WORKING PAPERS

N° IDEI-845

February 2015

"Venture Capital and Knowledge Transfer"

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Venture Capital and Knowledge Transfer*

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February 6, 2015

Abstract

This paper explores a new role for venture capitalists, as knowledge intermediaries. A venture capital investor can communicate valuable knowledge to an entrepreneur, facilitating innovation. The venture capitalist can also communicate the entrepreneur's innovative knowledge to other portfolio companies. We study the costs and benefits of these two forms of knowledge transfer, and their implications for investment, innovation, and product market competition. The model also sheds light on the choice between venture capital and other forms of finance, and the determinants of the decision to seek patent protection for innovations. Our analysis provides a rationale for the use of contingencies (specifically, patent approval) in VC contracts documented by Kaplan and Stromberg (2003), and for recent evidence on patterns of syndication among venture capitalists.

Keywords: venture capital, knowledge intermediaries, contracts, innovation, competition, patents.

JEL: D82, D86, G24, L22.

*We would like to thank Giacinta Cestone, Marco Da Rin, Thomas Hellmann, Bart Lambrecht, Raghavendra Rau, Mark Schankerman, Yossi Spiegel, Avanidhar Subrahmanyam, Otto Toivanen, participants at the Barcelona GSE Summer Forum, the Bank of Finland-CEPR workshop on Entrepreneurship and Innovation and the Cambridge Finance seminar, for many helpful comments. We would also like to thank Xundong Yin for his suggestions.

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1 Introduction

Innovative start-up firms often produce valuable new knowledge. Investors who are closely involved with the start-ups they finance, such as venture capitalists[1], typically have direct access to this innovative knowledge, while outsiders do not. These investors are therefore in a very favorable position to act as knowledge intermediaries, transferring knowledge between the different companies they are involved with[2]. This paper investigates the role of venture capitalists in knowledge transfer. Much of the theoretical literature has explored instead their role as monitors and/or providers of advice and support. We abstract from these to focus on knowledge transfer.

Evidence on knowledge transfer by venture capitalists is difficult to obtain, but several empirical studies suggest it plays an important role. Some direct evidence based on patent citations comes from Gonzalez-Uribe (2013), who finds that venture capitalists diffuse knowledge about their existing patented innovations among their portfolio companies. Evidence that venture capitalists also transfer valuable non-patented knowledge is presented by Pahnke et al. (2014), based on interviews with entrepreneurs, venture capitalists and industry experts. There is, moreover, indirect evidence, highlighting the importance of knowledge transfer in other, similar settings. Helmers et al. (2013) find that information transmission through interlocking boards of directors has a significant positive effect on innovation. Asker and Ljungqvist (2010) show that firms are disinclined to share investment banks with other firms in the same industry, but only when the firms engage in product-market competition (suggesting concern over the possibility of knowledge transfer to competitors)[3].

We develop a theoretical model to study the costs and benefits of knowledge transfer, as well as the implications for investment, firm performance and innovation. Our analysis identifies the circumstances in which innovative start-ups can benefit from venture capital finance, taking fully into account the likelihood that some of the innovative knowledge they generate will be transmitted by the venture capitalists to other portfolio companies, including competitors. The model has an *ex-ante* innovation stage, followed by an *ex-post* commercialization stage. An entrepreneur with an innovative project may develop a valuable innovation at the end of the

[1]Gorman and Sahlman (1989) find that lead venture investors visit each portfolio company an average of 19 times per year and spend 100 hours in direct contact (onsite or by phone) with the company. Sahlman (1990) highlights venture capitalists' involvement with their portfolio companies in a variety of ways, including the recruitment and compensation of key individuals, strategic decisions, and links with suppliers and customers. Bottazzi et al. (2008) provide further evidence of active involvement by venture capitalists and frequent interaction with their portfolio firms. Kaplan and Strömberg (2003) show that venture capitalists often hold seats on the board, as well as substantial voting and control rights.

[2]Many of these will be innovative start-ups, although it is worth noting that venture capitalists also often serve on boards of mature public firms (see Celikyurt et al. (2012)).

[3]Atanasov et al. (2008) find that 47% of a sample of VC-related lawsuits involve allegations of "tunneling" (wrongful transfers of assets, expropriation of profitable opportunities, etc.), suggesting that concern over reputation is not always sufficient to deter such behavior. Knowledge transfer is typically much harder to demonstrate, and hence easier to undertake.

first stage; the innovation then has to be commercialized in the second stage in order to yield financial returns at the end. We begin by studying the case where the valuable innovation cannot be protected through a patent. We analyze two forms of knowledge transfer by the venture capitalist (VC) who funds the project: *ex ante*, the VC may, by incurring a private cost C, communicate useful knowledge obtained from other firms to the entrepreneur. This *inward* knowledge transfer helps the entrepreneur to develop a valuable innovation. *Ex post*, once the entrepreneur has innovated successfully, the VC may communicate this innovative knowledge to other firms. We assume this *outward* knowledge transfer has a beneficial effect on the other companies, yielding a gain, G, for the VC. However, it also reduces the entrepreneur's expected profitability through a competition effect, parameterized by k. In general, the parameters C, G and k can vary across the firms in a VC portfolio, depending on the characteristics of the project and the resulting innovation. For example, some innovations may generate greater positive spillovers than others, affecting G, while the extent to which knowledge sharing leads to erosion of profits through competition may vary with industry and product characteristics, affecting k and C.[4]

We study optimal contracts between the entrepreneur and the VC. While inward knowledge transfer is always beneficial for the venture, outward knowledge transfer has several effects. It has a direct negative impact on profitability through increased competition, but also an indirect positive impact because it relaxes the venture capitalist's participation constraint. The first of these channels tends to reduce entrepreneurial effort, while the second tends to increase it. Moreover, outward knowledge transfer interacts with the venture capitalist's ex-ante incentives to engage in inward knowledge transfer. The interplay of these effects determines the optimal choice of VC contract. We find that, depending on parameter values, the two forms of knowledge transfer can emerge as substitutes or complements, with quite different implications for innovation and profitability. For intermediate values of potential spillovers (G), the optimal contract either gives the VC a low financial stake in the venture and induces outward knowledge transfer, or it gives the VC a higher financial stake and induces inward knowledge transfer. For higher values of potential spillovers, optimal contracts induce both forms of knowledge transfer.

We then explore the entrepreneur's choice between VC and non-VC (no knowledge transfer) finance. The main drawback of VC finance is due to the cost of inducing the VC *not* to transfer knowledge outwards when the spillover benefit G is below a critical threshold. We show that, as a consequence, the trade-off between the two forms of finance can be non-monotonic in G: for low and high values of G, VC finance is preferred; while for intermediate values of G, non-VC finance dominates. An interesting special case of our model occurs when outward knowledge transfer benefits (only) non-competitors ($k \geqslant 1$): VC finance then always dominates non-VC

[4] In empirical work, heterogeneity among portfolio companies in terms of their positions in technology space and product market space could be used to investigate some of our model's predictions, in the spirit of Bloom et al. (2013)

finance. In practice, however, evidence on indirect ties among VC portfolio companies suggests that the transfer of knowledge to competitors is an important phenomenon: Pahnke et al. (2014) find that 53% of the VC-backed start-ups in their sample share a VC investor with a competitor. They also present interview evidence on the nature of knowledge transfer, highlighting flows of information about product design as well as regulatory experiences.

Our model sheds light on the costs and benefits of knowledge transfer between competitors, implemented by venture capitalists. One implication is that, other things held equal, venture capitalists will prefer to syndicate their investments with a relatively small and stable set of partners, so as to internalize knowledge spillovers between portfolio companies and the associated financial externalities. This is consistent with evidence on syndication patterns among venture capitalists in Bubna et al. (2014).

In section 4, we go on to study the case of patentable innovations. We allow for some uncertainty over the outcome of patent applications, and for the fact that the patent application process can disclose information to competitors. One of our main objectives in this section is to investigate the determinants of the decision to apply for patent protection. We find that these differ depending on how the firm is financed, with VC-funded firms exhibiting a greater propensity to apply for a patent (holding constant the quality of the innovation). This is not due to fear of expropriation by the VC, but rather to the fact that the VC's role as knowledge intermediary offers protection against the loss associated with expropriation by competitors following information disclosure and patent rejection. Our results provide a rationale for the empirical evidence showing that venture capital has a significant positive effect on innovation measured by patent counts[5]. In our model, this is due to two effects: first, inward knowledge transfer by the VC increases the probability of a valuable innovation; second, VC-funded firms are more likely to apply for patent protection. Teasing out the relative importance of these two effects is an interesting avenue for future empirical research[6].

Section 4 also studies the use of contingencies in venture capital contracts; specifically, whether and how optimal contracts condition on the approval or rejection of a patent application. Our results imply that, in general, optimal contracts will not condition on patent approval, with one exception: for some parameter values, contracts designed to induce inward knowledge transfer and deter outward knowledge transfer will optimally offer a lower (higher) financial stake to the VC when a patent is (not) granted. This is consistent with evidence from Kaplan and Stromberg (2003): they find that contingent contracts rewarding the entrepreneur on the basis of non-financial performance are used in almost 9% of financing rounds in their sample - with

[5]Kortum and Lerner (2000) , Mollica and Zingales (2007), Ueda and Hirukawa (2008).

[6]In a different context, Helmers et al. (2013) are able to exploit the occurrence of an exogenous corporate governance reform and an exogenous change to the patent system in India to identify a positive effect of board interlocks on R&D spending, as well as a separate positive effect on patenting propensity.

patent approval being one of the main contingencies[7].

The remainder of the paper is organized as follows. We complete this section by discussing the related theoretical literature. Section 2 presents the baseline model. We study the case of innovations that cannot be patented in section 3, and patentable innovations in section 4. Section 5 concludes.

1.1 Relationship to theoretical literature

There is a large theoretical literature on the role of venture capitalists, which focuses primarily on monitoring[8] and advice/support[9]. We add a new role, as knowledge intermediaries. In this respect, the closest papers to ours are Bhattacharya and Chiesa (1995), Ueda (2004) and Yosha (1995). Bhattacharya and Chiesa consider an economy with many industries: in each industry, two rival firms engage in an R&D race. There are two banks in the economy. Bhattacharya and Chiesa compare bilateral financing, in which each bank finances only one of the rivals in each industry, with multilateral financing, in which each bank provides half of the funding of each rival in each industry. Being one of the financiers gives access to any knowledge produced by the firm at the interim stage. At this stage, financiers decide whether to disclose the knowledge produced by one firm to its rival: this is the link with our paper. The setting is completely different though, and the main focus of Bhattacharya and Chiesa is the effect of a commitment to knowledge sharing on firms' *ex-ante* incentives to invest in R&D. Yosha (1995) also studies the choice between bilateral and multilateral financing, under the assumption that the latter entails a lower cost but greater leakage of information to competitors[10,11].

Ueda (2004) explores the trade-off between bank and VC finance under the assumption that venture capitalists, unlike banks, may steal an entrepreneur's idea at the *ex-ante* financing stage (before the project is undertaken); on the other hand, venture capitalists have greater ability to evaluate projects.[12] We focus instead on knowledge transfer after the project has been funded and undertaken.

A few other papers have studied the choice between venture capital and bank finance, focusing

[7] An example is the payment of committed funding (by the VC) when a patent is approved. In such cases, the VC may still provide funding if the patent is not approved, but will typically do so on terms more favorable to the VC.

[8] See, for example, Dessí (2005) and Holmstrom and Tirole (1997).

[9] See, among others, Bottazzi et al. (2009),Casamatta (2003), Cestone (2014) , Cumming et al. (2005) , De Bettignies and Brander (2007), Dessí (2010), Hellmann (1998), Jeng and Wells (2000), Lerner and Schoar (2005), Repullo and Suarez (2000, 2004), Riyanto and Schwienbacher (2006), and Schmidt (2003).

[10] Thus higher quality firms, who have more to lose from information leakage, prefer bilateral financing.

[11] See also Bhattacharya and Ritter (1983), who examine the trade-off between information disclosure to competitors and raising finance on better terms on capital markets.

[12] See also Biais and Perotti (2008), who study an entrepreneur's decision to hire experts when different forms of expertise are valuable but experts may steal a good idea, and Hellmann and Perotti (2011), who examine the costs and benefits of circulating initially incomplete ideas (completion versus appropriation).

on quite different trade-offs from those examined in our paper. Winton and Yerramilli (2008) assume that venture capitalists have a greater ability to evaluate possible continuation strategies for the firm. A trade-off arises because VCs are also assumed to have a higher cost of capital. Landier (2003) views the choice between VC and bank finance as determined by a hold-up problem: when investors need protection against hold-up by the entrepreneur, venture capital with staged financing is preferred; when the entrepreneur needs protection against hold-up by investors, long-term bank finance is preferred.

2 The Baseline Model

The model has two stages, with three corresponding dates, $t = 0, 1, 2$. All agents (entrepreneur and investors) are assumed to be risk neutral and protected by limited liability.

2.1 Project

Consider an entrepreneur (start-up firm) endowed with an innovative investment project. The project starts with an innovative idea and requires a contractible initial investment I (money) at the beginning of the first stage (date 0). During the first stage, the idea may be developed into a valuable innovation. For example, we can think of the entrepreneur as having an idea for a new product to begin with; he then undertakes some initial production and carries out the tests/trials required to establish that it works well and satisfies appropriate quality standards. If the first stage is successful, the innovation then needs to be commercialized: here the entrepreneur's effort is crucial, key strategic decisions have to be made, new personnel may need to be recruited, and so on[13]. We assume that if the innovation has been developed successfully (at date 1), and in the absence of knowledge transfer (see below), the project will finally succeed at date 2 with probability e, where e captures the entrepreneur's effort during the second stage. Irrespective of the entrepreneur's effort, success is never certain, thus $e < 1$. If the initial innovative idea fails to be developed into a valuable innovation[14], the project's success probability is reduced; for simplicity, we assume it is equal to zero. If the project succeeds at date 2, it yields verifiable returns R; if it fails, it yields nothing ($R > 0$).

2.2 Entrepreneur

The entrepreneur has no initial monetary wealth, and needs to raise finance from outside investors. If he is able to secure outside funding and undertake the project (and absent knowledge

[13] We focus here on entrepreneurs, who will manage the business and try to make it succeed, rather than pure inventors, who may prefer to exit as soon as they have developed a valuable innovation.

[14] For expositional convenience, we will refer to this as the "no innovation" outcome.

transfer, see below), he develops a successful innovation with probability π. He then chooses his effort level e, where $0 \leq e < 1$, and the cost of effort is given by $c(e) \equiv \frac{1}{2}e^2$. To make the analysis interesting, we assume that $R > \frac{I}{\pi}$, otherwise the project would not be worth financing (absent knowledge transfer). Given our assumptions about effort, we normalize both R and I to be less than one[15].

2.3 Investors

Investors provide the initial funding I for the project. We assume they are competitive, earning zero expected profits in equilibrium.

In our model the main difference between venture capitalists and other investors lies in the venture capitalists' close connections[16] with their portfolio firms, implying that venture capitalists (henceforth VCs) can transfer knowledge relatively easily between the firms they fund. In particular, we assume that VCs would find it easier to transfer knowledge than any outsiders, including other, arm's length investors, since they interact closely and repeatedly with the entrepreneur, and have privileged access to information throughout the time in which the innovation is being developed. For simplicity, we capture this difference by assuming that VCs, unlike other investors, can transfer knowledge. As we shall see, this brings about both benefits and costs. To focus on the trade-off between these costs and benefits, we abstract from other roles played by venture capitalists, such as monitoring or screening, which have been studied extensively in the theoretical literature on venture capital.

2.4 Knowledge transfer

We consider two forms of knowledge transfer. The VC may communicate valuable knowledge to the entrepreneur (e.g. information acquired through his involvement with other portfolio firms) during the first stage, while the innovation is being developed. We model this as increasing the probability of a valuable innovation, from π to $\pi + \tau$ ($\tau > 0$). The VC incurs a private cost C in doing this (e.g. opportunity cost of time, effort, or lower expected returns on his investment in other portfolio firms). We refer to this as inward knowledge transfer, or *ex ante* knowledge transfer because it occurs in the first stage of our model. The second form of knowledge transfer is outward, or *ex post*, knowledge transfer, whereby the VC transfers knowledge to another firm once the entrepreneur has successfully developed an innovation, in a way that is beneficial to the other firm (and to the VC), but has an adverse effect on the entrepreneur's profitability, due to greater competition. We model this as bringing a private benefit of value $G > 0$ to the

[15] In the simplest case and absent knowledge transfer considerations, the socially optimal effort is given by $e^* = \arg\max_e eR - \frac{1}{2}e^2$. The first order condition tells us $e^* = R$. Since we have assumed $e < 1$, we must also have $R < 1$. Given that $R > \frac{I}{\pi}$, this further implies $I < 1$.

[16] See footnote 1.

VC, reflecting the value of positive spillovers, while decreasing the success probability of the entrepreneur's project from e to ke, with $0 < k < 1$. As noted in the Introduction, we shall also briefly discuss the interesting and analytically simpler case where outward knowledge transfer benefits only non-competitors (i.e. $k \geqslant 1$).

We assume that the entrepreneur does not observe whether the VC transfers knowledge outward, and that both forms of knowledge transfer cannot be contracted on explicitly. The VC will therefore engage in one, or both, if, and only if, this is in his interest. Finally, we allow for the possibility that, when the VC does not expropriate the entrepreneur's innovative knowledge, some of his competitors may later succeed in doing so (e.g. reverse engineering), or may independently develop an equivalent innovation, which also reduces the success probability of the entrepreneur's project from e to ke. We shall treat these two possibilities together, assuming they occur with probability μ, where $1 > \mu > 0$. For expositional convenience we will refer to them simply as expropriation (by competitors).

2.5 Contract design

Contracts specify the investor's (venture capitalist's) financial contribution at the beginning (I), and a sharing rule for final returns, R.

2.6 Patent protection

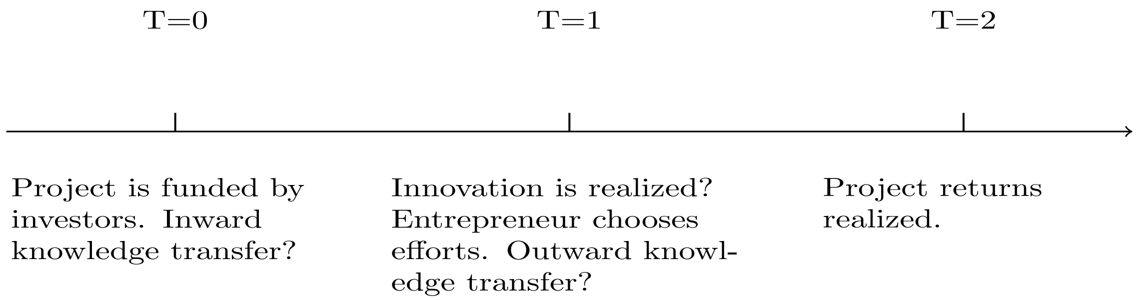
Section 3 focuses on innovative knowledge that cannot, by its very nature, be protected from expropriation by a patent. In section 4 we go on to examine patentable innovations. We assume that, once he has successfully developed an innovation, the entrepreneur can apply for a patent. The application is approved with probability $\beta < 1$.[17] If the application is approved, expropriation is no longer feasible, and knowledge transfer to other firms can only occur through licensing. If the application is rejected, the innovation remains vulnerable to expropriation. Moreover, we allow for a higher probability of expropriation by competitors in this case, $\alpha > \mu$, reflecting leakage of information through the patent application.

2.7 Time line

Figure 1 shows the timeline for the baseline model.

[17] We treat β as a parameter of the model, capturing the efficiency of the patent system, and/or the characteristics of the product or process.

Figure 1: Timeline for the Baseline Model



3 Non-patentable innovations

We begin by considering innovative knowledge that cannot, by its very nature, obtain patent protection. Section 4 will study patentable innovations. We examine first the case where the entrepreneur raises the required external funding from a non-VC investor, then go on to analyze the case of VC funding. In each case, we study optimal contracts between the entrepreneur and the investor. Finally, we examine the entrepreneur's optimal choice between VC and non-VC finance.

3.1 Non-VC investor

At date 0, the entrepreneur secures external funding for his project from a non-VC investor. The contract signed with the investor maximizes the entrepreneur's expected payoff, subject to guaranteeing zero expected profits to the investor (since we are assuming that investors are competitive). The contract specifies the investor's capital contribution, I, and the share of final returns going to each party: R_e^N for the entrepreneur, $R - R_e^N$ for the investor. To study the optimal contracting problem, we apply backward induction and start with the effort decision of the entrepreneur at the second stage. The optimal effort level exerted by the entrepreneur is given by[18] $e^N = argmax_e \;\; e(1-\mu+\mu k)R_e^N - \frac{1}{2}e^2$. The first order condition gives us $e^N = \omega R_e^N$, where $\omega = (1 - \mu + \mu k)$.

[18] Recall that expropriation is not observed by the entrepreneur: he therefore chooses his effort knowing that other firms will expropriate with probability μ, and that when this happens his probability of success will be reduced to ke.

Thus, the optimal contract solves:

$$\begin{aligned}
\max_{R_e^N} \quad & \pi[e^N(1-\mu+\mu k)R_e^N - \frac{1}{2}(e^N)^2] \\
s.t. \;\; e^N &= \omega R_e^N \quad (IC_e) \\
\pi e^N \omega(R - R_e^N) &\geq I \quad (PC_i) \\
&\Longleftrightarrow \\
\max \quad & R_e^N \\
s.t. \;\; y &\geq \frac{I}{\pi\omega^2} \\
where \;\; y &= R_e^N(R - R_e^N), \omega = 1-\mu+\mu k
\end{aligned}$$

When condition ($I \leq \frac{\pi\omega^2 R^2}{4}$) [19] is satisfied[20], the optimal contract is given by $R_e^N \geq \frac{R}{2}$, where R_e^N is the largest root of $\pi\omega^2 R_e^N(R - R_e^N) = I$.

3.2 VC investor

We now study how the contracting problem differs when the entrepreneur obtains external finance from a venture capitalist. As discussed earlier, we focus on one, so far under-explored difference between venture capitalists and other investors: by virtue of their close involvement with portfolio firms, VCs can more easily transfer knowledge between them. From the perspective of the entrepreneur in our model, knowledge transfer can take two forms. The first is inward (ex-ante) knowledge transfer, whereby the VC communicates valuable knowledge to him during the innovation stage. The second is outward (ex-post) knowledge transfer, whereby the VC transfers the entrepreneur's knowledge to other firms once he has developed a valuable innovation, in a way that reduces the entrepreneur's profitability (expropriation). Recall from section 2 that outward knowledge transfer reduces the entrepreneur's success probability from e to ke ($k < 1$), because of greater competition, while yielding a private benefit of value G to the venture capitalist, reflecting the value of positive spillovers.

We model inward knowledge transfer as increasing the probability of a valuable innovation from π to $\pi + \tau$, where $\tau > 0$. The VC incurs a private cost $C > 0$ (e.g. opportunity cost of time, effort, or lower expected returns on his investments in other portfolio firms). Formally, our modeling of inward knowledge transfer is analogous to models of "advice and support" in the theoretical literature on venture capital. We differ from these models in considering also the role of outward knowledge transfer, and the interaction between the two.

[19] Note that from PC_i, we have $\pi\omega^2 R_e^N(R - R_e^N) = I$ at the optimum,and the maximum value of $R_e^N(R - R_e^N)$ is $\frac{R^2}{4}$ when R_e^N equals $\frac{R}{2}$. Thus it is never optimal for the entrepreneur to set $R_e^N < \frac{R}{2}$.

[20] If this condition is not satisfied, the entrepreneur cannot raise the funding needed to undertake his project.

When the entrepreneur turns to a VC for external finance, he can choose between four different contracting possibilities. He can design the contract to induce the VC to engage in both types of knowledge transfer, only one type, or no knowledge transfer. In what follows, we characterize the optimal contract for each of these possible choices. We then study the entrepreneur's optimal choice.

3.2.1 Outward (ex-post) knowledge transfer, or expropriation

We begin by considering the case where the VC only transfers knowledge outward. This reduces the entrepreneur's probability of success from e to ke, while yielding a private benefit $G > 0$ for the VC. The optimal contract solves the following problem (P1):

$$
\begin{aligned}
\max_{R_e^{VN}} \quad \pi[ke^{VN}R_e^{VN} &- \frac{1}{2}(e^{VN})^2] \\
s.t.\ e^{VN} &= kR_e^{VN} \quad (IC_e) \\
\pi[ke^{VN}(R - R_e^{VN}) + G] &\geq I \quad (PC_{VC}) \\
\tau[ke^{VN}(R - R_e^{VN}) + G] &\leq C \quad (IC_{VC}\ \textit{ex ante}) \\
G + ke^{VN}(R - R_e^{VN}) &\geq \omega e^{VN}(R - R_e^{VN}) \quad (IC_{VC}\ \textit{ex post})
\end{aligned}
$$

Comparing this with the equivalent problem for the non-VC investor case, we see that the entrepreneur's incentive constraint, (IC_e), is modified to allow for the fact that the VC always expropriates ex post, reducing the probability of success. On the other hand, the private benefit G relaxes the venture capitalist's participation constraint, (PC_{VC}), making it possible to offer more high-powered monetary incentives to the entrepreneur (higher R_e^{VN}). In addition, we have two new constraints. Since we are considering the case without inward knowledge transfer, it must be the case that the VC has no incentive to transfer knowledge to the entrepreneur; i.e. the private cost C is greater than the expected financial return to the VC (IC_{VC} *ex ante*). Finally, it must be the case that the VC expects a net gain from transferring the entrepreneur's knowledge to competitors (IC_{VC} *ex post*); i.e. the private benefit G is greater than the reduction in the VC's expected return on his investment in the entrepreneur's project.

The solution to P1 is described in the following lemma.

Lemma 1 *Inducing the VC investor to transfer knowledge outwards (to other firms) but not inwards (to the entrepreneur) requires that* $\frac{I}{\pi} \leq \frac{C}{\tau}$ *and* $\frac{C}{\tau} \geqslant G \geq \frac{\omega - k}{\omega}\frac{I}{\pi}$. *If* $G \geq \frac{I}{\pi}$, *the optimal contract sets* $R_e^{VN} = R$. *When the inequality holds strictly, the VC will make an additional payment* F *ex ante, beyond* I, *so that the participation constraint holds as an equality; i.e.* $\pi G = I + F$. *If* $G < \frac{I}{\pi}$, *the optimal contract,* R_e^{VN}, *is determined by the largest root of the* *following equation:*

$$\pi[k^2 R_e^{VN}(R - R_e^{VN}) + G] = I;$$

The problem has a solution only when condition $\pi[\frac{k^2R^2}{4} + G] \geq I$ is satisfied.

Proof. See Appendix. ∎

The intuition for Lemma 1 is as follows: If the cost C is too low ($\frac{C}{\tau} < \frac{I}{\pi}$), it is not possible to induce the VC to participate (which requires that his expected gain from innovative success be sufficiently large) without transferring knowledge inwards (which increases the probability of innovative success). Similarly, it is not possible to induce the VC to participate and to expropriate ex post if the private benefit from expropriation is too low. The final condition simply requires the investment cost, I, not to be too high relative to the expected benefits from the project, which include its financial returns as well as the venture capitalist's private benefit from expropriation. When the private benefit G and the cost C are not too low, the optimal contract is determined by the participation constraint of the VC.

Thus contracts inducing (only) outward knowledge transfer may be used when the potential spillovers from knowledge transfer are significant.

3.2.2 Inward (ex ante) and outward (ex post) knowledge transfer

When the VC transfers knowledge both inwards ("advice") and outwards ("expropriation"), we know that the entrepreneur's effort level e^{VN} is determined by $argmax_e \; keR_e^{VN} - \frac{1}{2}e^2 = kR_e^{VN} \;\; (IC_e)$, since the probability of success is reduced to ke by expropriation. The venture capitalist's participation constraint is given by:

$$(\pi + \tau)[ke^{VN}(R - R_e^{VN}) + G] \geq I + C \quad (PC_{VC})$$

reflecting the higher probability of innovation success $(\pi + \tau)$ due to advice, as well as the private benefit G due to expropriation. There are two incentive constraints for the VC. First, he has to be induced to advise ex ante:

$$\tau[ke^{VN}(R - R_e^{VN}) + G] \geq C \quad (IC_{VC} \; ex \; ante)$$

Second, he has to be induced to expropriate ex post:

$$G + ke^{VN}(R - R_e^{VN}) \geq \omega e^{VN}(R - R_e^{VN}) \quad (IC_{VC} \; ex \; post)$$

The optimal contract that induces the venture capitalist to advise ex ante and expropriate ex post is determined by the following optimization problem (P2):

$$\max_{R_e^{VN}} \ (\pi+\tau)[ke^{VN}R_e^{VN} - \frac{1}{2}(e^{VN})^2]$$
$$s.t. \ (IC_e)$$
$$(PC_{VC})$$
$$(IC_{VC} \ ex \ ante)$$
$$(IC_{VC} \ ex \ post)$$

The solution to P2 is provided in Lemma 2.

Lemma 2 *When $G < \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, it is not possible to induce the VC to transfer knowledge ex post. When $G \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, the optimal contract is $R_e^{VN} = R$; when $max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} > G \geq \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, the optimal contract is the largest root of the following equation:*

$$k^2 R_e^{VN}(R - R_e^{VN}) + G = max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\};$$

The optimal contract will also entail an ex ante fee when the VC participation constraint is slack, to ensure the VC earns zero expected rents. The solution holds only when condition $(\pi+\tau)[\frac{1}{4}k^2R^2 + G] \geq I + C$ is satisfied, otherwise, the problem has no solution.

Proof. See Appendix. ■

Comparing this with the result for the optimal contract that induces only outward knowledge transfer reveals that when both contracts are feasible (requiring the condition $\frac{I}{\pi} \leq \frac{C}{\tau}$ to hold), the optimal contract which induces both inward and outward knowledge transfer in general offers a lower stake in the project's financial returns to the entrepreneur (lower R_e^{VN}). Specifically, this will be the case when the inequality holds strictly ($\frac{I}{\pi} < \frac{C}{\tau}$). Thus the project's probability of final success, once a valuable innovation has been developed, is lower in this case. On the other hand, the probability of a successful innovation is higher. Essentially, when the cost of advice is relatively high, the entrepreneur has to relinquish a higher share of final returns to the VC to induce him to transfer knowledge inwards: this increases the likelihood of innovating successfully ex ante, but reduces entrepreneurial effort ex post.

When the cost of inward knowledge transfer is relatively low ($\frac{I}{\pi} > \frac{C}{\tau}$), on the other hand, the only feasible contract is the one that induces both types of knowledge transfer.

3.2.3 Inward (ex ante) knowledge transfer, or advice

We now study the optimal contract when the entrepreneur chooses to induce only inward knowledge transfer by the VC. Following a successful innovation, the project's success probability is given by e if there is no expropriation by others (with probability $1-\mu$), and ke otherwise. The entrepreneur's expected probability of success when he chooses his effort level is therefore equal to ωe where $\omega = 1-\mu+\mu k$, implying that effort is given by $e^{VN} = argmax_e \ \omega e R_e^{VN} - \frac{1}{2}e^2 = \omega R_e^{VN} \quad (IC_e)$. The optimal contract with ex-ante knowledge transfer but no expropriation ex post is determined by the following program (P3):

$$\begin{aligned}
\max_{R_e^{VN}} \ (\pi+\tau)[\omega e^{VN} R_e^{VN} \ &- \ \frac{1}{2}(e^{VN})^2] \\
s.t.\ e^{VN} \ &= \ \omega R_e^{VN} \quad (IC_e) \\
(\pi+\tau)[\omega e^{VN}(R-R_e^{VN})] \ &\geq \ I+C \quad (PC_{VC}) \\
\tau[\omega e^{VN}(R-R_e^{VN})] \ &\geq \ C \quad (IC_{VC}\ ex\ ante) \\
G + k e^{VN}(R-R_e^{VN} \ &\leq \ \omega e^{VN}(R-R_e^{VN}) \quad (IC_{VC}\ ex\ post)
\end{aligned}$$

Comparing this program with those studied earlier, we see that the private benefit G no longer appears in the venture capitalist's participation constraint or in his ex-ante incentive constraint. His ex-post incentive constraint now induces him *not* to transfer knowledge ex post. The solution to P3 is described by Lemma 3.

Lemma 3 *If the entrepreneur seeks to induce the VC to transfer knowledge ex ante but not ex post:*

- *when $G < \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, the optimal contract is the largest root of the following equation:*

$$\omega^2 R_e^{VN}(R-R_e^{VN}) = max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\};$$

- *when $G \geq \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, the optimal contract is the largest root of the following equation:*

$$(\omega-k)\omega R_e^{VN}(R-R_e^{VN}) = G$$

The optimal contract will entail a fee ex ante if the VC participation constraint is slack. The problem has a solution only when the following conditions are satisfied: $\frac{1}{4}\omega^2 R^2 \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ and $G \leq \frac{1}{4}(\omega-k)\omega R^2$.

Proof. See Appendix. ■

Lemma 3 shows that inducing only inward knowledge transfer requires the venture capitalist's private benefit from outward knowledge transfer to be below a critical threshold value. When the advice cost is relatively low ($\frac{C}{\tau} < \frac{I+C}{\pi+\tau}$), there are two possibilities: either the VC participation constraint binds, or his ex-post incentive constraint (requiring him to refrain from expropriation) binds. Conversely, when the advice cost is relatively high ($\frac{C}{\tau} > \frac{I+C}{\pi+\tau}$), either his ex-ante incentive constraint (requiring him to transfer knowledge inwards) binds, or his ex-post incentive constraint binds.

3.2.4 No Knowledge Transfer

Finally, we study under what conditions the venture capitalist chooses not to engage in any form of knowledge transfer. In this case, the VC acts in the same way as the non-VC investor: the difference lies in the constraints that must be satisfied for the VC to refrain from transferring knowledge, yielding a different optimization problem for the entrepreneur and a different resulting contract. The optimal contract solves the following program (P4):

$$\max_{R_e^{NN}} \quad \pi[\omega e^{NN} R_e^{NN} - \frac{1}{2}(e^{NN})^2]$$

$$\begin{aligned}
s.t.\ e^{NN} &= \omega R_e^{NN} \quad (IC_e) \\
\pi\omega e^{NN}(R - R_e^{NN}) &\geq I \quad (PC_{VC}) \\
\tau\omega e^{NN}(R - R_e^{NN}) &< C \quad (IC_{VC}\ ex\ ante) \\
G + ke^{NN}(R - R_e^{NN}) &\leq \omega e^{NN}(R - R_e^{NN}) \quad (IC_{VC}\ ex\ post)
\end{aligned}$$

The solution to P4 is described in the following lemma.

Lemma 4 *The VC chooses not to transfer knowledge ex ante or ex post in the following two cases:*

- *when $\frac{I}{\pi} \leq \frac{C}{\tau}$ and $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$. The optimal contract is determined by the largest root of the following equation:*

$$\pi\omega^2 R_e^{NN}(R - R_e^{NN}) = I;$$

- *when $\frac{I}{\pi} \leq \frac{C}{\tau}$ and $\frac{\omega-k}{\omega}\frac{C}{\tau} \geqslant G \geq \frac{\omega-k}{\omega}\frac{I}{\pi}$. The optimal contract is the largest value such that IC_{VC} ex post is binding:*

$$(\omega - k)\omega R_e^{NN}(R - R_e^{NN}) = G$$

The optimal contract will entail a fee ex ante if the VC participation constraint is slack. The problem has a solution only when conditions $\frac{\pi\omega^2 R^2}{4} \geq I$ and $G \leq \frac{1}{4}(\omega - k)\omega R^2$ are satisfied.

Proof. See Appendix. ■

Lemma 4 shows that there are two cases of interest. Both require the cost of advice C to be relatively high, to deter inward knowledge transfer by the VC. In the first case, the venture capitalist's private benefit from expropriation G is sufficiently low not to tempt him, given his stake in the financial returns of the entrepreneur's project (required to satisfy his participation constraint). In the second case, the private benefit from expropriation is larger, and the VC has to be offered a higher share of financial returns to ensure he does not expropriate. Thus in the first case, the optimal contract with the VC is the same as with the non-VC investor, and the entrepreneur is indifferent between raising external finance from a VC or a non-VC investor. In the second case, the optimal contract with the VC differs from the one with the non-VC investor because of the binding ex-post incentive constraint for the VC: in this case, the entrepreneur will prefer to raise funding from a non-VC investor.

3.2.5 Choice of contract under VC finance

Using the results summarized by Lemmas 1 to 4, we can study the entrepreneur's optimal choice of contract when he raises external finance from a venture capitalist. We will then be able to examine the tradeoffs involved in obtaining funding from a VC relative to a non-VC investor. Optimal VC contracts have the following properties:

- intuitively, when the cost of inward knowledge transfer is low (so low that the VC ex-ante incentive constraint is never binding), the optimal contract always induces this form of transfer. In addition, it also induces outward knowledge transfer if, and only if, the spillover benefits G are above a critical threshold value.

- when the cost of inward knowledge transfer is higher, we find that

  1. for intermediate values of the spillover benefits G, the optimal contract induces **either** outward knowledge transfer (with a low financial stake for the VC), **or** inward knowledge transfer (with a higher financial stake for the VC);

  2. for higher values of G, the optimal contract induces **both** inward and outward knowledge transfer.

More formally, the following result describes the optimal choice of contract under VC finance.

Proposition 1 *The entrepreneur's choice of VC contract is determined as follows:*

- *when $\frac{C}{\tau} < \frac{I}{\pi}$, the optimal contract will always induce the VC to transfer knowledge ex ante. There exists a cutoff value $G^* > \frac{\omega - k}{\omega}\frac{I+C}{\pi+\tau}$, such that*

  - *when $G > G^{*}$, the optimal contract will be the one that induces knowledge transfer ex ante and ex post;*
  - *when $G < G^{*}$, the optimal contract will be the one yielding only knowledge transfer ex ante.*

- *when $\frac{C}{\tau} > \frac{I}{\pi}$, there exist two cutoff values, G^{**}, where $G^{**} > \frac{\omega-k}{\omega}\frac{C}{\tau}$, and G^{***}, where $G^{***} < \frac{\omega-k}{\omega}\frac{C}{\tau}$, such that*
  - *when $G > G^{**}$, the optimal contract will always induce the VC to transfer knowledge ex post. For $\frac{C}{\tau}$ below a cutoff value, the contract will also induce the VC to transfer knowledge ex ante.*
  - *when $G^{**} \geq G \geqslant G^{***}$, the optimal contract will either induce knowledge transfer ex ante or it will induce knowledge transfer ex post (depending on the magnitude of $\frac{C}{\tau}$, G, k and ω).*
  - *when $G < G^{***}$, the optimal contract may induce knowledge transfer ex ante or ex post, or no knowledge transfer. For lower values of G, there will be no ex post knowledge transfer.*

Proof. See Appendix. ■

The intuition for the second part of Proposition 1 is as follows. When the spillover benefit G is sufficiently high, the optimal contract will always induce expropriation; it may also induce advice provided the advice cost is not too high. Conversely, when the spillover benefit is sufficiently low, the optimal contract will never induce expropriation; it may again induce advice provided the advice cost is not too high. For intermediate values of G, two possibilities emerge. The optimal contract may entail advice without expropriation: inducing the VC to transfer knowledge ex ante means he has to be given a relatively high share of financial returns, which deters expropriation ex post, given that the private benefit from expropriation is not so large. Alternatively, the optimal contract may entail expropriation without advice: this implies that the VC is given a relatively low share of financial returns, which leads him to transfer knowledge ex post, but does not induce him to advise ex ante. Thus for intermediate values of G, the two forms of knowledge transfer are substitutes. They become complements for higher values of G: the anticipation of spillover benefits then induces the VC to advise, while advice increases the probability of a successful innovation and hence also spillover benefits.

3.2.6 Choosing between VC and non-VC finance

We can now study the trade-offs faced by the entrepreneur in choosing between VC and non-VC finance. Our analysis reveals that:

- when the cost of inward knowledge transfer is lower than a critical threshold,

  (i) either VC finance is always preferred,

  (ii) or the choice between VC and non-VC finance is non-monotonic in the spillover benefit G: VC finance is preferred for higher and lower G, while non-VC finance is preferred for intermediate G.

- when the cost of inward knowledge transfer is higher, VC finance is preferred if, and only if, the spillover benefit G is above a critical threshold.

The intuition for the non-monotonicity property is straightforward: relying on VC finance entails a cost when expropriation is inefficient but may nevertheless be tempting for the VC, since he bears only part of the cost (the remainder is borne by the entrepreneur). This occurs for intermediate values of G. The optimal VC contract may either allow inefficient expropriation, or deter such expropriation - at a cost (the distortionary effect on entrepreneurial effort due to the need to increase the VC financial stake in the venture). Non-VC finance may then be preferred.

Formally, the optimal choice between VC and non-VC finance is summarized by the following result, and described in detail in the Appendix.

Proposition 2 *The entrepreneur' s choice between VC and non-VC finance is determined below.*

- *When $\frac{C}{\tau} < \frac{I}{\pi}$, there are two threshold values, $\ddot{G} > \frac{\omega - k}{\omega}\frac{I+C}{\pi+\tau}$, and $\dot{\mu}$, such that:*

  1. *when $\mu > \dot{\mu}$, VC finance is preferred. There exists a threshold $\check{G}$, where $\frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau} < \check{G} < \ddot{G}$, such that for $G < \check{G}$, case III is the optimal choice, while when $G > \check{G}$, case II is preferred.*
  2. *when $\mu < \dot{\mu}$, we have: for $G < \ddot{G}$, VC finance (case III) is preferred. For $\ddot{G} < G < \dddot{G}$, Non-VC finance is preferred. And for $G > \dddot{G}$, VC finance (case II) is preferred. The threshold $\dddot{G}$ decreases with μ.*

- *When $\frac{C}{\tau} > \frac{I}{\pi}$, there exists a threshold value $\frac{\tilde{C}}{\tau} > \frac{I}{\pi}$ such that:*

  1. *when $\frac{C}{\tau} > \frac{\tilde{C}}{\tau}$, there is a threshold, $G1$, such that Non-VC finance is preferred for $G \leqslant G1$, and VC finance (case I or II) is preferred otherwise. The threshold $G1$ decreases with μ.*
  2. *When $\frac{C}{\tau} < \frac{\tilde{C}}{\tau}$, there are two cutoff values, $G2$ and $G3$, with $G3 \geqslant G2$, such that VC finance (case III) is preferred for $G \leqslant G2$, non-VC finance is preferred for $G2 < G < G3$, and VC finance (case I or II) is preferred for $G \geqslant G3$. The threshold $G3$ decreases with μ.*

Proof. See Appendix. ■

Thus when the cost of ex ante knowledge transfer is sufficiently high, the entrepreneur will prefer non-VC finance as long as the benefit from ex post knowledge transfer is below a threshold value, and otherwise he will prefer VC finance. When the cost of ex ante knowledge transfer is lower, on the other hand, two possibilities emerge: either the entrepreneur always chooses VC finance, or there will be a non-monotonic relationship between financing choice and G, in the sense that non-VC finance is preferred for intermediate values of G, while VC finance is preferred for higher or lower values of G.

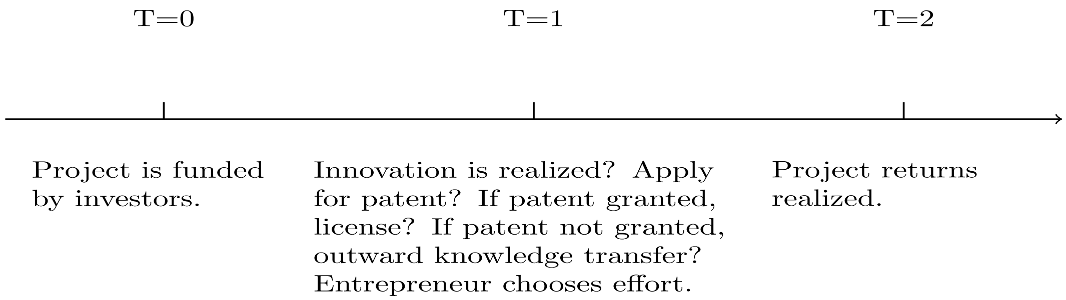
4 Patentable innovations and the decision to seek patent protection

In this section, we extend the analysis to patentable innovations. We incorporate a crucial feature of the way patent systems work in practice: typically there is some uncertainty as to whether a patent application will be successful, even for commercially valuable innovations. Moreover, the patent application itself often reveals information that may be beneficial to competitors. We model this by assuming that, following the development of a valuable innovation, the entrepreneur can apply for a patent: this application will be approved with probability $\beta < 1$. The parameter $\beta > 0$ captures the efficiency of the patent system, industry characteristics, and the characteristics of the innovation. We also assume that, if the patent application is rejected, the leakage of information from the patenting application increases the probability of subsequent expropriation by competitors from μ to α, with $1 > \alpha > \mu$. This assumption is motivated by empirical evidence from the 2008 Berkeley Patent Survey: Graham, Merges, Samuelson and Sichelman (2010) analyze the responses from 1332 early stage companies founded since 1998 and find that 35% cite "Did not want to disclose information" as a reason for not seeking patent protection for their innovations[21]. If a patent is granted, there are two possibilities. Either the patent is used to exclude competitors: in this case the entrepreneur's project succeeds with probability e. Alternatively, the intellectual property can be licensed: this yields revenue $L \geqslant G$ for the firm, while the project succeeds with reduced probability ke. This captures the idea that private knowledge transfer by the VC may yield a lower benefit than licensing, as it cannot be done through an explicit legal contract.

Our main interest in what follows is to explore the decision to seek patent protection, and how it differs depending on whether the entrepreneur raises external finance from a VC or a non-VC investor. For simplicity, this part of the analysis abstracts from ex-ante (inward) knowledge

[21] The survey highlights substantial differences across industries, with the proportion of respondents citing information disclosure as a reason not to seek patent protection varying from 59% in biotechnology to 25% in software.

Figure 2: Timeline for the Patentable Innovation Model



transfer, and focuses on ex-post (outward) knowledge transfer by the VC, which is the one directly affected (ruled out) when the innovation is protected by a patent. We bring back inward knowledge transfer later, when we examine the use of contingencies in venture capital contracts. The timing of the model is illustrated in Figure 2.

4.1 Non-VC investor

We begin by studying optimal contracts between the entrepreneur and a non-VC investor. We examine each of the two cases of interest: first, the case where the patent is used to exclude competitors and no entry occurs. Second, the case where the entrepreneur licenses the patented innovation.

4.1.1 Non-VC investor: patent used to exclude competitors

When the firm chooses to use the patent to exclude competitors, the effort level exerted by the entrepreneur following patent approval is given by $e^P = argmax_e \;\; eR_e^P - \frac{1}{2}e^2$, and in case of patent rejection it is $e^R = argmax_e \;\; ezR_e^R - \frac{1}{2}e^2$, where $z \equiv 1 - \alpha + \alpha k$. The first order conditions give us: $e^P = R_e^P, e^R = zR_e^R$. The optimal contract solves the following maximization problem (P5):

$$\begin{aligned}
\max_{R_e^P, R_e^R} \quad & \pi\{\beta[e^P R_e^P - \frac{1}{2}(e^P)^2] + (1-\beta)[e^R z R_e^R - \frac{1}{2}(e^R)^2]\} \\
s.t. \quad & \pi\{\beta e^P (R - R_e^P) + (1-\beta) e^R z (R - R_e^R)\} \geq I \quad (PC_i) \\
& e^P = R_e^P, e^R = zR_e^R \quad (IC_e)
\end{aligned}$$

The solution to P5 is described by Lemma 5.

Lemma 5 *The optimal contract satisfies $R_e^P = R_e^R = \hat{R}$, and $\hat{R}$ is the largest root of $\pi[\beta + (1-\beta)z^2]\hat{R}(R - \hat{R}) = I$ provided that $I \leq \frac{R^2}{4}\pi[\beta + (1-\beta)z^2]$.*

Proof. See Appendix. ■

4.1.2 Non-VC investor: licensing

When the firm licenses its intellectual property, the probability of project success decreases from e to ke. Therefore, the effort level of the entrepreneur following patent approval is altered: $e^L = argmax_e keR_e^L - \frac{1}{2}e^2 = kR_e^L$. The effort level in case of patent rejection is unchanged, i.e., $e^R = zR_e^R$.

We can see that in general it is optimal to allocate all the license revenue L to the investor, since this relaxes his participation constraint, making it possible to maximize the share of the final project return given to the entrepreneur, which induces higher entrepreneurial effort. The two channels through which the licensing decision affects the entrepreneur's payoff are: on the one hand, licensing reduces the probability of project success, which decreases the expected payoff of the entrepreneur; on the other hand, licensing relaxes the investor's participation constraint, giving a higher share of the final returns to the entrepreneur, which increases his expected return.

The optimization problem of the entrepreneur (P6) is:

$$\begin{aligned} \max_{R_e^L, R_e^R} \quad & \pi\{\beta[ke^L R_e^L - \frac{1}{2}(e^L)^2] + (1-\beta)[e^R z R_e^R - \frac{1}{2}(e^R)^2]\} \\ & = \frac{\pi}{2}[\beta(kR_e^L)^2 + (1-\beta)(zR_e^R)^2] \\ s.t. \quad & \pi\{\beta[k^2 R_e^L(R - R_e^L) + L] + (1-\beta)z^2 R_e^R(R - R_e^R)\} \geq I \end{aligned}$$

The solution to P6 is given by Lemma 6.

Lemma 6 *When $L = \frac{I}{\pi\beta}$, the optimal contract is $R_e^L = R_e^R = R$, and the VC earns the license fee L. When $L < \frac{I}{\pi\beta}$, the problem has the following interior solution: 1. $R_e^L = R_e^R \equiv \hat{\hat{R}}$; 2. $\hat{\hat{R}}$ is the largest root of $\pi(\beta k^2 + (1-\beta)z^2)\hat{\hat{R}}(R - \hat{\hat{R}}) = I - \pi\beta L$; 3. The condition $I \leq \frac{\pi R^2}{4}(\beta k^2 + (1-\beta)z^2) + \pi\beta L$ must be satisfied. When $L > \frac{I}{\pi\beta}$, the investor is willing to provide more initial capital than the required amount I, i.e., $I + Z = L\pi\beta$, where Z denotes the difference between the initial investment I and the investor's initial capital contribution.*

Proof. See Appendix. ■

4.1.3 The patenting decision with non-VC finance

Comparing Lemma 6 with Lemma 5, we see that if the license fee L were reduced to zero, using the patent to exclude competitors would clearly be preferred, since the benefit from licensing disappears, while the project's probability of success is reduced by licensing. As L increases, the entrepreneur's expected utility from the licensing contract increases monotonically, while the expected utility from the patent to exclude competitors contract is unchanged. Thus for L above some threshold value, the entrepreneur's preference switches in favor of the licensing contract. Comparing Lemma 5 with our earlier results for non-VC finance without patents, we also see that there is a clear trade-off between applying for a patent with which to exclude competitors, and not applying for a patent at all. Specifically, it is optimal to apply for a patent to exclude competitors only if the expected benefit from applying for the patent, due to the ability to protect the innovation if the patent is approved, outweighs the expected cost, due to information disclosure (i.e., $\beta + (1-\beta)z^2 > \omega^2$).

The following result describes the entrepreneur's optimal choice between the three possible options with non-VC finance: apply for a patent and, if approved, use it to exclude competitors; apply for a patent and, if approved, license the innovation; do not apply for patent protection.

Proposition 3 *There exist two cutoff values L^N and L^P,*[22] *such that $L^P > L^N > L^*$, where $L^* = \frac{(1-k^2)I}{\pi[\beta+(1-\beta)z^2]}$ is the licensing value such that $\hat{\hat{R}} = \hat{R}$, and:*

1. *When $\beta + (1-\beta)z^2 > \omega^2$, it is optimal to apply for a patent. When $L \leq L^N$, it is also optimal to use the patent to exclude competitors, while when $L \geq L^N$, it is optimal to license.*
2. *When $\beta + (1-\beta)z^2 < \omega^2$, applying for a patent and licensing is preferred if $L > L^P$. Otherwise, if $L < L^P$, it is optimal not to apply for a patent.*

Proof. See Appendix. ■

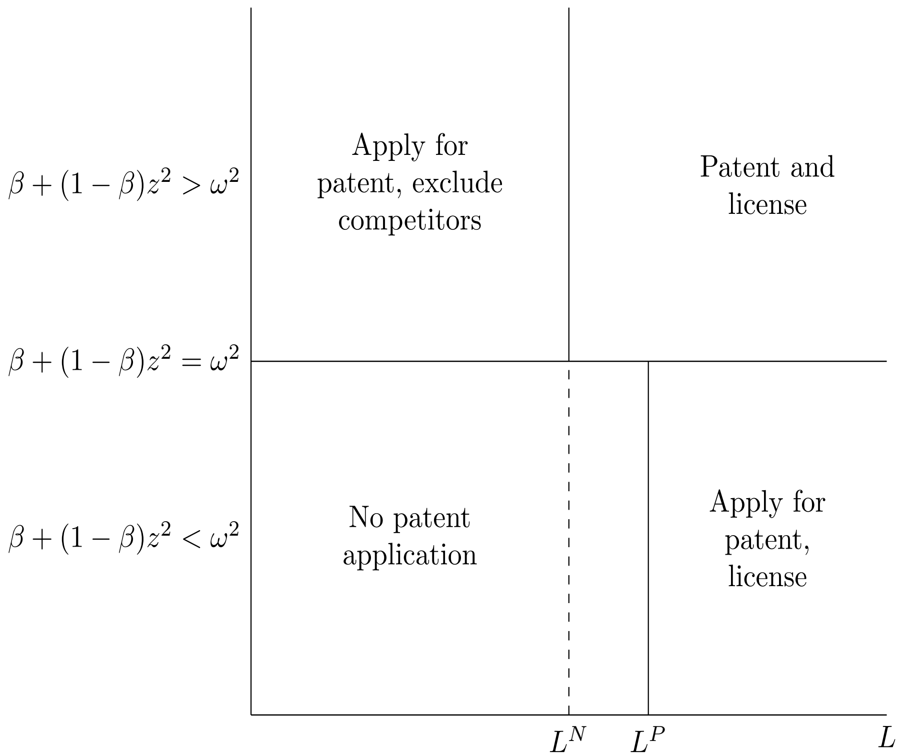
The tradeoffs described by the Proposition are illustrated in Figure 3.

4.2 VC investor

The entrepreneur's choice is somewhat more complicated when he raises external finance from a venture capitalist, and is studied below. There are in principle six possible options: (1) apply for a patent, use it to exclude competitors if the patent is approved; otherwise induce the VC to transfer knowledge; (2) apply for a patent, use it to exclude competitors if the patent is approved; otherwise induce the VC not to transfer knowledge; (3) apply for a patent, license if the patent is approved; otherwise induce the VC to transfer knowledge; (4) apply for a patent, license if the patent is approved; otherwise induce the VC not to transfer knowledge; (5) do not apply for a patent; induce the VC to transfer knowledge; (6) do not apply for a patent; induce the VC not to transfer knowledge. However, the options where the VC does not transfer knowledge yield the same outcome in terms of knowledge transfer as non-VC finance, and a lower expected utility for

[22] The values of L^N and L^P are given in the appendix.

Figure 3: Patent and License Decision



the entrepreneur if the VC incentive constraint (ensuring that he does not transfer knowledge) is binding. Thus non-VC finance is preferred. Without loss of generality, we can therefore focus on the three options that entail knowledge transfer by the VC.

For expositional convenience we assume that $G < \frac{I}{\pi}$, i.e., the expected gain from expropriation would never be sufficient, on its own, to induce the VC to fund the entrepreneur, and similarly $L < \frac{I}{\pi}$,implying that the licensing fee is not enough to recover all the investment cost of the project.[23].

4.2.1 VC investor: Patent used to exclude competitors

When the patent is used to exclude competitors, the entrepreneur's effort level will be $e^P = argmax_e \; eR_e^{VP} - \frac{1}{2}e^2 = R_e^{VP}$ if the patent is granted, and $e^R = argmax_e \; ekR_e^{VR} - \frac{1}{2}e^2 = kR_e^{VR}$ if the patent is rejected, since in the latter case the VC will expropriate.

[23]These assumptions reduce the number of cases to be considered, without affecting the main insights from our results.

The optimal contract is defined by the following problem (P7):

$$
\begin{aligned}
\max_{R_e^{VP},R_e^{VR}} \quad & \pi\{\beta[e^P R_e^{VP} - \frac{1}{2}(e^P)^2] + (1-\beta)[e^R k R_e^{VR} - \frac{1}{2}(e^R)^2]\} \\
& s.t. \;\; e^P = R_e^{VP}, e^R = kR_e^{VR} \;\; (IC_e) \\
\pi\{\beta e^P(R - R_e^{VP}) + (1-\beta)[e^R k(R - R_e^{VR}) + G]\} \geq I \;\; & (PC_{VC}) \\
G + ke^R(R - R_e^{VR}) \geq ze^R(R - R_e^{VR}) \;\; & (IC_{VC})
\end{aligned}
$$

The solution to problem (P7) is given by Lemma 7.

Lemma 7 *Define the threshold values* $C_2 = \frac{k(z-k)}{\beta+(1-\beta)kz}\frac{I}{\pi}$, $C_3 = \frac{z-k}{(1-\beta)z}[\frac{I}{\pi} - \frac{1}{4}\beta R^2]$. *Then:*

- *when* $G \geq C_2$, *the optimal contract specifies* $R_e^{VP} = R_e^{VR} = \hat{\hat{R}}$, *where* $\hat{\hat{R}}$ *is the largest value such that* PC_{VC} *is binding;*
- *when* $C_2 > G \geq C_3$, *the optimal contract specifies* $\hat{R}_e^{VR} > \hat{R}_e^{VP}$, *where* $\hat{R}_e^{VR}$ *is the largest value such that* IC_{VC} *is binding, and given* $\hat{R}_e^{VR}$, $\hat{R}_e^{VP}$ *is the largest value such that* PC_{VC} *is binding;*
- *when* $G < C_3$, *it is not possible to induce the VC to participate and transfer knowledge.*

Proof. See Appendix. ■

Lemma 7 tells us that if the expropriation benefit G is large enough, then it is optimal to give the entrepreneur the same share of final returns if the patent is granted and if the patent is rejected; this share is determined by the binding participation constraint for the VC. As G decreases, the incentive constraint of the VC can no longer be satisfied. Therefore, the share of returns going to the VC when the patent is rejected needs to be reduced, while his share of returns when the patent is approved increases to satisfy the participation constraint as an equality. Finally if G is too low, it is not possible to induce the venture capitalist to participate and transfer knwoledge.

4.2.2 VC investor: licensing

The entrepreneur's effort when a patent is granted and then licensed is given by $e^L = argmax_e \, ekR_e^{VL} - \frac{1}{2}e^2 = kR_e^{VL}$. The optimal contract in this case solves the following problem (P8):

$$\max_{R_e^{VL}, R_e^{VR}} \quad \pi\{\beta[ke^L R_e^{VL} - \frac{1}{2}(e^L)^2] + (1-\beta)[ke^R R_e^{VR} - \frac{1}{2}(e^R)^2]\}$$
$$s.t. \;\; e^L = kR_e^{VL}, \; e^R = kR_e^{VR} \;\; (IC_e)$$
$$\pi\{\beta[L + ke^L(R - R_e^{VL})] + (1-\beta)[G + ke^R(R - R_e^{VR})]\} \geq I \;\; (PC_{VC})$$
$$G \geq (z-k)kR_e^{VR}(R - R_e^{VR}) \;\; (IC_{VC})$$

The solution to (P8) is summarized in Lemma 8.

Lemma 8 *Define the threshold values* $H_2 \equiv \frac{z-k}{\beta k + (1-\beta)z}(\frac{I}{\pi} - \beta L)$, *and* $H_3 \equiv \frac{z-k}{(1-\beta)z}[\frac{I}{\pi} - \beta L - \frac{\beta k^2 R^2}{4}]$. *then:*

when $G \geq H_2$, *it is optimal to specify the same share of returns for the entrepreneur when the patent is granted or rejected, determined as the largest share that satisfies the binding VC participation constraint;*

when $H_2 > G \geq H_3$, *it is optimal to set* $R_e^{VR} > R_e^{VL}$. *Here* R_e^{VR} *is the largest value such that* IC_{VC} *is binding; while* R_e^{VL} *is the value such that* PC_{VC} *is binding given* R_e^{VR}*;*

when $G < H_3$, *it is not possible to induce the VC to participate and expropriate.*

Proof. See Appendix. ■

4.2.3 The patenting decision with VC finance

We first investigate the decision to apply for patent protection under VC finance:

Lemma 9 *Under VC finance, it is always optimal to apply for patent protection.*

Proof. See Appendix. ■

We now examine the entrepreneur's choice between licensing and excluding competitors when a patent is granted. This is described by the following result.

Lemma 10 *When VC finance is obtained and a patent is granted, the choice between licensing and excluding competitors is determined as follows.*

(i) if $H_2 > G \geq C_2$, *the patent is used to exclude competitors;*

(ii) if $C_2 > G \geq H_2$, *the patent is licensed;*

(iii) if $G \geq \max\{C_2, H_2\}$, *there is a cutoff value* $L^{\#}$, *where* $L^{\#} > \hat{L} \equiv \frac{(1-k^2)[I-(1-\beta)\pi G]}{\pi[\beta+(1-\beta)k^2]}$, *such that the patent is licensed when* $L > L^{\#}$ *and used to exclude competitors otherwise.*

Proof. See Appendix. ■

4.3 The decision to seek patent protection

It is clear from our analysis so far that the decision to seek patent protection differs depending on whether the entrepreneur is financed by a venture capitalist or a non-VC investor. In particular, we have shown that:

(i) it is always optimal to apply for patent protection under VC finance;

(ii) it can be optimal not to apply for patent protection under non-VC finance. This will be the case if, and only if, the expected benefit from applying, due to the ability to protect the innovation from expropriation if the patent is approved, is lower than the expected cost, due to information disclosure.

This difference means that, holding the probability of a successful innovation constant (here exogenously equal to π), we should expect to see a greater propensity to patent among VC-funded firms. Interestingly, this is not due to entrepreneurs' fear of being expropriated by their VC investors: the result in our model is driven instead by the reluctance of non-VC-funded firms to apply for patent protection when there is sufficient uncertainty over the outcome of the application, combined with information disclosure that makes expropriation by competitors more likely if the patent application is unsuccessful. This reluctance is not shared by VC-funded firms, since they can rely on the venture capitalist to transfer knowledge profitably when the patent application is rejected, pre-emptying subsequent expropriation by competitors. Moreover, the venture capitalists' expected gains from such transfers are taken into account at the contracting stage, relaxing financing constraints so that entrepreneurs who would otherwise be denied funding can obtain the external finance needed to undertake their projects. This result is consistent with the finding by Mollica and Zingales (2007) that venture capital firms tend to increase both patents *and* the number of new businesses.

4.4 Contingencies in venture capital contracts

Our analysis of patentable innovations so far has shown that in general optimal VC contracts will not condition the share of financial returns going to the VC (entrepreneur) on whether a patent is granted or not. There is one important exception: this occurs when the VC incentive constraint is binding. It is interesting then to extend our analysis to study the implications for the use of contingencies in venture capital contracts. To do this, we investigate the form of optimal VC contracts when innovations are patentable, allowing for both forms of knowledge transfer. In the interest of brevity, we simply summarize here our key findings. Details of all the results and proofs are given in the Appendix. Optimal VC contracts for patentable innovations have the following properties:

- in general, the share of final returns going to the VC (entrepreneur) is not contingent on patent approval;

- for some parameter values, however, the VC ex post incentive constraint will be binding: in this case, contracts will be contingent on patent approval. Specifically,

  1. when the contract induces outward knowledge transfer (with or without inward knowledge transfer), the share of final returns going to the VC (entrepreneur) will be higher (lower) when a patent is granted;
  2. when the contract induces inward knowledge transfer without outward knowledge transfer, the share of final returns going to the VC (entrepreneur) will be lower (higher) when a patent is granted.

However, it can be verified that when the VC ex post incentive constraint is binding, the following holds:

(1) the optimal VC contract with outward knowledge transfer (and no inward transfer) is dominated by non-VC finance;

(2) the optimal VC contract with both outward and inward knowledge transfer is dominated by the optimal VC contract with only inward knowledge transfer.

Thus in equilibrium we can expect to observe two types of VC contract: contracts that are not contingent on patent approval, and (less frequently) contracts that offer a lower (higher) share of final returns to the VC (entrepreneur) when a patent is granted. As discussed in the Introduction, this is consistent with the evidence presented by Kaplan and Stromberg (2003).

4.5 Robustness and extensions

Our results on the decision to apply for patent protection were obtained, for tractability as well as ease of exposition, under the assumption that the VC could only engage in *ex-post* knowledge transfer, or equivalently that the cost C of *ex-ante* knowledge transfer was very high. Allowing for a lower cost C can modify our analysis in two ways. First, if the spillover benefit G is low, VC finance may nevertheless be preferred, with the optimal VC contract designed to induce knowledge transfer ex ante, but not ex post. In this case, the decision to apply for a patent under VC finance is based on the same trade-off as under non-VC finance, namely the trade-off between protection against expropriation by competitors if the patent is granted, and a higher probability of expropriation by competitors if the patent is not granted, because of information disclosure. Second, for higher values of G, VC finance may be preferred with contracts inducing both forms of knowledge transfer. In this case, the patenting decision under VC finance remains the same as above; i.e. it remains optimal to always apply for patent protection following a successful innovation.

An interesting extension of our analysis is to consider the case where $k \geqslant 1$. Transferring knowledge to other firms in this case leaves the entrepreneur's probability of success unaffected, or better still, it increases his chances of success. This case is not without practical interest: for example, there can be circumstances when transmitting private knowledge to other firms helps to generate new complementary products and services and profitable opportunities. Financing and patenting decisions then become very straightforward: the entrepreneur will always prefer VC finance, and under VC finance it will always be optimal to apply for a patent following the development of a successful innovation (as long as $L \geqslant G$).

5 Conclusions

This paper has studied the role of venture capitalists as knowledge intermediaries. We focused exclusively on this role because it has been under-researched until now, and yet the limited empirical evidence available so far suggests it is important. Indeed, we view our model as a first step towards understanding its implications for financing constraints and new business creation, for innovation, and for product market competition, leading to promising empirical research.

There is also much theoretical analysis of venture capitalists to be done in the future, notably to explore the interaction of knowledge transfer with other roles, and the implications for the wider economy.

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6 Appendix

6.1 Proof of Lemma 1

Proof. From PC_{VC} and IC_{VC} *ex ante*, we can see that $\frac{C}{\tau} \geq ke^{VN}(R - R_e^{VN}) + G \geq \frac{I}{\pi}$. It holds only when $\frac{C}{\tau} \geq \frac{I}{\pi}$.

From IC_{VC} *ex post* and PC_{VC}, we have

$$G \geq (\omega - k)e^{VN}(R - R_e^{VN})$$
$$e^{VN}(R - R_e^{VN}) \geq \frac{\frac{I}{\pi} - G}{k}$$

Therefore, we have $G \geq \frac{\omega-k}{k}(\frac{I}{\pi} - G)$. Rearrange the above inequality, we have $G \geq \frac{\omega-k}{\omega}\frac{I}{\pi}$.

If $G \geq \frac{I}{\pi}$, PC_{VC} can be satisfied easily by setting $R_e^{VN} = R$, which maximizes the expected payoffs to the entrepreneur. When the inequality holds strictly, the VC will make an additional payment F ex ante, beyond I, so that the participation constraint holds as an equality; i.e. $F = \pi G - I$ because the VC market is competitive. Moreover, IC_{VC} *ex ante* requires that $\frac{C}{\tau} \geqslant G$.

If $G < \frac{I}{\pi}$, as the participation constraint will be binding in optimum, the optimal contract, R_e^{VN}, is determined by the largest root of the following equation: $\pi[k^2 R_e^{VN}(R - R_e^{VN}) + G] = I$. The condition for the range of I must be satisfied: $\pi[\frac{k^2R^2}{4} + G] \geq I$. ■

6.2 Proof of Lemma 2

Proof. To induce VC to transfer knowledge ex post, it implies that

$$G \geq (\omega - k)kR_e^{VN}(R - R_e^{VN}) \tag{1}$$

(from IC_{VC} *ex post*). By rewriting PC_{VC} and IC_{VC} *ex ante*, we have

$$kR_e^{VN}(R - R_e^{VN}) \geq \frac{1}{k}[\frac{I+C}{\pi+\tau} - G]$$
$$kR_e^{VN}(R - R_e^{VN}) \geq \frac{1}{k}[\frac{C}{\tau} - G]$$

Combining these two inequalities, we have

$$kR_e^{VN}(R - R_e^{VN}) \geq \frac{1}{k}[\max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} - G] \tag{2}$$

Substitute the above inequality into the expression (1), we have

$$G \geq \frac{\omega - k}{\omega} \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}.$$

That's to say, when $G < \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, it is not possible to induce the VC to transfer knowledge ex post.

From expression (2), it's easy to find out that when $G \geq \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, the optimal contract is $R_e^{VN} = R$. In this case, IC_{VC} *ex post* is always satisfied. PC_{VC} and IC_{VC} *ex ante* are also satisfied as inequality (2) holds as well. If PC_{VC} is slack, the VC needs to pay an additional fee ex ante $F = (\pi+\tau)[k^2 R_e^{VN}(R - R_e^{VN}) + G] - I - C = (\pi+\tau)G - I - C$ to the entrepreneur such that PC_{VC} is binding.

When $\max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} > G \geq \frac{\omega-k}{\omega} \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, IC_{VC} *ex post* is always satisfied. Expression (2) must be binding, which implies that either PC_{VC} or IC_{VC} *ex post* will be binding in optimum, depending on the relative size between $\frac{I+C}{\pi+\tau}$ and $\frac{C}{\tau}$ (If$\frac{I+C}{\pi+\tau} > \frac{C}{\tau}$, then PC_{VC} will be binding; and vice versa.). The optimal contract is the largest root of (2) when (2) holds in equality.

If $\frac{I+C}{\pi+\tau} < \frac{C}{\tau}$, such that the participation constraint of VC is slack, then VC would pay an extra fee ex ante, F, to the entrepreneur such that $(\pi+\tau)[ke^{VN}(R - R_e^{VN}) + G] = I + C + F$. VCs always earn zero expected rents as they are competitive.

The participation constraint under which VC will invest in the project could be rewritten as:

$$R_e^{VN}(R - R_e^{VN}) \geq \frac{1}{k^2}\{\frac{I+C}{\pi+\tau} - G\}$$

which entails that the problem has a solution iff

$$(\pi+\tau)[\frac{1}{4}k^2R^2 + G] \geq I + C. \quad \blacksquare$$

■

6.3 Proof of Lemma 3

Proof. Similar to the Proof of Lemma 2, from PC_{VC} and IC_{VC} *ex ante* , we have

$$\omega^2 R_e^{VN}(R - R_e^{VN}) \geq \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} \tag{3}$$

From IC_{VC} *ex post*, we have

$$G \leq (\omega - k)\omega R_e^{VN}(R - R_e^{VN}) \tag{4}$$

If expression (4) holds with inequality, then at optimum, expression (3) must hold with equality, i.e. $\omega^2 R_e^{VN}(R - R_e^{VN}) = \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$. Then substitute it into expression (4), we have $G < \frac{\omega-k}{\omega}\max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$. In this case, either PC_{VC} or IC_{VC} *ex ante* is binding, depending on the relative size between $\frac{C}{\tau}$ and $\frac{I+C}{\pi+\tau}$ (If $\frac{C}{\tau} > \frac{I+C}{\pi+\tau}$, IC_{VC} *ex ante* is binding and PC_{VC} is slack; vice versa.) When the VC participation constraint is slack, then VC would pay an extra fee ex ante, F, to the entrepreneur such that $(\pi+\tau)\omega e^{VN}(R - R_e^{VN}) = I + C + F$. VCs always earn zero expected rents as they are competitive. The optimal contract is the largest root of the following equation: $\omega^2 R_e^{VN}(R - R_e^{VN}) = \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$. The problem has a solution only when $\frac{1}{4}\omega^2 R^2 \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ as $R_e^{VN}(R - R_e^{VN}) \leq \frac{1}{4}R^2$.

If expression (4) holds with equality, subsititute it into expression (3), it implies that $G \geq \frac{\omega-k}{\omega}\max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$. The optimal contract is the largest root of the following equation:

$$G = (\omega - k)\omega R_e^{VN}(R - R_e^{VN}).$$

The problem has a solution only when $G \leq \frac{1}{4}(\omega - k)\omega R^2$. In this case, VC_{PC} may also be slack and therefore VC would pay an extra fee ex ante to the entrepreneur, $F = (\pi+\tau)\frac{\omega}{\omega-k}G - I - C$ such that participation constraint is binding. ■

6.4 Proof of Lemma 4

Proof. Condition PC_{VC} and IC_{VC} *ex ante* implies that $\frac{C}{\tau} > \omega e^{NN}(R - R_e^{NN}) \geq \frac{I}{\pi}$. It holds only when $\frac{C}{\tau} > \frac{I}{\pi}$. Moreover, from IC_{VC} *ex post*, we must have $G \leqslant \frac{\omega-k}{\omega}\frac{C}{\tau}$.

If IC_{VC} *ex post* holds with inequality, it implies that $G < (\omega - k)\omega R_e^{NN}(R - R_e^{NN})$. Then at optimum, R_e^{NN} should be as large as possible, which implies that $\omega e^{NN}(R - R_e^{NN})$ should be as small as possible. Therefore, PC_{VC} is binding while IC_{VC} *ex ante* is slack at optimum. Substitute $\omega e^{NN}(R - R_e^{NN}) = \frac{I}{\pi}$ into IC_{VC} *ex post*, we have $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$. In short, we can say that when $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$, the optimal contract exists, which is the largest root of $\pi\omega^2 R_e^{NN}(R - R_e^{NN}) = I$. The condition for the range of I must be satisfied: $\frac{\omega^2 R^2}{4} \geq \frac{I}{\pi}$.

If IC_{VC} *ex post* holds with equality, it implies that $G = (\omega - k)\omega R_e^{NN}(R - R_e^{NN})$. Substitute it into PC_{VC}, we have $G \geq \frac{\omega-k}{\omega}\frac{I}{\pi}$. In this case, the optimal contract is the largest value such that IC_{VC} *ex post* is binding. The condition for the range of G must be satisfied: $G \leq \frac{1}{4}(\omega - k)\omega R^2$. In this case, if PC_{VC} is slack, then VC would pay an ex ante fee $F = \frac{\pi\omega G}{\omega-k} - I$ to the entrepreneur such that his expected rent is zero, similar to the above situations. ■

6.5 Proof of Lemma 5

Proof. By plugging in IC_e into the objective function and PC_i, the optimization problem for the entrepreneur when facing non-VC investor and patent application without license can be rewritten as:

$$\max_{R_e^P, R_e^R} \quad \pi[\frac{\beta}{2}(R_e^P)^2 + \frac{1-\beta}{2}z^2(R_e^R)^2]$$
$$s.t. \quad \pi[\beta R_e^P(R - R_e^P) + (1-\beta)z^2 R_e^R(R - R_e^R)] \geq I \quad (PC_i)$$

The Lagrangian function could expressed in this form:

$$L = \pi[\frac{\beta}{2}(R_e^P)^2 + \frac{1-\beta}{2}z^2(R_e^R)^2] + \lambda\{\pi[\beta R_e^P(R - R_e^P) + (1-\beta)z^2 R_e^R(R - R_e^R)] - I\}$$

The first order conditions are:

$$\frac{\partial L}{\partial R_e^P} = \pi\beta R_e^P + \lambda\pi\beta(R - 2R_e^P) = 0 \tag{5}$$

$$\frac{\partial L}{\partial R_e^R} = \pi(1-\beta)z^2 R_e^R + \lambda\pi(1-\beta)z^2(R - 2R_e^R) = 0 \tag{6}$$

$$\frac{\partial L}{\partial \lambda} = \pi[\beta R_e^P(R - R_e^P) + (1-\beta)z^2 R_e^R(R - R_e^R)] - I = 0 \tag{7}$$

Equation (7) is simply the participation constraint PC_i. Divide equation (7) by (6), we have

$$\frac{\beta R_e^P}{(1-\beta)z^2 R_e^R} = \frac{\beta(R - R_e^P)}{(1-\beta)z^2(R - R_e^R)} \tag{8}$$

Equation (8) finally gives us

$$R_e^P = R_e^R = \hat{R} \tag{9}$$

Combine (9) and (7), the participation constraint of non-VC investor can be simplified as

$$\pi[\beta + (1-\beta)z^2]\hat{R}(R - \hat{R})] = I \tag{10}$$

The largest root of equation (10) is the optimal payment to the entrepreneur when facing non-VC and patent protection without license.

As we all know that $\hat{R} \in [0, R]$, then $\hat{R}(R - \hat{R}) \leq \frac{R^2}{4}$. And from (10), we have $I \leq \frac{R^2}{4}\pi[\beta + (1-\beta)z^2]$. ■

6.6 Proof of Lemma 6

Proof. The Lagrangien function for the problem (P2) can be written as

$$L = \beta k^2 (R_e^L)^2 + (1-\beta)(zR_e^R)^2 + \lambda\{\frac{I}{\pi} - \beta[L + k^2 R_e^L (R - R_e^L)] - (1-\beta) z^2 R_e^R (R - R_e^R)\}$$

The first order conditions give us:

$$\frac{\partial L}{\partial R_e^L} = 2\beta k^2 R_e^L - \lambda \beta k^2 (R - 2R_e^L) = 0 \tag{11}$$

$$\frac{\partial L}{\partial R_e^R} = 2(1-\beta) z^2 R_e^R - \lambda (1-\beta) z^2 (R - 2R_e^R) = 0 \tag{12}$$

$$\frac{\partial L}{\partial \lambda} = \frac{I}{\pi} - \beta L - \beta k^2 R_e^L (R - R_e^L) - (1-\beta) z^2 R_e^R (R - R_e^R) \leq 0 \tag{13}$$

1. When $\pi\beta L \geq I \Longleftrightarrow L \geq \frac{I}{\pi\beta}$;
   Then it's possible to set $R_e^L = R_e^R = R$ and still satisfy the investor's participation constraint; If the condition holds as a strictly inequality, the investor can provide additional capital ex ante above I, i.e., $L = \frac{I+Z}{\pi\beta}$, where Z denotes the difference between the initial investment and the willingness to fund of VC as VC market is competitive. Therefore, investor's PC will always be binding and the initial investment becomes $I + Z$.

2. When $\pi\beta L < I \Longleftrightarrow L < \frac{I}{\pi\beta}$;
   The investor's participation constraint is binding. Interior solutions for R_e^L and R_e^R satisfy:

   $$\frac{2\beta k^2 R_e^L}{2(1-\beta) z^2 R_e^R} = \frac{\lambda \beta k^2 (R - 2R_e^L)}{\lambda (1-\beta) z^2 (R - 2R_e^R)} \Longrightarrow R_e^L = R_e^R$$

   Let $R_e^L = R_e^R \equiv \hat{\hat{R}}$, the problem becomes

$$\max \quad \hat{\hat{R}}$$
$$s.t.\ \pi(\beta k^2 + (1-\beta) z^2) \hat{\hat{R}} (R - \hat{\hat{R}}) = I - \pi\beta L$$

So it has a solution iff

$$\frac{\pi R^2}{4} (\beta k^2 + (1-\beta) z^2) \geq I - \pi\beta L$$

If this condition holds, the optimal contract is $R_e^L = R_e^R = \hat{\hat{R}} \geq \frac{1}{2} R$, where $\hat{\hat{R}}$ is the largest root of $(\beta k^2+(1-\beta)z^2)\hat{\hat{R}}(R-\hat{\hat{R}})=\frac{I}{\pi}-\beta L$. ■

6.7 Proof of Lemma 7

Proof. Suppose at optimum, IC_{VC} is always satisfied. Since PC_{VC} must be binding, the similar routine of Lagrangian function as in Proof of Lemma 5 gives us, at optimum, $R_e^{VP}=R_e^{VR}\equiv\hat{\hat{R}}$. Then $\hat{\hat{R}}(R-\hat{\hat{R}})=\frac{\frac{I}{\pi}-(1-\beta)G}{\beta+(1-\beta)k^2}$. Plug it into IC_{VC}, we have $[\beta+(1-\beta)k^2]G\geq(z-k)k[\frac{I}{\pi}-(1-\beta)G]$ $\Longrightarrow[\beta+(1-\beta)k^2+(1-\beta)(z-k)k]G\geq(z-k)k\frac{I}{\pi}$.

Therefore, we could discuss optimal contract by the following cases:

1. If $\pi(1-\beta)G\geq I$, it's possible to set $R_e^{VP}=R_e^{VR}=R$ and it satisfies PC_{VC} and IC_{VC}. However, due to our assumption that $G<\frac{I}{\pi}$, we will ignore this case in our analysis.

2. If $\pi(1-\beta)G<I$, and $G[\beta+(1-\beta)zk]\geq(z-k)k\frac{I}{\pi}$ ($\Longrightarrow G\geq\frac{k(z-k)}{\beta+(1-\beta)zk}\frac{I}{\pi}$) The optimal contract is $R_e^{VP}=R_e^{VR}=\hat{\hat{R}}$, where $\hat{\hat{R}}$ is largest root of $\pi\{(1-\beta)G+[\beta+(1-\beta)k^2]\hat{\hat{R}}(R-\hat{\hat{R}})\}=I$

3. If $\frac{z-k}{(1-\beta)z}[\frac{I}{\pi}-\frac{1}{4}\beta R^2]\leq G<\frac{k(z-k)}{\beta+(1-\beta)kz}\frac{I}{\pi}$, the PC_{VC} and IC_{VC} are both binding. The optimal contract is $\hat{R}_e^{VP}$ and $\hat{R}_e^{VR}$, where $\hat{R}_e^{VR}$ is the largest root of $G=(z-k)k\hat{R}_e^{VR}(R-\hat{R}_e^{VR})$. And given $\hat{R}_e^{VR}$, $\hat{R}_e^{VP}$ is the largest root of $\pi\{\beta\hat{R}_e^{VP}(R-\hat{R}_e^{VP})+(1-\beta)[G+k^2\hat{R}_e^{VR}(R-\hat{R}_e^{VR})]\}=I$. It's easy to see that $\hat{R}_e^{VR}\geq\hat{R}_e^{VP}$ since when G become smaller than $\frac{k(z-k)}{\beta+(1-\beta)kz}\frac{I}{\pi}$, we must give the entrepreneur higher share of return in case of patent rejection such that the IC_{VC} could be easily satisfied. Therefore, $\hat{R}_e^{VR}\geq\hat{R}_e^{VP}$.

4. If $G<\frac{z-k}{(1-\beta)z}[\frac{I}{\pi}-\frac{1}{4}\beta R^2]$, it's not possible to induce the VC to participate and expropriate. Because if the maximum possible level of R_e^{VP}, $\frac{R}{2}$, together with the maximum feasible level of R_e^{VR} that satisfies the IV_{VC}, are not sufficient to satisfy the PC_{VC}, i.e., if

$$\begin{aligned}\beta\hat{R}_e^{VP}(R-\hat{R}_e^{VP})+(1-\beta)[G+k^2\hat{R}_e^{VR}(R-\hat{R}_e^{VR})]&<\frac{I}{\pi}\\ \Rightarrow\beta\frac{R^2}{4}+(1-\beta)[G+k^2\frac{G}{(z-k)k}&<\frac{I}{\pi}\\ \Rightarrow G&<\frac{z-k}{(1-\beta)z}[\frac{I}{\pi}-\frac{1}{4}\beta R^2]\end{aligned}$$

In this case, the optimal contract is the same as the non-VC case.

■

6.8 Proof of Lemma 8

Proof.

1. Suppose $\pi[\beta L + (1-\beta)G] \geq I$, then we can set $R_e^{VL} = R_e^{VR} = R$, PC_{VC} and IC_{VC} are all satisfied. Note that we assume that $L < \frac{I}{\pi}$, $G < \frac{I}{\pi}$, therefore, this case is ruled out.

2. If $\pi[\beta L + (1-\beta)G] < I$, then PC_{VC} will be binding. Suppose first the IC_{VC} is slack, the problem gives:

$$L = \frac{2}{\pi}k^2[\beta(R_e^{VL})^2 + (1-\beta)(R_e^{VR})^2]$$
$$+\lambda\{\frac{I}{\pi} - \beta[L + k^2 R_e^{VL}(R - R_e^{VL})] - (1-\beta)[G + k^2 R_e^{VR}(R - R_e^{VR})]\}$$

For interior solution, we have

$$\frac{\partial L}{\partial R_e^{VL}} = \pi k^2 \beta R_e^{VL} - \lambda\beta k^2(R - 2R_e^{VL}) = 0$$
$$\frac{\partial L}{\partial R_e^{VR}} = \pi k^2(1-\beta)R_e^{VR} - \lambda(1-\beta)k^2(R - 2R_e^{VR}) = 0$$
$$\Longrightarrow \frac{\beta R_e^{VL}}{(1-\beta)R_e^{VR}} = \frac{\beta(R - 2R_e^{VL})}{(1-\beta)(R - 2R_e^{VR})}$$
$$\Longrightarrow R_e^{VL} = R_e^{VR} = \hat{\hat{R}}^V$$

the problem becomes

$$\max \quad \hat{\hat{R}}^V$$
$$s.t. \quad \beta L + (1-\beta)G + k^2\hat{\hat{R}}^V(R - \hat{\hat{R}}^V) \geq \frac{I}{\pi}$$
$$\Longrightarrow k^2\hat{\hat{R}}^V(R - \hat{\hat{R}}^V) \geq \frac{I}{\pi} - \beta L - (1-\beta)G$$

So it has a solution iff $\frac{k^2}{4}R^2 \geq \frac{I}{\pi} - \beta L - (1-\beta)G$. If this condition holds, and IC_{VC} is satisfied, the optimal contract is $R_e^{VL} = R_e^{VR} = \hat{\hat{R}}^V$, where $\hat{\hat{R}}^V$ is the largest root of $k^2\hat{\hat{R}}^V(R - \hat{\hat{R}}^V) = \frac{I}{\pi} - \beta L - (1-\beta)G$, plug it into IC_{VC}, we have $G \geq \frac{z-k}{k}[\frac{I}{\pi} - \beta L - (1-\beta)G]$, i.e., $G[1 + \frac{z-k}{k}(1-\beta)] \geq \frac{z-k}{k}[\frac{I}{\pi} - \beta L]$, $\Longrightarrow G \geq \frac{z-k}{\beta k + (1-\beta)z}[\frac{I}{\pi} - \beta L]$.

3. If $G < \frac{z-k}{\beta k + (1-\beta)z}[\frac{I}{\pi} - \beta L]$, IC_{VC} and PC_{VC} will be binding. The optimal contract will be $\hat{\hat{R}}_e^{VR}, \hat{\hat{R}}_e^{VL}$, s.t. $\hat{\hat{R}}_e^{VR}$ is the largest root of $G = (z-k)kR_e^{VR}(R - R_e^{VR})$, $\hat{\hat{R}}_e^{VL}$ is the largest root of $\beta L + (1-\beta)G + k^2[\beta R_e^{VL}(R - R_e^{VL}) + (1-\beta)\hat{\hat{R}}_e^{VR}(R - \hat{\hat{R}}_e^{VR})] = \frac{I}{\pi}$ given $\hat{\hat{R}}_e^{VR}$. It's easy to see that $\hat{\hat{R}}_e^{VR} \geq \hat{\hat{R}}_e^{VL}$ since when G become smaller than $\frac{z-k}{\beta k + (1-\beta)z}[\frac{I}{\pi} - \beta L]$, we must

give the entrepreneur higher share of return in case of patent rejection such that the IC_{VC} could be easily satisfied. Therefore, $\hat{\hat{R}}_e^{VR} \geq \hat{\hat{R}}_e^{VL}$.

4. Finally, it's not possible to induce the VC to participate and expropriate if the maximum possible level of R_e^{VL}, $\frac{1}{2}R$, together with the maximum feasible level of R_e^{VR} that satisfies the IC_{VC}, are not together sufficient to satisfy the PC_{VC}, i.e., if

$$\beta L + (1-\beta)G + k^2[\frac{\beta R^2}{4} + (1-\beta)\frac{G}{k(z-k)}] < \frac{I}{\pi}$$
$$\Longrightarrow (1-\beta)G[1 + \frac{k^2}{k(z-k)}] < \frac{I}{\pi} - \beta L - \frac{\beta k^2 R^2}{4}$$
$$\Longrightarrow G < \frac{z-k}{(1-\beta)z}[\frac{I}{\pi} - \beta L - \frac{\beta k^2 R^2}{4}].$$

■

6.9 Proof of Lemma 9

The proof is straightforward. We are interested in the case where VC finance is chosen. If no patent application is made, the VC is induced to transfer knowledge ex post. If a patent application is made, it is always possible to do at least as well by licensing when the patent is granted (since $L \geqslant G$) and by inducing the VC to transfer knowledge when the patent is not granted.

6.10 Proof of Lemma 10

Proof. We focus on the case where VC finance is obtained; i.e. it is preferred to non-VC finance. This implies $G \geq C_2$ and/or $G \geq H_2$. To see this, note that when the patent is used to exclude competitors, non-VC finance is preferred for $G < C_2$: specifically, for $G < C_3$ it is not possible to induce the VC to participate and transfer knowledge (hence, there is no difference between VC and non-VC finance), while for $C_2 > G \geq C_3$, non-VC finance is preferred.[24]

Similarly, when the patent is licensed, non-VC finance is preferred for $G < H_2$: specifically, for $G < H_3$ it is not possible to induce the VC to participate and transfer knowledge (hence, there is no difference between VC and non-VC finance), while for $H_2 > G \geq H_3$ the VC incentive constraint is binding, implying that non-VC finance is preferred. Clearly then if $G \geq C_2$ and

[24] Consider problem P7, $C_2 > G \geq C_3$. Let the solution be S, V, where S is given by $G = (z-k)kS(R-S)$, and then V is given by $\beta V(R-V) + (1-\beta)[k^2 S(R-S) + G] = \frac{I}{\pi}$. These two conditions imply $\beta V(R-V) + (1-\beta)zkS(R-S) = \frac{I}{\pi}$. The participation constraint for non-VC finance can be written as $\beta V(R-V) + (1-\beta)z^2 S(R-S) \geqslant \frac{I}{\pi}$, implying that for the same values of S and V (the ones that solve problem P7) the constraint is slack, since $z^2 > zk$. Moreover, the expected utility for the VC contract is $U^{VC} = \pi\{\beta\frac{V^2}{2} + (1-\beta)\frac{k^2 S^2}{2}\}$, while for the non-VC contract it is $U^{NVC} = \pi\{\beta\frac{V^2}{2} + (1-\beta)\frac{z^2 S^2}{2}\}$. Since $z^2 > k^2$, we have $U^{NVC} > U^{VC}$.

$G < H_2$ there will be no licensing under VC finance; similarly, if $G < C_2$ and $G \geq H_2$ the patent will not be used to exclude competitors under VC finance.

When $G \geq \max\{C_2, H_2\}$, the optimal contracts are the largest root of the following equations,

$$\begin{aligned} for\ patent\ and\ no\ license &: \quad [\beta + (1-\beta)k^2]\hat{\hat{R}}(R - \hat{\hat{R}}) = \frac{I}{\pi} - (1-\beta)G \\ for\ patent\ and\ license &: \quad k^2 \hat{\hat{R}}^V (R - \hat{\hat{R}}^V) = \frac{I}{\pi} - \beta L - (1-\beta)G \end{aligned}$$

Therefore, when $\frac{\frac{I}{\pi}-(1-\beta)G}{\beta+(1\beta)k^2} = \frac{\frac{I}{\pi}-\beta L-(1-\beta)G}{k^2}$, the optimal contracts with and without licensing provide the same share of final returns to the entrepreneur, that is, $L = \frac{(1-k^2)[I-(1-\beta)\pi G]}{\pi[\beta+(1-\beta)k^2]} \equiv \hat{L}$.

The condition $G \geq \max\{C_2, H_2\}$ implies that

$$G \geq \frac{k(z-k)}{\beta + (1-\beta)kz}\frac{I}{\pi} \tag{14}$$

$$G \geq \frac{z-k}{\beta k + (1-\beta)z}\left(\frac{I}{\pi} - \beta L\right), \tag{15}$$

where $L = \hat{L}$. Inequality (14) implies that

$$G \geq \frac{z-k}{\beta k + (1-\beta)z}\frac{I}{\pi} - \frac{(z-k)\beta}{\beta k + (1-\beta)z}\frac{(1-k^2)[\frac{I}{\pi} - (1-\beta)G]}{\beta + (1-\beta)k^2},$$

which gives us

$$G \geq \frac{(z-k)k^2}{[\beta k + (1-\beta)z][\beta + (1-\beta)k^2] - (z-k)\beta(1-k^2)(1-\beta)}\frac{I}{\pi} \tag{16}$$

Therefore, as long as

$$\begin{aligned} G &\geq \max\{C_2, H_2(L = \hat{L})\} \\ &= \max\left\{\frac{k(z-k)}{\beta + (1-\beta)kz}\frac{I}{\pi}, \frac{(z-k)k^2}{[\beta k + (1-\beta)z][\beta + (1-\beta)k^2] - (z-k)\beta(1-k^2)(1-\beta)}\frac{I}{\pi}\right\}, \end{aligned}$$

i.e., $G \in \Phi$, where $\Phi = [\max\{C_2, H_2(L = \hat{L})\}, +\infty)$ then when $L = \hat{L}$, the optimal contracts with and without licensing provide the same share of final returns to the entrepreneur, while the licensing contract implies a lower success probability, thus the contract without licensing is preferred.

Then we have

$$U_P(G \geq \max\{C_2, H_2(L = \hat{L})\}) > U_{PL}(G \geq \max\{C_2, H_2(L = \hat{L})\})$$

Cases	Contract Equation	Condition for I, G
I. Outward knowledge transfer	$k^2 R_e^{VN}(R - R_e^{VN}) + G = \frac{I}{\pi}$ (e) $R_e^{VN} = R$ (r)	$\frac{I}{\pi} > G \geq \frac{\omega - k}{\omega}\frac{I}{\pi}$ (o) $G \geq \frac{I}{\pi}$ (m)
II. Inward and outward knowledge transfer	$k^2 R_e^{VN}(R - R_e^{VN}) + G = max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ (a) $R_e^{VN} = R$ (b)	$max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} > G \geq \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ $G \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$
III. Inward knowledge transfer	$\omega^2 R_e^{VN}(R - R_e^{VN}) = max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ (c) $(\omega - k)\omega R_e^{VN}(R - R_e^{VN}) = G$ (d)	$G < \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ $G \geq \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$
IV. No knowledge transfer	$\omega^2 R_e^{NN}(R - R_e^{NN}) = \frac{I}{\pi}$ (f) $(\omega - k)\omega R_e^{NN}(R - R_e^{NN}) = G$ (g)	$G < \frac{\omega-k}{\omega}\frac{I}{\pi}$ (p) $G \geq \frac{\omega-k}{\omega}\frac{I}{\pi}$ (q)

Cases	Condition for I, G	Utility	F
I.Outward knowledge transfer	$\pi[\frac{1}{4}k^2R^2 + G] \geq I \& \ \frac{I}{\pi} \leq \frac{C}{\tau}$ (k) $\frac{I}{\pi} \leq \frac{C}{\tau}$ (n)	$U_1 = \frac{\pi}{2}k^2(R_e^{VN})^2$ $U_1 = \frac{\pi}{2}k^2(R_e^{VN})^2 + F$	 $F = \pi G - I$
II. Inward and outward	$(\pi + \tau)[\frac{1}{4}k^2R^2 + G] \geq I + C$ (i)	$U_2 = \frac{(\pi+\tau)}{2}k^2(R_e^{VN})^2$ $+F \cdot 1\{\frac{C}{\tau} > \frac{I}{\pi}\}$	$F = (\pi + \tau)[k^2 R_e^{VN}(R - R_e^{VN})$ $+G] - I - C$
knowledge transfer		$U_2 = \frac{(\pi+\tau)}{2}k^2(R_e^{VN})^2 + F$	$F = (\pi + \tau)G - I - C$
III. Inward	$\frac{1}{4}\omega^2R^2 \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ (j)	$U_3 = \frac{(\pi+\tau)}{2}\omega^2(R_e^{VN})^2$ $+F \cdot 1\{\frac{C}{\tau} > \frac{I}{\pi}\}$	$F = (\pi + \tau)\omega^2 R_e^{VN}(R - R_e^{VN})$ $-I - C$
knowledge transfer	$G \leq \frac{1}{4}(\omega - k)\omega R^2$	$U_3 = \frac{(\pi+\tau)}{2}\omega^2(R_e^{VN})^2 + F$	$F = (\pi + \tau)\frac{\omega}{\omega-k}G - I - C$
IV. No knowledge transfer	$\frac{\pi\omega^2R^2}{4} \geq I \ \& \ \frac{I}{\pi} \leq \frac{C}{\tau}$ (l) $G \leq \frac{1}{4}(\omega - k)\omega R^2 \ \& \ \frac{I}{\pi} \leq \frac{C}{\tau}$ (s)	$U_4 = \frac{\pi\omega^2}{2}(R_e^{NN})^2$ $U_4 = \frac{\pi\omega^2}{2}(R_e^{NN})^2 + F$	 $F = \frac{\pi\omega G}{\omega-k} - I$

However, $\frac{\partial U_{PL}}{\partial L} > 0$, $\frac{\partial U_P}{\partial L} = 0$. Therefore, there exists a cutoff value $L^{\#}$, where $L^{\#} > \hat{L}$, such that when $L > L^{\#}$, $U_{PL} > U_P$. ■

6.11 Proof of Proposition 1

Proof. The optimal contracts for VC-finance with non-patentable knowledge in different cases are listed in the above table. For simplification, in what follows, we will refer to "Outward knowledge transfer", "Inward and outward knowledge transfer", "Inward knowledge transfer", and "No knowledge transfer" as Case I, Case II, Case III, and Case IV, respectively. We will denote by F any ex- ante fee paid by the VC to the entrepreneur as part of the contract.

Then:

1. When $\frac{C}{\tau} < \frac{I}{\pi}$ $(i.e., \frac{c}{\tau} < \frac{I+C}{\pi+\tau})$, $\Rightarrow max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} = \frac{I+C}{\pi+\tau}$, Cases I & IV will not happen. We therefore need to consider the choice between Case II and Case III.

   At $G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, we have to compare Case II(a) and Case III(c). From II(a), we have $R_e^{VN}(R - R_e^{VN}) = \frac{I+C}{\omega k(\pi+\tau)}$. From III(c), we have $R_e^{VN}(R - R_e^{VN}) = \frac{I+C}{\omega^2(\pi+\tau)}$. Therefore,

R_e^{VN} would be higher in case III(c). Hence $U_3 > U_2$ as $\omega > k$. Therefore, at point $G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, case III is preferred to case II.

As G increases, the expected payoff from case III decreases, since the VC ex post incentive constraint becomes binding, distorting the optimal contract. The expected payoff from case II is increasing in G.

Therefore, there exists a cutoff value $G^* > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, such that when $G > G^*$, the optimal choice switches from case III to case II.

2. When $\frac{C}{\tau} > \frac{I}{\pi}$ $(i.e., \frac{C}{\tau} > \frac{I+C}{\pi+\tau})$, $\Rightarrow max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} = \frac{C}{\tau}$, we have the following pattern:

   for $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$, cases I and II are not relevant. The choice is between cases III and IV.

   for $\frac{\omega-k}{\omega}\frac{I}{\pi} \leqslant G < \frac{\omega-k}{\omega}\frac{C}{\tau}$, case II is not relevant. The choice is between cases I, III and IV.

   for $G \geqslant \frac{\omega-k}{\omega}\frac{C}{\tau}$, case IV is not relevant. The choice is between cases I, II and III.

   The following comparisons will help us examine these choices.

   - We start by comparing cases II and III. At point $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$, we have $R_e(R-R_e) = \frac{C}{\tau\omega k}$ for case II and $R_e(R-R_e) = \frac{C}{\tau\omega^2}$ for case III. Therefore, R_e is higher in case III. And $F_2 = (\pi+\tau)\frac{C}{\tau} - I - C = F_3$. Hence, $U_3 > U_2$ at $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$. As argued in part 1, as G increases the expected payoff from case III decreases while the expected payoff from case II increases. Therefore, there must exist a cutoff value for G, say, $G^+ > \frac{\omega-k}{\omega}\frac{C}{\tau}$, such that when $G > G^+$, case II dominates case III, and vice versa. The threshold G^+ decreases wtih μ.

   - We now compare cases I and IV. At point $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$, $R_e(R-R_e) = \frac{I}{\pi\omega k}$ for case I and $R_e(R-R_e) = \frac{I}{\pi\omega^2}$ for case IV. Therefore, R_e is higher in case IV. Hence, $U_4 > U_1$ at $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$. For $G > \frac{\omega-k}{\omega}\frac{I}{\pi}$, as G increases, the expected payoff from case I increases. The expected payoff from case IV decreases because the VC ex post incentive constraint becomes binding.

     Therefore, there must exist a cutoff value for G, say, $G^- > \frac{\omega-k}{\omega}\frac{I}{\pi}$, such that when $G > G^-$, case I dominates case IV, and vice versa.

   - Now compare cases III and IV. At point $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$, Case III implies that $\omega^2 R_e^{VN}(R - R_e^{VN}) = \frac{C}{\tau}$, $U_3 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 + F_3$, $F_3 = \frac{\omega}{\omega-k}(\pi+\tau)G - I - C$. Case IV implies that $\omega(\omega-k)R_e^{NN}(R - R_e^{NN}) = G$, $U_4 = \frac{\pi}{2}\omega^2(R_e^{NN})^2 + F_4$, $F_4 = \frac{\omega}{\omega-k}\pi G - I$. As $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$, $R_e^{NN} = R_e^{VN}$ and $F_3 = F_4$. Therefore, we have $\triangle U_{34} = U_3 - U_4 = \frac{\pi+\tau}{2}\omega^2(R_e)^2 - \frac{\pi}{2}\omega^2(R_e)^2 > 0$. Thus at point $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$, case III dominates case IV.

   - Continuing the comparison of cases III and IV, consider the point $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$. Case III implies that $\omega^2 R_e^{VN}(R - R_e^{VN}) = \frac{C}{\tau}$, $F_3 = (\pi+\tau)\frac{C}{\tau} - I - C$, $U_3 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 + F_3$.

Case IV implies that $\omega^2 R_e^{NN}(R - R_e^{NN}) = \frac{I}{\pi},\ F_4 = 0,\ U_4 = \frac{\pi}{2}\omega^2(R_e^{NN})^2$. Therefore, we have

$$\triangle U_{34} = U_3 - U_4 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^{NN})^2 + F_3$$

For $\frac{C}{\tau}$ just above $\frac{I}{\pi}$, suppose $\frac{C}{\tau} = \frac{I}{\pi} + \epsilon$, where $\epsilon > 0$ can be as small as we want. Then, $R_e^{VN} \sim R_e^{NN}$, and

$$\begin{aligned}\triangle U_{34} &= \frac{\tau}{2}\omega^2(R_e^{VN})^2 + (\pi+\tau)\frac{C}{\tau} - I - C \\ &= \frac{\tau}{2}\omega^2(R_e^{VN})^2 + \pi\frac{C}{\tau} - I \\ &= \frac{\tau}{2}\omega^2(R_e^{VN})^2 + \pi\epsilon > 0\end{aligned}$$

Therefore, for $\frac{C}{\tau}$ just above $\frac{I}{\pi}$, case III is preferred to case IV. In case III, increasing $\frac{C}{\tau}$ decreases the expected payoff from the contract because the IC_{VC} ex ante is binding. Therefore, as $\frac{C}{\tau}$ increases, U_3 decreases while U_4 is constant. Thus there exists a cutoff value $\frac{\tilde{C}}{\tau}$, with $\frac{\omega^2 R^2}{4} \geqslant \frac{\tilde{C}}{\tau} > \frac{I}{\pi}$, such that when $\frac{C}{\tau} > \frac{\tilde{C}}{\tau}$, the choice switches to case IV at $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$. This implies that there exists a cutoff value $\hat{G} < \frac{\omega-k}{\omega}\frac{C}{\tau}$, such that for $G < \hat{G}$ case IV is preferred to case III, and for $G > \hat{G}$ case III is preferred to case IV. The threshold $\hat{G}$ increases with $\frac{C}{\tau}$.

- Now compare cases I and III. At point $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$, Case I implies that $k^2 R_e^{VN}(R - R_e^{VN}) = \frac{I}{\pi} - G = \frac{k}{\omega}\frac{I}{\pi}$, $U_1 = \frac{\pi}{2}k^2(R_e^{VN})^2$. Case III implies that $\omega^2 R_e^{VN}(R - R_e^{VN}) = \frac{C}{\tau}$, $F_3 = (\pi+\tau)\frac{C}{\tau} - I - C,\ U_3 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 + \pi\frac{C}{\tau} - I$.

  Consider the value of $\frac{C}{\tau}$ at which R_e^{VN} is the same in case I and case III. It is straightforward to derive this as $\frac{C}{\tau} = \frac{\omega}{k}\frac{I}{\pi} > \frac{I}{\pi}$. For this value, it is easy to see that case III is preferred to case I. This will also be true for smaller values, since the expected payoff from case I does not change while the expected payoff from case III decreases with $\frac{C}{\tau}$. For the same reason, there exists a cutoff value $\frac{\tilde{C'}}{\tau} > \frac{\omega}{k}\frac{I}{\pi}$, such that when $\frac{C}{\tau} > \frac{\tilde{C'}}{\tau}$, the choice switches from case III to case I.

- Continuing the comparison of cases I and III, consider the point $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$. Suppose that $\frac{\omega-k}{\omega}\frac{C}{\tau} \leq \frac{I}{\pi}$.

  Case I implies that $k^2 R_e^{VN}(R - R_e^{VN}) = \frac{I}{\pi} - G = \frac{I}{\pi} - \frac{\omega-k}{\omega}\frac{C}{\tau}$, $U_1 = \frac{\pi}{2}k^2(R_e^{VN})^2$. Case III implies that $\omega^2 R_e^{VN}(R - R_e^{VN}) = \frac{C}{\tau}$, $F_3 = \pi\frac{C}{\tau} - I,\ U_3 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 + \pi\frac{C}{\tau} - I$. We proceed as above. Consider the value of $\frac{C}{\tau}$ at which R_e^{VN} is the same in case I and case III. It is straightforward to derive this as $\frac{C}{\tau} = \frac{\omega^2}{\omega^2 - k(\omega-k)}\frac{I}{\pi} > \frac{I}{\pi}$. For this value, it is easy to see that case III is preferred to case I. This will also be true for smaller values, since the expected payoff from case I increases with G while the expected payoff from case III decreases with $\frac{C}{\tau}$. For the same reason, there exists a cutoff value $\frac{\tilde{C''}}{\tau} > \frac{\omega}{k}\frac{I}{\pi}$, such that when $\frac{C}{\tau} > \frac{\tilde{C''}}{\tau}$, the choice switches from case III to case I.

- Finally compare cases I and II. At the point $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$, suppose that $\frac{\omega-k}{\omega}\frac{C}{\tau} = \frac{I}{\pi}$. For case I, we have $U_1 = \frac{\pi}{2}k^2R^2$; for case II, we have $k^2R_e(R-R_e) = \frac{C}{\tau} - G = \frac{k}{\omega}\frac{C}{\tau}$, $F_2 = \pi\frac{C}{\tau} - I,\ U_2 = \frac{\pi+\tau}{2}k^2R_e^2 + F_2$. Therefore, we have

$$
\begin{aligned}
\triangle U_{21} &= \frac{\pi+\tau}{2}k^2R_e^2 + \frac{k}{\omega-k}I - \frac{\pi}{2}k^2R^2 \\
&\geq \frac{\pi+\tau}{2}k^2\frac{R^2}{4} + \frac{k}{\omega-k}I - \frac{\pi}{2}k^2R^2 \\
&= \frac{\tau}{8}k^2R^2 - \frac{3}{8}\pi k^2R^2 + \frac{k}{\omega-k}I
\end{aligned}
$$

  As τ increases, $\triangle U_{21}$ increases. When $\tau \geq 3\pi$, $\triangle U_{21} > 0$. Therefore, there must exist a cutoff value, $\tilde{\tau} < 3\pi$, such that

  - when $\tau > \tilde{\tau}$, case II is preferred to case I at $G = \frac{\omega-k}{\omega}\frac{C}{\tau} = \frac{I}{\pi}$;
  - when $\tau < \tilde{\tau}$, case I is preferred to case II at $G = \frac{\omega-k}{\omega}\frac{C}{\tau} = \frac{I}{\pi}$.

  The expected payoff from case I is independent of $\frac{C}{\tau}$, while for case II it decreases as $\frac{C}{\tau}$ increases, i.e., for $\frac{\omega-k}{\omega}\frac{C}{\tau} > \frac{I}{\pi}$. Therefore, the cutoff value $\tilde{\tau}$ must be higher for higher $\frac{C}{\tau}$ ($\frac{\omega-k}{\omega}\frac{C}{\tau} > \frac{I}{\pi}$), and must be lower for lower $\frac{C}{\tau}$ ($\frac{\omega-k}{\omega}\frac{C}{\tau} < \frac{I}{\pi}$) .

- We can conclude from the discussion above that:

- 1. when $\frac{C}{\tau} < \frac{I}{\pi}$, there exists a cutoff value $G^* > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, such that when $G > G^*$, the optimal choice is case II, and when $G < G^*$, the optimal choice is case III.

  2. When $\frac{C}{\tau} > \frac{I}{\pi}$, there are two cutoff values, $G^+ > \frac{\omega-k}{\omega}\frac{C}{\tau}$, such that when $G > G^+$, case II dominates case III, and vice versa; and $\hat{G} < \frac{\omega-k}{\omega}\frac{C}{\tau}$, such that for $G < \hat{G}$ case IV is preferred to case III, and for $G > \hat{G}$ case III is preferred to case IV. The threshold G^+ decreases wtih μ, while the threshold $\hat{G}$ increases with $\frac{C}{\tau}$.

     We then obtain:

     for $G > \max[G^+, \frac{C}{\tau}]$, the choice is case II

     for $G^+ \leqslant G < \frac{C}{\tau}$, the choice is between I and II

     for $\frac{C}{\tau} \leqslant G < G^+$, the choice is III

     for $\frac{\omega-k}{\omega}\frac{C}{\tau} \leqslant G < \min[G^+, \frac{C}{\tau}]$, the choice is between I and III

     for $\frac{\omega-k}{\omega}\frac{I}{\pi} < \hat{G} \leqslant G < \frac{\omega-k}{\omega}\frac{C}{\tau}$, the choice is between I and III

     for $\frac{\omega-k}{\omega}\frac{I}{\pi} < G < \hat{G}$, the choice is between IV and I

     for $G \leqslant \frac{\omega-k}{\omega}\frac{I}{\pi} < \hat{G}$, the choice is IV

     for $\hat{G} < \frac{\omega-k}{\omega}\frac{I}{\pi} < G < \frac{\omega-k}{\omega}\frac{C}{\tau}$, the choice is between I and III for $\hat{G} \leqslant G \leqslant \frac{\omega-k}{\omega}\frac{I}{\pi}$, the choice is III.

To prove Proposition 1, let $G^{**} = G^{+}$ and $G^{***} = \frac{\omega-k}{\omega}\frac{C}{\tau}$.

■

6.12 Proof of Proposition 2

Proof. Denote non-VC finance as "case O". For $G \leqslant \frac{\omega-k}{\omega}\frac{I}{\pi}$, case O is equivalent to case IV under VC finance, while for larger values of G case O dominates case IV since there is no binding VC incentive constraint in case O. We assume that when the entrepreneur is indifferent between VC and non-VC finance, he chooses non-VC finance. Thus we ignore case IV from now on and compare case O to cases I, II and III. We know that for case O, we have $\omega^2 R_e(R-R_e) = \frac{I}{\pi}$, $U_0 = \frac{\pi}{2}\omega^2 R_e^2$.

1. When $\frac{C}{\tau} < \frac{I}{\pi}$, we know from Proposition 1 that under VC finance there exists a cutoff value $G^{*} > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, such that when $G > G^{*}$, case II is preferred, and otherwise, case III is preferred.

   (a) At point $G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, compare case III and case O. Since $\frac{I+C}{\pi+\tau} < \frac{I}{\pi}$, R_e is higher in case III than in case O. Hence, $U_3 > U_0$.

   For $G < \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, as G decreases, the expected payoffs from case III and case O remain constant, therefore, we can say that for $G \leq \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, case III dominates case O.

   For $G > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, as G increases, the expected payoff from case III decreases because the VC ex post incentive constraint is binding, while the expected payoff from case O does not change. Then there must exist a cutoff value for G, $\ddot{G} > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, such that when $G > \ddot{G}$, case O is preferred to case III, otherwise, vice versa.

   (b) At point $G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, compare case II and case O. For case II, we have $k^2 R_e(R-R_e) = \frac{k}{\omega}\frac{I+C}{\pi+\tau}$, $U_2 = \frac{\pi+\tau}{2}k^2 R_e^2$, while for case O, we have $\omega^2 R_e(R-R_e) = \frac{I}{\pi}$, $U_0 = \frac{\pi}{2}\omega^2 R_e^2$. Therefore, we have

   $$\triangle U_{20}(G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}) = \frac{\pi+\tau}{2}k^2 R_e^2 - \frac{\pi}{2}\omega^2 (R_e^0)^2$$

   Now suppose μ is close to 1, such that $\omega \to k$, i.e., assume $\omega \sim k$. Then $\triangle U_{20}(G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}) > 0$ case II is preferred to case O. As μ decreases, ω increases, implying that there exists a cutoff value $\hat{\mu}$ such that

   - when $\mu > \hat{\mu}$, $\triangle U_{20}(G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}) > 0$, therefore case II dominates case O. Moreover, since the expected payoff from case II increases with G while the expected payoff from case O does not change, the difference in expected payoffs will increase with G for $G > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$.

- when $\mu < \hat{\mu}$, $\triangle U_{20}(G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}) < 0$, therefore case O dominates case II.

We have proved above that case III dominates case O for $G < \ddot{G}$, and otherwise, vice versa. Let $\dot{\mu}$ denote the cutoff value at which $U_3(G = \ddot{G}) = U_2(G = \ddot{G}) = U_0$. Since U_0 does not change with G while U_2 increases with G, we must have $\dot{\mu} < \hat{\mu}$. Therefore,

- when $\mu > \dot{\mu}$, case O is always dominated. There exists a cutoff value $\check{G}$, where $\frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau} < \check{G} < \ddot{G}$, such that for $G < \check{G}$, case III is the optimal choice, while when $G > \check{G}$, case II is preferred.
- when $\mu < \dot{\mu}$, we have: for $G < \ddot{G}$, case III is preferred. For $\ddot{G} < G < \dddot{G}$, case O is preferred. And for $G > \dddot{G}$, case II is preferred. The cutoff value $\dddot{G}$ increases as μ decreases.

2. When $\frac{C}{\tau} > \frac{I}{\pi}$,

- At point $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$, we know that the results for case IV from Proposition 1 apply to case O. Thus for $G \leqslant \frac{\omega-k}{\omega}\frac{I}{\pi}$ we know that there exists a cutoff value $\frac{\tilde{C}}{\tau}$, with $\frac{\omega^2 R^2}{4} \geqslant \frac{\tilde{C}}{\tau} > \frac{I}{\pi}$, such that when $\frac{C}{\tau} > \frac{\tilde{C}}{\tau}$, case O is preferred to case III, and otherwise case III is preferred to case O.
- At point $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$, the comparison between case III and case O is the same as at point $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$, since the expected payoffs from case O and from case III do not change with G over the range $\frac{\omega-k}{\omega}\frac{I}{\pi} < G < \frac{\omega-k}{\omega}\frac{C}{\tau}$.
- For $\frac{\omega-k}{\omega}\frac{I}{\pi} < G < \frac{I}{\pi}$, compare case I with case O.

  Case O implies that $\omega^2 R_e(R-R_e) = \frac{I}{\pi}$, $U_0 = \frac{\pi}{2}\omega^2(R_e)^2$. Case I implies that $k^2 R_e^{VN}(R-R_e^{VN}) = \frac{I}{\pi} - G$, $U_1 = \frac{\pi}{2}k^2(R_e^{VN})^2$. Therefore, we have

$$\triangle U_{10} = U_1 - U_0 = \frac{\pi}{2}k^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e)^2$$

  We know from Proposition 1 that at $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$, $\triangle U_{10} < 0$; then U_1 increases with G while U_0 does not change. Thus we have two possibilities:

  Either case O still dominates at $G = \frac{I}{\pi}$, which would require that $kR_e^{VN} \leqslant \omega R_e$. This implies

$$\frac{k}{\omega} \leqslant \frac{1+\sqrt{1-4\frac{\frac{I}{\pi}}{\omega^2 R^2}}}{2}$$

  and in this case case O is preferred to case I throughout the range.

  Or

$$\frac{k}{\omega} > \frac{1+\sqrt{1-4\frac{\frac{I}{\pi}}{\omega^2 R^2}}}{2}$$

then there exists a cutoff value of G, $\mathring{G}$, such that when $\frac{\omega-k}{\omega}\frac{I}{\pi} < G < \mathring{G}$, case O is preferred to case I, and when $\mathring{G} < G < \frac{I}{\pi}$, case I is preferred to case O. Denote by μ^* the cutoff value for μ such that

$$\frac{k}{\omega} = \frac{1 + \sqrt{1 - 4\frac{\frac{I}{\pi}}{\omega^2 R^2}}}{2}$$

Then for $\mu < \mu^*$, case O is preferred throughout the range. For $\mu > \mu^*$, the preference switches to case I at $G = \mathring{G} < \frac{I}{\pi}$.

From the above discussion, we can conclude that

- 1. when $\frac{C}{\tau} > \frac{\tilde{C}}{\tau}$, case III is dominated by case O for all values of G. At $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$, we know from Proposition 1 that $U_3 > U_2$. Thus we must have $U_0 > U_2$. Moreover, this must be the case for lower values of G as well, since U_2 is increasing in G and U_0 does not change with G. We have proved that case O is also preferred to case I for $G < \mathring{G}$. Thus we must have two thresholds, $G1$ and $G2$, with $G2 \geqslant G1$, such that case O is preferred for $G \leqslant G1$, case I is preferred for $G1 < G < G2$, and case II is preferred for $G > G2$. The threshold $G1$ decreases with μ.

  2. When $\frac{C}{\tau} < \frac{\tilde{C}}{\tau}$, case III is preferred to case O for $G \leqslant \frac{\omega-k}{\omega}\frac{C}{\tau}$. We know from Proposition 1 that case II is not relevant in this range, and that at $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$ case O is preferred to case I. Thus case III must be the optimal choice for $G \leqslant \frac{\omega-k}{\omega}\frac{I}{\pi}$. If $\mu > \mu^*$ and $\mathring{G} \leqslant \frac{\omega-k}{\omega}\frac{C}{\tau}$, the optimal choice will never be case O. Then there will be two cutoff values, $G1$ and $G2$, with $G2 \geqslant G1$, such that case III is preferred for $G \leqslant G1$, case I is preferred for $G1 < G < G2$, and case II is preferred for $G > G2$. Otherwise, there may be a a range of G where case O is preferred, for $G > \frac{\omega-k}{\omega}\frac{C}{\tau}$, since for $G > \frac{\omega-k}{\omega}\frac{C}{\tau}$ the payoff from case III decreases with G. There will then be three cutoff values, $G1$, $G2$, and $G3$, with $G3 \geqslant G2 \geqslant G1$, such that case III is preferred for $G \leqslant G1$, case 0 is preferred for $G1 < G < G2$, case I is preferred for $G2 < G < G3$, and case II is preferred for $G > G3$. The threshold $G2$ decreases with μ.

■

6.13 Proof of Proposition 3

Proof. The optimal contracts for not to patent, patent without licensing, and patent with licensing are respectively the largest root of the following equations:

$$\pi\omega^2 R_e^N(R - R_e^N) = I \tag{17}$$
$$\pi[\beta + (1-\beta)z^2]\hat{R}(R - \hat{R}) = I \tag{18}$$
$$\pi[\beta k^2 + (1-\beta)z^2]\hat{\hat{R}}(R - \hat{\hat{R}}) = I - \pi\beta L \tag{19}$$

The expected profit of entrepreneur for not to patent, patent without licensing, and patent with licensing can be expressed respectively as:

$$U_{NP} = \frac{\pi}{2}\omega^2(R_e^N)^2 \tag{20}$$
$$U_P = \frac{\pi}{2}[\beta + (1-\beta)z^2]\hat{R}^2 \tag{21}$$
$$U_{PL} = \frac{\pi}{2}[\beta k^2 + (1-\beta)z^2]\hat{\hat{R}}^2 \tag{22}$$

From (17) and (18), we can see that as long as $\omega^2 < \beta + (1-\beta)z^2$, $\hat{R}$ is larger than R_e^N. In this situation, (20) and (21) tell us that patent without licensing is strictly favorable than not to patent.

Therefore, when $\omega^2 < \beta + (1-\beta)z^2$, not to patent is dominated and can be ignored, we only need to focus on the two cases patent without licensing and patent with licensing: When $\hat{\hat{R}} = \hat{R}$, subtracting these two equations in both sides gives us $(1-k^2)\pi\beta\hat{R}(R-\hat{R}) = \pi\beta L$. From equation (18), we know that $\hat{R}(R-\hat{R}) = \frac{I}{\pi[\beta+(1-\beta)z^2]}$, therefore, it gives us

$$L^* = \frac{(1-k^2)I}{\pi[\beta + (1-\beta)z^2]}$$

The utility from patenting without licensing is given as $U_P = \frac{\pi}{2}[\beta + (1-\beta)z^2]\hat{R}^2$, while the utility from patenting with licensing is given as $U_{PL} = \frac{\pi}{2}[\beta k^2 + (1-\beta)z^2]\hat{\hat{R}}^2$. Since $k < 1$, if licensing is preferable, it must be $\hat{\hat{R}} > \hat{R}$. We can see from (19) that $\hat{\hat{R}}$ is monotonically increasing with L until $L = \frac{I}{\pi\beta}$. Therefore, if $U_{PL}(L = \frac{I}{\pi\beta}) < U_P$, patent without licensing will always be preferred to patent with licensing; Otherwise, there exists a cutoff value, L^N, under which $U_{PL}(L = L^N) = U_P$, and when $L > L^N$, $U_{PL} > U_P$. Define $a = [\beta + (1-\beta)z^2]$, $b = [\beta k^2 + (1-\beta)z^2]$, when $L = L^N$,

$$U_{PL} = \frac{\pi}{2}b\hat{\hat{R}}^2 = U_P = \frac{\pi}{2}a\hat{R}^2 \tag{23}$$

Equation (18) and (19) can be rewritten as

$$\pi a R\hat{R} - \pi a\hat{R}^2 = I \tag{24}$$

$$\pi b R\hat{\hat{R}} - \pi b\hat{\hat{R}}^2 = I - \pi\beta L^N \tag{25}$$

Plug equation (23) in to the above equations, and subtract them from both sides, we have $a\hat{R} - b\hat{\hat{R}} = \frac{\beta L}{R}$. Then $\hat{\hat{R}} = \frac{a}{b}\hat{R} - \frac{\beta L^N}{bR}$, where $\hat{R}$ and $\hat{\hat{R}}$ are the largest root of equation (24) and (25).

When $\omega^2 > \beta + (1-\beta)z^2$, patent without license is dominated by not to patent, and can be ignored, we only need to focus on the two cases not to patent and patent with licensing:

Similar to the above situation, we can see from (19) that $\hat{\hat{R}}$ is monotonically increasing with L until $L = \frac{I}{\pi\beta}$. Therefore, if $U_{PL}(L = \frac{I}{\pi\beta}) < U_{NP}$, not to patent will always be preferred to patent with licensing; Otherwise, there exists a cutoff value, L^P , under which $U_{PL}(L = L^P) = U_{NP}$, and when $L > L^P$, $U_{PL} > U_{NP}$.

Define $c = \omega^2$, $b = [\beta k^2 + (1-\beta)z^2]$, when $L = L^P$,

$$U_{PL} = \frac{\pi}{2}b\hat{\hat{R}}^2 = U_{NP} = \frac{\pi}{2}c(R_e^N)^2 \tag{26}$$

Equation (17) and (19) can be rewritten as

$$\pi c R R_e^N - \pi c(R_e^N)^2 = I \tag{27}$$

$$\pi b R\hat{\hat{R}} - \pi b\hat{\hat{R}}^2 = I - \pi\beta L^P \tag{28}$$

Plug equation (26) in to the above equations, and subtract them from both sides, we have $a\hat{R} - b\hat{\hat{R}} = \frac{\beta L}{R}$. Then $\hat{\hat{R}} = \frac{c}{b}R_e^N - \frac{\beta L^P}{bR}$, where R_e^N and $\hat{\hat{R}}$ are the largest root of equation (27) and (28). Since $c > b$ & $R_e^N > \hat{R}$, therefore $L^P > L^N$. ■

6.14 Proofs for Section 4.4

Proof. To prove the results summarized in section 4.4, we first derive the optimal financial contracts between the entrepreneur and the VC for the following three cases:

- Case I: Outward knowledge transfer when patent is rejected
- Case II: Inward knowledge transfer and outward knowledge transfer when patent is rejected
- Case III: Inward knowledge transfer

For Case I (Outward), the optimization problem can be written as:

$$
\begin{aligned}
\max_{R_e^{AP}, R_e^{AR}} \ \pi\{\beta[e^{AP}R_e^{AP} - \frac{1}{2}(e^{AP})^2] &+ (1-\beta)[e^{AR}kR_e^{AR} - \frac{1}{2}(e^{AR})^2]\} \\
s.t.\ e^{AP} = R_e^{AP},\ e^{AR} &= kR_e^{AR}\ (IC_e) \\
\pi\{\beta e^{AP}(R - R_e^{AP}) + (1-\beta)[e^{AR}k(R - R_e^{AR}) + G]\} &\geq I\ (PC_{VC}) \\
\tau\{\beta e^{AP}(R - R_e^{AP}) + (1-\beta)[e^{AR}k(R - R_e^{AR}) + G]\} &\leq C\ (IC_{VC}\ ex\ ante) \\
ke^{AR}(R - R_e^{AR}) + G &\geq ze^{AR}(R - R_e^{AR})(IC_{VC}\ ex\ post)
\end{aligned}
$$

From (IC_{VC} *ex ante*) and (PC_{VC}), we see that $\frac{C}{\tau} \geq \beta e^{AP}(R - R_e^{AP}) + (1-\beta)[e^{AR}k(R - R_e^{AR}) + G] \geq \frac{I}{\pi}$. It holds only when $\frac{C}{\tau} \geq \frac{I}{\pi}$.

Comparing to Problem 7 (P7), this problem (P9) differs in two ways: first, there is the additional (IC_{VC} *ex ante*) to be satisfied. Second, in section 4.2(P7), we assume $G \leq \frac{I}{\pi}$ for simplicity, while in P9, we remove this assumption so that the results are comparable with those for non-patentable innovations in section 3.

To meet the constraint IC_{VC} *ex ante*, we must have$\frac{C}{\tau} \geq (1-\beta)G$. Therefore,

- when $\frac{C}{\tau} \geq (1-\beta)G \geq \frac{I}{\pi}$, it's possible to set $R_e^{AP} = R_e^{AR} = R$ and all the constraints are satisfied. If PC_{VC} holds with stictly inequality, the VC can provide ex ante additional fee, F, to the entrepreneur, such that $\pi(1-\beta)G = I + F$.

- when $(1-\beta)G \leq \frac{I}{\pi}$, as we know that PC_{VC} must be binding, as long as $\frac{C}{\tau} \geq \frac{I}{\pi}$, ($IC_{VC}$ *ex ante*) will always be satisfied. Thus the results are the same as in Lemma 7 (cases 2, 3 and 4).

For Case II (Inward + Outward), the optimization problem is defined as follows:

$$
\begin{aligned}
\max_{R_e^{AP}, R_e^{AR}} (\pi+\tau)\{\beta[e^{AP}R_e^{AP} - \frac{1}{2}(e^{AP})^2] &+ (1-\beta)[e^{AR}kR_e^{AR} - \frac{1}{2}(e^{AR})^2]\} \\
s.t.\ e^{AP} = R_e^{AP},\ e^{AR} &= kR_e^{AR}\ (IC_e) \\
(\pi+\tau)\{\beta e^{AP}(R - R_e^{AP}) + (1-\beta)[e^{AR}k(R - R_e^{AR}) + G]\} &\geq I + C\ (PC_{VC}) \\
\tau\{\beta e^{AP}(R - R_e^{AP}) + (1-\beta)[e^{AR}k(R - R_e^{AR}) + G]\} &\geq C\ (IC_{VC}\ ex\ ante) \\
ke^{AR}(R - R_e^{AR}) + G &\geq ze^{AR}(R - R_e^{AR})(IC_{VC}\ ex\ post)
\end{aligned}
$$

From (IC_{VC} *ex post*), we have

$$G \geq (z-k)kR_e^{AR}(R - R_e^{AR}) \tag{29}$$

By rewriting PC_{VC} and IC_{VC} *ex ante*, we have

$$\begin{aligned}
\beta R_e^{AP}(R - R_e^{AP}) + (1-\beta)k^2 R_e^{AR}(R - R_e^{AR}) &\geq \frac{I+C}{\pi+\tau} - (1-\beta)G \\
\beta R_e^{AP}(R - R_e^{AP}) + (1-\beta)k^2 R_e^{AR}(R - R_e^{AR}) &\geq \frac{C}{\tau} - (1-\beta)G
\end{aligned}$$

Combining these two inequalities, we have

$$\beta R_e^{AP}(R - R_e^{AP}) + (1-\beta)k^2 R_e^{AR}(R - R_e^{AR}) \geq \max\{\frac{I+C}{\pi+\tau}, \frac{C}{\tau}\} - (1-\beta)G \tag{30}$$

Suppose (29) is satisfied. Then at optimum, (30) must be binding. Therefore, the Lagrangian function gives us, at optimum, $R_e^{AP} = R_e^{AR} \equiv \tilde{\tilde{R}}$, where $\tilde{\tilde{R}}$ is the largest root of (30) when it is binding.

Substituting this back into (30)(binding) and (29), we have

$$G \geq \frac{k(z-k)}{z(1-\beta)k+\beta}\Gamma$$

where $\Gamma \equiv \max\{\frac{I+C}{\pi+\tau}, \frac{C}{\tau}\}$.

When $G < \frac{k(z-k)}{z(1-\beta)k+\beta}\Gamma$, (29) will not be satified. By increasing R_e^{AR}, it is possible to make (29) binding. Therefore, the optimal contract R_e^{AR} is the largest root of the following equation:

$$G = (z-k)kR_e^{AR}(R - R_e^{AR}) \tag{31}$$

and R_e^{AP} is the largest root of the following equation, given that R_e^{AR} has been determined by equation (31).

$$\beta R_e^{AP}(R - R_e^{AP}) + (1-\beta)k^2 R_e^{AR}(R - R_e^{AR}) = \max\{\frac{I+C}{\pi+\tau}, \frac{C}{\tau}\} - (1-\beta)G$$

From expression (30), it is easy to find out that when $G \geq \frac{\Gamma}{1-\beta}$, the optimal contract is $R_e^{AP} = R_e^{AR} = R$. In this case, IC_{VC} *ex post* is alwary satisfied. PC_{VC} and IC_{VC} *ex ante* are also satisfied as inequality (30) holds.

When $\frac{\Gamma}{1-\beta} > G \geq \frac{k(z-k)}{z(1-\beta)k+\beta}\Gamma$,$IC_{VC}$ *ex post* is alwary satisfied. Expression (30) must be binding, which implies that either PC_{VC} or IC_{VC} *ex ante* will be binding in optimum, depending on the relative size between $\frac{I+C}{\pi+\tau}$ and $\frac{C}{\tau}$ (If $\frac{I+C}{\pi+\tau} > \frac{C}{\tau}$, then PC_{VC} will be binding; and vice versa.). The optimal contract is the largest root of (30) when (30) holds in equality and satisfies $R_e^{AP} = R_e^{AR} \equiv \tilde{\tilde{R}}$.

When $\frac{k(z-k)}{z(1-\beta)k+\beta}\Gamma > G \geq \frac{\Gamma(z-k)}{z(1-\beta)}$, IC_{VC} *ex post* is binding. The optimal contract R_e^{AR} is the largest root of the binding IC_{VC} *ex post*. And R_e^{AP} is the largest root of the binding (30) given that R_e^{AR} has been determined above. And it is easy to see that $R_e^{AR} > R_e^{AP}$ as G becomes smaller than $\frac{k(z-k)}{z(1-\beta)k+\beta}\Gamma$.

If $\frac{I+C}{\pi+\tau} < \frac{C}{\tau}$, such that the participation constraint of VC is slack, then VC would pay an extra fee ex ante, F, to the entrepreneur such that $(\pi+\tau)\{\beta e^{AP}(R-R_e^{AP}) + (1-\beta)[e^{AR}k(R-R_e^{AR}) + G]\} = I + C + F$. VCs always earn zero expected rents as they are competitive.

The participation constraint under which VC will invest in the project can be rewritten as:

$$\tilde{R}(R-\tilde{R}) \geq \frac{\frac{I+C}{\pi+\tau} - (1-\beta)G}{\beta + (1-\beta)k^2}$$

which entails that the problem has a solution iff

$$\frac{1}{4}k^2R^2 \geq \frac{\frac{I+C}{\pi+\tau} - (1-\beta)G}{\beta + (1-\beta)k^2}$$

For Case III (Inward), the optimization problem can be written as:

$$\begin{aligned}
\max_{R_e^{AP}, R_e^{AR}} (\pi+\tau)\{\beta[e^{AP}R_e^{AP} - \frac{1}{2}(e^{AP})^2] \quad &+ \quad (1-\beta)[e^{AR}zR_e^{AR} - \frac{1}{2}(e^{AR})^2]\} \\
s.t.\ e^{AP} = R_e^{AP},\ e^{AR} \quad &= \quad zR_e^{AR}\ (IC_e) \\
(\pi+\tau)\{\beta e^{AP}(R-R_e^{AP}) + (1-\beta)e^{AR}z(R-R_e^{AR})\} \quad &\geq \quad I + C\ (PC_{VC}) \\
\tau\{\beta e^{AP}(R-R_e^{AP}) + (1-\beta)e^{AR}z(R-R_e^{AR})\} \quad &\geq \quad C\ (IC_{VC}\ ex\ ante) \\
ke^{AR}(R-R_e^{AR}) + G \quad &\leq \quad ze^{AR}(R-R_e^{AR})(IC_{VC}\ ex\ post)
\end{aligned}$$

Similar as the Proof for Case II, from PC_{VC} and IC_{VC} *ex ante*, we have

$$\beta R_e^{AP}(R-R_e^{AP}) + (1-\beta)z^2R_e^{AR}(R-R_e^{AR}) \geq \Gamma \tag{32}$$

From IC_{VC} *ex post*, we have

$$G \leq (z-k)zR_e^{AR}(R-R_e^{AR}) \tag{33}$$

If expression (33) holds with inequality, then at optimum, expression (32) must hold with equality, i.e. $\beta R_e^{AP}(R-R_e^{AP}) + (1-\beta)z^2R_e^{AR}(R-R_e^{AR}) = \Gamma$. The Lagrangian method gives us, $R_e^{AP} = R_e^{AR} \equiv \bar{R}$, which is the largest root of the following equation

$$[\beta + (1-\beta)z^2]\bar{R}(R-\bar{R}) = \Gamma. \tag{34}$$

Substituting it back into (33), we have

$$G \leq \frac{(z-k)z}{\beta+(1-\beta)z^2}\Gamma$$

Then at optimum, either PC_{VC} or IC_{VC} *ex ante* is binding, depending on the relative size between $\frac{C}{\tau}$ and $\frac{I+C}{\pi+\tau}$ (If $\frac{C}{\tau} > \frac{I+C}{\pi+\tau}$, IC_{VC} *ex ante* is binding and PC_{VC} is slack; vice versa.) When the VC participation constraint is slack, then VC would pay an extra fee ex ante, F, to the entrepreneur such that $(\pi+\tau)[\beta+(1-\beta)z^2]\bar{R}(R-\bar{R}) = I+C+F$. VCs always earn zero expected rents as they are competitive. The optimal contract is the largest root of the equation (34). The problem has a solution only when $\frac{1}{4}R^2[\beta+(1-\beta)z^2] \geq \Gamma$ as $\bar{R}(R-\bar{R}) \leq \frac{1}{4}R^2$.

If expression (33) holds with equality (i.e., $G \geq \frac{(z-k)z}{\beta+(1-\beta)z^2}\Gamma$), the optimal contract for R_e^{AR} is the largest root of the following equation:

$$G = (z-k)zR_e^{AR}(R-R_e^{AR}). \tag{35}$$

It is easy to see that in this case, $R_e^{AR} < R_e^{AP}$ as G becomes greater. At optimum, as long as $G \leq \frac{z-k}{(1-\beta)z}\Gamma$, R_e^{AP} is determined by

$$\beta R_e^{AP}(R-R_e^{AP}) = \Gamma - \frac{(1-\beta)z}{z-k}G$$

The problem has a solution only when $G \leq \frac{1}{4}(z-k)zR^2$. If $G > \frac{z-k}{(1-\beta)z}\Gamma$, the optimal contract implies $R_e^{AP} = R$, R_e^{AR} is determined by (35), and VC pays an ex-ante fee to ensure PC_{VC} holds as an equality.

Now that we have derived optimal VC contracts for Cases I, II and III, we need to show that when the VC ex post incentive constraint is binding, the following holds: ■

(1) the optimal VC contract with outward knowledge transfer (and no inward transfer), i.e. Case I, is dominated by non-VC finance;

(2) the optimal VC contract with both outward and inward knowledge transfer, i.e. Case II, is dominated by the optimal VC contract with only inward knowledge transfer, i.e. Case III.

Proof.

- Comparing Case I with non-VC finance when IC_{VC} *ex post* is binding.

  Suppose that IC_{VC} *ex post* and IC_{VC} *ex ante* are not binding, the optimal contract is $R_e^{AP} = R_e^{AR} = \hat{\hat{R}}$, which satisfies the PC_{VC} as an equality.

  Consider the value of G such that IC_{VC} *ex post* holds as an equality: $\underline{G} = (z-k)k\hat{\hat{R}}(R-\hat{\hat{R}})$.

Substituting this into binding PC_{VC} we have

$$\hat{\hat{R}}(R - \hat{\hat{R}}) = \frac{I}{\pi[\beta + (1-\beta)kz]}$$

Now consider the optimal contract for non-VC finance: from Lemma 5 we have $R_e^P = R_e^R = \hat{R}$, which satisfies the investor's participation constraint as an equality. Thus,

$$\hat{R}(R - \hat{R}) = \frac{I}{\pi[\beta + (1-\beta)z^2]}$$

Since $z > k$, we find that $\hat{R} > \hat{\hat{R}}$, therefore non-VC finance is better.

For any value of G lower than $\underline{G} = (z-k)k\hat{\hat{R}}(R-\hat{\hat{R}})$, the IC_{VC} *ex post* becomes binding in the VC contract, distorting the contract and decreasing the entrepreneur's expected utility, while his expected utility from non-VC finance is unchanged.

Hence, the expected utility from the VC contract will be lower than from non-VC finance whenever the IC_{VC} *ex post* is binding in the former.

- Comparing Case II with Case III when IC_{VC} *ex post* is binding.

  As in the previous case, suppose in Case II that IC_{VC} *ex post* and IC_{VC} *ex ante* are not binding, the optimal contract for case II is $R_e^{AP} = R_e^{AR} = \tilde{\tilde{R}}$, which satisfies the (30) as an equality.

  Consider the value of G such that IC_{VC} *ex post* holds as an equality: $\bar{G} = (z-k)k\tilde{\tilde{R}}(R-\tilde{\tilde{R}})$. Substituting this value into binding (30), we have

$$\tilde{\tilde{R}}(R - \tilde{\tilde{R}}) = \frac{\Gamma}{[\beta + (1-\beta)zk]}$$

  And $\bar{G} = \frac{k(z-k)}{z(1-\beta)k+\beta}\Gamma$.

  Now consider the optimal contract for Case III, as we could see from the Proof of Proposition 4, when $G = \bar{G} < \frac{z(z-k)}{z^2(1-\beta)+\beta}\Gamma$, IC_{VC} is satisfied, and therefore $R_e^{AP} = R_e^{AR} = \bar{R}$, which satisfies (32) as an equality. Thus

$$\bar{R}(R - \bar{R}) = \frac{\Gamma}{[\beta + (1-\beta)z^2]}$$

  It is easy to see that $\bar{R} > \tilde{\tilde{R}}$. Therefore Case III dominates Case II at $G = \bar{G}$.

  For any value of G lower than $\bar{G}$, the IC_{VC} *ex post* is binding in Case II, distoring the contract and decreasing the entrepreneur's expected utility, while IC_{VC} *ex post* is always satisfied in Case III, and the expected utility from Case III is unchanged.

Hence, the expected utility from Case II (inward+outward knowledge transfer) will be lower than that from case III (inward knowledge transfer) when the IC_{VC} *ex post* for Case II is binding.

■