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"Risk and Choice: A Research Saga"

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Abstract

As in any research field, risk theory has its important questions, results, and paradoxes, as well as its seminal papers and key authors. Louis Eeckhoudt has been a key author in the field of risk theory. To celebrate his many contributions and continue the development of theories of decision making under risk, the Toulouse School of Economics hosted "Risk and choice: A conference in honor of Louis Eeckhoudt" July 12-13, 2012. This paper presents some of Eeckhoudt's main contributions to the literature, and provides some illustrations of the remarkable research saga in risk theory over the last 50 years since Pratt's (1964) characterization of risk aversion under expected utility.

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For as long as homo sapiens has existed, people have made decisions without being certain of the consequences. Arguably the first normatively compelling theory of decision making under uncertainty was developed in response to the St. Petersburg paradox, which was proposed just 300 years ago in a letter from Nicolas Bernoulli to Pierre Raymond de Montmort dated September 9, 1713. The St. Petersburg paradox posited a wager with infinite expected value, for which no individual seems prepared to pay more than a modest amount. In its best-known form, the gambler is offered a monetary prize of 2^{n-1} monetary units if a series of coin tosses produces its first "heads" on the n^{th} toss.

Several "solutions" to the paradox were offered in the early 18th century (see Seidl 2013 for an interesting review). These include neglecting outcomes whose probability falls below some threshold or prizes larger than can realistically be paid, concepts which later reappear in the guise of "acceptable risk" (e.g., a mortality risk smaller than one in a million; Starr 1969, Kelley 1991) and "worst-case analysis" (Sunstein 2009). What has been accepted as the standard solution was proposed by Daniel Bernoulli (Nicolas' cousin) in 1738. He proposed that the value of a prize to an individual was an increasing but strictly concave function of the prize. If the degree of concavity is sufficient, the expected utility of the gamble is finite and the certainty equivalent may even be quite modest. For example, an individual whose utility function is logarithmic with an initial wealth of 10 monetary units would not be ready to pay more than 2.88 monetary units to play the St. Petersburg game.

The three centuries following the introduction of the St. Petersburg paradox have provided a wealth of development in theories and applications of decision making under uncertainty, most of it occurring in the lifetime of Louis Eeckhoudt (a correlation that reflects partial causation). Expected utility theory (EUT) was proposed and axiomatized by von Neumann and Morgenstern (1944, the year of Louis' birth). A critical limitation of EUT is that it assumes the probabilities of the alternative consequences are known. While this seems acceptable for classical games of chance such as tossing dice and spinning roulette wheels, it is inappropriate for betting on horse races and other sporting contests, and more significantly for financial investment, insurance, business, health, and public policy decisions. It is often written that Frank Knight (1921) first drew attention to this distinction, calling the first case (where probabilities are "objective") one of "risk" and the second one of "uncertainty." For cases of Knightian uncertainty, EUT was generalized to subjective expected utility by L. James Savage (1954). In the most recent decades, EUT has been further generalized to nonlinearity in probabilities (Quiggin 1982, Tversky and Kahneman 1992) and ambiguity (Gilboa and Schmeidler 1989, Klibanoff et al. 2005).

Bernstein (1996) in his "remarkable story of risk" provides an entertaining summary of these three centuries of research on risk. We focus on the last 50 years, developed mostly under the EUT hypothesis. A key and simple property under EUT is that risk aversion is equivalent to the concavity of the utility function. Another important property is the equivalence between the statistical concepts of first-order and second-order stochastic dominance and, respectively, the positivity of the first and negativity of the second derivative of the utility function. Many other properties associating the form of the derivative(s) of utility function have been derived under EUT, and we will present more of them later.

EUT is not unanimously accepted as a normative model of how people ought to choose and there is much evidence of violation, especially in laboratory experiments. Nevertheless, it remains the most widely used theory under risk and uncertainty. This is especially true outside the field of pure decision theory, for instance in public economics, macroeconomics, game theory and information economics.

It is a good place here to recall that some of the most prominent economists have contributed to the research on risk. To illustrate, let us cite some theoretical works produced by recipients of the Nobel Prize in Economic Science. The list includes: Milton Friedman and Harry Markowitz on the convex-concave utility function explaining both insurance and gambling choices (Friedman and Savage 1948, Markowitz 1952), James Tobin on the preference for liquidity under risk aversion (Tobin 1958), Paul Samuelson on the benefit of diversification under risk aversion (Samuelson 1963, 1967), Kenneth Arrow on the measure of risk aversion and analysis of risk taking (Arrow 1965), Gary Becker on the substitutability of self-insurance and self-protection with market insurance (Ehrlich and Becker 1972), Joseph Stiglitz (Rothshild and Stiglitz 1970) and more recently Robert Aumann (Aumann and Serrano 2007) on measures of risk, Maurice

Allais (Allais 1953) and Daniel Kahneman (Kahneman and Tversky 1979) for their criticism of the EUT paradigm, and the list could go on.

A starting point to summarize the development and the influence of the research under EUT in the last 50 years is the seminal paper by Pratt (1964), "Risk aversion in the small and in the large," published in *Econometrica*. This paper introduces an index of risk aversion "in the small," namely using a second order approximation of the risk premium, and then obtains a result about comparative risk aversion across agents "in the large," namely for all risks. Pratt's paper has more than 5000 citations on GoogleScholar. But it has in reality many more citations, as this paper became so famous that it is often not even cited when Pratt's risk aversion index is used.[1] (Pratt's paper continues to have great influence. In summer 2013, *Econometrica* lists a forthcoming paper entitled "Ambiguity in the small and in the large" (which, interestingly, does not cite Pratt 1964).

Looking back at the last 50 years of research on risk, it is interesting to investigate why some papers received attention and not others. The first paper by Louis (coauthored with Philippe Caperaa) was published in 1975 in the *Journal of Financial Economics*. This paper provides an analysis of risk premiums for delayed risks and is a comment on the paper by Drèze and Modigliani published in 1972 in the *Journal of Economic Theory* (note that Franco Modigliani also received the Nobel Prize in Economic Science). The paper by Drèze and Modigliani provides a thorough analysis of precautionary savings with and without complete markets. Interestingly, a previous version of this paper had already been published in French in 1966 (Drèze and Modigliani 1966), and a first version (in French) was circulating in the late 1950s.[2] Therefore this paper is a precursor to the analysis of precautionary savings, before Leland (1968) and Sandmo (1970), and much before Kimball (1990).[3] However, this paper is almost never cited nowadays.

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[1] The index was developed independently by Arrow (1965) and is often described as the "Arrow-Pratt index."

[2] Another example in which pioneering work was published in French and hence not widely recognized in the profession is Drèze's (1962) development of the standard model for valuing mortality risk, independently developed by Schelling (1968) and Jones-Lee (1974).

[3] The paper by Kimball (1990), entitled "Precautionary savings in the small and in the large", is by far the most cited in the literature on precautionary savings.

A plausible reason for why Drèze and Modigliani (1972)'s paper has not been very influential, despite the fame of its authors, might be because of the use of a nonseparable intertemporal utility function. Indeed most results in Drèze and Modigliani (1972) depend on complex properties of the (cross-)derivatives of this function. At that time, however, it was not clear how to interpret those complex properties and thus how to provide a meaningful economic interpretation of these early technical results on the precautionary savings motive.[4]

Interestingly, the set of papers published by Louis more than 40 years after the French publication of Drèze and Modigliani (Eeckhoudt and Schlesinger 2006, Eeckhoudt et al. 2007, Eeckhoudt et al. 2009) precisely provide such an intuitive interpretation to the (cross-)derivatives of the utility function. Louis has contributed in various ways to the literature on risk and choice, and the rest of this paper presents more of his contributions. This provides in turn other illustrations of the remarkable research saga in risk theory over the last 50 years.

1. Saga 1: Increases in risk and risk taking

In the 1970's and early 1980's, economists were busy trying to make general equilibrium models more realistic by including risk in their models. One of the many relevant questions there was the question of the impact of uncertain output prices on firms' production decisions. Sandmo (1971) conjectured that a marginal increase in uncertainty will decrease output, but concluded that this conjecture cannot be proved categorically. Similarly, while intuition suggests that an increase in risk should reduce the demand for risky assets, this is not true in general. To see that; consider a portfolio choice problem with a safe asset with a zero return and a risky asset. The risky asset has a return of +100%, 0% and -100% respectively with probability 3/5, 1/5 and 1/5. Consider an increase in risk with a new distribution of return (+150%, 3/10; +50%, 7/20; -16.67%, 3/20; -100%, 1/5). A straightforward analysis shows that a risk-averse investor with

[4] Interestingly, Dionne and Eeckhoudt (1984) develops, as Drèze and Modigliani (1972), a comparative statics analysis in a model with two risky decisions and derives results that depend on complex conditions on the utility function. This paper was nevertheless rejected by several economics journals (personal communication) and was published in a technical insurance journal, where it has received very limited attention.

concave utility function $u(c) = \min(c, 3 + 0.3(c-3))$ and an initial wealth $w = 2$ will double her investment in the risky asset (from 1 to 2) due to this large increase in risk on the return of this asset. Thus, in the framework of expected utility with two assets, one safe and one risky, a Rothschild-Stiglitz increase in risk in the return of the risky asset may increase the demand for it by some risk-averse agents. Similarly, a risk-averse policyholder can reduce her demand for insurance when the insurable risk increases, and some risk-averse entrepreneurs will increase their output after an increase in uncertainty on the output price. Let us explore the evolution of this old literature. Rothschild and Stiglitz (1970) define an increase in risk as follows: Lottery Y is riskier than lottery X with the same mean if and only if all risk-averse agents prefer X to Y: $EX = EY$, and for all u increasing and concave,

$$Eu(w+Y) \le Eu(w+X) \qquad (1)$$

Because any concave function can be expressed as a convex combination of min functions $v_\theta(z) = \min(z, \theta)$, inequality (1) holds for all u concave if it holds for all min functions. This observation yields the well-known integral condition for increases in risk. In a companion paper (Rothschild and Stiglitz 1971), the same authors explore another problem. Suppose that agents can decide how many lottery tickets to purchase. When making the lottery riskier from X to Y, should all risk-averse agents reduce the number of tickets they purchase? In other words, can we guarantee that when Y is riskier than X, the following inequality holds for all u increasing and concave,

$$\arg\max_\alpha Eu(w+\alpha Y) \le \arg\max_\alpha Eu(w+\alpha X)? \qquad (2)$$

As said before, the answer is no. Because Rothschild and Stiglitz (1971) found this result counter-intuitive, they looked for restrictions on the set of acceptable utility functions that eliminate the paradox. This line of research has also been explored by Fishburn and Porter (1976), Cheng, Magill and Shafer (1987) and Hadar and Seo (1990). But the outcome of this line of research may be judged disappointing, because its main conclusion is a sufficient condition that is complex and quite stringent. [5]

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[5] Relative risk aversion must be less than 1 and increasing, and absolute risk aversion must be decreasing.

Louis Eeckhoudt and many co-authors were attracted by this puzzle as early as the mid 1970's. Eeckhoudt and Hansen (1980) demonstrated that a mean-preserving price "squeeze" through an increase in the minimum price and a reduction of the maximum price always raises the optimal production, with some interesting applications to the European agricultural policy for example. During the next fifteen years, this initial contribution generated a tsunami of incremental results, most often producing very specific restrictions on the increase in risk (IR), and more generally on the second-order stochastic dominance order, to yield the desired result: strong IR, relatively strong IR, relatively weak IR, portfolio dominance, monotone likelihood ratio order, monotone probability ratio order, and many others. With the exception of the last two restrictions, these results did not retain much attention from the economics profession.

The development of this literature culminated with the publication of the *necessary* and sufficient condition on the change in distribution from X to Y that implies that property (2) holds for all concave utility functions (Gollier 1995). This property can be rewritten as follows:

$$EXu'(w+X)=0 \quad \Rightarrow \quad EYu'(w+Y)\leq 0. \tag{3}$$

In words, if it is optimal to purchase one unit of X, it is optimal to purchase less than one unit of Y. It is tempting to rewrite this condition as follows: For all u increasing and concave,

$$EYu'(w+Y)\leq EXu'(w+X). \tag{4}$$

Because any concave function is a convex combination of min functions, it is easy to simplify this condition in a similar fashion to that for the integral condition for IR. That was done by Rothschild and Stiglitz (1971), who claimed that they obtained the necessary and sufficient condition. But this was a mistake. Indeed, condition (4) is more restrictive than (3), hence so is the Rothschild-Stiglitz condition coming from (4). Indeed, this condition needs to hold only for the utility functions u that yield $EXu'(w+X)=0$.

It is interesting to observe that the seminal paper that launched this literature was fundamentally wrong, and that it took 25 years to detect the mistake and to characterize the right solution. Interestingly enough, Georges Dionne and Louis Eeckhoudt had a paper accepted in the *International Economic Review* in 1992 whose main result was using the wrong Rothschild-Stiglitz (1971) condition. Christian Gollier, who was visiting Dionne at that time, found a counterexample to the main result in the Dionne-Eeckhoudt paper. The team spent an entire month trying to find the error in that paper, before realizing that the problem was in Rothschild and Stiglitz (1971)! This led to a new version of their paper that was eventually published in the same journal by Dionne, Eeckhoudt and Gollier (1993).[6]

Currently, the line of research described above is still used in a few strands of the economics literature. Perhaps most significantly, it is used for topics at the frontier between decision theory and behavioral economics. Indeed, these results are instrumental to understanding the effect on risk taking of pessimism and heterogeneity in beliefs (Abel 2002, Barro 2006), anticipatory feelings (Brunnermeier et al. 2007), and ambiguity aversion (Gollier 2011).

2. Saga 2: Self-protection and risk aversion

Self-protection is another important model of decision under risk. It refers to reduction of the probability of a loss. Ehrlich and Becker (1972), in a well-known paper published in the *Journal of Political Economy*, were the first to develop a rigorous economic analysis of self-protection, which they distinguished from self-insurance (reducing the magnitude of the possible loss), which is a special topic of our first saga. They provided examples of self-protection like investment in a burglar alarm to reduce the probability of theft. Many daily decisions, like reducing speed on the road, wearing a bicycle helmet, quitting smoking, and hiring a good lawyer, which all reduce the probability of a bad outcome, can be viewed as self-protection (many of these decisions also reduce the magnitude of potential loss, and hence incorporate an element of self-insurance). Also, many societal choices about risk management (e.g., nuclear safety investments, chemical regulation) have a dimension of self-protection. The issue of self-protection is fundamental in tort law (Kaplow and Shavell 2002), as in the concepts of contributory negligence and comparative negligence.

[6] This also led to the production of Gollier (1995) which played a key role in the recruitment of that author by the Toulouse School of Economics.

Interestingly, self-protection is at the origin of the important concept of moral hazard. Indeed moral hazard is typically illustrated by a situation where an agent reduces self-protection effort after purchasing an insurance contract (Arrow 1963). Nevertheless, it is interesting to note that Ehrlich and Becker (1972) proved that moral hazard is not an inevitable consequence of market insurance, as self-protection and market insurance can act as complements rather than substitutes.

Economic models of self-protection examine the ex-ante trade-off between the benefit of a reduction in the probability of a bad outcome and the cost of a sure reduction in the utility in *all states*. Self-protection is one of the fundamental schemes to mitigate risk and is presented as such in risk theory textbooks (e.g., Eeckhoudt et al. 2005, Handbook of Insurance 2013). Following Pratt (1964), one expects that more risk aversion leads to a decrease in risk taking. One would thus expect that (more) risk aversion increases self-protection as well. But, is that really the case?

This question was first formally studied in a note published in *Economics Letters* by Dionne and Eeckhoudt (1985). They consider the following simple model of self-protection. An agent can exert a self-protection effort x that reduces the probability $p(x)$ of loss L (but does not affect the size of the loss L if it occurs). The probability of loss is a decreasing function of the level of self-protection, i.e. $p'(x) < 0$ for all $x \geq 0$. In this case, the agent's expected utility is:

$$p(x)u(w-c(x)-L)+\left[1-p(x)\right]u(w-c(x)). \qquad (5)$$

A fundamental observation from (5) is that when the self-protection effort x increases it also decreases the final wealth in the state where the loss occurs, i.e. $w-c(x)-L$. This is not good news for a risk averse agent, and this helps explain why Dionne and Eeckhoudt conclude from this simple model that "contrary to intuition, self-protection is not monotonically related to risk aversion" (Dionne and Eeckhoudt 1985, page 39). They provide a simple example with a quadratic utility function showing that self-protection is increased due to risk aversion if and only if the probability of loss is smaller than ½.

This early comparative statics result was viewed as a paradox in risk theory and needed more explanation. The saga could then begin. Here follow a sample of results in the economic literature on self-protection, among many others (see Courbage et al. 2013 for a recent overview). Briys and Schlesinger (1990) show that more self-protection neither leads to a mean-preserving contraction nor a mean-preserving spread of final wealth distribution. Briys *et al*. (1991) examine whether the conditions under which the paradox holds are robust to the case where the effectiveness of self-protection is uncertain. Sweeney and Beard (1992) show that the effect of an initial wealth increase depends on both the probability of loss and the characteristics of the agent's absolute risk aversion. Lee (1998) shows that the relationship between self-protection and risk aversion depends on precise conditions on the probability function. Jullien et al. (1999) prove that self-protection increases with risk aversion if and only if the initial probability is less than a "utility-dependent" threshold. Dachraoui et al. (2004) use the concept of mixed risk aversion (MRA), introduced by Caballé and Pomansky (1996), and show that if an agent is more risk averse in the sense of MRA than another then this agent will select a higher level of self-protection and have a higher WTP than the other individual when the loss probability is smaller than ½.

The production of this set of technical results during the two decades after the publication of the Dionne and Eeckhoudt (1985) paradox illustrates the high activity in the field of risk theory. However, it also illustrates that it is difficult to relate self-protection and risk preferences in an interpretable fashion. An important step forward was taken by Eeckhoudt and Gollier (2005) who show how prudence enters into the analysis of self-protection. Their main result is that if a risk neutral agent selects a self-protection effort that yields a probability of loss equal to ½ then *all* prudent agents select a lower level of effort, whereas all imprudent agents select a greater level of effort than the risk neutral agent. The intuition for this result is based on the idea of downside risk aversion (Menezes et al. 1980, Chiu 2005, Dionne and Li 2011) which can be illustrated through the simple lotteries displayed in Figure 1.

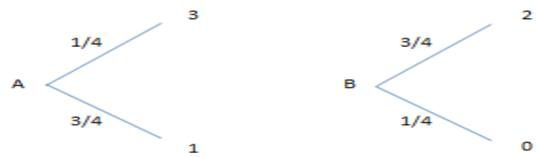

Figure 1: Mao's lotteries

Louis named the lotteries A and B in Figure 1 "Mao's lotteries," as they were introduced by Mao (1970). Observe that going from lottery A to lottery B reflects the basic trade-off induced by a self-protection choice, as it increases the probability of the good state (from ¼ to ¾) but decreases wealth in both states by one unit. Importantly, observe that the mean and the variance of lotteries A and B are equal. That is, only the downside risk is changed (Menezes et al. 1980).[7] That implies the following equivalence. Consider an initial lottery yielding either 2 or 1 with equal probability. Suppose that an agent prefers the lottery A over lottery B. This is equivalent to saying that this agent prefers that a background risk yielding 1 or -1 with equal probability is added contingent to the good outcome of the initial lottery (i.e., 2) rather than added contingent to the bad outcome of the initial lottery (i.e., 1). In other words, an agent who prefers lottery A to B, and who thus dislikes the self-protection choice, is also averse to a downside risk. This explains why prudence, or equivalently downside risk aversion, is instrumental in signing the effect of risk aversion on self-protection.

The result of Eeckhoudt and Gollier (2005) thus provides a simple benchmark where the intuitive idea that risk aversion should increase self-protection is always mistaken, and also provides a simple explanation based on prudence for why this is not the case. Note that we are essentially saying here that "prudence" and "self-protection" go in opposite direction, which seems disturbing as it goes against the intuition provided

[7] Neither of the two lotteries dominates the other in the sense of Rothschild and Stiglitz (1970).

by common language. The ambiguity dissipates a bit if one realizes that self-protection is very much like purchasing a raffle ticket, which increases the chances of winning the raffle (i.e., reduces the probability of the bad outcome, which is not winning) at a specific financial cost. If one views self-protection as gambling in a raffle, it should not be a surprise that it goes in the opposite direction to risk aversion and prudence.

The reader may also have noticed that virtually all results in the literature on self-protection (including that of Eeckhoudt and Gollier 2005) turn out to depend on a threshold probability of loss set at the value of ½. There is a simple intuition for that. Indeed the probability of ½ is the critical threshold below which an increase in the probability increases the variance (which is proportional to $p(x) \times (1 - p(x))$) of the binary distribution of final wealth. This explains why this threshold probability condition appears systematically for "if and only if" results; indeed it is in fact necessary for small risks, where the variance of the risk becomes essentially all that matters for the comparative statics analysis of risk aversion. This illustrates that the early example of Dionne and Eeckhoudt (1985) obtained with a simple quadratic utility function captures an important insight that has carried over in all the subsequent literature.

We must observe at this stage that most common examples of self-protection, like reducing speed on the road, involve a health risk, not (only) a financial risk. However, all the literature cited above has considered only financial risks. Eeckhoudt and Hammitt (2001) consider a health risk and examine the effects of an independent health risk or a financial risk on the value of self-protection; Eeckhoudt and Hammitt (2004) define conditions under which aversion to a financial risk increases self-protection effort.

Also, we observe that in many situations the effort of self-protection is not contemporaneous to its effect on the probability. Consider the example of the purchase of a burglar alarm, or that of the decision to stop smoking. In both examples, self-protection precedes its effect on risk reduction. Menegatti (2009) emphasizes this point and considers a two-period model where the self-protection effort occurs in the first period while its effect occurs in the second period.Interestingly, he finds that the effect of prudence is exactly opposite to that in Eeckhoudt and Gollier (2005). Indeed, self-protection becomes very similar to precautionary savings in Menegatti (2009)'s two-period model, and thus increases with prudence. We note however that the notion of "more risk aversion" is not well defined in a two-period model, or in a model with state-dependent preferences, as it may change other aspects of preferences beyond pure attitude toward risk. This issue is not fully solved in the literature; see Kihlstrom and Mirman (1974) and Karni (1983) for early approaches. Relatedly, Meyer and Meyer (2011) generalize some previous results about self-protection by considering a more general notion of increased risk aversion.

We note finally that these results on the relationship between self-protection and risk preferences have been generalized in various directions. For instance, Jewitt (1989), Shogren and Crocker (1991), Jullien et al. (1999), and Athey (2002) examine the effect of risk preferences on effort for general (not only binary) distributions which include self-protection and self-insurance as special cases. Konrad and Skaperdas (1993) as well as Bleichrodt and Eeckhoudt (2006) generalize the analysis of self-protection to non-expected utility models. Finally, recent research efforts have examined the effect of ambiguity aversion on self-protection (Treich 2010, Snow 2011, and Alary et al. 2013). These papers consider a model of ambiguity aversion à la Klibanoff et al. (2005) and show that the effect of ambiguity aversion depends on how ambiguity affects the probability of loss, and need not reinforce that of risk aversion.

3. Saga 3: Higher (and lower) order derivatives of utility

Even as the degree (and sign) of risk aversion remains the most important property of a utility function, recent years have seen a rapid growth in understanding the importance and implications of derivatives of the utility function other than the second. As described in Saga 2, the third derivative of the utility function was identified as important for self-protection by Eeckhoudt and Gollier (2005). It had previously been associated with the concepts of downside risk aversion (Menezes et al. 1980) and precautionary savings (Kimball 1990). Indeed, it was Kimball (1990) who coined the term "prudence" and associated it with a positive third derivative of the utility function.[8]

[8] Kimball (1990, p. 54) defines prudence as the "propensity to prepare and forearm oneself in the face of uncertainty, in contrast to 'risk aversion', which is how much one dislikes uncertainty."

Recognition of the importance of the fourth derivative grew out of the search for conditions under which an agent who would not accept either of two independent risks alone could be guaranteed to not accept them together. Common intuition suggests that being subjected to one risk should decrease a risk-averse decision maker's willingness to assume another (independent) risk. However, this result requires much stronger conditions.

Pratt (1964) showed that a decision maker who is more risk-averse than another is willing to pay more to eliminate a risk, i.e., he has a larger risk premium. However, it does not follow that the more risk-averse decision maker is willing to pay more for a partial reduction in risk (i.e., to exchange the initial risk for a mean-preserving contraction). Intuitively, the more risk-averse decision maker has larger risk premia for both the larger and the smaller risks; that does not imply that the difference between his risk premia is larger than the corresponding difference for the other decision maker. Ross (1981) provided a necessary condition: that the smallest value (over the possible wealth values that may be realized) of the Arrow-Pratt index for the more risk-averse decision maker is larger than the largest value of the index over that interval for the other decision maker.

During the 1980's another puzzle attracted attention in the economics profession, namely the "equity premium puzzle". Indeed, Mehra and Prescott (1985) argued that sensible levels of risk aversion can hardly explain the low demand for risky assets (note that this paper has almost 5000 citations on Google Scholar, and that Edward Prescott is another recipient of the Nobel Prize in Economic Science). A possible explanation may be that investors face in reality many other sources of risks beyond financial risks. Yet, if these other sources of risk are not diversifiable, they may lead investors to behave as if they are more risk averse, hence the apparent puzzle. But is that true theoretically?

A number of authors havesearched for theoretical conditions under which imposing an undiversifiable "background risk" increases risk aversion. Pratt and Zeckhauser (1987) defined "proper risk aversion" as the condition that an undesirable risk can never be made desirable by the imposition of an independent, undesirable risk (an undesirable risk is one the decision maker would reject if it were offered alone). Kimball (1993) defined "standard risk aversion" as the condition that an undesirable risk can never be made desirable by the imposition of an independent loss-aggravating risk (a loss-aggravating risk increases the decision maker's expected marginal utility; it makes a sure loss more undesirable and is itself more undesirable when a sure loss is imposed). Gollier and Pratt (1996) defined "risk vulnerability" as the condition that an undesirable risk can never be made desirable by the imposition of an independent, unfair risk (i.e., a risk with non-positive expected value).[9]

The three concepts are related: standardness implies properness which implies risk vulnerability. Necessary conditions involve both the third and fourth derivatives of the utility function. Building from Pratt's (1964) approximation that the risk premium for a small lottery is proportional to the Arrow-Pratt index of risk aversion, $r(w) = -\frac{u''(w)}{u'(w)}$, Gollier and Pratt (1987) show that the coefficient of local prudence, $\rho(w) = -\frac{u'''(w)}{u''(w)}$, measures how much the risk premium for a small risk increases in the presence of a sure loss and the coefficient of local temperance[10], $t(w) = -\frac{u''''(w)}{u'''(w)}$, measures how much the risk premium increases in the presence of a small zero-mean risk. Risk vulnerability is characterized by the condition that local prudence and local temperance both exceed local risk aversion, i.e, $\rho \geq r$ and $t \geq r$. A necessary condition for proper risk aversion is that $r'' \geq r'r$, or $t \geq r\left[2 - \left(r/\rho\right)\right]$. Standard risk aversion is characterized by decreasing absolute risk aversion and decreasing absolute prudence ($r' \leq 0$ and $\rho' \leq 0$) or $t \geq \rho \geq r$.

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[9] A follow-on to this work is Eeckhoudt et al. (1996), also published in *Econometrica*, which examines the conditions under which a risk deterioration (in the sense of first and second order stochastic dominance) increases risk aversion. In fact, Louis noticed in the late 1980's a dissertation in progress at Harvard by Miles Kimball, and showed to his two coauthors how this could be applied to insurance and risky asset demand with background risks (personal communication).

[10] Kimball (1992) coined the term "temperance" to denote a tendency toward moderation in accepting risk. At the Toulouse conference, he also proposed that utility functions having a negative sixth derivative be called "bent."

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While risk aversion addresses the question of whether an individual will accept a risk (i.e., how risk affects utility), Kimball (1990) addressed the question of how risk affects optimization of a continuous variable (precautionary savings), and hence how risk affects marginal utility. Analogously, while the concepts of proper risk aversion, standard risk aversion, and risk vulnerability address the question of whether background risk can make an individual more tolerant of a new risk, Lajeri-Chaherli (2004) investigated how precautionary savings are affected by the presence of not one but two independent future risks. She introduced the concepts of proper prudence, standard prudence, and precautionary vulnerability and showed how reactivity of behavior to multiple risks is influenced by the fifth derivative of the utility function. She defined a measure of local absolute "edginess," $e(w) = -\frac{u'''''(w)}{u''''(w)}$, and showed its value in characterizing these conditions; e.g., standard prudence is equivalent to $e \geq t \geq \rho$.

Following this explosion of results turning on the signs and magnitudes of higher-order derivatives (see Gollier 2001 for many other results), Eeckhoudt and Schlesinger (2006) developed an integrated approach to understanding the effects of successive derivatives. They adopt an old concept, the utility premium, discussed by Friedman and Savage (1948) but not often used after Pratt's (1964) results for the risk premium. The utility premium measures the loss in expected utility (or "pain") associated with a change in risk. Eeckhoudt and Schlesinger show that the pattern of alternating signs ($u' > 0$, $u'' < 0$, $u''' > 0$, $u'''' < 0$, $u''''' > 0$) associated with monotonicity, risk aversion, prudence, temperance, edginess, etc. can be characterized as a preference for "combining good with bad" or for disaggregating harms in a series of binary lotteries in which the two outcomes are equally likely. In this framework, prudence ($u''' > 0$) is equivalent to a preference for assigning a zero-mean risk to the better outcome in a binary lottery between two wealth levels, and temperance ($u'''' < 0$) is equivalent to a preference for assigning a zero-mean risk to the better outcome in a binary lottery between a certain wealth level and that wealth level plus an independent zero-mean risk.

Eeckhoudt and Schlesinger (2006) propose the terminology "risk apportionment of order n" and show that concept is equivalent to the condition that the sign

$u^{(n)} = (-1)^{n+1}$. They observe that a utility function with alternating signs for its first through nth derivatives is consistent with nth order stochastic dominance (see Ekern 1980 for an early analysis).

In related work, Denuit and Rey (2010) show that the alternating sign pattern corresponds to the intuition that sensitivity to adverse conditions and to probability spreads should decrease with wealth. Eeckhoudt et al. (2007) extend the approach to multivariate utility and show how the signs of various cross-derivatives are associated with a preference for locating risks where circumstances are otherwise most favorable.

While the standard measures of higher-order properties are of the form $-\frac{u^{(n)}}{u^{(n-1)}}$ (i.e., the inverse of the ratio of the nth to the (n – 1)th derivative, the utility-premium approach suggests an alternative measure, the ratio of the nth to the 1st derivative. This ratio can be interpreted as the effect of the nth derivative on utility normalized to a measure of wealth by dividing, as usual, by the marginal utility of wealth (u'). Modica and Scarsini (2005) and Crainich and Eeckhoudt (2008) use $\frac{u'''}{u'}$ as a measure of downside risk aversion (see Keenan and Snow 2010 for an alternative measure). Note that the Arrow-Pratt measure of risk aversion can be interpreted in the same way: u'' measures the expected utility loss which is divided by the marginal utility of income to convert it to a monetary measure, which is proportional to the risk premium (Pratt 1964).

Going in the other direction, the “fear of ruin” coefficient $\left[u(w) - u(0)\right]/u'(w)$ and its reciprocal “boldness” were introduced by Aumann and Kurz (1977).[11] Fear of ruin measures a decision maker’s willingness to risk his fortune for a marginal gain, or equivalently his willingness to pay to marginally reduce the chance of losing all his wealth. Fear of ruin appears in the conventional model for the marginal rate of substitution between wealth and mortality risk (i.e., the value per statistical life; Drèze 1962, Jones-Lee 1974). When $u(0)$ equals the utility of death and the marginal utility of wealth in the state of death is zero (i.e., there is no bequest motive), the value per

[11] Aumann and Kurz (1977) credit Kenneth Arrow for the name.

statistical life equals fear of ruin divided by the survival probability. Foncel and Treich (2005) provide a systematic analysis of fear of ruin and describe its application to value per statistical life, first-price auctionsand comparisons with the Arrow-Pratt and other measures of risk aversion (e.g., asymptotic risk aversion; Jones-Lee 1980).

4. The saga continues

These three research sagas illustrate the evolution of knowledge in the field of risk theory. Obviously, research is ongoing, and we briefly give now two examples of recent developments. We presented Mao's lotteries in Figure 1. Interestingly, Mao (1970) presented these lotteries to a sample of professional risk managers, who largely displayed an aversion to downside risk. This may be seen as an early experiment on risk preferences. A few papers have recently followed these ideas and provide experimental evidence on higher order risk preferences (Deck and Schlesinger 2010, Ebert and Wiesen 2013). Another application concerns game theory. Usually, models of game theory assume risk neutrality. But it has been well known that risk aversion plays a role in some games, as in auctions for instance (Milgrom and Weber 1982, Harrison 1990). Some recent research has studied the implications of risk aversion and higher order derivatives of the utility function in other games like public goods (Bramoullé and Treich 2009), bargaining (White 2008), and rent-seeking games (Treich 2010).

As in other fields, current research ideas are developed and shared through presentation at conferences, such as the annual seminar of the European Group of Risk and Insurance Economists (EGRIE). As the field has grown, so has this meeting. We have been told that in the first EGRIE meetings in the early 1970s, the practice of English was not so common for many participants and some researchers had to provide translation during the presentations.[12] Despite these modest beginnings, the EGRIE association has substantially developed. In September 2013, the EGRIE is having its 40th conference in Paris and will gather several contributors that we have cited above (including Miles Kimball who provides the keynote lecture this year) together with a set of young researchers working in the area of risk theory. Louis Eeckhoudt was present in

[12] Moreover, the supply of discussants was so limited that some researchers, like Karl Borch and Henri Loubergé, had to discuss a handful of papers by themselves.

the early EGRIE conferences and still actively participates every year (in particular through his collaboration with younger researchers), and he has been instrumental in the development of the EGRIE association.

To celebrate the contributions of Louis and continue the development of theories of decision making under risk, the Toulouse School of Economics hosted a conference "Risk and choice: A conference in honor of Louis Eeckhoudt" July 12-13, 2012. A total of 23 papers were presented, covering the measurement of risk and risk attitudes, intertemporal optimization, and applications to health and insurance. Authors were invited to submit their papers for publication in this special issue. After a rigorous selection process, including two anonymous reviewers for each submitted paper, we the guest editors selected four papers for publication in this special issue.

The papers published in this issue contribute to the research sagas we described above. The paper by Jouini, Napp and Nocetti studies the effect of change in (background) risk on risk taking, while that of Liu and Meyer extends the Ross characterization of risk aversion. Noussair, Trautmann, Van de Kuilen and Vellekoop provide some experimental results on risk preferences and their links with variables like religiosity. Finally, the paper by Drèze and Schokkaert studies moral hazard in health insurance.

We mentioned in the Introduction the first publication of Louis in 1975. As we write this (in summer 2013), his most recently published paper "Even (mixed) risk lovers are prudent" appears in the *American Economic Review* (Crainich et al. 2013). This paper studies the behavior of risk lovers. Note however that risk lovers have been forgotten in modern risk analysis since the attention has focused almost exclusively on risk averters, as illustrated by the three research sagas described above. This was not the case in the early papers on risk, as for instance Tobin (1958) devotes a specific treatment of risk lovers, or "plungers" as he coins them, and Friedman and Savage (1948) suggested that typical individuals are risk averse over low and high wealth levels and risk seeking over intermediate wealth levels. This reminds us that there is often something to gain in going back to the old papers in a field and having a fresh look at an "old" idea. The utility premium of Friedman and Savage (1948) is another good example. This last observation illustrates perfectly the role that Louis Eeckhoudt has played "in the small" circle of coauthors and friends who participated in the recent Toulouse conference as well as "in the large" community of researchers on risk: a curious, non-conventional researcher raising important and difficult questions, and offering simple and elegant answers to many of them.

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