendix F.

Figure 8 illustrates the equilibrium price and cash level upon the value of security with low quality R_L. The above equilirbium characterization yields 3 different regimes in terms of relationship between dispersion of return on securities and equilibrium cash holding level.

Proposition 13. *1. When dispersion of return on securities is low ($R_L > \tilde{R}_x$), there is no relationship between dispersion of return on securities and the equilibrium cash-holding level.*

2. When dispersion of return on securities is intermediate $\hat{R}_x < R_L < \tilde{R}_x$, there is negative relationship between dispersion of return on securities and the equilibrium cash-holding level.

3. When dispersion of return on securities becomes high enough such that $R_L < \hat{R}_x$, there is positive relationship between dispersion of return on securities and the equilibrium cash-holding level.

The above proposition indicates that increases in dispersion of return on securities can inverse abruptly the relationship between dispersion of return on securities and cash-holding level shifting from negative correlation to positive correlation.

B. No existence of equilibrium with $pR_H < 1$ or with $pR_L > 1$

In the main part of this paper, we only focused on the case with $pR_H \geq 1 \geq pR_L$. In this appendix, we will demonstrate that there is no equilibrium when $pR_H < 1$ and when $pR_L > 1$.

1. When $pR_L > 1$

In this case, the bank is not cash-constrained independent of the type of securities, so the marginal return on cash is simply

$$\frac{\partial \pi}{\partial s} = \frac{1}{p} - \bar{R}$$

s maximizing individual profit requires $\frac{\partial \pi}{\partial s} = 0$, therefore $p = \frac{1}{\bar{R}}$, which implies $pR_L = \frac{R_L}{\bar{R}} \leq 1$ since $\bar{R} \geq R_L$. This is contradictory with $pR_L > 1$. Therefore, there does not exist an equilibrium with $pR_L > 1$.

2. When $pR_H < 1$

In this case, the bank is cash-constrained when it is affected by liquidity shock independent of the type of securities. The marginal return conditional on the type of security θ

is

$$\left[\lambda y + (1-\lambda)\frac{1}{p}\right] - \left[\lambda p R_\theta y + (1-\lambda) R_\theta\right]$$

yielding the expected marginal return on cash as following:

$$\frac{\partial \pi}{\partial s} = \left(\frac{1}{p} - \bar{R}\right) + \lambda\left(y - \frac{1}{p}\right)\left(1 - p\bar{R}\right)$$

We know that $p\bar{R} < 1$ since $p\bar{R} \leq pR_H < 1$. From lemma 1, $\frac{1}{p} \leq y$ in equilibrium. As a consequence, $\frac{\partial \pi}{\partial s} > 0$. This is contradictory with $\frac{\partial \pi}{\partial s} = 0$ required by profit maximizing s. Therefore, there does not exist an equilibrium with $pR_H < 1$.

C. Proof of proposition 3

In this appendix, we prove the equilibria with $pR_H \geq 1 \geq pR_L$. For the convenience, we consider the two following cases separately: i) equilibria with $S = D$ and ii) equilibria with $S > D$.

1. Equilibrium with $S = D$

$D = S$ gives

$$s^* = \frac{\lambda\left(\mu + (1-\mu) pR_L\right)}{\lambda\left(\mu + (1-\mu) pR_L\right) + (1-\lambda)} \tag{26}$$

We consider two subcases:

Equilibrium p^* and s^* are determined by the individual profit maximization yielding $\frac{\partial \pi}{\partial s} = 0$ and market equilibrium condition $S = D$.

i) When $pR_L = 1$:

The equation (26) gives

$$s^* = \lambda$$

and the marginal return on cash is given by

$$\frac{\partial \pi}{\partial s} = \frac{1}{p} - \bar{R}$$

s maximizing individual profit requires $\frac{\partial \pi}{\partial s} = 0$, which yields

$$p = \frac{1}{\bar{R}}.$$

Combining is expression with $pR_L = 1$ implies $\frac{R_L}{\bar{R}} = 1$, which can only hold if $R_H = R_L = \bar{R}$.

We can conclude that there exist an equilibrium with $s^* = \lambda$ and $p^* = \frac{1}{\bar{R}}$ when $R_L = R_H$.

ii) When $pR_H \geq 1 > pR_L$:

s maximizing individual profit requires $\frac{\partial \pi}{\partial s} = 0$, which yields

$$\frac{1}{p^*} = \bar{R} - (1-\mu)\lambda\left(y - \frac{1}{p^*}\right)(1 - p^* R_L) \tag{27}$$

Since $p^* \leq 1$, we have

$$R_L \geq 1 - \frac{\mu(R_H - 1)}{(1-\mu)(\lambda y + 1 - \lambda)} = \hat{R}$$

Equilibrium s^* and p^* are determined by (26) and (27) under the condition that p^*satisfies $pR_H \geq 1 > pR_L$. p^* determined by (27) satisfies $pR_H \geq 1 > pR_L$ whenever $\hat{R} \leq R_L < \bar{R}$. We can easily check that s^* is lower than λ from (26) when $\hat{R} \leq R_L < \bar{R}$. From (26) and (3), we can check that s^* is lower than $\hat{s}$ under $p^* < 1$, which implies that $s^* < \hat{s}$ when $R_L > \hat{R}$.

2. Equilibria with $S > D$

In such equilibrium, if exist, $p^* = 1$ from proposition 2. Since $R_H > 1$ by definition, the case that we consider $pR_H \geq 1 \geq pR_L$ becomes $R_L \leq 1 < R_H$. We consider two subcases:

i) When $R_L = 1$:

In such case, the condition (26) gives $s^* = \lambda$. However, the marginal return on cash is $\frac{\partial \pi}{\partial s} = 1 - \bar{R} < 0$, which implies that $s = 0$ is profitable deviation. Therefore, there does not exist an equilibrium with $R_L = 1$ and $S > D$.

ii) When $R_L < 1$:

In this case, we have always $s + (1-s)R_L \leq 1$. We consider separately the case with $s + (1-s)R_L < 1$ and that with $s + (1-s)R_L = 1$:

When $s + (1-s)R_L < 1$: This always holds under $R_L < 1$ except at $s = 1$. On the one hand, no profitable deviation condition gives

$$\frac{\partial \pi}{\partial s} = 1 - \bar{R} + (1-\mu)\lambda(y-1)(1 - R_L) = 0$$

yielding

$$R_L = 1 - \frac{\mu(R_H - 1)}{(1-\mu)(\lambda y + 1 - \lambda)} = \hat{R}$$

On the other hand, from $S > D$,

$$s^* > \frac{\lambda(\mu + (1-\mu)R_L)}{\lambda(\mu + (1-\mu)R_L) + 1 - \lambda} = \hat{s}$$

We can conclude that when $R_L = \hat{R}$ given $\hat{R} \geq 0$, a pair of p and s such that $p = 1$ and $s \in (\hat{s}, 1)$ can be an equilibrium. If $\hat{R} < 0$, there does not exist such an equilibrium.

When $s+(1-s)R_L=1$: Given $R_L<1$, the only case satisfying the above is $s^*=1$ and

$$\frac{\partial \pi}{\partial s}(s<1)=1-\bar{R}+(1-\mu)\lambda(y-1)(1-R_L)\geq 0$$

should hold in order for $s=1$ to be an equilibrium.

We can conclude that if $R_L<\hat{R}$ given that $\hat{R}>0$, $p^*=1$ and $s^*=1$ is an equilibrium. If $\hat{R}\leq 0$, there does not exist such an equilibrium.

D. Proof of proposition 6

Deriving (14) by $\bar{R}$ yields

$$\frac{\partial s^*}{\partial \bar{R}}=\frac{\lambda(1-\lambda)(1-\mu)k}{\left[\lambda\left(\mu+(1-\mu)kp^*\bar{R}\right)+(1-\lambda)\right]^2}\left[\frac{\partial p*}{\partial \bar{R}}\bar{R}+p^*\right]$$

The sign of $\frac{\partial s^*}{\partial \bar{R}}$ depends on that of $\frac{\partial p*}{\partial \bar{R}}\bar{R}+p^*$. Deriving both sides of (15) by $\bar{R}$ yields

$$\frac{\partial p^*}{\partial \bar{R}}=-\frac{\left[1+\lambda(1-\mu)(p^*y-1)k\right]p^*}{(1-\lambda(1-\mu))\frac{1}{p}+\lambda(1-\mu)p^*R_Ly}$$

Plugging this into $\frac{\partial p*}{\partial \bar{R}}\bar{R}+p^*$ gives

$$\frac{\partial p*}{\partial \bar{R}}\bar{R}+p^*=-\frac{\lambda(1-\mu)(1-p^*R_L)py}{(1-\lambda(1-\mu))\frac{1}{p}+\lambda(1-\mu)p^*R_Ly}$$

Since p^*R_L is always lower than 1 in equilibrium, $\frac{\partial p*}{\partial \bar{R}}\bar{R}+p^*$ is always negative. Therefore, $\frac{\partial s^*}{\partial \bar{R}}$ is negative as well.

E. Proof of proposition 11

When $R_L\geq x$, social planner is never constrained by the value of securities when it transfers cash. The problem is same as that of unconstrained social planner. $s_x^{SB}=s_x^{FB}=\lambda x$. Now we turn to the case with $R_L<x$. In such case, The aggregate amount of redistributed cash is given by

$$C_x=\begin{cases}(1-\lambda)s & \text{if} \quad s+(1-s)R_L \geq x, \\ \lambda\left[\mu(x-s)+(1-\mu)(1-s)R_L\right] & \text{if} \quad s+(1-s)R_L < x.\end{cases} \tag{28}$$

When $s+(1-s)R_L\geq x$, the constrained social planner can attain first-best. Now, consider the case with $s+(1-s)R_L<x$. Let $\hat{s}_x$ denote the level of s such that there is no cash left over after transfers. It is the value of s that solves $\lambda\left[\mu(x-s)+(1-\mu)(1-s)R_L\right]=(1-\lambda)s$. It is easy to verify that $\frac{\partial W}{\partial s}>0$ if $s<\hat{s}_x$. Indeed, when $s<\hat{s}_x$ an additional

unit of cash can be used to continue a larger share of the project without resulting in any leftover cash after transfers. When $s \geq \hat{s}_x$, we have

$$\frac{\partial W}{\partial s} = (1-\lambda) + \lambda\mu + \lambda(1-\mu)\left(\frac{y}{x} - \left(\frac{y}{x} - 1\right)R_L\right) - \bar{R}$$

The sign of this expression depends on R_L, but not on s. Let $\hat{R}_x$ denote the value of R_L such that $\frac{\partial W}{\partial s} = 0$. If $R_L > \hat{R}_x$, then $\partial W/\partial s < 0$ and if $R_L < \hat{R}$, then $\partial W/\partial s > 0$. From the above consideration, the constrained optimal level of cash when $R_L < 1$ depends on the two threshold levels of s: s such that $s + (1-s)R_L = x$, or $s = \frac{x-R_L}{1-R_L}$on the one hand and $\hat{s}_x$ on the other hand. Two following cases should be considered separately:

1. When $\frac{x-R_L}{1-R_L} < \hat{s}_x$, or $R_L > \frac{(1-\lambda)x}{1-\lambda x}$:

Note that in such case, $\frac{x-R_L}{1-R_L} < \lambda x < \hat{s}_x$: i) When $s < \frac{x-R_L}{1-R_L}$, $\frac{\partial W}{\partial s} > 0$ since $s < \frac{x-R_L}{1-R_L}$ and $s < \hat{s}_x$; ii) When $\frac{x-R_L}{1-R_L} < s < \lambda x$, $\frac{\partial W}{\partial s} > 0$; iii) When $s > \lambda x > \frac{x-R_L}{1-R_L}$, $\frac{\partial W}{\partial s} < 0$. Welfare maximizing cash holding level under the constraint is therefore $s_x^{SB} = \lambda x$ which equal to the first-best level.

2. When $\frac{x-R_L}{1-R_L} > \hat{s}_x$, or $R_L < \frac{(1-\lambda)x}{1-\lambda x}$:

Note that in such case, $\hat{s}_x < \lambda x < \frac{x-R_L}{1-R_L}$: i) When $s < \hat{s}_x$, $\frac{\partial W}{\partial s} > 0$. ii) When $s > \hat{s}_x$, the sign of $\frac{\partial W}{\partial s}$ depends on R_L. If $R_L > \hat{R}_x$, then $\frac{\partial W}{\partial s} < 0$. Therefore, $s_x^{SB} = \hat{s}_x$. If $R_L < \hat{R}$, then $\frac{\partial W}{\partial s}$ is positive as far as $s > \frac{x-R_L}{1-R_L}$ and becomes negative when $s < \frac{x-R_L}{1-R_L}$. Therefore, $s_x^{SB} = \frac{x-R_L}{1-R_L}$.

We can conclude that i) when $R_L > \frac{(1-\lambda)x}{1-\lambda x}$, $s_x^{SB} = s_x^{FB} = \lambda x$; ii) when $\hat{R}_x < R_L < \frac{(1-\lambda)x}{1-\lambda x}$, $s^{SB} = \hat{s}_x < s_x^{FB}$; iii) when $R_L < \hat{R}_x$, $s^{SB} = \frac{x-R_L}{1-R_L} > s_x^{FB}$.

F. Proof of proposition 12

For the convenience, we consider the two following cases separately: i) equilibria with $S = D$ and ii) equilibria with $S > D$.

1. Equilibrium with $S = D$

Equilibrium p^* is determined by equilibrium condition (21) and / or (22).

We consider two subcases:

i) When $s + (1-s)pR_L \geq x$:

$S = D$ gives

$$s^* = \lambda x$$

and the marginal return on cash is given by

$$\frac{\partial \pi}{\partial s} = \frac{1}{p} - \bar{R}$$

a) Equilibrium condition (21) yields

$$p^* = \frac{1}{\bar{R}}.$$

Combining this expression with $s + (1-s)\, pR_L \geq x$ implies

$$R_L \geq \frac{x - \lambda x}{1 - \lambda x} \bar{R}$$

We can conclude that there exist an equilibrium with $s^* = \lambda x$ and $p^* = \frac{1}{\bar{R}}$ when $R_L \geq \frac{x-\lambda x}{1-\lambda x}\bar{R}$.

b) Equilibrium condition (22) is relevant only when $s + (1-s)\, pR_L = x$. Plugging $s^* = \lambda x$ into $s + (1-s)\, pR_L = x$ yields

$$p^* = \frac{x - \lambda x}{1 - \lambda x} \frac{1}{R_L}$$

Equilibrium condition (22) yields

$$\frac{\partial \pi}{\partial s}\left(s < s^*\right) = \left(\frac{1}{p} - \bar{R}\right) + (1-\mu)\,\lambda \left(\frac{y}{x} - \frac{1}{p}\right)(1 - pR_L) \geq 0$$

and

$$\frac{\partial \pi}{\partial s}\left(s > s^*\right) = \frac{1}{p} - \bar{R} \leq 0$$

Plugging p^*into the above two conditions, they yield respectively $R_L \geq \tilde{R}_x$ given by (23) and $R_L \leq \frac{x-\lambda x}{1-\lambda x}\bar{R}$.

We can conclude that $s^* = \lambda x$ and $p^* = \frac{x-\lambda x}{1-\lambda x}\frac{1}{R_L}$ is an equilibrium when $\tilde{R}_x \leq R_L \leq \frac{x-\lambda x}{1-\lambda x}\bar{R}$.

ii) When $s + (1-s)\, pR_L < x$:

$D = S$ yields s^* given by (24). Note that given $s + (1-s)\, pR_L < x$, $s^* < \lambda x$. The individual incentive condition for equilibrium $\frac{\partial \pi}{\partial s} = 0$ yields p^* given by (25). Since $p^* \leq 1$ should hold, we have

$$R_L \geq \hat{R}_x$$

The upper bound of R_L is given by $s + (1-s)\, pR_L = x$ and $s^* = \lambda x$ as well as (25) , which yields

$$R_L < \tilde{R}_x$$

We can therefore conclude that the pair of s^* and p^* given by (24) and (25) is an equilibrium when $\hat{R}_x \leq R_L < \tilde{R}_x$.

2. Equilibria with $S > D$

In such equilibrium, if exist, $p^* = 1$, implying that the price of the security is its fundamental value since there exist excess supply in the market. We consider three subcases:

i) When $s + (1 - s) R_L > x$:

In such case, $S > D$ requires $s^* > \lambda x$. However, the marginal return on cash is $\frac{\partial \pi}{\partial s} = 1 - \bar{R} < 0$, which implies that $s = 0$ is profitable deviation. Therefore, there does not exist an equilibrium with $s + (1 - s) R_L \geq x$ and $S > D$.

ii) When $s + (1 - s) R_L = x$:

$s + (1 - s) R_L = x$ gives

$$s^* = \frac{x - R_L}{1 - R_L}$$

$S > D$ requires $s^* > \lambda x$. This yields

$$R_L < \frac{x - \lambda x}{1 - \lambda x}$$

We know that $\frac{\partial \pi}{\partial s} = 1 - \bar{R} < 0$ at s such that $s + (1 - s) R_L = x$. Therefore, the equilibrium condition (22) requires:

$$\frac{\partial \pi}{\partial s} (s < s^*) = \left(1 - \bar{R}\right) + (1 - \mu) \lambda \left(\frac{y}{x} - 1\right) (1 - R_L) \geq 0$$

because $\frac{\partial \pi}{\partial s} (s > s^*) = 1 - \bar{R}$ is always negative. This yields

$$R_L < \hat{R}_x$$

Note that $\hat{R} < \frac{x - \lambda x}{1 - \lambda x}$. We can therefore conclude that $p^* = 1$ and $s^* = \frac{x - R_L}{1 - R_L}$ is an equilibrium when $R_L < \hat{R}$ given that $\hat{R} > 0$. If $\hat{R} \leq 0$, there does not exist such an equilibrium. Note that as R_L becomes smaller, s^* becomes bigger.

iii) When $s + (1 - s) R_L < x$:

On the one hand, no profitable deviation condition gives

$$\frac{\partial \pi}{\partial s} = 1 - \bar{R} + (1 - \mu) \lambda (y - 1) (1 - R_L) = 0$$

yielding

$$R_L = \hat{R}_x$$

On the other hand, from $S > D$,

$$s^* > \frac{\lambda (\mu + (1 - \mu) R_L)}{\lambda (\mu + (1 - \mu) R_L) + 1 - \lambda} = \hat{s}_x$$

We can conclude that a pair of $p^* = 1$ and any value of $s \in \left(\hat{s}_x, \frac{x - \hat{R}_x}{1 - \hat{R}_x}\right)$ can be an equilibrium when $R_L = \hat{R}$ given $\hat{R} > 0$, . If $\hat{R} \leq 0$, there does not exist such an equilibrium.

References

Acharya, V., D. Gromb and T. Yorulmazer (2012), 'Imperfect competition in the interbank market for liquidity as a rationale for central banking', *American Economic Journal: Macroeconomics* **4**, 184–217.

Acharya, V., H.-S. Shin and T. Yorulmazer (2010), 'Crisis resolution and bank liquidity', *Review of Financial Studies* **24**(6), 2166 – 2205.

Acharya, V. and O. Merrouche (2012), 'Precautionary hoarding of liquidity and inter-bank markets: Evidence from the sub-prime crisis', *Review of Finance* **28**(4), 1103–1152.

Allen, F. and D. Gale (2004), 'Financial fragility, liquidity, and asset prices', *Journal of the European Economic Association* **2**, 1015–1048.

Allen, F. and D. Gale (2005), 'From cash-in-the-market-pricing to financial fragility', *Journal of the European Economic Association* **3**, 535–546.

Allen, F., E. Carletti and D. Gale (2009), 'Interbank market liquidity and central bank interventions', *Journal of Monetary Economics* **56**, 639–652.

Ashcraft, A., J. McAndrews and D. Skeie (2011), 'Precautionary reserves and the interbank market', *Journal of Money, Credit and Banking* **43**, 311–348.

Bolton, P., T. Santos and J. Scheinkman (2011), 'Outside and inside liquidity', *Quarterly Journal of Economics* **126**(1), 259–321.

de Haan, L. and J. van den End (2013), 'Banks' responses to funding liquidity shocks: Lending adjustment, liquidity hoarding and fire sales', *Journal of International Financial Markets, Institutions and Money* **26**, 152–174.

Diamond, D. and P. Dybvig (1983), 'Bank runs, deposit insurance and liquidity', **91**(3), 401–419.

Diamond, D. and R. Rajan (2001), 'Liquidity risk, liquidity creation, and financial fragility: A theory of banking', *Journal of Political Economy* **109**(2), 287–327.

ECB (2013), 'The eurosystem collateral framework throughout the crisis', *ECB Monthly Bulletin* pp. 71–86.

ECB (2014), 'Recent developments in excess liquidity and money market rates', *ECB Monthly Bulletin* pp. 69–82.

Freixas, X., A. Martin and D. Skeie (2011), 'Bank liquidity, interbank markets, and monetary policy', *Review of Financial Studies* **24**, 2656–2692.

Gale, D. and T. Yorulmazer (2013), 'Liquidity hoarding', *Theoretical Economics* **8**, 291–324.

Gorton, G. (1985), 'Clearinghouses and the origin of central banking in the u.s.', *Journal of Economic History* **45**(2), 277 – 283.

Gorton, G. (2010), *Slapped by the Invisible Hand: The Panic of 2007*, Oxford Univ. Press.

Heider, F., M. Hoerova and C. Holthahsen (2015), 'Liquidity hoarding and interbank market spreads: The role of counterparty risk', *Journal of Financial Economics* **Forthcoming**.

Holmstrom, B. and J. Tirole (1998), 'Private and public supply of liquidity', *Journal of Political Economy* **106**(1), 1–40.

Malherbe, F. (2014), 'Self-fulfilling liquidity dry-ups', *The Journal of Finance* **69**(2), 947–970.

Sauerzopf, B. (2007), 'Credit claims as eligible collateral for eurosystem credit operations', *Monetary Policy and the Economy* **Q2/07**, 80–92.

Swanson, W. (1908), 'The crisis of 1860 and the first issue of clearing-house certificates: Ii', *Journal of Political Economy* **16**(4), 212 – 226.

Tamura, K. and E. Tabakis (2013), The use of credit claims as collateral for eurosystem credit operations, *in* 'ECB occasional papers', number 148.