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"Competitive Cross-Subsidization"

Zhijun Chen and Patrick Rey

INSTITUT D'ECONOMIE INDUSTRIELLE

Toulouse School of Economics

Competitive Cross-Subsidization*

Zhijun Chen[†] Patrick Rey[‡]

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Abstract

This paper analyzes competitive pricing policies by multiproduct firms facing heterogeneous buying patterns. We show that cross-subsidization arises when firms have comparative advantages on different products but are equally efficient overall: Firms earn a profit from multi-stop shoppers by charging positive margins on their strong products but, as price competition for one-stop shoppers drives total margins down to zero, they price weaker products below cost. Banning below-cost pricing leads to higher profits and higher prices for one-stop shoppers, and may reduce consumer surplus as well as total social welfare.

JEL Classification: L11, L41

Keywords: Bertrand competition, cross-subsidization, buying patterns, one-stop and multi-stop shopping.

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[†]University of Auckland. Email: chenzj1219@gmail.com

[‡]Toulouse School of Economics (UT1C, GREMAQ and IDEI). Email: patrick.rey@tse-fr.eu

1 Introduction

Multiproduct firms often engage in cross-subsidization, pricing some products below cost and subsidizing the resulting loss by the profits from other products. In telecommunications, energy, and postal markets, for instance, historical incumbents may subsidize their activity in the newly liberalized segments with the profits achieved in protected segments. This has been a key issue for the theory and practice of regulation for decades,[1] and has prompted structural or behavioral remedies.[2]

Cross-subsidization is also commonly observed in competitive markets, however. For instance, supermarkets often price staples such as milk and bakery below cost, and recoup the loss on other products.[3] Retail banks provide some services below cost (e.g., zero account fees or free travel insurance) to attract customers on other services.

Although below-cost pricing could be treated as predatory,[4] in many cases there is not such a thing as a "predatory phase," followed by a "recoupment phase" (e.g., once rivals have been driven out of the market). Loss leading and related below-cost pricing strategies appear instead to be adopted consistently over time; this led the UK Competition Commission to conclude, in its 2008 report, that "We find that the pattern of below-cost selling that we observed by large grocery retailers does not represent behavior that was predatory in relation to other grocery retailers."[5]

[1] The seminal paper of Faulhaber (1975) defines rigorously the concept of cross-subsidy and introduces two tests for subsidy-free pricing, which have been widely applied in both regulation and antitrust enforcement. See also Faulhaber (2005) for a recent survey.

[2] Such concerns led for instance to the break-up of *AT&T* and the imposition of lines of business restrictions on local telephone companies (*U.S. v. AT&T 1982*). More recently, the European Commission required the German postal operator to separate the competitive parcel services from the letter monopoly (*Deutsche Post, 2001*).

[3] Such practice is known as loss leading in grocery retailing. In its recent investigation of grocery markets, the UK Competition Commission (2008) notes that most large retailers in the UK engage in loss leading, and finds that the sales of loss leaders represent up to 6% of a retailer's total sales.

[4] See e.g., Bolton, Brodley and Riordan (2000) and Eckert and West (2003) for detailed discussions of how predatory-pricing tests should be designed. Rao and Klein (1992) and Berg and Weisman (1992) examine the treatment of cross-subsidization under U.S. antitrust laws.

[5] See Competition Commission (2008) at p. 98.

Yet, below-cost pricing has triggered hot policy debates in antitrust circles. For instance, in 2000 the German Federal Cartel Office (FCO) ordered two supermarket chains, Wal-Mart and Aldi, to stop selling staples below cost, arguing that this could impair competition and force smaller retailers to exit the market. Wal-Mart appealed the decision to the Düsseldorf Court of Appeals, arguing that below-cost pricing was driven by the competitive pressure from other large retailers. The Court of Appeals ruled for Wal-Mart and reversed the decision of FCO, but in 2002 the German Supreme Court finally upheld the FCO's decision,[6] noting that pricing strategies that are permissible under EU competition law may still violate Germany's below-cost sales statute, even in the absence of a dominant position.

This debate is also reflected in the discrepancy of statutes on below-cost pricing. In the EU, below-cost resale is banned in Belgium, France, Ireland, Luxembourg, Portugal, and Spain, restricted in Austria, Denmark, Germany, Greece, Italy, Sweden and Switzerland, but generally allowed in the Netherlands and the UK. In the United States, under federal antitrust law below-cost pricing is only illegal when it is predatory (under Section 2 of the Sherman Act, which deals with monopolization strategies), but many states have adopted specific laws against certain forms of below-cost resale.[7]

This begs several related questions: What is the rationale for cross-subsidization in competitive markets, if it is not predatory? What is the impact on consumers and society, then? Should below-cost pricing be banned by competition laws?

Unfortunately, the existence of cross-subsidization in competitive markets is somewhat at odds with conventional economic theory, according to which cross-subsidization arises only when a firm has substantial market power – as argued by Faulhaber (2005, pp.442), "under competitive conditions, the issue of cross-subsidy simply does not arise." Alternatively, cross-subsidization has been viewed as an advertising strategy adopted to attract consumers who are imperfectly informed of prices;[8] however, in markets such as

[6] See http://www.wilmerhale.com/files/Publication/d3382527-7acd-45c6-bae6-4ef14157e415/Presentation/PublicationAttachment/007af760-21b4-46b0-b356-a96612caf3cf/International%20Competition%20Law%20Update12-2-02.pdf

[7] See Calvani (2001) for a detailed discussion.

[8] Lal and Matutes (1994), for example, consider a situation where multi-product firms compete for

the grocery retailing industry, characterized by frequent purchases, consumers appear to be well informed of prices and unlikely to be misled by below-cost pricing.

This paper highlights a different rationale for competitive cross-subsidization. We develop a model of price competition between multiproduct firms, offering the same product line to the same customers, who are perfectly informed of prices. Our key modelling feature is to account for consumers' heterogeneous transaction costs: some customers face higher costs, e.g., when visiting a store or adopting a new technology, and may thus have a strong preference for "one-stop shopping," whereas others, having lower costs, may be more prone to engage in "multi-stop shopping." For the sake of exposition, we will focus on consumers (i.e., final users) and, following Klemperer (1992), will refer to their real or perceived costs of dealing with a supplier as "shopping costs."[9]

We develop a simple setting with two firms in two markets. Each firm enjoys a comparative advantage in one of the markets – e.g., due to lower costs and/or higher quality. For the sake of exposition, we initially assume that firms have similar comparative advantages; that is, their assortments generate the same total surplus.[10] Yet, in each market there is asymmetric competition, as one firm has a stronger product than its rival.

In the absence of shopping costs, in each market the stronger firm would be able to earn a profit, reflecting its comparative advantage. If instead all consumers had substantial shopping costs, and thus favor one-stop shopping, neck and neck competition would lead firms to offer their assortments at cost. Hence, in both cases there is no need for cross-subsidization.[11] But when consumers have heterogeneous shopping costs, so that both

consumers who are initially unaware of prices, and find that in equilibrium firms may indeed choose to advertise a few loss leaders in order to increase store traffic.

[9]This convention is widely adopted in the literature of multiproduct competition – see e.g., Klemperer and Padilla (1997) and Armstrong and Vickers (2010).

[10]We show in Section 6 that the analysis carries over when this symmetry assumption is relaxed.

[11]Ambrus and Weinstein (2008) study Bertrand competition among symmetric firms competing for one-stop shoppers. They first show that below-cost pricing cannot arise when consumers have inelastic demand. When demand is elastic, pricing below cost can occur but only under rather specific forms of demand complementarity; in particular, below-cost pricing cannot arise when consumer demand is sufficiently diverse. The scope for below-cost pricing in these settings, as well as its impact on consumers and welfare, still needs to be assessed.

"one-stop" and "multi-stop" shopping patterns arise in equilibrium, we show that firms price their weak products below cost: This allows them to earn a profit on their strong products, while still supplying their assortments globally at cost. Cross-subsidization thus appears here as a way to screen consumers: Firms derive positive profits from multi-stop shoppers, even though price competition dissipates profits from one-stop shoppers.

To evaluate the welfare effect of this cross-subsidization, we then study the impact of a ban on below-cost pricing. We find that banning below-cost pricing forces firms to price weak products at cost, and results in higher prices for one-stop shoppers and greater (expected) profits for the firms.[12] By the same token, banning below-cost pricing fosters multi-stop shopping but reduces total demand. Finally, the impact of a ban on below-cost pricing on multi-stop shoppers and social welfare is ambiguous, and depends on the relative importance of firms' competitive advantage, comparing with the total value of their assortment.

To study the robustness of our insights, we show that the analysis applies more generally, as long as two ingredients are present: (i) Both types of shopping patterns are present, which is likely to be the case when consumers' transaction costs are sufficiently diverse; and (ii) Firms want to charge higher prices to multi-stop shoppers; we note that this is consistent with the fact that firms' assortments are (possibly imperfect) substitutes for one-stop shoppers (and so competition for these consumers tends to drive down total prices), whereas their strong products are complements for multi-stop shoppers (thus generating double-marginalization problems for strong products).

Our paper is related to DeGraba (2006), who also shows that below-cost pricing could serve as a screening device. In that paper, firms face two types of consumers (who are all one-stop shoppers), and one good is specifically purchased by the more profitable consumers. As a result, firms compete for these more profitable customers by selling that good below cost. By contrast, here customers differ in their transaction costs (and thus in their buying patterns), and the more profitable consumers (namely, the multi-stop shoppers) only buy the goods sold with a positive margin.

The present paper is a complementary piece to Chen and Rey (2012), which shed a first

[12] When below-cost pricing is banned, the equilibrium involves mixed strategies: Firms sell weak products at cost, but randomize prices for strong products.

light on the exploitative use of loss leading and cross-subsidization. This previous paper however focuses on competition with asymmetric product ranges – e.g., a supermarket competing with specialized stores or hard-discounters; the larger firm may then price below cost the products on which it competes with smaller rivals, and raise the price on the other products; in this way, it maintains the profit earned on one-stop shoppers but increases that on multi-stop shoppers. In that situation, banning below-cost pricing reduces the profit of the larger firm, but benefits both consumers and smaller rivals, and enhances social welfare. By contrast, the present paper analyzes competition on the same product line and shows that below-cost pricing can still arise even when Bertrand-like competition for one-stop shoppers drives total prices down to total costs. This provides a conceptual framework for the assessment of cross-subsidization in competitive markets, but also calls for a cautious use of regulation in such markets, as banning below-cost pricing has here a more ambiguous impact on multi-stop shoppers and social welfare.

The paper is organized as follows. Section 2 illustrates the main intuition by way of a simple example. Section 3 develops our baseline framework, in which consumers' shopping costs span the entire possible range. Section 4 presents our main insights – in equilibrium, both types of shopping patterns coexist, and firms engage in cross-subsidization even though overall they sell their assortments at cost. Section 5 then studies the impact of a ban on below-cost pricing. Finally, Section 6 discusses the robustness of the insights, and Section 7 concludes.

2 A simple example

A numerical example illustrates the main intuition. Two firms, 1 and 2, sell two products, A and B, which consumers value at $u^A = u^B = \$50$; firm 1 has a cost advantage in market A, whereas firm 2 enjoys a lower cost in market B: $c_1^A = c_2^B = \$10 < c_2^A = c_1^B = \$30$. Finally, consumers face a shopping cost s, encompassing the opportunity cost of the time spent in traffic, parking, selecting products, checking out, and so forth; it may also account for the consumer's taste for shopping.

Suppose first that all consumers face a "high" shopping cost, equal to or exceeding the production cost differential: $s \geq \Delta c = \$20$. In equilibrium they then behave as "one-stop shoppers," buying both products from the same firm; thus only the total price for the bundle $A - B$ matters. And as both firms can provide the same overall value of \$100 at the same total cost of \$40, fierce price competition drives down profit to zero: Both firms sell their bundle at a total price equal to total cost,

$$p_1^A + p_1^B = p_2^A + p_2^B = \$40,$$

where p_i^A (resp. p_i^B) represents firm i's price for product A_i (resp. B_i).

Suppose instead that consumers' shopping cost s is sufficiently low that, in equilibrium, they all behave as "multi-stop shoppers," buying each product at the lowest available price. Asymmetric Bertrand competition then leads firms to sell weak products at cost, $p_2^A = p_1^B = \$30$, and strong products at a price equal to (or just below) the rival's cost, minus consumers' shopping cost: $p_1^A = p_2^B = \$30 - s$.

Note that cross-subsidization does not arise in these two situations, where consumers have homogeneous shopping costs. By contrast, suppose now that half of the consumers face a "high" shopping cost: $s_H = \$20$, whereas the others have a "low" shopping cost: $s_L = \$2$. As before, fierce price competition dissipates profits from one-stop shoppers, and drives the prices for the assortment $A - B$ down to total cost:

$$p_1^A + p_1^B = p_2^A + p_2^B = \$40.$$

Yet, as each firm has a cost advantage in one market, it can sell its strong product at a lower price than its rival; keeping the total price constant for one-stop shoppers, it suffices to undercut the rival's weak product by $s_L = \$2$ to attract multi-stop shoppers. It follows that the equilibrium prices are:

$$\begin{aligned} p_1^A &= \$19,\ p_1^B = \$21, \\ p_2^A &= \$21,\ p_2^B = \$19. \end{aligned}$$

That is, each firm sells its weak product below cost, $p_2^A = p_1^B = \$21 < \$30$, and compensates the loss with the profit from the strong product. This pricing strategy does not affect the shopping behavior of high-cost consumers (who still face a total price of \$40), but generates a positive profit from multi-stop shoppers, who buy A from 1 and B from 2 as $p_1^A + p_2^B = \$38 < \$40$.

3 The model

We now consider more general supply and demand conditions and assume that in market A (resp., B), firm i can produce a variety A_i (resp., B_i) at constant unit cost c_i^A (resp., c_i^B). Consumers are willing to buy one unit of A and one unit of B, with reservation prices u_i^A and u_i^B, respectively.[13] Let $w_i^A \equiv u_i^A - c_i^A > 0$, $w_i^B \equiv u_i^B - c_i^B > 0$, and $w_i^{AB} \equiv w_i^A + w_i^B$ denote the social value generated, respectively, by product A_i, product B_i, and the assortment $A_i - B_i$. Firm 1 is more efficient in supplying A, whereas firm 2 is more efficient in supplying B: $w_1^A > w_2^A$ and $w_1^B < w_2^B$. This efficiency gain may be derived from specializing in different product markets, and can be driven by either quality or cost conditions. For the sake of exposition we initially focus on the "symmetric" case where firms' assortments offer the same total value: $w_1^{AB} = w_2^{AB} \equiv w$, implying that the two firms enjoy the same comparative advantage on their strong products: $w_1^A - w_2^A = w_2^B - w_2^A \equiv \delta$.

Our key modelling feature is that consumers incur a shopping cost s to visit a firm, and that this cost varies across consumers, reflecting the fact they may be more or less time-constrained, or value shopping experience in different ways. We allow for general distributions of the shopping cost s, characterized by a cumulative distribution function $F(\cdot)$ with a continuous, positive density function $f(\cdot)$ over $\mathbb{R}_+$. Intuitively, consumers with a high shopping cost favor one-stop shopping, whereas those with a lower shopping cost can take advantage of multi-stop shopping; the mix of multi-stop and one-stop shoppers is however endogenous and depends on firms' prices.

We model price competition as follows: (i) The two firms simultaneously set their prices, (p_1^A, p_1^B) and (p_2^A, p_2^B);[14] (ii) Consumers then observe all prices and make their shopping decisions.

[13] While we focus here on independent demands for A and B, the analysis carries over when there is partial substitution or complementarity, that is, when the value of consuming both products is either lower or higher than $u_i^A + u_j^B$.

[14] We first consider stand-alone prices, and show later that allowing for bundled discounts cannot increase firms' profits; see Remark 4 in section 4.

4 Competitive cross-subsidization

A consumer is willing to buy both products A and B from firm i if the value of the assortment $A_i - B_i$ exceeds the shopping cost, that is, if:

$$v_i \equiv u_i^{AB} - p_i^A - p_i^B = w_i^A - m_i^A + w_i^B - m_i^B = w - m_i^A - m_i^B \geq s,$$

where $m_i^A \equiv p_i^A - c_i^A$ and $m_i^B \equiv p_i^B - c_i^B$ denote the margins that firm i charges on A and B.

One-stop shoppers patronize firm i only if it offers a better value than its rival, firm j, that is, if it charges a lower total margin $m_i \equiv m_i^A + m_i^B$:

$$v_i = w - m_i > v_j = w - m_j \Leftrightarrow m_i < m_j.$$

Consumers may however prefer buying both strong products, that is, purchasing A_1 from firm 1 and B_2 from firm 2,[15] rather than patronizing one firm only. Such multi-stop shopping yields a total value

$$v_{12} \equiv w_1^A - m_1^A + w_2^B - m_2^B = w + \delta - m_1^A - m_2^B,$$

at an extra shopping cost s. Thus, consumers favor multi-stop shopping than patronizing only firm 1 if

$$s \leq \tau_1 \equiv v_{12} - v_1 = \delta + m_1^B - m_2^B.$$

The threshold τ_1 reflects the value difference that firm 2 offers on its strong product, B. Similarly, consumers prefer multi-stop shopping to patronizing only firm 2 if

$$s \leq \tau_2 \equiv v_{12} - v_2 = \delta - m_1^A + m_2^A,$$

where τ_2 reflects the value difference of firm 1 on product A. It follows that the number of multi-stop shoppers is $F(\tau)$, where $\tau = \min\{\tau_1, \tau_2\}$.

We show in Appendix A that there are both multi-stop shoppers and one-stop shoppers in equilibrium. Furthermore, multi-stop shoppers buy the strong products, A_1 and B_2, and competition for one-stop shoppers dissipates firms' profits from selling the assortment:

[15] We show in the appendix that mixing-and-matching weak products, A_2 and B_1, never arises in equilibrium.

Lemma 1 *In equilibrium:*

- *(i) Multi-stop shoppers and one-stop shoppers are both active.*
- *(ii) Multi-stop shoppers buy strong products.*
- *(iii) Firms sell their assortments at cost:* $m_i = m_j = 0$.

Proof. See Appendix A. ■

It follows that, in equilibrium, firms only make a profit by selling their strong products to multi-stop shoppers; furthermore, as $m_1 = m_2 = 0$, multi-stop shoppers are those consumers with $s < \tau = \tau_1 = \tau_2 = \delta - m_1^A - m_2^B$. Hence, firm 1's profit, say, can be expressed as

$$\pi_1 = m_1^A F(\tau).$$

It is obviously optimal for firm 1 to charge a positive margin on its strong product: $m_1^A > 0$; but as the bundle is offered at cost (i.e., $m_1^A + m_1^B = m_1 = 0$), this implies $m_1^B < 0$: the weak product is sold *below cost.* We thus obtain our first insight:

Corollary 1 *In equilibrium, firms sell their weak products below cost.*

As expected, firms charge a positive margin on their strong products; but as fierce price competition for one-stop shoppers drives total margin to zero, in equilibrium strong products cross-subsidize weak ones. The intuition is quite simple. Suppose that initially firm i sells both products at cost, and consider the following "cross-subsidization" deviation: Keeping the total margin equal to zero, firm i slightly raises the margin on its strong product, reducing that on its weak product by the same amount. This deviation does not affect the profit generated by one-stop shoppers, but generates a profit from multi-stop shoppers, who now pay a higher price for the strong product. To be sure, this deviation also decreases the value of multi-stop shopping, and may thus induce some consumers to opt instead for one-stop shopping. But this does not affect firm i, which still obtains zero profit from these consumers, regardless of which firm they go to. Hence cross-subsidization is profitable.

For expositional purposes, it is convenient to denote by ρ_i the margin that firm i charges on its strong product; that is: $\rho_1 = m_1^A$ and $\rho_2 = m_2^B$. Firm i's profit can then be expressed as

$$\pi_i = \rho_i F(\tau) = \rho_i F(\delta - \rho_1 - \rho_2).$$

To ensure the quasi-concavity of this profit function, we will assume that the inverse hazard rate, $h(\cdot) \equiv F(\cdot)/f(\cdot)$, is strictly increasing. The equilibrium then satisfies:

$$\rho_i = \rho^* \equiv h(\tau^*) > 0,$$

where the equilibrium threshold τ^* satisfies $\tau^* = \delta - 2\rho^* = \delta - 2h(\tau^*)$ and is thus determined by

$$\tau^* = j^{-1}(\delta),$$

where $j(s) = s + 2h(s)$ is strictly increasing in s.

In equilibrium, firms subsidize their weak products ($m_2^{A*} = m_1^{B*} = -\rho^* < 0$), and obtain a positive profit from multi-stop shoppers, given by

$$\pi^* = h(\tau^*)F(\tau^*).$$

Summarizing the above analysis leads to:

Proposition 1 *If the inverse hazard rate $h(\cdot)$ is strictly increasing, there exists a unique Nash equilibrium, in which:*

(i) firms supply one-stop shoppers at cost, but make a profit on multi-stop shoppers;

(ii) both firms sell their weak products below cost and cross-subsidize them by their strong products.

Proof. See Appendix B. ■

Cross-subsidization allows here firms to extract some rents from multi-stop shoppers, despite the fierce competition for one-stop shoppers. Yet, this competition dissipates some of the profits; indeed, as $h(\tau^*) < \delta$, the equilibrium margin is strictly lower than the efficiency gain: $m_1^{A*} = m_2^{B*} = \rho^* < \delta$.

Below-cost pricing stems from the tension between two forces: Firms have market power over multi-stop shoppers, who buy their strong products, but compete fiercely for one-stop shoppers, for whom the two assortments are perfect substitutes. As a result, total prices are driven down to cost, and earning a profit on strong products implies selling weak products below cost.[16]

Remark: Bundling. Firms cannot gain here from engaging in tying or bundling. For instance, if one firm ties both products together physically, consumers are forced to engage in one-stop shopping, and price competition for one-stop shoppers leads to zero profit. A similar reasoning applies to pure bundling when products are costly, so that it does not pay to add one's favorite variety to a bundle. In principle, a firm may also engage in mixed bundling, and offer three prices: one for its strong product, one for the weak product, and one (involving a discount) for the bundle. However, as one-stop shoppers only purchase the bundle, and multi-stop shoppers only buy the strong product, no consumer will ever pick the weak product on a stand-alone basis. Hence only two prices matter here: the total price for the bundle, and the stand-alone price for the strong product. As these prices can be implemented as well using the stand-alone prices for the two products, offering a bundled discount, in addition to these stand-alone prices, cannot generate any additional profit.

5 Banning below-cost pricing

To study the welfare effects of cross-subsidization, we now examine the impact of a ban on below-cost pricing.

Thus, suppose that firms are required to charge $m_1^B \geq 0$ and $m_2^A \geq 0$. We first note that there is no pure-strategy equilibrium. Indeed, fierce price competition for one-stop shoppers would drive total margins down to zero, implying that they would offer *both* products at cost. However, this cannot be a Nash equilibrium as, in response to such aggressive strategy, any firm could secure its minmax profit by slightly increasing the margin on its strong product. Indeed, suppose that firm j offers both products at cost,

[16]Note however that, although firms compete for one-stop shoppers, the prices of strong products are subject to double marginalization problems: These products being complements for multi-stop shoppers, the lack of coordination leads firms to charge higher prices than what would maximize industry profits, keeping constant the total margin for the assortments; see the discussion in section 6.

and consider firm i's best response:

- It cannot profitably supply one-stop shoppers, as firm j supplies them at cost.
- It cannot sell its weak product to multi-stop shoppers, as firm j offers a better variety at cost.
- But firm i can make a profit from selling its strong product to multi-stop shoppers; as this profit is equal to $\pi_i = \rho_i F(\tau) = \rho_i F(\delta - \rho_i)$, this leads firm i to charge a margin $\rho_i = \bar{\rho}$ on its strong product, where $\bar{\rho}$ is such that

$$\bar{\rho} \equiv h\left(\bar{\tau}\right), \tag{1}$$

where, using $\bar{\tau} = \delta - \bar{\rho} = \delta - h(\bar{\tau})$, the multi-stop shopping threshold $\bar{\tau}$ is determined by

$$\bar{\tau} \equiv l^{-1}(\delta),$$

where $l(x) \equiv x + h(x)$ is strictly increasing in x.

Firms' minmax profit is therefore equal to

$$\bar{\pi} = \bar{\rho} F\left(\bar{\tau}\right) = h\left(\bar{\tau}\right) F\left(\bar{\tau}\right).$$

Although there is no pure-strategy equilibrium, intuitively, banning below-cost pricing should lead the firms to offer their weak product at cost. We show in the Appendix that there indeed exists a symmetric mixed-strategy equilibrium, yielding expected profits equal to the minmax level, $\bar{\pi}$, in which each firm supplies its weak product at cost but randomizes over the margin on its strong product, according to some distribution $K\left(\rho\right)$. The upper bound of the support of this equilibrium distribution is moreover equal to $\bar{\rho}$, as by construction a firm charging the upper bound is undercut for sure by its rival, and thus derives its profit only from multi-stop shoppers: Its profit is thus equal to $\pi = \rho F\left(\tau\right) = \rho F\left(\delta - \rho\right)$, and $\pi = \bar{\pi}$ implies $\rho = \bar{\rho}$. When instead a firm charges the lower bound of the distribution, it is sure to undercut its rival and to supply all active consumers; its profit is thus equal to $\rho F\left(v\right) = \rho F\left(w - \rho\right)$, implying that the lower bound $\underline{\rho}$ is such that

$$\underline{\rho} F\left(w - \underline{\rho}\right) = \bar{\pi} = \bar{\rho} F\left(\delta - \bar{\rho}\right). \tag{2}$$

Banning below-cost pricing thus induces positive total margins: Even the lower bound of the distribution satisfies $\underline{m} = \underline{\rho} > 0$. It follows that one-stop shoppers now always face higher prices. Furthermore, from (2), the lower bound $\underline{\rho}$ decreases as w increases, and converges towards $\bar{\rho}$ as w tends to δ;[17] hence the expected margin, $E[\rho]$, also tends to $\bar{\rho}$ when w tends to δ. As $\bar{\rho} > \rho^*$,[18] it follows that multi-stop shoppers, too, now face higher prices when w is close to δ, that is, when strong products are relatively much better than weak ones. Finally, even if banning below-cost pricing may lower expected prices for multi-stop shoppers (when w is large compared with δ, so that competition for one-stop shoppers tends to prevail),[19] the overall impact on prices always yields higher profits:

$$\bar{\pi} = \max_{\rho} \rho F(\delta - \rho) > \pi^* = \max_{\rho} F(\delta - \rho^* - \rho).$$

We thus have:

Proposition 2 *If the inverse hazard rate $h(\cdot)$ is strictly increasing then, when banning below-cost pricing:*

- *There exists a symmetric mixed-strategy equilibrium, in which firms sell weak products at cost and randomize the margins on strong products over the range of $\left[\underline{\rho}, \bar{\rho}\right]$, where the bounds $\bar{\rho}$ and $\underline{\rho}$ are respectively given by* (1) *and* (2)*;*
- *This equilibrium yields higher expected profit than in the absence of a ban;*
- *Total margins are always positive, and thus one-stop shoppers face higher prices than in the absence of a ban;*
- *Multi-stop shoppers also face higher prices when strong products are relatively much better than weak ones (i.e., when w is close to δ).*

Proof. See Appendix C. ■

[17] Note that $w_1^B, w_2^A > 0$ implies $w = w_1^A + w_1^B = w_2^A + \delta + w_1^B > \delta$.

[18] This stems from $\bar{\rho} = h(\bar{\tau})$ and $\rho^* = h(\rho^*)$, where $\bar{\tau} = l^{-1}(\delta) > \tau^* = j^{-1}(\delta)$, as $j(s) = s + 2h(s)$ and $l(s) = s + h(s)$ are both increasing in s, and $j(s) > l(s)$.

[19] It can for instance be checked that, if below-cost pricing is forbidden, then for a uniform distribution of the shopping cost the expected price of strong products tends to their costs when w goes to infinity.

Ex post, the shopping cost threshold below which consumers prefer multi-stop shopping is $\tau^b(\rho_1, \rho_2) \equiv \delta - \max\{\rho_1, \rho_2\}$, and thus satisfies:

$$\tau^b(\rho_1, \rho_2) > \delta - \bar{\rho} = \bar{\tau} > \tau^*,$$

$\bar{\tau} = l^{-1}(\delta) > \tau^* = j^{-1}(\delta)$. Thus, more consumers engage in multi-stop shopping when below-cost pricing is banned. By contrast, as total margins are now higher, the value $v^b(\rho_1, \rho_2) \equiv w - \min\{\rho_1, \rho_2\}$ offered to one-stop shoppers decreases; it follows that the number of active consumers (and thus, *a fortiori*, that of one-stop shoppers) decreases: $F\left(v^b\right) < F(v^*)$.

We now compare the impact of a ban on social welfare. When firms are allowed to price below-cost, equilibrium social welfare can be expressed as

$$W^* = \int_0^w (w-s) f(s)ds + \int_0^{\tau^*} (\delta - s) f(s)ds,$$

where the first term is the surplus that would be generated if all consumers were one-stop shoppers (and thus bought the bundles at cost), and the second term represents the additional surplus from multi-stop shopping. When below-cost pricing is banned, *ex post* social welfare can be expressed as

$$W^b(\rho_1, \rho_2) = \int_0^{v^b(\rho_1,\rho_2)} (w-s) f(s)ds + \int_0^{\tau^b(\rho_1,\rho_2)} (\delta - s) f(s)ds.$$

Thus, the impact on *ex post* social welfare is equal to

$$\Delta W(\rho_1, \rho_2) = W^b(\rho_1, \rho_2) - W = \int_{\tau^*}^{\tau^b(\rho_1,\rho_2)} (\delta - s) f(s)ds - \int_{v^b(\rho_1,\rho_2)}^{w} (w-s) f(s)ds.$$

The first term is the gain from increased multi-stop shopping, whereas the second term is the loss from the reduced value offered to one-stop shoppers. Total welfare decreases if the second effect dominates the first, which depends on w and δ as well as on the distribution of consumers' shopping cost. When for instance the shopping cost is uniformly distributed (i.e., $F(s) = s$), banning below-cost pricing leads to higher expected social welfare if and only if w is sufficiently large:

Proposition 3 *If the inverse hazard rate $h(\cdot)$ is strictly increasing then, when banning below-cost pricing:*

- *The number of multi-stop shoppers increases, whereas the total number of consumers decreases.*
- *When shopping costs are uniformly distributed, there exists some $\hat{w} > \delta$ such that expected social welfare increases if $w > \hat{w}$, and decreases if $w < \hat{w}$.*

Proof. See Appendix D. ■

Banning below-cost pricing thus increases firms' profits but appears to have a mixed impact on consumers and society: It hurts one-stop shoppers (and possibly multi-stop shoppers as well), and may either increase or decrease total welfare.

6 Discussion

In our baseline model, below-cost pricing emerges as the result of two conflicting forces: head-to-head competition for one-stop shoppers drives the total price of the assortments down to cost, whereas imperfect competition for multi-stop shoppers yields positive margins on strong products; it follows that weak products are subsidized. The insights thus rely on two key ingredients: (i) both types of shopping patterns arise in equilibrium, which is likely to be the case when consumers' shopping costs are sufficiently diverse; and (ii) firms have more market power over multi-stop than one-stop shoppers. We now show that the insights remain indeed valid when these two ingredients are present.

6.1 Bounded dispersion of shopping costs

The baseline model assumes a widespread heterogeneity in consumers' shopping cost, spanning the entire range from "pure multi-stop shoppers" (e.g., consumers with $s = 0$ will always choose the best value offered for each product) to "pure one-stop shoppers" (e.g., consumers with $s \geq \delta$ will never visit a second firm). In order to provide a robustness check, we consider here bounded distributions of consumers' shopping costs, and show that the previous insights remain valid as long as both types of shopping patterns arise.

To see this, suppose now that consumers' shopping costs range from $\underline{s} \geq 0$ to $\overline{s} \leq w$,[20] say, and consider a candidate equilibrium with active one-stop and multi-stop shoppers.

[20] Consumers with shopping cost exceeding w will never visit any firm.

It is straightforward to check that, as before, (i) competition for one-stop shoppers drives total margins down to zero: $m_1^* = m_2^* = 0$, and (ii) multi-stop shoppers buy the strong products. Hence, firms derive their profits only from multi-stop shoppers, and firm i's profit is equal to $\rho_i^* F(\tau) = \rho_i^* F(\delta - \rho_1^* - \rho_2^*)$, where ρ_i^* denotes firm i's equilibrium margin on its strong product. Ruling out local deviations in ρ_i then leads to the same characterization as before: $\rho_1^* = \rho_2^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$.

The following propositions confirm that this equilibrium exists whenever consumers' shopping costs are sufficiently heterogeneous. By contrast, when shopping costs are all low enough, active consumers systematically visit both stores and only buy strong products, which firms price above cost. Conversely, when shopping costs are all high enough, consumers visit at most one firm, and symmetric Bertrand competition leads both firms to offer their bundle $A - B$ at cost.

Formally, we have:

Proposition 4 *Suppose that consumers' shopping costs are distributed over* $[0, \overline{s}]$*, where* $\overline{s} > 0$*. Then:*

- *If* $\overline{s} > j^{-1}(\delta)$*, there exists a unique equilibrium, with both types of shopping patterns and the same prices as in the baseline model.*
- *If instead* $\overline{s} \leq j^{-1}(\delta)$*, there exist multiple equilibria, and: (i) only multi-stop shopping arises, (ii) weak products are priced below cost, and (iii) on strong products, firms earn a positive margin, which ranges from* $h(\overline{s})$ *to* $\delta - l(\overline{s})$*.*

Proof. See Appendix E. ■

Firms therefore always price their weak products below cost. However, cross-subsidization actually occurs only when some consumers have high enough shopping costs, namely, when $\overline{s} > j^{-1}(\delta)$; otherwise, there is no one-stop shopping, and consumers only buy strong products – in the limit case $\overline{s} = 0$, where consumers incur no shopping cost, each firm earns a margin of up to δ on its strong product, reflecting its comparative advantage, as in standard asymmetric Bertrand competition.

Proposition 5 *Suppose that consumers' shopping costs are distributed over* $[\underline{s}, +\infty)$*, where* $\underline{s} < w$*. Then:*

- *If $\underline{s} < \delta/3$, there exists a unique equilibrium, with both types of shopping patterns and the same prices as in the baseline model.*
- *If instead $\underline{s} > \delta$, there exist multiple equilibria, and: (i) only one-stop shopping arises, and (ii) firms make zero profit.*
- *Finally, if $\delta/3 \leq \underline{s} \leq \delta$, both types of equilibria exist.*

Proof. See Appendix F. ■

Thus, cross-subsidization can arise in equilibrium as long as some consumers' shopping cost is lower than the extra value δ brought by strong products, and it does arise for sure when some consumers have a low enough shopping cost (namely, below $\delta/3$).

6.2 Market power

The second key ingredient underlying the above analysis is that firms want to charge higher prices to multi-stop shoppers. We first note that it is likely to be the case, even if firms compete less aggressively for one-stop shoppers. Indeed, for one-stop shoppers the assortments offered by the two firms are (possibly imperfect) substitutes, whereas for multi-stop shoppers firms' strong products are complements – cutting the price on either product fosters multi-stop shopping, thereby boosting the sales of the other strong product. Hence, even imperfect competition for one-stop shoppers tends to curb total margins on firms' assortments, whereas double-marginalization tends to induce high prices for strong products.

To see this more formally, suppose for instance that firms have asymmetric comparative advantages: $w_1^A - w_2^A \equiv \bar{\delta} > \underline{\delta} \equiv w_2^B - w_1^B$, which implies that firm 1 is more efficient in serving one-stop shoppers: $w_1^{AB} - w_2^{AB} = \bar{\delta} - \underline{\delta} > 0$. Even if the difference is small, firm 1 now enjoys some market power over one-stop shoppers: In equilibrium, firm 2 offers its assortment at cost ($m_2 = 0$) but firm 1 attracts all one-stop shoppers and charges a total margin reflecting its competitive advantage $m_1 = \bar{\delta} - \underline{\delta}$ (and so $v_1 = v_2 = w_2^{AB}$). Firm 1 now attracts one-stop shoppers, buying both products, as well as multi-stop shoppers purchasing its strong product only; its profit can then be expressed as

$$\begin{aligned}\pi_1 &= \rho_1 F(\tau) + m_1 [F(v_1) - F(\tau)] \\ &= m_1 F\left(w_2^{AB}\right) - \mu_1 F(\tau),\end{aligned}$$

where $\mu_1 \equiv m_1 - \rho_1$ denotes firm 1's margin on its weak product. Obviously, it is again optimal to set $\mu_1 < 0$: Firm 1 sells its weak product below cost.[21]

The insight remains valid even if firm 1 is so much more efficient that firm 2 no longer exerts a competitive pressure on one-stop shoppers. In equilibrium, firm 2 then still offers its assortment at cost but firm 1 offers a better value, $v_1 = w_1^{AB} - m_1 > v_2 = w_2^{AB}$, and the multi-stop shopping threshold becomes

$$\tau = v_{12} - v_1 = \left(w_1^{AB} + \underline{\delta} - \rho_1 - \rho_2\right) - \left(w_1^{AB} - m_1\right) = \underline{\delta} - \mu_1 - \rho_2.$$

Hence firm 1 maximizes

$$\begin{aligned}\pi_1 &= \rho_1 F(\tau) + m_1 [F(v_1) - F(\tau)] \\ &= m_1 F\left(w_1^{AB} - m_1\right) - \mu_1 F(\underline{\delta} - \mu_1 - \rho_2),\end{aligned}$$

which leads it to charging a "monopoly margin" m_1^m on its assortment but still subsidizing its weak product ($\mu_1 < 0$).

The scope for below-cost pricing would however disappear if firms could coordinate their pricing decisions, e.g., through tacit or explicit collusion. Consider for instance our baseline setup but suppose now that firms interact repeatedly over time and are so "patient" (that is, their discount factors is close to 1) that they can perfectly collude and maximize their joint profits. Using as decision variables the total margin, m, and the margin differential between strong and weak products, $t = \rho - \mu$, total industry profit can be expressed as

$$\Pi = mF(v) + tF(\tau) = mF(w - m) + tF(\delta - t).$$

[21] Firm 2 only sells its strong product – to multi-stop shoppers – and thus charges a positive margin on it; as $m_2 = 0$, firm 2 thus still prices its weak product below-cost, but consumers do not buy it in equilibrium.

It is thus separable in m and t, and a revealed preference argument shows[22] that the industry-profit maximizing margins satisfy $m = \rho + \mu > t = \rho - \mu$, and thus $\mu > 0$: There is no below-cost pricing. Hence, cross-subsidization arises here precisely when firms are strongly competing against each other for one-stop shoppers, and/or face double-marginalization problems on the demand from multi-stop shoppers.

7 Conclusion

Our analysis provides a rationale for below-cost pricing in competitive markets and identifies key factors underlying it, namely, heterogeneous shopping patterns and comparative advantages on different sets of products: Competitive cross-subsidization then arises as a way to better discriminate consumers according to their shopping patterns, in order to extract more surplus from multi-stop shoppers. We also show that banning below-cost pricing leads to greater profits at the expense of higher prices for one-stop shoppers (and possibly for multi-stop shoppers as well), and may reduce both consumer surplus and total social welfare.

Cross-subsidization has been the subject of hot debates in antitrust policy circles, reflected in the discrepancy of statutes on below-cost sales. Our analysis provides a conceptual framework for its assessment, from which we can derive policy implications. In Chen and Rey (2012), we show that a firm that dominates a market has an incentive to subsidize its products in more competitive adjacent segments. In such a case, banning below-cost pricing can discipline the dominant firm and improve consumer surplus as well as social welfare. By contrast, in markets such as those studied here, where competition limits firms' market power on all products, cross-subsidization need not be harmful to consumers, and regulation against below-cost pricing is then counter-productive. This suggests to limit regulation on below-cost sales to situations of market dominance.

We have developed these insights using a simple setup, with individual unit demands and homogeneous consumer valuations for the goods; allowing for more general market environments, e.g., where individual demands are downward sloping, or where underlying characteristics (e.g., wealth) affect consumers' willingness to pay as well as their shopping

[22] Recall that, by construction, $w > \delta$.

costs, is left to future research.

Appendix

Recall that $v_i = w - m_i$ for $i = 1, 2$, and $v_{12} = w + \delta - m_1^A - m_2^B$; and denote by $\hat{v}_{12} = w - \delta - m_1^B - m_2^A$ the value from multi-stop shopping when consumers pick the weak products. The shopping cost thresholds, below which consumers favor picking both strong products rather than patronizing only firm 1 or firm 2 are respectively $\tau_1 = v_{12} - v_1 = \delta + m_1^B - m_2^B$ and $\tau_2 = v_{12} - v_2 = \delta - m_1^A + m_2^A$, and $\tau = \min\{\tau_1, \tau_2\}$; likewise, the thresholds for picking weak products are $\hat{\tau}_1 \equiv \hat{v}_{12} - v_1 = m_1^A - m_2^A - \delta$, $\hat{\tau}_2 \equiv \hat{v}_{12} - v_2 = m_2^B - m_1^B - \delta$, and $\hat{\tau} = \min\{\hat{\tau}_1, \hat{\tau}_2\}$. Note that $\hat{\tau}_1 = -\tau_2$, $\hat{\tau}_2 = -\tau_1$, and thus $\hat{\tau} = -\tau$.

A Proof of Lemma 1

To prove the lemma, we first establish the following claims.

Claim 1 *Some consumers are active in equilibrium.*

Proof. Suppose there is no active consumer. It must be the case that $\max\{v_1, v_2, v_{12}, \hat{v}_{12}\} \leq 0$, and firms make no profit. Consider the following deviation for firm 1: charge $\tilde{m}_1^B > 0$ and $\tilde{m}_1^A > 0$ such that $\tilde{m}_1 = \tilde{m}_1^A + \tilde{m}_1^B = w - \varepsilon$, for some $\varepsilon \in (0, w)$. Firm 1 then attracts consumers with shopping cost $s \leq \tilde{v}_1 = \varepsilon$ and earns a positive profit, a contradiction. Thus some consumers must be active in equilibrium. ■

Claim 2 *If there are active one-stop shoppers in equilibrium, then $m_1 = m_2 = 0$.*

Proof. Consider a candidate equilibrium in which some one-stop shoppers are active, which requires $\max\{v_1, v_2\} > 0$. We first show that no firm charges a negative total margin. To see this, suppose firm 1 sets $m_1 < 0$ (and thus $v_1 > w > 0$), say, then:

- If $m_1 < m_2$, firm 1 incurs a loss by attracting one-stop shoppers; then consider the following deviations:
    - If there is no multi-stop shopper, or if firm 1 does not make a profit on multi-stop shoppers, then firm 1 could avoid all losses by increasing both of its prices.

– If some multi-stop shoppers buy the strong products, and firm 1 makes a profit on them (that is, $m_1^A > 0$), then firm 1 would benefit from raising the margin on its weak product: Keeping m_1^A constant, raising the margin on the weak product to $\tilde{m}_1^B = -m_1^A = m_1^B - m_1 > m_1^B$ (i) yields $\tilde{m}_1 = 0$, thus avoiding the loss from one-stop shoppers, and (ii) moreover increases the demand from multi-stop shoppers (on which firm 1 makes a positive margin), as it reduces the value from one-stop shopping without affecting that of multi-stop shopping.

– If some multi-stop shoppers buy the weak products, and firm 1 makes a profit on them (that is, $m_1^B > 0$), then firm 1 could avoid the loss from one-stop shoppers by raising the margin on its strong product to $\tilde{m}_1^A = -m_1^B$ (yielding $\tilde{m}_1 = 0$), which would also increase the demand from multi-stop shoppers, by reducing the value from one-stop shopping without affecting that of multi-stop shopping.

- If instead $m_1 \geq m_2$ (and thus $m_2 < 0$), then the same argument applies to any firm that attracts one-stop shoppers (firm 2 if $m_1 > m_2$, and at least one of the firms if $m_1 = m_2$).

Next, we show that both firms charging a positive total margin cannot be an equilibrium. Suppose firms set $m_1, m_2 > 0$. Then:

- If any firm, say firm 1, charges a higher margin than its rival ($m_1 > m_2 > 0$ and $v_2 > \max\{v_1, 0\}$), it faces no demand from one-stop shoppers; then consider the following deviations:

  – If there is no multi-stop shopper, which requires $\max\{v_{12}, \hat{v}_{12}\} \leq v_2$, firm 1 can make a positive profit by undercutting both of its rival's "quality-adjusted" prices by $\varepsilon/2$: for ε positive but small enough, charging $\tilde{m}_1^A = m_2^A + \delta - \varepsilon/2$ and $\tilde{m}_1^B = m_2^B - \delta - \varepsilon/2$ profitably attracts one-stop shoppers (as $\tilde{v}_1 = v_2 + \varepsilon > v_2$ and $\tilde{m}_1 = m_2 - \varepsilon > 0$), without transforming them into multi-stop shoppers (as $\tilde{v}_1 = v_2 + \varepsilon > \max\{\tilde{v}_{12}, \widetilde{\hat{v}}_{12}\} = v_2 + \varepsilon/2$).

– If some multi-stop shoppers are active, which requires $\max\{v_{12}, \hat{v}_{12}\} > v_2\,(> v_1)$, firm 1 can make a profit by keeping constant its margin on the product purchased by multi-stop shoppers, and reducing its other margin so as to yield $\tilde{m}_1 = m_2 - \varepsilon$, with $\varepsilon > 0$: Doing so attracts all one-stop shoppers (as $\tilde{v}_1 = v_2 + \varepsilon > v_2$ and $\tilde{m}_1 = m_2 - \varepsilon > 0$), at the cost of slightly reducing the demand of multi-stop shoppers (as it does not affect the value from multi-stop shopping, $\max\{v_{12}, \hat{v}_{12}\}$, and only increases the value of one-stop shopping by ε, from v_2 to $\tilde{v}_1 = v_2 + \varepsilon$), and is obviously profitable for ε small enough.

- If both firms charge the same total margin ($m_1 = m_2 > 0$), then $v_2 = v_1$ and $\tau_1 = \tau_2$. At least one firm, say firm 1, does not obtain more than half of the demand from one-stop shoppers; but then, this firm can attract all one-stop shoppers using the deviations described above for the case $m_1 > m_2$, and the gain from doing so offsets the loss from the slight reduction in demand, if any, from multi-stop shoppers.

Finally, we show that no firm charges a positive total margin in equilibrium. Suppose for instance that $m_1 > m_2 = 0$; firm 2 then makes zero profit from one-stop shoppers. If it makes a loss on multi-stop shoppers, then it could profitably deviate by raising all of its prices, so as to avoid the loss. If instead it supplies multi-stop shoppers at or above cost, then it could profitably deviate by increasing by some $\varepsilon > 0$ the margin of the product *not picked* by multi-stop shoppers, keeping constant its other margin: For ε small enough, firm 2 still supplies all one-stop shoppers, but now makes a profit on them; and doing so moreover *increases* the demand from multi-stop shoppers and thus the profit on them.

We conclude that firms must charge $m_1 = m_2 = 0$ in any equilibrium with active one-stop shoppers. ■

Claim 3 *In equilibrium, active multi-stop shoppers buy the strong products.*

Proof. Suppose that some multi-stop shoppers buy the weak products. Each firm must then offer on its weak product a better value than the rival's strong product; that is, each firm must sell its strong product with a margin that exceeds its rival's "quality-adjusted" margin: $m_2^B \geq m_1^B + \delta$ and $m_1^A \geq m_2^A + \delta$. We show that such configuration cannot be an equilibrium. We consider two cases:

- Suppose first that there are only multi-stop shoppers (buying the weak products). To make profits, firms must charge non-negative margins on their weak products, i.e., m_1^B, $m_2^A \geq 0$. From the above, this implies that each firm sells its strong product with a margin that exceeds its comparative advantage δ: $m_2^B \geq \delta$ and $m_1^A \geq \delta$. But then, any firm could profitably undercut its rival. For instance, keeping m_1^B unchanged, by charging $\tilde{m}_1^A = m_2^A + \delta - \varepsilon > 0$ firm 1 would sell its strong product as well to all previously active consumers, as it now offers a better value on A: $\tilde{v}_1^A = v_2^A + \varepsilon$; the deviation may also attract additional one-stop shoppers, on which the firm makes a profit as $\tilde{m}_1^A > 0$ and $m_1^B \geq 0$.

- Suppose instead that there are both one-stop shoppers and multi-stop shoppers. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$. As firms make no profit from one-stop shoppers, they must charge non-negative margins on their weak products, i.e., m_1^B, $m_2^A \geq 0$. But this implies that margins on strong products are non-positive, say, $m_1^A = m_1 - m_1^B \leq 0$, which contradicts the condition $m_1^A \geq m_2^A + \delta \geq \delta$.

Therefore multi-stop shoppers must buy strong products in equilibrium. ■

Claim 4 *Some multi-stop shoppers are active in equilibrium.*

Proof. Suppose all active consumers are one-stop shoppers, which requires $\max\{v_1, v_2\} > 0$ and $\max\{v_1, v_2\} \geq \max\{v_{12}, \hat{v}_{12}\}$. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$, and thus firms make zero profit. We show that this configuration cannot be an equilibrium.

By construction, $v_1 + v_2 = v_{12} + \hat{v}_{12}$, as it corresponds to the total value of buying one unit of both products from both firms. Here, we moreover have $v_1 = v_2 \geq \max\{v_{12}, \hat{v}_{12}\}$, it follows that $v_1 = v_2 = v_{12} = \hat{v}_{12}$; that is, firms must offer the same value on both products, by charging a margin $\delta/2$ on strong products, and subsidizing weak products by the same amount. It follows that it is profitable for any firm to encourage some consumers to buy only its strong product: For instance, increasing m_1^B by $\varepsilon > 0$ and decreasing m_1^A by the same amount raise both τ_1 and τ_2 by ε, which triggers some multi-stop shopping as $\tau = \varepsilon > 0$; and as $\tilde{m}_1^A = \delta/2 - \varepsilon > 0$ for ε small enough, firm 1 now makes a positive profit on these multi-stop shoppers. ■

Claim 5 *Some one-stop shoppers are active in equilibrium.*

Proof. Suppose there are only multi-stop shoppers, who from Claim 3 buy the strong products. Consumers are willing to visit both firms if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$), but would prefer one-stop shopping if $s > \tau = v_{12} - \max\{v_1, v_2\}$; hence, we must have $\tau \geq v_{12}/2$, and the demand from multi-stop shoppers is $F(v_{12}/2)$. As consumers only buy strong products, firms must charge non-negative margins on these products. Note also that the condition

$$\frac{v_{12}}{2} \leq \tau = v_{12} - \max\{v_1, v_2\}$$

implies $\max\{v_1, v_2\} \leq v_{12}/2$.

Without loss of generality, suppose $m_2^B \geq m_1^A$ (≥ 0), and consider the following deviation for firm 1: Keeping m_1^A constant, change m_1^B to

$$\tilde{m}_1^B = \frac{w - \delta + m_2^B - m_1^A}{2} - \varepsilon \geq \frac{w - \delta}{2} - \varepsilon > 0,$$

so as to *increase* the value offered to one-stop shoppers to

$$\tilde{v}_1 = w - m_1^A - \tilde{m}_1^B = \frac{w + \delta - m_1^A - m_2^B}{2} + \varepsilon = \frac{v_{12}}{2} + \varepsilon.$$

This deviation does not affect v_{12} nor τ_2 (which only depends on the prices for A), but it decreases τ_1 to $\tilde{\tau}_1 = \delta + \tilde{m}_1^B - m_2^B = v_{12}/2 - \varepsilon$; as initially $\tau \geq v_{12}/2$, it follows that the multi-stop shopping threshold becomes $\tilde{\tau} = \tilde{\tau}_1$ $(< v_{12}/2) < \tilde{v}_1$. This adjustment thus induces some of the initial multi-stop shoppers to buy both products from firm 1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $v_{12}/2$), on which firm 1 earns an extra profit from selling its weak product (as $\tilde{m}_1^B > 0$), and it moreover attracts some additional one-stop shoppers (those whose shopping cost lies between $v_{12}/2$ and $\tilde{v}_1$), which generates additional profit (as $m_1^A \geq 0$ and $\tilde{m}_1^B > 0$). ■

Claims 4 and 5 establish part (i) of the Lemma. Part (ii) then follows from Claim 3, whereas part (iii) follows from Claim 2.

B Proof of Proposition 1

From Lemma 1, the equilibrium is interior and such that consumers whose shopping cost lies below $\tau^* > 0$ patronize both firms, whereas those whose shopping cost lies between τ^* and w patronize a single firm. The monotonicity of the inverse hazard rate $h(\cdot)$ furthermore ensures that the first-order conditions characterize a unique candidate equilibrium, satisfying $m_1^* = m_2^* = 0$ and $m_1^{A*} = m_2^{B*} = \rho^*$, such that

$$\rho^* = h(\tau^*),$$

where

$$\tau^* = j^{-1}(\delta).$$

We show now firms cannot benefit from any deviation. Suppose firm 1 charges m_1^A and m_1^B instead of $m_1^{A*} = \rho^*$ and $m_1^{B*} = -\rho^*$. Then:

- It cannot make a profit from one-stop shoppers, as it would have to charge $m_1 \leq m_2^* = 0$ to attract them.
- It cannot make a profit either by offering the weak product to multi-stop shoppers, as it would have to charge $m_1^B \leq m_2^{B*} - \delta = \rho^* - \delta < 0$ (as $\rho^* < \delta$) to attract them.
- Thus, it can only make a profit from multi-stop shoppers, and this profit is equal to $m_1^A F(\tau)$, where $\tau = \min\{\delta + m_2^{A*} - m_1^A, \delta + m_1^B - m_2^{B*}\}$; but then

  $$\pi_1^A \leq m_1^A F(\tau) \leq m_1^A F\left(\delta + m_2^{A*} - m_1^A\right) = m_1^A F(\delta - \rho^* - m_1^A) \leq \pi^*,$$

  where the second inequality comes from the fact that the profit function $m_1^A F(\delta - \rho^* - m_1^A)$ is quasi-concave, from the monotonicity of $h(\cdot)$, and by construction maximal for $m_1^A = \rho^* = m_1^{A*}$.

C Proof of Proposition 2

We first characterize the mixed-strategy equilibrium. Consider a candidate equilibrium in which each firm i: (i) sells its weak product at cost; (ii) randomizes the margin ρ_i on its strong product, according to a distribution $K(\rho)$ over some interval, with continuous density $k(\rho)$; and obtains an expected profit equal to the minmax, $\bar{\pi}$. By construction, the bounds of the support of the distribution must be given by (1) and (2).

Consider consumers' response to given margins ρ_i and ρ_j:

- Consumers buy both goods from firm i if
  - firm i undercuts its rival:
$$\rho_j \geq \rho_i;$$
  - one-stop shopping is valuable:
$$s \leq v_i = w - \rho_i;$$
  - and more so than multi-stop shopping:
$$s \geq v_{12} - v_i = \delta - \rho_j.$$
- Consumers instead engage in multi-stop shopping if
$$s \leq v_{12} - \max\{v_1, v_2\},$$
  which boils down to
$$s \leq \delta - \rho_i \text{ and } s \leq \delta - \rho_j.$$

Figure 1 depicts consumers' response.

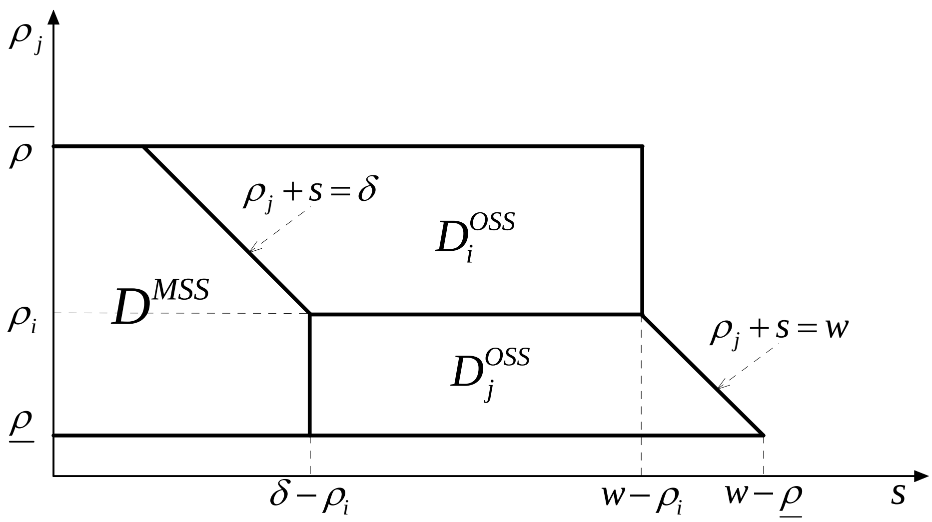

Figure 1

Firm i's expected profit can then be expressed as
$$\rho_i E\left[D_i^{OSS} + D^{MSS}\right],$$

where D_i^{OSS} represents the demand from one-stop shoppers going to firm i and D^{MSS} is the demand from multi-stop shoppers. As firm j's margin is distributed according to the distribution function $K\left(\rho_j\right)$, firm i's expected profit can be written as:

$$\begin{aligned}\pi\left(\rho_i\right) &= \rho_i\left[\left(1-K\left(\rho_i\right)\right)F(w-\rho_i)+K\left(\rho_i\right)F\left(\delta-\rho_i\right)\right]\\ &= \rho_i\left[F(w-\rho_i)-K\left(\rho_i\right)\left(F(w-\rho_i)-F\left(\delta-\rho_i\right)\right)\right].\end{aligned}$$

Hence, for a firm to obtain its minmax profit $\bar{\pi}$ we must have, for all ρ:

$$\rho\left[F(w-\rho)-K\left(\rho\right)\left(F(w-\rho)-F\left(\delta-\rho\right)\right)\right]=\bar{\pi},$$

or

$$K\left(\rho\right)\equiv\frac{\rho F\left(w-\rho\right)-\bar{\pi}}{\rho F\left(w-\rho\right)-\rho F\left(\delta-\rho\right)}. \tag{3}$$

By construction, the function $K\left(\cdot\right)$ defined by (3) is such that $K\left(\underline{\rho}\right)=0$ and $K\left(\bar{\rho}\right)=1$; it remains to check that it is increasing in ρ in the range $\left[\underline{\rho},\bar{\rho}\right]$. Differentiating (3) with respect to ρ, we have

$$K'\left(\rho\right)=\frac{\left[\bar{\pi}-\rho F\left(\delta-\rho\right)\right]\left[F(w-\rho)-\rho f(w-\rho)\right]+\left[\rho F(w-\rho)-\bar{\pi}\right]\left[F\left(\delta-\rho\right)-\rho f\left(\delta-\rho\right)\right]}{\left[\rho F(w-\rho)-\rho F\left(\delta-\rho\right)\right]^2}$$

As $w>\delta$, and given (1) and (2), the functions $\rho F(w-\rho)$ and $\rho F(\delta-\rho)$ are both increasing in the range $\left[\underline{\rho},\bar{\rho}\right]$, and moreover satisfy $\underline{\rho}F(w-\underline{\rho})=\bar{\rho}F(\delta-\bar{\rho})=\bar{\pi}$ and $\rho F(w-\rho)>\bar{\pi}>\rho F(\delta-\rho)$ for $\underline{\rho}<\rho<\bar{\rho}$. It follows that $K'\left(\bar{\rho}\right)=0$ and $K'\left(\rho\right)>0$ for $\underline{\rho}\leq\rho<\bar{\rho}$.

We now show that the function $K\left(\cdot\right)$ supports a symmetric mixed strategy equilibrium. To see this, consider firm i's best response when its rival, firm j, adopts the above strategy. If firm i were to charge a total margin $m_i>\bar{\rho}$, one-stop shoppers would go to the rival and multi-stop shoppers are those consumers whose shopping cost is lower than $v_{12}-v_j=\delta-\rho_i$; hence, firm i would earn a profit equal to $\rho_i F\left(\delta-\rho_i\right)\leq\bar{\pi}$. Thus, without loss of generality, we can restrict to the deviations such that $m_i\leq\bar{\rho}$.

Suppose first that firm i prices its weak product above cost (i.e., its total margin satisfies $m_i>\rho_i$), and consider the impact of an increase in the margin on the strong product, ρ_i, keeping constant the total margin m_i:

• For realizations of the rival's margins such that $m_j\left(=\rho_j\right)>m_i$, one-stop shoppers (if any) favor firm i, and thus the multi-stop shopping threshold is $\tau=v_{12}-v_i=\delta+m_i-\rho_i-\rho_j$; two cases may then arise:

- If $\tau = v_{12} - v_i \leq v_i$, which amounts to $v_i \geq v_{12}/2$, consumers whose shopping cost lies below τ engage in multi-stop shopping and buy strong products, whereas those with s between τ and v_i buy both products from firm i. Hence, increasing ρ_i:

  – Increases the profit earned from selling the strong product to all active consumers (that is, those with $s \leq v_i = w - m_i$);

  – and also induces some multi-stop shoppers to buy firm i's weak product as well, which further enhances firm i's profit.

- If instead $v_i < v_{12}/2$, consumers whose shopping cost lies below $v_{12}/2$ engage in multi-stop shopping and buy strong products, and all other consumers are inactive. Hence, firm i's profit is equal to

$$\pi_i(\rho_i) = \rho_i F\left(\frac{v_{12}}{2}\right) = \rho_i F\left(\frac{w + \delta - \rho_1 - \rho_2}{2}\right),$$

  which increases with ρ_i: The derivative is equal to

$$\pi_i'(\rho_i) = F\left(\frac{v_{12}}{2}\right) - \frac{\rho_i f\left(\frac{v_{12}}{2}\right)}{2} = \left[2h\left(\frac{v_{12}}{2}\right) - \rho_i\right]\frac{f\left(\frac{v_{12}}{2}\right)}{2},$$

  where the term in bracket is positive, as $v_i < v_{12}/2$ implies $2h(v_{12}/2) > h(v_{12}/2) > h(v_i) = h(w - m_i) > m_i > \rho_i$ (where the penultimate inequality stems from $m_i \leq \bar{\rho}$, the function $m_i F(w - m_i)$ being increasing in m_i in that range).

• For realizations of the rival's margins such that $m_j \left(= \rho_j\right) < m_i$, one-stop shoppers (if any) favor firm j; hence, firm i only sells (its strong product) to multi-stop shoppers, and the multi-stop shopping threshold is $\tau = v_{12} - v_j = \delta - \rho_i$; two cases may again arise:

- If $\tau = v_{12} - v_j \leq v_i$, which amounts to $v_j \geq v_{12}/2$, all consumers whose shopping cost lies below τ engage in multi-stop shopping, and so firm i's profit is equal to:

$$\pi_i(\rho_i) = \rho_i F(\tau) = \rho_i F(\delta - \rho_i),$$

  which increases with ρ_i on the relevant range $\rho_i \leq \bar{\rho}$.

- If instead $v_j < v_{12}/2$, only those consumers with s below $v_{12}/2$ engage in multi-stop shopping, and so firm i's profit is equal to $\pi_i(\rho_i) = \rho_i F\left(\frac{v_{12}}{2}\right)$. The same reasoning as above then shows that this profit again increases with ρ_i.

Therefore, it is never optimal for a firm to price its weak product above cost: Starting from $\rho_i < m_i$, raising ρ_i would always increase firm i's *ex post* profit, and would thus increase its expected profit as well.

Suppose now that firm i sells its weak product at cost: $m_i = \rho_i$. By construction, choosing any ρ_i in the range $\left[\underline{\rho}, \bar{\rho}\right]$ yields the same expected profit, $\bar{\pi}$. It remains to check that it cannot be profitable to pick a margin ρ_i outside the support of K:

- Choosing $\rho_i < \underline{\rho}$ attracts all one-stop shoppers and thus yields an expected profit equal to $\pi_i(\rho_i) = \rho_i F(w - \rho_i)$, which increases in ρ_i for $\rho_i \leq \bar{\rho}$, and is thus lower than $\pi_i\left(\underline{\rho}\right) = \bar{\pi}$.

- Choosing $\rho_i > \bar{\rho}$ attracts no one-stop shoppers, and thus the expected profit must be lower than $\rho_i F(\delta - \rho_i) \leq \max_\rho \rho F(\delta - \rho) = \bar{\pi}$;

This establishes the first part of the proposition; the rest has been established in the main text.

D Proof of Proposition 3

The first part of the Proposition has been established in the text. We establish here the second part, assuming that consumers' shopping costs are uniformly distributed: $F(s) = s$, in which case $\tau^* = \delta/3$, $\bar{\rho} = \delta/2$, and

$$K(\rho) = \frac{w - \rho - \frac{\bar{\pi}}{\rho}}{w - \delta}, k(\rho) = \frac{\frac{\bar{\pi}}{\rho^2} - 1}{w - \delta}.$$

We then have:

$$\begin{aligned}
\Delta W(\rho_1, \rho_2) &= \int_{\tau^*}^{\tau^b(\rho_1,\rho_2)} (\delta - s) f(s) ds - \int_{v^b(\rho_1,\rho_2)}^{w} (w - s) f(s) ds \\
&= \frac{(\delta - \tau^*)^2 - (\max\{\rho_1, \rho_2\})^2 - (\min\{\rho_1, \rho_2\})^2}{2} \\
&= \frac{2}{9}\delta^2 - \frac{\rho_1^2 + \rho_2^2}{2}.
\end{aligned}$$

Hence

$$E[\Delta W] = \frac{2}{9}\delta^2 - E\left[\rho^2\right],$$

where, using $K\left(\underline{\rho}\right)=0$ and $K\left(\bar{\rho}\right)=1$:

$$E\left[\rho^2\right]=\int_{\underline{\rho}}^{\bar{\rho}}\rho^2 dK\left(\rho\right)=\left[\rho^2K\left(\rho\right)\right]_{\underline{\rho}}^{\bar{\rho}}-2\int_{\underline{\rho}}^{\bar{\rho}}\rho K\left(\rho\right)d\rho=\bar{\rho}^2-2\int_{\underline{\rho}}^{\bar{\rho}}\rho K\left(\rho\right)d\rho.$$

Furthermore, the upper bound $\bar{\rho}$ only depends on δ; and although the lower bound $\underline{\rho}$ depends also on w, as $K\left(\underline{\rho}\right)=0$ and

$$\frac{\partial K\left(\rho\right)}{\partial w}=\frac{\bar{\pi}-\rho\left(\delta-\rho\right)}{\rho\left(w-\delta\right)^2}>0,$$

we have:

$$\frac{\partial E\left[\rho^2\right]}{\partial w}=-2\int_{\underline{\rho}}^{\bar{\rho}}\rho\frac{\partial K\left(\rho\right)}{\partial w}d\rho<0.$$

It follows that $E\left[\Delta W\right]$ increases with w.

The lower bound $\underline{\rho}$, determined by (2), decreases as w increases. Furthermore, it tends to $\bar{\rho}$ as w tends δ, in which case the change in welfare tends to:

$$\begin{aligned}\Delta W|_{w=\delta} &= \int_{\tau^*}^{\bar{\tau}}\left(\delta-s\right)ds-\int_{\bar{\tau}}^{\delta}\left(\delta-s\right)ds\\ &= \frac{\left(\delta-\tau^*\right)^2}{2}-\left(\delta-\bar{\tau}\right)^2\\ &= \frac{2\delta^2}{9}-\frac{\delta^2}{4}<0.\end{aligned}$$

Conversely, as w goes to infinity, then

$$E\left[\rho^2\right]=\int_{\underline{\rho}}^{\bar{\rho}}\rho^2\frac{\frac{\bar{\pi}}{\rho^2}-1}{w-\delta}d\rho=\int_{\underline{\rho}}^{\bar{\rho}}\frac{\bar{\pi}-\rho^2}{w-\delta}d\rho$$

converges to zero, and thus $E\left[\Delta W\right]$ tends to $2\delta^2/9>0$.

Hence, as $E\left[\Delta W\right]$ increases with w, is negative for w close to δ and positive for w large enough, it follows that there exists $\hat{w}>\delta$ such that $E\left[\Delta W\right]>0$ if $w>\hat{w}$, and $E\left[\Delta W\right]<0$ if $w<\hat{w}$.

E Proof of Proposition 4

Suppose that consumers' shopping costs are distributed over $[0,\overline{s}]$, where $\overline{s}>0$. It is straightforward to check that the first four claims in the proof of Lemma 1 still hold; that is, in any equilibrium, there exist active multi-stop shoppers who buy the strong products; in addition, if there are active one-stop shoppers, then $m_1 = m_2 = 0$.

We first note that the equilibrium identified in the baseline model still exists when $\overline{s}$ is large enough:

Claim 6: When $\overline{s} > j^{-1}(\delta)$, then there exists an equilibrium with both types of shoppers: Consumers with shopping cost lower than $\tau^* = j^{-1}(\delta)$ engage in multi-stop shopping, and face a margin $\rho^* = h(\tau^*)$ on each strong product, whereas those with higher cost favor one-stop shopping.

Proof: As shown in the text, there is a unique candidate equilibrium where both types of shopping patterns arise, and it is as described in the Claim. The existence of one-stop shopping however requires $\overline{s} > \tau^* = j^{-1}(\delta)$. Conversely, when this condition holds, the margins $m_1^* = m_2^* = 0$ and $\rho_1^* = \rho_2^* = h(\tau^*)$ do support an equilibrium: The reasoning of the proof of Proposition 1 indeed ensures that no deviation is profitable.

Next, we show that one-stop shopping cannot arise if $\overline{s}$ is too low:

Claim 7: When $\overline{s} \leq j^{-1}(\delta)$, then one-stop shopping does not arise in equilibrium.

Proof: Suppose there exist some one-stop shoppers, which requires $\tau < \min\{\max\{v_1, v_2\}, \overline{s}\}$. Competition for these one-stop shoppers leads to $m_1 = m_2 = 0$, and thus $\tau_1 = \tau_2 = \delta - m_1^A - m_2^B < \overline{s}$, which implies $m_1^A + m_2^B > \delta - \overline{s} > 2h(\overline{s})$. Therefore, at least one firm's margin of the strong product must exceed $h(\overline{s})$. Suppose $m_1^A > h(\overline{s})$; then $m_1^A > h(\overline{s}) > h(\tau)$, as $\overline{s} > \tau$ and $h(\cdot)$ is strictly increasing. Consider now the following deviation: decrease m_1^A to $\tilde{m}_1^A$ and increase m_1^B by the same amount, so as to maintain the total margin. This does not affect the profit from one-stop shoppers (which remains equal to zero), but yields a profit from multi-stop shoppers, equal to $\tilde{\pi}_1 = \tilde{m}_1^A F(\tilde{\tau})$, where $\tilde{\tau} = \delta - \tilde{m}_1^A - m_2^B$. As $d\tilde{\pi}_1/d\tilde{m}_1^A|_{\tilde{m}_1^A = m_1^A} = -f(\tau)(m_1^A - h(\tau))$, which is strictly negative as $m_1^A > h(\tau)$, such deviation is profitable. Hence, one-stop shopping does not arise in equilibrium. **Q.E.D.**

Claims 6 and 7 together establish the first part of the Proposition. We now characterize the equilibria where all consumers are multi-stop shoppers.

Claim 8: When $\overline{s} \leq j^{-1}(\delta)$, any margin profile such that $m_1^A, m_2^B \in [h(\overline{s}), \delta - \overline{s} - h(\overline{s})]$, $m_2^A = m_1^A - \delta + \overline{s}$ and $m_1^B = m_2^B - \delta + \overline{s}$, constitutes an equilibrium in which all active consumers are multi-stop shoppers.

Proof: Suppose there are only multi-stop shoppers, who from Claim 3 buy the strong products. Consumers are willing to visit both firms if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$), but would prefer one-stop shopping if $s > \tau = v_{12} - \max\{v_1, v_2\}$; hence, we must have $\tau \geq \min\{v_{12}/2, \overline{s}\}$, and the demand from multi-stop shoppers is $F(\min\{v_{12}/2, \overline{s}\})$. As consumers only buy strong products, firms must charge non-negative margins on these products: $m_1^A, m_2^B \geq 0$.

If $\overline{s} < \min\{v_{12}/2, \tau\}$, each firm can profitably deviate by slightly raising the price for its strong product: This increases the margin without affecting the demand, equal to $F(\overline{s})$. Hence, without loss of generality, we can assume $\overline{s} \geq \min\{v_{12}/2, \tau\}$. The condition $\tau \geq \min\{v_{12}/2, \overline{s}\}$ then implies that either $v_{12}/2 \leq \min\{\tau, \overline{s}\}$, or $v_{12}/2 \geq \tau = \overline{s}$. We consider these two cases in turn.

Consider the first case, and note that the condition

$$\frac{v_{12}}{2} \leq \tau = v_{12} - \max\{v_1, v_2\}$$

then implies $\max\{v_1, v_2\} \leq v_{12}/2$. Without loss of generality, suppose $m_2^B \geq m_1^A (\geq 0)$, and consider the following deviation for firm 1: Keeping m_1^A constant, reduce m_1^B so as to offer $\tilde{v}_1 = v_{12}/2 + \varepsilon$, which amounts to charging

$$\tilde{m}_1^B = \frac{w - \delta + m_2^B - m_1^A}{2} - \varepsilon \geq \frac{w - \delta}{2} - \varepsilon > 0.$$

This deviation does not affect v_{12} nor $\tau_2 = v_{12} - v_2$, but it decreases τ_1 to $\tilde{\tau}_1 = v_{12} - \tilde{v}_1 = v_{12}/2 - \varepsilon$; as initially $\tau_2 \geq \tau \geq v_{12}/2$, it follows that the multi-stop shopping threshold becomes $\tilde{\tau} = \tilde{\tau}_1 (< v_{12}/2) < \tilde{v}_1$. This adjustment thus induces some multi-stop shoppers to buy everything from firm 1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $v_{12}/2$), on which firm 1 earns an extra profit from selling its weak product (as $\tilde{m}_1^B > 0$), and it moreover attracts some additional one-stop shoppers (those whose shopping cost lies between $v_{12}/2$ and $\tilde{v}_1$), generating additional profit (as $m_1^A \geq 0$ and $\tilde{m}_1^B > 0$).

Hence, we cannot have an equilibrium of the type $v_{12}/2 \leq \min\{\tau, \overline{s}\}$.

Consider now the second case: $\overline{s} = \tau \leq v_{12}/2$. Note first that, if $\tau = \tau_i = v_{12} - v_i < \tau_j = v_{12} - v_j$, then firm i could again profitably deviate by increasing the margin on its strong product without affecting the demand (as τ_i does not depend on ρ_i). Hence, we must have $\overline{s} = \tau = \tau_1 = \tau_2$, and thus $v_1 = v_2$, or $m_1 = m_2 = m$.

We now show that firms' margins on weak products must moreover satisfy $m_1^B, m_2^A \leq -h(\overline{s})$, and margins on strong products must satisfy $m_1^A, m_2^B \geq h(\overline{s})$. To see this, note that firm 1, say, could induce some multi-stop shoppers to buy its weak product B as well, by reducing the margin on its weak product, so that $\tilde{\tau}_1 = \delta + \tilde{m}_1^B - m_2^B < \tau_1 \left(= \delta + m_1^B - m_2^B\right) = \overline{s}$, keeping the total margin constant: $\tilde{m}_1^A + \tilde{m}_1^B = m_1$. By so doing, firm 1 would earn a profit equal to

$$\begin{aligned}\pi_1 &= \tilde{m}_1^A F(\tilde{\tau}_1) + m_1(F(\overline{s}) - F(\tilde{\tau}_1)) \\ &= m_1 F(\overline{s}) - \tilde{m}_1^B F\left(\delta + \tilde{m}_1^B - m_2^B\right).\end{aligned}$$

To rule out such deviation, m_1^B must satisfy

$$m_1^B \in \arg\max_{\tilde{m}_1^B \leq m_1^B} -\tilde{m}_1^B F\left(\delta + \tilde{m}_1^B - m_2^B\right),$$

which, given the monotonicity of $h(\cdot)$, amounts to

$$m_1^B \leq -h(\overline{s}).$$

Alternatively, firm 1 could discourage some multi-stop shoppers by increasing $\tilde{m}_1^A$, so that $\tilde{\tau}_2 = \delta + m_2^A - \tilde{m}_1^A < \tau_2 \left(= \delta + m_2^A - m_1^A\right) = \overline{s}$, keeping $\tilde{m}_1^B$ unchanged. Doing so yields a profit equal to

$$\pi_1 = \tilde{m}_1^A F(\tilde{\tau}_2).$$

Ruling out this deviation thus requires

$$m_1^A \in \arg\max_{\tilde{m}_1^A \leq m_1^A} \tilde{m}_1^A F\left(\delta + m_2^A - \tilde{m}_1^A\right),$$

or:

$$m_1^A \geq h(\overline{s}).$$

The conditions $m_2^A \leq -h(\overline{s})$ and $m_2^B \geq h(\overline{s})$ can be derived using the same logic.

Therefore, the margins for any candidate equilibria must satisfy (using $\tau = \delta + m_1^B - m_2^B = \overline{s}$): $-h(\overline{s}) \geq m_1^B = m_2^B - \delta + \overline{s} \geq h(\overline{s}) - \delta + \overline{s}$, implying $\overline{s} + 2h(\overline{s}) \leq \delta$. Hence, an equilibrium with multi-stop shopping only exists only when $\overline{s} \leq j^{-1}(\delta)$. Conversely, when this condition holds any margins satisfying m_1^A, $m_2^B \in [h(\overline{s}), \delta - \overline{s} - h(\overline{s})]$, $m_2^A = m_1^A - \delta + \overline{s}$ and $m_1^B = m_2^B - \delta + \overline{s}$ constitute an equilibrium, in which all consumers are multi-stop shoppers.

Claims 7 and 8 together establish the second part of the Proposition. **Q.E.D.**

F Proof of Proposition 5

Suppose that consumers' shopping costs are distributed over $[\underline{s}, +\infty)$, where $\underline{s} < w$. We first show that part of Lemma 1 still applies:

Lemma 2 *Suppose that consumer shopping costs are distributed over $[\underline{s}, +\infty)$, where $\underline{s} < w$. Then, in equilibrium:*

- *(i) Some one-stop shoppers are active.*
- *(ii) $m_1 = m_2 = 0$;*
- *(iii) Active multi-stop shoppers buy strong products.*

Proof. It is straightforward to check that the first three claims of the proof of Lemma 1 remain valid: In equilibrium, some consumers are active (Claim 1); $m_1 = m_2 = 0$ whenever there are active one-stop shoppers (Claim 2), and active multi-stop shoppers buy the strong products (Claim 3). Furthermore, Claim 3 establishes part (iii) of the Lemma, whereas Claim 2 implies that part (ii) follows from part (i). To complete the proof, it thus remains to validate part (i).

To establish this, suppose that all active consumers are multi-stop shoppers, buying strong products from Claim 3. Consumers are willing to visit both firms if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$), but would prefer one-stop shopping if $s > \tau = v_{12} - \max\{v_1, v_2\}$; hence, we must have $\tau \geq v_{12}/2$, and the demand from multi-stop shoppers is $F(v_{12}/2)$. As consumers only buy strong products, firms must charge non-negative margins on these products. Note also that the condition

$$\frac{v_{12}}{2} \leq \tau = v_{12} - \max\{v_1, v_2\}$$

implies $\max\{v_1, v_2\} \leq v_{12}/2$.

Without loss of generality, suppose $m_2^B \geq m_1^A$ (≥ 0), and consider the following deviation for firm 1: Keeping m_1^A constant, change m_1^B to

$$\tilde{m}_1^B = \frac{w - \delta + m_2^B - m_1^A}{2} - \varepsilon \geq \frac{w - \delta}{2} - \varepsilon > 0,$$

so as to *increase* the value offered to one-stop shoppers to

$$\tilde{v}_1 = w - m_1^A - \tilde{m}_1^B = \frac{w + \delta - m_1^A - m_2^B}{2} + \varepsilon = \frac{v_{12}}{2} + \varepsilon.$$

This deviation does not affect v_{12} nor τ_2 (which only depends on the prices for A), but it decreases τ_1 to $\tilde{\tau}_1 = \delta + \tilde{m}_1^B - m_2^B = v_{12}/2 - \varepsilon$; as initially $\tau \geq v_{12}/2$, it follows that the multi-stop shopping threshold becomes $\tilde{\tau} = \tilde{\tau}_1 \,(< v_{12}/2) < \tilde{v}_1$. This adjustment thus induces some of the initial multi-stop shoppers to buy both products from firm 1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $v_{12}/2$), on which firm 1 earns an extra profit from selling its weak product (as $\tilde{m}_1^B > 0$), and it moreover attracts some additional one-stop shoppers (those whose shopping cost lies between $v_{12}/2$ and $\tilde{v}_1$), which generates additional profit (as $m_1^A \geq 0$ and $\tilde{m}_1^B > 0$). ■

We now proceed to show the three results of the proposition. We first note that multi-stop shopping must arise when some consumers have low enough shopping costs:

Lemma 3 *If $\underline{s} < \delta/3$, some multi-stop shoppers are active in equilibrium.*

Proof. Suppose all active consumers are one-stop shoppers. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$. Ruling out multi-stop shopping requires $v = w \geq \hat{v}_{12} - \underline{s} = w - \delta - m_1^B - m_2^A - \underline{s}$, or (using $m_1 = m_2 = 0$) $m_1^A + m_2^B \leq \delta + \underline{s}$. If firm 2, say, is the one that charges less on its strong product (i.e., $m_2^B \leq m_1^A$), then we must have $m_2^B \leq (\delta + \underline{s})/2$. Consider the following deviation for firm 1: charge $\tilde{m}_1^A = \varepsilon > 0$ and $\tilde{m}_1^B = -\varepsilon$ such that the total margin remains zero. The multi-stop shopping threshold becomes

$$\tilde{\tau} = \delta - \tilde{m}_1^A - m_2^B \geq \delta - \varepsilon - \frac{\delta + \underline{s}}{2} = \frac{\delta - \underline{s}}{2} - \varepsilon.$$

As $\delta > 3\underline{s}$ (implying $(\delta - \underline{s})/2 > \underline{s}$), it follows that $\tilde{\tau} > \underline{s}$ for ε sufficiently small. Hence, firm 1 can induce some consumers to engage in multi-stop shopping, and make a profit on them. ■

Next, we show that there exists indeed an equilibrium with multi-stop shopping as long as some consumers have not too large shopping costs:

Lemma 4 *If $\underline{s} < \delta$, there exists an equilibrium exhibiting both types of shopping patterns, in which firms' total margin is zero ($m_i^* = 0$) and their margin on strong products is equal to $\rho_i^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$.*

Proof. Suppose $\underline{s} < \delta$. As seen in the text, the unique candidate equilibrium exhibiting both types of shopping patterns is such that: (i) both firms charge zero total margins ($m_i^* = 0$) and a positive margin on their strong products equal to $\rho_i^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$; and (ii) consumers with shopping cost lies between $\underline{s}$ and τ^* engage in multi-stop shopping and those with shopping cost lies between τ^* and w are one-stop shoppers. Therefore, this type of equilibrium exists when $\underline{s} < \tau^* = j^{-1}(\delta)$. As the function $j(\cdot)$ is strictly increasing and satisfies $j(\underline{s}) = \underline{s} + 2h(\underline{s}) = \underline{s}$, the condition $\underline{s} < \tau^*$ amounts to $\underline{s} < \delta$.

Conversely, these margins constitute indeed an equilibrium. By construction, no firm can make a profit on one-stop shoppers due to fierce price competition, and these margins maximize each firm's profit earned from multi-stop shoppers. ■

It follows that the analysis of the baseline model still applies when the lower bound is small enough, namely, when $\underline{s} < \delta/3$. From Lemmas 2 and 3, both types of shopping patterns must arise in equilibrium; Lemma 4 then ensures that the unique candidate identified in the text is indeed an equilibrium. This establishes the first part of the Proposition.

We now turn to the second part of the Proposition, and first note that multi-stop shopping cannot arise when all consumers have high shopping costs:

Lemma 5 *If $\underline{s} > \delta$, there are no multi-stop shoppers in equilibrium.*

Proof. Suppose to the contrary there are some active multi-stop shoppers. From Lemma 2, $m_1 = m_2 = 0$ and multi-stop shoppers must buy strong products; hence, $\tau = \delta - m_1^A - m_2^B > \underline{s}$. As $\underline{s} > \delta$, it follows that $m_1^A + m_2^B < 0$; hence, at least one firm must charge a negative margin on its strong product and incur a loss from serving multi-stop shoppers. Obviously this cannot be an equilibrium as that firm would then increase its prices to avoid the loss. ■

Finally we show that, when all consumers have large enough shopping costs, there exists equilibria with no multi-stop shoppers.

Lemma 6 *There exist equilibria with one-stop shopping if and only if $\underline{s} \geq \delta/3$. In these equilibria, margins satisfy (i) $m_1^A + m_1^B = m_2^A + m_2^B = 0$, (ii) $\delta - \underline{s} \leq m_1^A, m_2^B, m_1^A + m_2^B \leq \delta + \underline{s}$, and (iii) $-w_1^B \leq m_1^A \leq w_1^A$ and $-w_2^A \leq m_2^B \leq w_2^B$.*

Proof. Consider a candidate equilibrium with only one-stop shopping. From Lemma 2, $m_1 = m_2 = 0$ and thus $\tau = \delta - m_1^A - m_2^B$. Firm 1, say, cannot be profitable to deviate by attracting one-stop shoppers, as this would require a negative total margin $\tilde{m}_1 < 0$. Firm 1 could however deviate so as to induce some consumers to engage in multi-stop shopping; more specifically:

- (i) It could induce some consumers to buy both strong products by charging $\tilde{m}_1^A$ such that $\tilde{\tau}_2 = \delta - \tilde{m}_1^A + m_2^A = \delta - \tilde{m}_1^A - m_2^B > \underline{s}$, or $\tilde{m}_1^A < \delta - \underline{s} - m_2^B$.
- (ii) Alternatively, it could induce some consumers to buy both weak products by charging $\tilde{m}_1^B$ such that $\widetilde{\tau}_2 = -\delta + m_2^B - \tilde{m}_1^B > \underline{s}$, or $\tilde{m}_1^B < m_2^B - \delta - \underline{s}$.

Ruling out the first type of deviation requires $m_2^B \geq \delta - \underline{s}$, while preventing the second type of deviation requires $m_2^B \leq \delta + \underline{s}$. Therefore, the equilibrium margin m_2^B must lie between $\delta - \underline{s}$ and $\delta + \underline{s}$. Applying the same logic to rule out firm 2's deviations requires the equilibrium margin m_1^A to lie between $\delta - \underline{s}$ and $\delta + \underline{s}$ as well. Moreover, the margins cannot exceed the social values, which requires $-w_1^B \leq m_1^A \leq w_1^A$ and $-w_2^A \leq m_2^B \leq w_2^B$.

Conversely, any margins that satisfy (i) $m_1^A + m_1^B = m_2^A + m_2^B = 0$, (ii) $\delta - \underline{s} \leq m_1^A, m_2^B, m_1^A + m_2^B \leq \delta + \underline{s}$, and (iii) $-w_1^B \leq m_1^A \leq w_1^A$ and $-w_2^A \leq m_2^B \leq w_2^B$ constitute an equilibrium, in which all active consumers are one-stop shoppers and both firms earn zero profits.

The above analysis shows that equilibrium margins must satisfy: (i) $\delta - \underline{s} \leq m_1^A, m_2^B$, implying $m_1^A + m_2^B \geq 2\delta - 2\underline{s}$; and (ii) $m_1^A + m_2^B \leq \delta + \underline{s}$. These two conditions then leads to $2\delta - 2\underline{s} \leq \delta + \underline{s}$, which amounts to $\delta/3 \leq \underline{s}$. It thus follows that such equilibrium exists if and only if $\delta/3 \leq \underline{s}$. ■

Combining Lemmas 5, 6 and 2 yields the second part of the Proposition, whereas Lemmas 4 and 6 together yield the last part.

References

Ambrus, Attila, and Jonathan Weinstein (2008), "Price Dispersion and Loss Leaders," *Theoretical Economics,* 3:525-537.

Armstrong, Mark, and John Vickers (2010), "Competitive Non-linear Pricing and Bundling," *Review of Economic Studies,* 76:1-31.

Berg, Sanford V., and Dennis L. Weisman (1992), "A Guide to Cross-Subsidization and Price Predation," *Telecommunication Policy*, August:447-459.

Bolton, Patrick, Joseph Brodley and Michael Riordan (2000), "Predatory Pricing: Strategic Theory and Legal Policy," *Georgetown Law Journal,* 88:2239-2330.

Calvani, Terry (2001), "Predatory Pricing and State Below-Cost Sales Statutes in the United States: An Analysis," *Report for the Competition Bureau, Canada*; the report is available at http://www.bureaudelaconcurrence.gc.ca/eic/site/cb-bc.nsf/eng/01292.html.

Chen, Zhijun, and Patrick Rey (2012), "Loss Leading as an Exploitative Practice," *American Economic Review,*102(7):3462-82.

Competition Commission, UK (2008), "Market Investigation into the Supply of Groceries in the UK," Competition Commission Report available at http://www.competition-commission.org.uk/rep_pub/reports/2008/538grocery.htm.

DeGraba, Patrick (2006), "The loss leader is a turkey: Targeted discounts from multi-product competitors," *International Journal of Industrial Organization,* 24: 613– 628.

Eckert, Andrew, and Douglas S. West (2003), "Testing for Predation by A Multiproduct Retailer," in *The Pros and Cons of Low Prices*, Stockholm: Swedish Competition Authority.

Faulhaber, Gerald R. (1975), "Cross-Subsidization: Pricing in Public Enterprises," *American Economic Review*, 65(5):966-977.

Faulhaber, Gerald R. (2005), "Cross-Subsidy Analysis with More Than Two Services," *Journal of Competition Law and Economics,* 1(3):441-448.

Klemperer, Paul (1992), "Equilibrium Product Lines: Competing Head-to-Head May Be Less Competitive," *American Economic Review*, 82:740-755.

Klemperer, Paul and Jorge Padilla (1997) "Do Firms' Product Lines Include Too Many Varieties?" *RAND Journal of Economics*, 28: 472-488.

Lal, Rajiv, and Carmen Matutes (1994), "Retail Pricing and Advertising Strategies," *Journal of Business,* 67:345-370.

OECD (2007), "Resale Below Cost Laws and Regulations," *OECD Journal of Competition Law and Policy*, 9:169-255.

Rao, P.M., and Joseph A. Klein (1992), "Antitrust issues concerning R&D cross-subsidization: A case study of US telecommunications," *Telecommunication Policy,* July:415-424.