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Abstract

This article is the first to examine electric power producers' investment decisions when competition is imperfect and the transmission grid congested. This analysis yields numerous original insights. First, congestion on the grid is transient, and may disappear when demand is highest. Second, transmission capacity increases have complex impacts on generation: they may increase, decrease, or have no impact on the marginal value of generation, and may have similar or opposite impacts on the marginal value of different technologies. Third, the true social value of transmission, including its impact on investment, may be significantly lower than is commonly assumed.

Keywords: electric power markets, imperfect competition, investment, transmission constraints

JEL Classification: L11, L94, D61

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1 Introduction

The electricity industry has been restructured for about twenty years in many countries. Former regional or national monopolies have been dismantled. Electricity production and supply (retail) have been opened to competition. One essential objective of the restructuring was to push to the market decisions and risks associated with investment in electric power production (Joskow (2008)). It was expected that efficiency gains from competitive pressure would more than compensate for the loss of coordination in planning electricity generation and transmission infrastructure.

Twenty years later, the perspective is rather different: policy makers in Europe and the United States are concerned that generation and transmission investments are poorly coordinated. To examine this issue, this article develops a model of investment in generation assets, that incorporates imperfect competition among producers and constraints on the transmission grid. As discussed below, these are essential features of the investment decision.

In most countries, only a handful of companies compete to develop and operate electric power plants. While their number varies by country, less than ten in most European markets, more in most North American markets, no observer argues that the industry is perfectly competitive. An analysis of investment in power generation must therefore incorporate imperfect competition.

Constraints on the transmission grid split power markets into sub-markets. This is not surprising: historically, incumbents developed the transmission grid to move power within their service area. Interconnections were built primarily to provide reliability, not to facilitate trade. Maybe more surprising has been the difficulty faced by would-be developers of new transmission lines. Two reasons explain this quasi-impossibility: first, Not In My Back Yard (NIMBY) opposition by local communities and general environmental constraints and limitations. Second, economic difficulty in apportioning the costs and benefits of transmission expansion among all stakeholders (Hogan (2013)).

Investors therefore incorporate their competitors' strategies and constraints on the transmission grid as they analyze possible generation investment: most energy companies develop and run power flow models that predict prices in different markets, taking into account transmission constraints and confirmed and planned generation and transmission expansion.

As will be discussed in Section 2, previous articles have examined the impact of transmission constraints in imperfectly competitive spot markets, while another branch of the literature has examined investment decisions in a single market. This article is the first to examine investment decisions under imperfect competition in the presence of transmission constraints. Using a simple network topology, presented later in this introduction, this analysis yields four main insights of relevance for policy making, which were not available using the previous analyses.

First, *congestion is dynamic and potentially transient.* Consider the simple case of two markets, linked by an interconnection. Demand varies across states of the world. Marginal cost in market $i = 1, 2$ is c_i. Without loss of generality, assume $c_1 < c_2$. Suppose that the line becomes congested from market 1 to market 2: producers in market 1 would like to export their cheaper power into market 2, but are limited by the interconnection capacity. Previous spot-market analyses, that ignored investment in generation, concluded that the line remains congested. However, the analysis presented in this article proves this intuition wrong: to cover capital cost, the price in market 1 must rise above c_1 for some states of the world, and reaches c_2, at which point the interconnection is no longer congested: congestion on the interconnection is thus transient.

Second, *transmission constraints modify generation investment in a non-trivial way.* Transmission and generation can be complements or substitutes, i.e., an increase in transmission capacity may increase, decrease, or have no impact on the marginal value of generation capacity. It may have similar or opposite impacts on the marginal value of baseload (low marginal cost) and peaking (high marginal cost) technologies.

These first two observations highlight the complex interaction between transmission and generation. To fully understand the impact of policies they propose, policy makers cannot simply rely on general economic principles. They must develop detailed models of the industry, that include the transmission network.

Third, *the social value of transmission is not solely the difference in marginal costs*, as is commonly assumed, but also includes the impact of transmission on investment in generation and on competitive intensity. This observation is crucial to evaluate the benefits of new transmission projects. The resulting value may be lower than the simple difference in marginal costs. On a simple example, the article shows that the standard approach, that uses only marginal cost, overstates the social value of the interconnection capacity by almost 330%.

Finally, *the impact of an increase in interconnection capacity on producers' profits is unclear.* Thus suggests producers may not be best positioned to advocate or finance grid re-inforcement. This conclusion had been reached from the analysis of short-term competitive interactions (e.g., Léautier (2001)). It is now confirmed when long-term investment incentives are taken into account. This strengthens the policy objective of vertical separation between producers and transmission grid owners.

This article uses the simplest network topology: two markets, linked by one interconnection. Demand varies across states of the world. One technology is available in each market. The baseload technology, located in market 1, has lower marginal cost and higher investment cost than the peaking technology, located in market 2. This simple setup is more realistic than it seems. Real power networks consists of course of multiple interconnected zones, but to a first approximation, many can be represented by two zones: for example in Britain, north (gas fired production) and south (high London demand); upstate and downstate New York (separated by the Central East constraint); northern and southern California; and in Germany, north (off shore wind mills) and south (industrial Bavaria). Furthermore, constraints exist precisely because production costs differ, thus assuming a single technology by region is an adequate first step.

This article also assumes congestion on the grid is managed via Financial Transmission Rights ($FTRs$, a precise definition is provided later). Since $FTRs$ are used in most US markets and are progressively implemented in Europe, this assumption provides a reasonable description of reality.

Finally, I consider N symmetric generation firms, present in both markets, hence having access to both generation technologies. This assumption is not always met in practice, since firms are rarely exactly symmetric. However, it is consistent with the long-term equilibrium, which is the focus on this article: with free entry, firms enter each market as long as it remains profitable, and develop, in each market, the available generation technology.

With these assumptions, the transmission-constrained Cournot equilibrium can be easily compared to the transmission-constrained social optimum, and to the unconstrained Cournot equilibrium.

This article's scientific contribution is threefold: first, it characterizes the imperfectly competitive investment in the presence of transmission constraints (Proposition 1). If the interconnection is "large" (but not so that large that is never congested), it is congested from the baseload market 1 to the peaking market 2 for some states of the world. The aggregate cumulative capacity in equilibrium is not affected by the congestion, while the equilibrium baseload capacity is the uncongested baseload capacity, weighted by the size of its domestic market, plus the interconnection capacity.

If the interconnection is "thin", it is first congested from market 1 to market 2, then for higher-demand states of the world, from market 2 to market 1. The equilibrium peaking capacity is the cumulated uncongested capacity, weighted by the size of its domestic market, plus the capacity of the interconnection. The equilibrium baseload capacity is the solution of a simple first-order condition.

The impact of an interconnection capacity increase on installed generation capacity in each market is shown to have counterintuitive. properties. In particular, the impact is reversed as the line moves from "thin" to "large".

These effects are derived analytically and illustrated using a stylized representation of the French and British markets.

Second, this article determines the marginal social value of interconnection capacity (Proposition 2): an increase in interconnection capacity reduces the short-term cost of congestion, but also modifies the equilibrium generation investment and the competitive intensity. While the net welfare impact is always positive, it may be much lower than is generally assumed.

Third, this article shows that an increase in interconnection capacity has an ambiguous impact on producers profits (Proposition 3): it increases the FTR payment, but it also modifies generation investment. The net effect may be positive or negative.

This article is structured as follows. Section 2 relates this article to the academic literature. Section 3 presents the setup and the equilibrium investment without transmission constraints, that closely follows Zöttl (2011). Section 4 derives the equilibrium investment when the interconnection is congested. Section 5 derives the marginal social value of interconnection capacity. Section 6 derives the marginal value of interconnection capacity for the producers. Finally, Section 7 presents concluding remarks and avenues for further research. Technical proofs are presented in the Appendix.

2 Review of the academic literature

This article brings together three distinct streams of literature. First, electrical engineering and operations research scientists, for example Schweppe et al. (1988), have determined the optimal vertically integrated investment plan from an engineering/economics perspective.

A second series of articles has examined imperfect competition in the spot market when transmission constraints are present (for example, Borenstein and Stoft (2001), Cardell et al. (1997), Léautier (2001), Willems (2002), and more recently the empirical analysis by Wolak (2013)). This article's setup is almost identical to Borenstein and Stoft (2001): two markets linked an interconnection, and two production technologies. The main difference is that producers here are present in both markets, and own $FTRs$. As will be shown later, this considerably simplifies the analysis of the spot market equilibrium.

Finally, other articles have examined the investment decision for a single market. This literature started with the peak-load pricing analysis of Boiteux (1949), and Crew and Kleindorfer (1976), that examine the economic optimum. Borenstein and Holland (2005) determine the perfectly competitive outcome. Joskow and Tirole (2006) examine the perfect and imperfect competition cases. Zöttl (2011) develops a model of Cournot competition and investment in a single market. This article extends Zöttl (2011) analysis to include multiple markets, separated by a congested interconnection.

Ruderer and Zöttl (2012) is the closest to this work, that examines the impact of transmission pricing rules on investment, under perfect competition. This work thus extends Ruderer and Zöttl (2012) by incorporating imperfect competition.

3 Uncongested investment

3.1 Assumptions and definitions

Demand All customers are homogenous. Individual demand is $D(p,t)$, where p is the electricity price, and $t \geq 0$ is the state of the world, distributed according to cumulative distribution $F(.)$, and probability distribution $f(.) = F'(.)$.

Assumption 1 *$\forall t \geq 0, \forall q \leq Q$, the inverse demand $P(Q,t)$ satisfies*[1]

1.

$$P_q(Q,t) < 0 \text{ and } P_q(Q,t) < -qP_{qq}(Q,t).$$

[1] Using the usual notation: for any function $g(x,y)$, $g_x = \frac{\partial g}{\partial x}$, $g_y = \frac{\partial g}{\partial y}$, and g_{xx}, g_{xy}, and g_{yy} are the second derivatives.

2.

$$P_t(Q,t) > 0 \text{ and } P_t(Q,t) > q\left|P_{qt}(Q,t)\right|.$$

$P_q < 0$ requires inverse demand to be downward sloping. $P_q(Q,t) < -qP_{qq}(Q,t)$ implies that the marginal revenue is decreasing with output

$$\frac{\partial^2}{\partial q^2}(qP(Q,t)) = 2P_q(Q,t) + qP_{qq}(Q,t) < 0,$$

and guarantees existence and unicity of a Cournot equilibrium.

$P_t > 0$ orders the states of the world, $P_t(Q,t) > q\left|P_{qt}(Q,t)\right|$ implies that the marginal revenue is increasing with the state of the world

$$\frac{\partial^2}{\partial t \partial q}(qP(Q,t)) = P_t(Q,t) + qP_{qt}(Q,t) > 0,$$

and that the Cournot output and profit (defined later) are increasing.

Assumption 1 is met for example if demand is linear with constant slope $P(Q,t) = a(t) - bQ$, with $b > 0$ and $a'(t) > 0$.

Customers are located in two markets, indexed by $i = 1, 2$. Total mass of customers is normalized to 1, a fraction $\theta_i \in (0,1)$ of customers is located in market i. Demands in both markets are thus perfectly correlated.

Supply Two production technologies are available, indexed by $i = 1, 2$, and characterized by variable cost c_i and capital cost r_i, expressed in $€/MWh$. Technology 1 is the baseload technology: $c_1 < c_2$ and $r_1 > r_2$. For example, technology 1 is nuclear generation, while technology 2 is gas-fired generation. Investing and using both technologies is assumed to be economically efficient. Precise sufficient conditions are provided later in this Section.

Technology 1 (resp. 2) can be installed in market 1 (resp. 2) only. This is not unrealistic: the mix of technologies chosen to produce electricity depends on the resource endowment of a market. For example, market 1 could be France, which uses nuclear generation, and market 2 could be Britain, which uses gas-fired generation, or market 1 could be the western portion of the PJM market (coal), and market 2 could be the eastern sea shore of PJM (gas).

Each producer has access to both technologies. N symmetric producers compete à la Cournot in both markets.

Firms profits In state t, firm n produces $q_i^n(t)$ using technology i. Its cumulative production is $q^n(t)$. Aggregate production using technology i is $Q_i(t)$, which is also the aggregate production in market i. $Q(t)$ is the aggregate cumulative production. If both markets are perfectly connected, firm n operating profit in state t is

$$\pi^n(t) = q^n(t) P(Q(t), t) - c_1 q_1^n(t) - c_2 q_2^n(t) = q^n(t) (P(Q(t), t) - c_2) + q_1^n(t)(c_2 - c_1).$$

For $i = 1, 2$, firm n capacity invested in technology i is k_i^n, aggregate capacity invested technology i is $K_i = \sum_{n=1}^N k_i^n$, also the aggregate capacity in market i. Producer n cumulative capacity is k^n, and aggregate cumulative capacity is $K = \sum_{n=1}^N k^n$.

Critical states of the world and value functions The equilibrium output of a symmetric N-firm Cournot equilibrium with cost c is $Q^C(c, t)$, uniquely defined by

$$P\left(Q^C(c,t), t\right) + \frac{Q^C(c,t)}{N} P_q\left(Q^C(c,t), t\right) = c.$$

Consider a producer with marginal cost $c > 0$ and capacity $z > 0$, while aggregate capacity is $Z > 0$. The first state of the world for which the marginal revenue of this producer is equal to c is $\hat{t}(z, Z, c)$, uniquely defined by

$$P\left(Z, \hat{t}(z,Z,c)\right) + z P_q\left(Z, \hat{t}(z,Z,c)\right) = c.$$

As will be proven below, the marginal value of capacity is $\Psi(z, Z, c)$, defined by

$$\Psi(z, Z, c) = \int_{\hat{t}(z,Z,c)}^{+\infty} (P(Z,t) + z P_q(Z,t) - c) f(t) \, dt.$$

3.2 Equilibrium investment absent congestion

Producers play a two-stage game. In the first-stage, they decide on their baseload and peaking capacities. In the second stage, they compete à la Cournot in each state of the world, constrained by their installed capacities.

Lemma 1 *(Zöttl (2011)) The unique symmetric equilibrium* $\left(K_1^U, K^U\right)$ *of the investment-then-production game is characterized by*

$$\Psi\left(K^U, c_2\right) = \int_{\hat{t}(K^U, c_2)}^{+\infty} \left(P\left(K^U, t\right) + \frac{K^U}{N} P_q\left(K^U, t\right) - c_2\right) f(t)\, dt = r_2 \tag{1}$$

and

$$\begin{aligned}\Psi\left(K_1^U, c_1\right) - \Psi\left(K_1^U, c_2\right) &= \int_{\hat{t}\left(K_1^U, c_1\right)}^{\hat{t}\left(K_1^U, c_2\right)} \left(P\left(K_1^U, t\right) + \frac{K_1^U}{N} P_q\left(K_1^U, t\right) - c_1\right) f(t)\, dt + \int_{\hat{t}\left(K_1^U, c_2\right)}^{+\infty} (c_2 - c_1) f(t)\, dt \\ &= r_1 - r_2. \end{aligned} \tag{2}$$

Proof. *The reader is referred to Zöttl (2011) for the proof. Intuition for the result can be obtained by assuming firms play a symmetric equilibrium, and deriving the necessary first-order conditions. Suppose firms play a symmetric strategy: for all* $n = 1, ..., N$, $k_1^n = \frac{K_1}{N}$ *and* $k^n = \frac{K}{N}$. *Firms first play a* N*-firm Cournot game with cost* c_1. *For* $t \geq \hat{t}(K_1, c_1)$, *all firms produce at their baseload capacity. Price is thus determined by the intersection of the (vertical) supply and the inverse demand curves. For* $t \geq \hat{t}(K_1, c_2)$, *all firms start using peaking technology, and play a* N*-firm Cournot game with cost* c_2. *Finally, for* $t \geq \hat{t}(K, c_2)$, *all firms produce at their cumulative capacity, and the price is again set by the intersection of the (vertical) supply and the inverse demand curves. This yields expected profit for firm* n

$$\begin{aligned}\Pi^U\left(k^n, k_1^n\right) &= \int_0^{\hat{t}(K_1, c_1)} \frac{Q^C(c_1, t)}{N} \left(P\left(Q^C(c_1, t), t\right) - c_1\right) f(t)\, dt + \int_{\hat{t}(K_1, c_1)}^{\hat{t}(K_1, c_2)} k_1^n \left(P(K_1, t) - c_1\right) f(t)\, dt \\ &+ \int_{\hat{t}(K_1, c_2)}^{\hat{t}(K, c_2)} \left(\frac{Q^C(c_2, t)}{N} \left(P\left(Q^C(c_2, t), t\right) - c_2\right) + k_1^n (c_2 - c_1)\right) f(t)\, dt \\ &+ \int_{\hat{t}(K, c_2)}^{+\infty} \left(k^n \left(P(K, t) - c_2\right) + k_1^n (c_2 - c_1)\right) f(t)\, dt - (r_1 - r_2)\, k_1^n - r_2 k^n,\end{aligned}$$

which can be rewritten as

$$\Pi^U \left(k^n, k_1^n\right) = B\left(k_1^n, K_1, c_1, c_2\right) - \left(r_1 - r_2\right) k_1^n + A\left(k^n, K, c_2\right) - r_2 k^n, \tag{3}$$

where

$$A(z, Z, c) = \int_{\hat{t}(z,Z,c)}^{+\infty} \left(z\left(P(Z,t) - c\right) - \left(\frac{Q^C(c,t)}{N} \left(P\left(Q^C(c,t), t\right) - c\right)\right)\right) f(t)\, dt,$$

and

$$B(z, Z, c_1, c_2) = A(z, Z, c_1) - A(z, Z, c_2) + \int_0^{+\infty} \frac{Q^C(c_1,t)}{N} \left(P\left(Q^C(c_1,t), t\right) - c_1\right) f(t)\, dt.$$

$\Pi^U \left(k^n, k_1^n\right)$ is separable in $\left(k^n, k_1^n\right)$. This is a fundamental economic property of the problem: the determination of the cumulative and the baseload capacities are independent.

Observe that $\Psi(z, Z, c)$ is the derivative of $A(z, Z, c)$ at a symmetric equilibrium, i.e., if $z = \frac{Z}{N}$:

$$\left. \frac{\partial A}{\partial z}(z, Z, c) + \frac{\partial A}{\partial Z}(z, Z, c) \right|_{z=\frac{Z}{N}} = \Psi\left(\frac{Z}{N}, Z, c\right).$$

To simplify the notation, I use $\hat{t}(Y,c) \equiv \hat{t}\left(\frac{Y}{N}, Y, c\right)$, $\Psi(Y,c) \equiv \Psi\left(\frac{Y}{N}, Y, c\right)$, $A(Y,c) \equiv A\left(\frac{Y}{N}, Y, c\right)$, and $B(Y, c_1, c_2) \equiv B\left(\frac{Y}{N}, Y, c_1, c_2\right)$ to characterize symmetric equilibria. Then, maximizing equation (3) with respect to k^n (resp. k_1^n), then setting $k^n = \frac{K}{N}$ (resp. $k_1^n = \frac{K_1}{N}$) yields the first-order condition (1) (resp. (2)). The structure of the equilibrium is illustrated on Figure 1. By considering upward and downward deviations, Zöttl (2011) proves that $\left(\frac{K^U}{N}, \frac{K_1^U}{N}\right)$ is indeed the unique symmetric equilibrium, if c_2 and c_1 are sufficiently different. ■

Cumulative capacity has value only when it is constrained, hence only states of the world $t \geq \hat{t}\left(K^U, c_2\right)$ appear in equation (1). As usual with Cournot games, a marginal capacity increase generates incremental margin $\left(P(K,t) - c_2\right)$ and reduces margin on all inframarginal units. Equilibrium capacity balances this expected gain against the marginal investment cost r_2.

Similarly, only states of the world $t \geq \hat{t}\left(K_1^U, c_1\right)$ appear in equation (2). A marginal substitution of baseload for peaking capacity increases the marginal revenue when baseload capacity is constrained but not yet marginal, and reduces the cost of production by $(c_2 - c_1)$ in all of states where the peaking technology is marginal. Equilibrium capacity exactly balances this gain against the marginal cost of the substitution $(r_1 - r_2)$. An alternative interpretation is that a marginal substitution of one unit of baseload for peaking capacity substitutes $(\Psi(K_1, c_1) - r_1)$ for $(\Psi(K_1, c_2) - r_2)$. At the equilibrium, both values are equal.

Equations (1) and (2) are closely related to the expressions defining the welfare maximizing capacity. Define $\hat{t}_0(Z, c)$ and $\Psi_0(Z, c)$ by

$$P\left(Z, \hat{t}_0(Z, c)\right) = c \text{ and } \Psi_0(Z, c) = \int_{\hat{t}_0(Z,c)}^{+\infty} (P(Z, t) - c) f(t) \, dt.$$

The peak load pricing literature (for example, Léautier (2013)), proves that the optimal cumulative capacity K^* and baseload capacity K_1^* are respectively defined by

$$\Psi_0(K^*, c_2) = r_2 \text{ and } \Psi_0(K_1^*, c_1) - \Psi_0(K_1^*, c_2) = r_1 - r_2.$$

The equilibrium capacities are simply obtained by replacing inverse demand by marginal revenue in the first-order conditions. This result arises because producers invest in both technologies, thus fully internalize the value of the substitution between baseload and peaking technologies, which coincides with the social optimum.

I have sofar assumed existence and unicity of $\left(K_1^U, K^U\right)$. A set of necessary and sufficient conditions is:

Assumption 2 Necessary and sufficient conditions for existence of $\left(K_1^U, K^U\right)$

1. *In every state of the world, the first unit produced is worth more than its marginal cost:* $P(0, t) > c_2 \; \forall t \geq 0$*; on average, the first unit produced is worth more than its long-term marginal cost:* $\mathbb{E}[P(0, t)] > c_2 + r_2$.

2. *Technology* 2 *exhibits higher long-term marginal cost than technology* 1*:* $c_2 + r_2 > c_1 + r_1$.

3. *Equilibrium cumulative capacity is higher using technology* 2 *than using technology* 1*:*

$$\chi(c_2, r_2) > \chi(c_1, r_1),$$

where $\chi(c,r)$ is the unique solution to $\Psi(\chi(c,r),c) = r$.

4. c_2 and c_1 are sufficiently different.

The first part of Assumption 2 guarantees existence of $K^U > 0$ solution of first-order condition (1), its second part guarantees existence of $K_1^U > 0$ solution of first-order condition (2), and its third part guarantees that $K^U > K_1^U$. Zöttl (2011) proves that the last part guarantees that there is no incentive for an upward deviation from K_1^U, hence that K_1^U is indeed an equilibrium. As will be shown below, the latter condition is not required when the interconnection is congested, thus I do not explicit it further.

4 Equilibrium investment when the interconnection is congested

We now introduce the possibility that the interconnection may be congested.

Congestion, Financial transmission Rights, and firms profits $\varphi(t)$ is the flow on the interconnection from market 1 to market 2 in state t. The power flowing on the interconnection is limited by the technical characteristics of the line and reliability operating standards. The maximum flow on the interconnection from market 1 to market 2 (resp. from market 2 to market 1) is Φ^+ (resp. Φ^-). Since reliability the constraints imposed by operating standards are not symmetrical, maximum admissible flows are not in general symmetrical i.e., $\Phi^+ \neq \Phi^-$. The transmission constraints are thus

$$-\Phi^- \leq \varphi(t) \leq \Phi^+.$$

Congestion on the interconnection is managed using Financial Transmission Rights ($FTRs$, Hogan (1992)). Each firm owns (or has rights to) $\frac{1}{N}th$ of the available $FTRs$. I assume producers do not include the acquisition cost of $FTRs$ in their analysis. For example, they are granted $FTRs$, as was the case in the Mid Atlantic market in the United States. Further work will examine how the equilibrium is modified when this assumption is relaxed.

If the line is not congested, each firm receives the single market price for its entire production, and no congestion revenue, as was the case in Section 3. Uncongested flows, prices, and quantities are illustrated on Figure 2.

If the interconnection is congested, $p_i(t)$, the price in market 1 reflects local supply and demand conditions. For example, if the interconnection is congested from market 1 to market 2,

$$\begin{cases} \theta_1 D\left(p_1\left(t\right), t\right) = Q_1\left(t\right) - \Phi^+ \\ \theta_2 D\left(p_2\left(t\right), t\right) = Q_2\left(t\right) + \Phi^+ \end{cases} \Leftrightarrow \begin{cases} p_1\left(t\right) = P\left(\frac{Q_1(t) - \Phi^+}{\theta_1}, t\right) \\ p_2\left(t\right) = P\left(\frac{Q_2(t) + \Phi^+}{\theta_2}, t\right) \end{cases}.$$

This is illustrated on Figure 3.

Each firm receives the local market price for its production in each market, plus the FTR payment: $\left(p_2\left(t\right) - p_1\left(t\right)\right)\frac{\Phi^+}{N}$ if the interconnection is congested from market 1 to market 2, $\left(p_1\left(t\right) - p_2\left(t\right)\right)\frac{\Phi^-}{N}$ if the interconnection is congested from market 2 to market 1.

If the interconnection is congested from market 1 to market 2, firm's n operating profit in state t is thus

$$\begin{aligned} \pi^n &= q_1^n\left(p_1 - c_1\right) + q_2^n\left(p_2 - c_2\right) + \frac{\Phi^+}{N}\left(p_2 - p_1\right) \\ &= q_1^n\left(P\left(\frac{Q_1 - \Phi^+}{\theta_1}, t\right) - c_1\right) + q_2^n\left(P\left(\frac{Q_2 + \Phi^+}{\theta_2}, t\right) - c_2\right) + \frac{\Phi^+}{N}\left(P\left(\frac{Q_2 + \Phi^+}{\theta_2}, t\right) - P\left(\frac{Q_1 - \Phi^+}{\theta_1}, t\right)\right) \\ &= \theta_1\frac{q_1^n - \frac{\Phi^+}{N}}{\theta_1}\left(P\left(\frac{Q_1\left(t\right) - \Phi^+}{\theta_1}, t\right) - c_1\right) + \theta_2\frac{q_2^n + \frac{\Phi^+}{N}}{\theta_2}\left(P\left(\frac{Q_2 + \Phi^+}{\theta_2}, t\right) - c_2\right) + \frac{\Phi^+}{N}\left(c_2 - c_1\right). \end{aligned}$$

Define the adjusted outputs $\gamma_1^n = \frac{q_1^n - \frac{\Phi^+}{N}}{\theta_1}$, $\gamma_2^n = \frac{q_2^n + \frac{\Phi^+}{N}}{\theta_2}$, $X^+ = \frac{\Phi^+}{\theta_2}$and $\Gamma_i = \sum_{n=1}^N \gamma_i^n$ for $i = 1, 2$. Then,

$$\pi^n = \theta_1\gamma_1^n\left(P\left(\Gamma_1, t\right) - c_1\right) + \theta_2\gamma_2^n\left(P\left(\Gamma_2, t\right) - c_2\right) + \theta_2\frac{X^+}{N}\left(c_2 - c_1\right). \quad (4)$$

Adjusted output γ_i^n is firm n decision variable in market i, that incorporates market size, the impact of imports (exports), and the FTR payment. When the interconnection is congested, dynamics in each market are independent, thus firms optimize separately in each market. Equation (4) shows that the profit function is the sum of two "standard" Cournot profit functions, were adjusted output γ_i^n replaces output q_i^n. The equilibrium of the congested spot market game is therefore easily obtained.

The simplicity of the solution to the spot market game is due to the inclusion of the FTR payment in the profit function and the symmetry of generators. These assumptions are the main difference with Borenstein, Bushnell and Stoft (2000). Since most electricity markets use $FTRs$, the first feature is realistic, while the second corresponds to the long-term equilibrium with free entry in each market.

Adjusted baseload capacity for producer n is defined by $x_1^n = \frac{k_1^n - \Phi^+}{\theta_1}$, and the aggregate adjusted baseload capacity by $X_1 = \frac{K_1 - \Phi^+}{\theta_1}$.

Similarly, if the interconnection is constrained from market 2 to market 1, producer n adjusted baseload (resp. peaking) capacity is $y_1^n = \frac{k_1^n + \frac{\Phi^-}{N}}{\theta_1}$ (resp. $y_2^n = \frac{k_2^n - \frac{\Phi^-}{N}}{\theta_2}$), and the aggregate adjusted baseload (resp. peaking) capacity is $Y_1 = \frac{K_1 + \Phi^-}{\theta_1}$ (resp. $Y_2 = \frac{K_2 - \Phi^-}{\theta_2}$).

Congestion regimes Analysis presented in Section 3 shows that the maximum flow from market 1 to market 2 occurs when baseload technology produces at capacity, and peaking technology is not yet turned on, and is equal to $\varphi(t) = \theta_2 K_1$. The maximum flow from market 2 to market 1 occurs when both technologies produce at capacity, and is equal to $\varphi(t) = K_1 - \theta_1 K$. Thus, different situations must be analyzed, represented in the (Φ^+, Φ^-) plane on Figure 4.

Suppose first $\theta_1 K_2^U \leq \theta_2 K_1^U$. Then, the interconnection is never congested if $\Phi^+ \geq \theta_2 K_1^U$, and congested from market 1 to market 2 if $\Phi^+ < \theta_2 K_1^U$ and $-\Phi^- < K_1 - \theta_1 K$. Analysis presented in Proposition 1 shows that this latter condition is equivalent to $(\Phi^+ + \Phi^-) \geq \theta_1 K_2^U$. If $(\Phi^+ + \Phi^-) < \theta_1 K_2^U$ the interconnection is successively congested in both directions.

If $\theta_1 K_2^U > \theta_2 K_1^U$, there also exist a region of the plan (Φ^+, Φ^-) for which the interconnection is congested from market 2 to market 1.

To simplify the exposition, I assume $\theta_1 K_2^U \leq \theta_2 K_1^U$, which leads to all relevant cases: interconnection not congested, congested in one direction only, and congested successively in both directions.

Equilibrium investment The equilibrium is summarized in the following:

Proposition 1 *Equilibrium generation mix* $\left(K_1^C, K^C\right)$.

1. *If* $\Phi^+ \geq \theta_2 K_1^U$, *the transmission line is never congested,* $K^C = K^U$ *and* $K_1^C = K_1^U$.

2. *If* $\Phi^+ < \theta_2 K_1^U$ *and* $(\Phi^+ + \Phi^-) \geq \theta_1 K_2^U$, *the transmission line is congested from market 1 to market 2 for some states of the world. The cumulative installed capacity* K^C *is the cumulative uncongested capacity:*

$$K^C = K^U, \tag{5}$$

and the baseload capacity is the uncongested baseload capacity scaled down by its domestic market *size $\theta_1 K_1^U$, plus the interconnection capacity Φ^+:*

$$X_1^C = K_1^U \Leftrightarrow K_1^C = \theta_1 K_1^U + \Phi^+. \tag{6}$$

3. *If $(\Phi^+ + \Phi^-) < \theta_1 K_2^U$, the transmission line is congested from market 1 to market 2 for some states of the world, then from market 2 to market 1 for higher-demand states of the world. The peaking capacity is the total capacity scaled down by its domestic market size $\theta_2 K^U$, plus the interconnection capacity Φ^-:*

$$Y_2^C = K^U \Leftrightarrow K_2^C = \theta_2 K^U + \Phi^-, \tag{7}$$

while baseload capacity is determined implicitly as the unique solution of

$$\Psi\left(X_1^C, c_1\right) - \Psi\left(X_1^C, c_2\right) + \Psi\left(Y_1^C, c_2\right) = r_1. \tag{8}$$

Proof. *The first point is evident. In the remainder of this proof, suppose $\Phi^+ < \theta_2 K_1^U$. We first need to prove that the line is indeed congested, i.e., that $\Phi^+ < \theta_2 K_1^C$. The proof proceeds by contradiction. If $\Phi^+ > \theta_2 K_1^C$, the line would never be congested, hence $K_1^C = K_1^U$, and $\Phi^+ > \theta_2 K_1^U$, which contradicts the hypothesis.*

Then, to obtain intuition for the equilibrium profits, suppose firms play a symmetric strategy: for all $n = 1, ..., N$, $k_1^n = \frac{K_1}{N}$ and $k^n = \frac{K}{N}$. As long as the interconnection is not congested, firms use the baseload technology, and play a symmetric N-firm Cournot game with cost c_1. Power flows from market 1, where production is located, to market 2.

For $t \geq \hat{t}\left(X^+, c_1\right)$, the transmission constraint is binding, before technology 1 is at capacity. Power flow from market 1 to market 2 is equal to the interconnection capacity Φ^+. Both markets are independent. Consider first market 1. Applying equation (4) to market 1, firms play a symmetric Cournot game with cost c_1, which sets the price in market 1. For $t \geq \hat{t}\left(X_1, c_1\right)$, technology 1 reaches capacity, and price in market 1 is determined by the intersection of the vertical supply curve at $\left(K_1 - \Phi^+\right)$ and the demand curves $\theta_1 D\left(p, t\right)$. Consider now market 2. First, price in market 2 is determined by the intersection of the vertical supply curve at Φ^+ and the demand curves $\theta_2 D\left(p, t\right)$. Then, for

$t \geq \hat{t}\left(X^{+}, c_2\right)$, both technologies produce. Applying equation (4) to market 2, firms play a symmetric Cournot game with cost c_2, which sets the price in market 2.

For $t \geq \hat{t}\left(X_1, c_2\right)$, prices in both markets are equal. The interconnection is no longer constrained, and we are back to the unconstrained case. Algebraic manipulations presented in Appendix A prove that expected profit can be expressed as

$$\Pi^n = \theta_1 \left(B\left(x_1^n, X_1, c_1, c_2\right) - \left(r_1 - r_2\right) x_1^n\right) + \left(A\left(k^n, K, c_2\right) - r_2 k^n\right) + \theta_2 \left(B\left(X^{+}, c_1, c_2\right) - \left(r_1 - r_2\right) \frac{1}{N} X^{+}\right). \tag{9}$$

Profits are again separable in $\left(x_1^n, k^n\right)$. If a symmetric equilibrium exists, it satisfies equations (5) and (6). This is illustrated on Figures 5a, 5b, and 5c. By considering deviations from the equilibrium candidate, Appendix A shows that $\left(\frac{X_1^C}{N}, \frac{K^C}{N}\right)$ for all $n = 1, ..., N$ is indeed an equilibrium.

The above analysis has assumed that the line is never congested from market 2 to market 1. This is true if and only if

$$\varphi(t) = K_1^C - \theta_1 K^C \geq -\Phi^{-} \Leftrightarrow \left(\theta_1 K_1^U + \Phi^{+}\right) - \theta_1 K^U \geq -\Phi^{-} \Leftrightarrow \Phi^{+} + \Phi^{-} \geq \theta_1 \left(K^U - K_1^U\right) = \theta_1 K_2^U$$

as announced. Suppose now $\left(\Phi^{+} + \Phi^{-}\right) < \theta_1 K_2^U$. Nothing changes until $t = \hat{t}\left(X_1, c_2\right)$. For $t \geq \hat{t}\left(X_1, c_2\right)$, prices in both markets are equal, the interconnection is no longer constrained, firms play a symmetric Cournot game with cost c_2.

For $t \geq \hat{t}\left(Y_1, c_2\right)$, the interconnection from market 2 to market 1 is congested before *cumulative capacity is reached. Markets are again separated. Price in market 1 is determined by the intersection of the vertical supply curve at $\left(K_1 + \Phi^{-}\right)$ and the demand curves $\theta_1 D(p, t)$.*

In market 2, taking their FTR revenue into account, producers play a symmetric Cournot game with cost c_2. Finally, for $t \geq \hat{t}\left(Y_2, c_2\right)$, technology 2 produces at capacity. Price in market 2 is determined by the intersection of the vertical supply curve at $\left(K_2 - \Phi^{-}\right)$ and the demand curves $\theta_2 D(p, t)$. This is illustrated on Figure 6a, and 6b.

Appendix B proves that a firm expected profit is

$$\Pi^n = \theta_1 \left(B\left(x_1^n, X_1, c_1, c_2\right) + A\left(y_1^n, Y_1, c_2\right)\right) - r_1 k_1^n + \theta_2 \left(A\left(y_2^n, Y_2, c_2\right) - r_2 y_2^n\right) + \theta_2 B\left(X^{+}, c_1, c_2\right) - r_2 \frac{\Phi^{-}}{N}. \tag{10}$$

Then,

$$\begin{aligned}
\left.\frac{\partial \Pi^n}{\partial k_2^n}\right|_{k_2^n=\frac{K_2}{N}} &= \left.\frac{\partial A}{\partial x_2^n}\left(x_2^n, X_2, c_2\right) + \frac{\partial A}{\partial X_2}\left(x_2^n, X_2, c_2\right)\right|_{x_2^n=\frac{X_2}{N}} - r_2 \\
&= \Psi\left(X_2, c_2\right) - r_2,
\end{aligned}$$

and

$$\begin{aligned}
\left.\frac{\partial \Pi^n}{\partial k_1^n}\right|_{k_1^n=\frac{K_1}{N}} &= \left.\frac{\partial B}{\partial x_1^n}\left(x_1^n, X_1, c_1, c_2\right) + \frac{\partial B}{\partial X_1}\left(x_1^n, X_1, c_1, c_2\right) + \frac{\partial A}{\partial y_1^n}\left(y_1^n, Y_1, c_2\right) + \frac{\partial A}{\partial Y_1}\left(y_1^n, Y_1, c_2\right)\right|_{y_1^n=\frac{Y_1}{N}} - r_1 \\
&= \Psi\left(X_1, c_1\right) - \Psi\left(X_1, c_2\right) + \Psi\left(Y_1, c_2\right) - r_1.
\end{aligned}$$

If a symmetric equilibrium exists, it satisfies conditions (7) *and* (8). *By considering upward and downward deviations, Appendix B proves that* $\left(\frac{K_1^C}{N}, \frac{K_2^C}{N}\right)$ *defined by equations* (7) *and* (8) *is in fact the unique symmetric equilibrium.* ■

Proposition 1 calls for a few observations. Suppose first interconnection is congested in one direction only, $\Phi^+ < \theta_2 K_1^U$ *and* $(\Phi^+ + \Phi^-) \geq \theta_1 K_2^U$. Congestion stops on peak. This appears counterintuitive. One would argue that, since peaking technology (located in market 2) has higher marginal cost than the baseload technology (located in market 1), once the interconnection becomes congested, it always remains so. This intuition turns out to be invalid, as it ignores the necessary recovery of investment cost: when the baseload technology produces at capacity, price in market 1 increases, and eventually reaches the marginal cost of the peaking technology.

As a consequence of the previous observation, congestion has no impact on the oligopolists' choice of total installed capacity. This may again appear surprising. The intuition is that total capacity is determined by its marginal value when total capacity is constrained. In these states of the world, the interconnection is no longer congested, and the peaking technology is price-setting. Thus congestion no longer matters.

For this reason, this result is robust to changes in the ownership structure of generation assets, the allocation of $FTRs$, or the method for congestion management (as long as no transmission charge is levied when the interconnection is not congested).

Let us now turn to the baseload technology. By assumption, baseload generation reaches capacity after the interconnection is congested (otherwise, there would never be congestion). Equation (9) shows that the economics of the adjusted baseload capacity X_1 are identical to those of the baseload capacity K_1 when the interconnection is not congested.

Congestion on the transmission line reduces the baseload capacity installed at market 1, and increases the peaking capacity installed at market 2. Equation (6) simple relationship between K_1^C and K_1^U results from the symmetry of asset ownership and the FTR allocation. However, the general insight should be robust to other specifications.

Consider now the heavily congested line, $(\Phi^+ + \Phi^-) < \theta_1 K_2^U$. In equilibrium, congestion from market 2 to market 1 depends not only on Φ^-, the interconnection capacity in that direction, but on the sum of interconnection capacities. This result may appear surprising. The intuition is that, as Φ^+ increases, so does the installed baseload capacity, hence the flow from market 1 to market 2. Thus, both Φ^+ and Φ^- contribute to reducing congestion from market 2 to market 1.

The peaking technology reaches capacity after the line is congested (similar to the baseload technology in the previous case). Thus, as equation (10) illustrates, the economics of the adjusted peaking capacity Y_2 are identical to those of the total capacity K when the interconnection is not congested. An increase in Φ^- raises K_2^C one for one. This result is robust to a change of ownership, as long as the N generators located in market 2 are entitled to the FTR payments from market 2 to market 1.

Baseload technology reaches capacity after the interconnection is congested in one direction, but before it gets congested in the other direction. Marginal value is thus $(\Psi(X_1, c_1) - \Psi(X_1, c_2))$ when the interconnection is congested into market 2, plus $\Psi(Y_1, c_2)$ when the interconnection is congested into market 1. At the equilibrium, the total marginal value is equal to the marginal cost r_1, as described by equation (8).

Formally, baseload capacity K_1^C and peaking capacity K_2^C are functions of interconnection capacities (Φ^+, Φ^-). A few properties of $K_i^C(\Phi^+, \Phi^-)$ for $i = 1, 2$ are summarized below:

Corollary 1 *Suppose* $(\Phi^+ + \Phi^-) \leq \theta_2 K_2^U$, *then*

$$K_i^C(0,0) = \theta_i \chi(c_i, r_i),\ i = 1, 2,$$

and

$$0 < \frac{\partial K_1^C}{\partial \Phi^+} < 1 \textit{ and } \frac{\partial K_1^C}{\partial \Phi^-} = \frac{\partial K_1^C}{\partial \Phi^+} - 1 < 0.$$

Proof. *The proof is presented in Appendix B.* ■

When both markets are isolated, only technology i is available to serve demand in market i, hence equilibrium capacity is $K_i(0,0) = \theta_i \chi(c_i, r_i)$.

An increase in Φ^+, the interconnection capacity from market 1 to market 2, leads to a less than one for one increase in the capacity installed in market 1: an increase in K_1 reduces $\Psi(Y_1, c_2)$, the marginal value of K_1 once the interconnection is congested into market 1, hence, *ceteris paribus*, leads to lower K_1. Similarly, an increase in Φ^-, the interconnection capacity from market 2 to market 1 reduces the capacity installed in market 1 (and increases the capacity installed in market 2 one for one).

This analysis highlights the counter-intuitive impact interconnection expansion has on installed generation capacity. If both markets are isolated, imperfectly competitive producers install the autarky capacity $\theta_i \chi(c_i, r_i)$ in each market. If capacity is increased, for example by $\delta\Phi^+ = \delta\Phi^- = \delta\Phi$ such that $\delta\Phi^+ + \delta\Phi^- = 2\delta\Phi < \theta_2 K_2^U$, producers install $\delta K_2 = \delta\Phi$ additional capacity in market 2. They install less capacity in market 1 if and only if

$$\delta K_1 = \frac{\partial K_1^C}{\partial \Phi^+}\delta\Phi^+ + \frac{\partial K_1^C}{\partial \Phi^-}\delta\Phi^- = \left(2\frac{\partial K_1^C}{\partial \Phi^+} - 1\right)\delta\Phi < 0 \Leftrightarrow \frac{\partial K_1^C}{\partial \Phi^+} < \frac{1}{2}.$$

This condition may or may not be met, depending on the value of the parameters. Thus, increasing the interconnection capacity has an ambiguous impact on installed capacity in market 1. This is slightly surprising as one would have expected that additional export capability would have led to higher baseload capacity.

Increased interconnection capacity also increases cumulative capacity, since

$$\delta K_1 + \delta K_2 = 2\frac{\partial K_1^C}{\partial \Phi^+}\delta\Phi > 0.$$

Again, this is slightly surprising, as one would have expected that additional exchanges possibility lead to greater trade, hence to lower installed capacity. Furthermore, if $\frac{\partial K_1^C}{\partial \Phi^+} > \frac{1}{2}$, the cumulative capacity increase is more than 1 for 1.

If $2\delta\Phi \geq \theta_2 K_2^U$, the impact is almost opposite: aggregate capacity remains constant, and baseload capacity substitutes for peaking capacity.

Even in the simplest setting, the impact of increasing interconnection capacity on installed generation is sometimes surprising, and has opposite impacts depending on the level of congestion. In a real power grid, characterized by multiple technologies and multiple nodes, the complexity is much higher.

This suggests that policy makers should be extremely careful when assessing the impact of transmission capacity increase on installed generation.

Finally, as in the unconstrained case, the equilibrium capacities are obtained by replacing inverse demand by marginal revenue in the first-order conditions (see for example Léautier (2013)). This remarkable properties is due to the symmetry of generators and the use of $FTRs$.

Numerical illustration The analysis can be illustrated on a stylized description of France (market 1) and Britain (market 2). Demand is assumed to be identical in Britain and France, up to the market size. Maximum demand is 90 GW in France, and 60 GW in Britain[2]. Thus, $\theta_1 = 3/5$ and $\theta_2 = 2/5$. The interconnection capacity is $\Phi^+ = \Phi^- = 2\ GW$.

Technology 1 is nuclear generation, while technology 2 is Combined Cycle Gas Turbine ($CCGT$). Using data from International Energy Agency (IEA (2010)), reported in Hansen and Percebois (2010, page 324), CO_2 emissions rate from the US Environmental Protection Agency, and a carbon price of 30 €/ton, the variable and investment costs, expressed in €/MWh are:

	1	2
c_n	10.3	55.5
r_n	41.0	10.5

Considering only one generation technology in each country is of course a first approximation: nuclear generation produces around 80% of the electricity consumed in France, while $CCGT$ contributed

[2] Data for year 2010 from the Transmission System Owners and Operators, Réseau de transport d'électricité (Rte) in France, and National Grid in Britain.

46% of the electricity produced in Britain[3] in 2010. More problematic is the assumption of symmetric generators. While at least three firms have significant presence in both markets (EDF, $E.ON$, and $GdF - Suez$), the symmetry assumption is clearly not met today. Thus, the analysis is illustrative, that reflects the long term equilibrium, assuming free entry in each market. I run two scenarii: $N = 3$, and $N = 6$.

Demand and uncertainty are specified as follows (Léautier (2014)): (i) inverse demand is linear with constant slope: $P(Q;t) = a(t) - bQ$ where $a(t) = a_0 - a_1 e^{-\lambda_2 t}$, (ii) and states of the world are distributed according to $f(t) = \lambda_1 e^{-\lambda_1 t}$. This specification provides an adequate representation of actual demand (shape of load duration curve, and elasticity), while leading to simple expressions, as illustrated below.

The parameters to be estimated are a_0, a_1, $\lambda = \frac{\lambda_1}{\lambda_2}$, and b. Maximum Likelihood on the load duration curve for France in 2010 is used to estimate λ. The same load duration curve provides an expression of a_0 and a_1 as a function of bQ^∞ where $Q^\infty = \frac{a_0 - p_0}{b}$ is the maximum demand for price p_0. As previously mentioned, $Q^\infty = 150\ GW$. The average demand elasticity η is then used to estimate b. Of course, estimates of the short-run elasticity of demand are very uncertain. I use $\eta = -0.1$ at price $p_0 = 100$ €$/MWh$, which corresponds to the upper bound of estimates reported by Lijesen (2007). Following this procedure, Léautier (2014) estimates

$$\begin{cases} bQ^\infty = 1\ 873 \text{ €}/MWh \\ a_0 = 1\ 973 \text{ €}/MWh \\ a_1 = 1\ 236 \text{ €}/MWh \\ \lambda = 1.78 \end{cases}.$$

It is easier to express all capacity as fractions of Q^∞. The functional form selected leads a simple form for $\Psi(K,c)$. Integrating by parts yields

$$\Psi(K,c) = \frac{a_1}{(1+\lambda)} \left(\frac{a_0 - c - \frac{N+1}{N} bQ^\infty \frac{K}{Q^\infty}}{a_1} \right)^{1+\lambda}.$$

Suppose first $N = 3$. Solving numerically equations (1) and (2) yields $K^U = 63.7\% \times Q^\infty$ and

[3] Source: https://www.gov.uk/government/statistical-data-sets/historical-electricity-data-1920-to-2011?? ??

$K_1^U = 38.3\% \times Q^\infty$, thus $K_2^U = 25.4\% \times Q^\infty$.

Then, $\theta_1 K_2^U = 15.26\% \times Q^\infty < 15.30\% \times Q^\infty = \theta_2 K_1^U$, which is the situation described in this article. The sum of interconnection capacities in both directions is $\Phi^+ + \Phi^- = 4\ GW = 1.33\% \times Q^\infty$. Thus, $\Phi^+ + \Phi^- < \theta_1 K_2^U$: the model predicts the interconnection is successively congested in both directions. This is verified empirically, as illustrated on Figure 7. Britain imported from France 73% of the time in 2010, and the interconnection was congested from France into Britain 8% of the time. However, Britain also exported into France, suggesting that the price in France rose higher than the price in Britain. The exports were so high 4% of the time that the interconnection was congested from Britain into France.

We then solve numerically equation (8). The functions K_1^C and K^C are presented on Figure 8 for $\frac{\Phi^+}{Q^\infty} = \frac{\Phi^-}{Q^\infty} \in [0, 20\%]$. For $\Phi^+ \leq \frac{\theta_1 K_2^U}{2} = 7.6\% \times Q^\infty$, K_1^C decreases from $K_1^C(0) = 34.4\% \times Q^\infty$ to $K_1^C\left(\frac{\theta_1 K_2^U}{2}\right) = 30.6\% \times Q^\infty$, while K^C increases from $K^C(0) = 59.9\% \times Q^\infty$ to $K^C\left(\frac{\theta_1 K_2^U}{2}\right) = K^U = 63.7\% \times Q^\infty$.

The impact of an increase in interconnection capacity on generation capacity is completely reversed depending on the congestion level: if the line is highly congested $\left(\Phi^+ \leq \frac{\theta_1 K_2^U}{2}\right)$, increasing its capacity yields lower baseload and higher peaking capacities in equilibrium, while it yields higher baseload and lower peaking capacities if the line is lightly congested $\left(\frac{\theta_1 K_2^U}{2} \leq \Phi^+ \leq \theta_2 K_1^U\right)$.

The structure of the solution is identical for $N = 6$. As expected, the unconstrained capacities are higher: $K^U(N=6) = 72.8\% \times Q^\infty$, $K_1^U(N=6) = 43.7\% \times Q^\infty$, and $K_2^U(N=6) = 29.1\% \times Q^\infty$. For $\Phi^+ \leq \frac{\theta_1 K_2^U}{2} = 8.7\% \times Q^\infty$, K_1^C decreases from $K_1^C(0) = 39.3\% \times Q^\infty$ to $K_1^C\left(\frac{\theta_1 K_2^U}{2}\right) = 35.0\% \times Q^\infty$, while K^C increases from $K^C(0) = 68.4\% \times Q^\infty$ to $K^C\left(\frac{\theta_1 K_2^U}{2}\right) = K^U = 72.8\% \times Q^\infty$.

5 Marginal value of transmission capacity

The net social surplus is defined as

$$W\left(\Phi^+, \Phi^-\right) = \mathbb{E}\left[\theta_1 S\left(p_1(t), t\right) + \theta_2 S\left(p_2(t), t\right) - c_1 Q_1(t) - c_2 Q_2(t)\right] - r_1 K_1 - r_2 K_2$$

where $S(p,t)$ is the gross consumer surplus at price p, i.e., $S(p,t) = \int_0^{D(p,t)} P(x,t)\, dx$.

We now determine the social value of an increase in transmission capacity. We consider separately an increase in Φ^+ and an increase in Φ^-, since actions that may increase the maximum admissible flow in one direction may not increase the limit in the other direction.

Proposition 2 *Marginal value of transmission capacity*

1. *If* $\Phi^+ \geq \theta_2 K_1^U$, *the interconnection is never congested, hence its marginal value is equal to zero:* $\frac{\partial W}{\partial \Phi^+} = \frac{\partial W}{\partial \Phi^-} = 0.$

2. *If* $\Phi^+ < \theta_2 K_1^U$ *and* $(\Phi^+ + \Phi^-) \geq \theta_1 K_2^U$, *the marginal value of interconnection capacity from market* 1 *to market* 2 *is*

$$\frac{\partial W}{\partial \Phi^+} = \int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} \left(P\left(X^+,t\right) - c_1\right) f(t)\,dt + \int_{\hat{t}(X^+,c_2)}^{+\infty} (c_2 - c_1) f(t)\,dt - (r_1 - r_2). \tag{11}$$

The marginal value of interconnection from market 2 *to market* 1 *is equal to zero:* $\frac{\partial W}{\partial \Phi^-} = 0.$

3. *If* $(\Phi^+ + \Phi^-) < \theta_1 K_2^U$, *the marginal value of interconnection from market* 1 *to market* 2 *also includes the impact of* Φ^+ *on* X_1^C *and* K_1^C :

$$\begin{aligned}\frac{\partial W}{\partial \Phi^+} &= \int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} \left(P\left(X^+,t\right) - c_1\right) f(t)\,dt + \int_{\hat{t}(X^+,c_2)}^{+\infty} (c_2 - c_1) f(t)\,dt \\ &\quad - \left(\int_{\hat{t}(X_1^C,c_1)}^{\hat{t}(X_1^C,c_2)} \left(P\left(X_1^C,t\right) - c_1\right) f(t)\,dt + \int_{\hat{t}(X_1^C,c_2)}^{+\infty} (c_2 - c_1) f(t)\,dt\right) \\ &\quad - \left(\int_{\hat{t}(X_1^C,c_1)}^{\hat{t}(X_1^C,c_2)} \frac{X_1^C}{N} P_q\left(X_1^C,t\right) f(t)\,dt + \int_{\hat{t}(Y_1^C,c_2)}^{+\infty} \frac{Y_1^C}{N} P_q\left(Y_1^C,t\right) f(t)\,dt\right) \frac{\partial K_1^C}{\partial \Phi^+}.\end{aligned}$$

The marginal value of interconnection from market 2 *to market* 1 *includes the increased cost of peaking capacity and the impact of* Φ^- *on* K_1^C :

$$\begin{aligned}\frac{dW}{d\Phi^-} &= \int_{\hat{t}(Y_1^C,c_2)}^{+\infty} \left(P\left(Y_1^C,t\right) - c_2\right) f(t)\,dt - r_2 \\ &\quad - \left(\int_{\hat{t}(X_1^C,c_1)}^{\hat{t}(X_1^C,c_2)} \frac{X_1^C}{N} P_q\left(X_1^C,t\right) f(t)\,dt + \int_{\hat{t}(Y_1^C,c_2)}^{+\infty} \frac{Y_1^C}{N} P_q\left(Y_1^C,t\right) f(t)\,dt\right) \frac{\partial K_1^C}{\partial \Phi^-}.\end{aligned}$$

Proof. *The proof is presented in Appendix C.* ■

If an interconnection is unconstrained, the marginal value of capacity is equal to zero.

If the interconnection is congested in one direction only, the marginal value of interconnection capacity is the value of the substitution between technologies it enables. Equation (11) extends to the imperfect competition case the optimal transmission capacity derived in the engineering literature (see for example Schweppe et al. (1988)). It includes imperfect competition (and differs from the engineering value) in the boundaries of the expectations. The congestion starts in state $\hat{t}\left(X^{+}, c_1\right)$ and stops in state $\hat{t}\left(X^{+}, c_2\right)$, both later than if competition was perfect, since $\hat{t}_0\left(X^{+}, c_n\right) < \hat{t}\left(X^{+}, c_n\right)$.

Observe that

$$\lim_{\Phi^{+} \to \left(\theta_2 K_1^U\right)^{-}} \frac{\partial W}{\partial \Phi^{+}} = -\int_{\hat{t}\left(K_1^U, c_1\right)}^{\hat{t}\left(K_1^U, c_2\right)} \frac{P_q\left(K_1^U, t\right)}{N} f(t)\, dt > 0,$$

i.e., the marginal value of interconnection is discontinuous at $\Phi^{+} = \theta_2 K_1^U$: strictly positive on the left, equal to zero on the right. By contrast, the engineering marginal value of interconnection is continuous (and equal to zero at the boundary). This difference is the strategic effect: an increase in transmission capacity not only increases the technical efficiency, by allowing substitution of cheaper for more expensive power, but it also increases competitive intensity.

Numerical simulation using the specification described above shows that $\frac{\partial W}{\partial \Phi^{+}}\left(\left(\theta_2 K_1^U\right)^{-}\right) = 13.08$ €$/MWh$ for $N = 3$. The strategic effect is far from insignificant.

For $N = 6$, $\frac{\partial W}{\partial \Phi^{+}}\left(\left(\theta_2 K_1^U\right)^{-}\right) = 13.99$ €$/MWh$: transmission is more valuable if the industry is less concentrated! To understand this surprising result, start from the expression of $\frac{\partial W}{\partial \Phi^{+}}\left(\left(\theta_2 K_1^U\right)^{-}\right)$ when demand is linear:

$$\frac{\partial W}{\partial \Phi^{+}}\left(\left(\theta_2 K_1^U\right)^{-}\right) = \frac{b}{N}\left[F\left(\hat{t}\left(K_1^U, c_2\right)\right) - F\left(\hat{t}\left(K_1^U, c_1\right)\right)\right].$$

"Differentiating[4]" with respect to N and denoting $\hat{t}_i = \hat{t}\left(K_1^U, c_i\right)$ yields

$$\frac{\partial^2 W}{\partial N \partial \Phi^{+}}\left(\left(\theta_2 K_1^U\right)^{-}\right) = \frac{b}{N}\left[-\frac{F\left(\hat{t}_2\right) - F\left(\hat{t}_1\right)}{N} + \left(f\left(\hat{t}_2\right)\frac{\partial \hat{t}_2}{\partial N} - f\left(\hat{t}_1\right)\frac{\partial \hat{t}_1}{\partial N}\right)\right].$$

The first term is clearly negative, and corresponds to the "normal" economic intuition: as N increases, the primary distortion arising from imperfect competition, expressed as $\frac{b}{N}$, decreases. This effect reduces the marginal value of transmission capacity. However, a second effect is present: as N increases,

[4] Technically, differentiation with respect to an integer is not defined, and one should take finite differences. Nevertheless, the analysis provides the intuition.

$\hat{t}_i$ is modified. In the illustrative example we use, this second effect is positive, and larger than the first.

Finally, as indicated in equation (11), the marginal value from a welfare perspective includes the full value of the substitution: both the reduction in marginal cost and the increase in investment cost. This last term is often ignored by practitioners and policy makers. For example, the European Network of Transmission System Operators for Electricity Guideline for Cost Benefit Analysis of Grid Development Projects ($ENTSO-E$, (2013, pp. 31-35) appears to include only the gain in short-term variable costs, and to ignore the increase in investment costs. In the United States, merchant transmission lines requested to receive the value of their contribution to generation adequacy in the importing market, for example by being allowed to participate in capacity markets[5]. In this case, if the capacity price was set at r_2, the capital cost of the peaking technology, the marginal value of the line would be estimated as the short-term congestion cost plus r_2, thus overstating the true value by the entire capital cost of the baseload technology r_1.

The increase in capital cost is far from insignificant in practice, as illustrated using the previous example. The marginal value of transmission capacity arising from operating costs is

$$\Delta\left(\Phi^{+}\right)=\int_{\hat{t}(X^{+},c_1)}^{\hat{t}(X^{+},c_2)}\left(P\left(X^{+},t\right)-c_1\right)f\left(t\right)dt+\int_{\hat{t}(X^{+},c_2)}^{+\infty}\left(c_2-c_1\right)f\left(t\right)dt.$$

Numerical estimation shows that, for $N=3$, $\Delta\left(\left(\theta_2 K_1^U\right)^-\right)=43.53$ €$/MWh$, slightly lower than the difference in marginal costs $c_2-c_1=45.21$€$/MWh$ since the line is not congested all the time. The true marginal value of transmission capacity is

$$\frac{\partial W}{\partial \Phi^{+}}\left(\left(\theta_2 K_1^U\right)^-\right)=\Delta\left(\left(\theta_2 K_1^U\right)^-\right)-\left(r_2-r_1\right)=43.53-30.45=13.08 \text{ €}/MWh.$$

The marginal value arising from operating costs is 3.3 times larger than true marginal value. This suggests that, by ignoring that producers take the transmission grid into account when deciding on generation expansion, policy makers vastly overstate the value of transmission expansion.

Consider now the interconnection congested in both directions. Increasing Φ^{+} has three effects.

[5] Capacity markets are not included in this analysis. I assume that $\frac{\partial W}{\partial \phi^{+}}$ is not modified when capacity markets are introduced.

First, higher interconnection capacity enables the substitution of cheap for expensive power, as in the previous case. Second, for a given K_1, increasing Φ^+ reduces X_1, thus reduces net surplus. Finally, increasing Φ^+ increases baseload capacity (less than one for one), which then in turns increases net surplus.

Similarly, increasing Φ^- has three effects: for a given K_1, it increases Y_1, thus increases net surplus by $(P(Y_1, t) - c_2)$. Second, it leads to higher peaking capacity, at capital cost r_2. Finally, it leads to a reduction in K_1^C, which reduces net surplus.

6 Marginal impact on interconnection capacity on producers profits

We now examine the marginal impact of increasing transmission capacity on producers profits. Consider first the case $\Phi^+ < \theta_2 K_1^U$ and $(\Phi^+ + \Phi^-) \geq \theta_1 K_2^U$. Differentiating equation (9) yields

$$\begin{aligned} \frac{d\Pi^n}{d\Phi^+} &= \frac{1}{N}\left(\int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} \left(P\left(X^+,t\right) + X^+ P_q\left(X^+,t\right) - c_1\right) f(t)\,dt + \int_{\hat{t}(X^+,c_2)}^{+\infty} (c_2 - c_1) f(t)\,dt - (r_1 - r_2)\right) \\ &= \frac{\partial B}{\partial X^+}\left(X^+, c_1, c_2\right) - \frac{r_1 - r_2}{N}. \end{aligned}$$

Increasing Φ^+ modifies the FTR revenue: it increases the volume, but it also reduces the price differential. The oligopolists take this revenue reduction effect into account, which is absent from the social value. The resulting impact is unclear:

$$\frac{\partial B}{\partial X^+}\left(X^+, c_1, c_2\right) = \frac{1}{N}\begin{pmatrix} \int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} \left(P\left(X^+,t\right) + \frac{X^+}{N} P_q\left(X^+,t\right) - c_1\right) f(t)\,dt + \int_{\hat{t}(X^+,c_2)}^{+\infty} (c_2 - c_1) f(t)\,dt \\ + \frac{N-1}{N} \int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} X^+ P_q\left(X^+,t\right) f(t)\,dt \end{pmatrix}.$$

The first two terms are positive, while the last one is negative.

Furthermore, increasing Φ^+ lead to a substitution of cheap for dear capacity, at cost $\left(\frac{r_1 - r_2}{N}\right)$ for firm n.

If $N = 1$, the FTR payment increases, and this effect is high enough to compensate for the increased investment cost. Formally, for $N = 1$, $\frac{d\Pi^n}{d\Phi^+} = \Psi\left(X^+, c_1, c_2\right) - (r_1 - r_2)$, thus $\frac{d\Pi^n}{d\Phi^+}\left(\theta_2 K_1^U\right) = 0$. Since $\Psi(., c_1, c_2)$ is decreasing in its first argument, $\frac{d\Pi^n}{d\Phi^+} > 0$ for $\Phi^+ < \theta_2 K_1^U$.

This may not be the case for $N > 1$, at least when the interconnection is lightly congested:

$$\lim_{\Phi^+ \to \left(\theta_2 K_1^U\right)^-} \frac{d\Pi^n}{d\Phi^+} = \frac{N-1}{N} \int_{\hat{t}\left(K_1^U, c_1\right)}^{\hat{t}\left(K_1^U, c_2\right)} K_1^U P_q\left(K_1^U, t\right) f(t)\, dt < 0.$$

Suppose now $(\Phi^+ + \Phi^-) < \theta_1 K_2^U$. Differentiation of equation (10), presented in Appendix D yields:

$$\begin{aligned}\frac{\partial \Pi^n}{\partial \Phi^+} &= \frac{\partial B}{\partial X}\left(X^+, c_1, c_2\right) - \frac{\partial B}{\partial X}\left(X_1^C, c_1, c_2\right) \\ &+ \left(\int_{\hat{t}\left(Y_1^C, c_2\right)}^{+\infty} \frac{Y_1^C}{N} P_q\left(Y_1^C, t\right) f(t)\, dt + \int_{\hat{t}\left(X_1^C, c_1\right)}^{\hat{t}\left(X_1^C, c_2\right)} \frac{X_1^C}{N} P_q\left(X_1^C, t\right) f(t)\, dt\right) \frac{N-1}{N} \frac{\partial K_1^C}{\partial \Phi^+},\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \Pi^n}{\partial \Phi^-} &= \frac{\partial A}{\partial Y_1^C}\left(Y_1^C, c_2\right) - \frac{r_2}{N} \\ &+ \left(\int_{\hat{t}\left(Y_1^C, c_2\right)}^{+\infty} \frac{Y_1^C}{N} P_q\left(Y_1^C, t\right) f(t)\, dt + \frac{X_1^C}{N} P_q\left(X_1^C, t\right) f(t)\, dt\right) \frac{N-1}{N} \frac{\partial K_1^C}{\partial \Phi^-}.\end{aligned}$$

If $(\Phi^+ + \Phi^-) < \theta_1 K_2^U$, increasing Φ^+ has a direct impact: the change in the value of the FTR (term $\frac{\partial B}{\partial X}\left(X^+, c_1, c_2\right)$) minus the change in the value of operating profits through the direct impact of Φ^+ on X_1^C (term $\frac{\partial B}{\partial X}\left(X_1^C, c_1, c_2\right)$). This direct impact cannot be signed in general. Increasing Φ^+ also has an indirect impact: the change in competitors baseload investment (term $\frac{N-1}{N}\frac{\partial K_1^C}{\partial \Phi^+}$) times its impact on own profit (term $\int_{\hat{t}\left(Y_1^C, c_2\right)}^{+\infty} \frac{Y_1^C}{N} P_q\left(Y_1^C, t\right) f(t)\, dt + \int_{\hat{t}\left(X_1^C, c_1\right)}^{\hat{t}\left(X_1^C, c_2\right)} \frac{X_1^C}{N} P_q\left(X_1^C, t\right) f(t)\, dt$). Since an increase in Φ^+ increases K_1^C, and an increase in competitors capacity reduces profits in Cournot games, the indirect impact is negative.

Similarly, increasing Φ^- has a direct impact, including increased investment cost, and an indirect impact. Since increasing Φ^- reduces competitors' baseload investment, the indirect impact is positive.

These observations are summarized in the following:

Proposition 3 *A marginal increase in interconnection capacity has an ambiguous impact on producers profits. It modifies the FTR payment, but also the investment profile.*

7 Conclusion

This article is the first to examine investment decisions for electric power generation under imperfect competition and in the presence of transmission constraints. This analysis yields four original insights relevant for policy makers. First, it shows that congestion is dynamic and potentially transient, for example, congestion may disappear when demand is the highest. Second, it finds that transmission constraints modify generation investment in a non-trivial way: an increase in transmission capacity may increase, decrease, or have no impact on the marginal value of generation capacity. It may have similar or opposite impacts on the marginal value of baseload (low marginal cost) and peaking (high marginal cost) technologies. These two observations suggest that, to evaluate the impact of their policies, decision makers must develop detailed models of the industry, that include the transmission network. Third, the social value of transmission is not solely the difference in marginal costs, as is commonly assumed, but also includes the impact of transmission on investment in generation and on competitive intensity. The resulting value may be significantly lower than the simple difference in marginal costs. Finally, the impact of an increase in interconnection capacity on producers' profits is unclear, which strengthens the policy objective of vertical separation between producers and transmission grid owners.

The analysis presented here can be expanded in several directions. One can examine different transmission pricing rules, different ownership structures, and more general network topologies and technology mixes. It would be interesting to see how the results would change. For example, one would like to establish sufficient conditions for the Cournot investment to be obtained from the optimal investment by replacing demand by marginal revenues.

In addition, the analysis presented here can be used to examine various policy issues involving two interconnected markets. For example, one can determine the impact of introducing a capacity market in one market, while the other one remains energy-only.

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A Equilibrium investment when $\Phi^+ < \theta_2 K_1^U$ and $(\Phi^+ + \Phi^-) \geq \theta_1 K_2^U$

A.1 Expected profits for a symmetric strategy

For $t \in \left[0, \hat{t}\,(X^+, c_1)\right]$, only baseload technology is producing and serving the entire market. Firms play a N-player Cournot game with marginal cost c_1, hence the aggregate production in state t is $Q^C(c_1, t)$. Since $\Phi^+ < \theta_2 K_1^U$, the transmission line becomes congested *before* baseload generation produces at capacity:

$$\theta_2 Q^C(c_1, t) = \Phi^+ \Leftrightarrow Q^C(c_1, t) = X^+ \Leftrightarrow t = \hat{t}\left(X^+, c_1\right).$$

For $t \geq \hat{t}\,(X^+, c_1)$, as long as the interconnection is congested, both markets are independent. We first examine market 2. As long as the peaking technology is not turned on, price in market 2 is determined by the intersection of the vertical supply curve at Φ^+, and the demand curves $\theta_2 D(p, t)$, thus $p_2(t) = P(X^+, t)$.

From equation (4), technology 2 is turned on as soon as

$$\left.\frac{\partial \pi^n}{\partial q_2^n}\right|_{q_2^n(t)=0} = 0 \Leftrightarrow \left.\frac{\partial \pi^n}{\partial \gamma_2^n}\right|_{\gamma_2^n(t)=\frac{X^+}{N}} = P\left(X^+, t\right) - c_2 + \frac{X^+}{N} P_q\left(X^+, t\right) = 0 \Leftrightarrow t = \hat{t}\left(X^+, c_2\right).$$

As expected, the decision to turn-on the peaking technology is independent of the conditions in market

1. Thus, for $t \in \left[\hat{t}(X^+, c_1), \hat{t}(X^+, c_2)\right]$, $p_2(t) = P(X^+, t)$.

For $t \in \left[\hat{t}(X^+, c_2), \hat{t}(X_1, c_2)\right]$, the peaking technology produces $\Gamma_2 = Q^C(c_2, t)$.

To understand the upper bound $\hat{t}(X_1, c_2)$, we now turn to market 1. For $t \geq \hat{t}(X^+, c_1)$, producers in market 1 compete à la Cournot, thus $\gamma_1^n = \frac{Q^C(c_1,t)}{N}$. This lasts until the baseload technology reaches capacity:

$$Q^C(c_1, t) = X_1 \Leftrightarrow t = \hat{t}(X_1, c_1).$$

For $t \geq \hat{t}(X_1, c_1)$, price in market 1 is determined by the intersection of the vertical supply curve $(K_1 - \Phi^+)$ and the demand curves $\theta_1 D(p, t)$, thus $p_1(t) = P(X_1, t)$.

$p_1(t)$ increases until it reaches the price is market 2, $p_2(t) = P\left(Q^C(c_2, t), t\right)$:

$$X_1 = Q^C(c_2, t) \Leftrightarrow t = \hat{t}(X_1, c_2).$$

This characterization of equilibria assumes that the price in the baseload market reaches c_2 before the peaking technology reaches capacity, i.e., that $\hat{t}(X_1, c_2) \leq \hat{t}(X_2, c_2)$. This is proven below:

Lemma 2 *Assumption 2 implies that $\hat{t}(X_1, c_2) \leq \hat{t}(X_2, c_2)$.*

Proof. *The proof proceeds by contradiction. Suppose $\hat{t}(X_2, c_2) < \hat{t}(X_1, c_2)$: peaking technology reaches capacity before price in market 1 reaches c_2. Then, the line is always congested and*

$$X_2 = \chi(c_2, r_2) \text{ and } X_1 = \chi(c_1, r_1).$$

Thus,

$$\hat{t}(X_2, c_2) < \hat{t}(X_1, c_2) \Leftrightarrow X_2 < X_1 \Leftrightarrow \chi(c_2, r_2) < \chi(c_1, r_1)$$

which is contrary to assumption 2. Thus, $\hat{t}(X_2, c_2) < \hat{t}(X_1, c_2)$ leads to a contradiction, which proves the lemma. ■

For $t \geq \hat{t}(X_1, c_2)$, the interconnection is no longer constrained, and we are back to the unconstrained case.

To simplify the expressions, it is useful to introduce the expected Cournot profits over an interval

$$I^C(c, a, b) = \int_a^b \frac{Q^C(c, t)}{N}\left(P\left(Q^C(c, t), t\right) - c\right) f(t)\, dt \text{ and } I^C(c, a) = \int_a^{+\infty} \frac{Q^C(c, t)}{N}\left(P\left(Q^C(c, t), t\right) - c\right) f(t)\, dt.$$

Using this notation, it is sometimes more convenient to express $B(z, Z, c_1, c_2)$ as

$$B(z, Z, c_1, c_2) = \int_{\hat{t}(z,Z,c_1)}^{\hat{t}(z,Z,c_2)} z(P(Z,t) - c_1) f(t)\, dt + \int_{\hat{t}(z,Z,c_2)}^{+\infty} z(c_2 - c_1) f(t)\, dt + I^C(c_1, 0) + I^C\left(c_2, \hat{t}(z, Z, c_2)\right).$$

Expected profits can be expressed as

$$\begin{aligned}
\Pi^n &= I^C\left(c_1, 0, \hat{t}\left(X^+, c_1\right)\right) \\
&+\theta_1\left(I^C\left(c_1, \hat{t}\left(X^+, c_1\right), \hat{t}(X_1, c_1)\right) + \int_{\hat{t}(X_1,c_1)}^{\hat{t}(X_1,c_2)} x_1^n (P(X_1,t) - c_1) f(t)\, dt\right) \\
&+\theta_2 \int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} \frac{X^+}{N}\left(P\left(X^+, t\right) - c_1\right) f(t)\, dt \\
&+\theta_2\left(I^C\left(c_2, \hat{t}\left(X^+, c_2\right), \hat{t}(X_1, c_2)\right) + \int_{\hat{t}(X^+,c_2)}^{\hat{t}(X_1,c_2)} \frac{X^+}{N}(c_2 - c_1) f(t)\, dt\right) \\
&+I^C\left(c_2, \hat{t}(X_1, c_2), \hat{t}(K, c_2)\right) + \int_{\hat{t}(X_1,c_2)}^{\hat{t}(K,c_2)} k_1^n (c_2 - c_1) f(t)\, dt \\
&+\int_{\hat{t}(K,c_2)}^{+\infty} \left(k^n (P(K,t) - c_2) + k_1^n (c_2 - c_1)\right) f(t)\, dt - (r_1 - r_2) k_1^n - r_2 k^n.
\end{aligned}$$

Observing that $k_1^n = \theta_1 x_1^n + \Phi^+$, then rearranging terms yields

$$\begin{aligned}
\Pi^n &= \theta_1 x_1^n \left(\int_{\hat{t}(X_1,c_1)}^{\hat{t}(X_1,c_2)} x_1^n (P(X_1,t) - c_1) f(t)\, dt + \int_{\hat{t}(X_1,c_2)}^{+\infty} (c_2 - c_1) - (r_1 - r_2)\right) \\
&+\int_{\hat{t}(K,c_2)}^{+\infty} k^n (P(K,t) - c_2) f(t)\, dt - r_2 k^n \\
&+\theta_2 \frac{X^+}{N}\left(\int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} \left(P\left(X^+, t\right) - c_1\right) f(t)\, dt + \int_{\hat{t}(X^+,c_2)}^{+\infty} (c_2 - c_1) f(t)\, dt - (r_1 - r_2)\right) \\
&+I^C\left(c_1, 0, \hat{t}\left(X^+, c_1\right)\right) + \theta_1 I^C\left(c_1, \hat{t}\left(X^+, c_1\right), \hat{t}(X_1, c_1)\right) \\
&+\theta_2 I^C\left(c_2, \hat{t}\left(X^+, c_2\right), \hat{t}(X_1, c_2)\right) + I^C\left(c_2, \hat{t}(X_1, c_2), \hat{t}(K, c_2)\right).
\end{aligned}$$

Introducing the necessary integrals I^C yields

$$\begin{aligned}\Pi^n &= \theta_1\left(\int_{\hat{t}(X_1,c_1)}^{\hat{t}(X_1,c_2)} x_1^n\left(P\left(X_1,t\right)-c_1\right)f\left(t\right)dt - I^C\left(c_1,\hat{t}\left(X_1,c_1\right),\hat{t}\left(X_1,c_2\right)\right)\right)\\ &\quad +\theta_1\left(\int_{\hat{t}(X_1,c_2)}^{+\infty}\left(c_2-c_1\right)x_1^n-\left(r_1-r_2\right)x_1^n\right)\\ &\quad +\int_{\hat{t}(K,c_2)}^{+\infty} k^n\left(P\left(K,t\right)-c_2\right)f\left(t\right)dtf\left(t\right)dt - I^C\left(c_2,\hat{t}\left(K,c_2\right)\right)-r_2k^n\\ &\quad +\theta_2\left(\int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)}\frac{X^+}{N}\left(P\left(X^+,t\right)-c_1\right)f\left(t\right)dt - I^C\left(c_1,\hat{t}\left(X^+,c_1\right),\hat{t}\left(X^+,c_2\right)\right)-\left(r_1-r_2\right)\frac{X^+}{N}\right)\\ &\quad +I^C\left(c_1,0\right),\end{aligned}$$

thus

$$\Pi^n = \theta_1\left(B\left(x_1^n,X_1,c_1,c_2\right)-\left(r_1-r_2\right)x_1^n\right)+\left(A\left(k^n,K,c_2\right)-r_2k^n\right)+\theta_2\left(B\left(X^+,c_1,c_2\right)-\left(r_1-r_2\right)\frac{1}{N}X^+\right),$$

which is equation (9).

A.2 Proof of equilibrium, peaking technology

Since the interconnection is no longer saturated when the peaking technology reaches capacity, the proof of the unconstrained case (Zöttl (2011)) applies.

A.3 Proof of equilibrium baseload technology, downward deviation

Consider a downward deviation by producer 1: for all $n \geq 1$, $k^C = \frac{K^C}{N}$, for all $n > 1$, $x_1^C = \frac{K_1^C - \Phi^+}{N} = \frac{K_1^U}{N}$, while $x_1^1 \leq x_1^C$.

As we consider downward (and later upward) deviations, we introduce two additional functions. The symmetric equilibrium strategy for $(N-1)$ firms competing à la Cournot for marginal cost c in state t, while the last firm produces y is $\xi^{N-1}(y,c,t)$, uniquely defined by

$$P\left(y+(N-1)\xi^{N-1},t\right)+\xi^{N-1}P_q\left(y+(N-1)\xi^{N-1},t\right)=c.$$

Similarly, the monopoly output for a firm with marginal cost c in state t, while the $(N-1)$ other firms each produces y is $\xi^M(y, c, t)$ uniquely defined by

$$P\left((N-1)y + \xi^M, t\right) + \xi^M P_q\left((N-1)y + \xi^M, t\right) = c.$$

A.3.1 Small downward deviation

Consider first small downward deviations, such that the interconnection is congested before the base-load technology produces at capacity, as illustrated on Figure 9:

$$X^+ \leq X_1 \Leftrightarrow X^+ \leq K_1.$$

As previously, for $t \leq \hat{t}(X^+, c_1)$, firms compete à la Cournot for marginal cost c_1. Each produces $\frac{Q^C(c_1,t)}{N}$.

Consider now $t \geq \hat{t}(X^+, c_1)$. Consider first market 1. For $t \geq \hat{t}(X^+, c_1)$, firms play a symmetric equilibrium $\gamma_1^n = \frac{Q^C(c_1,t)}{N}$. This lasts until

$$\gamma_1^n = \frac{Q^C(c_1, t)}{N} = x_1^1 = t = \hat{t}\left(x_1^1, Nx_1^1, c_1\right).$$

For $t \geq \hat{t}\left(x_1^1, Nx_1^1, c_1\right)$, firm 1's adjusted production is x_1^1. Adjusted production for the $(N-1)$ larger firms is $\gamma_1^n = \xi^{N-1}\left(x_1^1, c_1, t\right)$. Then, these firms produce at their baseload capacity when

$$\gamma_1^n(t) = x_1^C \Leftrightarrow P\left(x_1^1 + (N-1)x_1^C, t\right) + x_1^C P\left(x_1^1 + (N-1)x_1^C, t\right) = c_1 \Leftrightarrow t = \hat{t}\left(x_1^C, X_1, c_1\right).$$

For $t \in \left[\hat{t}\left(x_1^C, X_1, c_1\right), \hat{t}(X_1, c_2)\right]$, all firms produce at baseload capacity. To understand the upper bound $\hat{t}(X_1, c_2)$, we now turn to market 2.

For $t \in \left[\hat{t}(X^+, c_1), \hat{t}(X^+, c_2)\right]$, peaking technology is not yet turned on.

For $t \geq \hat{t}(X^+, c_2)$, all firms turn on peaking technology and play the symmetric equilibrium $\gamma_2^n = \frac{Q^C(c_2,t)}{N}$. Then, prices in both markets are equal when

$$P(X_1, t) = P\left(Q^C(c_2, t), t\right) \Leftrightarrow X_1 = Q^C(c_2, t) \Leftrightarrow t = \hat{t}(X_1, c_2).$$

For $t \geq \hat{t}(X_1, c_2)$, nothing changes compared to the symmetric equilibrium.

This yields expected profit

$$\begin{aligned}
\Pi^1 &= I^C\left(c_1, 0, \hat{t}\left(X^+, c_1\right)\right) \\
&+\theta_1 I^C\left(c_1, \hat{t}\left(X^+, c_1\right), \hat{t}\left(x_1^1, N x_1^1, c_1\right)\right) \\
&+\theta_1 \int_{\hat{t}\left(x_1^1, N x_1^1, c_1\right)}^{\hat{t}\left(x_1^C, X_1, c_1\right)} x_1^1\left(P\left(x_1^1+(N-1)\,\xi^{N-1}, t\right)-c_1\right) f(t)\, dt \\
&+\theta_1 \int_{\hat{t}\left(x_1^C, X_1, c_1\right)}^{\hat{t}\left(X_1, c_2\right)} x_1^1\left(P\left(X_1, t\right)-c_1\right) f(t)\, dt \\
&+\theta_2 \frac{X^+}{N} \int_{\hat{t}\left(X^+, c_1\right)}^{\hat{t}\left(X^+, c_2\right)}\left(P\left(X^+, t\right)-c_1\right) f(t)\, dt \\
&+\theta_2\left(I^C\left(c_2, \hat{t}\left(X^+, c_2\right), \hat{t}\left(X_1, c_2\right)\right)+\frac{X^+}{N} \int_{\hat{t}\left(X^+, c_2\right)}^{\hat{t}\left(X_1, c_2\right)}\left(c_2-c_1\right) f(t)\, dt\right) \\
&+I^C\left(c_2, \hat{t}\left(X_1, c_2\right), \hat{t}\left(K^U, c_2\right)\right)+\int_{\hat{t}\left(X_1, c_2\right)}^{\hat{t}\left(K^U, c_2\right)} k_1^1\left(c_2-c_1\right) f(t)\, dt \\
&+\int_{\hat{t}\left(K^U, c_2\right)}^{+\infty}\left(k^U\left(P\left(K^U, t\right)-c_2\right)+k_1^1\left(c_2-c_1\right)\right) f(t)\, dt-\left(r_1-r_2\right) k_1^1-r_2 k^U .
\end{aligned}$$

Since output is continuous with respect to t, all functions are also continuous with respect to t. Thus, only the derivative of the integrands matters. Then,

$$\begin{aligned}
\frac{\partial \Pi^1}{\partial k_1^1} &= \int_{\hat{t}\left(x_1^1, N x_1^1, c_1\right)}^{\hat{t}\left(x_1^C, X_1, c_1\right)}\left(P\left(x_1^1+(N-1)\,\xi^{N-1}, t\right)+x_1^1\left(1+(N-1) \frac{\partial \xi^{N-1}}{\partial x_1^1}\right) P_q-c_1\right) f(t)\, dt \\
&+\int_{\hat{t}\left(x_1^C, X_1, c_1\right)}^{\hat{t}\left(X_1, c_2\right)}\left(P\left(X_1, t\right)+x_1^1 P_q\left(X_1, t\right)-c_1\right) f(t)\, dt+\int_{\hat{t}\left(X_1, c_2\right)}^{+\infty}\left(c_2-c_1\right) f(t)\, dt-\left(r_1-r_2\right).
\end{aligned}$$

The first-order condition defining ξ^{N-1} is

$$P\left(x_1^1+(N-1)\,\xi^{N-1}, t\right)+\xi^{N-1} P_q\left(x_1^1+(N-1)\,\xi^{N-1}, t\right)=c_1,$$

thus

$$P+x_1^1\left(1+(N-1) \frac{\partial \xi^{N-1}}{\partial x_1^1}\right) P_q-c_1=-\left(\xi^{N-1}-x_1^1-(N-1) \frac{\partial \xi^{N-1}}{\partial x_1^1}\right) P_q \geq 0$$

since $\xi^{N-1} \geq x_1^1$, $\frac{\partial \xi^{N-1}}{\partial x_1^1}<0$ since quantities are substitutes, and $P_q<0$. Thus, the first integral is positive.

Substituting the first-order condition (8) defining x_1^C in the last three terms yields

$$\begin{aligned}
E_1 &= \int_{\hat{t}\left(x_1^C, X_1, c_1\right)}^{\hat{t}(X_1, c_2)} \left(P\left(X_1, t\right) + x_1^1 P_q\left(X_1, t\right) - c_1\right) f\left(t\right) dt + \int_{\hat{t}(X_1, c_2)}^{+\infty} \left(c_2 - c_1\right) f\left(t\right) dt - \left(r_1 - r_2\right) \\
&= \int_{\hat{t}\left(x_1^C, X_1, c_1\right)}^{\hat{t}(X_1, c_2)} \left(P\left(X_1, t\right) + x_1^1 P_q\left(X_1, t\right) - c_1\right) f\left(t\right) dt - \int_{\hat{t}\left(X_1^C, c_1\right)}^{\hat{t}\left(X_1^C, c_2\right)} \left(P\left(X_1^C, t\right) + x_1^C P_q\left(X_1^C, t\right) - c_1\right) f\left(t\right) dt \\
&\quad + \int_{\hat{t}(X_1, c_2)}^{\hat{t}\left(X_1^C, c_2\right)} \left(c_2 - c_1\right) f\left(t\right) dt \\
&= \int_{\hat{t}\left(x_1^C, X_1, c_1\right)}^{\hat{t}\left(X_1^C, c_2\right)} \left(P\left(X_1, t\right) + x_1^1 P_q\left(X_1, t\right) - c_1\right) f\left(t\right) dt - \int_{\hat{t}(X_1, c_2)}^{\hat{t}\left(X_1^C, c_2\right)} \left(P\left(X_1^C, t\right) + x_1^C P_q\left(X_1^C, t\right) - c_2\right) f\left(t\right) dt \\
&\quad + \int_{\hat{t}\left(X_1^C, c_1\right)}^{\hat{t}\left(X_1^C, c_2\right)} \left(P\left(X_1, t\right) + x_1^1 P_q\left(X_1, t\right) - \left(P\left(X_1^C, t\right) + x_1^C P_q\left(X_1^C, t\right)\right)\right) f\left(t\right) dt.
\end{aligned}$$

Since $x_1^C \geq x_1^1$, $\hat{t}\left(x_1^C, X_1, c_1\right) \geq \hat{t}\left(x_1^1, X_1, c_1\right)$, thus the first integral is positive. The second integral is negative since $t \leq \hat{t}\left(X_1^C, c_2\right)$. Finally, the last integral is positive since $x_1^C \geq x_1^1$ and the marginal revenue is decreasing. Thus $E_1 > 0$, hence $\frac{\partial \Pi}{\partial k_1^1} > 0$: no downward deviation is profitable.

A.3.2 Large downward deviation

Suppose now the downward deviation is so large that firm 1 reaches baseload capacity before the line is constrained, $\hat{t}\left(k_1^1, N k_1^1, c_1\right) < \hat{t}\left(X^+, c_1\right)$.

If the $(N-1)$ other firms reach baseload capacity before the line is constrained, the line is never constrained. Thus, applying the analysis of the unconstrained case, no downward deviation is profitable.

Suppose now the line is constrained before the peaking technology is turned-on (and before the $(N-1)$ other firms produce at baseload capacity). The structure of the profit function is illustrated on Figure 10. For $t \in \left[\hat{t}\left(k_1^1, N k_1^1, c_1\right), \hat{t}\left(X^+, c_1\right)\right]$, firm 1 produces at its capacity k_1^1, while the $(N-1)$ other firms play a symmetric Cournot equilibrium $\xi^{N-1}\left(k_1^1, c_1, t\right)$. For $t \geq \hat{t}\left(X^+, c_1\right)$, the interconnection is constrained, and we are back to the previous case. To simplify the exposition, I present only the relevant terms, i.e., terms that include x_1^1 (or k_1^1) in the integrand. D_2, the sum of the relevant terms is

$$
\begin{aligned}
D_2 &= \int_{\hat{t}\left(k_1^1,Nk_1^1,c_1\right)}^{\hat{t}\left(X^+,c_1\right)} k_1^1 \left(P\left(k_1^1+(N-1)\,\xi^{N-1}\left(k_1^1,c_1,t\right),t\right)-c_1\right) f(t)\,dt \\
&+\theta_1 x_1^1 \int_{\hat{t}(X^+,c_1)}^{\hat{t}\left(x_1^C,X_1,c_1\right)} \left(P\left(x_1^1+(N-1)\,\xi^{N-1}\left(x_1^1,c_1,t\right),t\right)-c_1\right) f(t)\,dt \\
&+\theta_1 x_1^1 \left(\int_{\hat{t}\left(x_1^C,X_1,c_1\right)}^{\hat{t}(X_1,c_2)} \left(P\left(X_1,t\right)-c_1\right) f(t)\,dt + \int_{\hat{t}(X_1,c_2)}^{+\infty} \left(c_2-c_1\right) f(t)\,dt - \left(r_1-r_2\right)\right).
\end{aligned}
$$

Then,

$$
\begin{aligned}
\frac{\partial \Pi^1}{\partial k_1^1} &= \int_{\hat{t}\left(x_1^1,Nx_1^1,c_1\right)}^{\hat{t}\left(X^+,c_1\right)} \left(P\left(k_1^1+(N-1)\,\xi^{N-1},t\right)+k_1^1\left(1+(N-1)\frac{\partial \xi^{N-1}}{\partial k_1^1}\right)P_q-c_1\right) f(t)\,dt \\
&+\int_{\hat{t}(X^+,c_1)}^{\hat{t}\left(x_1^C,X_1,c_1\right)} \left(P\left(x_1^1+(N-1)\,\xi^{N-1},t\right)+x_1^1\left(1+(N-1)\frac{\partial \xi^{N-1}}{\partial x_1^1}\right)P_q-c_1\right) f(t)\,dt \\
&+\theta_1 \left(\int_{\hat{t}\left(x_1^C,X_1,c_1\right)}^{\hat{t}(X_1,c_2)} \left(P\left(X_1,t\right)+x_1^1 P_q\left(X_1,t\right)-c_1\right) f(t)\,dt + \int_{\hat{t}(X_1,c_2)}^{+\infty} \left(c_2-c_1\right) f(t)\,dt - \left(r_1-r_2\right)\right).
\end{aligned}
$$

As for small downward deviations, inserting the first-order conditions defining $\xi^{N-1}\left(k_1^1,c_1,t\right)$, $\xi^{N-1}\left(x_1^1,c_1,t\right)$, and X_1^C, then re-arranging proves that $\frac{\partial \Pi}{\partial k_1^1}>0$: no downward deviation is profitable.

Suppose now the downward deviation is so large that the interconnection is constrained after firm 1 turns on the peaking technology (but still before the $(N-1)$ other firms produce at baseload capacity). For $t \geq \hat{t}\left(k_1^1,Nk_1^1,c_1\right)$, the $(N-1)$ larger firms produce $\xi^{N-1}\left(k_1^1,c_1,t\right)$. Firm 1 turns on the peaking technology for $t^*\left(k_1^1,c_1,c_2\right)$ defined by

$$
P\left(k_1^1+(N-1)\,\xi^{N-1}\left(k_1^1,c_1,t^*\right),t^*\right)+k_1^1 P_q\left(k_1^1+(N-1)\,\xi^{N-1}\left(k_1^1,c_1,t^*\right),t^*\right)=c_2.
$$

For $t \geq t^*\left(k_1^1,c_1,c_2\right)$ firms play an asymmetric Cournot equilibrium, firm 1 with marginal cost c_2, the $(N-1)$ firms with marginal cost c_1. Denote $\zeta^C\left(c_2,c_1,t\right)$ firm's 1 strategy, and $\zeta^{N-1}\left(c_1,c_2,t\right)$ the strategy of the $(N-1)$ other firms. Observe that neither $\zeta^C\left(c_2,c_1,t\right)$ nor $\zeta^{N-1}\left(c_1,c_2,t\right)$ depend on k_1^1. The flow on the interconnection is

$$
\begin{aligned}
\varphi(t) &= \theta_2 Q_1(t)-\theta_1 Q_2(t)=\theta_2\left(k_1^1+(N-1)\,\zeta^{N-1}\left(c_1,c_2,t\right)\right)-\theta_1\left(\zeta^C\left(c_2,c_1,t\right)-k_1^1\right) \\
&= k_1^1+\theta_2(N-1)\,\zeta^{N-1}\left(c_1,c_2,t\right)-\theta_1\zeta^C\left(c_2,c_1,t\right).
\end{aligned}
$$

Depending on the values of θ_1 and θ_2, $\varphi(t)$ may be increasing or decreasing. If $\varphi(t)$ is decreasing or if $\varphi(t)$ is increasing and

$$k_1^1 + \theta_2 (N-1) k_1^C - \theta_1 \zeta^C (c_2, c_1, t) \leq \Phi^+,$$

the line is never congested, and we are back to the uncongested case. If $\varphi(t)$ is increasing, the line may be congested for $\bar{t}(X^+, c_1, c_2)$ uniquely defined by

$$k_1^1 + \theta_2 (N-1) \zeta^{N-1} (c_1, c_2, \bar{t}) - \theta_1 \zeta^C (c_2, c_1, \bar{t}) = \Phi^+.$$

This situation is described on Figure 11. D_3, the sum of the relevant terms, is

$$\begin{aligned} D_3 &= \int_{\hat{t}\left(k_1^1, N k_1^1, c_1\right)}^{t^*\left(k_1^1, c_1 c_2\right)} k_1^1 \left(P\left(k_1^1 + (N-1)\xi^{N-1}\left(k_1^1, c_1, t\right), t\right) - c_1\right) f(t)\, dt \\ &\quad + \theta_1 x_1^1 \int_{\bar{t}(X^+, c_1, c_2)}^{\hat{t}\left(x_1^C, X_1, c_1\right)} \left(P\left(x_1^1 + (N-1)\xi^{N-1}\left(x_1^1, c_1, t\right), t\right) - c_1\right) f(t)\, dt \\ &\quad + \theta_1 x_1^1 \left(\int_{\hat{t}\left(x_1^C, X_1, c_1\right)}^{\hat{t}(X_1, c_2)} \left(P(X_1, t) - c_1\right) f(t)\, dt + \int_{\hat{t}(X_1, c_2)}^{+\infty} (c_2 - c_1) f(t)\, dt - (r_1 - r_2)\right). \end{aligned}$$

Analysis similar to the previous cases shows that $\frac{\partial \Pi^1}{\partial k_1^1} > 0$ for $k_1^1 < k_1^C$: no negative deviation is profitable.

A.4 Proof of equilibrium, baseload technology upward deviation

Consider an upward deviation by firm 1: for all $n \geq 1$, $k^C = \frac{K^C}{N}$, for all $n > 1$, $k_1^C = \frac{K_1^C}{N}$, while $k_1^1 \geq k_1^C$. x_1^1 and X_1 are defined as previously.

For $t \geq \hat{t}(X^+, c_1)$, firms in market 1 play a symmetric equilibrium $\gamma_1^n = \frac{Q_1^C(c_1,t)}{N}$, up until the $(N-1)$ smallest firms reach their baseload capacity:

$$\frac{Q_1^C(c_1, t)}{N} = x_1^C \Leftrightarrow t = \hat{t}\left(x_1^C, N x_1^C, c_1\right).$$

For $t \geq \hat{t}\left(x_1^C, N x_1^C, c_1\right)$, firm 1 is a monopolist on residual demand, hence $\gamma_1^1(t) = \xi^M\left(x_1^C, c_1, t\right)$ up until

$$\xi^M\left(x_1^C, c_1, t\right) = x_1^1 \Leftrightarrow t = \hat{t}\left(x_1^1, X_1, c_1\right).$$

For $t \in \left[\hat{t}\left(x_1^1, X_1, c_1\right), \hat{t}\left(X_1, c_2\right)\right]$, all firms produce at baseload capacity, while the interconnection remains congested.

For $t \geq \hat{t}\left(X_1, c_2\right)$, the interconnection is no longer congested. This situation is represented on Figure 12.

The relevant terms in the profit function are thus

$$U_1 = \theta_1 x_1^1 \left(\int_{\hat{t}\left(x_1^1, X_1, c_1\right)}^{\hat{t}(X_1, c_2)} \left(P\left(X_1, t\right) - c_1\right) f(t)\, dt + \int_{\hat{t}(X_1, c_2)}^{+\infty} \left(c_2 - c_1\right) f(t)\, dt - \left(r_1 - r_2\right) \right).$$

Then,

$$\frac{\partial \Pi^1}{\partial k_1^1} = \int_{\hat{t}\left(x_1^1, X_1, c_1\right)}^{\hat{t}(X_1, c_2)} \left(P\left(X_1, t\right) + x_1^1 P_q\left(X_1, t\right) - c_1\right) f(t)\, dt + \int_{\hat{t}(X_1, c_2)}^{+\infty} \left(c_2 - c_1\right) f(t)\, dt - \left(r_1 - r_2\right),$$

and

$$\theta_1 \frac{\partial^2 \Pi^1}{\left(\partial x_1^1\right)^2} = \int_{\hat{t}\left(x_1^1, X_1, c_1\right)}^{\hat{t}(X_1, c_2)} \left(2P_q\left(X_1, t\right) + x_1^1 P_{qq}\left(X_1, t\right) - c_1\right) f(t)\, dt < 0:$$

an upward deviation is never profitable.

Paradoxically, including the constraint on the interconnection simplifies the analysis of upwards deviations. The decision to turn on the peaking technology in market 2 is independent of the conditions in market 1, hence the second-order derivative has a very simple expression.

B Equilibrium investment when $\left(\Phi^+ + \Phi^-\right) < \theta_1 K_2^U$

B.1 Expected profits for a symmetric strategy

Nothing changes for $t \leq \hat{t}\left(X_1, c_2\right)$. For $t \in \left[\hat{t}\left(X_1, c_2\right), \hat{t}\left(Y_1, c_2\right)\right]$, all firms play a symmetric Cournot equilibrium for marginal cost c_2, thus each produces $\frac{Q^C(c_2, t)}{N}$. The transmission constraint from market 2 to market 1 becomes binding when

$$K_1 - \theta_1 Q^C\left(c_2, t\right) = -\Phi^- \Leftrightarrow Q^C\left(c_2, t\right) = \frac{K_1 + \Phi^-}{\theta_1} = Y_1 \Leftrightarrow t = \hat{t}\left(Y_1, c_2\right).$$

For $t \geq \hat{t}(Y_1, c_2)$, the markets split again. Firm's n profits are

$$\begin{aligned}
\pi^n &= q_1^n (p_1 - c_1) q_1^n + q^n (p_2 - c_2) + \frac{\Phi^-}{N}(p_1 - p_2) \\
&= q_1^n \left(P\left(\frac{Q_1 + \Phi^-}{\theta_1}, t\right) - c_1 \right) + q_2^n \left(P\left(\frac{Q_2 - \Phi^-}{\theta_2}, t\right) - c_2 \right) + \frac{\Phi^-}{N}(p_1 - p_2) \\
&= \theta_1 \frac{q_1^n + \frac{\Phi^-}{N}}{\theta_1} \left(P\left(\frac{Q_1 + \Phi^-}{\theta_1}, t\right) - c_1 \right) + \theta_2 \frac{q_2^n - \frac{\Phi^-}{N}}{\theta_2} \left(P\left(\frac{Q_2 - \Phi^-}{\theta_2}, t\right) - c_2 \right) - \frac{\Phi^-}{N}(c_2 - c_1) \\
&= \theta_1 \delta_1^n (P(\Delta_1, t) - c_1) + \theta_2 \delta_2^n (P(\Delta_2, t) - c_2) - \frac{\Phi^-}{N}(c_2 - c_1),
\end{aligned}$$

where $\delta_1^n = \frac{q_1^n + \frac{\Phi^-}{N}}{\theta_1}$, $\delta_2^n = \frac{q_2^n - \frac{\Phi^-}{N}}{\theta_2}$, and $\Delta_i = \sum_{n=1}^N \delta_i^n$ for $i = 1, 2$.

For $t \geq \hat{t}(Y_1, c_2)$, firms in market 1 produce at baseload capacity, while firms in market 2 compete à la Cournot, for marginal cost c_2, thus $\delta_2^n = \frac{Q^C(c_2, t)}{N}$. This lasts until

$$\delta_2^n = y_2^n \Leftrightarrow Y_2 = Q^C(c_2, t) \Leftrightarrow t = \hat{t}(Y_2, c_2).$$

Finally, for $t \geq \hat{t}(Y_2, c_2)$, firms in market 2 produce at capacity.

This yields expected profits

$$\begin{aligned}
\Pi^n &= I^C\left(c_1, 0, \hat{t}\left(X^+, c_1\right)\right) \\
&+\theta_1 \left(I^C\left(c_1, \hat{t}\left(X^+, c_1\right), \hat{t}(X_1, c_1)\right) + \int_{\hat{t}(X_1, c_1)}^{\hat{t}(X_1, c_2)} x_1^n (P(X_1, t) - c_1) f(t)\, dt \right) \\
&+\theta_2 \int_{\hat{t}(X^+, c_1)}^{\hat{t}(X^+, c_2)} \frac{X^+}{N} \left(P\left(X^+, t\right) - c_1\right) f(t)\, dt \\
&+\theta_2 \left(I^C\left(c_2, \hat{t}\left(X^+, c_2\right), \hat{t}(X_1, c_2)\right) + \int_{\hat{t}(X^+, c_2)}^{\hat{t}(X_1, c_2)} \frac{X^+}{N} (c_2 - c_1) f(t)\, dt \right) \\
&+I^C\left(c_2, \hat{t}(X_1, c_2), \hat{t}(Y_1, c_2)\right) + \int_{\hat{t}(X_1, c_2)}^{\hat{t}(Y_1, c_2)} k_1^n (c_2 - c_1) f(t)\, dt \\
&+\theta_1 \int_{\hat{t}(Y_1, c_2)}^{+\infty} y_1^n (P(Y_1, t) - c_1) f(t)\, dt - \int_{\hat{t}(Y_1, c_2)}^{+\infty} \frac{\Phi^-}{N} (c_2 - c_1) f(t)\, dt \\
&+\theta_2 \left(I^C\left(c_2, \hat{t}(Y_1, c_2), \hat{t}(Y_2, c_2)\right) + \int_{\hat{t}(Y_2, c_2)}^{+\infty} y_2^n (P(Y_2, t) - c_2) f(t)\, dt_2^n \right) - r_1 k_1^n - r_2 k_2^n.
\end{aligned}$$

Observe that

$$\begin{aligned}
\theta_1 \int_{\hat{t}(Y_1,c_2)}^{+\infty} y_1^n \left(P\left(Y_1,t\right)-c_1\right) f\left(t\right) dt &= \theta_1 \int_{\hat{t}(Y_1,c_2)}^{+\infty} y_1^n \left(P\left(Y_1,t\right)-c_2\right) f\left(t\right) dt + \int_{\hat{t}(Y_1,c_2)}^{+\infty} \theta_1 y_1^n \left(c_2-c_1\right) f\left(t\right) dt \\
&= \theta_1 \int_{\hat{t}(Y_1,c_2)}^{+\infty} y_1^n \left(P\left(Y_1,t\right)-c_2\right) f\left(t\right) dt \\
&\quad + \int_{\hat{t}(Y_1,c_2)}^{+\infty} \theta_1 x_1^n \left(c_2-c_1\right) f\left(t\right) dt + \int_{\hat{t}(Y_1,c_2)}^{+\infty} \frac{\Phi^+ + \Phi^-}{N} \left(c_2-c_1\right) f\left(t\right) dt
\end{aligned}$$

since

$$k_1^n = \theta_1 x_1^n + \frac{\Phi^+}{N} = \theta_1 y_1^n - \frac{\Phi^-}{N} \Rightarrow \theta_1 y_1^n = \theta_1 x_1^n + \frac{\Phi^+ + \Phi^-}{N}.$$

Then, rearranging terms yields

$$\begin{aligned}
\Pi^n &= \theta_1 \int_{\hat{t}(Y_1,c_2)}^{+\infty} y_1^n \left(P\left(Y_1,t\right)-c_2\right) f\left(t\right) dt \\
&\quad +\theta_1 \left(\int_{\hat{t}(X_1,c_1)}^{\hat{t}(X_1,c_2)} x_1^n \left(P\left(X_1,t\right)-c_1\right) f\left(t\right) dt + \int_{\hat{t}(X_1,c_2)}^{+\infty} x_1^n \left(c_2-c_1\right) f\left(t\right) dt \right) - r_1 k_1^n \\
&\quad +\theta_2 \left(\int_{\hat{t}(Y_2,c_2)}^{+\infty} y_2^n \left(P\left(Y_2,t\right)-c_2\right) f\left(t\right) dt_2^n - r_2 y_2^n \right) \\
&\quad +\theta_2 \frac{X^+}{N} \left(\int_{\hat{t}(X^+,c_1)}^{\hat{t}\left(X^+,c_2\right)} \left(P\left(X^+,t\right)-c_1\right) f\left(t\right) dt + \int_{\hat{t}(X^+,c_2)}^{+\infty} \left(c_2-c_1\right) f\left(t\right) dt \right) - r_2 \frac{\Phi^-}{N} \\
&\quad +I^C\left(c_1, 0, \hat{t}\left(X^+,c_1\right)\right) + \theta_1 I^C\left(c_1, \hat{t}\left(X^+,c_1\right), \hat{t}\left(X_1,c_1\right)\right) \\
&\quad +\theta_2 I^C\left(c_2, \hat{t}\left(X^+,c_2\right), \hat{t}\left(X_1,c_2\right)\right) + I^C\left(c_2, \hat{t}\left(X_1,c_2\right), \hat{t}\left(Y_1,c_2\right)\right) \\
&\quad +\theta_2 I^C\left(c_2, \hat{t}\left(Y_1,c_2\right), \hat{t}\left(Y_2,c_2\right)\right),
\end{aligned}$$

thus

$$\Pi^n = \theta_1 \left(A\left(y_1^n, Y_1, c_2\right) + B\left(x_1^n, X_1, c_1, c_2\right)\right) - r_1 k_1^n + \theta_2 \left(A\left(y_2^n, Y_2, c_2\right) - r_2 y_2^n\right) + \theta_2 B\left(X^+, c_1, c_2\right) - r_2 \frac{\Phi^-}{N},$$

which is equation (10).

B.2 Proof of equilibrium, peaking technology

The peaking capacity has no impact on the transmission constraints, which are solely determined by the baseload capacity. The unconstrained analysis thus applies, and $y_2^n = \frac{K^U}{N}$ is an equilibrium.

B.3 Proof of equilibrium, baseload technology

Consider a small downward deviation by firm 1. For $t \geq \hat{t}(X^+, c_1)$, $\gamma_1^n = \frac{Q^C(c_1,t)}{N}$. Firm 1 reaches baseload capacity when

$$x_1^1 = \frac{Q^C(c_1,t)}{N} \Leftrightarrow t = \hat{t}\left(x_1^1, Nx_1^1, c_1\right).$$

For $t \geq \hat{t}\left(x_1^1, Nx_1^1, c_1\right)$, firm 1 produces at its baseload capacity, and the other firms produce $\gamma_1^n = \xi^{N-1}\left(x_1^1, c_1, t\right)$. The $(N-1)$ other firms reach baseload capacity when

$$x_1^C = \xi^{N-1}\left(x_1^1, c_1, t\right) \Leftrightarrow t = \hat{t}\left(x_1^C, X_1, c_1\right).$$

When the interconnection was constrained in one direction only $x_1^C = k_1^U$. This is no longer the case when the interconnection is constrained in both directions.

For $t \in \left[\hat{t}\left(x_1^C, X_1, c_1\right), \hat{t}(X_1, c_2)\right]$, all firms produce at their baseload capacity.

For $t \in \left[\hat{t}(X_1, c_2), \hat{t}(Y_1, c_2)\right]$, the interconnection is not congested, and $q_n = \frac{Q^C(c_2,t)}{N}$.

For $t \geq \hat{t}(Y_1, c_2)$, the interconnection is congested from market 2 to market 1. This is illustrated on Figure 13.

The relevant terms in the profit function are

$$\begin{aligned} D_4 &= \theta_1 x_1^1 \int_{\hat{t}\left(x_1^1, Nx_1^1, c_1\right)}^{\hat{t}\left(x_1^C, X_1, c_1\right)} \left(P\left(x_1^1 + (N-1)\xi^{N-1}, t\right) - c_1\right) f(t)\, dt \\ &\quad + \theta_1 x_1^1 \left(\int_{\hat{t}\left(x_1^C, X_1, c_1\right)}^{\hat{t}(X_1,c_2)} \left(P(X_1,t) - c_1\right) f(t)\, dt + \int_{\hat{t}(X_1,c_2)}^{+\infty} (c_2 - c_1) f(t)\, dt\right) \\ &\quad + \int_{\hat{t}(Y_1,c_2)}^{+\infty} \theta_2 y_1^1 \left(P(Y_1,t) - c_2\right) f(t)\, dt - r_1 k_1^1 \end{aligned}$$

Then

$$\begin{aligned} \frac{\partial \Pi^1}{\partial k_1^1} &= \int_{\hat{t}\left(x_1^1, Nx_1^1, c_1\right)}^{\hat{t}\left(x_1^C, X_1, c_1\right)} \left(P\left(x_1^1 + (N-1)\xi^{N-1}, t\right) + x_1^1 P_q \times \left(1 + (N-1)\frac{\partial \xi^{N-1}}{\partial x_1^1}, t\right) - c_1\right) f(t)\, dt \\ &\quad + \int_{\hat{t}\left(x_1^C, X_1, c_1\right)}^{\hat{t}(X_1,c_2)} \left(P(X_1,t) + x_1^1 P_q(X_1,t) - c_1\right) f(t)\, dt + \int_{\hat{t}(X_1,c_2)}^{+\infty} (c_2 - c_1) f(t)\, dt \\ &\quad + \int_{\hat{t}(Y_1,c_2)}^{+\infty} \left(P(Y_1,t) + y_1^1 P_q(Y_1,t) - c_2\right) f(t)\, dt - r_1. \end{aligned}$$

The familiar argument shows that the firm term is positive since $x_1^1 \leq \xi^{N-1}$. Inserting the first-order condition (8) yields two terms. The first term is

$$\begin{aligned} E_4 &= \int_{\hat{t}(Y_1,c_2)}^{+\infty} \left(P\left(Y_1,t\right)+y_1^1 P_q\left(Y_1,t\right)-c_2\right) f\left(t\right) dt - \int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty} y_1^C \left(P\left(Y_1^C,t\right)+y_1^C P_q\left(Y_1^C,t\right)-c_2\right) f\left(t\right) dt \\ &= \int_{\hat{t}(Y_1,c_2)}^{\hat{t}\left(Y_1^C,c_2\right)} y_1^1 \left(P\left(Y_1,t\right)+y_1^1 P_q\left(Y_1,t\right)-c_2\right) f\left(t\right) dt \\ &+ \int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty} \left(P\left(Y_1,t\right)+y_1^1 P_q\left(Y_1,t\right)-\left(P\left(Y_1^C,t\right)+y_1^C P_q\left(Y_1^C,t\right)\right)\right) f\left(t\right) dt, \end{aligned}$$

which is positive since $\hat{t}\left(Y_1,c_2\right) \geq \hat{t}\left(y_1^1,Y_1,c_2\right)$, and $y_1^1 \leq y_1^C$ and the marginal revenue is decreasing. The second term is

$$\begin{aligned} F_4 &= \int_{\hat{t}\left(x_1^C,X_1,c_1\right)}^{\hat{t}(X_1,c_2)} \left(P\left(X_1,t\right)+x_1^1 P_q\left(X_1,t\right)-c_1\right) f\left(t\right) dt - \int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}\left(X_1^C,c_2\right)} \left(P\left(X_1^C,t\right)+x_1^C P_q\left(X_1,t\right)-c_1\right) f\left(t\right) dt \\ &+ \int_{\hat{t}(X_1,c_2)}^{\hat{t}\left(X_1^C,c_2\right)} \left(c_2-c_1\right) f\left(t\right) dt \\ &= \int_{\hat{t}\left(x_1^C,X_1,c_1\right)}^{\hat{t}\left(X_1^C,c_1\right)} \left(P\left(X_1,t\right)+x_1^1 P_q\left(X_1,t\right)-c_1\right) f\left(t\right) dt \\ &+ \int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}(X_1,c_2)} \left(P\left(X_1,t\right)+x_1^1 P_q\left(X_1,t\right)-\left(P\left(X_1^C,t\right)+x_1^C P_q\left(X_1,t\right)\right)\right) f\left(t\right) dt \\ &- \int_{\hat{t}(X_1,c_2)}^{\hat{t}\left(X_1^C,c_2\right)} \left(P\left(X_1^C,t\right)+x_1^C P_q\left(X_1,t\right)-c_2\right) f\left(t\right) dt, \end{aligned}$$

which is positive since $x_1^1 \leq x_1^C$, thus $\hat{t}\left(x_1^1,X_1,c_1\right) \geq \hat{t}\left(x_1^C,X_1,c_1\right)$; the marginal revenue is decreasing; and $P\left(X_1^C,t\right)+x_1^C P_q\left(X_1,t\right)-c_2 < 0$ for $t \leq \hat{t}\left(X_1^C,c_2\right)$. Thus, $\frac{\partial \Pi^1}{\partial k_1^1}$ is positive: no downward deviation is profitable.

A similar argument can be applied to a larger downward deviation, and to upward deviations.

B.4 Proof of Corollary 1: properties of $K_1^C\left(\Phi^+,\Phi^-\right)$

First observe that $\Psi\left(.,.\right)$ is decreasing in both arguments by inspection, and that, for $c_1 < c_2$, $\left(\Psi\left(.,c_1\right)-\Psi\left(.,c_2\right)\right)$ is decreasing since

$$\Psi_q\left(Z,c_1\right)-\Psi_q\left(Z,c_2\right) = \frac{(N+1)}{N} \int_{\hat{t}(Z,c_1)}^{\hat{t}(Z,c_2)} \left(P_q\left(Z,t\right)+\frac{Z}{N+1} P_{qq}\left(Z,t\right)\right) f\left(t\right) dt < 0.$$

Implicit differentiation of equation (8) with respect to Φ^+ yields

$$\Psi_q\left(X_1^C, c_1\right)\left(\frac{\partial K_1^C}{\partial \Phi^+} - 1\right) - \Psi_q\left(X_1^C, c_2\right)\left(\frac{\partial K_1^C}{\partial \Phi^+} - 1\right) + \Psi_q\left(Y_1^C, c_2\right)\frac{\partial K_1^C}{\partial \Phi^+} = 0$$

$\Leftrightarrow$

$$\frac{\partial K_1^C}{\partial \Phi^+} = \frac{\Psi_q\left(X_1^C, c_1\right) - \Psi_q\left(X_1^C, c_2\right)}{\Psi_q\left(X_1^C, c_1\right) - \Psi_q\left(X_1^C, c_2\right) + \Psi_q\left(Y_1^C, c_2\right)} > 0.$$

Then, implicit differentiation of equation (8) with respect to Φ^- yields

$$\Psi_q\left(X_1^C, c_1\right)\frac{\partial K_1^C}{\partial \Phi^-} - \Psi_q\left(X_1^C, c_2\right)\frac{\partial K_1^C}{\partial \Phi^-} + \Psi_q\left(Y_1^C, c_2\right)\left(\frac{\partial K_1^C}{\partial \Phi^-} + 1\right) = 0$$

$\Leftrightarrow$

$$\frac{\partial K_1^C}{\partial \Phi^-} = -\frac{\Psi_q\left(Y_1^C, c_2\right)}{\Psi_q\left(X_1^C, c_1\right) - \Psi_q\left(X_1^C, c_2\right) + \Psi_q\left(Y_1^C, c_2\right)} = \frac{\partial K_1^C}{\partial \Phi^+} - 1.$$

Finally, for $\Phi^+ = \Phi^- = 0$, equation (8) simplifies to

$$\Psi\left(\frac{K_1^C}{\theta_1}, c_1\right) - \Psi\left(\frac{K_1^C}{\theta_1}, c_2\right) + \Psi\left(\frac{K_1^C}{\theta_1}, c_2\right) = r_1 \Leftrightarrow \Psi\left(\frac{K_1^C}{\theta_1}, c_1\right) = r_1 \Leftrightarrow K_1^C(0,0) = \theta_1\chi\left(c_1, r_1\right).$$

Setting $\Phi^- = 0$ in equation (7) shows that $K_2^C(0,0) = \theta_2\chi\left(c_2, r_2\right)$.

C Proof of Proposition 2: marginal value of interconnection capacity

For $\Phi^+ < \theta_2 K_1^U$ and $(\Phi^+ + \Phi^-) \geq \theta_1 K_2^U$, substituting in the optimal values yields

$$
\begin{aligned}
W\left(\Phi^+\right) = & \int_0^{\hat{t}\left(X^+,c_1\right)} \left(S\left(P\left(Q^C\left(c_1,t\right),t\right),t\right) - c_1 Q^C\left(c_1,t\right)\right) f(t)\,dt \\
& + \int_{\hat{t}\left(X^+,c_1\right)}^{\hat{t}\left(K_1^U,c_1\right)} \left(\theta_1 S\left(P\left(Q^C\left(c_1,t\right),t\right),t\right) - c_1\left(\theta_1 Q_1^C(t) + \Phi^+\right)\right) f(t)\,dt \\
& + \int_{\hat{t}\left(K_1^U,c_1\right)}^{\hat{t}\left(K_1^U,c_2\right)} \left(\theta_1 S\left(P\left(K_1^U,t\right),t\right) - c_1\left(\theta_1 K_1^U + \Phi^+\right)\right) f(t)\,dt \\
& + \int_{\hat{t}\left(X^+,c_1\right)}^{\hat{t}\left(X^+,c_2\right)} \theta_2 S\left(P\left(X^+,t\right),t\right) f(t)\,dt \\
& + \int_{\hat{t}\left(X^+,c_2\right)}^{\hat{t}\left(K_1^U,c_2\right)} \left(\theta_2 S\left(P\left(Q^C\left(c_2,t\right),t\right),t\right) - c_2\left(\theta_2 Q_2^C(t) - \Phi^+\right)\right) f(t)\,dt \\
& + \int_{\hat{t}\left(K_1^U,c_2\right)}^{\hat{t}\left(K^U,c_2\right)} \left(S\left(P\left(Q^C\left(c_2,t\right),t\right),t\right) - c_2 Q^C\left(c_2,t\right) + \left(c_2 - c_1\right)\left(\theta_1 K_1^U + \Phi^+\right)\right) f(t)\,dt \\
& + \int_{\hat{t}\left(K^U,c_2\right)}^{+\infty} \left(S\left(P\left(K^U,t\right),t\right) - c_2 K^U + \left(c_2 - c_1\right)\left(\theta_1 K_1^U + \Phi^+\right)\right) f(t)\,dt \\
& - r_2 K^U - \left(r_1 - r_2\right)\left(\theta_1 K_1^U + \Phi^+\right).
\end{aligned}
$$

Introducing the expected Cournot surplus

$$
J^C(c,a,b) = \int_a^b \left(S\left(P\left(Q^C(c,t),t\right),t\right) - cQ^C(c,t)\right) f(t)\,dt,
$$

and rearranging yields

$$
\begin{aligned}
W\left(\Phi^+\right) = & \theta_1 \int_{\hat{t}\left(K_1^U,c_1\right)}^{\hat{t}\left(K_1^U,c_2\right)} \left(S\left(P\left(K_1^U,t\right),t\right) - c_1 K_1^U\right) f(t)\,dt - \left(r_1 - r_2\right) K_1^U \\
& + \int_{\hat{t}\left(K^U,c_2\right)}^{+\infty} \left(S\left(P\left(K^U,t\right),t\right) - c_2 K^U\right) f(t)\,dt - r_2 K^U \\
& + \theta_2 \left(\int_{\hat{t}\left(X^+,c_1\right)}^{\hat{t}\left(X^+,c_2\right)} \left(S\left(P\left(X^+,t\right),t\right) - c_1 X^+\right) f(t)\,dt + \int_{\hat{t}\left(X^+,c_2\right)}^{+\infty} X^+\left(c_2 - c_1\right) f(t)\,dt - \left(r_1 - r_2\right) X^+\right) \\
& + J^C\left(c_1, 0, \hat{t}\left(X^+,c_1\right)\right) + \theta_1 J^C\left(c_1, \hat{t}\left(X^+,c_1\right), \hat{t}\left(K_1^U,c_1\right)\right) \\
& + \theta_2 J^C\left(c_2, \hat{t}\left(X^+,c_2\right), \hat{t}\left(K_1^U,c_2\right)\right) + J^C\left(c_2, \hat{t}\left(K_1^U,c_2\right), \hat{t}\left(K^U,c_2\right)\right).
\end{aligned}
$$

Since output hence surplus are continuous with respect to the state of the world, only the derivatives of the integrands matter. Thus,

$$\frac{dW}{d\Phi^+} = \int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} \left(P\left(X^+,t\right) - c_1\right) f(t)\, dt + \int_{\hat{t}(X^+,c_2)}^{+\infty} (c_2 - c_1) - (r_1 - r_2)\,.$$

We immediately verify that

$$\begin{aligned}\frac{d^2W}{(d\Phi^+)^2} &= \int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} P_q\left(X^+,t\right) f(t)\, dt + \frac{X^+}{N} P\left(X^+,\hat{t}\left(X^+,c_1\right)\right) \frac{\partial \hat{t}\left(X^+,c_1\right)}{\partial X^+} f\left(\hat{t}\left(X^+,c_1\right)\right) \\ &< 0.\end{aligned}$$

For $(\Phi^+ + \Phi^-) < \theta_1 K_2^U$,

$$\begin{aligned} W\left(\Phi^+,\Phi^-\right) &= J^C\left(c_1, 0, \hat{t}\left(X^+,c_1\right)\right) \\ &+\theta_1 J^C\left(c_1, \hat{t}\left(X^+,c_1\right), \hat{t}\left(X_1^C,c_1\right)\right) \\ &+\theta_1 \int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}\left(X_1^C,c_2\right)} \left(S\left(P\left(X_1^C,t\right),t\right) - c_1 X_1^C\right) f(t)\, dt \\ &+\theta_2 \left(\int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} S\left(P\left(X^+,t\right),t\right) f(t)\, dt + \int_{\hat{t}(X^+,c_2)}^{\hat{t}\left(X_1^C,c_2\right)} X^+ (c_2 - c_1)\right) f(t)\, dt \\ &+\theta_2 J^C\left(c_2, \hat{t}\left(X^+,c_2\right), \hat{t}\left(X_1^C,c_2\right)\right) \\ &+J^C\left(c_2, \hat{t}\left(X_1^C,c_2\right), \hat{t}\left(Y_1^C,c_2\right)\right) + \int_{\hat{t}\left(X_1^C,c_2\right)}^{\hat{t}\left(Y_1^C,c_2\right)} (c_2 - c_1)\left(\theta_1 X_1^C + \Phi^+\right) f(t)\, dt \\ &+\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty} \left(\theta_1 S\left(P\left(Y_1^C,t\right),t\right) - c_1 K_1^C\right) f(t)\, dt \\ &+\int_{\hat{t}\left(Y_1^C,c_2\right)}^{\hat{t}\left(Y_2^C,c_2\right)} \left(\theta_2 S\left(P\left(Q^C(c_2,t),t\right),t\right) - c_2\left(\theta_2 Q^C(c_2,t) + \Phi^-\right)\right) f(t)\, dt \\ &+\int_{\hat{t}\left(K^U,c_2\right)}^{+\infty} \left(\theta_2 S\left(P\left(K^U,t\right),t\right) - c_2\left(\theta_2 K^U + \Phi^-\right)\right) f(t)\, dt \\ &-r_2\left(\theta_2 K^U + \Phi^-\right) - r_1 K_1^C. \end{aligned}$$

Observing that

$$\begin{aligned}
\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}\left(\theta_1 S\left(P\left(Y_1^C,t\right),t\right)-c_1K_1^C\right)f\left(t\right)dt &= \int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}\left(\theta_1 S\left(P\left(Y_1^C,t\right),t\right)-c_2\left(\theta_1 Y_1^C-\Phi^-\right)\right)f\left(t\right)dt \\
&\quad +\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}\left(c_2-c_1\right)\left(\theta_1 X_1^C+\Phi^+\right)f\left(t\right)dt \\
&= \theta_1\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}\left(S\left(P\left(Y_1^C,t\right),t\right)-c_2Y_1^C\right)f\left(t\right)dt \\
&\quad +\theta_1\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}X_1^C\left(c_2-c_1\right)f\left(t\right)dt \\
&\quad +\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}c_2\Phi^- f\left(t\right)dt+\theta_2\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}X^+\left(c_2-c_1\right)f\left(t\right)dt,
\end{aligned}$$

then rearranging yields

$$\begin{aligned}
W\left(\Phi^+,\Phi^-\right) &= \theta_1\left(\int_{\hat{t}(X_1,c_1)}^{\hat{t}\left(X_1^C,c_2\right)}\left(S\left(P\left(X_1^C,t\right),t\right)-c_1X_1^C\right)f\left(t\right)dt+\int_{\hat{t}\left(X_1^C,c_2\right)}^{+\infty}X_1^C\left(c_2-c_1\right)f\left(t\right)dt\right) \\
&\quad +\theta_1\left(\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}\left(S\left(P\left(Y_1^C,t\right),t\right)-c_2Y_1^C\right)f\left(t\right)dt\right)-r_1K_1^C \\
&\quad +\theta_2\left(\int_{\hat{t}(K^U,c_2)}^{+\infty}\left(S\left(P\left(K^U,t\right),t\right)-c_2K^U\right)f\left(t\right)dt-r_2K^U\right) \\
&\quad +\theta_2\left(\int_{\hat{t}(X^+,c_1)}^{\hat{t}\left(X^+,c_2\right)}\left(S\left(P\left(X^+,t\right),t\right)-c_1X^+\right)f\left(t\right)dt+\int_{\hat{t}(X^+,c_2)}^{+\infty}X^+\left(c_2-c_1\right)f\left(t\right)dt\right) \\
&\quad -r_2\Phi^- \\
&\quad +\theta_2J^C\left(c_1,0,\hat{t}\left(X^+,c_1\right)\right)+\theta_1J^C\left(c_1,0,\hat{t}\left(X_1^C,c_1\right)\right) \\
&\quad +\theta_2J^C\left(c_2,\hat{t}\left(X^+,c_2\right),\hat{t}\left(Y_2^C,c_2\right)\right)+\theta_1J^C\left(c_2,\hat{t}\left(X_1^C,c_2\right),\hat{t}\left(Y_1^C,c_2\right)\right).
\end{aligned}$$

Thus,

$$
\begin{aligned}
\frac{dW}{d\Phi^+} &= \frac{\partial W}{\partial \Phi^+} + \frac{\partial W}{\partial K_1^C}\frac{\partial K_1^C}{\partial \Phi^+} \\
&= -\left(\int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}\left(X_1^C,c_2\right)} \left(P\left(X_1^C,t\right),t-c_1\right) f(t)\,dt + \int_{\hat{t}\left(X_1^C,c_2\right)}^{+\infty} (c_2-c_1) f(t)\,dt\right) \\
&\quad + \int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} \left(P\left(X^+,t\right)-c_1\right) f(t)\,dt + \int_{\hat{t}(X^+,c_2)}^{+\infty} (c_2-c_1) f(t)\,dt \\
&\quad + \left(\begin{array}{c} \int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}\left(X_1^C,c_2\right)} \left(P\left(X_1^C,t\right)-c_1\right) f(t)\,dt + \int_{\hat{t}\left(X_1^C,c_2\right)}^{+\infty} (c_2-c_1) f(t)\,dt - r_1 \\ + \int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty} \left(P\left(Y_1^C,t\right)-c_2 Y_1^C\right) f(t)\,dt \end{array}\right) \frac{\partial K_1^C}{\partial \Phi^+} \\
&= \int_{\hat{t}(X^+,c_1)}^{\hat{t}(X^+,c_2)} \left(P\left(X^+,t\right)-c_1\right) f(t)\,dt + \int_{\hat{t}(X^+,c_2)}^{+\infty} (c_2-c_1) f(t)\,dt \\
&\quad - \left(\int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}\left(X_1^C,c_2\right)} \left(P\left(X_1^C,t\right),t-c_1\right) f(t)\,dt + \int_{\hat{t}\left(X_1^C,c_2\right)}^{+\infty} (c_2-c_1) f(t)\,dt\right) \\
&\quad - \frac{1}{N}\left(\int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}\left(X_1^C,c_2\right)} P_q\left(X_1^C,t\right) f(t)\,dt + \int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty} P_q\left(Y_1^C,t\right) f(t)\,dt\right) \frac{\partial K_1^C}{\partial \Phi^+}
\end{aligned}
$$

which proves the result. Finally,

$$
\begin{aligned}
\frac{dW}{d\Phi^-} &= \frac{\partial W}{\partial \Phi^-} + \frac{\partial W}{\partial K_1^C}\frac{\partial K_1^C}{\partial \Phi^-} \\
&= \int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty} \left(P\left(Y_1^C,t\right)-c_2 Y_1^C\right) f(t)\,dt - r_2 \\
&\quad - \frac{1}{N}\left(\int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}\left(X_1^C,c_2\right)} P_q\left(X_1^C,t\right) f(t)\,dt + \int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty} P_q\left(Y_1^C,t\right) f(t)\,dt\right) \frac{\partial K_1^C}{\partial \Phi^+}
\end{aligned}
$$

D Proof of Proposition 3

Differentiating equation (10) with respect to Φ^+ yields

$$
\begin{aligned}
\frac{\partial \Pi^n}{\partial \Phi^+} &= \frac{\partial B}{\partial X}\left(X^+,c_1,c_2\right) - \frac{\partial B}{\partial X}\left(X_1^C,c_1,c_2\right) \\
&\quad + \left(\frac{\partial A}{\partial Y}\left(Y_1^C,c_2\right) + \frac{\partial B}{\partial X}\left(X_1^C,c_1,c_2\right) - \frac{r_1}{N}\right)\frac{\partial K_1^C}{\partial \Phi^+}.
\end{aligned}
$$

Observing that

$$\begin{aligned}
\frac{\partial A}{\partial Y}(Y,c) &= \frac{1}{N}\int_{\hat{t}(Y,c)}^{+\infty}\left(P(Y,t)+YP_q(Y,t)-c\right)f(t)\,dt \\
&= \frac{1}{N}\left(\Psi(Y,c)+(N-1)\int_{\hat{t}(Y,c)}^{+\infty}\frac{Y}{N}P_q(Y,t)\,f(t)\,dt\right)
\end{aligned}$$

yields

$$\begin{aligned}
\frac{\partial \Pi^n}{\partial \Phi^+} &= \frac{\partial B}{\partial X}\left(X^+,c_1,c_2\right)-\frac{\partial B}{\partial X}\left(X_1^C,c_1,c_2\right) \\
&\quad +\frac{1}{N}\left(\Psi\left(Y_1^C,c_2\right)+\Psi\left(X_1^C,c_1\right)-\Psi\left(X_1^C,c_2\right)-r_1+\right)\frac{\partial K_1^C}{\partial \Phi^+} \\
&\quad +\frac{N-1}{N}\left(\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}\frac{Y_1^C}{N}P_q\left(Y_1^C,t\right)f(t)\,dt+\int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}\left(X_1^C,c_2\right)}\frac{X_1^C}{N}P_q\left(X_1^C,t\right)f(t)\,dt\right)\frac{\partial K_1^C}{\partial \Phi^+},
\end{aligned}$$

which then yields the result.

Similarly, differentiating equation (10) with respect to Φ^- yields

$$\begin{aligned}
\frac{\partial \Pi^n}{\partial \Phi^-} &= \frac{\partial A}{\partial Y}\left(Y_1^C,c_2\right)-\frac{r_2}{N} \\
&\quad +\left(\frac{\partial A}{\partial Y}\left(Y_1^C,c_2\right)+\frac{\partial B}{\partial X}\left(X_1^C,c_1,c_2\right)-\frac{r_1}{N}\right)\frac{\partial K_1^C}{\partial \Phi^-} \\
&= \frac{\partial A}{\partial Y}\left(Y_1^C,c_2\right)-\frac{r_2}{N} \\
&\quad +\frac{N-1}{N}\left(\int_{\hat{t}\left(Y_1^C,c_2\right)}^{+\infty}\frac{Y_1^C}{N}P_q\left(Y_1^C,t\right)f(t)\,dt+\int_{\hat{t}\left(X_1^C,c_1\right)}^{\hat{t}\left(X_1^C,c_2\right)}\frac{X_1^C}{N}P_q\left(X_1^C,t\right)f(t)\,dt\right)\frac{\partial K_1^C}{\partial \Phi^-},
\end{aligned}$$

which yields the result.

Figure 1: Unconstrained Cournot equilibrium

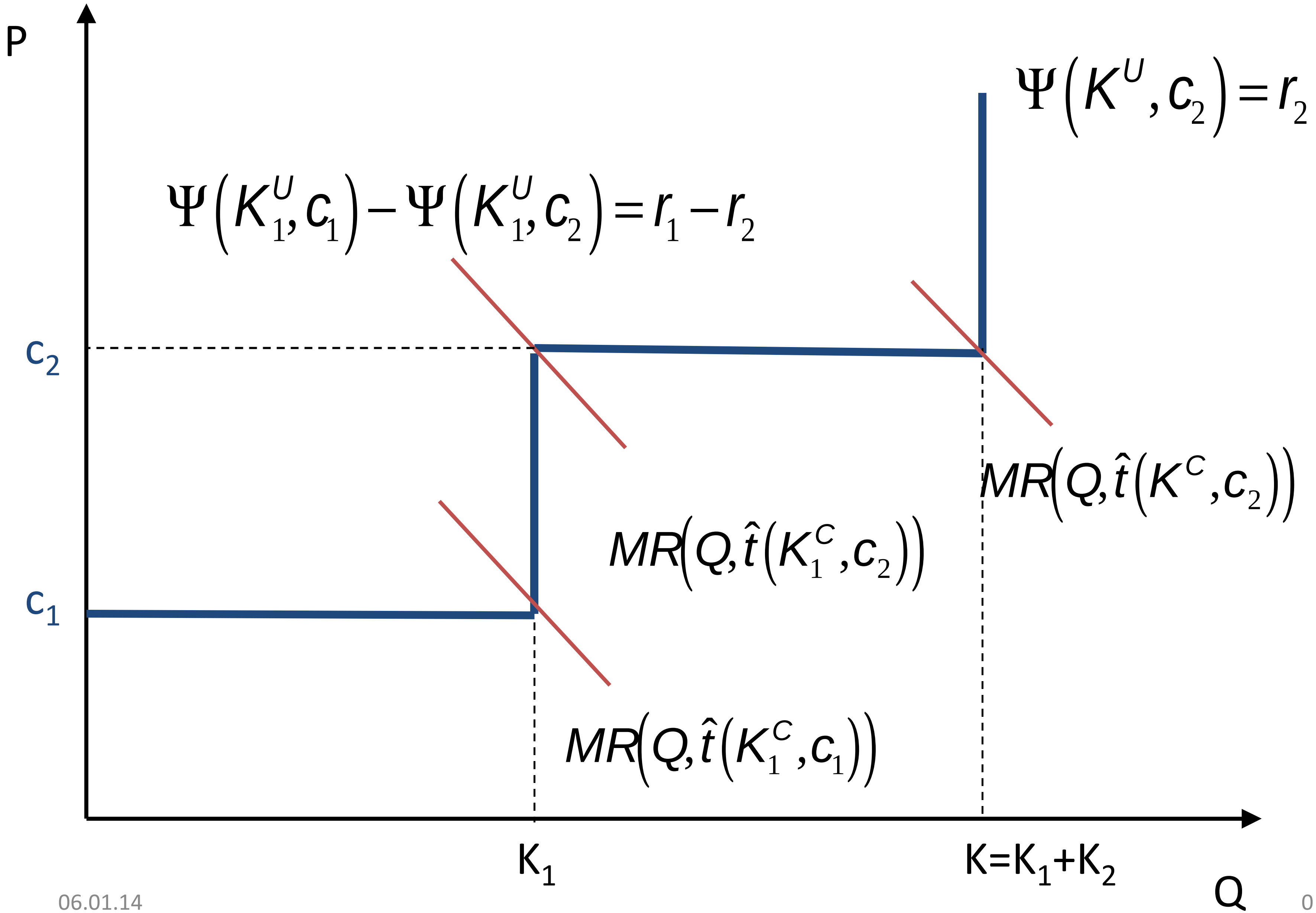


Figure 2: Unconstrained prices and quantities

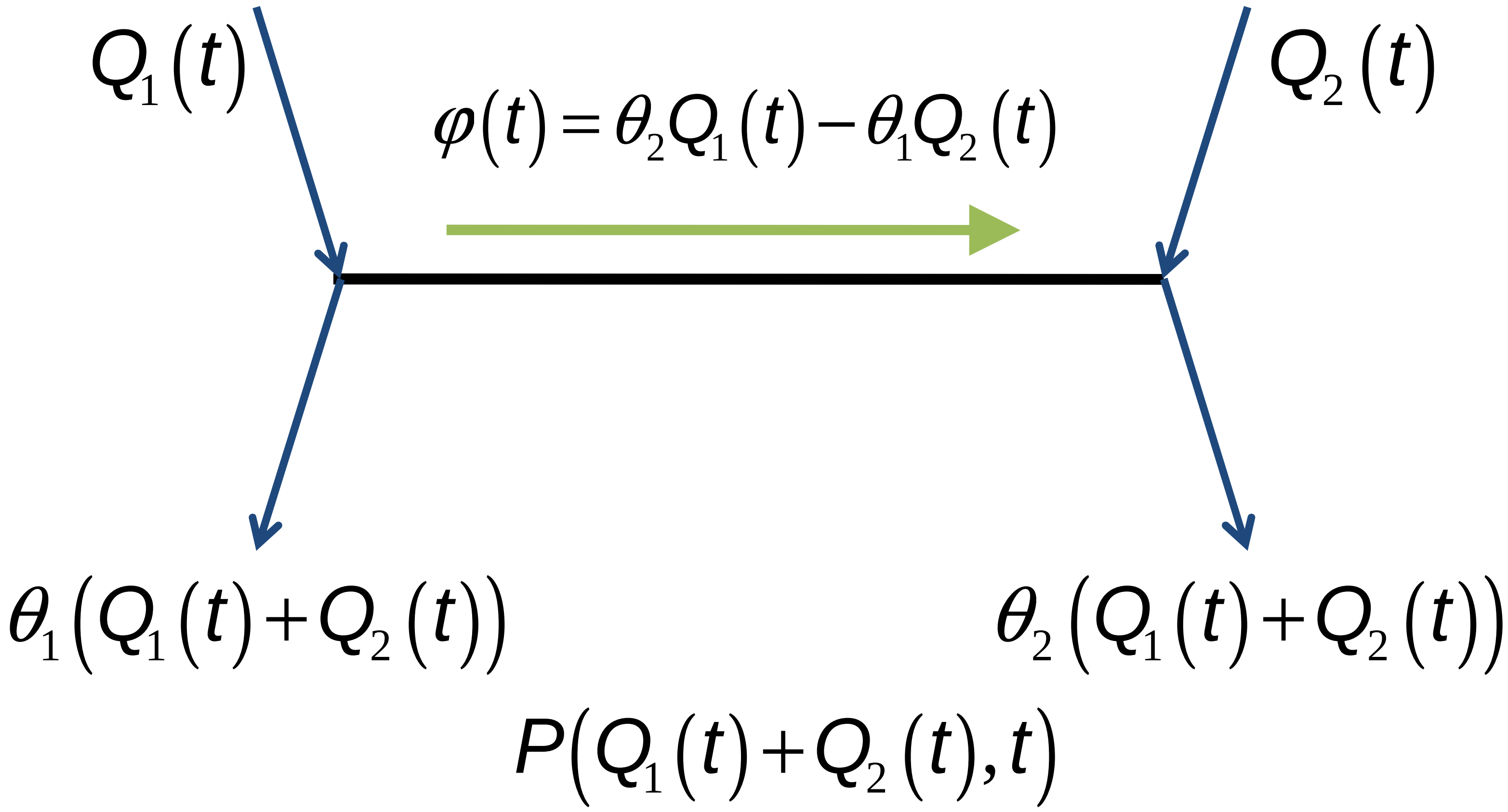
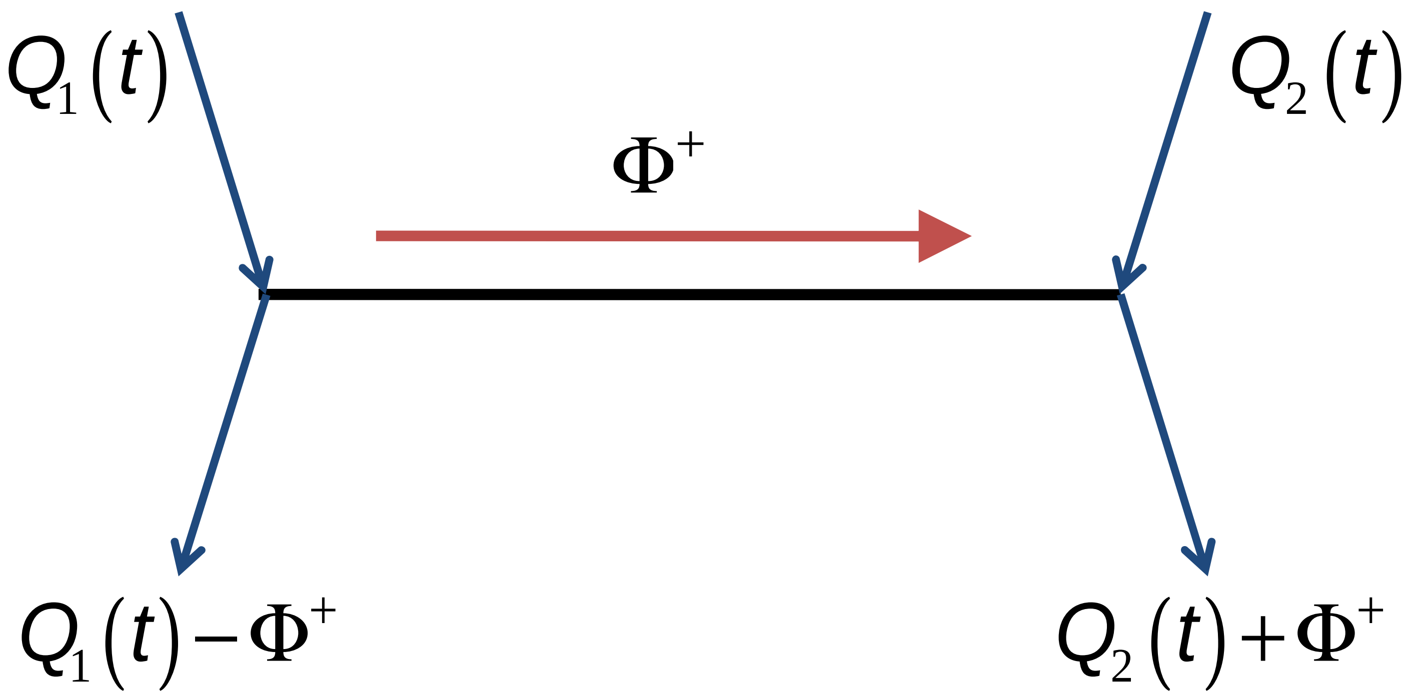


Figure 3: Prices and quantities under congestion from market 1 to market 2



$$P\left(\frac{Q_1(t)-\Phi^+}{\theta_1},t\right)<P\left(\frac{Q_2(t)+\Phi^+}{\theta_2},t\right)$$

Figure 4: congestion regimes if $\theta_1 K_2^U \leq \theta_2 K_1^U$

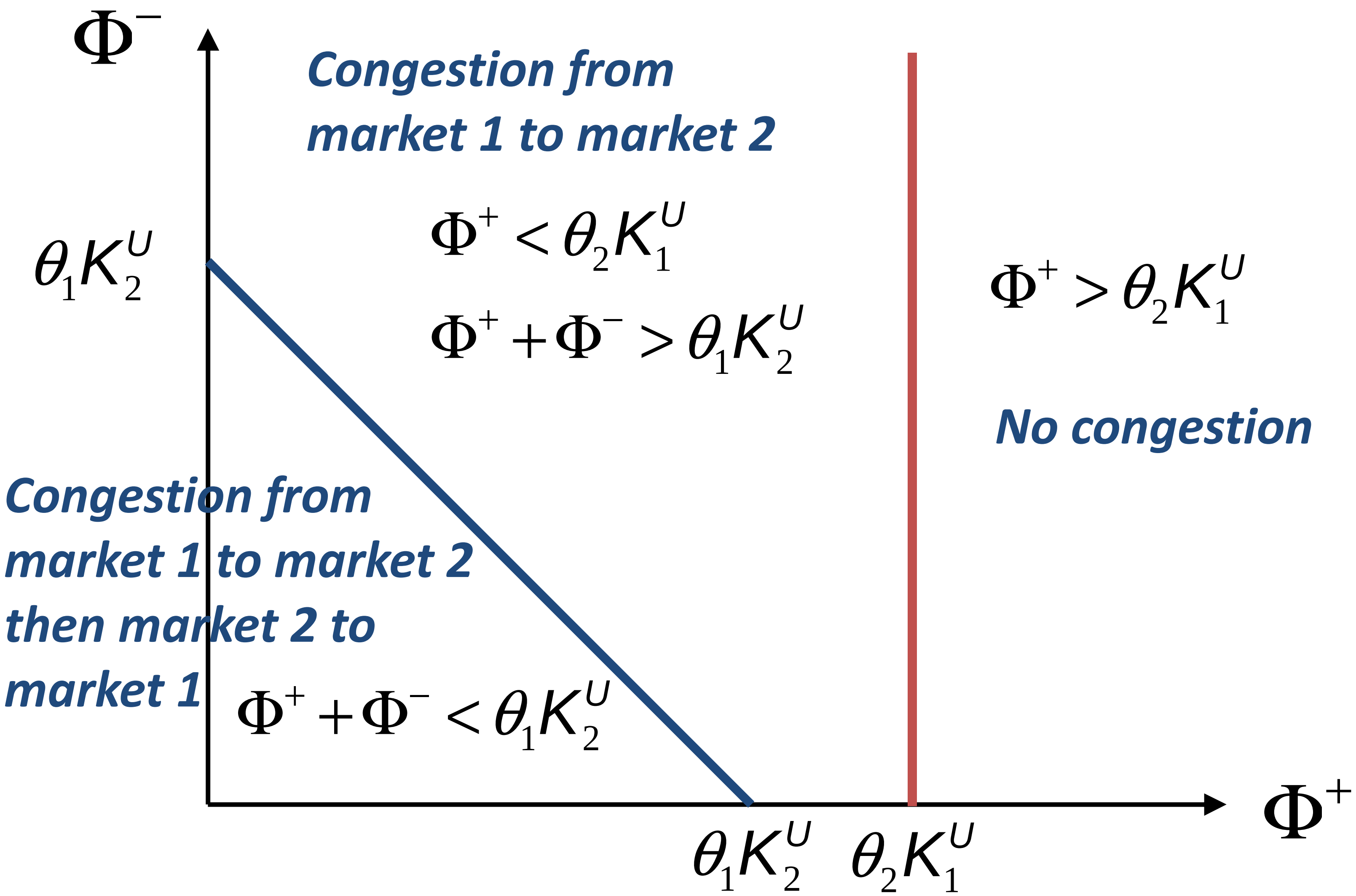


Figure 5a: Sequence of Cournot equilibria, market 1

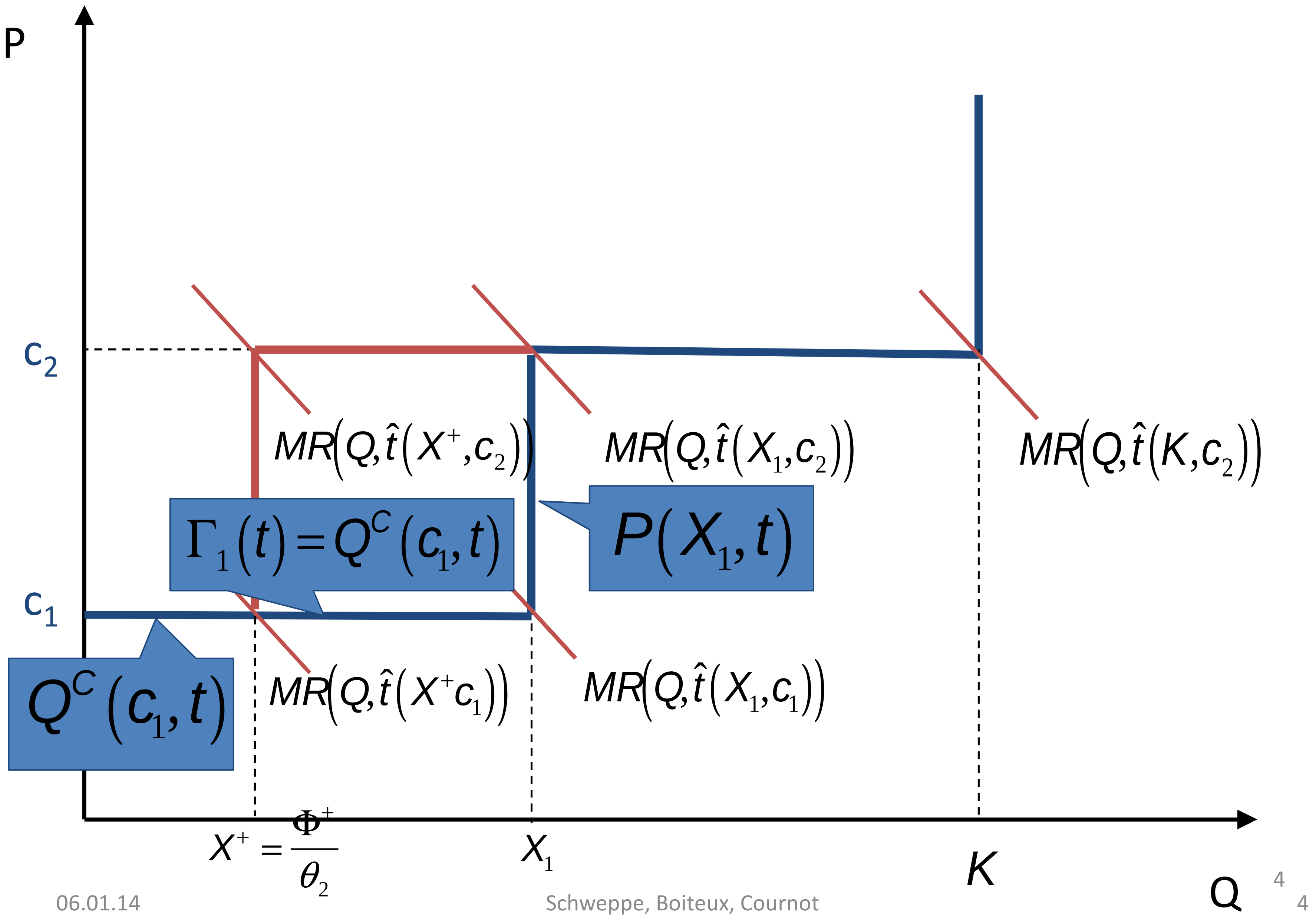


Figure 5b: Sequence of Cournot equilibria, market 2

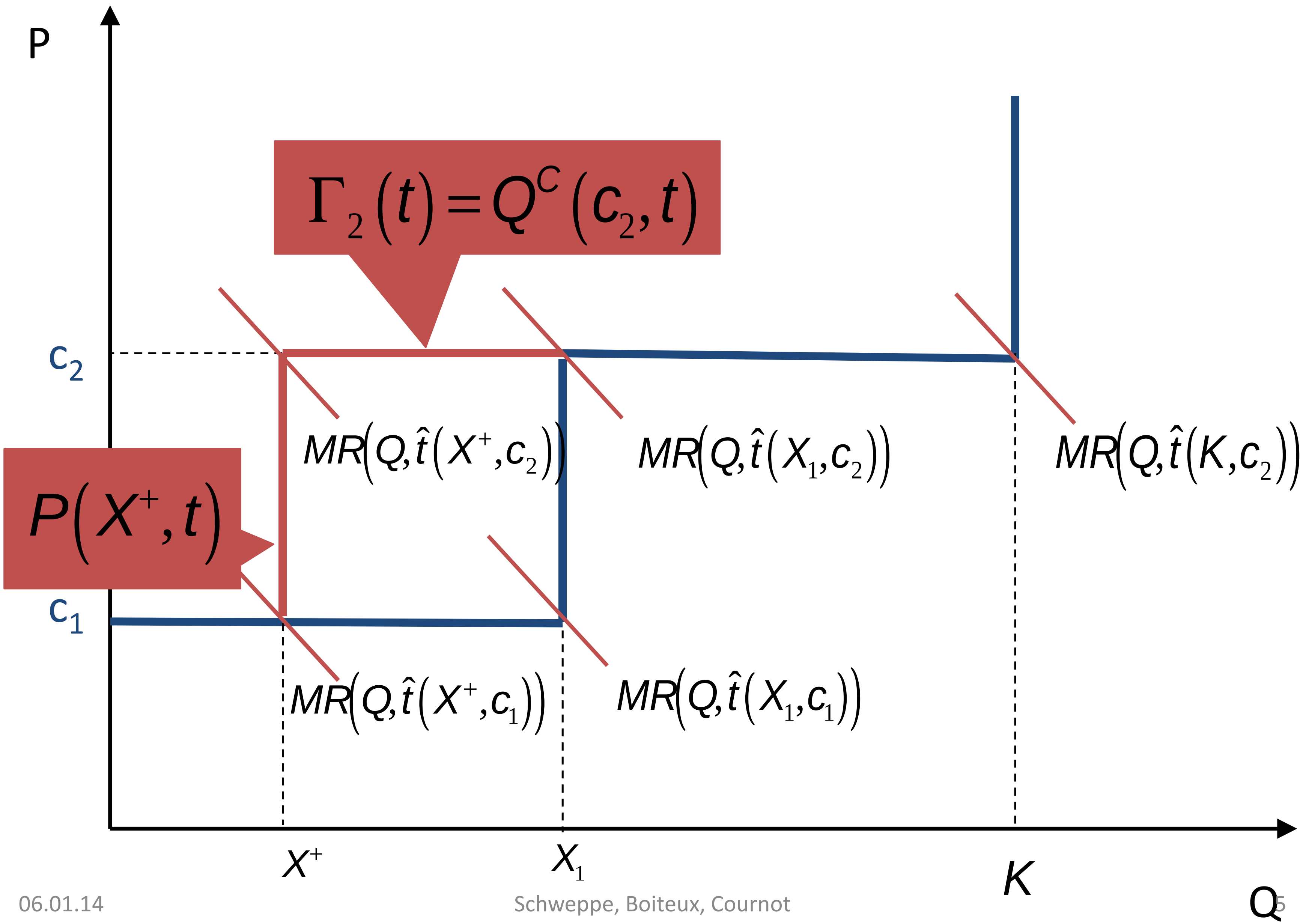


Figure 5c: Sequence of Cournot equilibria, congestion has stopped

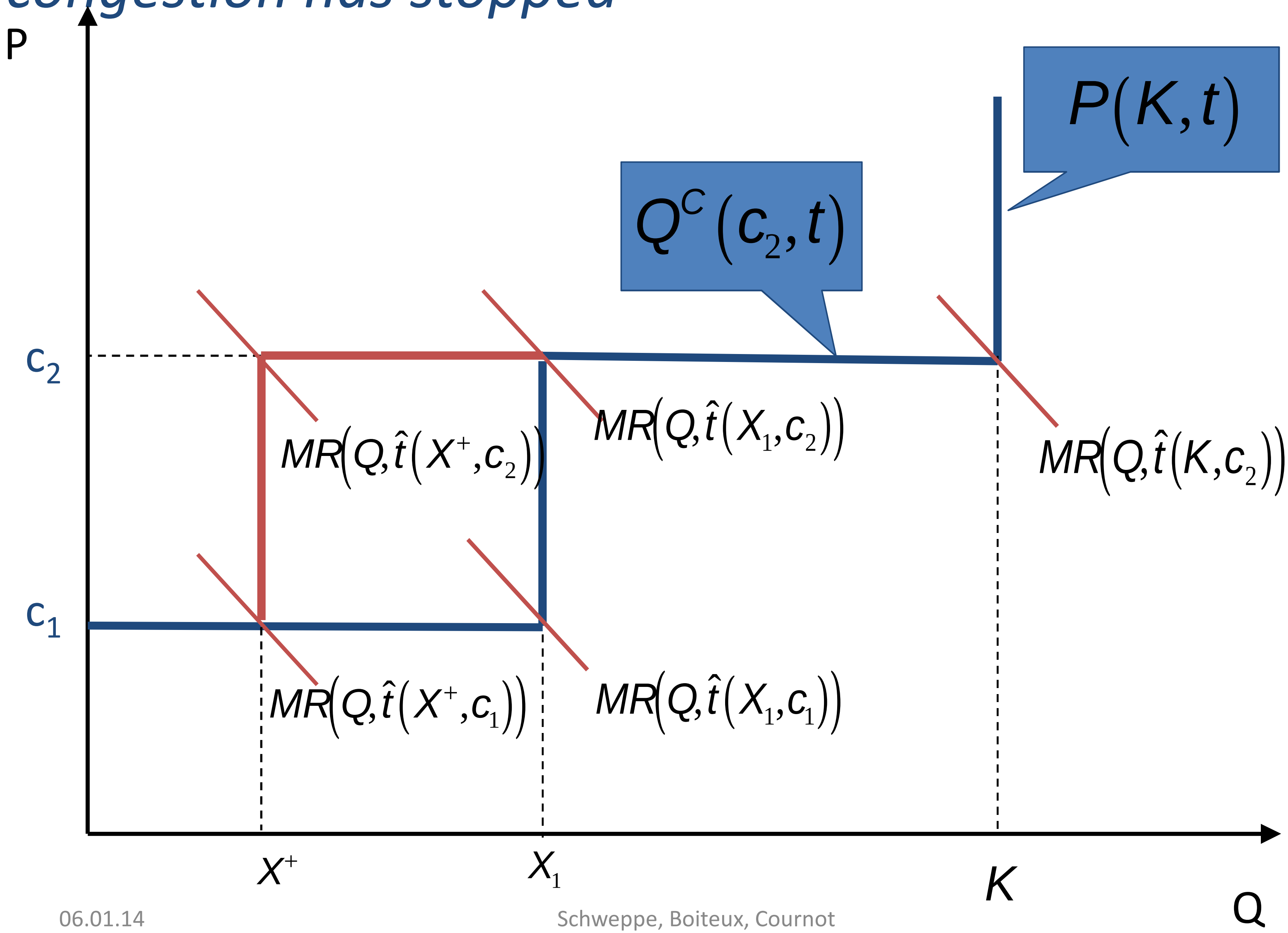


Figure 6a: sequence of Cournot equilibria, market 1

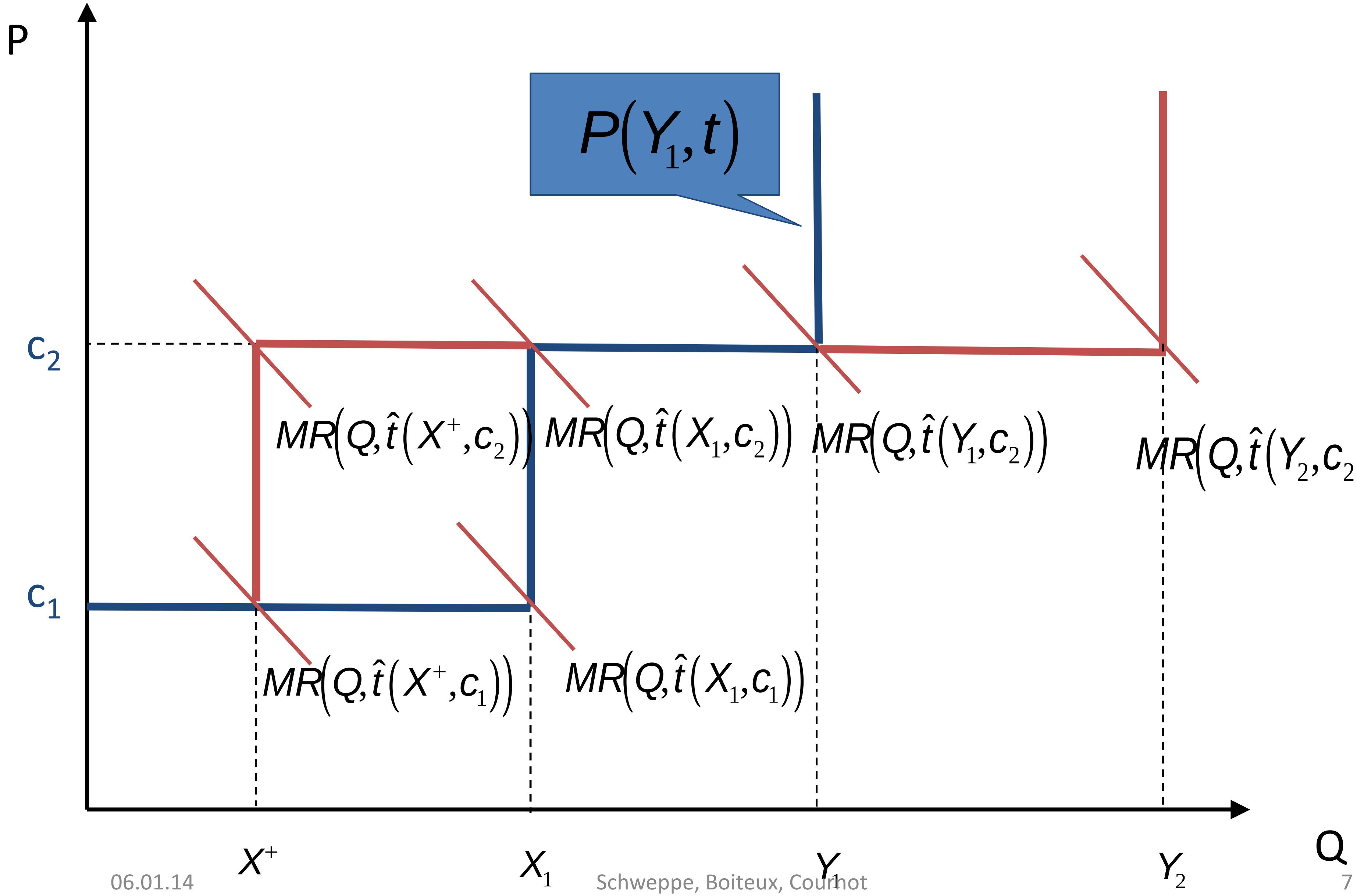


Figure 6b: sequence of Cournot equilibria, market 2

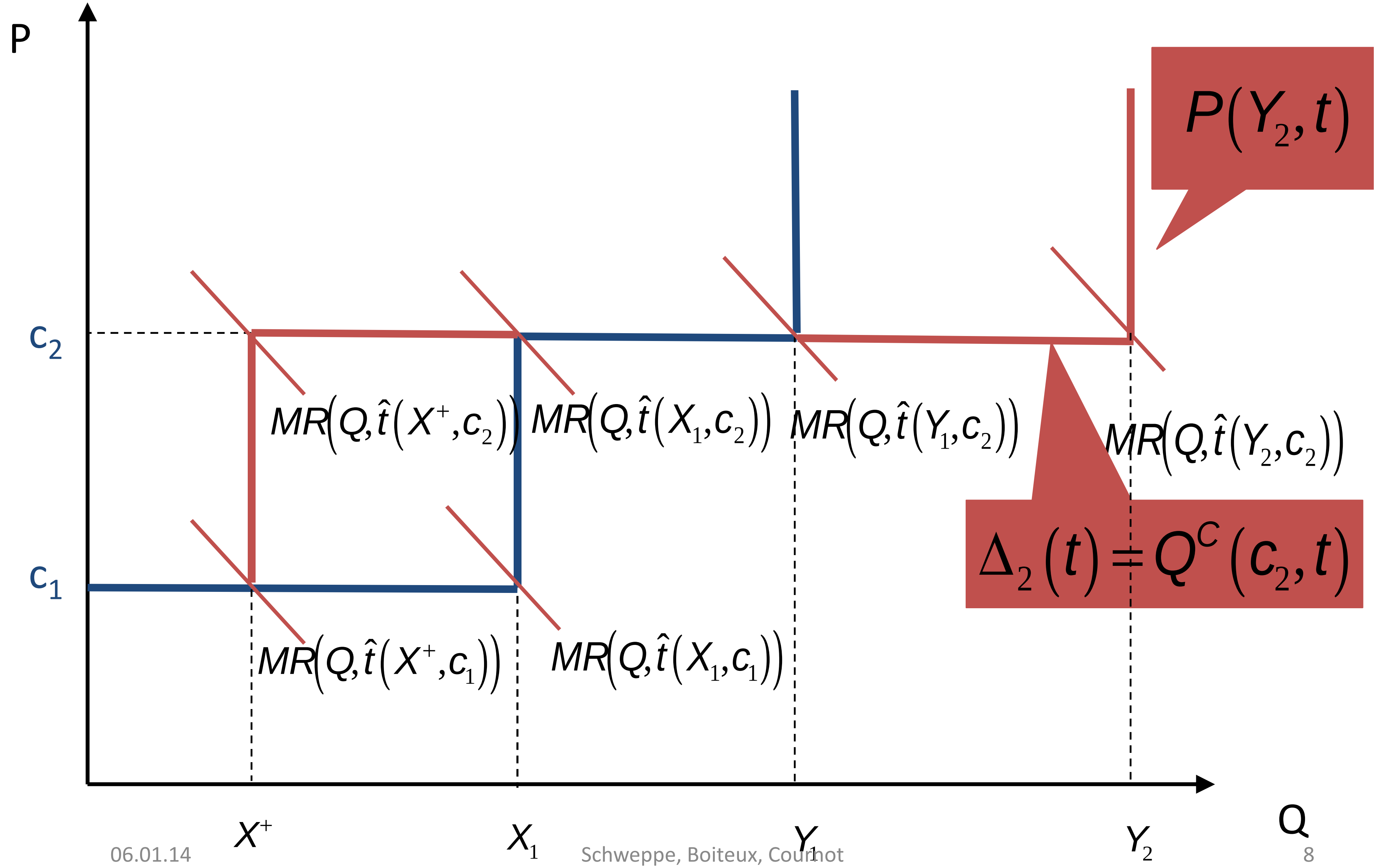
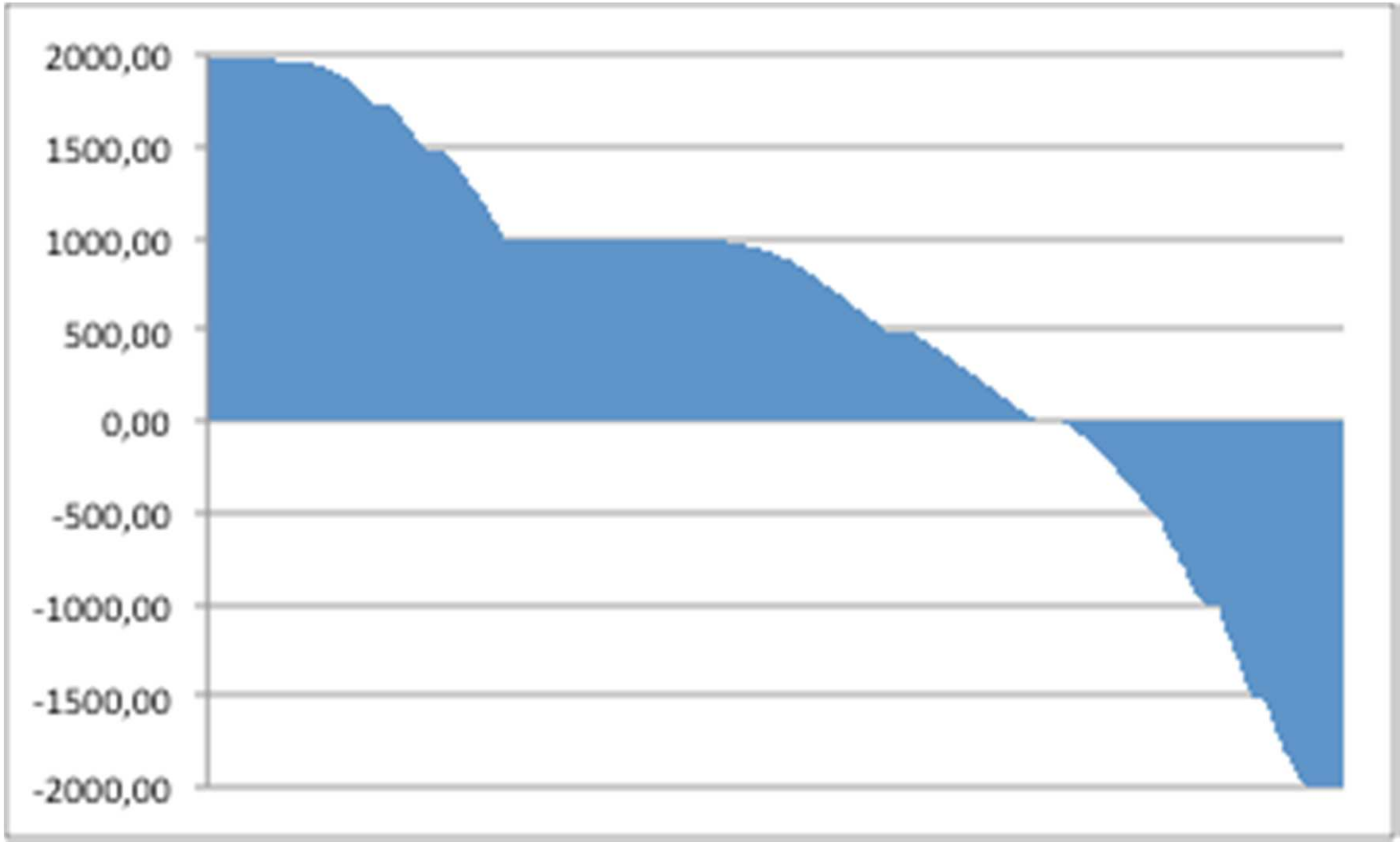


Figure 7: Britain to France interconnection flows

Imports from France into Britain (+), exports from Britain into France (-), year 2010, hours ordered from highest to lowest imports, MW



hours

Source:National Grid

Figure 8: Equilibrium capacities

K^C (burgundy), K_1^C (blue), % peak demand,

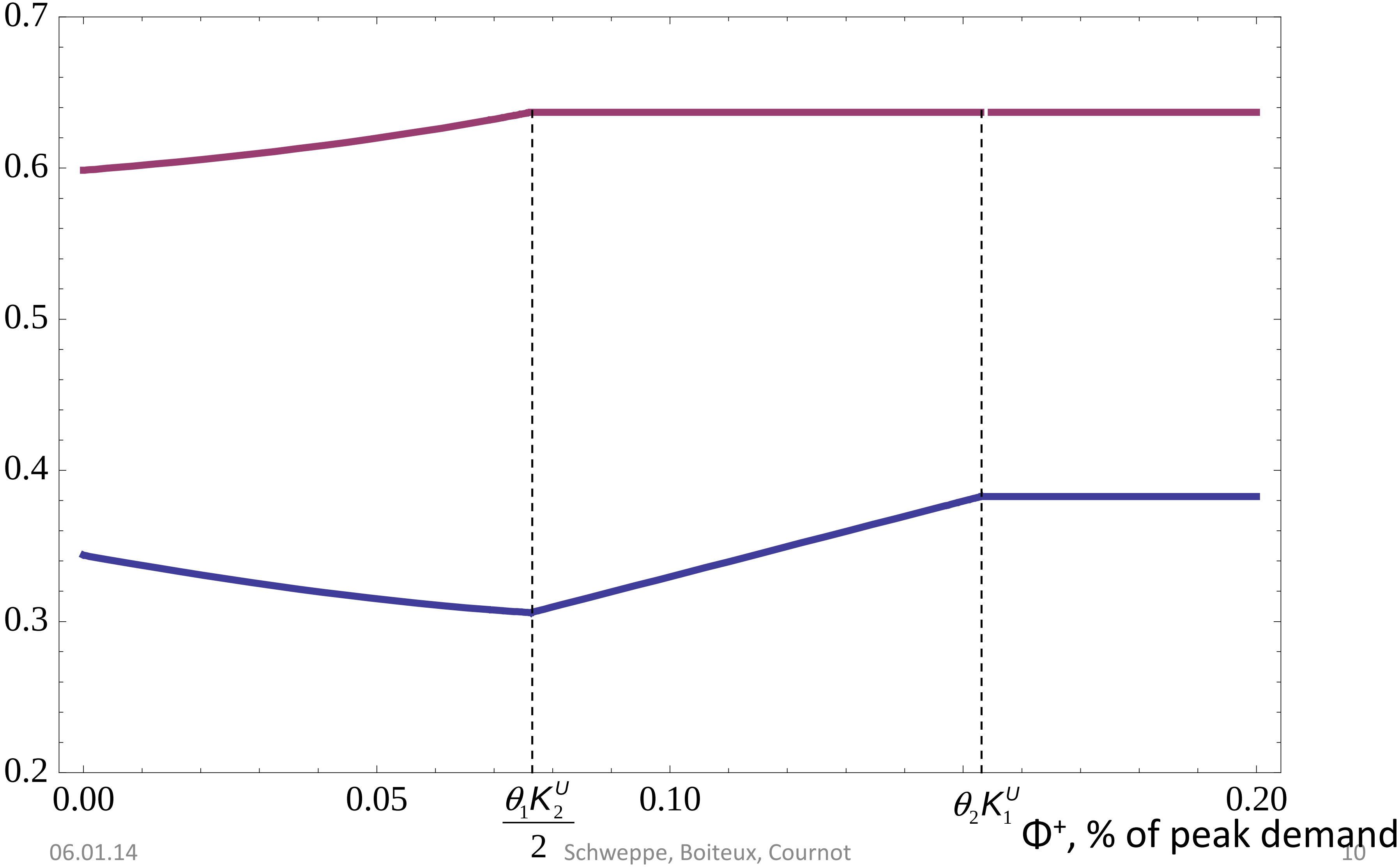


Figure 9: small downward deviation

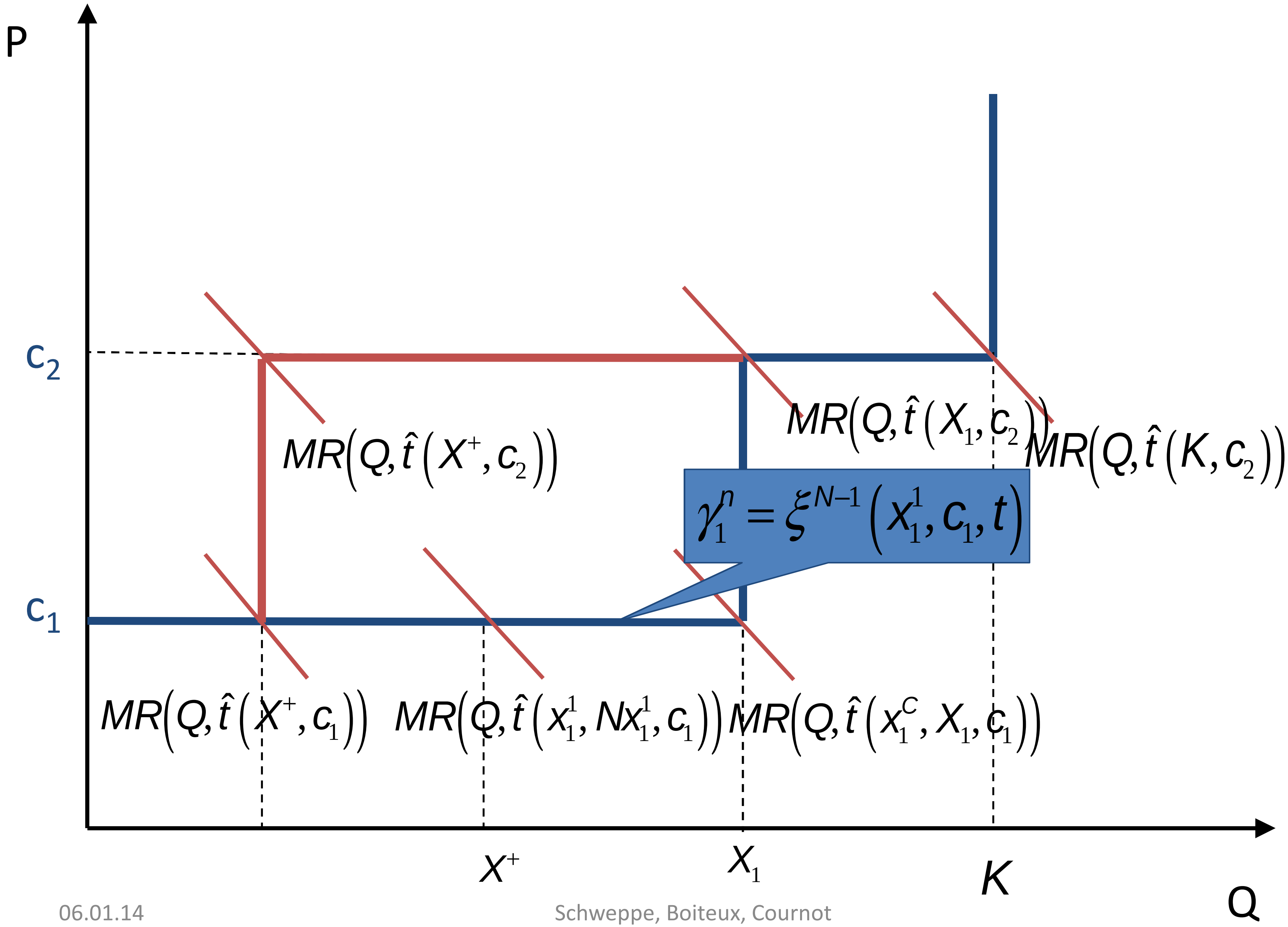


Figure 10: large downward deviation

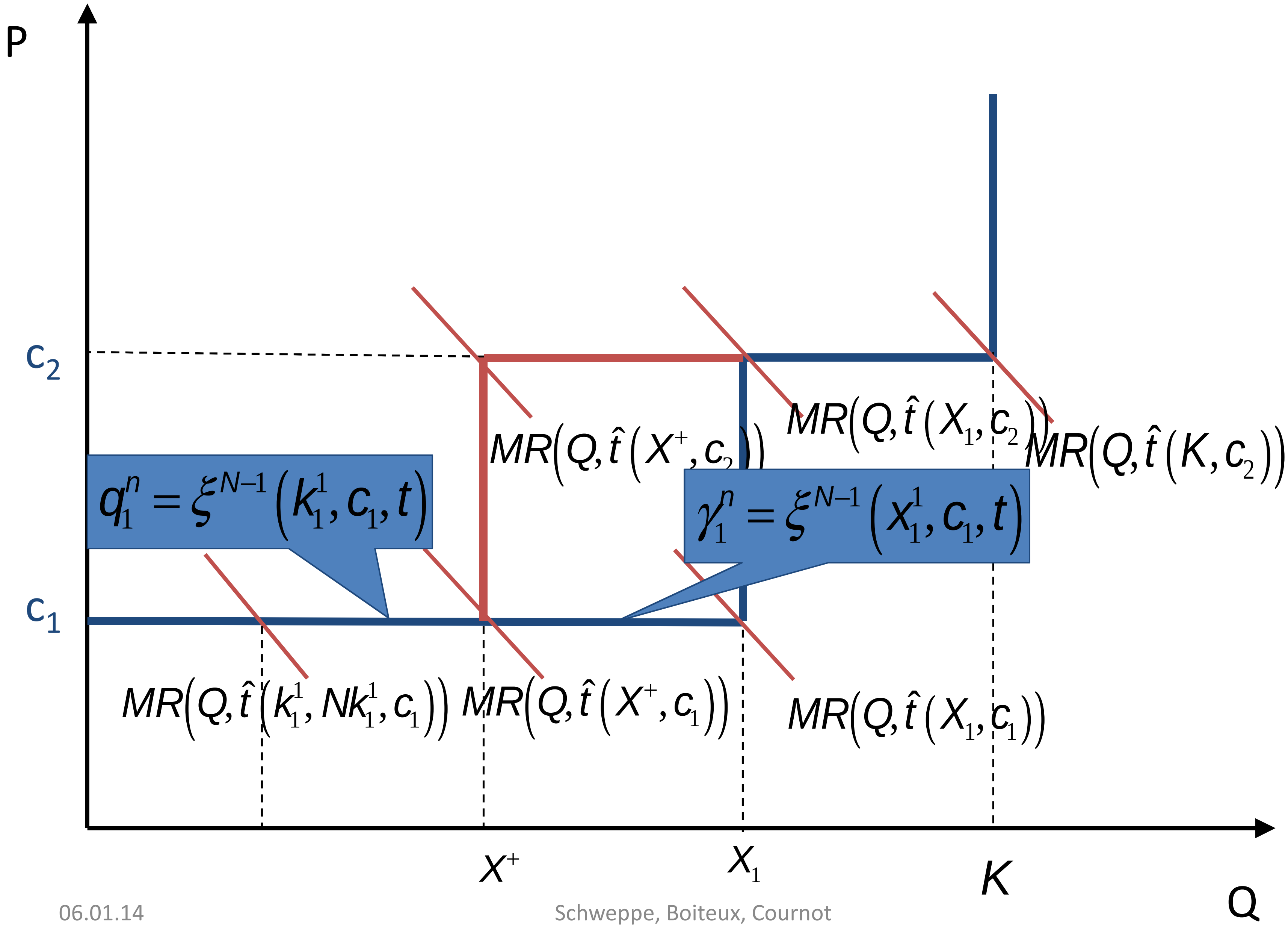


Figure 11: very large downward deviation

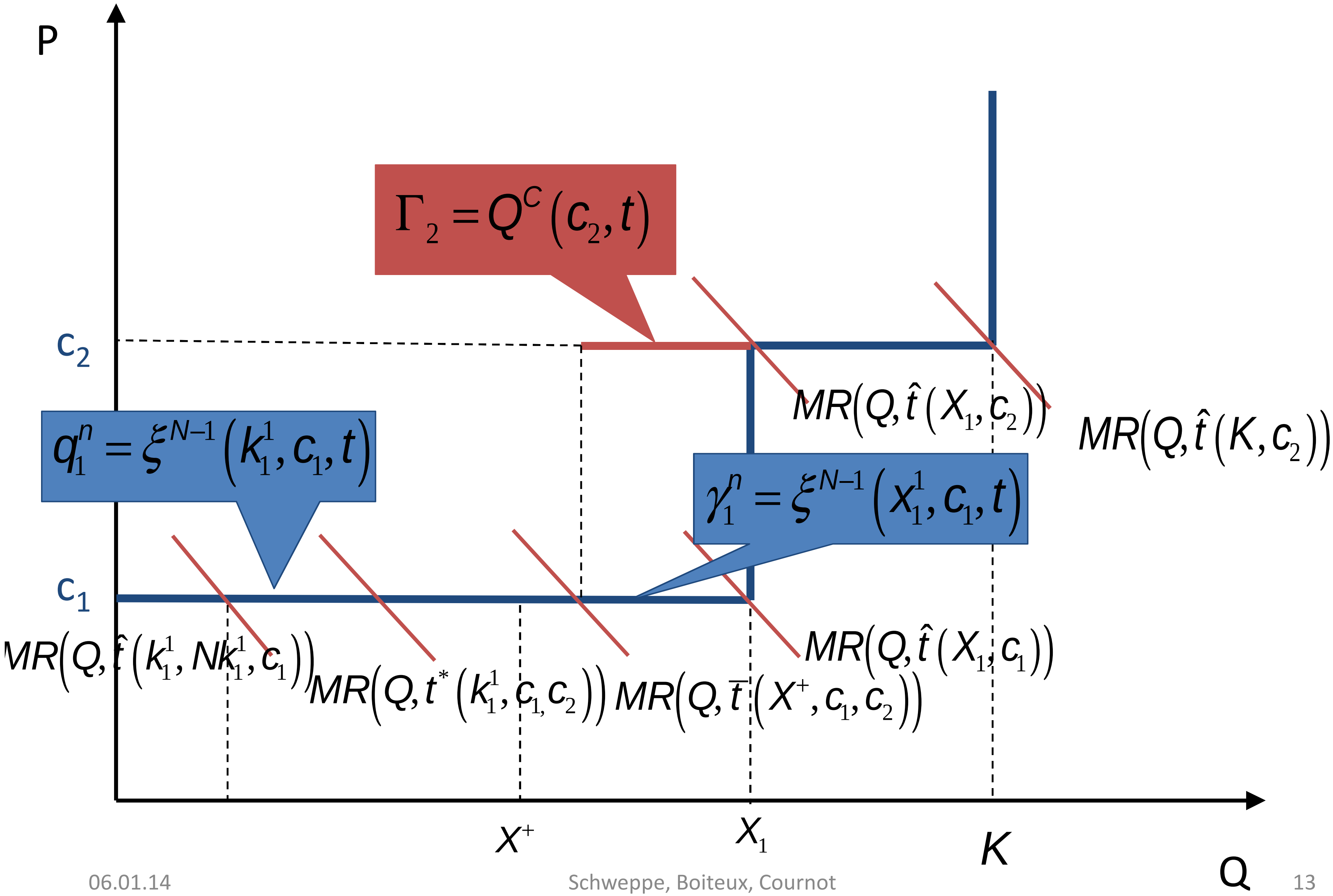


Figure 12: upward deviation

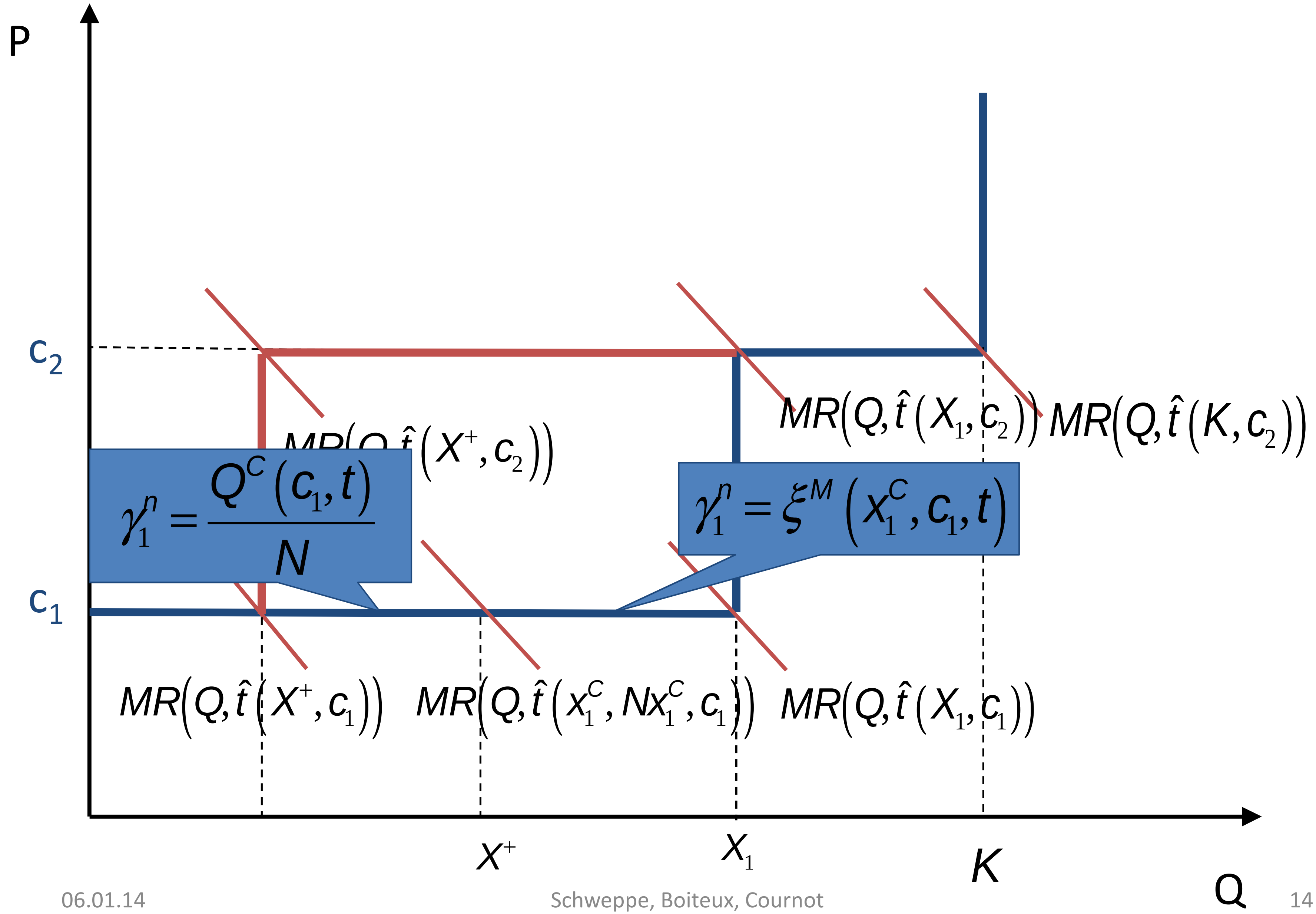


Figure 13: small downward deviation – 2 way congestion