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# Cyclicalty and term structure of Value-at-Risk within a threshold autoregression setup

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**Abstract:** This paper explores empirically the link between stocks returns Value-at-Risk (VaR) and the state of financial markets across various holding horizons. The econometric analysis is based on a self-exciting threshold autoregression setup. Using quarterly French and US data from 1970Q4 to 2012Q4, it turns out that the  $k$ -year VaR of equities is actually dependent on the state of the market: the expected losses as measured by the VaR are smaller in bear market than in normal or bull market, whatever the horizon. These results suggest that the rules regarding the solvency capital requirements should adapt to the state of the financial market.

**Keywords:** Expected equities returns, Value at Risk, Financial cycle, Investment horizon, Threshold Autoregression.

**JEL classification:** G11.

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# Introduction

After the renewal of banks regulatory framework with the Basel II agreement in 2003, the European Commission has developed new capital standards for insurance companies which are referred to as "Solvency 2", to be implemented in early 2016. This new solvency regulation of insurers differs markedly from earlier standards by aiming to determine capital requirements on the basis of the net risk position of their balance sheet, as is now the case for banks. However, banking and insurance activities are different by nature, in terms of liquidity, maturity transformation and guarantees. A crucial question for life insurers and collective defined-contribution pension funds is to determine whether the new prudential regulation in Europe should recognize the long maturity of life insurance liabilities and the insurers' role as intermediaries promoting intergenerational risk sharing (see Gollier [2008]), two aspects of insurance activities that are irrelevant in the banking sector. If policyholders have themselves a long-term perspective for their saving, it is important that these financial intermediaries get the right incentives to select assets portfolios that fit best their customers' interests, both in risk and in maturity. The current Solvency 2 regulation is based on the hypothesis that the equity risk is stable through time and is independent of the duration of the holding period. This hypothesis justifies using solvency rules in insurance that are similar to those used in the banking sector in spite of the important differences in the duration and liquidity of their liabilities. In this paper, we reevaluate this hypothesis. More specifically, we quantify the relationship between the equity risk and both the holding duration and the state the financial market is in.

Actually, the risk-based capital requirements now prevailing in the banking sector have themselves been widely criticized because they could exacerbate financial cycles, or more generally business cycle fluctuations (see e.g. Kashyap and Stein [2004], Adrian and Shin [2008, 2010], Plantin, Sapra and Shin [2008], Rochet [2008]). Basically, these authors claim that solvency capital requirements (SCR hereafter) rules which do not depend on the state of the business/financial cycle may lead to large pro-cyclical leverage

effects. As a result of such rules, investors demand of securities increases during financial booms, thereby reinforcing them. Conversely, investors have to sell securities during financial downturns in order to restore their solvency ratios, which exacerbates the financial recession.<sup>1</sup> Yet, a cyclical SCR rule allowing for smaller capital requirements during downturns could at least dampen, if not completely eliminate, this procyclical leverage effect.

Providing further support to such a cyclical SCR rule, a growing empirical literature points to predictability and mean-reversion in stocks returns (see e.g. Campbell [1991], Campbell [1996], Barberis [2000], Campbell and Viceira [2002], Bec and Gollier [2007], Campbell and Thompson [2008] or Jondeau and Rockinger [2009]). More precisely, excess stock returns risk is found to be mean reverting in the sense that the risk associated with long holding periods is lesser than the one associated with short holding horizons as e.g. the widely scrutinized one-year horizon. Beyond this potential investment horizon effect, returns mean reversion may also imply a cyclical effect. In other words, the financial cycle's position could help predicting future returns and future risk.

Our contribution to this literature is twofold. First, we assess empirically the importance of these cyclical and investment horizon effects for French stock price data. For comparison purpose, the widely studied US data are also considered. The question is explored by modelling the dynamics of excess return of equities from a self-exciting threshold autoregression (hereafter SETAR) model. This setup aims at disentangling bear and bull markets dynamics. The choice of this representation is basically motivated by the fact that it allows for straightforward computation of the conditional first and second-order moments matrices, namely the conditional mean and variance-covariance matrices. Hence, two crucial variables for dynamic portfolio allocation optimization are obtained easily — the time- $t$  conditional expectation (forecast) and conditional variance (risk measure) for asset returns at horizon  $t + h$ . Our second contribution is then to propose a measure of the Value-at-Risk based on the SETAR estimates which takes the

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<sup>1</sup>See Adrian and Shin [2010] for a very clear presentation of this procyclical leverage effect.

influence of the recent cycle conditions into account. It is in line with existing measures in that it derives from the empirical distribution of the expected  $k$ -period returns. Nevertheless, it has the advantage of not imposing any assumption regarding the law of distribution of the sample but relies on bootstrapped quantiles instead.<sup>2</sup> Finally, we take advantage of this analysis to propose a cycle-dependent measure of the Solvency Capital Requirement which accounts for the illiquidity risk.

Using quarterly French and US data from 1970Q4 on, it turns out that both cyclical and horizon effects do influence the Value-at-Risk: the expected future returns are higher (and the Value-at-Risk more favorable) from a crisis than from a normal or bull market, and lower for long than for short investment horizons. These findings suggest that if the prudential regulation aims at maintaining a constant yearly default risk, SCR rules should be flexible enough so as to take these cyclical and horizon effects into account. More precisely, for the countries considered here, the SCR rules should induce intermediaries to be more conservative in long phases of normal or high returns and to be more risk-taking in downturns. Failing to recognize these features of financial markets would induce financial intermediaries with long and illiquid liabilities to overreact to downturns on the equity markets, and to excessively reduce their equity holdings over the entire financial cycle.

The paper is organized as follows. Section 1 presents the econometric methodology. Section 2 describes the data used for the threshold autoregression presented in Section 3. In Section 4, estimated stocks returns VaR are compared across investment horizons and phases of financial cycle. Section 5 concludes.

## 1 SETAR modelling of VaR

### 1.1 The SETAR model

Let  $R_{0t}$  denote the nominal short rate and  $r_{0t} = \log(1 + R_{0t})$  the log (or continuously compounded) return on this asset that is used as a benchmark to compute excess re-

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<sup>2</sup>See e.g. Feunou and Meddahi [2007] for a different approach to derive the term structure of risk.

turns on equities. Then, with  $r_{et}$  the log stock return, let  $x_{et} = r_{et} - r_{0t}$  denote the corresponding log excess returns that we wish to model as a Self-Exciting Threshold AutoRegression. In what follows, two kinds of SETAR will be considered for  $x_{et}$ . The first one is a two-regime SETAR model which aims at capturing high *vs* low regimes and is given by:

$$x_{et} = \left(\mu_\ell + \sum_{i=1}^n \rho_{\ell i} x_{e,t-i}\right) s_t + \left(\mu_u + \sum_{i=1}^n \rho_{ui} x_{e,t-i}\right) (1 - s_t) + v_t, \quad (1)$$

where  $s_t$  is a zero-one valued transition function defined by:

$$s_t = \begin{cases} 1 & \text{if } x_{e,t-1} \leq \lambda, \\ 0 & \text{otherwise.} \end{cases}$$

Hence, we expect the dynamics of the excess returns to be regime-switching with  $x_{e,t-1}$  as the transition variable. More precisely, for values of  $x_{e,t-1}$  smaller than or equal to the threshold  $\lambda$ , the dynamics is governed by  $\mu_\ell$  and  $\rho_{\ell i}$ : this corresponds to the bear market regime. In the bull market regime, where values of  $x_{e,t-1}$  are greater than  $\lambda$ , the dynamics is governed by  $\mu_u$  and  $\rho_{ui}$ .<sup>3</sup> It is assumed that the roots of the characteristic polynomials  $\rho_j(z) = 1 - \rho_{j1}z - \dots - \rho_{jn}z^n$ ,  $j = \ell, u$ , lie strictly outside the unit circle in absolute value, a condition which rules out nonstationary or explosive behavior in  $x_{e,t}$ . Finally, the innovations  $v_t$  are assumed to be *i.i.d.* distributed with mean zero and variance  $\sigma_v^2$ . The second SETAR specification of interest is a 3-regime SETAR of the form:

$$x_{et} = \left(\mu_\ell + \sum_{i=1}^n \rho_{\ell i} x_{e,t-i}\right) s_{\ell t} + \left(\mu_m + \sum_{i=1}^n \rho_{mi} x_{e,t-i}\right) s_{mt} + \left(\mu_u + \sum_{i=1}^n \rho_{ui} x_{e,t-i}\right) s_{ut} + v_t, \quad (2)$$

where  $s_{\ell t} = 1$  if  $x_{e,t-1} \leq \lambda_\ell$  and 0 otherwise,  $s_{ut} = 1$  if  $x_{e,t-1} > \lambda_u$  and 0 otherwise, with  $\lambda_u > \lambda_\ell$  and  $s_{mt} = 1 - s_{\ell t} - s_{ut}$ . There are now two real-valued thresholds,  $\lambda_\ell$  and  $\lambda_u$ . In this 3-regime SETAR, we allow for a middle regime on top of the lower

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<sup>3</sup>Of course, this model is retained as a first approach of threshold modeling of the excess returns and cannot capture the fact that high expected returns do not have the same interpretation in periods of high or low volatility. Nevertheless, we believe that at a quarterly frequency, this simplification is not too misleading.

and upper regimes. This middle regime could correspond *i.e.* to a “normal” regime, in between the bear and bull markets. Contrary to the 2-regime model, the specification given in Equation (2) could also capture a distinction between large absolute values of the returns (in the lower and upper regimes) and smaller ones (in the middle regime).

## 1.2 From SETAR to Value-at-Risk

Following Campbell and Viceira [2004], the one-period log excess returns are added over  $k$  successive periods in order to get the cumulative  $k$ -period log excess returns on equities, denoted  $x_{et}^k \equiv (x_{e,t+1} + \dots + x_{e,t+k})$ . Thanks to its autoregressive nature, the SETAR model is particularly well suited for forecasting purposes. Indeed, the  $k$  successive one-period returns used to define  $x_{et}^k$  are obtained by forward recursion of models (1) or (2).

Hence, the value-at-risk obtains straightforwardly from this model. The VaR is basically defined as a number such that there is a probability  $p$  that a worse excess (log-)return occurs over the next  $k$  periods. Throughout this paper, the VaR is defined from the left tail of the loss and profit distribution function. Hence, a negative (respectively positive) VaR denotes a loss (resp. profit), contrary to the alternative definition based on the profit and loss distribution. Hence, the VaR over the time horizon  $k$  with probability  $p$  may be defined from:

$$p = Pr [x_{et}^k \leq VaR_k(p)] = F_k(VaR_k(p)), \quad (3)$$

where  $F_k(\cdot)$  denotes the cumulative distribution function of  $x_{et}^k$ . The quantile function is the inverse of the cumulative distribution function from which the VaR obtains:

$$VaR_k(p) = F_k^{-1}(p). \quad (4)$$

Since  $x_{et}^k$  is the sum of log excess returns over  $k$  periods, it is also the log of the product of the excess returns (not taken in log) over  $k$  periods. Hence, the VaR of the corresponding portfolio value simply obtains as:

$$VaR_k^{cr}(p) = \exp(VaR_k(p)) - 1$$

Since we are interested in the Value-at-Risk for various time horizons, it is desirable to keep an equivalent risk level over all the horizons, which means adjusting  $p$  with  $k$ . For instance, the  $1 - p = 95\%$  level retained in VaR analysis is chosen on a yearly basis. In order to maintain the same yearly probability, the corresponding probability for horizon  $k$  should be adjusted accordingly. Unfortunately, the question of the correct adjustment regarding returns on equities is still open so far. Consequently, just to give an idea of how the time-horizon may affect this probability, we propose a very crude approximation instead, that is  $1 - p = (95\%)^k$ .<sup>4</sup> All the VaR computations below will be made by fixing  $p$  to 5%, 10%, etc... for all horizons on the one hand and by using this horizon-adjusted probability on the other hand.

## 2 The data

The benchmark asset from which the excess returns on equities will be calculated is a short rate. For France, the 3-month PIBOR rate obtained from Datastream is retained from 1970M11 to 1998M12. It is then continued using the 3-month EURIBOR rate from 1999M1 to 2012M12. For the US, we use the three-month Treasury Bills rate. The end-of-quarter values from these monthly series are retained to get quarterly observations, and  $r_{0t}$  denotes the log return on the 3-month rate.

French and US data for stock prices and returns come from Morgan Stanley Capital International (MSCI) database and are available since December 1969. More precisely, using the monthly MSCI National Price and Gross Return Indices in local currency, a quarterly stock total return series and a quarterly dividend series are obtained following the methodology described in Campbell [1999]<sup>5</sup>. Note that we depart from Campbell's approach by not including the tax credits on dividends. Indeed, MSCI calculates returns

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<sup>4</sup>This horizon correction is usually retained in default models with time independency. Note that even with *i.i.d.* returns, it wouldn't be correct. This crude approximation consists basically in assuming that the survival function of the mean of  $x_1$  and  $x_2$  is the square of the survival function of  $x_1$ .

<sup>5</sup>See also Campbell's "Data Appendix for *Asset Prices, Consumption and the Business Cycle*", March 1998, downloadable from Campbell's homepage.



from the perspective of US investors, so it excludes from its indices these tax credits which are available only to local investors. For France, Campbell chooses to add back the tax credits quite roughly, by applying the 1992 rate of 33.33% to all the sample. Nevertheless, this rate hasn't remained fixed over the sample considered here (1970Q1—2012Q4). On top of this, the way dividends are taxed has also changed during that period. We couldn't find exact tax rate data for our sample and have chosen to work with data excluding tax credits. The equities excess return,  $x_{et}$ , is then obtained by subtracting  $r_{0t}$  from the log return on equities.

Figure 4 in Appendix reports annualized French and US log excess return on equities data.

### 3 Empirical assessment of the influence of the financial market cycle on equities log excess returns

The lag order  $n$  of the SETAR models is chosen so as to eliminate residuals serial correlation, which leads to retain one lag. For both models (1) and (2), we have also considered a constrained version in which the intercept is assumed to be the same across regimes, i.e.  $\mu_\ell = \mu_u (= \mu_m)$ . The threshold estimates were obtained by grid search so as to *i*) leave at least 5% of the observations in each regime and *ii*) minimize the model's sum of squared residuals. First, we performed linearity tests. Since the thresholds are unidentified nuisance parameters under the null hypothesis, we use the SupLR statistic whose non standard asymptotic distribution obtains from Hansen [1996]. The residual bootstrap method described in Hansen and Seo [2002] is used to compute the p-value. The results of these linearity tests, based on 5000 simulations, are reported in Table 1 below.

In France, the two- and three-regime models with a constrained intercept reject the null of linearity at the 5%-level. Since the comparison of these models involves a (second) threshold which is an unidentified nuisance parameter under the two-regime null hypothesis, a standard LR test cannot be performed to this end. To circumvent this

Table 1: SupLR linearity tests  $p$ -values

	Model (1) unconstrained	Model (1) constrained	Model (2) unconstrained	Model (2) constrained
France	0.21	0.05	0.18	0.05
US	0.08	0.03	0.01	0.03

issue, a SupLR statistics can be used again: it leads to a bootstrap  $p$ -value of 11% and hence does not reject the null of a constrained two-regime model at conventional levels. The latter will accordingly be used in the subsequent empirical analysis. By contrast, only the unconstrained intercept two-regime model does not reject the linear null in the US: three SETAR candidates remain possible. Since the constrained and unconstrained versions of the three-regime model are nested with the linear restrictions  $\mu_\ell = \mu_u = \mu_m$ , these constraints can be tested using a standard LR test. The corresponding LR statistics is equal to 16.20 and is  $\chi^2(2)$  distributed: the constrained three-regime model is strongly rejected. Then, according to the SupLR test performed to compare the unconstrained three-regime model and the two-regime model with a constrained intercept, the null of a constrained two-regime model can be rejected with a bootstrap  $p$ -value of 1%. Consequently, the unconstrained three-regime model is retained for the US data.

The OLS estimates of the selected SETAR models are reported in Table 2, see Appendix. For these SETAR(1) models, the null of no residuals serial correlation up to order 4 is not rejected according to the Portmanteau and LM test statistics. It is also worth noticing that, likely due to the quarterly frequency of our observations sample, both ARCH and White F tests do not reject the homoskedastic null hypothesis. Actually, returns data heteroskedasticity is mainly a high frequency phenomenon.

In the two-regime model retained for French data, the threshold estimate is -22.56%, which is strongly negative. As can be seen from Figure 4 in Appendix, which plots the log excess returns together with the threshold value, the lower regime basically accounts for the financial crisis which happened during the period: the two oil price shocks with

the financial market troughs in 1974 and 1977, the election of the left-hand President François Mitterrand in the second quarter of 1981, the stocks markets Black Monday in the fourth quarter of 1987, the Asian financial crisis at the end of 1990 and 1998, the internet bubble burst in 2001 and 2002, and finally the subprimes crisis which makes the French returns on equities visit the lower regime in 2008. Since only nine observations out of 168 are classified in this low regime<sup>6</sup>, it seems to capture the troughs, or crisis, rather than bear market times: The average of the log excess returns in this regime is -29.50%. The autoregressive coefficient estimate  $\rho_\ell$  is -0.26 — the nullity of this coefficient is rejected at the 6%-level — which implies strong mean reversion in this regime. In the upper regime, where most of the observations lie,  $\rho_u$  is 0.21 which suggests a slight persistence. The average of the observations lying in the upper regime is 3.39%.

The same troughs are captured by the lower regime in the unconstrained three-regime model retained for the US data. However, this model fails to capture a “normal” regime: While  $\lambda_\ell$  is estimated at -9.95%, the second threshold is also found strongly negative with an estimate of -6.81%. Looking at Figure 4, where  $\lambda_\ell$  and  $\lambda_u$  are plotted, it appears that the middle regime basically captures the returns fall which forgoes a large trough. As can be seen from Table 2,  $\rho_m = 13.21$ , which means that a negative observation in this regime will have a negative impact on the next observation, whereas there is still strong mean reversion in the lower regime through the corresponding positive intercept  $\mu_\ell$ .

Overall, these two SETAR models yield quite similar results: There is strong mean-reversion in the lower regime and there is some persistence in the upper regime, even though rather weak. If the dynamics of the log returns is regime-dependent, so should be the dynamics of the Value-at-Risk as will be checked below.

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<sup>6</sup>Note however that the estimated threshold is not the lower boundary of the grid-search interval.

## 4 The dynamics of Value-at-Risk

### 4.1 The $\text{VaR}_k$ based on filtered historical simulations

The bootstrap method described below belongs to the filtered historical simulation (FHS) method presented in Chirstoffersen [2009]. This method consists in simulating future returns from a model using historical return innovations. It is qualified by “filtered” because it does not use simulations from the set of returns directly, but from the set of shocks, which are basically returns such as filtered here by the SETAR model.

The FHS method described in Chirstoffersen [2009] would amount in our case to the following: First, using random draws from a uniform distribution, the estimated residuals of model (1) or (2) are resampled  $S$  times. Using these  $S$  series of  $v^s$  together with the estimated parameters of the corresponding model and the observed value of  $x_{e,t-1}$ ,  $S$  hypothetical sequences of  $x_{et}^k$  are obtained by forward recursion of the SETAR model. Finally, a *conditional*  $\text{VaR}_k(p)$  is calculated by retaining — amongst these  $S$  simulated sequences — the value of return such that there is a probability  $p$  that a worse value occurs at horizon  $k$ . This method clearly accounts for the uncertainty of the shocks realization. However, by setting  $x_{e,t-1}^s = x_{e,t-1}$ , it makes the VaR measure strongly dependent on the last available observations: contrary to the definition given earlier in Equation (4), this method defines the Value-at-Risk conditionally to past returns values. In order to illustrate this, Figures 1 and 2 report this time-dependent VaR measure calculated from 200,000 simulations for the one- to three-year investment horizons and for all  $t$  from 1980Q1 on. For each date  $t$ , the corresponding SETAR is estimated from 1971Q1 until  $t$ , which yields the  $k$ -year VaRs by the bootstrap method described above. These figures also plot the ex-post observed values of  $\exp(x_{et}^k) - 1$ . Even though time-dependent, these conditional VaRs are still much less volatile than the corresponding expected returns: the latter’s volatility is around four times larger than the conditional VaR at the one-year horizon and a little bit more than ten times larger at the 3-year horizon. Then, for all these investment horizons, the VaRs underestimates the stock return risk during the 2001-2002 and 2007-2008 episodes. Table 3

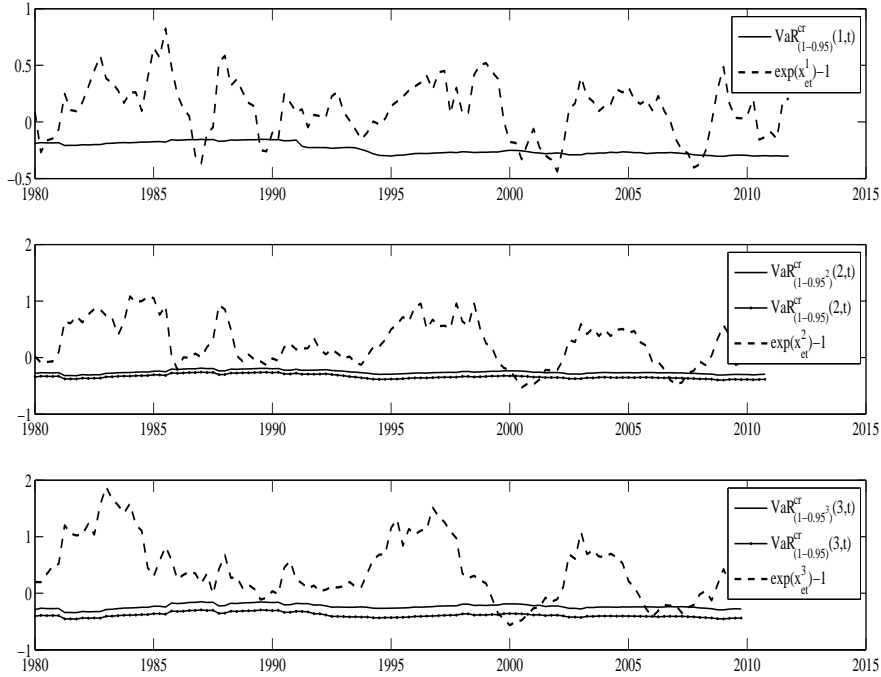


Figure 1: French VaR and corresponding observed returns

in appendix reports out-of-sample tests of predictive accuracy of the models considered here for the  $VaR(5\%)$ 's up to five years. Following the lines of e.g. Guidolin and Timmermann [2006], we consider the unconditional coverage probability which is the percentage of VaRs above the corresponding ex-post observed return, as well as the  $S_P$  test statistic given in Escanciano and Olmo [2009], Equation (5) therein. This  $S_P$  statistic refers to the so-called unconditional backtesting which tests whether or not the unconditional expectation of the 'hits' or 'exceedances' is equal to the theoretical one.<sup>7</sup> From the unconditional coverage probabilities, it turns out that the SETAR model is

<sup>7</sup>Under the null,  $(p(1-p))^{-0.5}S_P$  has a standard normal distribution given the DGP is known. In practice, the DGP parameters are estimated but Escanciano and Olmo [2009] show that this unconditional S-test still possesses rather good finite-sample power properties even in presence of estimation risk. See also Giacomini and White [2006] for a comparison of unconditional and conditional backtesting.

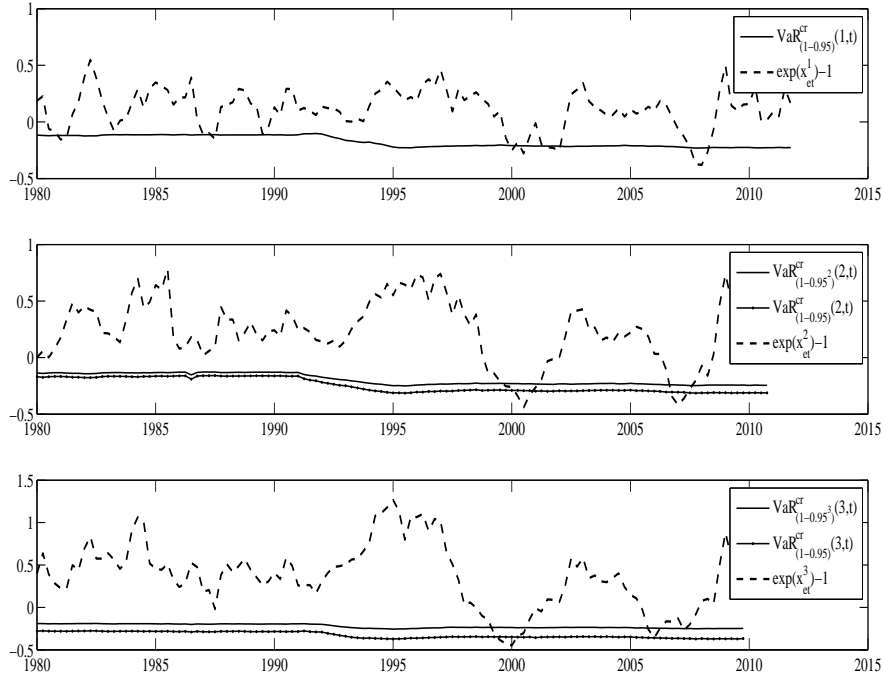


Figure 2: US VaR and corresponding observed returns

slightly too liberal at the one-year horizon. As expected, it tends to become much too conservative at longer horizons, particularly when the probability  $p$  is held constant over horizons. In France, the unconditional backtesting does not reject the null of forecasting accuracy for most horizons. In the US, the null is rejected after the three-year horizon only.

## 4.2 Empirical measures of $\text{VaR}_k$ across investment horizon and financial market state

Finally, since we aim at evaluating the impact of the state the financial market is in on the VaR for various investment horizons, we would rather control for its position. This is done by initializing the value of the excess return for the forward recursion of model

(1) or (2) with the average of observations lying respectively in the lower, middle (if any) and upper regimes. In this variant, we also adapt the bootstrap procedure to account for possibly neglected residuals heteroskedasticity following the lines described in e.g. Cavaliere, Rahbek and Taylor [2010]: instead of being resampled, the estimated SETAR residuals are multiplied by a Gaussian i.i.d.  $\mathcal{N}(0,1)$  sequence so that the resulting simulated residuals keep the same heteroskedastic features as the estimated ones.

The results reported below were obtained for  $S = 200,000$  simulations for each  $k = 1, \dots, 15$  years, from which the  $p \in \{0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5\}$  quantiles are picked up for each  $VaR_k^{cr}$ . Figure 3 plots the regime-dependent measures of  $VaR_k^{cr}$  described above against holding horizons up to fifteen years<sup>8</sup>.

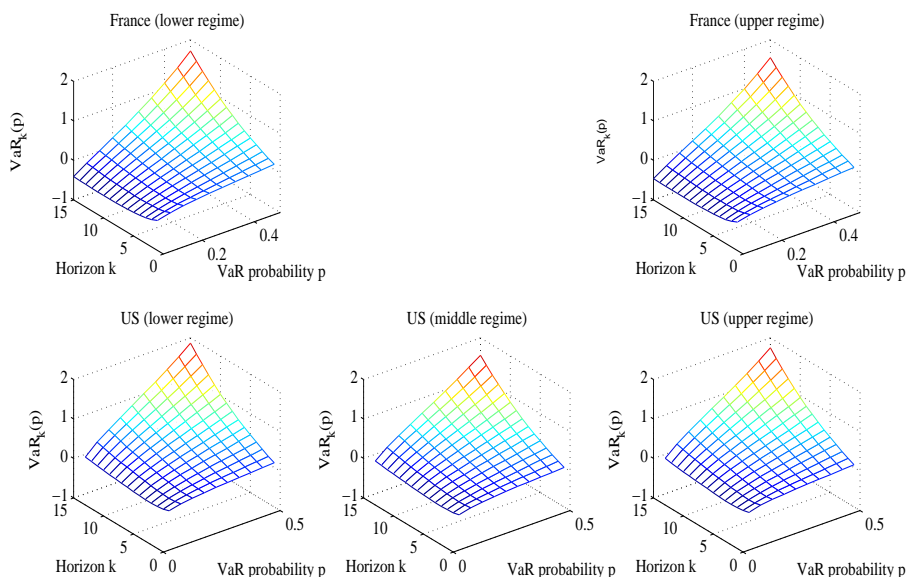


Figure 3: French Value-at-Risk( $95^k\%$ ) across cycle and horizons

The first important result emerging from this figure is that whatever the investment horizon, the VaR depends on the state the financial market is in. For all horizons, the VaR is worst in upper regime than in the lower one. The VaR's gap between these two

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<sup>8</sup>The corresponding figures are available upon request.

regimes at the one-year horizon is around 5% for both countries. This gap widens up to about 15% at the 15-year horizon, but such long-horizons results must be cautiously interpreted since the model is estimated using thirty-seven years only. Note that for all investment horizons in the US, the largest VaR in absolute value appears in the middle regime. This confirms our interpretation of this regime as the one capturing the returns fall which forgoes a large trough. The VaR's gap between the lower and middle regimes reaches 7% at the one-year horizon. Overall, these results suggest that a rule imposing the same solvency capital requirement whatever the state of the financial market could actually be pro-cyclical.

The second important result regards the dynamics of the VaR across investment horizons. In a previous study (see Bec and Gollier [2007]), mean-reversion was found in log returns on French equities relatively to other assets returns: their relative risk was found decreasing with the holding period. The same result was found by Campbell and Viceira [2002] for US quarterly data. This is confirmed by our results as can be seen in Figure 3.

As a further check, the simulations were also performed fitting distribution laws that allow for larger tails, since extreme risks can hardly be captured from such a short period as the one considered here. This is a shortcoming of the bootstrap approach retained above.

As can be seen in Figures 5 and 6 reported in appendix, only an extreme value distribution<sup>9</sup> is able to capture the fat left tail of the residuals distribution. Figures 7 and 8 in appendix show the resulting regime and horizon dependent Value-at-Risk when using 100,000 random draws from the fitted extreme value distribution (the Student-t distribution is also used for comparison purpose): As expected, the VaRs corresponding to small  $p$  worsen when residuals are drawn from the extreme value distribution, but both financial market state and horizon effects are confirmed.

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<sup>9</sup>The probability density function for the extreme value distribution with location parameter  $\mu$  and scale parameter  $\sigma$  is  $f(x|\mu, \sigma) = \sigma^{-1} \exp\left(\frac{x-\mu}{\sigma}\right) \exp\left(-\exp\left(\frac{x-\mu}{\sigma}\right)\right)$ .



## 5 Concluding remarks

The SETAR modelling of French and US stocks excess returns provides evidence of the regime-dependent nature of their dynamics: large downturns are strongly corrected whereas a slight persistence is found otherwise. Since the Value-at-Risk is evaluated from the expected excess returns, it is also influenced by the state the financial market is in. The VaR calculated from a trough corresponds to smaller losses than the ones evaluated in the bull market for all investment horizons. Our results provide support to the claim that fixed solvency capital requirements may have important procyclical consequences for the dynamic investment strategies of the financial intermediaries. They also suggest some predictability in equities returns since they point to a decrease in the absolute value of the VaR as the holding period increases. One limit of the approach retained here is that it assumes the existence of financial markets cycles without explaining their origins. A better understanding of this phenomenon is a challenging question on our research agenda.

## References

- Adrian, T. and H.S. Shin, Liquidity and financial contagion, *Financial Stability Review*, *Banque de France*, 2008, 11. February.
- and — , Liquidity and leverage, *Journal of Financial Intermediation*, 2010, 19, 418-437.
- Barberis, N., Investing for the long run when returns are predictable, *Journal of Finance*, 2000, 55, 225–264.
- Bec, F. and C. Gollier, *Assets Returns Volatility and Investment Horizon*, Working paper 467, IDEI, University of Toulouse 1 2007.
- Campbell, J., A variance decomposition for stock returns, *Economic Journal*, 1991, 101, 157–179.

- , Understanding risk and return, *Journal of Political Economy*, 1996, *104*, 298–345.
- , Asset Prices, Consumption, and the Business Cycle, in J. Taylor and M. Woodford, editors, *Handbook of Macroeconomics, vol.1*, North-Holland: Amsterdam, 1999, pp. 1231–1303.
- and L. Viceira, *Strategic Asset Allocation*, Oxford University Press, 2002.
- and —, *Long-Horizon Mean-Variance Analysis: A User Guide*, Manuscript, Dept of Economics, Harvard University, Cambridge MA 2004.
- Campbell, J.Y. and S. Thompson, Predicting the Equity Premium Out Of Sample: Can Anything Beat the Historical Average?, *Review of Financial Studies*, 2008, *21*, 1509–1531.
- Cavaliere, G., A. Rahbek, and R. Taylor, Testing for Co-integration in Vector Autoregressions with Non-Stationary Volatility, *Journal of Econometrics*, 2010, *158*, 7–24.
- Christoffersen, P., Value-at-Risk models, in T. Andersen, R. Davis, J.-P. Kreiss, and T. Mikosch, editors, *Handbook of Financial Time Series, vol.1*, Springer Verlag, 2009. Forthcoming.
- Escanciano, J. and J. Olmo, Backtesting parametric Value-at-Risk with estimation risk, *Journal of Business and Economic Statistics*, 2009.
- Feunou, B. and N. Meddahi, *Realized term structure of risk*, Manuscript, CIRANO, Canada 2007.
- Giacomini, R. and H. White, Tests of Conditional Predictive Ability, *Econometrica*, 2006, *74*, 1545–1578.
- Gollier, C., Intergenerational risk-sharing and risk-taking of a pension fund, *Journal of Public Economics*, 2008, *92*, 1463–1485.

- Guidolin, M. and A. Timmermann, Term Structure of Risk under Alternative Econometric Specifications, *Journal of Econometrics*, 2006, *131*, 285–308.
- Hansen, B. and B. Seo, Testing for Two-Regime Threshold Cointegration in Vector Error-Correction Models, *Journal of Econometrics*, 2002, *110*, 293–318.
- Hansen, B.E., Inference when a Nuisance Parameter Is Not Identified Under the Null Hypothesis, *Econometrica*, 1996, *64* (2), 413–430.
- Jondeau, E. and M. Rockinger, *Portfolio Allocation for European markets with Predictability and Parameter Uncertainty*, Mimeo, University of Lausanne 2009.
- Kashyap, A. and J. Stein, Cyclical implications of the Basel II capital standards, *Federal Reserve Bank of Chicago Economic Perspectives*, 2004, *Q1*, 18–31.
- Plantin, G., H. Saprà, and H.S. Shin, Marking-to-market: Panacea or Pandora’s box?, *Journal of Accounting Research*, 2008, *46*.
- Rochet, J.-C., Procyclicality of financial systems: Is there a need to modify current accounting and regulatory rules?, *Financial Stability Review, Banque de France*, 2008, *12*.

## Appendix

Table 2: SETAR estimates

	France	US
$\mu$	0.809 [0.39]	—
$\mu_\ell$	—	11.52 [0.04]
$\mu_m$	—	107.17 [0.00]
$\mu_u$	—	0.86 [0.30]
$\rho_\ell$	-0.257 [0.06]	0.372 [0.27]
$\rho_m$	—	13.21 [0.00]
$\rho_u$	0.215 [0.03]	0.234 [0.04]
$\lambda_\ell$	-22.56	-9.95
$\lambda_u$	—	-6.81
R-squared	0.05	0.17
ARCH(1) p-val.	0.39	0.68
ARCH(4) p-val.	0.58	0.16
Q(4) p-val.	0.94	0.56
White F p-val.	0.25	0.53

$p$ -values of  $t$ -statistics in [ ].

Table 3: Out-of-sample tests of predictive accuracy

	1 year		2 years		3 years		4 years		5 years	
	$p$	$p$	$p_{adj}$	$p$	$p_{adj}$	$p$	$p_{adj}$	$p$	$p_{adj}$	
Expected % of violations	5%	5%	9.7%	5%	14.3%	5%	18.5%	5%	22.6%	
France	8.6%	6.4%	7.3%	4.2%	10.8%	2.6%	13.8%	0%	15.2%	
U.S.	8.6%	4.8%	8.1%	2.5%	7.5%	0%	6%	0%	1.8%	
	Unconditional backtesting $((p(1-p))^{-0.5}S_P)$									
France	1.86	0.74	-0.93	-0.42	-1.07	-1.19	-1.32	-2.43*	-1.88	
U.S.	1.86	-0.08	-0.63	-1.26	-2.12*	-2.47*	-3.47*	-2.43*	-5.27*	

The adjusted expected % of violations,  $p_{adj}$ , is given by  $(1 - 0.95^k)$ .

$S_P$  is defined in Escanciano-Olmo (2009). ‘\*’ means rejection at 5%-level.

Table 4: France :  $\text{VaR}_k(p)$ , Extreme Value Simulations, Lower Regime

$p$	$(1 - 0.95^k)$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
Years											
1	-0.31	-0.31	-0.22	-0.16	-0.11	-0.07	-0.03	0.01	0.05	0.09	0.13
2	-0.30	-0.40	-0.30	-0.22	-0.15	-0.09	-0.04	0.02	0.08	0.13	0.19
3	-0.26	-0.45	-0.34	-0.25	-0.17	-0.10	-0.03	0.04	0.11	0.18	0.25
4	-0.21	-0.49	-0.37	-0.27	-0.18	-0.10	-0.02	0.06	0.14	0.23	0.32
5	-0.13	-0.52	-0.39	-0.28	-0.18	-0.09	0.00	0.09	0.19	0.29	0.39
6	-0.05	-0.54	-0.40	-0.29	-0.18	-0.08	0.02	0.13	0.24	0.35	0.47
7	0.05	-0.55	-0.41	-0.29	-0.18	-0.07	0.05	0.16	0.28	0.41	0.55
8	0.17	-0.57	-0.42	-0.29	-0.17	-0.05	0.08	0.20	0.33	0.48	0.64
9	0.30	-0.57	-0.42	-0.29	-0.16	-0.03	0.11	0.25	0.39	0.55	0.72
10	0.45	-0.58	-0.43	-0.28	-0.14	-0.01	0.14	0.29	0.45	0.63	0.81
11	0.63	-0.59	-0.43	-0.27	-0.13	0.02	0.18	0.34	0.51	0.70	0.91
12	0.83	-0.60	-0.42	-0.27	-0.11	0.05	0.21	0.39	0.58	0.79	1.02
13	1.06	-0.60	-0.42	-0.26	-0.09	0.08	0.25	0.44	0.65	0.88	1.13
14	1.32	-0.60	-0.42	-0.25	-0.08	0.10	0.29	0.50	0.72	0.97	1.25
15	1.62	-0.61	-0.41	-0.23	-0.05	0.13	0.33	0.55	0.79	1.06	1.37

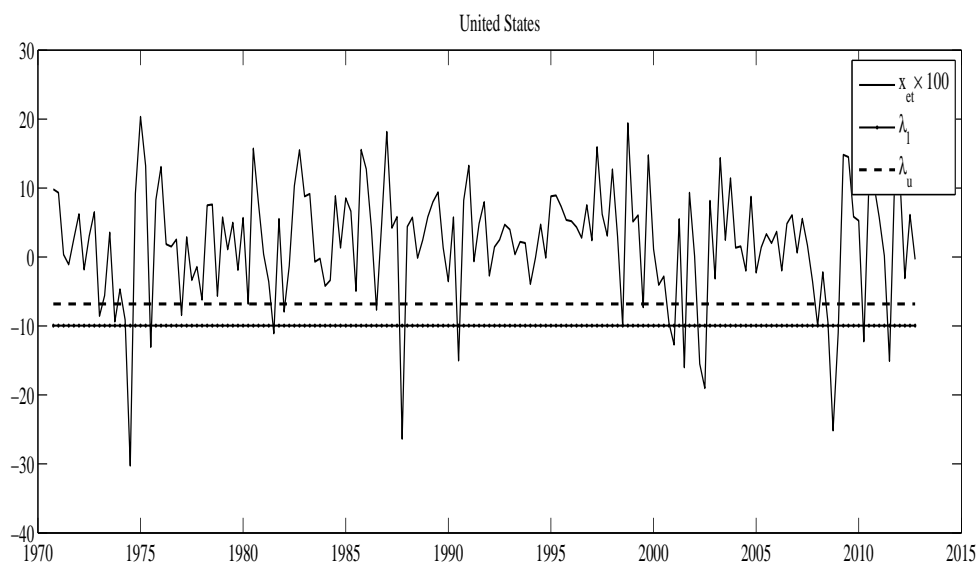
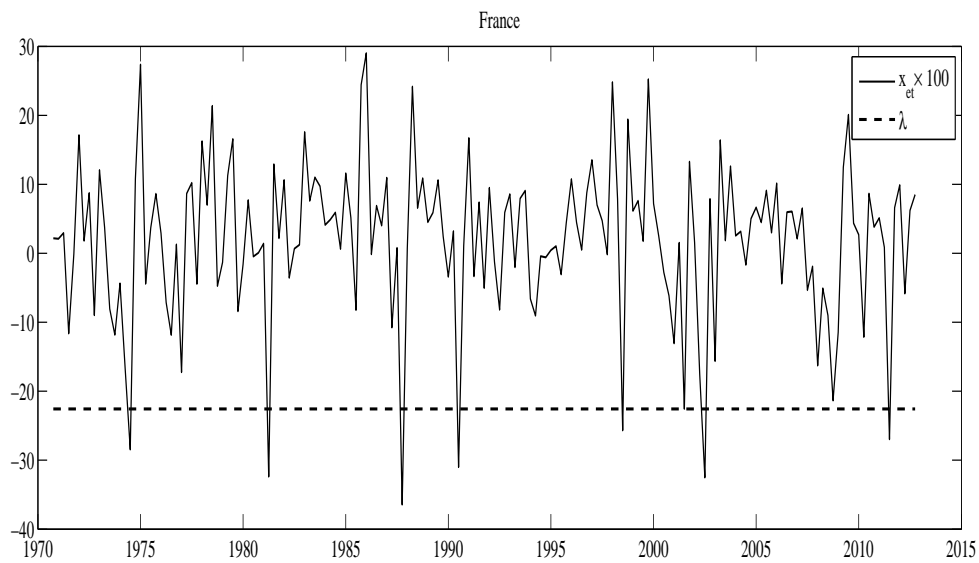


Figure 4: Data (1970Q4—2012Q4)

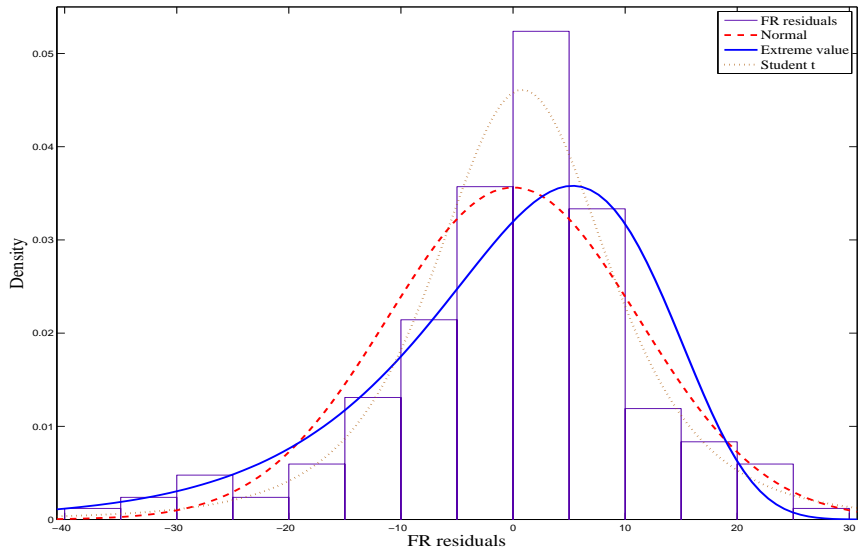


Figure 5: French residuals distribution fits

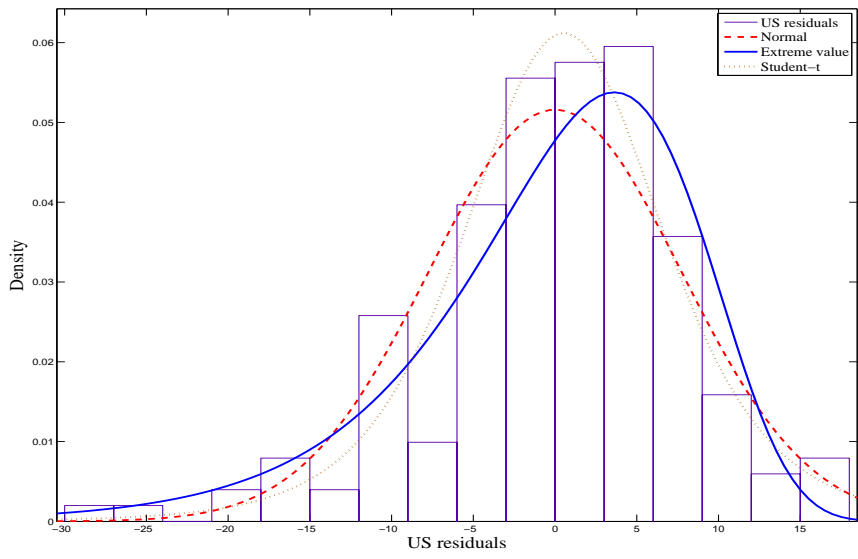


Figure 6: US residuals distribution fits

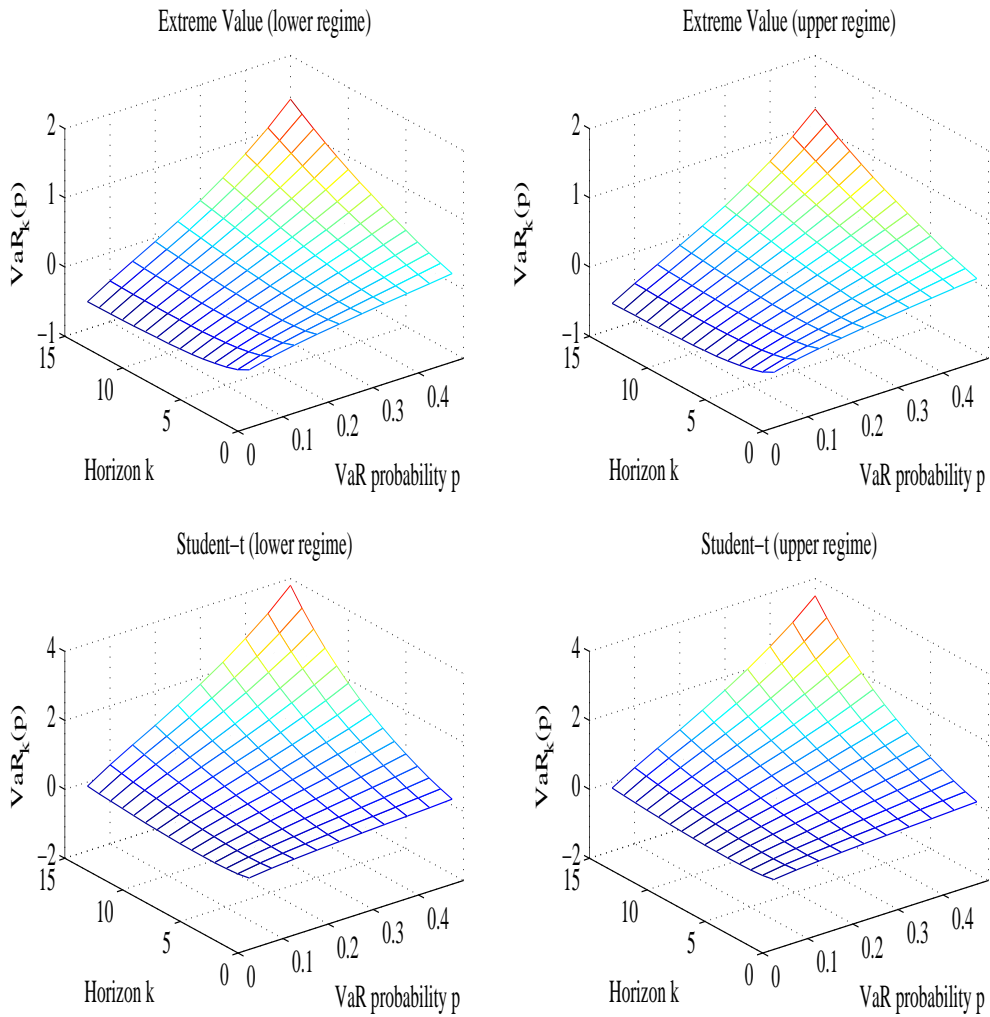


Figure 7: Regime and horizon dependent VaR in France: EV and  $t$  distributions



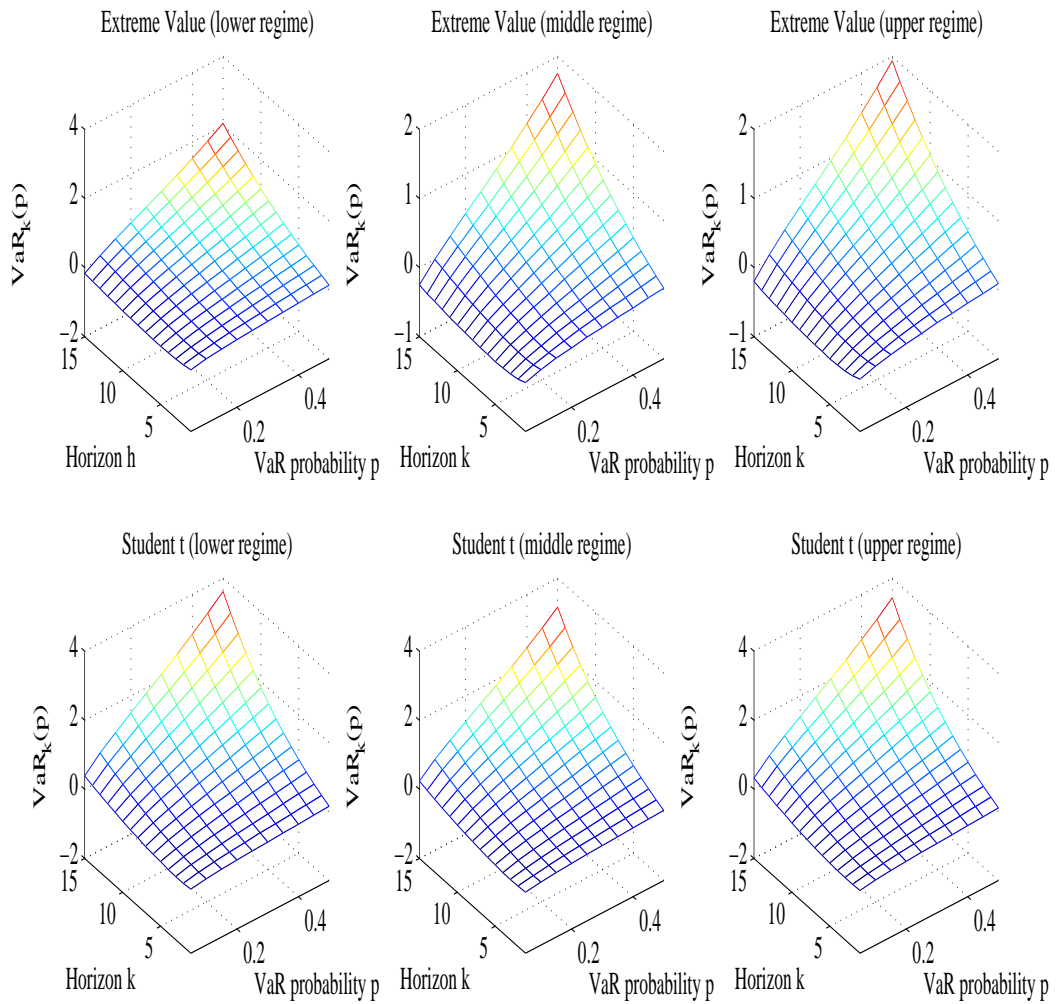


Figure 8: Regime and horizon dependent VaR in the US: EV and  $t$  distributions

Table 5: France :  $\text{VaR}_k(p)$ , Extreme Value Simulations, Upper Regime

$p$	$(1 - 0.95^k)$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
Years											
1	-0.34	-0.34	-0.27	-0.21	-0.16	-0.12	-0.08	-0.05	-0.01	0.03	0.06
2	-0.34	-0.43	-0.34	-0.27	-0.20	-0.15	-0.09	-0.04	0.01	0.06	0.12
3	-0.30	-0.48	-0.38	-0.29	-0.22	-0.15	-0.09	-0.03	0.04	0.11	0.17
4	-0.25	-0.52	-0.40	-0.31	-0.23	-0.15	-0.08	0.00	0.07	0.15	0.24
5	-0.19	-0.54	-0.42	-0.32	-0.23	-0.14	-0.06	0.03	0.12	0.21	0.31
6	-0.10	-0.57	-0.44	-0.33	-0.23	-0.13	-0.04	0.06	0.16	0.27	0.38
7	-0.01	-0.58	-0.45	-0.33	-0.23	-0.12	-0.02	0.09	0.21	0.33	0.46
8	0.10	-0.59	-0.45	-0.33	-0.22	-0.10	0.01	0.13	0.25	0.39	0.54
9	0.22	-0.60	-0.46	-0.33	-0.21	-0.08	0.04	0.17	0.31	0.45	0.62
10	0.37	-0.61	-0.46	-0.32	-0.19	-0.07	0.07	0.21	0.36	0.53	0.70
11	0.53	-0.62	-0.46	-0.32	-0.18	-0.04	0.11	0.26	0.42	0.60	0.80
12	0.72	-0.62	-0.46	-0.31	-0.17	-0.02	0.14	0.31	0.48	0.68	0.90
13	0.93	-0.62	-0.46	-0.30	-0.15	0.01	0.18	0.36	0.55	0.76	1.00
14	1.18	-0.63	-0.45	-0.29	-0.13	0.04	0.21	0.41	0.62	0.85	1.11
15	1.46	-0.63	-0.45	-0.28	-0.11	0.06	0.25	0.46	0.68	0.94	1.23

Table 6: France :  $\text{VaR}_k(p)$ , Student-t Simulations, Lower Regime

$p$	$(1 - 0.95^k)$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
Years											
1	-0.22	-0.22	-0.14	-0.09	-0.04	0.00	0.03	0.07	0.10	0.14	0.17
2	-0.18	-0.28	-0.18	-0.11	-0.04	0.02	0.07	0.12	0.18	0.23	0.29
3	-0.11	-0.31	-0.19	-0.10	-0.01	0.06	0.13	0.20	0.28	0.35	0.42
4	0.00	-0.32	-0.18	-0.07	0.02	0.12	0.21	0.30	0.39	0.48	0.58
5	0.13	-0.32	-0.17	-0.04	0.07	0.18	0.29	0.40	0.51	0.62	0.75
6	0.30	-0.32	-0.14	0.00	0.13	0.26	0.38	0.51	0.65	0.78	0.93
7	0.49	-0.31	-0.12	0.05	0.20	0.34	0.48	0.64	0.79	0.96	1.13
8	0.72	-0.30	-0.08	0.10	0.27	0.43	0.60	0.77	0.95	1.15	1.36
9	1.00	-0.28	-0.05	0.16	0.35	0.54	0.72	0.92	1.13	1.36	1.61
10	1.34	-0.26	-0.01	0.22	0.44	0.65	0.86	1.09	1.33	1.59	1.89
11	1.75	-0.24	0.05	0.30	0.54	0.77	1.01	1.27	1.56	1.87	2.20
12	2.22	-0.21	0.10	0.37	0.64	0.90	1.18	1.48	1.80	2.15	2.54
13	2.79	-0.18	0.15	0.46	0.76	1.05	1.37	1.70	2.07	2.46	2.91
14	3.47	-0.15	0.22	0.55	0.88	1.21	1.57	1.94	2.35	2.81	3.33
15	4.30	-0.12	0.29	0.66	1.01	1.39	1.78	2.21	2.68	3.21	3.81