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"On the strategic value of risk management»

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On the strategic value of risk management[1]

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Abstract

This article examines how firms facing volatile input prices and holding some degree of market power in their product market link their risk management and their production or pricing strategies. This issue is relevant in many industries ranging from manufacturing to energy retailing, where risk averse firms decide on their hedging strategies *before* their product market strategies. We find that hedging modifies the pricing and production strategies of firms. This strategic effect is channelled through the risk-adjusted expected cost, i.e., the expected marginal cost under the probability measure induced by shareholders' risk aversion. It has opposite effects depending on the nature of product market competition: hedging toughens quantity competition while it softens price competition. Finally, if firms can decide not to commit on their hedging position, this can never be an equilibrium outcome: committing is always a best response to non committing. In the Hotelling model, committing is a dominant strategy for all firms.

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1 Introduction

Most formal analyses of corporate risk management decisions consider price-taking firms that face volatile cash flows. For example, small firms producing commodities or raw materials (e.g., metals and minerals, oil and gas, electric power) face output price volatility. They can use derivatives contracts to hedge against fluctuations of the output prices. This standard, "non-strategic" risk management logic also applies to firms facing input price volatility, provided they do not exert market power in either their input or product markets.

However, when firms facing input price volatility have some degree of market power in their product market, their strategies become more elaborate. A firm's hedging modifies its realized input cost, hence its product market strategy. Thus, the firm alters the competitive dynamics in its industry, and must take into account the behavior of its competitors.

This situation occurs in many industries. For example, electricity retailers purchase power on wholesale markets and resell it to their retail customers. In Britain, the electricity and gas regulatory agency (Ofgem (2008), page 10) indicates that: "there is evidence that the (6 largest suppliers) seek to benchmark their hedging strategies against each other in order to minimize the risk of their wholesale costs diverging materially from the competition". Suppliers thus appear to include their competitors' hedging strategy in their own hedging strategy, and ultimately their product market strategy.

Airlines also constitute a relevant example. Carter et al. (2006) report that, over the period $1992-2003$, fuel price represented more than 13% of airlines operating costs, and exhibited annualized volatility of 27%. Airlines do not exert market power in the fuel market, yet they are an oligopoly on specific routes (see for example Gerardi and Shapiro (2009)).

The food processing industry provides another example. Food processing firms are exposed to volatile feedstock prices (e.g., grains, tobacco). They may not exert market power in the feedstock markets, however most empirical studies document market power in their product markets: see for example the survey by Sheldon and Sperling (2001).

As the examples above suggest, the interaction between hedging and product market strategies is relevant for multiple industries. Yet, the academic literature on the strategic aspects of hedging is small, for, in most cases, a separation (or dichotomy) property exists: hedging is found to have no impact on product market strategy (see for example the references in Dionne and Santugini (2013)). This article is among the few that explicitly establishes a link between hedging and product market strategy when firms compete in quantity (Cournot) and in price (differentiated Bertrand).

We focus the analysis on risk-averse firms that hedge before deciding their product market strategies. The empirical relevance of this choice is justified in Section 3. Formally, we use two-stage games: firms first determine their hedging strategy, then determine their product market strategy (quantity or price), conditional on their first-stage choice.

We first analyze quantity competition. We prove that the first-order conditions characterizing the equilibrium of the (second-stage) production game are similar to the standard Cournot case, except that risk-adjusted expected costs replace marginal costs. The intuition is that investors value a marginal cost increase using the probability measure induced by their marginal utility of wealth in each state of the world, and not the physical probability measure. This risk-adjusted expected marginal cost is determined in equilibrium, and is decreasing in own hedging. Thus, a firm that increases its hedging becomes more aggressive (Lemma 2).

An equilibrium of the production game always exists. If firms' absolute risk aversion is constant (or does not vary too much), this equilibrium is unique, and an increase in own hedging reduces the other firm's equilibrium output (Proposition 2). If a symmetric equilibrium of the (first-stage) hedging game exists, hedging *toughens* quantity competition: firms hedge more than their (anticipated) equilibrium production, thus committing to produce more than if their costs were constant and equal to the expected cost under the physical probability measure (Proposition 3).

We establish similar results for price competition, although we reach the opposite conclusion: hedging *softens* price competition. As with quantity competition, risk-adjusted expected marginal costs, determined in equilibrium, replace constant marginal costs in the first-order conditions characterizing the equilibrium of the pricing game. Since risk-adjusted expected costs are decreasing in own hedging, a firm that increases its hedging becomes more aggressive (Lemma 3).

As with quantity competition, an equilibrium of the pricing game always exists. If absolute risk aversion is constant, the equilibrium is unique, and an increase in own hedging reduces the other firm's equilibrium price. The crucial difference with quantity competition is that hedging *softens* pricing competition: firms hedge less than their (anticipated) equilibrium production, thus committing to a price higher than if their cost was constant and equal to the expected cost under the physical probability measure (Proposition 5).

Finally, we examine the strategic incentives to commit to a hedging position (Proposition 6). The above results are derived under the assumption that Boards of Directors impose that firms commit to their hedging position. This is usually meant to limit speculation by traders. Ignoring that objective, does commitment arise in equilibrium?

We show that, whether firms compete in quantity or in price, committing is a firm's best response to the other not committing. Thus universal non commitment never arises as an equilibrium. Furthermore, in the particular case of the Hotelling model (where firms compete in price and total demand is inelastic) commitment is a dominant strategies for all firms.

This article is structured as follows: Section 2 discusses the links with the literature. Section 3 presents the model. Section 4 analyzes quantity competition. Section 5 analyzes price competition. Section 6 examines incentives to commit to hedging decisions. Technical proofs are presented in the Appendix.

2 Literature Review

As mentioned previously, only a few articles examine the interaction between hedging and product market strategies. They are reviewed below.

Dionne and Santugini (2013) examine a two-stage game related to ours. In the first stage, firms decide to enter the market (or not). In the second stage, risk averse firms facing a volatile input price compete à la Cournot in their output market, and simultaneously determine their hedging strategy. As they solve the game backwards, Dionne and Santugini (2013) find that both hedging and product market strategies depend on the number of firms in the market. The latter is then determined in equilibrium, and is a function of the volatility of input price, the level of forward prices, and firms' risk aversion. There are two main differences with our model: we consider a mature market where the number of active firms is fixed, while Dionne and Santugini (2013) look at markets where new firms can enter. The second difference is that in our model firms commit ex ante to their hedging strategies (more on this in the next section) while in Dionne and Santugini (2013) hedging and quantities are jointly determined: firms use hedging as a strategic commitment device in our model, while they use market entry in Dionne and Santugini (2013).

Allaz and Villa (1993) look at firms that are large enough to exert market power on both the spot and the forward markets. There is no uncertainty. In the spot market (stage 2), a firm that has already sold a share of its output faces lower incentives to withhold output. In the forward market (stage 1), firms face a prisoner dilemma, and cannot resist selling output forward. Thus the existence of forward contracts reduces firms' market power. This result is very similar to our Proposition 6, even though the setting is different: firms in Allaz and Villa sell output in spot and forward markets where they exert market power, while in ours, firms exert no market power in the spot and forward markets for input.

Adam, Dasgupta, and Titman (2007) examine two-period games in the presence of financial constraints: firms' hedging decision in the first-period affects their investment capacity, hence their profitability in the second period. They show that asymmetric equilibria arise: in equilibrium, some firms hedge, while others do not. In their model, the presence of financial constraints and the resulting potential underinvestment is the conduit for strategic interaction. Similarly, Loss (2012) examines the interaction between hedging demand and the characteristics of investment opportunities in the presence of financial constraints. Using a reduced form for profit functions, he finds that a firm's hedging demand is high when investments are strategic substitutes, and low when they are strategic complements. In this article, by contrast, prices and outputs are endogenized together with hedging positions. Bodnar, Dumas, and Marston (2002) consider a duopoly with asymmetric exposure to an exchange rate, and determine the optimal pass-through and related exposure. While the problem is related to the one examined here, the analytical approach is very different: they treat the exchange rate as a fixed input price, not as a stochastic variable. Recently, Nocke and Thanassoulis (forthcoming) examine how credit constraints, making firms endogenously risk averse, impact vertical relationships in the supply chain. They find that, in the short run, the optimal supply contract involves risk sharing and double marginalization.

This article also is methodologically related to the literature on the strategic impact of firms' financial structure[1], that was initiated by Brander and Lewis (1986). Brander and Lewis (1986) examine a two stage game. In the first stage, firms determine their debt level. In the second stage, they compete à la Cournot, facing uncertainty on their profitability. Brander and Lewis (1986) find that, in the states of the world where marginal profitability is higher, an increase in debt increases the equilibrium output, which echoes our Proposition 2. They also find that the equilibrium level of debt is excessive from the industry's point of view, which echoes our Proposition 6.

Our analytical approach, namely the use of two-stage games, is identical to Brander and Lewis (1986). However, we examine a different decision by firms: hedging and not capital structure. Specifically, we show that firms can use risk management as a strategic instrument, as powerful as debt.

[1] A review of this literature can be found in Tirole (2006), Chapter 7.

3 Quantity Competition: the Model

Consider two identical firms, indexed by $i = 1, 2$, competing à la Cournot. Firm i produces q_i units of output, total production is $Q = q_1 + q_2$, and inverse demand $P(Q)$. The technology is linear: each unit of input, that costs $\tilde{c}$, is transformed into one unit of output. Firm i's commercial profit is thus:

$$\pi^C(q_i, q_j; \tilde{c}) = q_i (P(Q) - \tilde{c}).$$

3.1 Uncertainty on input costs and risk management

Ex ante, the input cost $\tilde{c}$ is a random variable, with a cumulative distribution function $G(.)$ on a bounded support. Firms can hedge some of their risk on input cost $\tilde{c}$ by buying forward contracts at (unit) price F. To eliminate speculative motives for hedging, we assume that $F = \mathbb{E}[\tilde{c}]$. There are no transaction costs associated with hedging. Thus, at $t = 2$ (i.e., once the input cost $\tilde{c}$ is known), the profit function of firm i that has purchased forward quantity H_i at price F is:

$$\begin{aligned}\pi_i &= \pi(q_i, q_j, H_i; \tilde{c}) = q_i (P(Q) - \tilde{c}) + H_i (\tilde{c} - F) \\ &= q_i (P(Q) - F) - (q_i - H_i)(\tilde{c} - F).\end{aligned}$$

The first expression for firm $i\prime s$ profit corresponds to adding to its commercial profit the net gain (or loss) on its forward position H_i. The second expression shows an equivalent decomposition: firm i can consider that its production cost is F, and its net (unhedged) exposure to input price fluctuations $(q_i - H_i)$.

Firms do not have market power in the spot and forward markets for input, even though they do exert market power in their product market. We also assume that firms choose their outputs (or prices) *before* input costs are realized. This is the case for most manufacturing industries[2]: producers (e.g., car manufacturers) commit to a product price or volume for

[2] This assumption does not apply to industries where output price is flexible. For example delivery services

the relevant period (typically one or two quarters). During that period, input prices (e.g., aluminum and steel prices) vary. This assumption also holds for the electricity supply industry. For example in the United Kingdom, electricity retail rates typically change only 3 or 4 times a year, while wholesale power prices vary continuously.

Two other assumptions are crucial for risk management to have any strategic value. First, each firm must be able to publicly commit to its hedging decision. Second, firms must be "risk averse", in the sense that shareholder value can be measured by the expectation of some concave function of profit. We now motivate these assumptions.

3.2 Public commitment to the hedging decision

Except for Section 6, we assume throughout this article that firms publicly commit to their hedging decisions (H_1, H_2) before they select their output (or prices). This assumption can be justified as follows.

First, financial regulations require firms to publish, in their quarterly statements, a description of their portfolio of forward purchases and sales. While some discretion still exists in disclosure, an outside party can get a close picture of a firm's hedging portfolio. For example, Jin and Jorion (2007) were able to compute the delta-equivalent of the forward portfolio for US oil and gas companies. Also, as previously mentioned, electricity suppliers in Britain infer each other's hedging portfolio from financial statements and other public information.

Second, industrial firms can – and in practice do – commit to a hedging strategy through their risk management policy. Forward sales and purchases, that require the use of derivatives, are usually handled with extreme caution by Board of Directors, concerned about

(e.g., Fedex and UPS) explicitly include in their published rate a fuel surcharge schedule, that depends on the price of an oil index. Similarly, electricity suppliers in Norway offer retail contracts explicitly adjusted to the wholesale power price.

When firms set production after the input price is realized, the profit from the hedge is known before the production decision is made, thus has no impact on it. Firms cannot do any better than standard deterministic profit maximization. Knowing that, when firms make the hedging decision, they follow the "standard" non-strategic risk management logic.

potential speculative behavior by traders. Boards then require management to define and follow a clear hedging strategy. As mentioned earlier, this position is communicated to investors and regulators. Management has then limited discretion to deviate from it.

Without commitment on hedging, no strategic interaction would arise and there would be a dichotomy between hedging and production. Suppose indeed that firms select output, then hedge. Reasoning backwards, consider first the hedging decision, once production is known. Since (i) firms are risk averse, and (ii) there are no transaction costs nor expected gain from hedging (i.e., $\mathbb{E}[\tilde{c}] - F = 0$), full hedging is the optimal strategy. Consider now the production decision. Knowing that input costs will be perfectly covered at the forward price, firms play a symmetric Cournot game with constant marginal costs equal to F. The same reasoning holds for price competition.

Thus, this article is focussed on situations where risk-averse firms hedge *before* making their production (or price) decision.

3.3 Objective function

To obtain a strategic impact of risk management, a crucial ingredient is that firms be risk-averse. Thus, we assume that each firm i maximizes some expected utility $U(\pi_i)$ of its profits π_i, where $U(.)$ is increasing, concave and exhibits non-increasing absolute risk aversion $\rho(\pi) \equiv \frac{-U''}{U'}(\pi)$. This is a natural assumption when the firm is owned by a small number of shareholders who cannot fully diversify. This is also a convenient reduced form for widely held firms when there are financial frictions[3]. These financial frictions can take the form of a wedge between the costs of external and internal finance (Froot, Scharfstein, and Stein (1993)), transaction costs on primary security markets (Décamps et al. (2011)), or agency costs (DeMarzo and Sannikov (2006), Biais et al. (2007)). In each of these cases, shareholder

[3]If financial markets were complete and frictionless, the Modigliani and Miller theorem would apply. In particular risk management would not create any value for shareholders.

value can be represented as the expectation of a concave function[4] of future profits. For simplicity, we consider a symmetric model where $U(.)$ is the same for all firms. At date $t = 0$, the shareholder value of firm i is then

$$v_i = v(q_i, q_j, H_i) = \mathbb{E}\left[U\left(\pi\left(q_i, q_j, H_i; \tilde{c}\right)\right)\right] = \mathbb{E}\left[U\left(q_i\left(P(Q) - F\right) + \left(H_i - q_i\right)\left(\tilde{c} - F\right)\right)\right].$$

We look for subgame perfect equilibria of the two-stage game played by firms. We use backward induction: we first determine the second-stage equilibrium $(q^*(H_1, H_2), q^*(H_2, H_1))$, and then compute the first-stage payoff functions $V_i = V(H_i, H_j) \equiv v_i(q^*(H_i, H_j), q^*(H_j, H_i), H_i)$.

4 Strategic use of hedging

4.1 An illustrative example

Before turning to the general Cournot model, we illustrate the main insights with a simple example: (i) Constant Absolute Risk Aversion ($CARA$), i.e. $U(x) = 1 - \exp(-\rho x)$, (ii) linear inverse demand $P(Q) = 1 - Q$, and (iii) normally distributed[5] input cost $\tilde{c}$, with mean F and standard deviation σ. In this case,

$$v(q_i, q_j, H_i) = \mathbb{E}\left[1 - \exp\left(-\rho\left(q_i\left(P(Q) - F\right) + \left(H_i - q_i\right)\left(\tilde{c} - F\right)\right)\right)\right] = 1 - \exp\left(-\rho m\left(q_i, q_j, H_i\right)\right)$$

where

$$m_i = m(q_i, q_j, H_i) = q_i(1 - Q - F) - \frac{\rho\left(H_i - q_i\right)^2 \sigma^2}{2}$$

is the certainty equivalent of firm $i's$ profit. Maximizing v_i is equivalent to maximizing m_i. Now

$$\frac{\partial m_i}{\partial q_i} = 1 - 2q_i - q_j - F + \rho\sigma^2\left(H_i - q_i\right).$$

First note that $\frac{\partial m_i}{\partial q_i}$ is decreasing in q_i (which means that m_i is concave in q_i) and in q_j

[4] We take U as exogenous. A fully dynamic model, with explicit modelling of financial frictions, would allow to endogenize U. This is done by Rochet and Villeneuve (2011) in a non-strategic context. Extending it to a strategic context would be difficult and is outside the scope of the present paper.

[5] Assuming normal distributions has the usual drawback that costs can take arbitrarily large (or negative) values, and thus equilibrium prices or quantities can be negative, which of course is meaningless from the economic viewpoint. However, for reasonable values of the parameters, the probability of negative prices and quantities is essentially zero, and our equilibria are essentially identical to the fully correct ones that would have been obtained with appropriately truncated normal distributions.

(which means that quantities are strategic substitutes: firm i produces less if its competitor produces more). Note again that if hedging H_i was determined simultaneously with output q_i, firms would choose complete hedging $H_i = q_i$ and we would be back to the Cournot model with deterministic cost F. Thus our assumption that hedging is determined *before* output is decided is crucial for risk management to play any strategic role.

The first-order necessary and sufficient conditions characterizing the equilibrium are

$$q_i\left(2+\rho\sigma^2\right)+q_j = 1-F+\rho\sigma^2 H_i \quad (i=1,2), \tag{1}$$

which yields the unique equilibrium of the second-stage game:

$$q_i^* = q^*\left(H_i, H_j\right) = \frac{1-F+\frac{\rho\sigma^2}{1+\rho\sigma^2}[\left(2+\rho\sigma^2\right)H_i - H_j]}{3+\rho\sigma^2} \quad (i=1,2).$$

Thus $V\left(H_i, H_j\right) = 1-\exp\left(-\rho M\left(H_i, H_j\right)\right)$, where

$$M_i = M\left(H_i, H_j\right) = m\left(q^*\left(H_i, H_j\right), q^*\left(H_j, H_i\right), H_i\right),$$

is the certainty equivalent of firm $i's$ profit in the production game (stage 1). Now[6],

$$\frac{\partial M_i}{\partial H_i} = \frac{\partial m_i}{\partial q_i}\frac{\partial q_i^*}{\partial H_i} + \frac{\partial m_i}{\partial q_j}\frac{\partial q_j^*}{\partial H_i} + \frac{\partial m_i}{\partial H_i} = \left(q_i^* P'\left(Q\right)\frac{\partial q_j^*}{\partial H_i} - \left(H_i - q_i^*\right)\rho\sigma^2\right)$$
$$= -\rho\sigma^2[H_i - \left(1+\frac{1}{\left(3+\rho\sigma^2\right)\left(1+\rho\sigma^2\right)}\right)q_i^*].$$

The necessary first-order conditions of the hedging game (stage 1) are then:

$$H_i^* = \left(1+\frac{1}{\left(3+\rho\sigma^2\right)\left(1+\rho\sigma^2\right)}\right)q_i^* \quad (i=1,2).$$

Since M is concave in its first argument, these conditions are also sufficient. Replacing q_i^*

[6]Note that $\frac{\partial m_i}{\partial q_i} = 0$ by the equilibrium condition of the second-stage game.

by $q^*(H_i, H_j)$ and solving for the first stage equilibrium yields a unique solution, which is symmetric ($q_i^* = q_j^* = q^*$, $H_i^* = H_j^* = H^*$), characterized by

$$H^* = q^*(1 + \frac{1}{(3+\rho\sigma^2)(1+\rho\sigma^2)}) > q^*, \text{ and } q^* = \frac{1-F}{3 - \frac{\rho\sigma^2}{(3+\rho\sigma^2)(1+\rho\sigma^2)}} > \frac{1-F}{3}.$$

Note that $\frac{1-F}{3}$ would be the equilibrium output of each firm in the absence of commitment on hedging. These results are summarized in the next proposition.

Proposition 1 *Assume that firms compete in quantities, have a constant absolute risk aversion, demand is linear, and costs are normally distributed. Then the two-stage game has a unique subgame perfect equilibrium, which is symmetric. Firms over hedge and produce more than in the absence of commitment on hedging.*

Thus, with our simple specification, committing ex-ante on their hedging positions makes Cournot competitors more aggressive. If hedging was done after (or together with) output decisions, shareholders would be collectively better-off: prices and profits would be higher and firms would be perfectly hedged against input price fluctuations[7]. As we now see, these features also hold under more general conditions on utility, demand, and cost distribution.

4.2 The general case

In order to extend the analysis to a more general specification, we first need to guarantee the existence and unicity of a Cournot equilibrium in the second stage of the game, for any couple of hedging strategies (H_i, H_j) of the first stage. For this we impose classical conditions that give existence and uniqueness of a Cournot equilibrium when firms have different costs (see for example Vives (2001)).

[7] However, we show in Section 6 that it is never privately optimal for the shareholders of each of the firms not to commit on their hedging positions. This is a prisoners' dilemma type of situation.

Assumption 1 *For all $Q \geq 0$, the inverse demand function $P(.)$ satisfies*

$$\frac{QP''(Q)}{(-P'(Q))} < 1 \tag{2}$$

and

$$\lim_{Q\to\infty} A(Q) = 0 \text{ and } \lim_{Q\to 0} A(Q) = \lim_{Q\to 0} P(Q) = +\infty$$

where

$$A(Q) = 2P(Q) + QP'(Q).$$

Under Assumption 1 the Cournot game with deterministic (but different) costs (c_i, c_j) has natural properties that we will use in the sequel. In this game, firm i's profit function is:

$$\pi^C(q_i, q_j; c_i) = q_i(P(Q) - c_i).$$

Thus:

$$\frac{\partial \pi_i^C}{\partial q_i} = (P(Q) - c_i) + q_i P'(Q) = 0, \tag{3}$$

and

$$\frac{\partial^2 \pi_i^C}{(\partial q_i)^2} = 2P'(Q) + q_i P''(Q).$$

Lemma 1 *Assumption 1 implies that, for all (c_1, c_2) there exists a unique Cournot equilibrium $\left(q^C(c_1, c_2), q^C(c_2, c_1)\right)$. When this equilibrium is interior $\left(q_i^C = q^C(c_i, c_j) > 0 \text{ for } i = 1, 2\right)$, it satisfies $\frac{\partial q_i^C}{\partial c_i} < 0$ and $\frac{\partial q_j^C}{\partial c_i} > 0$ for $i = 1, 2$.*

Proof. *The result is standard. For the reader's convenience, the proof is presented in Appendix A.1.* ■

Thus, an increase in firm i/s marginal cost induces a reduction in its Cournot equilibrium output q_i^C and an increase in the equilibrium output of its competitor q_j^C. This property will be crucial for our results.

We now return to the random cost case. The shareholder value of firm i is

$$v_i = \mathbb{E}\left[U\left(\pi_i\right)\right] = \mathbb{E}\left[U\left(q_i\left(P\left(Q\right) - F\right) + \left(H_i - q_i\right)\left(\tilde{c} - F\right)\right)\right],$$

and thus

$$\begin{aligned}\frac{\partial v_i}{\partial q_i} =& \mathbb{E}\left[U'\left(\pi_i\right)\frac{\partial \pi_i}{\partial q_i}\right] = \mathbb{E}\left[U'\left(\pi_i\right)\left(P\left(Q\right) - \tilde{c} + q_i P'\left(Q\right)\right)\right] \\ =& \mathbb{E}\left[U'\left(\pi_i\right)\right]\left(P\left(Q\right) + q_i P'\left(Q\right) - \widehat{c}\left(q_i, q_j, H_i\right)\right),\end{aligned}$$

where

$$\widehat{c} \equiv \widehat{c}(q_i, q_j, H_i) = \frac{\mathbb{E}\left[U'\left(\pi\left(q_i, q_j, H_i; \tilde{c}\right)\right)\tilde{c}\right]}{\mathbb{E}\left[U'\left(\pi\left(q_i, q_j, H_i; \tilde{c}\right)\right)\right]} = F + \frac{cov\left[U'\left(\pi\left(q_i, q_j, H_i; \tilde{c}\right)\right), \tilde{c}\right]}{\mathbb{E}\left[U'\left(\pi\left(q_i, q_j, H_i; \tilde{c}\right)\right)\right]} \tag{4}$$

is the risk-adjusted expected cost[8] of firm i. Note that $v\left(q_i, q_j, H_i\right)$ is concave in q_i since

$$\frac{\partial^2 v_i}{\left(\partial q_i\right)^2} = \mathbb{E}\left[U''\left(\pi_i\right)\left(\frac{\partial \pi_i}{\partial q_i}\right)^2 + U'\left(\pi_i\right)\frac{\partial^2 \pi_i}{\left(\partial q_i\right)^2}\right] < 0.$$

Therefore, if an interior Nash equilibrium $\left(q_i^*, q_j^*\right)$ exists, it is determined by the first order conditions

$$P\left(Q^*\right) + q_i^* P'\left(Q^*\right) - \widehat{c}\left(q_i^*, q_j^*, H_i\right) = 0, (i = 1, 2). \tag{5}$$

Before proving existence of a Nash equilibrium and deriving sufficient conditions for unicity, we examine system (5). The interaction between hedging and production is channelled through the expected risk-adjusted cost $\widehat{c}\left(q_i, q_j, H_i\right)$, determined in equilibrium. If firm i produces one more unit at random cost $\tilde{c}$, the impact on shareholder value is $\mathbb{E}\left[U'\left(\pi_i\right)\tilde{c}\right]$. More generally, the marginal certainty equivalent of a random cash flow $x\left(\tilde{c}\right)$ for the shareholders

[8] The notion of risk-adjusted expectation is borrowed from mathematical finance, where it is used in particular to price derivative contracts.

of firm i is given by its risk-adjusted expectation:

$$\widehat{\mathbb{E}}_i\left[x\left(\tilde{c}\right)\right]=\frac{\mathbb{E}\left[U'\left(\pi\left(q_i,q_j,H_i;\tilde{c}\right)\right)x\left(\tilde{c}\right)\right]}{\mathbb{E}\left[U'\left(\pi\left(q_i,q_j,H_i;\tilde{c}\right)\right)\right]}.$$

For example in the case of constant absolute risk aversion and normal distribution of costs, the risk adjusted expected cost of firm i does not depend on its competitor output. It is just equal to the expected cost F plus a risk premium that increases linearly with unhedged output[9] $(q_i - H_i)$:

$$\widehat{c}(q_i,H_i)\equiv\frac{\mathbb{E}\left[U'\left(\pi_i\right)\tilde{c}\right]}{\mathbb{E}\left[U'\left(\pi_i\right)\right]}=F+\rho\sigma^2\left(q_i-H_i\right)$$

It is increasing in q_i, decreasing in H_i, and lower than the expected cost F if and only if $H_i > q_i$. When firms' absolute risk aversion $\rho(\pi)=\frac{-U''(\pi)}{U'(\pi)}$ is not constant, $\widehat{c_i}$ also depends on the output of the other firm, which complicates the analysis[10]: $\widehat{c_i}\equiv\widehat{c}(q_i,q_j,H_i)$. However the properties derived above are robust:

Lemma 2 *:*

$$\frac{\partial\widehat{c}}{\partial q_i}\left(q_i^*,q_j^*,H_i\right)\geq 0,\ \ and\ \frac{\partial\widehat{c_i}}{\partial H_i}<0. \tag{6}$$

$$\widehat{c_i}\leq F\Leftrightarrow H_i\geq q_i.$$

$$\frac{\partial\widehat{c_i}}{\partial q_j}=-q_iP'\left(Q\right)\widehat{cov}_i\left[\rho\left(\pi_i\right),\tilde{c}\right], \tag{7}$$

thus

$$\begin{cases}\frac{\partial\widehat{c}_i}{\partial q_j}=0 & \text{if } \rho\left(\pi\right) \text{ is constant}\\ \frac{\partial\widehat{c}_i}{\partial q_j}<0\Leftrightarrow H_i>q_i & \text{if } \rho\left(\pi\right) \text{ is decreasing}\end{cases}. \tag{8}$$

Proof. *See Appendix A.2.* ■

[9] This can be seen by computing $v_i=\mathbb{E}[U(\pi_i)]=U(m_i)$ in two different ways:
$\frac{\partial U_i}{\partial H_i}=\mathbb{E}[U'(\pi_i)\frac{\partial\pi_i}{\partial H_i}]=\mathbb{E}[U'(\pi_i)(\widetilde{c}-F)]=\mathbb{E}[U'(\pi_i)][\widehat{c}_i-F]$ and
$\frac{\partial U_i}{\partial H_i}=U'(m_i)\frac{\partial m_i}{\partial H_i}=\mathbb{E}[U'(\pi_i)]\rho\sigma^2(q_i-H_i)$.

[10] Dependency of $\widehat{c_i}$ with respect to q_j is indirect. It is channelled through variations in absolute risk aversion.

At equilibrium, expected risk-adjusted marginal cost increases in q_i as if there were decreasing returns to scale. Increasing H_i reduces firm $i's$ risk-adjusted expected marginal cost. This comes from the fact that a marginal increase in hedging decreases ex-post marginal cost when $\tilde{c}_i > F$ and increases it when $\tilde{c}_i < F$. Since favorable realizations are weighted by a lower marginal utility, overall the risk-adjusted expected cost decreases.

We now turn to existence and unicity of the equilibrium of the production game. Since we ultimately focus on symmetric equilibria (where $H_1^* = H_2^* = H$, $q_1^* = q_2^* = q^*$, $\widehat{c}_1 = \widehat{c}_2$), we restrict our attention to the case where $|H_1 - H_2|$ is small enough that the equilibrium of the production game is interior.

(q_1^*, q_2^*) is thus a fixed point of the function Φ, defined from $\mathbb{R}^2$ into $\mathbb{R}^2$ by

$$\begin{cases} \Phi_1(q_1, q_2) = q^C\left(\widehat{c}(q_1, q_2, H_1), \widehat{c}(q_2, q_1, H_2)\right) \\ \Phi_2(q_1, q_2) = q^C\left(\widehat{c}(q_2, q_1, H_2), \widehat{c}(q_1, q_2, H_1)\right) \end{cases}.$$

Proposition 2 *For any (H_1, H_2) close enough to the diagonal, an equilibrium of the production game exists. If absolute risk aversion is constant, this equilibrium is unique, and denoted*[11] *by $q_i^* = q^*(H_i, H_j)$ for $i = 1, 2$. Furthermore, a marginal increase in firm i's hedging reduces firm j's equilibrium output: $\frac{\partial q_j^*}{\partial H_i} < 0$.*

Proof. *For existence, we apply Brouwer's fixed point theorem to the mapping Φ. Since by assumption $\widetilde{c}$ is bounded above by some constant $\overline{c}$,*

$$\widehat{c}(q_i, q_j, H_i) = \frac{\mathbb{E}\left[U'\left(\pi\left(q_i, q_j, H_i; \tilde{c}\right)\right)\tilde{c}\right]}{\mathbb{E}\left[U'\left(\pi\left(q_i, q_j, H_i; \tilde{c}\right)\right)\right]} \le \overline{c}$$

for all (q_i, q_j, H_i). Now, since $\frac{\partial q^C}{\partial c_j}(c_i, c_j) \ge 0$ and $\frac{\partial q^C}{\partial c_i}(c_i, c_j) \le 0$:

$$q^C\left(\widehat{c}(q_i, q_j, H_i), \widehat{c}(q_j, q_i, H_j)\right) \le q^C(0, \overline{c}) \equiv \overline{q}^C.$$

Thus, we can limit our search to $(q_i, q_j) \in \left[0, \overline{q}^C\right]^2$. Since $q^C(x, y)$ and $\widehat{c}(x, y, .)$ are contin-

[11] Unicity of the equilibrium and symmetry of the game imply symmetry of the equilibrium.

uous in (x, y), and defined on a compact and convex set of $\mathbb{R}^2$, Brouwer's theorem implies the existence of an equilibrium.

If absolute risk aversion is constant, we prove in Appendix A.3 that the real parts of the eigenvalues of the Jacobian matrix

$$J\left(q_1^*, q_2^*, H_1, H_2\right) = \begin{bmatrix} \frac{\partial q_1^C}{\partial q_1} - 1 & \frac{\partial q_1^C}{\partial q_2} \\ \frac{\partial q_2^C}{\partial q_1} & \frac{\partial q_2^C}{\partial q_2} - 1 \end{bmatrix}$$

are always negative, which implies that the equilibrium is unique. Finally, constant absolute risk aversion also implies $\frac{\partial \widehat{c}}{\partial q_j}\left(q_i^*, q_j^*, H_i\right) = 0$ by Lemma 2. Firms play a familiar Cournot game with marginal costs $\widehat{c}_i$ increasing in q_i at the equilibrium, and decreasing in H_i. We prove in Appendix A.4 that, since increasing hedging reduces a firm's cost, it makes it more aggressive, and reduces its competitor's output. ■

Proposition 2 shows that a marginal increase in H_i reduces q_j^*, and thus increases q_i^* , since quantities are strategic substitutes. Another way to see this is that a marginal increase in H_i reduces the risk-adjusted expected cost, because it decreases the unhedged input $(q_i - H_i)$, and thus increases q_i^*. Thus a marginal increase in H_i commits firm i to a higher output.

If risk aversion varies with profit, a "revenue effect" arises. Equilibrium of the quantity game may not always be unique. However, this effect is of second-order importance if risk aversion does not vary too much, and our basic conclusions remain valid[12].

4.3 Equilibrium of the hedging game

Suppose a symmetric interior equilibrium of the hedging game (H^*, H^*) exists. Then we have:

Proposition 3 *1. H^* is characterized by the first-order condition:*

$$\widehat{c}\left(q^*, q^*, H^*\right) - F + P'\left(Q^*\right) q^* \frac{\partial q_j^*}{\partial H_i}\left(H^*, H^*\right) = 0. \tag{9}$$

[12] Proofs are available from the authors upon request.

2. *If absolute risk aversion is constant, hedging toughens quantity competition: firms over-hedge ($H^* > q^*$), and produce more than if they did not commit to their hedging position: $q^* > q^C(F,F)$.*

Proof.

1. *For $i = 1, 2$,the first-order conditions characterizing equilibrium hedging volumes H_i^* are:*

$$\mathbb{E}\left[U'(\pi_i^*)\left(\frac{\partial \pi_i}{\partial H_i} + \frac{\partial \pi_i}{\partial q_j}\frac{\partial q_j^*}{\partial H_i} + \frac{\partial \pi_i}{\partial q_i}\frac{\partial q_i^*}{\partial H_i}\right)\right] = 0.$$

Since $\frac{\partial \pi_i}{\partial H_i} = \tilde{c} - F$, $\frac{\partial \pi_i}{\partial q_j} = P'(Q)\, q_i$, and $\frac{\partial \pi}{\partial q_i}\left(q_i^*, q_j^*, H_i\right) = 0$ by definition of q_i^*, this condition becomes

$$\mathbb{E}[U'(\pi_i^*)\left(\tilde{c} - F + P'(Q^*)\, q_i^* \frac{\partial q_j^*}{\partial H_i}\right)] = 0.$$

Dividing by $E\left[U'(\pi_i^*)\right] > 0$ and setting $H_i^* = H_j^* = H^*$ yields equation (9).

2. *Since $\frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0$, equation (9) yields $\widehat{c}(q^*, q^*, H) < F \Leftrightarrow H^* > q^*$.Thus, at a symmetric equilibrium, equation (5) yields*

$$P(2q^*) + q^* P'(2q^*) = \widehat{c}(q^*, q^*, H) < F = P\left(2q^C(F,F)\right) + q^C(F,F)\, P'\left(2q^C(F,F)\right).$$

Assumption 2 implies that $P(2q) + qP'(2q)$ is decreasing: $\left[P(2q) + qP'(2q)\right]' = 3P'(2q) + qP''(2q) = [-P'][\frac{qP''}{-P'} - 3] < 0$. Thus the above condition is equivalent to $q^* > q^C(F,F)$.

■

A marginal increase in H_i has two effects on firm i's expected utility. First, a direct effect on expected cost : the firm substitutes input at known cost F for input at uncertain cost $\tilde{c}$. When taking the risk-adjusted expectation, this substitution is worth $\widehat{c}(q_i, q_j, H_i) - F$. Second, an indirect effect, through the change in the other firm's production: $P'(Q^*)\, q_i^* \frac{\partial q_j^*}{\partial H_i}$.

At equilibrium, both effects exactly cancel out for both firms, which produces equilibrium conditions (9).

Thus, since $\frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0$ and $P'(Q^*)\, q_i^* < 0$, firms set $\widehat{c}\left(q_i^*, q_j^*, H_i^*\right) < F$. They over hedge, i.e., hedge more than their (anticipated) production, so that their risk-adjusted expected marginal cost is lower than their "physical" expected marginal cost $\mathbb{E}[\tilde{c}] = F$. This leads them to become more aggressive, and produce more than if they were perfectly hedged.

Finally, combining first-order conditions (5) and (9) yields:

$$P(Q^*) + q^* P'(Q^*) = F - P'(Q^*)\, q^* \frac{\partial q_j^*}{\partial H_i}(H^*, H^*), \quad (i = 1, 2).$$

Comparing with first-order condition (3) for $c_i = F$, an additional term $\left(-P'(Q^*)\, q^* \frac{\partial q_j^*}{\partial H_i} < 0\right)$ is added, that captures the strategic impact of firm i's hedging on firm j's production decision. Since this term is negative, each firm will be "tougher" in the second-stage. In the subgame perfect equilibrium, firms "invest" too much in hedging, in order to be tough in the quantity game[13].

5 Price competition

We now turn to price competition. As in the previous Section, we analyze the subgame perfect equilibria of a two-stage game. In the first stage, firms choose their hedging positions (H_i, H_j). In the second stage, they compete in prices. We will show that the strategic impact of hedging on price competition is exactly opposite to the one it has on quantity competition. Since the method of resolution is similar in both cases, results are stated briefly. We only emphasize the differences with quantity competition. As in the previous Section, we start by illustrating the basic properties on a simple case.

[13] This is reminiscent of the "top dog effect" in the taxonomy of Fudenberg and Tirole (1984).

5.1 An illustrative example

Consider a standard Hotelling model, where the demand faced by firm i is $D_i = D(p_i, p_j) = \frac{1}{2} + \frac{p_j - p_i}{2t}$. The profit functions are

$$\begin{aligned}\pi_i &= \left(\frac{1}{2} + \frac{p_j - p_i}{2t}\right)(p_i - \tilde{c}) + H_i(\tilde{c} - F) \\ &= \left(\frac{1}{2} + \frac{p_j - p_i}{2t}\right)(p_i - F) + (H_i - \frac{1}{2} + \frac{p_j - p_i}{2t})(\tilde{c} - F).\end{aligned}$$

Shareholder value of firm i is $v_i = v(p_i, p_j, H_i) = \mathbb{E}[U(\pi_i)]$. If absolute risk aversion ρ is constant and costs are normally distributed with mean F and standard deviation σ, the certainty equivalent of firm i's profit is

$$m_i = (\frac{1}{2} + \frac{p_j - p_i}{2t})(p_i - F) - \frac{\rho\sigma^2}{2}(H_i - \frac{1}{2} - \frac{p_j - p_i}{2t})^2.$$

Note again that if hedging was selected simultaneously with price, each firm would hedge completely and we would be back to the Hotelling model with deterministic costs F. The situation is different here since firms commit on their hedging positions before competing in prices. We have:

$$\frac{\partial m_i}{\partial p_i} = \frac{1}{2} + \frac{p_j - p_i}{2t} + \frac{F - p_i}{2t} - \frac{\rho\sigma^2}{2t}(H_i - \frac{1}{2} - \frac{p_j - p_i}{2t}).$$

$\frac{\partial m_i}{\partial p_i}$ is decreasing in p_i (indicating that m_i is concave in p_i) and increasing in p_j (indicating that prices are strategic complements: firm i charges a higher price if firm j does). Introducing the notation $\frac{\rho\sigma^2}{2t} = \varepsilon$, the first order condition characterizing the price equilibrium is

$$p_i = F + t + \varepsilon t[1 - 2H_i] - (1 + \varepsilon)(p_j - p_i), (i = 1, 2).$$

Taking the difference between these two equations and simplifying, we obtain

$$p_i - p_j = \frac{2\varepsilon t}{3+2\varepsilon}(H_j - H_i),$$

and thus

$$p_i^* \equiv p^*(H_i, H_j) = F + t + \varepsilon t[1 - 2H_i - \frac{2(1+\varepsilon)}{3+2\varepsilon}(H_j - H_i)], (i = 1, 2).$$

To compute the certainty equivalent M_i of firm i's profit in the hedging game, we replace p_i^* by its expression in the formula giving m_i. We obtain:

$$M_i = t\left(\frac{1}{2} - \frac{\varepsilon}{3+2\varepsilon}(H_j - H_i)\right)\left(1 + \varepsilon\left(1 - 2H_i - \frac{2(1+\varepsilon)}{3+2\varepsilon}(H_j - H_i)\right)\right) - \varepsilon t\left(H_i - \frac{1}{2} + \frac{\varepsilon}{3+2\varepsilon}(H_j - H_i)\right)$$

M_i is quadratic in (H_i, H_j) and strictly concave in H_i. Thus, there exists a unique Nash equilibrium of the first-stage game, characterized by the first order conditions. It is easy to see that this equilibrium is symmetric: $H_i^* = H_j^* = H^*$, where H^*satisfies: $1 - 2H^* = \frac{1+\varepsilon}{3+2\varepsilon} > 0$.

Compared with the case where firms would hedge perfectly, equilibrium price and shareholder value are higher:

$$p_i^* = p_j^* = F + t + \varepsilon t\frac{1+\varepsilon}{3+2\varepsilon} > F + t.$$

$$M_i^* = M_j^* = \frac{t}{2}[1 + \frac{\varepsilon(5 + 8\varepsilon + 3\varepsilon^2)}{2(3+2\varepsilon)^2}] > \frac{t}{2}.$$

These results are summarized in the next proposition:

Proposition 4 *In the Hotelling model with normally distributed costs and constant absolute risk aversion, there is a unique subgame perfect equilibrium. It is symmetric ($H_i^* = H_j^* = H^*$, $p_i^* = p_j^* = p^*$). Firms under hedge ($H^* < \frac{1}{2}$), charge a higher price ($p^* > F + t$), and have higher shareholder value than if they did not commit on hedging.*

Thus in the normal-Constant Absolute Risk Aversion case, commitment on risk management allows Hotelling firms to compete less aggressively and secure higher margins. As we now see, these features also hold more generally for differentiated Bertrand competition.

5.2 The general model

Consider the general case of two symmetric firms that compete in prices. Firm i faces demand $D_i = D(p_i, p_j)$, decreasing in its own price and increasing in the other firm's price. As in the Cournot case, we make assumptions that ensure that the second stage game has a unique interior equilibrium. These assumptions bear on the asymmetric Bertrand game where firms have different costs c_i and c_j, and firm's i profit function is

$$\pi_i^B = \pi^B(p_i, p_j, c_i) = D(p_i, p_j)(p_i - c_i).$$

Assumption 2 *: $D(p_i, p_j)$ is such that:*

(i) π^B is concave in its first argument: $\frac{\partial^2 \pi^B}{(\partial p_i)^2} < 0$ for all (p_i, p_j, c_i),

(ii) for all (c_i, c_j) close enough to the diagonal[14], the pricing game has a unique interior equilibrium $\left(p^B(c_i, c_j), p^B(c_j, c_i)\right)$

(iii) prices are strategic complements: $\frac{\partial^2 \pi^B}{\partial p_i \partial p_j} > 0$,and

(iv) the own price effect on demand is stronger than the other firm's price effect: $\frac{\partial D_i}{\partial p_i} + \frac{\partial D_i}{\partial p_j} \leq 0$ and $\frac{\partial^2 D_i}{(\partial p_i)^2} + \frac{\partial^2 D_i}{\partial p_i \partial p_j} \leq 0$ for all (p_i, p_j).

Assumption 2 is met for example in the Hotelling model considered above. In this case equilibrium prices are: $p^B(c_i, c_j) = t + \frac{2c_i + c_j}{2}$.

Concavity of the objective function and strategic complementarity of prices are met by many demand functions. Unicity of equilibrium with deterministic input costs is required to establish unicity with stochastic input costs. We prove in Appendix B.1 that when the own price effect is stronger than the other's price effect, an increase in one firm's cost increases both prices: $\frac{\partial p^B}{\partial c_i}(c_i, c_j) > 0$ and $\frac{\partial p^B}{\partial c_i}(c_j, c_i) > 0$.

[14] As in the Cournot case, $|c_1 - c_2|$ must be small enough to avoid a corner equilibrium.

5.3 Strategic hedging

As before, we assume that the shareholder value of each firm equals the expected utility of its profit:

$$v_i = v(p_i, p_j, H_i) = \mathbb{E}[U(\pi_i)] = \mathbb{E}[U\{D(p_i, p_j)(p_i - \widetilde{c}) + H_i(\widetilde{c} - F)\}].$$

Thus

$$\frac{\partial v_i}{\partial p_i} = \mathbb{E}\left[U'(\pi_i)\frac{\partial \pi_i}{\partial p_i}\right] = \mathbb{E}\left[U'(\pi_i)\cdot\left(D_i + (p_i - \tilde{c})\frac{\partial D_i}{\partial p_i}\right)\right] = \mathbb{E}[U'(\pi_i)]\left(D_i + (p_i - \widehat{c}_i)\frac{\partial D_i}{\partial p_i}\right),$$

where

$$\widehat{c}_i = \widehat{c}(p_i, p_j, H_i) \equiv \frac{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c}))\,\tilde{c}]}{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c}))]} = F + \frac{cov[U'(\pi), \tilde{c}]}{\mathbb{E}[U'(\pi)]}$$

is the risk-adjusted expected cost of firm i. We prove in Appendix B.2 that this risk-adjusted expected cost has similar (but not identical) properties to the Cournot case:

Lemma 3 *:*

$$\frac{\partial \widehat{c}}{\partial p_i}\left(p_i^*, p_j^*, H_i\right) \leq 0, \; \textit{and} \; \frac{\partial \widehat{c}_i}{\partial H_i} < 0.$$

$$\widehat{c}_i \leq F \Leftrightarrow H_i \geq D(p_i, p_j).$$

$$\frac{\partial \widehat{c}_i}{\partial p_j} = \frac{\partial D_i}{\partial p_j}\left\{(p_i - \widehat{c}_i)\,\widehat{\mathbb{E}}[\rho(\pi_i)(\widehat{c}_i - \tilde{c})] + \widehat{\mathbb{E}}\left[\rho(\pi_i)(\widehat{c}_i - \tilde{c})^2\right]\right\}.$$

Since $\frac{\partial^2 v_i}{\partial p_i^2} = \mathbb{E}\left[U''(\pi_i)\left(\frac{\partial \pi_i}{\partial p_i}\right)^2 + U'(\pi_i)\frac{\partial^2 \pi_i}{\partial p_i^2}\right] < 0$, $v(p_i, p_j, H_i)$ is concave in p_i. Thus if an interior Nash equilibrium of the pricing game $(p_1^*, p_2^*)(H_1, H_2)$ exists, it is characterized by the system of necessary first-order conditions:

$$\left(p_i^* - \widehat{c}\left(p_i^*, p_j^*, H_i\right)\right)\frac{\partial D}{\partial p_i}\left(p_i^*, p_j^*\right) + D\left(p_i^*, p_j^*\right) = 0, (i = 1, 2). \tag{10}$$

As will be proven below, this equilibrium of the pricing game exists, and, under certain conditions, is unique, hence of the form $(p^*(H_i, H_j), p^*(H_j, H_i))$. The expected value of firm i for its shareholders is $V_i = V(H_i, H_j) = v(p^*(H_i, H_j), p^*(H_j, H_i), H_i)$. The equilibrium of the two-stage game is then characterized as follows:

Proposition 5 *1. For any (H_i, H_j) close enough to the diagonal, there exists an interior equilibrium $(p_1^*, p_2^*)(H_1, H_2)$ of the pricing game. It is characterized by system (10).*

2. If absolute risk aversion is constant, this equilibrium is unique, and a marginal hedging increase by firm i reduces firm j's equilibrium price: $\frac{\partial p_j^*}{\partial H_i} < 0$.

3. Any interior equilibrium $\left(H_i^*, H_j^*\right)$ of the hedging game satisfies

$$\widehat{c}\left(p_i^*, p_j^*, H_i\right) = F + \frac{\frac{\partial D}{\partial p_j}\left(p_i^*, p_j^*\right)}{\frac{\partial D}{\partial p_i}\left(p_i^*, p_j^*\right)} D\left(p_i^*, p_j^*\right) \frac{\partial p_j^*}{\partial H_i}\left(H_i^*, H_j^*\right), (i = 1, 2). \tag{11}$$

4. If a symmetric interior equilibrium exists, and absolute risk aversion is constant, hedging softens price competition: firms under-hedge in order to induce higher prices than if marginal costs were constant and equal to F:

$$H^* < D(p^*, p^*) \text{ and } p^* > p^B(F, F).$$

Proof. *The proof follows the steps of Propositions 2 and 3. The risk-adjusted costs are bounded, thus the set in which we look for a fixed point is compact and convex in $\mathbb{R}^2$. Since all functions are continuous, Brouwer's fixed point theorem guarantees the existence of an equilibrium. If absolute risk aversion is constant, then Assumption 2 guarantees unicity of the equilibrium and allows to sign the direction of the strategic effect. Equation (11) is derived similarly to equation (9). Comparison of equations (11) and (10) shows that hedging softens price competition. Detailed proofs are available from the authors upon request.* ■

Combining the first-order conditions yields:

$$\left(p^{*}-F\right) \frac{\partial D_{i}}{\partial p_{i}}\left(p^{*}, p^{*}\right)+D\left(p^{*}, p^{*}\right)-\frac{\partial D_{i}}{\partial p_{j}}\left(p^{*}, p^{*}\right) D\left(p^{*}, p^{*}\right) \frac{\partial p_{j}^{*}}{\partial H_{i}}\left(H^{*}, H^{*}\right)=0.$$

Hedging has indeed a strategic effect, captured by the term $\frac{\partial p_j^*}{\partial H_i}$. Keeping some input price exposure uncovered commits firms to be less aggressive. This commitment then yields a higher equilibrium price : $p^* > p^E(F, F)$. The direction of the strategic effect is reversed

compared to Cournot competition: here, firms under-hedge, hence the equilibrium price is increased. This stark difference is best understood by comparing the first-order conditions (after appropriate transformations):

$$\widehat{c}_{i}-F+\frac{\partial \pi_{i}}{\partial q_{j}} \frac{\partial q_{j}^{*}}{\partial H_{i}}=0,$$

in the Cournot case, and

$$\widehat{c}_{i}-F+\frac{\frac{\partial D_{i}}{\partial p_{j}}}{\left(-\frac{\partial D_{i}}{\partial p_{i}}\right)} D\left(p_{i}^{*}, p_{j}^{*}\right) \frac{\partial p_{j}^{*}}{\partial H_{i}}=0$$

in the case of differentiated Bertrand. In both cases, when firm i increases hedging, firm j reduces her strategic variable (quantity or price). If firms compete in quantity, when firm j increases output, this reduces firm i's profit $\left(\frac{\partial \pi_i}{\partial q_j} < 0\right)$. Therefore, at the equilibrium, firm i over-hedges to set her risk-adjusted expected cost *below* F, and thus becomes more aggressive. Conversely, if firms compete in price, when firm j raises his price, this increases the demand faced by firm $i's$ $\left(\frac{\partial D_i}{\partial p_j} > 0\right)$, hence firm i under-hedges to set her risk-adjusted expected cost *above* F, and thus becomes less aggressive.

6 Incentives to commit on a hedging position

We have argued that firms commit to their hedging strategy because their Boards of Directors do not want them to speculate: risk managers are not allowed to significantly deviate from their pre-announced hedging position. This restriction has clear advantages in terms of monitoring the activity of traders. However, we have seen that it is not always profitable for shareholders. In this Section, we set aside governance problems, and assume that firms are free to decide ex-ante whether or not they want to commit to the hedging positions. This is done by adding a prior stage to our sequential games.

The timing is now as follows: at $t = 0$, each firm decides either to Commit (C) or Not Commit (NC) to its hedging position. At $t = 1$, firms that have chosen C publicly announce their hedging position. At $t = 2$, firms compete (in quantities or in prices), and the firms that have chosen (NC) decide on their hedging position. Finally, at $t = 3$, input cost is realized, and profits are determined. We assume that there is a unique sub-game perfect equilibrium at $t = 1$, independently of the commitments decisions made at $t = 0$ (this is the case for example if firms have constant absolute risk aversion). The shareholder value of firm i that plays strategy $X_i \in \{C, NC\}$ while firm j plays strategy $X_j \in \{C, NC\}$ is denoted $S(X_i, X_j)$.

To focus on the strategic impact of hedging, we continue to assume that (i) there are no transaction costs associated with hedging, and (ii) the forward price F is equal to the expected spot price $\mathbb{E}[\tilde{c}]$.

Proposition 6 *1. Not Committing cannot be sustained in equilibrium. Whether firms compete in quantity or in price:* $S(C, NC) > S(NC, NC)$.

2. *If firms compete in quantity, universal Not Commitment dominates universal Commitment:* $S(NC, NC) > S(C, C)$.

3. *If firms compete à la Hotelling, have constant absolute risk aversion, and input costs are normally distributed, universal Commitment dominates universal Non Commitment:* $S(C, C) > S(NC, NC)$ *and is a dominant strategy for all firms* $S(C, C) > S(NC, C)$.

Proof. *We first prove point 1 if firms compete in quantity. Suppose firm 2 plays NC. If firm 1 also plays NC, the shareholder value of both firms is $S(NC, NC)$. Suppose now* *that firm 1 plays C. At $t = 2$, both firms select their output. Then, firm 2, which did not commit, optimally selects complete hedging, while firm 1 has committed to H_1. Assuming the equilibrium $(\overline{q}_1(H_1), \overline{q}_2(H_1))$ is interior, it is characterized by the first-order conditions:*

$$\begin{cases} \overline{q}_2 P'(\overline{q}_1 + \overline{q}_2) + P(\overline{q}_1 + \overline{q}_2) - F = 0 \\ \overline{q}_1 P'(\overline{q}_1 + \overline{q}_2) + P(\overline{q}_1 + \overline{q}_2) - \widehat{c}(\overline{q}_1, \overline{q}_2, H_1) = 0 \end{cases}$$

At $t = 1$, firm 1 selects $\overline{H}_1$ to maximize $Z(H_1) = V(\overline{q}_1(H_1), \overline{q}_2(H_1), H_1)$. If firm 1 selects $H_1 = q^C(F,F)$, then $q_1 = q_2 = q^C(F,F)$ is a solution of the system, hence is the unique Cournot equilibrium for $H_1 = q^C(F,F)$. It yields the expected payoff $S(NC, NC)$. Thus when firm 2 does not commit, firm 1 can guarantee itself at least $S(NC, NC)$ by committing to $H_1 = q^C(F,F)$. This implies that $S(C, NC) \geq S(NC, NC)$. We now show that this inequality is strict.

$$\frac{dZ}{dH_1}\left(q^C(F,F)\right) = \mathbb{E}\left[U'(\pi_1)\left(\overline{q}_1 P'(\overline{Q})\frac{d\overline{q}_2}{dH_1} + (\tilde{c} - F)\right)\right] = \overline{q}_1 P'(\overline{Q})\frac{d\overline{q}_2}{dH_1}\mathbb{E}\left[U'(\pi_1)\right]$$

since $\widehat{c}\left(q^C(F,F), q^C(F,F), q^C(F,F)\right) = F$. Then, since $H_1 = \overline{q}_1$, $\frac{\partial \widehat{c}_1}{\partial q_2} = 0$, hence $\frac{d\overline{q}_2}{dH_1} < 0$. Thus, $\frac{dZ}{dH_1}\left(q^C(F,F)\right) > 0$, which implies that $\max_{H_1} Z(H_1) > Z\left(q^E(F,F)\right) \geq S(NC, NC)$. Thus: $S(C, NC) > S(NC, NC)$.

The proof of point 1 proceeds along the same lines if firms compete in price, and is presented in Appendix C, along with the formal proof of the other points. As expected, when firms compete in quantity, (C, C) yields lower prices and higher volatility, hence lower expected utility than (NC, NC). However if firms compete à la Hotelling, the expected profit increase more than compensates for the loss coming from increased volatility, hence (i) universal Commitment dominates universal Non Commitment: $S(C, C) > S(NC, NC)$, and (ii) is a dominant strategy for all firms: $S(C, C) > S(NC, C)$. ■

Even though universal Non Commitment dominates when firms compete in quantity, each firm prefers to Commit when the other does not. Thus, whether firms compete in quantity or in price, universal non Commitment can never be an equilibrium.

7 Concluding remarks

This article examines how firms facing volatile input prices and holding some degree of market power in their product market link their risk management and their production or pricing strategies. This issue is relevant in many industries ranging from manufacturing to energy retailing, where risk averse firms decide on their hedging strategies *before* their product market strategies. We find that hedging modifies the pricing and production strategies of firms. This strategic effect is channelled through the risk-adjusted expected cost, i.e., the expected marginal cost under the probability measure induced by shareholders' risk aversion. It has opposite effects depending on the nature of product market competition: hedging toughens quantity competition while it softens price competition. Finally, if firms can decide not to commit on their hedging position, this can never be an equilibrium outcome: committing is always a best response to non committing. In the Hotelling model, committing is a dominant strategy for all firms.

This paper could be extended in different directions. For example it would be interesting to examine asymmetric situations, where one firm is market leader and announces its hedging strategy before the other, or when different firms have different costs. Another possibility would be to endogenize pricing flexibility, i.e., to determine when it is optimal for firms not to adjust their output prices to reflect the realization of their input costs.

Finally, another avenue of research is to bring the model to the data, and in particular to test the predictions as to how firms' hedging decisions influence their and their competitors' pricing strategies. For econometricians, this naturally leads to the question : which model of competition (Cournot vs. Bertrand) is best suited to describe the industry of interest? This is an empirical question. As clearly articulated by Fudenberg and Tirole (1984), these two models should not be taken literally as resulting from a different choice of strategies (price vs. quantity). Instead, they have to be interpreted as two reduced forms models for the joint determination of prices and outputs. The choice between the two must be guided by the best fit to the data in the particular industry under study.

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A Quantity competition

A.1 Deterministic input cost (Lemma 1)

Condition 2 guarantees that $\frac{\partial^2 \pi_i^C}{(\partial q_i)^2} < 0$. Thus, if an interior Cournot equilibrium exists, it is characterized by the necessary first-order conditions (3). Assumption 1 guarantees that, for all $c > 0$, the equation

$$A(Q) = 2P(Q) + QP'(Q) = c$$

admits a unique solution $Q^C(c)$. When the equilibrium is interior $\left(q_i^C > 0 \text{ for } i = 1, 2\right)$, the equilibrium quantities are:

$$q^C(c_i, c_j) = \frac{P\left(Q^C(c_i + c_j)\right) - c_i}{\left(-P'\left(Q^C(c_i + c_j)\right)\right)}.$$

Finally, we verify that:

$$\frac{\partial q_i^C}{\partial c_i} = \frac{\partial q^C(c_i, c_j)}{\partial c_i} = \frac{2P'\left(Q^C\right) + q_j^C P''\left(Q^C\right)}{P'\left(Q^C\right)\left(3P'\left(Q^C\right) + Q^C P''\left(Q^C\right)\right)} < 0 \tag{12}$$

and

$$\frac{\partial q_j^C}{\partial c_i} = \frac{\partial q^C(c_j, c_i)}{\partial c_i} = -\frac{P'\left(Q^C\right) + q_i^C P''\left(Q^C\right)}{P'\left(Q^C\right)\left(3P'\left(Q^C\right) + Q^C P''\left(Q^C\right)\right)} > 0. \tag{13}$$

A.2 Properties of the risk-adjusted expected cost (Lemma 2)

Note first that for any variable x :

$$\begin{aligned}
\frac{\partial \widehat{c}_i}{\partial x} &= \frac{\mathbb{E}\left[U'(\pi_i)\right]\mathbb{E}\left[U''(\pi_i)\frac{\partial \pi_i}{\partial x}\tilde{c}\right] - \mathbb{E}\left[U'(\pi_i)\tilde{c}\right]\mathbb{E}\left[U''(\pi_i)\frac{\partial \pi_i}{\partial x}\right]}{\left(\mathbb{E}\left[U'(\pi_i)\right]\right)^2} \\
&= -\frac{\mathbb{E}\left[U'(\pi_i)\left(-\frac{U''(\pi_i)}{U'(\pi_i)}\frac{\partial \pi_i}{\partial x}\tilde{c}\right)\right]}{\mathbb{E}\left[U'(\pi_i)\right]} + \frac{\mathbb{E}\left[U'(\pi_i)\tilde{c}\right]}{\mathbb{E}\left[U'(\pi_i)\right]}\frac{\mathbb{E}\left[U'(\pi_i)\left(-\frac{U''(\pi_i)}{U'(\pi_i)}\frac{\partial \pi_i}{\partial x}\right)\right]}{\mathbb{E}\left[U'(\pi_i)\right]} \\
&= -\widehat{\mathbb{E}}_i\left[\rho(\pi_i)\frac{\partial \pi_i}{\partial x}\tilde{c}\right] + \widehat{c}_i\widehat{\mathbb{E}}_i\left[\rho(\pi_i)\frac{\partial \pi_i}{\partial x}\right] = \widehat{\mathbb{E}}_i\left[\rho(\pi_i)\frac{\partial \pi_i}{\partial x}(\widehat{c}_i - \tilde{c})\right].
\end{aligned}$$

For any (H_i, H_j), the equilibrium $\left(q_i^*, q_j^*\right)(H_i, H_j)$ of the Cournot game satisfies

$$P\left(q_i^* + q_j^*\right) + q_i^* P'\left(q_i^* + q_j^*\right) = \widehat{c}\left(q_i^*, q_j^*, H_i\right).$$

Thus, $\frac{\partial \pi}{\partial q_i}\left(q_i^*, q_j^*, H_i; \tilde{c}\right) = P\left(q_i^* + q_j^*\right) + q_i^* P'\left(q_i^* + q_j^*\right) - \tilde{c} = \widehat{c}\left(q_i^*, q_j^*, H_i\right) - \tilde{c}$. Using the above formula with $x = q_i$,we obtain:

$$\frac{\partial \widehat{c}}{\partial q_i}\left(q_i^*, q_j^*, H_i\right) = \widehat{\mathbb{E}}_i\left[\rho\left(\pi\left(q_i^*, q_j^*, H_i\right)\right)\left(\widehat{c}\left(q_i^*, q_j^*, H_i\right) - \tilde{c}\right)^2\right] > 0,$$

which establishes the first part of (6).

Now

$$\widehat{c}_i - F = \frac{\mathbb{E}\left[U'\left(\pi_i\right)\tilde{c}\right]}{\mathbb{E}\left[U'\left(\pi_i\right)\right]} - \mathbb{E}\left[\tilde{c}\right] = \frac{cov\left[U'\left(\pi_i\right), \tilde{c}\right]}{\mathbb{E}\left[U'\left(\pi_i\right)\right]}$$

Since (i) $U'(.)$ is non-increasing in π_i, and (ii) π_i increases in $\tilde{c}$ if and only if $H_i > q_i$, we have $\widehat{c}_i \leq F \Leftrightarrow H_i \geq q_i$.

Moreover, $\frac{\partial \pi_i}{\partial H_i} = (\tilde{c} - F)$, thus using again the above formula with $x = H_i$

$$\begin{aligned}\frac{\partial \widehat{c}_i}{\partial H_i} &= \widehat{\mathbb{E}}_i\left[\rho\left(\pi_i\right)\left(\tilde{c} - F\right)\left(\widehat{c}_i - \tilde{c}\right)\right] = -\widehat{\mathbb{E}}_i\left[\rho\left(\pi_i\right)\left(\tilde{c} - \widehat{c}_i\right)^2\right] + \left(\widehat{c}_i - F\right)\widehat{\mathbb{E}}_i\left[\rho\left(\pi_i\right)\left(\widehat{c}_i - \tilde{c}\right)\right] \\ &= -\left(\widehat{\mathbb{E}}_i\left[\rho\left(\pi_i\right)\left(\tilde{c} - \widehat{c}_i\right)^2\right] + \frac{cov\left[U'\left(\pi_i\right), \tilde{c}\right] \cdot \widehat{cov}_i\left[\rho\left(\pi_i\right), \tilde{c}\right]}{\mathbb{E}\left[U'\left(\pi_i\right)\right]}\right)\end{aligned}$$

Now

$$\widehat{\mathbb{E}}_i\left[\rho\left(\pi_i\right)\left(\widehat{c}_i - \tilde{c}\right)\right] = \widehat{\mathbb{E}}_i\left[\rho\left(\pi_i\right)\left(\widehat{\mathbb{E}}_i\left[\tilde{c}\right] - \tilde{c}\right)\right] = -\widehat{cov}_i\left[\rho\left(\pi_i\right), \tilde{c}\right].$$

Since $\rho(.)$ is, like $U'(.)$, non-increasing in π_i, $cov\left[U'\left(\pi_i\right), \tilde{c}\right]$ and $\widehat{cov}_i\left[\rho\left(\pi_i\right), \tilde{c}\right]$ have the same sign. Hence $\frac{\partial \widehat{c}_i}{\partial H_i} < 0$. This establishes the second part of (6).

Similar algebra yields

$$\frac{\partial \widehat{c}_i}{\partial q_j} = \widehat{\mathbb{E}}_i\left[\rho\left(\pi_i\right)\frac{\partial \pi_i}{\partial q_j}\left(\widehat{c}_i - \tilde{c}\right)\right] = q_i P'\left(Q\right)\widehat{\mathbb{E}}_i\left[\rho\left(\pi_i\right)\left(\widehat{c}_i - \tilde{c}\right)\right] = -q_i P'\left(Q\right)\widehat{cov}_i\left[\rho\left(\pi_i\right), \tilde{c}\right].$$

which establishes (7). (8) follows from (7).

A.3 Unicity of equilibrium (Proposition 2)

As mentioned in the main text, the equilibrium of the production game $(q_1^*, q_2^*)(H_1, H_2)$ is unique if the real parts of the eigenvalues of the Jacobian $J(q_1^*, q_2^*, H_1, H_2)$ are negative, where

$$J(q_1^*, q_2^*, H_1, H_2) = \begin{bmatrix} \frac{\partial q_1^E}{\partial q_1} - 1 & \frac{\partial q_1^E}{\partial q_2} \\ \frac{\partial q_2^E}{\partial q_1} & \frac{\partial q_2^E}{\partial q_2} - 1 \end{bmatrix}.$$

This is an application of Lyapunov stability theorem (see for example Khalil (2002)). The eigenvalues are the roots of $\lambda^2 - \lambda Tr + Det = 0$,where Tr is the trace of J and Det its determinant. The roots are: $\lambda_{\pm} = \frac{Tr \pm \sqrt{Tr^2 - 4Det}}{2}$. If $Tr^2 - 4Det < 0$, the two roots are complex and conjugate. Their real part is negative if and only if $Tr < 0$. If $Tr^2 - 4Det \geq 0$, the two roots are real. $Tr + \sqrt{Tr^2 - 4Det} < 0$ requires $Tr < 0$ and $Det > 0$. Thus, we have to show that $Tr < 0$ and $Det > 0$. We have:

$$Tr = \left(\frac{\partial q_1^C}{\partial q_1} + \frac{\partial q_2^C}{\partial q_2} - 2 \right).$$

By definition, $q_i^C = q^C\left(\widehat{c}(q_i, q_j, H_i), \widehat{c}(q_j, q_i, H_j), H_i\right)$,for $i = 1, 2$, thus

$$\frac{\partial q_i^C}{\partial q_i} = \frac{\partial q_i^C}{\partial c_i}\frac{\partial \widehat{c}_i}{\partial q_i} + \frac{\partial q_i^C}{\partial c_j}\frac{\partial \widehat{c}_j}{\partial q_i}, \tag{14}$$

and

$$\frac{\partial q_i^C}{\partial q_j} = \frac{\partial q_i^C}{\partial c_i}\frac{\partial \widehat{c}_i}{\partial q_j} + \frac{\partial q_i^C}{\partial c_j}\frac{\partial \widehat{c}_j}{\partial q_j}. \tag{15}$$

Thus, $Tr = \frac{\partial q_1^C}{\partial c_1}\frac{\partial \widehat{c}_1}{\partial q_1} + \frac{\partial q_1^C}{\partial c_2}\frac{\partial \widehat{c}_2}{\partial q_1} + \frac{\partial q_2^C}{\partial c_2}\frac{\partial \widehat{c}_2}{\partial q_2} + \frac{\partial q_2^C}{\partial c_1}\frac{\partial \widehat{c}_1}{\partial q_2} - 2$.

Lemma 2 shows that, when ρ is constant, $\frac{\partial \widehat{c}_1}{\partial q2} = \frac{\partial \widehat{c}_2}{\partial q_1} = 0$. Thus

$$Tr(q_1^*, q_2^*, H_1, H_2) = \underset{-}{\frac{\partial q_1^C}{\partial c_1}}\underset{+}{\frac{\partial \widehat{c}_1}{\partial q_1}} + \underset{-}{\frac{\partial q_2^C}{\partial c_2}}\underset{+}{\frac{\partial \widehat{c}_2}{\partial q_2}} - 2 < -2.$$

We now examine $Det\left(q_1^*, q_2^*, H_1, H_2\right)$.

$$Det = \left(\frac{\partial q_1^C}{\partial q_1} - 1\right)\left(\frac{\partial q_2^C}{\partial q_2} - 1\right) - \frac{\partial q_1^C}{\partial q_2}\frac{\partial q_2^C}{\partial q_1} = \frac{\partial q_1^C}{\partial q_1}\frac{\partial q_2^C}{\partial q_2} - \frac{\partial q_1^C}{\partial q_2}\frac{\partial q_2^C}{\partial q_1} - \frac{\partial q_1^C}{\partial q_1} - \frac{\partial q_2^C}{\partial q_2} + 1$$

Substituting $\frac{\partial q_i^C}{\partial q_i}$ and $\frac{\partial q_i^C}{\partial q_j}$ from equations (14) and (15), and simplifying yields

$$\frac{\partial q_1^C}{\partial q_1}\frac{\partial q_2^C}{\partial q_2} - \frac{\partial q_1^C}{\partial q_2}\frac{\partial q_2^C}{\partial q_1} = \left(\frac{\partial q_1^C}{\partial c_1}\frac{\partial q_2^C}{\partial c_2} - \frac{\partial q_1^C}{\partial c_2}\frac{\partial q_2^C}{\partial c_1}\right)\left(\frac{\partial \widehat{c}_1}{\partial q_1}\frac{\partial \widehat{c}_2}{\partial q_2} - \frac{\partial \widehat{c}_1}{\partial q_2}\frac{\partial \widehat{c}_2}{\partial q_1}\right).$$

Now, substituting in $\frac{\partial q_i^C}{\partial c_i}$ and $\frac{\partial q_j^C}{\partial c_i}$ from equations (12) and (13), and simplifying yields

$$\frac{\partial q_1^C}{\partial c_1}\frac{\partial q_2^C}{\partial c_2} - \frac{\partial q_1^C}{\partial c_2}\frac{\partial q_2^C}{\partial c_1} = \frac{1}{P'\left(Q^E\right)\left(3P'\left(Q^E\right) + QP''\left(Q^E\right)\right)}.$$

Thus,

$$Det = \frac{\frac{\partial \widehat{c}_1}{\partial q_1}\frac{\partial \widehat{c}_2}{\partial q_2} - \frac{\partial \widehat{c}_1}{\partial q_2}\frac{\partial \widehat{c}_2}{\partial q_1}}{P'\left(Q^C\right)\left(3P'\left(Q^C\right) + Q^C P''\left(Q^C\right)\right)} - Tr - 1.$$

Then, with ρ constant, we know that $\frac{\partial \widehat{c}_2}{\partial q_1} = \frac{\partial \widehat{c}_1}{\partial q_2} = 0$. Moreover, $\frac{\partial \widehat{c}_1}{\partial q_1} < 0$ and $\frac{\partial \widehat{c}_2}{\partial q_2} < 0$. Thus $Det > -Tr - 1 > 1$.

A.4 Impact of H_i on q_j^* with constant absolute risk aversion (Proposition 2)

Define $\psi\left(q_i, q_j, H_i\right) = P\left(Q\right) - \widehat{c}\left(q_i, q_j, H_i\right) + q_i P'\left(Q\right).$

The first order conditions characterizing the unique equilibrium of the production game can be written as $\psi\left(q_i^*, q_j^*, H_i\right) = \psi\left(q_j^*, q_i^*, H_j\right) = 0$. Total differentiation of these conditions with respect to H_i yields:

$$\begin{cases} \left(\psi_1 \frac{\partial q_i^*}{\partial H_i} + \psi_2 \frac{\partial q_j^*}{\partial H_i} + \psi_3\right)\left(q_i^*, q_j^*, H_i\right) = 0 \\ \left(\psi_1 \frac{\partial q_j^*}{\partial H_i} + \psi_2 \frac{\partial q_i^*}{\partial H_i}\right)\left(q_j^*, q_i^*, H_j\right) = 0 \end{cases},$$

where

$$\begin{cases} \psi_1(q_i,q_j,H_i) = 2P'(q_i+q_j) + q_iP''(q_i+q_j) - \frac{\partial \widehat{c}_i}{\partial q_i} \\ \psi_2(q_i,q_j,H_i) = P'(q_i+q_j) + q_iP''(q_i+q_j) - \frac{\partial \widehat{c}_i}{\partial q_j} \\ \psi_3(q_i,q_j,H_i) = -\frac{\partial \widehat{c}_i}{\partial H_i} > 0 \end{cases}.$$

The determinant of the above linear system is

$$\Delta = \psi_1\left(q_i^*,q_j^*,H_i\right)\psi_1\left(q_j^*,q_i^*,H_j\right) - \psi_2\left(q_i^*,q_j^*,H_i\right)\psi_2\left(q_j^*,q_i^*,H_j\right),$$

thus

$$\begin{cases} \frac{\partial q^*}{\partial H_i}(H_i,H_j) = -\frac{\psi_1\left(q_j^*,q_i^*,H_j\right)}{\Delta}\psi_3\left(q_i^*,q_j^*,H_i\right) \\ \frac{\partial q^*}{\partial H_i}(H_j,H_i) = \frac{\psi_2\left(q_j^*,q_i^*,H_j\right)}{\Delta}\psi_3\left(q_i^*,q_j^*,H_i\right) \end{cases}.$$

If ρ is constant, $\frac{\partial \widehat{c}_i}{\partial q_j} = 0$, thus

$$\psi_2(q_i,q_j,H_i) = P'(q_i+q_j) + q_iP''(q_i+q_j) < 0.$$

Then:

$$\begin{aligned} \Delta &= \left(2P'(Q^*) + q_i^*P''(Q^*) - \frac{\partial \widehat{c}_i}{\partial q_i}\right)\left(2P'(Q^*) + q_j^*P''(Q^*) - \frac{\partial \widehat{c}_j}{\partial q_j}\right) \\ &\quad - \left(P'(Q^*) + q_iP''(Q^*)\right)\left(P'(Q^*) + q_jP''(Q^*)\right) \\ &= \left(P'(Q^*) - \frac{\partial \widehat{c}_i}{\partial q_i}\right)\left(P'(Q^*) - \frac{\partial \widehat{c}_j}{\partial q_j}\right) + \left(P'(Q^*) - \frac{\partial \widehat{c}_i}{\partial q_i}\right)\left(P'(Q^*) + q_j^*P''(Q^*)\right) \\ &\quad + \left(P'(Q^*) - \frac{\partial \widehat{c}_j}{\partial q_j}\right)\left(P'(Q^*) + q_i^*P''(Q^*)\right). \end{aligned}$$

Since all terms in parentheses are negative, $\Delta > 0$. Thus

$$\frac{\partial q_j^*}{\partial H_i} = \frac{\partial q^*}{\partial H_i}(H_j,H_i) = \frac{\psi_2\left(q_j^*,q_i^*,H_i\right)}{\Delta}\psi_3\left(q_i^*,q_j^*,H_i\right) < 0.$$

B Price competition

Define $\psi(p_i, p_j, H_i) = \frac{\partial D}{\partial p_i}(p_i, p_j)(p_i - \widehat{c}(p_i, p_j, H_i)) + D(p_i, p_j)$, where

$$\widehat{c}_i \equiv \widehat{c}(p_i, p_j, H_i) \equiv \frac{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c}))\tilde{c}]}{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c}))]}$$

is the risk-adjusted expected cost of firm i. Suppose a unique interior equilibrium of the pricing game $(p_1^*, p_2^*)(H_1, H_2)$ exists. The first-order conditions characterizing this equilibrium are

$$\psi\left(p_i^*, p_j^*, H_i\right) = \psi\left(p_j^*, p_i^*, H_j\right) = 0.$$

Assuming $\Delta = \psi_1\left(p_i^*, p_j^*, H_i\right)\psi_1\left(p_j^*, p_i^*, H_j\right) - \psi_2\left(p_i^*, p_j^*, H_i\right)\psi_2\left(p_j^*, p_i^*, H_j\right) \neq 0$,

$$\begin{cases} \frac{\partial p_i^*}{\partial H_i} = \frac{\partial p^*}{\partial H_i}(H_i, H_j) = -\frac{\psi_1\left(p_j^*, p_i^*, H_j\right)}{\Delta}\psi_3\left(p_i^*, p_j^*, H_i\right) \\ \frac{\partial p_j^*}{\partial H_i} = \frac{\partial p^*}{\partial H_i}(H_j, H_i) = \frac{\psi_2\left(p_j^*, p_i^*, H_j\right)}{\Delta}\psi_3\left(p_i^*, p_j^*, H_i\right) \end{cases},$$

where

$$\begin{cases} \psi_1(p_i, p_j, H_i) = 2\frac{\partial D_i}{\partial p_i} + (p_i - \widehat{c}_i)\frac{\partial^2 D_i}{(\partial p_i)^2} - \frac{\partial D_i}{\partial p_i}\frac{\partial \widehat{c}_i}{\partial p_i} \\ \psi_2(p_i, p_j, H_i) = \frac{\partial D_i}{\partial p_j} + (p_i - \widehat{c}_i)\frac{\partial^2 D_i}{\partial p_i \partial p_j} - \frac{\partial D_i}{\partial p_i}\frac{\partial \widehat{c}_i}{\partial p_j} \\ \psi_3(p_i, p_j, H_i) = -\frac{\partial D_i}{\partial p_i}\frac{\partial \widehat{c}_i}{\partial H_i}. \end{cases}.$$

B.1 Impact of c_i on p_i^C and p_j^C (constant input costs)

Suppose first the marginal costs are constant:

$$\psi(p_i, p_j, c_i) = \frac{\partial D(p_i, p_j)}{\partial p_i}(p_i - c_i) + D(p_i, p_j)$$

and

$$\begin{cases} \psi_1(p_i, p_j, c_i) = 2\frac{\partial D_i}{\partial p_i} + (p_i - c_i)\frac{\partial^2 D_i}{(\partial p_i)^2} \\ \psi_2(p_i, p_j, c_i) = \frac{\partial D_i}{\partial p_j} + (p_i - c_i)\frac{\partial^2 D_i}{\partial p_i \partial p_j} \\ \psi_3(p_i, p_j, c_i) = -\frac{\partial D_i}{\partial p_i} \end{cases}.$$

Assumption 2 guarantees (i) existence and unicity of an equilibrium $\left(p^B(c_i,c_j), p^B(c_j,c_i)\right)$, (ii) $\psi_1(p_i,p_j,c_i)<0$, since $\pi^B(p_i,p_j,c_i)$ is concave in p_i, (iii) $\psi_2\left(p^B(c_i,c_j), p^B(c_j,c_i), c_i\right)>0$, since prices are strategic complements, and (iv) $(\psi_1+\psi_2)\left(p^B(c_i,c_j), p^B(c_j,c_i), c_i\right)<0$ since the own price effect dominates. Thus

$$\Delta^B = \psi_1\left(p_i^B,p_j^B,c_i\right)\psi_1\left(p_j^B,p_i^B,c_j\right) - \psi_2\left(p_i^B,p_j^B,c_i\right)\psi_2\left(p_j^B,p_i^B,c_j\right) > 0$$

and

$$\begin{cases} \frac{\partial p_i^B}{\partial c_i} = \frac{\partial p^B}{\partial c_i}(c_i,c_j) = \frac{\partial D_i}{\partial p_i}\frac{\psi_1\left(p_j^B,p_i^B,c_j\right)}{\Delta^E} = \frac{\partial D_i}{\partial p_i}\frac{2\frac{\partial D_j}{\partial p_j}+(p_j-c_j)\frac{\partial^2 D_j}{\left(\partial p_j\right)^2}}{\Delta^E} > 0 \\ \frac{\partial p_j^B}{\partial c_i} = \frac{\partial p^B}{\partial c_i}(c_j,c_i) = -\frac{\partial D_i}{\partial p_i}\frac{\psi_2\left(p_j^B,p_i^B,c_j\right)}{\Delta^E} = -\frac{\partial D_i}{\partial p_i}\frac{\frac{\partial D_j}{\partial p_i}+(p_j-c_j)\frac{\partial^2 D_j}{\partial p_i \partial p_j}}{\Delta^E} > 0 \end{cases}.$$

B.2 Properties of the risk-adjusted expected cost (Lemma 3)

For any (p_i,p_j,H_i), the same derivation as for Cournot competition yields:

$$\begin{aligned} \frac{\partial \widehat{c}_i}{\partial H_i} &= \widehat{\mathbb{E}}_i\left[\rho(\pi_i)(\tilde{c}-F)(\widehat{c}_i-\tilde{c})\right] = -\widehat{\mathbb{E}}_i\left[\rho(\pi_i)(\widehat{c}_i-\tilde{c})^2\right] + (\widehat{c}_i - F)\,\widehat{\mathbb{E}}_i\left[\rho(\pi_i)(\widehat{c}_i-\tilde{c})\right]. \\ &= -\left(\widehat{\mathbb{E}}_i\left[\rho(\pi_i)(\widehat{c}_i-\tilde{c})^2\right] + \frac{cov\left[U'(\pi_i),\tilde{c}\right]\cdot \widehat{cov}_i\left[\rho(\pi_i),\tilde{c}\right]}{\mathbb{E}\left[U'(\pi_i)\right]}\right). \end{aligned}$$

Since (i) $\rho(.)$ and $U'(.)$ are both non-increasing, and (ii) π_i increases in $\tilde{c}$ if and only if $H_i > D(p_i,p_j)$, we have (i) $cov\left[U'(\pi_i),\tilde{c}\right]\cdot\widehat{cov}_i\left[\rho(\pi_i),\tilde{c}\right]\geq 0$, thus $\frac{\partial \widehat{c}_i}{\partial H_i}<0$, and (ii) and $\widehat{c}_i \leq F \Leftrightarrow H_i \geq D(p_i,p_j)$. Similarly,

$$\begin{aligned} \frac{\partial \widehat{c}_i}{\partial p_j} &= \widehat{\mathbb{E}}_i\left[\rho(\pi_i)\frac{\partial \pi_i}{\partial p_j}(\widehat{c}_i-\tilde{c})\right] = \frac{\partial D_i}{\partial p_j}\widehat{\mathbb{E}}_i\left[\rho(\pi_i)(p_i-\tilde{c})(\widehat{c}_i-\tilde{c})\right] \\ &= \frac{\partial D_i}{\partial p_j}\left\{(p_i-\widehat{c}_i)\,\widehat{\mathbb{E}}_i\left[\rho(\pi_i)(\widehat{c}_i-\tilde{c})\right] + \widehat{\mathbb{E}}_i\left[\rho(\pi_i)(\widehat{c}_i-\tilde{c})^2\right]\right\}. \end{aligned}$$

Thus: $\frac{\partial \widehat{c}}{\partial p_j}\left(p_i^*,p_j^*,H_i\right) = \frac{\partial D_i}{\partial p_j}\left\{\frac{D_i}{\frac{\partial D_i}{\partial p_i}}\widehat{cov}_i\left[\rho(\pi_i),\tilde{c}\right] + \widehat{\mathbb{E}}_i\left[\rho(\pi_i)(\widehat{c}_i-\tilde{c})^2\right]\right\}$. Then:

$$H_i > D\left(p_i^*,p_j^*\right) \Leftrightarrow \widehat{cov}_i\left[\rho(\pi_i),\tilde{c}\right] < 0 \Rightarrow \frac{\partial \widehat{c}}{\partial p_j}\left(p_i^*,p_j^*,H_i\right) > 0.$$

Finally:

$$\frac{\partial \widehat{c}_i}{\partial p_i} = \widehat{\mathbb{E}}_i \left[\rho(\pi_i) \frac{\partial \pi_i}{\partial p_i} (\widehat{c}_i - \tilde{c}) \right].$$

Then, $\frac{\partial \pi}{\partial p_i} = \left((\widehat{c}_i - \tilde{c}) \frac{\partial D_i}{\partial p_i} \right) \left(p_i^*, p_j^*, H_i \right)$ and thus $\frac{\partial \widehat{c}}{\partial p_i} = \frac{\partial D}{\partial p_i} \widehat{\mathbb{E}}_i \left[\rho(\pi_i) (\widehat{c}_i - \tilde{c})^2 \right] < 0$.

C Strategic incentives to commit (Proposition 6)

C.1 Comparing $S(C, NC)$ and $S(NC, NC)$ when firms compete in price

Suppose firm 2 plays NC, while firm 1 plays C. At $t = 2$, firm 2 chooses $\overline{H}_2 = D(p_2, p_1)$, after firms simultaneously select prices $(\overline{p}_1(H_1), \overline{p}_2(H_1))$ that solve:

$$\begin{cases} D(\overline{p}_1, \overline{p}_2) + (\overline{p}_1 - \widehat{c}(\overline{p}_1, \overline{p}_2, H_1)) \frac{\partial D}{\partial p_1}(\overline{p}_1, \overline{p}_2) = 0 \\ D(\overline{p}_2, \overline{p}_1) + (\overline{p}_2 - F) \frac{\partial D}{\partial p_2}(\overline{p}_2, \overline{p}_1) = 0 \end{cases}$$

At $t = 1$, firm 1 selects $\overline{H}_1$ that maximizes $Z(H_1) = \mathbb{E}[U(\pi(\overline{p}_1(H_1), \overline{p}_2(H_1), H_1))]$. As in the Cournot case, if firm 1 chooses $H_1 = D\left(p^B(F,F), p^B(F,F)\right) = D_0/2$, $p_1 = p_2 = p^B(F,F)$ is a solution of the system, hence the unique equilibrium. The shareholder value of both firms is $S(NC, NC)$. Thus firm 1 can guarantee itself at least $S(NC, NC)$,which implies that $S(C, NC) \geq S(NC, NC)$. To prove that the inequality is strict, it suffices to show that $\frac{dZ}{dH_1}(D_0/2) \neq 0$. This is easy, since

$$\frac{dZ}{dH_1}(D_0/2) = \frac{\partial D_1}{\partial p_2} \left(p^B(F,F) - F \right) \frac{\partial \overline{p}_2}{\partial H_1} \mathbb{E}\left[U'(\pi_1) \right] < 0.$$

Thus, if the firm hedges $\left(\frac{D_0}{2} - \varepsilon\right)$ where $\varepsilon > 0$ is arbitrarily small, it can obtain $Z\left(\frac{D_0}{2} - \varepsilon\right) > Z\left(\frac{D_0}{2}\right)$. Thus, $S(C, NC) > S(NC, NC)$.

C.2 Comparing $S(C,C)$ and $S(NC,NC)$ if firms compete in quantity

$$S(C,C)=\mathbb{E}\left[U\left((P(Q^*)-F)\frac{Q^*}{2}+(H^*-Q^*)(\omega-F)\right)\right]<U\left((P(Q^*)-F)\frac{Q^*}{2}\right)$$

since $U(.)$ is concave, and

$$S(NC,NC)=U\left(\left(P\left(2q^C(F,F)\right)-F\right)q^C(F,F)\right).$$

For $x\geq 0$, denote $f(x)=(P(2x)-F)x$. Condition 2 implies that $f(.)$ is globally concave and admits a unique maximum x^* defined by $f'(x^*)=P(2x^*)-F+2x^*P'(2x^*)=0$. Then,

$$f'\left(q^C(F,F)\right)=-q^C(F,F)P'\left(2q^C(F,F)\right)+2q^C(F,F)P'\left(2q^C(F,F)\right)=q^C(F,F)P'\left(2q^C(F,F)\right)<0,$$

hence $q^E(F,F)>x^*$. Then, $f\left(q^E(F,F)\right)>f(q^*)$ since $q^*>q^E(F,F)$ and $f(.)$ is decreasing for $x\geq x^*$. Thus: $S(C,C)<U(f(q^*))<U\left(f\left(q^C(F,F)\right)\right)=S(NC,NC)$.

C.3 Hotelling competition

We have seen in the text that

$$S(C,C)=\frac{t}{2}[1+\frac{\varepsilon(5+8\varepsilon+3\varepsilon^2)}{2(3+2\varepsilon)^2}]>\frac{t}{2}=S(NC,NC).$$

Suppose that firm 2 plays NC, while firm 1 plays C and chooses hedging H_1. We prove that $S(C,C)>S(NC,C)$. The equilibrium prices $(\overline{p}_1(H_1),\overline{p}_2(H_1))$ are given by the Hotelling formula:

$$\begin{cases}\overline{p}_1=t+\frac{1}{3}\left(2\left(F+\rho\sigma^2\left(\frac{\overline{p}_2-\overline{p}_1}{2t}+\frac{1}{2}-H_1\right)\right)+F\right)=t+F+\frac{4t\varepsilon}{3}\left(\frac{\overline{p}_2-\overline{p}_1}{2t}+\frac{1}{2}-H_1\right)\\ \overline{p}_2=t+\frac{1}{3}\left(2F+\left(F+\rho\sigma^2\left(\frac{\overline{p}_2-\overline{p}_1}{2t}+\frac{1}{2}-H_1\right)\right)\right)=t+F+\frac{2t\varepsilon}{3}\left(\frac{\overline{p}_2-\overline{p}_1}{2t}+\frac{1}{2}-H_1\right)\end{cases}.$$

Taking the difference, we obtain

$$\overline{p}_2 - \overline{p}_1 = -\frac{2t\varepsilon}{3}\left(\frac{\overline{p}_2 - \overline{p}_1}{2t} + \frac{1}{2} - H_1\right)$$

thus

$$\overline{p}_2 - \overline{p}_1 = -2t\frac{\varepsilon}{3+\varepsilon}\left(\frac{1}{2} - H_1\right)$$

and

$$D_1 - H_1 = \frac{\overline{p}_2 - \overline{p}_1}{2t} + \frac{1}{2} - H_1 = \frac{3}{3+\varepsilon}\left(\frac{1}{2} - H_1\right).$$

Equilibrium prices are given by

$$\overline{p}_1 - F = t\left(1 + \tfrac{4\varepsilon}{3+\varepsilon}\left(\tfrac{1}{2} - H_1\right)\right)$$
$$\overline{p}_2 - F = t\left(1 + \tfrac{2\varepsilon}{3+\varepsilon}\left(\tfrac{1}{2} - H_1\right)\right)$$

Note that

$$\frac{\partial \overline{p}_2}{\partial H_1} = -\frac{2t\varepsilon}{3+\varepsilon} < 0.$$

Maximization of M_1 over H_1 yields:

$$\frac{1}{2t}\left(\overline{p}_1 - F + \rho\sigma^2\left(H_1 - D_1\right)\right)\frac{\partial \overline{p}_2}{\partial H_1} - \rho\sigma^2\left(H_1 - D_1\right) = 0$$

or finally

$$\frac{1}{2} - \overline{H}_1 = \frac{3+\varepsilon}{2\left(9+4\varepsilon\right)}.$$

This implies that $\overline{p}_2 - F = t\left(1 + \frac{\varepsilon}{9+4\varepsilon}\right)$ and $D\left(\overline{p}_2, \overline{p}_1\right) = \frac{1}{2}\left(1 + \frac{\varepsilon}{9+4\varepsilon}\right)$ .Thus

$$S\left(NC, C\right) = \left(1 + \frac{\varepsilon}{9+4\varepsilon}\right)^2\frac{t}{2}.$$

Finally,

$$S\left(C,C\right) > S\left(NC,C\right) \Longleftrightarrow 1+\frac{\varepsilon(5+8\varepsilon+3\varepsilon^2)}{2(3+2\varepsilon)^2} > 1+\frac{9\varepsilon(2+\varepsilon)}{\left(9+4\varepsilon\right)^2}$$

$\Longleftrightarrow$

$$\left(5+8\varepsilon+3\varepsilon^2\right)\left(9+4\varepsilon\right)^2 > 18(2+\varepsilon)(3+2\varepsilon)^2$$

which is verified for all $\varepsilon \geq 0$.