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"Climate Change and Carbon Capture and Storage"

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Climate Change and Carbon Capture and Storage[1]

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Abstract:

We study optimal carbon capture and storage (CCS), taking into account damages incurred from the accumulation of carbon in the atmosphere and exhaustibility of fossil fuel reserves. High carbon concentrations call for full CCS. We identify conditions under which partial or no CCS is optimal. In the absence of CCS the CO2 stock might be inverted U-shaped. With CCS more complicated behavior may arise. It can be optimal to have full capture initially, yielding a decreasing stock, then partial capture while keeping the CO2 stock constant, and a final phase without capturing but with an inverted U-shaped CO2 stock.

Key words: Climate change, carbon capture and storage, non-renewable resources

JEL codes: Q32, Q43, Q54

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1. Introduction

"The Quest CCS Project could be part of the action Alberta and Canada is looking for – to develop valuable oil sands resources with less climate-changing CO2. Quest would capture more than one million tonnes of CO2 per year from Shell's Scotford Upgrader, located near Fort Saskatchewan, Alberta. This is the equivalent to taking 175,000 cars off the road. The CO_2 would be transported safely by pipeline up to 80 kilometers north of the facility to injection wells. It would then be injected more than two kilometres underground where it would be permanently and safely secured under multiple layers of impermeable geological formations" [4]

Carbon capture and storage (CCS) is generally expected to play a crucial future role in combating climate change. For example, J. Edmonds (Joint Global Change Research Institute) puts forward: "meeting the low carbon stabilization limits that are being explored in preparation for the IPCC 5th Assessment Report are only possible with CCS" (Edmonds, 2008). The main rationale for this view is that the economy is still depending on the use of fossil fuels to a large degree and that it might be too costly to introduce renewables in the short to medium run. CCS would then offer the opportunity to keep on using fossil fuels while limiting the emissions of CO2 into the atmosphere.
CCS consists of several stages. In the first stage the CO2 is captured[5], mainly at power plants, point sources. For this several processes are available, including post-combustion capture, pre-combustion capture (oxidizing fossil fuel) and oxy-fuel combustion. In the second phase the CO2 is transported to a reservoir, where in the third phase the captured carbon is stored in for example deep geological formations. A side effect of the latter could be the use of captured carbon for increasing the pressure in oil fields, thereby reducing the cost of future extraction, but at the same time increasing the profitability of enhanced oil extraction, with the subsequent release of carbon, unless captured. As a fourth phase there is monitoring what is going on, once CO2 is in the ground. Each of these phases brings along costs. The economic attractiveness of capturing depends on the cost of capture and storage and the climate change damage prevented by mitigation of emissions of carbon. Herzog (2011) and Hamilton et al. (2009) provide estimates of these costs and conclude that the capture cost are about $52 per metric ton ((from supercritical pulverized coal power plants), whereas for transportation and storage the costs will be in the range of $5-$15 per metric ton CO2. This leads to overall costs amounting to $60-$65 per metric ton[6]. At the present state of climate change policy CCS is obviously not profitable, but with a carbon price at present of $25 and rising by 4% per year, large scale CCS becomes a serious option before 2040. Nevertheless numerous obstacles remain. Many questions are

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[4] http://www.shell.ca/home/content/can-en/aboutshell/our_business_tpkg/business_in_canada/upstream/oil_sands/quest/about_quest/

[5] Herzog (2011) points out that already decades ago capturing took place, but then the objective was to enhance oil recovery by injecting CO2 in order to increase the pressure in the well.

[6] These numbers are more or less confirmed in ZEP (2011).

still unresolved. Some are of a regulatory and legal nature, for example the rights-of-way for pipelines[7], access to the formation where CO2 is injected[8], and how to make the transition from capturing megatons in the present to capturing gigatons in the future in order to have capture at a level that is substantial enough to combat climate change. Moreover, in Europe the success of CCS also depends of the prevailing CO2 permit price, which at present is low, and has induced Eon and GDF Suez to postpone investments in an EU funded demonstration project near Rotterdam, The Netherlands.

In the present paper we address not so much the development of the CCS technology but the optimal use of the technology once it is available. We only look at capturing at point sources, and thereby abstract from geo-engineering, where carbon is captured from the atmosphere. We also assume that a storage technology is available, but cannot be utilized for making fossil fuel reserves accessible at lower cost. Actually, we don't take into account the necessity of (costly) storage capacity that might be limited (see e.g., Lafforgue et al.,2008a and 2008b). We explicitly account for the fact that fossil fuels are extracted from nonrenewable resources. This implies that total extraction of fossil fuels is limited over time, but that the timing of extraction is crucial. An important question becomes what is the optimal extraction of fossil fuel over time, given the limited reserves of fossil fuel. The criterion for optimality that we use is discounted utilitarianism with instantaneous welfare being the difference between utility from energy use and the damage arising from accumulated CO2 in the atmosphere. In answering this question one needs to simultaneously determine optimal capture and storage of CO2. We make a distinction between constant marginal capturing cost and increasing marginal capturing costs (with marginal capturing costs at zero capturing zero or positive). Along the optimum a tradeoff has to be made between the direct instantaneous welfare of using fossil fuel on the one hand and the cost of capturing and damage caused by the accumulated CO2 on the other. It is found that different assumptions on capturing and storage cost lead to considerable differences in the combined optimal capturing and storage and extraction regime, in the case of abundant fossil fuel reserves as well as when reserves are limited. We identify cases where in the presence of the CCS it is still optimal to let the CO2 stock increase before partial capturing takes place. The core of the paper is section 4 where we derive the optimum for the pivotal case of a finite resource stock and the availability of a CCS technology. There we show that, perhaps surprisingly, it might be optimal to have full capture initially, then partial capture while keeping the CO2 stock constant, and a final phase with no capturing but in which the CO2 stock increases initially, before decreasing eventually. Hence the CO2 stock is not inverted U-shaped, as in Tahvonen (1997).

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[7] See N. Jaakkola (2012) for problems that may arise in case of imperfect competition on the transportation network (offshore, in northwestern Europe).

[8] Feenstra et al. (2010) report on the public outcry when plans for storage in the village of Barendrecht (The Netherlands) were revealed.

The related literature is large. First of all there is the literature that highlights the interrelationship between the use of fossil fuels and climate change (see Plourde (1972), D'Arge and Kogiku (1973), Ulph and Ulph (1994), Withagen (1994), Hoel and Kverndokk (1996), for early contributions). Recently this literature was enriched by explicitly introducing backstop technologies (see e.g., Tsur and Zemel (2003, 2005)) with due attention to the Green Paradox, the problem that may arise if for political economy reasons an optimal carbon tax is infeasible and policy makers rely on a subsidy of the renewable (see e.g. Van der Ploeg and Withagen (2012a and 2012b)). Another step has been set by explicitly incorporating CCS in models with nonrenewable natural resources. We start by sketching two recent contributions by Amigues et al. (2012 and 2013), who give a nice up to date survey of the state of affairs and offer a generalization of Chakravorty et al. (2006) and Lafforgue et al. (2008). These papers come close to ours in several respects but at the same time our discussion serves to highlight the essence of our work. Amigues et al. assume that there is a finite stock of fossil fuel, that can be extracted at constant marginal cost. In our case extraction is costless. This is without loss of generality, as the results also hold for constant average extraction costs. They also assume the existence of a backstop technology that is produced at constant marginal cost, which may be high or low. The backstop is perfectly malleable with the extracted fossil fuel and yields utility, together with fossil fuel. We abstract from a backstop technology, but we shall argue that in the case of abundant fossil fuel reserves capturing essentially functions as a backstop. Net accumulation of CO2 is the difference between on the one hand emissions, resulting from burning fossil fuel minus the amount captured and stored, and, on the other hand, the natural decay of the stock of CO2, which is a constant fraction of the existing stock. The average cost of capturing may take several forms. It may depend just on the amount captured, but, alternatively, one could allow for learning or for scarcity effects. In the former case the average CCS cost is a decreasing function of amount already captured. The latter case captures the fact that with more CCS done in the past it gets more difficult to find new CO2 deposits. We don't allow for stock dependent storage costs, but we do look at different capturing cost constellations. Since we concentrate on capturing at point sources and not on capturing from the atmosphere, net emissions are bound to be non-negative. Apart from the cost aspect, a major difference is in the assumption regarding damages. Amigues et al. put an upper bound, sometimes called a ceiling, on the accumulated CO2 stock, whereas we allow for the stock to take any value in principle, but work with a strictly convex damage function. Conceptually a damage function is more appealing, because it can be constructed in such a way that it includes the ceiling, by taking the damage function almost flat until just before the presupposed ceiling is reached, from where on damage increases steeply. More importantly, Amigues at al. (2012 and 2013) show that for all specifications considered it is optimal not to start with CCS until the threshold is reached. But the main and usual motivation for choosing a ceiling is that it represents a threshold beyond which a catastrophe takes place. Given the many uncertainties surrounding the phenomenon of climate change, this evokes the question whether it is optimal indeed to capture only at the critical level. One of the objectives of the present paper is to investigate this in detail. Our finding is that it might be optimal to do partial CCS at some threshold level, keeping the stock at this level. But after such a phase, the CO2 stock might increase for a while, without CCS taking place.

Other papers addressing CCS include Amigues et al. (2010) and Coulomb and Henriet (2010), who both acknowledge that demand for fossil fuel derives from different sectors of the economy. For example, one sector is the electricity production sector, whereas the other is the transport sector. In the latter capturing is far less attractive than in the former. Also in these papers, a ceiling on the CO2 stock is exogenously imposed and capturing only takes place at the ceiling in the most likely scenarios. We assume away the existence of a backstop in order to highlight these innovative aspects.

The outline of the paper is as follows. We set up the model in section 2. Section 3 deals with the case of an abundant resource, whereas section 4 treats the case of a limited resource. Section 5 concludes.

2. The model and preliminary results.

In this section we introduce the formal model and provide some first results.

2.1 The model and necessary conditions for optimality.

The social planner maximizes societal welfare, composed of three elements. First there is the utility of consuming a commodity produced from a nonrenewable natural resource, such as fossil fuel. The second element consists of the cost of capturing. Finally, there is the damage from the accumulated stock of pollutants. Social welfare is given by

$$\int_0^\infty e^{-\rho t}\left[u(x(t))-c(a(t))-h(Z(t))\right]dt.$$

The rate of fossil fuel use is $x(t)$. Extraction cost is zero. Fossil fuel use yields instantaneous utility $u(x(t))$. Capturing is denoted by $a(t)$, which brings a cost $c(a(t))$. The stock of accumulated CO2 is $Z(t)$ causing damage $h(Z(t))$. Damage appears directly in the social welfare function. Alternatively, damage occurs in production (Nordhaus, 2008, and Rezai et al., 2012), but here there is no production so that the direct approach is appropriate. Finally, ρ is the constant rate of time preference, assumed positive. Regarding the functions involved we make the following assumptions.

Assumption 1.

Instantaneous gross surplus u is strictly increasing, strictly concave and satisfies $\lim_{x\downarrow 0} u'(x)=\infty$ and $\lim_{x\to\infty} u'(x)=0$.

Assumption 2.

The damage function h is assumed strictly increasing and strictly convex with $h(0)=0$,

$\lim_{Z\downarrow 0} h'(Z)=0$ and $\lim_{Z\to\infty} h'(Z)=\infty$.

Assumption 3.

The capturing cost function c is strictly increasing and convex.

We allow for different alternative properties within the class defined in this assumption: linear capturing costs, as well as strictly convex capturing costs (with zero or positive marginal costs at zero capturing).

The accumulation of CO2 is described by

(1) $\dot{Z}(t)=\zeta x(t)-a(t)-\alpha Z(t), \quad Z(0)=Z_0.$

Here Z_0 is the given initial CO2 stock. Net emissions are $\zeta x(t)-a(t)$. Decay of CO2 is exponential at a constant and positive rate α. This is an heroic assumption[9]. Let $X(t)$ denote the stock of fossil fuel time t and denote the initial stock by X_0. Then

(2) $\dot{X}(t)=-x(t), \quad X(0)=X_0.$

(3) $X(t)\geq 0.$

Since marginal utility goes to infinity as consumption of fossil fuel goes to zero, we don't mention the nonnegativity constraint on oil extraction explicitly. A distinguishing feature of our approach is that we don't allow for capturing CO2 from the atmosphere. Hence, only current emissions can be abated. The idea is that CO2 capturing at electricity power plants is far less costly than capturing CO2 from transportation, for example. So, in addition to non-negativity of capturing we impose non-negativity of net emissions.

(4) $a(t)\geq 0.$

(5) $\zeta x(t)-a(t)\geq 0.$

The current-value Lagrangian corresponding with maximizing social welfare reads

$$L(Z,X,x,a,\lambda,\mu,\gamma_a,\gamma_{xa})=u(x)-c(a)-h(Z)-\lambda[\zeta x-a-\alpha Z]+\mu[-x]+\gamma_a a+\gamma_{xa}[\zeta x-a].$$

Here λ is the shadow *cost* of pollution and μ is the shadow value of the stock of fossil fuels. It vanishes in case of an abundant resource. Omitting the time argument when there is no danger of confusion, we have as the necessary conditions the equations (1)-(5) and

[9] The process of decay is much more complicated in reality, because of all kinds of possible feedbacks and because part of the CO2 stock stays in the atmosphere indefinitely. See Farzin and Tahvonen (1996) for an early economic contribution, basing themselves on Maier-Raimer and Hasselman (1987). For more recent work, see Archer (2005), Archer et al. (2009) and Allen et al. (2009). For a recent discussion of the carbon cycle and its potential consequences for economic policy, see Amigues and Moreaux (2011) and Gerlagh and Liski (2012).

(6) $\dfrac{\partial L}{\partial x}=0: u'(x)=\mu+\zeta(\lambda-\gamma_{xa}).$

(7) $\dfrac{\partial L}{\partial a}=0: \lambda=c'(a)+\gamma_{xa}-\gamma_a.$

(8) $\gamma_a a=0, \quad \gamma_a\geq 0, \quad a\geq 0.$

(9) $\gamma_{xa}[\zeta x-a]=0, \quad \gamma_{xa}\geq 0, \quad \zeta x-a\geq 0.$

(10) $\dfrac{\partial L}{\partial X}=-\dot{\mu}+\rho\mu: \dot{\mu}=\rho\mu.$

(11) $\dfrac{\partial L}{\partial Z}=\dot{\lambda}-\rho\lambda: \dot{\lambda}=(\rho+\alpha)\lambda-h'(Z).$

(12) $e^{-\rho t}[\lambda(t)Z(t)+\mu(t)X(t)] \quad \to \quad 0 \quad \text{as} \quad t\to\infty.$

Conditions (6)-(9) are necessary and sufficient to maximize the Hamiltonian with respect to fossil fuel use and capturing, yielding that marginal benefits from capturing equal marginal cost, if capturing is taking place. Equations (10) and (11) are the usual no-arbitrage conditions. Finally, equation (12) is the transversality condition.

In the sequel we make a distinction between cheap and expensive CCS technologies. The definition we employ makes use of the steady state of the economy endowed with an infinite resource stock but lacking the CCS technology. With an infinite resource, $X_0=\infty$, the shadow price of oil vanishes: $\mu=0$. Since without the CCS option we have $\zeta x-a=\zeta x>0$, due to $u'(0)=\infty$, implying $\gamma_{xa}=0$, the necessary condition (6) reduces to $u'(x)=\zeta\lambda.$ From this we derive $x=x(\lambda)$ with $x'<0$, $\lim_{\lambda\downarrow 0}x(\lambda)=\infty$ and $\lim_{\lambda\to\infty}x(\lambda)=0.$ Then $\dot{Z}=\zeta x(\lambda)-\alpha Z$. This yields the following phase diagram in $(\lambda,Z)-$space. See figure 1.[10] The isocline $\dot{\lambda}=0$ is increasing because $h''(Z)>0$. The isocline $\dot{Z}=0$ is decreasing because x is decreasing in λ. The steady state (λ^*,Z^*) for this economy is defined by

(13) $u'(\dfrac{\alpha Z^*}{\zeta})=\dfrac{\zeta h'(Z^*)}{\alpha+\rho}, \lambda^*=\dfrac{h'(Z^*)}{\alpha+\rho}.$

This follows from setting $\dot{\lambda}=\dot{Z}=0$, implying $\zeta x=\alpha Z, u'(x)=\zeta\lambda$ and $\lambda=h'(Z)/(\alpha+\rho).$ The steady state is a saddle point because $u''<0$ and $h''>0$. The steady state is illustrated in figure 1, where $\Lambda(Z)$ is the stable manifold.

INSERT FIGURE 1 ABOUT HERE

[10] For the time being c and Z^h appearing in the diagram can be ignored, because in a world without capturing technology they don't have a meaning.

For high initial CO2 stocks, the stock is decreasing towards the steady state. Initially, the use of oil is low (the shadow price λ is high) in order to reduce the CO2 stock, but it is increasing. For small initial CO2 stocks, the reverse scenario prevails.

Next we introduce the notion of cheap and expensive capturing technologies. The capturing technology is called *cheap (expensive)* if $\lambda^* = h'(Z^*)/(\alpha+\rho) \geq (<) c'(0).$ This definition is motivated by the insight that if a resource abundant economy finds itself in the steady state Z^* and the capturing technology would become available, the technology would not be used if it is expensive but it would be used when cheap. Indeed, total discounted marginal damages in the steady state are $h'(Z^*)/(\alpha+\rho)$ and capturing makes sense only if the marginal capturing cost at zero capture outweigh the total discounted marginal damages. Define $(\tilde{Z},\tilde{\lambda})$ by

$$(14) \qquad \frac{h'(\tilde{Z})}{\alpha+\rho} = \tilde{\lambda} = c'(0)$$

So, $\tilde{Z}$ is the level of the CO2 stock for which CCS is neither cheap nor expensive.

3. CCS available, abundant resource.

In this section we describe the optimum in the presence of an abundant oil stock. With an abundant resource stock, the CO2 stock is monotonic: $\dot{Z}(t_1) \geq (\leq) 0$ for some $t_1 \geq 0$ implies $\dot{Z}(t) \geq (\leq) 0$ for all $t > t_1$. If this were not true, then, given that the CO2 stock is the only state variable, there would exist a state from which it is optimal to increase emissions, whereas at some other instant of time, with the same CO2 stock, it would be optimal to let it decrease. But the optimum is unique given our convexity assumptions. We also have that $\dot{Z}(0) \geq 0$ implies $\dot{\lambda}(t) \geq 0$ for all $t \geq 0$ and $\dot{Z}(0) \leq 0$ implies $\dot{\lambda}(t) \leq 0$ for all $t \geq 0$. To prove the first claim, suppose $\dot{Z}(0) \geq 0$ and $\dot{\lambda}(t_1) < 0$ for some $t_1 \geq 0.$ It follows from the monotonicity of Z and from $\dot{\lambda}(t) = (\rho+\alpha)\lambda(t) - h'(Z(t))$ that λ becomes negative eventually, which is clearly suboptimal. To prove the second claim, suppose $\dot{Z}(0) \leq 0$ and $\dot{\lambda}(t_1) > 0$ for some $t_1 \geq 0.$ It follows that λ goes to infinity. Since $\lambda = c'(a) + \gamma_{xa} - \gamma_a$, this is incompatible with $a = 0$ because then $\gamma_{ax} = 0$ and $\lambda = c'(0) - \gamma_a < \infty$. It follows that $a > 0$ and $u'(x) = \zeta c'(a)$, which implies from $\zeta x \geq a$ that a is bounded from above. Therefore, for λ going to infinity, we need $\gamma_{ax} \to \infty$ as $t \to \infty$ so that eventually $u'(x) = \zeta c'(a)$ and $\zeta x = a$. Hence, Z approaches zero as time goes to infinity. This is suboptimal since $c'(a) > 0$ and $h'(0) = 0$.

3.1 Expensive CCS.

We first consider the case of expensive capturing: $\lambda^* = h'(Z^*) / (\alpha+\rho) < c'(0).$ This excludes $c'(0) = 0.$

For expensive CCS we define Z^h by $c'(0) = \Lambda(Z^h)$ where Λ is the stable manifold of the resource abundant economy that has no capturing technology. See figure 1. Hence, Z^h is located on the stable manifold at the point where $\lambda = c'(0) > \lambda^*$. Note that $Z^h < \infty$. To see this, suppose $Z^h = \infty$. Then in the resource abundant economy without the capture option it would for any large initial pollution stock be optimal to have an initial consumption rate bounded away from zero ($u'(x(0)) = \zeta\lambda(0) < c'(0)$). Hence, marginal utility is bounded from above, whereas marginal disutility of pollution is arbitrarily large. This is obviously suboptimal: it is better to give up some consumption, and therefore utility, and to benefit greatly from smaller damages.

If $Z_0 \leq Z^h$ it is optimal to never capture CO2, because the marginal cost of doing so is too high. If the economy just follows the program of the economy without the CCS technology, all necessary conditions are satisfied, so that this program is optimal. Let us therefore consider initial CO2 levels strictly larger than Z^h. If the marginal capturing cost is constant, $c(a) = c$, then the optimum is to have full capture until Z^h is reached, and thereafter no capturing will take place anymore (see figure 1). With strictly convex capturing cost the situation is slightly more complicated. The corresponding figure A1 can be found in appendix A. Clearly, there will be full capturing if the CO2 stock is large enough. Moreover, the stock will decrease monotonically and will eventually approach Z^*. But contrary to the case of constant marginal cost of capturing, there must now be a phase with partial capturing. To see this recall that $\lambda = c'(a) + \gamma_{xa} - \gamma_a$. A transition from full capturing to zero capturing requires a downward discontinuity in $c'(a)$. Also, γ_{ax} becomes zero at the transition, whereas γ_a will not decrease at the transition. As a consequence, λ exhibits a downward jump, which contradicts that the co-state is continuous. How can we determine the CO2 stocks at which the transitions take place? Let us denote the instant of time where the transition takes place from full capturing to partial capturing by T_1 and the instant of time of the transition to zero capturing by T_2. Of course $Z(T_2) = Z^h$ and $\lambda(T_2) = c'(0)$. For every given T_1 we can uniquely determine $Z(T_1)$ from $\dot{Z}(t)/Z(t) = -\alpha$ for all $0 \leq t \leq T_1$ and $Z(0) = Z_0$. Also $\lambda(T_1) = c'(a(T_1))$. These are the starting values for the partial capture phase, where $\lambda(t) = c'(a(t)), \zeta\lambda(t) = u'(x(t))$, so that Z and λ are fully determined by these initial conditions and $\dot{\lambda} = (\rho + \alpha)\lambda - h'(Z)$ and $\dot{Z} = \zeta x(\lambda) - a(\lambda) - \alpha Z$. The transition times are then determined by the requirement that $Z(T_2) = Z^h$ and $\lambda(T_2) = c'(0)$.
Summarizing, we have shown

Proposition 1.

Suppose capturing costs are strictly convex, capturing is expensive and the resource is abundant. Then with a low initial CO2 stock it is optimal not to capture CO2 at all. For a sufficiently high initial CO2 stock it is optimal to have full capturing initially, then follows a phase with partial capturing, and there is a final phase without capturing. The second phase collapses if the marginal capture cost is constant.

3.2 Cheap CCS

To tackle this case, the distinction between the types of capturing cost functions is important.

3.2.1. Constant marginal cost: $c(a) = ca, c > 0.$

Clearly, as long as $Z(t) < \tilde{Z}$ no capturing will take place, since $h'(Z)/(\rho + \alpha) < c'(0).$ However, the CO2 stock will monotonically increase until $\tilde{Z}$ is reached. The increase of CO2 will not go beyond this level, because, once at this level, it is optimal to have partial capturing forever. For initial CO2 stocks larger than $\tilde{Z}$ it is optimal to have full capturing. The corresponding figure A2 is given in Appendix A.

3.2.2. Strictly convex capturing cost with $c'(0) > 0.$

It is instructive to draw another phase diagram in (λ, Z) -space. See figure 2. The locus of points where λ is constant is the same as in figure 1. Let us, for the time being, assume that capturing is partial forever. Then $\lambda(t) = c'(a(t)), u'(x(t)) = \zeta\lambda(t)$. This yields a and x as functions of λ so that the locus of points for which the CO2 stock is constant, is given by $\zeta x(\lambda) - a(\lambda) = \alpha Z$. This is a downward sloping curve. But it has to be taken into account that for partial capturing to prevail, we need $\lambda \geq c'(0)$. Moreover, there is an upper bound on λ, denoted by $\bar{\lambda}$, in order to satisfy the condition $\zeta x(\lambda) - a(\lambda) \geq 0.$ Since it has been assumed that capturing is cheap, a steady state exists: $(\hat{\lambda}, \hat{Z})$. Then, for a large initial CO2 stock, larger than Z^m in figure 2, it is optimal to have full capturing initially, followed by a period of time with partial capturing. With a very small initial CO2 stock, smaller than Z_a in figure 2, it is optimal to have zero capturing initially, followed by partial capturing for the rest of time. If $Z_a < Z_0 < \hat{Z}$ partial capturing prevails from the initial instant of time on. Note that if we would neglect the condition $\zeta x - a \geq 0$ the optimal path would be like the one indicated by (1) in figure 2. The figure has been drawn for not too cheap CCS, so that one can be sure that there is an initial phase with zero capture if the initial CO2 stock is small. Otherwise, it is optimal to have partial CCS from the start.

INSERT FIGURE 2 ABOUT HERE

3.2.3. *Strictly convex capturing cost with* $c'(0)=0$.

In this case it is clearly optimal to have carbon capturing forever. The optimum is therefore partial capture for low CO2 stocks, and possibly full capturing with high initial CO2 stocks.

One final remark is in order. With an abundant resource, the problems that we have considered thus far, with constant and increasing marginal capture costs, are essentially equivalent to the optimal use of a costly backstop technology. If we define $y = x + a/\zeta$ as total consumption, originating from the natural resource x and from a backstop a, properly scaled, and if the cost of producing the backstop is given by $c(a)$ then we have utility $u(y)$ and accumulation of pollution is given by $\dot{Z} = \zeta(y-a) - \alpha Z$. Hence, in mathematical terms, the backstop problem is essentially the same as the capturing problem. From the propositions that have been established we can then infer that a cheaper backstop will always lead to less pollution, as long as the backstop cost is not prohibitively high. The equivalence result no longer holds if the natural resource is exhaustible, to which we turn now.

4. Optimal capturing with a finite resource stock

4.1 General approach

The next step is to consider a finite oil stock, which is the main contribution of this study. Tahvonen (1997) studies a world without CCS with a finite resource stock[11]. He proves two propositions that serve as important benchmarks for the results that we obtain in the sequel. For the sake of notation we denote variables of the Tahvonen economy by the superscript T. The first property of the optimum is that, given the initial resource stock, for a low enough initial CO2 stock the shadow price of CO2 (λ^T) is inverted U-shaped. Otherwise, it is monotonically decreasing. Second, given the initial resource stock, for a low enough initial CO2 stock the CO2 stock is inverted U-shaped, and monotonically decreasing otherwise. In our analysis we need a somewhat different but related result, namely that, for a given initial CO2 stock below Z^*, the steady state value in case of an abundant resource stock, the CO2 stock will initially increase if the resource stock is large enough, and the other way around. Formally, for all $Z_0 < Z^*$ there exists W, depending on Z_0, such that $x^T(0) > (<) \alpha Z_0/\zeta$ if and only if $X_0 > (<) W$. To prove this property suppose that there exists $Z_0 < Z^*$ such that for all X_0 we have $x^T(0) < \alpha Z_0/\zeta$. As was demonstrated by Tahvonen, since the CO2 stock decreases initially, it decreases forever, implying that $\lambda^T(0) \le \dfrac{h'(Z_0)}{\alpha+\rho}$. Hence

[11] Tahvonen (1997) allows for a backstop technology and for stock dependent extraction cost. In describing Tahvonen's contribution we abstract from these issues, because we don't have them in our model.

$\mu_0^T = u'(x^T(0)) - \lambda^T(0) > u'(\alpha Z_0 / \zeta) - \frac{h'(Z_0)}{\alpha + \rho}$. The right hand side of this expression is bounded away from zero, implying that the initial stock X_0 is finite, a contradiction. Since the extraction rate is continuous over time, it is clear that with a small initial resource stock, it cannot be optimal to have an initial increase of the CO2 stock. Then the claim follows from a continuity argument.

We aim at constructing a diagram in the space of resource stocks and CO2 stocks in which we can unambiguously indicate the optimal program, corresponding with each state where the economy finds itself. Three types of phases can be distinguished, as in the previous section: full capture, partial capture and zero capture. In principle many transitions from one phase to another could be optimal, leading to a large number of potentially optimal regimes. However, we will state several general lemmata that will enable us to restrict attention to specific sequences of phases. The proofs of the lemmata are given in appendix B.
First of all, an interval of time with full capturing can only occur at the outset.

Lemma 1

Suppose there exist $0 \le t_1 < t_2$ such that $\zeta x(t) = a(t)$ for all $t \in [t_1, t_2]$. Then $\zeta x(t) = a(t)$ for all $t \in [0, t_2]$.

The intuition behind this property of the optimal program is that full capture should take place with a large CO2 stock, because marginal damages are very high. After full capture the economy will never again build up CO2 to such a degree that full capture is needed again. Actually, contrary to the case of an infinite resource stock, extraction, capturing and the CO2 stock itself will converge to zero now.

Lemma 2

Suppose the resource stock is finite. Then $(x(t), a(t), Z(t)) \to (0,0,0)$ as $t \to \infty$

In the previous section we made use of the fact that the CO2 stock and the co-state variable were monotonic. This is no longer the case now, as will be shown later. However, it is the case, that once the co-state λ is decreasing, it will decrease forever thereafter. So, the co-state variable in principle has an inverted-U shape.

Lemma 3

Suppose $\dot{\lambda}(t_1) \le 0$ for some $t_1 \ge 0$. Then $\dot{\lambda}(t) \le 0$ for all $t \ge t_1$.

A stronger property was proven by Tahvonen (1997) in an economy without the CCS technology. There the shadow price keeps decreasing strictly once it starts decreasing strictly. This will not be so in our model. Nevertheless, in some instances we can use the Tahvonen economy as a benchmark..

Lemma 4

Suppose $\lambda^T(t_1) > c'(0)$ for some $t_1 \geq 0$ and $Z_0 > \tilde{Z}$. Then $a(0) > 0$.

The next lemma excludes a sequence of first partial capture, then zero capture and finally partial capture again.

Lemma 5

It is impossible to have the following sequence in an optimum. There exist $0 < t_1 < t_2 < t_3$ such that $\zeta x(t) - a(t) > 0, a(t) > 0$ for all $t \in [0, t_1)$, $a(t) = 0$ for all $t \in [t_1, t_2)$, and $\zeta x(t) - a(t) > 0, a(t) > 0$ for all $t \in [t_2, t_3)$.

This lemma doesn't exclude the potential optimality of having zero capturing first, then partial capturing and then zero capturing once more. A formal proof will be provided in due course. But the intuition is clear. Suppose that $c'(0) > 0$ (because otherwise there will always be some capturing). Consider the optimum in the Tahvonen economy without the CCS option. Assume the initial CO2 stock is relatively low. Then the optimum for the co-state λ^T is inverted U-shaped. Denote the maximum by $\hat{\lambda}^T$. If $c'(0) \geq \hat{\lambda}^T$ it is suboptimal to use CCS ever. Now suppose that $c'(0)$ is only slightly smaller than $\hat{\lambda}^T$. Then, the optimum in the economy with the CCS option will not differ much from the Tahvonen economy. But if it would be optimal to abate initially (and not just a bit later), we would have $\lambda(0) \geq c'(0)$. But $c'(0)$ is close to $\hat{\lambda}^T$ and $\hat{\lambda}^T$ is strictly larger than $\lambda^T(0)$. So, $\lambda(0) - \lambda^T(0) = \int_t^\infty e^{-(\rho+\alpha)s} h'(Z(s)) - h'(Z^T(s)) ds$ is much larger than zero. Therefore, discounted future damages are much greater in the economy with capturing than in the economy without the capturing option.

4.2 Constant marginal capturing cost

In first instance we assume that capturing is cheap, so that $c = h'(\tilde{Z}) / (\rho + \alpha) < h'(Z^*) / (\rho + \alpha)$. The optimum is depicted in figure 3. Let us first consider the case $Z_0 = \tilde{Z}$ in detail. In the absence of CCS and with an abundant resource the CO2 stock monotonically increases from $Z_0 = \tilde{Z}$ to the steady state Z^*, as we have seen in figure 1. With a finite resource stock, we are in the Tahvonen (1997) economy.

As explained at the outset of section 4.1, there exists a threshold level, denoted by $\tilde{X}^T$, such that if $Z_0 = \tilde{Z}$ and $X_0 < \tilde{X}^T$ then the CO2 stock monotonically decreases along the optimum, whereas if $X_0 > \tilde{X}^T$ the CO2 stock will initially monotonically increase and monotonically decrease after some instant of time. Note that if $X_0 \leq \tilde{X}^T$ then $\lambda^T(0) < c$ because Z^T is decreasing and will therefore always be below $\tilde{Z}$ so that

$$\lambda^T(0) - c = \int_0^\infty e^{-(\rho+\alpha)s}(h'(Z^T(s)) - h'(\tilde{Z}))ds < 0 .$$

There exists another threshold level $\tilde{X}^{MW} > \tilde{X}^T$ (MW indicating the authors of this paper), such that $\lambda^T(0) = c$ if $Z_0 = \tilde{Z}$ and $X_0 = \tilde{X}^{MW}$. Indeed $\tilde{X}^{MW} > \tilde{X}^T$ because otherwise $Z^T(t) \leq \tilde{Z}$ for all $t \geq 0$ and hence $\lambda^T(0) < c.$ The idea behind the construction of $\tilde{X}^{MW}$ is that in the Tahvonen economy a higher initial resource stock will trigger the economy to stay closer to the unconstrained optimum so that total discounted marginal damages are higher: $\lambda^T(0) > c$. We will denote the stable branch in $Z - X$ space, passing through $(\tilde{Z}, \tilde{X}^{MW})$ and leading to $(Z, X) = (0,0)$ by D.

INSERT FIGURE 3 ABOUT HERE

Now suppose that $Z_0 = \tilde{Z}$ and $X_0 = \tilde{X}^{MW}$. It is then optimal to have zero capturing forever. The optimum is just the optimum of the Tahvonen economy, starting from the same initial state. So, it is optimal to stay on the curve D. By construction $\lambda^T(0) = \lambda^{MW}(0) = c$ and $\dot{\lambda}^{MW}(t) = \dot{\lambda}^T(t) < 0$ for all $t > 0$. All the necessary conditions, that are sufficient conditions as well, are satisfied.

Suppose next that $Z_0 = \tilde{Z}$ and $X_0 < \tilde{X}^{MW}$. Then clearly it is optimal again to follow the Tahvonen optimal program again, without any use of CCS.

If $Z_0 = \tilde{Z}$ and $X_0 > \tilde{X}^{MW}$ then it is straightforward to see that the optimum consists of two phases. A first phase has partial capture. Along this phase the CO2 stock remains constant at the $\tilde{Z}$ level. The resource stock is reduced until $X(\tilde{T}) = \tilde{X}^{MW}$ at some instant of time $\tilde{T} \geq 0$. The path follows the curve denoted by E in figure 3. After $\tilde{T}$ we are in the Tahvonen economy with the property that the CO2 stock will first increase and then decrease. We should also have $a(\tilde{T}) = 0.$ Hence, the transition occurs in a smooth way. Along an interval with partial capture, the capture rate and resource use are both monotonically decreasing, as can be seen from (6) with $\lambda - \gamma_{xa} = c$ so that x decreases, and from (1) with Z constant and x decreasing. Hence the timing of the transition will guarantees continuity.

The next step is to look at the remaining cases of cheap capturing technologies. If the initial resource stock is below the curve D, the Tahvonen program is optimal. If the initial resource stock is above the

D-curve and above $\tilde{X}^{MW}$, but below E, so in the region delineated by the curves D and E, it is optimal to have an initial period of time with zero capturing, then a period of time with partial capturing, moving along E, and a final interval of time, starting at ($\tilde{Z}, \tilde{X}^{MW}$), with zero capturing again, following D from the moment of the transition on. Next, suppose $X_0 > \tilde{X}^{MW}$ and $Z_0 > \tilde{Z}$. There will be full abatement initially. But three possibilities exist afterwards. If the initial CO2 stock is relatively small, we will arrive on the curve E where it is optimal to have partial CCS for an interval of time before arriving at the Tahvonen path. However, with the initial CO2 stock very large it would take too long to get there and it is better to stop CCS after some instant of time altogether. The dividing curve between the two regimes is indicated by the curve F. In figure 3 also a curve G is drawn, which indicates the stocks at which it is optimal to switch from full CCS to abandon CCS. Hence, for $X_0 < \tilde{X}^{MW}$, it indicates that for small initial CO2 stocks there should be no CCS at all, whereas for large initial CO2 stocks it is optimal to have full deployment of CCS initially. Note that if we start below the G-locus the initial λ is then no greater than c and will decrease forever. Note also that this is feasible, because $\tilde{Z} < Z_0$ does by itself not imply that $\lambda^T > c$. However, if $\lambda^T(t_1) > c$ for some $t_1 \geq 0$, this possibility is excluded and initially there will be full capturing.
Thus we may conclude as follows.

Proposition 2.

Suppose constant marginal CCS cost ($c(a) = ca$) and a cheap capturing technology ($c < \lambda^*$ or, equivalently, $\tilde{Z} \leq Z^*$). Then, when (Z_0, X_0) lies:

-within zone 1, below and to the left of D and G, it is optimal to never capture CO2.

-within zone 2, between G and F, it is optimal first to fully capture and next, once a point on the locus G is attained, to stop capturing forever.

-within zone 3, between F and E, it is optimal first to fully capture in order to reduce the CO2 stock to the level $\tilde{Z}$, next switch to a partial capturing policy, maintaining the CO2 stock at this level until the instant of time where the resource stock has been reduced to $\tilde{X}^{MW}$, from where on capturing is no longer necessary.

-within zone 4, between E and D, no capturing is required initially and the CO2 stock increases up to the level $\tilde{Z}$, which, once attained, is maintained for a while thanks to a partial capturing policy, capture being given up forever once the resource stock has been reduced to $\tilde{X}^{MW}$.

We now move to the case of an expensive capturing technology: $\tilde{Z} > Z^*$. Partial capturing is excluded then. The reason is that with partial capturing we have $Z = \tilde{Z}, \lambda = \tilde{\lambda} = c$. Hence,

$u'(x) = \mu_0 e^{\rho t} + \zeta c = \mu_0 e^{\rho t} + \zeta h'(\tilde{Z}) / (\alpha + \rho) < u'(\alpha \tilde{Z} / \zeta),$ which is incompatible with $\tilde{Z} > Z^*$ since $\zeta h'(Z^*) / (\alpha + \rho) = u'(\alpha Z^* / \zeta)$.

If $Z_0 < \tilde{Z}$ then there is zero capture forever. The reason is that $a(0) = \zeta x(0)$ would imply $\lambda(0) \geq c$. Then, the co-state λ is increasing and it will never decrease. Hence there will be full capturing forever, which is suboptimal. So, it is optimal now to follow the Tahvonen economy.

If $Z_0 > \tilde{Z}$ and if we would start with zero capture, then CSS will never be used, because the co-state λ monotonically decreases. This occurs if the initial resource stock is small. But, with a large initial resource stock and a large initial CO2 stock, it is optimal to have an initial interval of time with full capturing, followed by an interval of zero capturing. Hence, another frontier exists between starting with full of zero capturing.

Proposition 3.

Suppose constant marginal CCS cost ($c(a) = ca$) and an expensive capturing technology ($c < \lambda^*$ or, equivalently, $\tilde{Z} > Z^*$). Then there exists a critical level of the CO2 stock, larger than $\tilde{Z}$, and decreasing with the resource endowment X_0, such that:

-for initial CO2 stocks smaller than the critical level there is zero capturing forever.

-for initial CO2 stocks larger than the critical level it is optimal to have full capturing initially, before switching to a zero capturing policy forever.

If climate change damages are incorporated in the model, not by means of a damage function in the social preferences but though a ceiling: $Z(t) \leq \bar{Z}$, then only necessary condition (11) changes. It becomes

(11') $\dot{\lambda} = (\rho + \alpha)\lambda - \pi$

where $\pi(t) \geq 0, \pi(t)[\bar{Z} - Z(t)] = 0$. In the case of constant marginal capture cost capturing only takes place at the ceiling. Indeed, suppose that at some instant of time we have $Z(t) < \bar{Z}$ and $a(t) > 0$. Then $\pi(t) = 0$ and $\dot{\lambda}(t) = (\rho + \alpha)\lambda(t)$, implying that λ is increasing. Since $\lambda(t) = c + \gamma_{ax}(t)$ it follows that $\gamma_{ax} > 0$ and increasing, so that there is full capturing and the CO2 stock declines. This process goes on, and the threshold will never be reached. Moreover, consumption and capturing both go to zero as time goes to infinity, whereas positive consumption, bounded away from zero, is feasible. Hence, there will only be capturing at the ceiling. This poses a danger, if the ceiling is motivated by interpreting it as a threshold level, beyond which a catastrophe occurs and if there is uncertainty regarding the effect of capturing. More importantly, our model without the ceiling allows for much more complicated behaviour of the CO2 stock, as outlined in proposition 2.

4.3 Strictly convex capturing cost with $c'(0)=0$

In this section we consider the case of increasing marginal capturing cost, with zero marginal capturing cost at zero capturing. Contrary to the case of constant marginal capturing there will always be some CO2 capture. Actually, it could be optimal to have full capturing indefinitely. The reason is that, because of the limited availability of the resource, the rate of extraction is necessarily becoming smaller over time, so that the effort needed to capture all emitted CO2 gets smaller over time as well, and therefore may be worthwhile. Let us study this possibility in some detail. In case of permanent full capturing we have $\zeta x(t)=a(t)$ for all $t\geq 0$. Also $\lambda(t)=c'(a(t))+\gamma_{ax}(t)$ for all $t\geq 0$ from (7) and (8). Hence, from (6), $u'(x(t))=\mu_0 e^{\rho t}+\zeta c'(\zeta x(t))$ for all $t\geq 0$. Therefore, the extraction rate $x(t)$ is a function of time and the shadow price μ_0. It is monotonically decreasing over time. The resource constraint $\int_0^\infty x(t)dt = X_0$ uniquely determines μ_0. Consequently, also $x(t)$ and $a(t)$ are determined for all $t\geq 0$ and capturing is monotonically decreasing. Moreover, with full capturing from the start

$$\lambda(t)=e^{(\rho+\alpha)t}\int_t^\infty e^{-(\rho+\alpha)s}h'(Z_0e^{-\alpha s})ds$$

for all $t\geq 0$. In order for full capturing to be optimal it must hold that $\gamma_{ax}(t)\geq 0$ for all $t\geq 0$. It is easy to construct an example where this condition is satisfied. Consider the following functions: $c(a)=\frac{1}{2}ca^2, u(x)=x^{1-\eta}/(1-\eta), h(Z)=\frac{1}{2}bZ^2$, where $c>0, b>0$ and $\eta\neq 1$ are constants. Then, in the proposed optimum we have $\lambda(t)=bZ_0e^{-\alpha t}/(\rho+2\alpha)$. Moreover, $\dot{x}(t)/x(t)\to -\rho/\eta$ as $t\to\infty$ and in the limit $c(a)$ will behave as $e^{-(\rho/\eta)t}$. If Z_0 is large enough and $\alpha<\rho/\eta$ there will always be full capturing, because $\lambda=c'(a)+\gamma_{ax}-\gamma_a$ so that $\gamma_{ax}(t)>0$ for all $t>0$. Intuitively this makes sense: with a high initial pollution stock, low decay and a large rate of time preference it is optimal to get rid of pollution as soon as possible, and there is not much care for the future. We conclude that it is well possible to have full capturing forever. This occurs for high initial pollution stocks and low decay rates.

Next the question arises in what circumstances there will always be partial capturing. Along any interval of time with partial capturing the following holds:

$$u'(x(t))=\mu_0e^{\rho t}+\zeta c'(a(t))$$

$$\lambda(t)=c'(a(t))$$

$$\dot{\lambda}(t)=(\alpha+\rho)\lambda(t)-h'(Z(t))$$

$$\dot{Z}(t)=\zeta x(t)-a(t)-\alpha Z(t)$$

For a small initial resource stock the shadow price will be high (μ_0 high). Hence, with a large initial CO2 stock, and therefore a high initial shadow price $\lambda(0)$, it is definitely not optimal to start with partial capturing, because it follows from the first two equations that net emissions will be negative then. So, in order to have partial capturing throughout the initial pollution stock should not be too large and the resource stock should not be too small. Hence, to conclude the discussion, we state

Proposition 4

Suppose capturing costs are strictly convex with $c'(0) = 0$.

There will always be some CO2 capturing. Full capturing throughout is warranted for high initial CO2 stocks and low decay rates. Partial capturing throughout is in order for a low initial CO2 stock and a relatively large resource stock.

4.4 Strictly convex capturing cost with $c'(0) > 0$

With increasing marginal capturing cost the optimal pattern of capturing takes four possible forms, which are summarized in

Proposition 5

Suppose capturing costs are strictly convex with $c'(0) > 0$.

For high enough Z_0 it is optimal to have an initial phase with full capturing. Then follows a phase with partial capturing, and there is a final phase with zero capturing.

For intermediate levels of Z_0 the optimal sequence is: zero capturing, then partial capturing and finally zero capturing, with the first phase possibly degenerate.

For low levels of Z_0 it is optimal to have zero capturing throughout.

Proof

It has been shown before that full capturing can only occur at the outset of the planning period. Given that the marginal capturing cost at zero capturing is bounded away from zero, there should be no capturing eventually. From full capturing there is no transition possible to zero capturing, because that would violate the continuity of the co-state λ. Clearly, for high initial CO2 stocks one should start with full capturing. For low initial values of the CO2 stock, we are in the Tahvonen world where capturing is not needed. For intermediate initial CO2 stocks it is optimal to build up the stock first, and then to have partial capturing. Q.E.D.

5. Conclusions

In this paper we have given a full account of optimal CCS under alternative assumptions regarding capture cost in the case of an abundant stock of fossil fuels, that cause emissions of CO2. It has been shown that depending on initial conditions and the specification of capturing costs optimal policies may differ considerably. In the most realistic case of marginal capturing cost bounded far away from zero, no capturing is warranted at all. Otherwise, we might have full capturing initially, if the initial CO2 stock is high. But eventually capturing is partial. If exhaustibility is taken into account, in this case of bounded marginal capture cost, the picture changes. Optimal capturing is zero eventually. Hence, any regime with partial capturing comes to an end within finite time. With a high initial CO2 stock it is optimal to have full use of CCS. The general picture that arises for the, most realistic, cases where the marginal capture costs is bounded from below, is that the CO2 stock is inverted-U shaped. With a large initial resource stock it will initially increase, CSS is not used, then CCS is used partially, whereas in a final phase no capturing will take place. With constant marginal capture cost, the CO2 stock is stabilized at a certain level as long as partial capture takes place, but then definitely the CO2 stock increases for a period of time before approaching zero in the end. Compared with a world where for one reason or another an exogenous upper bound is set for the pollution stock, we find that, if we would put such an upper bound in addition to the damage function, it is well possible to have CCS use before the upper bound is reached.

The implementation of the first-best outcome in a decentralized economy is simple, at least from a theoretical perspective. If the resource extracting sector is competitive and also generates the energy for the consumers and owns the CCS technology, then it suffices to impose a carbon tax corresponding with marginal damage, evaluated in the optimum: $\tau(t) = \lambda(t) = e^{(\rho+\alpha)t} \int_t^\infty e^{-(\rho+\alpha)s} h'(Z(s))ds$

If the extractive sector , the energy generating sector and the CCS sector are distinct industries, then the same tax needs to be imposed on the energy generating sector, but in addition, the profit maximizing CCS industry needs to face the constraint that it cannot capture more than the CO2 emitted.

Finally, a crucial question is where the world's actual initial position is. It should be possible to accurately assess the amount of CO2 that is in the atmosphere at present, as well as the CO2 in the crust of the earth. But, to take the simple case of constant marginal CCS cost amounting to approximately $60, we then still need to specify the global damage function and the estimates of marginal damages vary considerably among studies. Moreover, the model we consider lacks the complexity of the real world. We have treated energy as a commodity that yields utility directly, whereas it should play a role in production rather than in consumption. Finally, for the description of the carbon cycle, we have followed an approach that is well established in economics, but, as we have stressed before (see footnote 10), that could be modified according to new insights from climatologists, according to which part of current emissions stay in the atmosphere indefinitely. With an abundant resource this would not lead to outcomes that qualitatively differ from what we found in section 3. We also conjecture that our results go through in case of decay being a strictly increasing and strictly convex function off the existing pollution stock. Most likely, the case for early CCS becomes stronger.

References

Allen, M., Frame, D. and Mason, C. (2009): "The case for mandatory sequestration", Nature Geoscience 2, 813-814.

Amigues, J.-P. And Moreaux, M. (2011): "The atmospheric carbon resilience problem: A theoretical analysis", mimeo Toulouse School of Economics.

Amigues, J.-P., Lafforgue, G. and Moreaux, M. (2010): "Optimal capture and sequestration from the carbon emission flow and from atmospheric carbon stock with heterogeneous energy consuming sectors", Working paper 10.05.311, LERNA.

Amigues, J.-P., Lafforgue, G. and Moreaux, M. (2012): "Optimal timing of carbon capture policies under alternative CCS cost functions", mimeo, INRA and LERNA.

Archer, D. (2005): "Fate of fossil uel CO2 in geological time", Journal of Geophysical Research, 110, C09S05, doi: 10.1029/2004JC0002625.

Archer, D., Eby, M., Brovkin, V., Ridgwell, A., Cao, L., Mikolajewicz, U., Caldeira, K., Matsumoto, K., Munhoven, G., Montenegro, A. And Tokos, A. (2009): "Atmospheric lifetime of fossil fuel carbon dioxide", Annual Review of Earth and Planetary Sciences 37, 117-134.

D'Arge, R. and K. Kogiku (1973): "Economic growth and the environment", Review of Economic Studies, 40, 61-77.

Chakravorty, U., Magné, M. and Moreaux, M. (2006): "A Hotelling model with a ceiling on the stock of pollution", Journal of Economic Dynamics and Control 30, 2875-2904.

Coulomb, R. and Henriet, F. (2010): "Carbon price and optimal extraction of a polluting fossil fuel with restricted carbon capture", PSE Working Paper 2010-11, PSE.

Edmonds, J. (2008): "The potential role of CCS in climate stabilization", Key note address at the 9th International Conference of Greenhouse Gas Control Technologies, Washington, cited by Herzog (2011)

Farzin, H. and Tahvonen, O. (1996): "Global carbon cycle and the optimal time path of a carbon tax", Oxford Economic Papers 48, 515-536.

Feenstra, C., Mikunda, T. and Brunsting, S. (2010): "What happened in Barendrecht? Case study on the planned onshore carbon dioxide storage in Barendrecht, the Netherlands", Report by the Energy Research Centre of the Netherlands and Global CCS Institute.

Gerlagh, R. and Liski, M. (2012): "Carbon prices for the next thousand years", mimeo Tilburg University.

Hamilton, M., Herzog, H. and Parsons, J. (2009): "Cost and U.S. public policy for new coal power plants with carbon capture and sequestration", Energy Procedia, GHGT9 Procedia 1, pp. 4487-4494.

Herzog, H. (2011): "Scaling up carbon dioxide capture and storage: From megatons to gigatons", Energy Economics 33, pp. 597-604.

Hoel, M. and Kverndokk, S. (1996): "Depletion of fossil fuels and the impacts of global warming", Resource and Energy Economics, 18, 115-136.

Intergovernmental Panel on Climate Change (2005): "IPCC Special report on carbon dioxide capture and storage". Prepared by Working Group III of the IPCC, Cambridge University Press.

Jaakkola, N. (2012): "Monopolistic sequestration of European carbon emissions", mimeo, Department of Economics, University of Oxford.

Lafforgue, G., Magnė, B. and Moreaux, M. (2008a): "Optimal sequestration policy with a ceiling on the stock of carbon in the atmosphere", in R. Guesnerie and H. Tulkens (eds.), The design of climate policy, M.I.T. Press, Cambridge, pp. 273-304.

Lafforgue, G., Magnė, B. and Moreaux, M. (2008b): "Energy substitutions, climate change and carbon sinks", Ecological Economics 67, pp. 589-597.

Maier-Raimer, , E. and Hasselman, K. (1987): "Transport and storage of CO2 in the ocean-An inorganic ocean-circulation carbon cycle model", Climate Dynamics 2, 63-90.

Nordhaus (2008), A question of balance: Economic Models of Climate Change, Yale University Press, New haven CT.

Ploeg, F. van der and C. Withagen (2012a). “Is there really a Green Paradox?”, Journal of Environmental Economics and Management 64, pp. 342-363.

Ploeg, F. van der and C. Withagen (2012b): “Economic development, renewables and the optimal carbon tax”, Research Paper 55, Oxcarre, University of Oxford, Oxford, U.K. Forthcoming in International Economic Review.

Plourde, C. (1972): “A model of waste accumulation and disposal”, Canadian Journal of Economics, 5, 119-25.

Rezai, A., Van der Ploeg, F. and Withagen, C. (2012): “The Optimal Carbon Tax and Economic Growth: Additive versus multiplicative damages“, Research Paper 93, Oxcarre, University of Oxford, Oxford, U.K.

Tahvonen, O. and Withagen, C. (1996): “Optimality of irreversible pollution accumulation”, Journal of Economic Dynamics and Control, 20, pp. 1775-1795.

Tahvonen, O. (1997): “Fossil fuels, stock externalities, and backstop technology”, Canadian Journal of Economics 30, pp. 855-874.

Tsur, Y. and A. Zemel (2003): “Optimal transition to backstop substitutes for nonrenewable resources”, Journal of Economic Dynamics and Control, 27, 551-572.

Tsur, Y. and A. Zemel (2005): “Scarcity, growth and R&D”, Journal of Environmental Economics and Management, 49, 484-499.

Ulph A. and D. Ulph (1994): “The optimal time path of a carbon tax”, Oxford Economics Papers, 46, 857-868.

Withagen, C. (1994): “Pollution and exhaustibility of fossil fuels”, Resource and Energy Economics, 16, 235-242.

ZEP (2011): “The cost of CO2 capture, transport and storage: Post demonstration CCS in the EU”, Zero Emissions Platform: European Technology Platform for Zero Emission Fossil Fuel Power

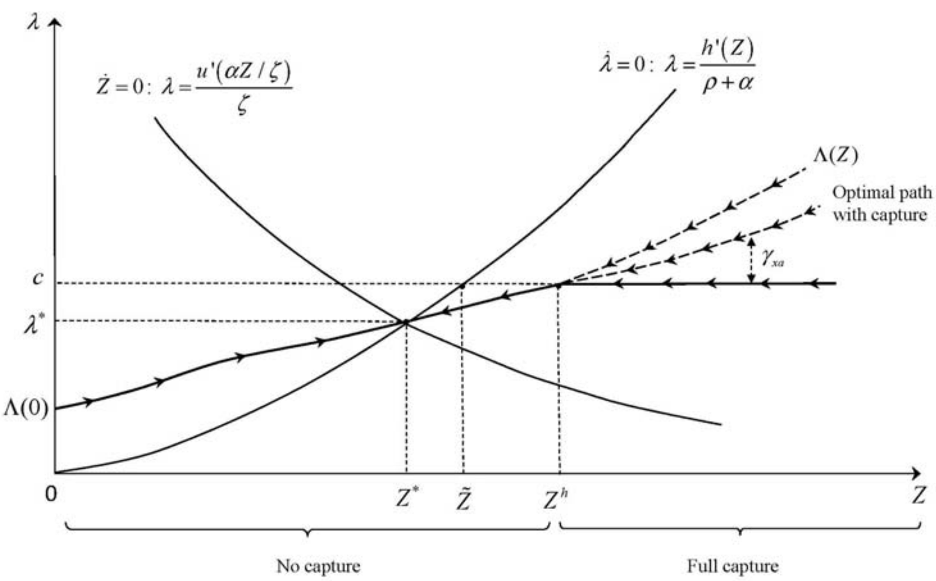

Figure 1: Phase diagram. Abundant resource and high marginal constant capture cost.

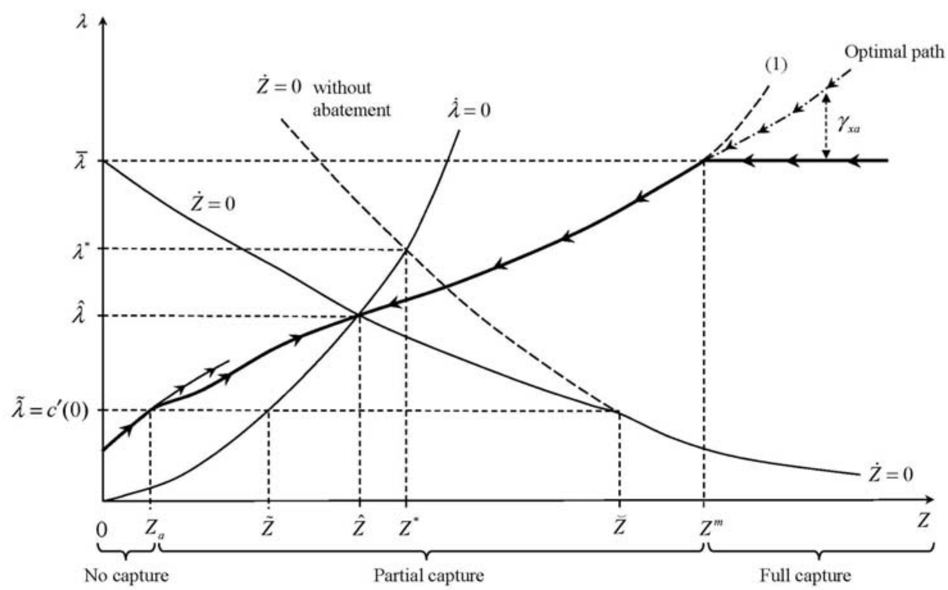

Figure 2: Phase diagram. Cheap strictly convex capture cost with $c'(0) > 0$ and an abundant resource.

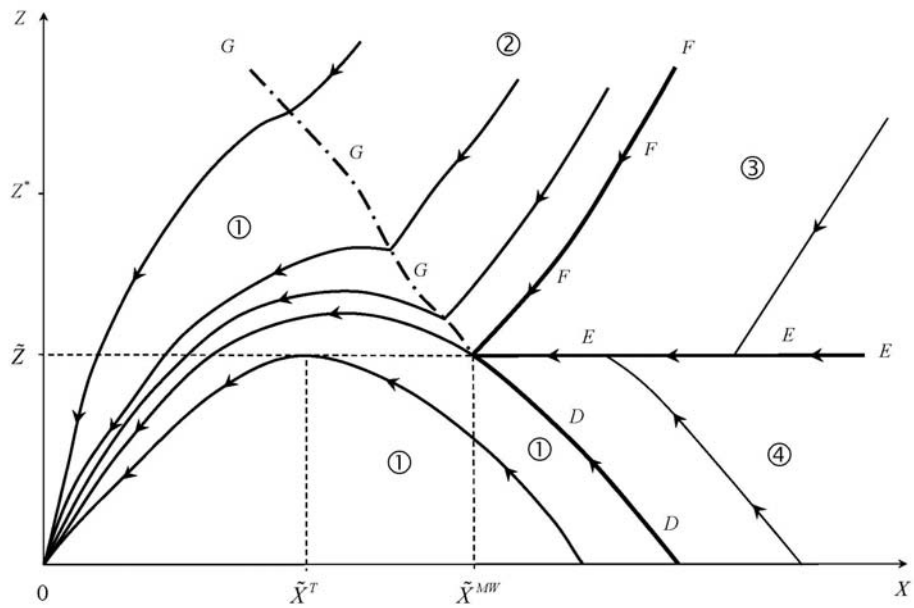

Figure 3: Phase diagram. Finite stock of non renewable resource and low constant marginal capture costs.

Appendix A. Figures A1 and A2

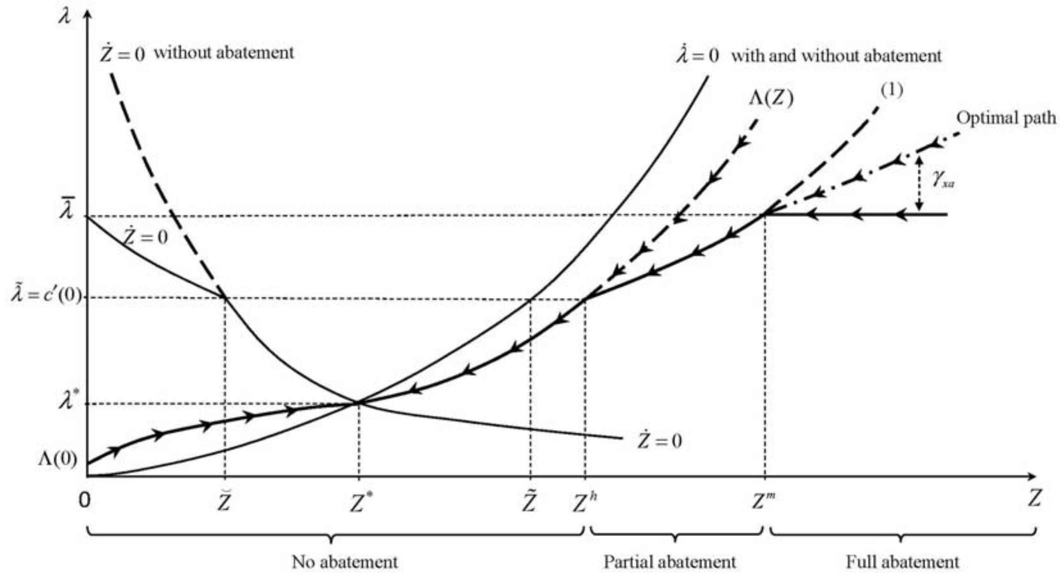

Figure A.1: Phase diagram. Convex abatement cost with large $c'(0)$: $c'(0) > \lambda^*$ and an abundant resource.
NB. (1) Path of λ with abatement neglecting the constraint $x - a \geq 0$.

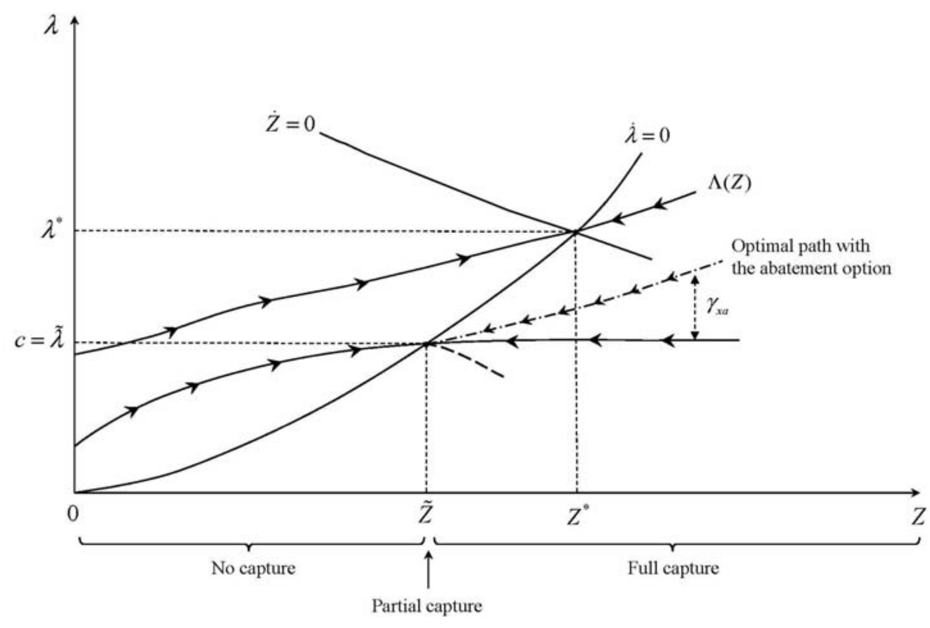

Figure A.2: Phase diagram. Abundant resource and low marginal constant capture costs.

Appendix B. Proofs

In this appendix the proofs of the lemmata of section 4 are provided.

Lemma 1

Suppose there exist $0 \le t_1 < t_2$ such that $\zeta x(t) = a(t)$ for all $t \in [t_1, t_2]$. Then $\zeta x(t) = a(t)$ for all $t \in [0, t_2]$.

Proof

Suppose the lemma doesn't hold. Let $t_1 > 0$ be the first instant of time where there is full capturing. Recall that $\lambda(t) = c'(a(t)) + \gamma_{ax}(t) - \gamma_a(t)$ and that $\lambda(t)$ is a continuous co-state variable. Hence, $\gamma_{ax}(t) = 0$ for all $0 \le t < t_1$ because before t_1 either zero or partial capturing prevails.

We show first that at t_1 we cannot have an upward jump in γ_{ax}. Suppose there were an upward jump. Then, with constant marginal capturing cost, and given the continuity of λ, this would require an upward jump in γ_a but this contradicts that $\gamma_a(t) \ge 0$ and $\lim_{t \downarrow t_1} \gamma_a(t) = 0$. With increasing marginal capturing cost, an upward jump in γ_{ax} requires a downward jump in capturing to preserve the continuity of λ. So, $\lambda - \gamma_{ax}$ jumps upwards, and, in view of $u'(x) = \mu_0 e^{\rho t} + \zeta(\lambda - \gamma_{ax})$, we have that x jumps downwards. But with x jumping upwards and a jumping downwards we cannot have $\zeta x - a \ge 0$ just before t_1 and $\zeta x - a = 0$ at t_1.

Second, it is impossible to have an interval of time, starting at t_1, with $\gamma_{ax} = 0$. In such an interval we would have $\lambda = c'(a)$ and $u'(a/\zeta) = \mu_0 e^{\rho t} + \zeta c'(a)$ so that we can write $a(t) = F(e^{\rho t})$, meaning that capturing is a function of $e^{\rho t}$ only. If we use this in $c''(a)\dot{a} = (\rho + \alpha)c'(a) - h'(Z(t_1)e^{-\alpha t})$ then we find a contradiction.

Hence, γ_{ax} increases from t_1 on. Along a path with full capturing, capturing decreases. Hence λ increases from t_1 on. But if it increases right after t_1 it will increase along the entire phase with full capturing since $\dot{\lambda} = (\alpha + \rho)\lambda - h'(Z)$ and $\dot{Z} < 0$. Hence γ_{ax} increases over the entire interval of full capturing. That requires a downward jump in γ_{ax} at the moment where the phase with full capturing comes to an end. But this is clearly not optimal in the case of constant marginal capturing cost. It is not optimal either with increasing marginal capturing cost because then it would require an upward jump in capturing and a downward jump in consumption of the resource. A final possibility is that the phase with full capturing does not come to an end. However, then λ goes to infinity at a rate that is $\rho + \alpha > \rho$ eventually, which means a violation of the transversality condition. Q.E.D.

Lemma 2

Suppose the resource stock is finite. Then $(x(t), a(t), Z(t)) \to (0,0,0)$ as $t \to \infty$

Proof

Suppose there exists $\varepsilon > 0$ such that for all $T > 0$ there exists $t > T$ with $a(t) \geq \varepsilon$. Then for all such $t's$ we have $u'(x(t)) = \mu(t) + \zeta c'(a(t))$, from (6), (7) and (8). With $\mu(t) \to \infty$ as $t \to \infty$ this implies from $u'(0) = \infty$ that there exists t for which $\zeta x(t) - a(t) < 0$, which is not allowed. Suppose there exists $\varepsilon > 0$ such that for all $T > 0$ there exists $t > T$ with $x(t) \geq \varepsilon$. Then, in view of the previous part of the lemma, there exists $\eta > 0$ and t large enough such that $\zeta x(t) - a(t) \geq \eta$. Hence $\gamma_{ax}(t) = 0$. We then also have $\lambda(t) = c'(a(t)) + \gamma_{ax}(t) - \gamma_a(t) = c'(a(t)) - \gamma_a(t) \geq 0$. Moreover, $u'(x) = \mu_0 e^{\rho t} + \zeta\lambda \geq \mu_0 e^{\rho t}$ so that we cannot have $x(t) \geq \varepsilon$ for t large enough. Hence,, $x(t) \to 0$. Then $Z(t) \to 0$ as $t \to \infty$ immediately follows. Q.E.D.

Lemma 3

Suppose $\dot{\lambda}(t_1) < 0$ for some $t_1 \geq 0$. Then $\dot{\lambda}(t) < 0$ for all $t \geq t_1$.

Proof

Suppose there exist $\varepsilon > 0$ and $T > 0$ such that $\lambda(t) \geq \varepsilon$ for all $t > T$. From (1) and (5) we have $\dot{Z}(t)/Z(t) \geq -\alpha$. From (11) we have $\dot{\lambda}/\lambda = (\rho + \alpha) - h'(Z)/\lambda$. From lemma 2, $Z(t) \to 0$ as $t \to \infty$ and hence $h'(Z(t)) \to 0$ as $t \to \infty$. Therefore, the transversality condition (12) is violated. So, if the lemma doesn't hold, there exist T_1 and T_2 such that $\dot{\lambda}(T_1) = \dot{\lambda}(T_2) = 0$, with $\dot{\lambda}(t) > 0$ for all $T_1 < t < T_2$. Moreover, $\ddot{\lambda}(T_1) = -h''(Z(T_1))\dot{Z}(T_1) > 0$ and $\ddot{\lambda}(T_2) = -h''(Z(T_2))\dot{Z}(T_2) < 0$. So, $\dot{Z}(T_1) < 0$ and $\dot{Z}(T_2) > 0$ implying that there exists $T_1 < \hat{T} < T_2$ such that $\dot{Z}(\hat{T}) = 0$. Hence, there exists $\hat{T} < \hat{T}_2 < T_2$ such that $Z(\hat{T}) < Z(\hat{T}_2)$, $0 = \dot{Z}(\hat{T}) < \dot{Z}(\hat{T}_2)$ and $\dot{Z}(t) > 0$ for $t \in [\hat{T}, \hat{T}_2]$. Therefore $\zeta x(\hat{T}) = \alpha Z(\hat{T}) + a(\hat{T})$ and $\zeta x(\hat{T}_2) > \alpha Z(\hat{T}_2) + a(\hat{T}_2)$. Since $\dot{Z}(t) > 0$ for $t \in [\hat{T}, \hat{T}_2]$ we have $\gamma_{ax}(t) = 0$ for $\hat{T} < t < \hat{T}_2$. Hence $\dot{x}(t) < 0$ for $\hat{T} \leq t < \hat{T}_2$. Moreover, since $\lambda(t) = c'(a(t)) - \gamma_a(t)$ and λ is increasing between $\hat{T}$ and $\hat{T}_2$ we have a non-decreasing. This yields a contradiction. Q.E.D.

Lemma 4

Suppose $\lambda^T(t_1) > c'(0)$ for some $t_1 \geq 0$ and $Z_0 > \tilde{Z}$. Then $a(0) > 0$.

Proof

Suppose $Z_0 > \tilde{Z}$ and $a(0) = 0$. Then $\lambda(0) \leq c'(0)$ and $\dot{\lambda}(0) < 0$ implying from the previous lemma that $\dot{\lambda}(t) \leq 0$ for all $t \geq 0$. This contradicts $\lambda^T(t_1) > c'(0)$. Q.E.D.

Lemma 5

It is impossible to have the following sequence in an optimum. There exist $0 < t_1 < t_2 < t_3$ such that $\zeta x(t) - a(t) > 0, a(t) > 0$ for all $t \in [0, t_1)$, $a(t) = 0$ for all $t \in [t_1, t_2)$, and $\zeta x(t) - a(t) > 0, a(t) > 0$ for all $t \in [t_2, t_3)$.

Proof

Along an interval of partial capturing we have $\lambda(t) = c'(a(t)) > c'(0)$. Zero capturing requires $\lambda(t) \le c'(0)$. Clearly, there is never zero capturing if $c'(0) = 0$. If $c'(0) > 0$ and $c''(a) > 0$ then, if we move from partial capturing to zero capturing, λ is decreasing, and according to lemma 3, it will keep decreasing, implying that we cannot have another phase with partial capturing. With constant marginal capturing cost, λ is non-increasing on the way to the first transition. But along that zero capturing phase we cannot have $\gamma_a = 0$ and therefore a constant $\lambda = c$, because then Z is constant and hence x is constant too along the zero capturing phase. But this contradicts $u'(x) = \mu_0 e^{\rho t} + \zeta c$ Q.E.D.