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Abstract

This paper studies oligopolistic competition in education markets when schools can be private and public and when the quality of education depends on "peer group" effects. In the first stage of our game schools set their quality and in the second stage they fix their tuition fees. We examine how the (subgame perfect Nash) equilibrium allocation (qualities, tuition fees and welfare) is affected by the presence of public schools and by their relative position in the quality range. When there are no peer group effects, efficiency is achieved when (at least) all but one school are public. In particular in the two school case, the impact of a public school is spectacular as we go from a setting of extreme differentiation to an efficient allocation. However, in the three school case, a single public school will lower welfare compared to the private equilibrium. We then introduce a peer group effect which, for any given school is determined by its student with the highest ability. These PGE do have a significant impact on the results. The mixed equilibrium is now never efficient. However, welfare continues to be improved if all but one school are public. Overall, the presence of PGE reduces the effectiveness of public schools as regulatory tool in an otherwise private education sector.

Keywords: Education, peer-group effects, mixed duopoly

***JEL*-Classification:** I2, L33

1 Introduction

This paper studies oligopolistic competition in education markets when education providers can be private and public and when the quality of educational achievement is jointly determined by the use of costly (private) inputs and by a school specific "peer group effect" (PGE). These different building blocks appear to represent realistic features of the education sector. In most countries there are public and private providers and there is some competition in the education market. Furthermore, the quality of the peer group seems to be an important determinant of education quality. Our concern is about the configuration of the market (size and "location" of schools) and the effect of competition on human capital formation and distribution. There are other possible objectives with we leave for future research such as the effect on income distribution.

Our paper brings together three subjects, which are central to the economics of education but have not been considered simultaneously. These issues have been studied separately, either in other education related settings or for different markets altogether. The main point we make is that these features interact and that putting them together leads to conclusions which cannot be understood by analyzing them separately.

First, our paper is related to the issue of competition between schools under alternative assumptions pertaining to the objective functions of schools and education outcomes. This subject has been examined by a number of authors, including Boadway, Marchand and Marceau (1996), De Fraja and Valbonesi (2012), De Fraja and Iossa (2002), MacLeod and Urquiola (2010) and Maldonado (2008). In particular these papers examine how the schools' objective (*e.g.*, maximizing "prestige", welfare or profits) and/or the presence of competition affects education outcomes. When competition is considered, *all schools have the same objective*. This brings us to the second subject covered by our paper namely that of mixed markets, *i.e.,* markets where *competing schools have different objectives*. Empirically, this is a widespread phenomena; see for instance Cellini and Goldin (2012) and Deming *et al.* (2012). These papers show that US education markets are effectively mixed. In particular, for-profit and non-for-profit universities coexist and, moreover, the proportion of for-profit universities has been steadily rising in the last decades.[1] Mixed markets in general have been studied for instance by Cremer, Marchand and Thisse (1991), while applications to the education markets have been examined by Brunello and Rocco (2008) and Romero and Del Rey (2004).

Finally, our paper also relates to the literature on the education production and the importance on the peer group effect on education quality. The empirical relevance of the peer group effect has been considered extensively by the empirical literature. The list of articles addressing this issue is long, some important references are Summers and Wolfe, (1977), Mayer and Jencks (1989), Evans, Oates and Schwab, (1992), Pong (1998), Hoxby (2000), Gaviria and Raphael (2001), Sacerdote, (2001), Hanushek (2003), Zimmerman (2003), Robertson and Symons (2003), Angrist and Lang (2004), Foster (2006), Bonesrønning (2008). The literature discusses the size, the form (linear or non-linear), the relevant variables (socioeconomic status of families of peers vs. GPA or the behavior of peers), and the outcomes that may be affected by the peer group eff ect (GPA, pregnancy rates, drug consumption, etc.).

In general these articles show that the peer group effect exists although some of them argue that it may be modest. The size or the importance of the peer group effect is not a concern for this paper since our objective is not to discuss policy issues but to discuss the effect of the existence of the peer group effect on school market configuration. Among this articles there is a group that discusses the form of the peer group effect and show that the peer group effect is not of the linear-in-means form; this is particularly important for our model since we will follow these articles in the way we model the peer group effect.

The implications of the peer group effect have also been considered by the theoretical literature. Most of the literature has concentrated on the implications of peer group effects on sorting and inequality (Bartolome, 1990 and Benabou, 1996), market efficiency and tuition fees (Epple and Romano, 1998) and the use of different types of policies like the presence of public schools vs. vouchers (Caucutt, 2002). Epple and

[1] These papers also show that for-profit universities have educacional outcomes that differ from those of other types of universities.

Romano (1998) and Caucutt (2002) are closest to our paper in spirit although. However, there are significant differences with our paper, and particularly the underlying type of competition. Both papers assume a large number of private schools which, in the absence of public schools, leads to perfect competition. In addition they do not account for strategic behavior of public schools which is of course a major ingredient in our oligopoly setting.[2]

In this paper we study mixed oligopoly equilibria in two- and three- schools settings allowing for different possible configurations regarding the number of public schools and their location. Private schools maximize profits and public schools maximize welfare. We consider a two stage game of which we determine the subgame perfect Nash equilibrium. In the first stage schools (simultaneously) set their quality and in the second stage they (simultaneously) fix their tuition fees. We examine how the equilibrium allocation (qualities, tuition fees and welfare) is affected by the presence of public schools and by their relative position in the quality range. To set a benchmark, we first consider a setting without peer group effects. Then we introduce peer group effects and reexamine the impact of public schools in this context. The peer group effect for any given school is determined by its student with the highest ability. Interestingly, the optimal allocation is unaffected by the presence of peer group effects. They do however, have a significant effect on the various equilibria we consider. When there are no peer group effects, efficiency is achieved when (at least) all but one school are public. In particular in the two school case, the impact of a public school is spectacular as we go from a setting of extreme differentiation to an efficient allocation. However, in the three school case, a single public school will lower welfare compared to the private equilibrium. Results turn out to be more complex (and less spectacular) when there are peer groups effects. In that case the mixed equilibrium is never efficient. However, welfare continues to be improved if all but one school are public. Overall, the presence of PGE reduces the effectiveness of public schools as regulatory tool in an otherwise private education sector. This may come as a surprise because PGE generate externalities which could be expected to enhance the case for public intervention. However, an effective public

[2]The behavioral foundations of PGE have been studied by Lazear (2001).

intervention has to rely on the appropriate instrument(s). And the lesson that emerges from our result is that the mere presence of public (welfare maximizing) schools may not be sufficient. As a matter of fact, in the absence of commitment, they may effectively be detrimental to social welfare. While this phenomenon can also occur in the absence of PGE, it appears to be exacerbated by the PGE. We first show that the peer group effects may be responsible for the asymmetry of the market. Second, we show that while without peer group effects the presence of public schools may restore efficiency, when there are peer group effects this is never the case. Finally we show that the optimal location of public schools differs according to whether there are peer group effects or not.

This paper is organized as follows. Section 2 introduces the model and presents the efficient allocation. Section 3 studies the case without peer group effects, while peer group effects are introduced in Section 4.

2 The Model

Consider a continuum of individuals with ability a distributed uniformly over $[\underline{a}, \overline{a}]$, where $0 \leq \underline{a} < \overline{a}$, and a finite number of schools, S, indexed by i. An individual of ability a who attends a school of quality q_i with a peer group effect g_i will realize a labor income $y(a, q_i, g_i)$ which is defined by

$$y(a, q_i, g_i) = (1 + a)q_i + \alpha g_i,$$

where $\alpha \geq 0$ measure the intensity of the PGE. Utility depends on consumption which equals labor income minus tuition fees. The utility of a student of ability a who attends school s is specified by

$$u(a, q_i, g_i, t_i) = \overline{U} + y(a, q_i, g_i) - t_i,$$

where $\overline{U}$ is a constant sufficiently large to assume that the market is covered in all the relevant equilibria.

Students attend the school providing them with the highest utility. The marginal student who is indifferent between attending school r and school s, is given by

$$a_{sr} = \frac{(t_s - t_r) - \alpha(g_s - g_r)}{q_s - q_r} - 1, \tag{1}$$

as long as this level belongs to $[\underline{a}, \overline{a}]$. Otherwise we have a corner solution where one of the schools is not active. For notational convenience all analytical expression assume that marginal students are interior; however, the possibility of corner solutions is explicitly accounted for in all the numerical examples.

A school's costs are linear in the number of students and convex in quality. We assume that the cost function is given by

$$c(n_i, q_i) = \frac{1}{2} n_i q_i^2$$

where n_i is the number of students attending school i.

Finally, we assume that the peer group effect provided by a school is determined by its most able student. Formally, for two successive schools in the quality range with $q_s < q_r$ and $s < S$ we have

$$g_s = a_{sr}. \tag{2}$$

For the highest quality school we have

$$g_S = \overline{a}.$$

Support for this specification of the peer group effect has been provided in the introduction, where we report evidence that the peer group effect is not of the "linear in means" type.

There are two types of schools: private and public. Private schools maximize profits

$$\pi_s = n_s t_s - c(n_s, q_s)$$

while public schools maximize (utilitarian) welfare. The precise writing of social welfare for an arbitrary number of schools and allowing for all possible cases regarding corner solutions for marginal students is rather tedious. In the remainder of the paper we shall concentrate on the case of 2 or 3 schools and we restrict ourselves to providing the expressions for the "well-behaved" cases.[3] With $S = 2$, $q_1 < q_2$ and $\underline{a} < a_{12} < \overline{a}$ we have

$$sw_2 = \int_{\underline{a}}^{a_{12}} [(1+a)q_1 + \alpha a_{12} - \frac{1}{2}q_1^2]da + \int_{a_{12}}^{\overline{a}} [(1+a)q_2 + \alpha\overline{a} - \frac{1}{2}q_2^2]da. \qquad (3)$$

When there are 3 schools with $q_1 < q_2 < q_3$ and $\underline{a} < a_{12} < a_{23} < \overline{a}$ we have

$$\begin{aligned} sw_3 \quad = \quad & \int_{\underline{a}}^{a_{12}} [(1+a)q_1 + \alpha a_{12} - \frac{1}{2}q_1^2]da + \int_{a_{12}}^{a_{23}} [(1+a)q_2 + \alpha a_{23} - \frac{1}{2}q_2^2]da \\ & + \int_{a_{23}}^{\overline{a}} [(1+a)q_3 + \alpha\overline{a} - \frac{1}{2}q_3^2]da. \end{aligned} \qquad (4)$$

Schools compete in quality and tuition fees. We consider a two stage game in which quality is chosen in the first stage and tuition fees in the second stage. The solution concept is the subgame perfect Nash equilibrium. A crucial element of this specification (following from the requirement of subgame perfection) is that schools cannot commit to tuition fees when choosing their qualities.

2.1 Optimal allocation

We start by determining the efficient allocation which is an interesting benchmark. For simplicity, we set (here and in the remainder of the paper) $\underline{a} = 0$ and $\overline{a} = 1$; this is purely a matter of normalization and has no impact on the qualitative results. The following proposition states the optimal allocation with 1, 2 or 3 schools.

Proposition 1 *a) The optimal allocation with a given number of schools does not depend on the peer group effect parameter α and is given by*
(i) $q_1^* = 3/2$ *for* $S = 1$;
(ii) $(q_1^*, q_2^*) = (5/4, 7/4)$ *and* $a_{12}^* = 1/2$ *for* $S = 2$;
(iii) $(q_1^*, q_2^*, q_3^*) = (7/6, 3/2, 11/6)$ *and* $(a_{12}^*, a_{23}^*) = (4/3, 5/3)$ *for* $S = 3$.

b) The tuition fees that decentralize this allocations satisfy
(i)

$$t_2^* - \frac{1}{2}(q_2^*)^2 = t_1^* - \frac{1}{2}(q_1^*)^2 + \alpha(1 - a_{12}^*) \qquad (5)$$

[3] We adopt this simplification for all our analytical expressions. However, the full expressions with all relevant constraint are used in all the numerical solutions.

for $S = 2$ *and*

(ii)

$$t_2^* - \frac{1}{2}(q_2^*)^2 = t_1^* - \frac{1}{2}(q_1^*)^2 + \alpha(a_{23}^* - a_{12}^*) \quad \textit{and} \quad t_3^* - \frac{1}{2}(q_3^*)^2 = t_2^* - \frac{1}{2}(q_2^*)^2 + \alpha(1 - a_{23}^*).$$

for $S = 3$.

In words, differences in tuition reflect differences in marginal cost and a Pigouvian term.

Proof. We prove this proposition for the two school case. The solution with three schools follows along the same lines. Differentiating social welfare specified by (3) yields

$$\begin{aligned}
\frac{\partial sw_2}{\partial q_1} &= \int_0^{a_{12}} [(1+a) - q_1]da = 0, \\
\frac{\partial sw_2}{\partial q_1} &= \int_{a_{12}}^{1} [(1+a) - q_2]da = 0, \\
\frac{\partial sw_2}{\partial a_{12}} &= [(1+a_{12})q_1 + \alpha a_{12} - \frac{1}{2}q_1^2] - [(1+a_{12})q_2 + \alpha - \frac{1}{2}q_2^2] + \int_0^{a_{12}} [\alpha]da = 0.
\end{aligned}$$

Solving we obtain $q_1 = 1.25$, $q_2 = 1.75$ and $a_{12} = 1.5$. Note that with these values the FOC with for a_{12} can be rewritten as

$$[(1+a_{12})q_1 - \frac{1}{2}q_1^2] - [(1+a_{12})q_2 - \frac{1}{2}q_2^2] + \alpha(a_{12} - 1) + \int_0^{a_{12}} [\alpha]da = 0 \qquad (6)$$

when $a_{12} = 1.5$ the last two terms cancel out:$-\alpha(0.5) + \alpha 0.5 = 0$. Now notice that the first two terms correspond to the difference between the net social utilities of the marginal individual when she attends school 1 and school 2 in the absence of peer group effects ($\alpha = 0$), this term is equal to zero when $a_{12} = 1.5$, $q_1 = 1.25$ and $q_2 = 1.75$; the two utilities are equal so that the first two terms in brackets also cancel out. ∎

To understand these results (as well as a number of arguments in the remainder of the paper) one has to observe that each individual has a "most preferred" level of school quality $q^*(a)$. Formally define

$$q^*(a) = \arg\max_q (1+a)q - \frac{1}{2}q^2 = 1 + a,$$

that is the quality level preferred by an individual with ability a, if tuition is set a marginal cost (and disregarding peer group effects). With $\underline{a} = 0$ and $\overline{a} = 1$, preferred qualities are then distributed over the interval $[1, 2]$ and we can think about the problem as one of optimal "locations" within this interval of preferred qualities.[4] Without peer group effects it is then plain that one school should be "located" at the center of this interval, two schools at the first and fourth quarter, three schools at $1/6$, $3/6$ and $5/6$; see Figure 1. In other words, it is optimal to have schools of equal size each offering a quality which corresponds to the most preferred quality of its median student. Interestingly this allocation remains optimal when peer group effects are accounted for. To understand this consider a marginal reallocation of students from a high quality to a low quality school. We have then two effects: on the one hand the reallocation increases the peer group effect of the low quality school, on the other hand it reduces the number of students who benefit from the high peer group effect in the high quality school. In our model, as a consequence of the uniform distribution of ability and the separability of the peer group effects these two effects cancel each other out exactly. This argument (along with expression (6)) shows that the second term on the RHS of (5) is effectively a "Pigouvian" term, measuring the marginal social damage of an individual's action. The point is that when the marginal individual switches from school 1 to school 2 he reduces the PGE of all students in school 1 by a total of $\int_0^{a_{12}}[\alpha]da$, which as we have seen equals $\alpha(1 - a_{12}^*)$ at the optimal solution.

The number of schools is taken as given throughout the paper. Nevertheless it is interesting to consider the efficient number of schools. This number depends on the value of α. In our model there are two main forces that determine the optimal number of schools. On the one hand the higher is the number of schools, the better will be the match between preferred and available qualities.[5] On the other hand, the peer group effect pushes towards a low number of schools. From that perspective, a single school performs best. For $\underline{a} = 0$ and $\overline{a} = 1$ one can check that if $\alpha \geq 1.125$ it is efficient to have only one school. If $\alpha \in [0.069, 0.125]$ it is efficient to have two school, and if $\alpha \in [0.0486, 0.069]$ it is efficient to have three schools.

[4] See Cremer and Thisse (1991).

[5] Fixed costs would also limit the optimal number of schools; they are ignored in our setting for simplicty.

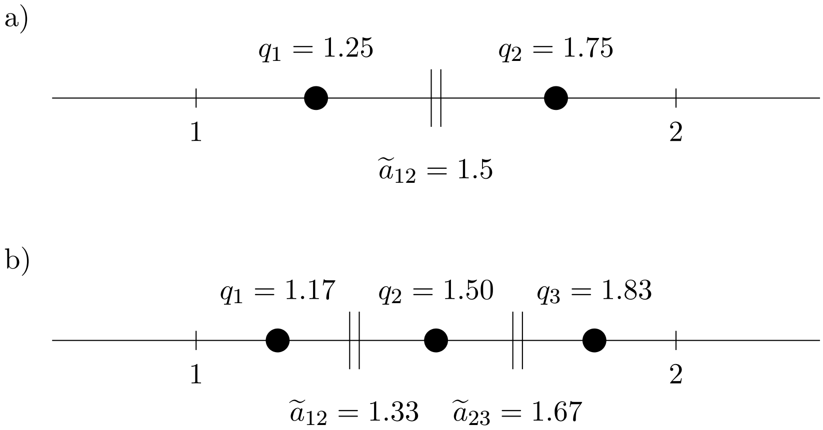

Figure 1: Efficient allocations: a) two schools b) three schools. Black dots represent school's localization and the double lines (||) represent the most preferred quality of the marginal students.

3 School competition without peer group effects

We first study school competition with two or three schools when there are no peer group effects. This provides a useful benchmark to assess the results with peer group effects derived below. We consider the possible configurations of mixed oligopolies, where each of the schools can be either public or private. When there are no peer group effects, the various equilibria can be determined analytically. However, this implies long and rather tedious derivations. We shall restrict our attention to the analytical arguments which are useful for interpreting the results. For instance it is interesting to look at the best-response functions of the different schools in the tuition subgame. For the rest, we provide the numerical results.[6]

3.1 Analytical results

Recall that school play a two stage game which we solve by backward induction. Consequently, we must first solve the equilibrium of the second stage conditional on arbitrary quality levels. We will first derive the best-reply functions of private and public schools

[6] With the normalization $\underline{a} = 0$ and $\overline{a} = 1$ and when $\alpha = 0$, the algebraic derivations will eventually yield a numerical solution.

located in positions 1, 2 and 3 (assuming $q_1 < q_2 < q_3$). We concentrate on the case of three schools; the expressions with two schools can be obtained as special cases from our results.[7] We will then use this reaction functions for the characterization of equilibria.

3.1.1 Tuition fee game

Private schools Using equation 1 with $\alpha = 0$ profits of a private school when offering the lowest quality (q_1) can be written as:

$$\pi_1 = \left(\frac{t_2 - t_1}{q_2 - q_1} - 1 \right) \left(t_1 - \frac{1}{2} q_1^2 \right).$$

From the first order condition with respect to t_1 we can find the best-reply in tuition fee game which is a function of t_2, q_1 and q_2:

$$t_1 = \frac{1}{2} t_2 - \frac{1}{2} (q_2 - q_1) + \frac{1}{4} q_1^2 \tag{7}$$

If the private school offers intermediate quality (q_2) profits are given by

$$\pi_2 = \left[\left(\frac{t_3 - t_2}{q_3 - q_2} - 1 \right) - \left(\frac{t_2 - t_1}{q_2 - q_1} - 1 \right) \right] \left(t_2 - \frac{1}{2} q_2^2 \right)$$

and the best reply in tuition fee game is given by

$$t_2 = \frac{t_1(q_3 - q_2) + t_3(q_2 - q_1)}{2(q_3 - q_1)} + \frac{1}{4} q_2^2$$

Finally, a private school that offers the highest quality (q_3) will have

$$\pi_3 = \left[1 - \left(\frac{t_3 - t_2}{q_3 - q_2} - 1 \right) \right] \left(t_3 - \frac{1}{2} q_3^2 \right)$$

as profits and the best reply in the tuition fee game will be given by

$$t_3 = \frac{1}{2} t_2 + (q_3 - q_2) + \frac{1}{4} q_3^2 \tag{8}$$

These expressions all have a familiar flavor and are rather standard in vertical differentiation models.

[7] The expression for the low quality school remains the same. With $S = 2$ the high quality school is indexed 2, but its best-response function is the same as that of school 3 with $S = 3$ (except that indexes have to be changed in an approriate way).

Public schools Public schools maximize social welfare which, with three schools, is given by (9). Substituting for a_{12} and a_{23} from (1) yields

$$\begin{aligned}\widetilde{sw}_3 &= \int_0^{\frac{t_2-t_1}{q_2-q_1}-1}[(1+a)q_1-\frac{1}{2}q_1^2]da+\int_{\frac{t_2-t_1}{q_2-q_1}-1}^{\frac{t_3-t_2}{q_3-q_2}-1}[(1+a)q_2-\frac{1}{2}q_2^2]da\\ &\quad+\int_{\frac{t_3-t_2}{q_3-q_2}-1}^{1}[(1+a)q_3-\frac{1}{2}q_3^2]da. \qquad (9)\end{aligned}$$

Differentiating with respect to t_1, setting equal to zero and solving we obtain

$$t_1 = t_2 - \frac{1}{2}\left(q_2^2 - q_1^2\right). \qquad (10)$$

This equation implies

$$t_1 - \frac{1}{2}q_1^2 = t_2 - \frac{1}{2}q_2^2,$$

so that the public school sets the same markup (above marginal cost) as the private school to achieve the appropriate level of a_{12}. When t_1 is given by (10) we have

$$a_{12} = \frac{1}{2}\frac{(q_2^2-q_1^2)}{q_2-q_1} - 1 = \frac{(q_1+q_2)}{2} - 1.$$

Consequently, the price is set to ensure that the marginal consumer has a preferred quality which is exactly in the center of the interval $[q_1, q_2]$. Observe that we get this result because with inelastic demand tuition fees are merely a transfer between households and school. The only impact of tuition fees on welfare is via the marginal consumer (the allocation of students to schools). The school with the highest quality uses exactly the same rule so that

$$t_3 = t_2 + \frac{1}{2}(q_3^2 - q_2^2).$$

Finally, a public school with intermediate quality (q_2), maximizes welfare with respect to t_2 and the best-reply function is given by

$$t_2 = \frac{1}{(q_3-q_2)}\left[\left(t_1 - \frac{q_1^2}{2}\right)(q_3-q_2) + \left(t_2 - \frac{q_2^2}{2}\right)(q_2-q_1)\right] + \frac{q_2^2}{2},$$

so that the markup of school 2 is a weighted average of the markups of the two other schools. The intuition is the same as before. If the other schools have the same markup the public school will match it to achieve an efficient allocation of students. When the other schools have different markups, this is not possible and it has to strike a compromise between the two marginal consumers.

3.1.2 Quality game and equilibrium

Using the best-reply functions above we can find equilibrium tuition fees conditional on the quality offered by each of the three schools. This tuition fees can be used to express (second stage equilibrium) profits or welfare as functions of qualities and then determine the equilibrium quality levels. To illustrate this we present the full analytical solution for one of the scenarios. This is the easiest case, yielding simple results and the expression are useful to understand the underlying intuition.

Schools 1 and 3 are public In that case, the tuition fee equilibrium implies

$$t_1 - \frac{1}{2}q_1^2 = t_2 - \frac{1}{2}q_2^2 = t_3 - \frac{1}{2}q_3^2$$

whatever the location. We thus also have

$$\begin{aligned} a_{12} &= \frac{q_1 + q_2}{2} - 1 && (11) \\ a_{23} &= \frac{q_2 + q_3}{2} - 1 && (12) \end{aligned}$$

More precisely, the equilibrium is obtained by solving

$$\begin{aligned} t_1 &= t_2 - \frac{1}{2}(q_2^2 - q_1^2) \\ t_2 &= \frac{t_1(q_3 - q_2) + t_3(q_2 - q_1)}{2(q_3 - q_1)} + \frac{1}{4}q_2^2 \\ t_3 &= t_2 + \frac{1}{2}(q_3^2 - q_2^2) \end{aligned}$$

which yields

$$t_i - \frac{q_i^2}{2} = \frac{1}{2}\left[q_1q_2 + q_2q_3 - q_1q_3\right] - \frac{1}{2}q_2^2 \qquad i = 1, 2, 3 \qquad (13)$$

Using these expressions we can express the profit of school 2 at the second stage equilibrium as

$$\pi_1^2 = \left[\frac{1}{2}\left(q_1q_2 + q_2q_3 - q_1q_3\right) - \frac{1}{2}q_2^2\right]\left[\frac{q_1 + q_3}{2}\right].$$

Observe that the market share does not depend on q_2. Differentiating this expression we obtain

$$\frac{\partial \pi_1^2}{\partial q_2} = \left[\frac{1}{2}q_1 - q_2 + \frac{1}{2}q_3\right]\left[\frac{q_1 + q_3}{2}\right].$$

Setting to zero and solving yields

$$q_2 = \frac{q_1 + q_3}{2}, \tag{14}$$

which implies that the solution is efficient, namely $(q_1, q_2, q_3) = (7/6, 3/2, 11/6)$. To see this observe that from (14) $q_2 = 3/2$ is the best reply to $q_1 = 7/6$ and $q_3 = 11/6$. As to the public schools, it is plain that they do not want to deviate from this (efficient) allocation.

Other scenarios The other cases with two public schools (1 an 2 or 2 and 3) are equally simple and also yield an efficient solution. The other scenarios involve more complex intermediate expressions, which are not very insightful. With our normalization, the solutions are in any event numerical and are presented in the next section.

3.2 Numerical results

The results of the two school cases equilibria are shown in Table 1 and Figure 2. It is well known from the product differentiation literature that the private equilibrium is not efficient. To decrease the intensity of tuition competition, profit maximizing schools will differentiate their quality in an excessive way. The main message emerging from our results is that the mixed oligopoly restores efficiency. The equilibrium is the same whether the public school offers highest or lowest quality. This conclusion is perfectly in line with the results obtained by Cremer Marchand and Thisse (1991) in a Hotelling setting with quadratic transportation cost.

Table 2 and Figure 3 show the results with three schools. In this case we can see that to restore efficiency there need be two public schools. It is important to note that in this case when there is only one public school the only possible equilibrium in which there are three active schools (a_{12} different than 1 and a_{23} different than 2) involves the public school offering intermediate quality; similarly if there are two public schools the only equilibria with three active schools involves the private school offering intermediate quality.

The main results of this section are summarized in the following proposition.

	Efficient	Private olig.	Mixed olig.
a_{12}	0.5000	0.5000	0.5000
q_1	1.2500	0.7500	1.2500
q_2	1.7500	2.2500	1.7500
t_1	-	1.0313	1.0313
t_2	-	3.2813	1.7813
π_1	-	0.3750	0.1250
π_2	-	0.3750	0.1250
sw	1.1563	1.0313	1.1563

Table 1: Two schools without peer group effect ($\alpha = 0$)

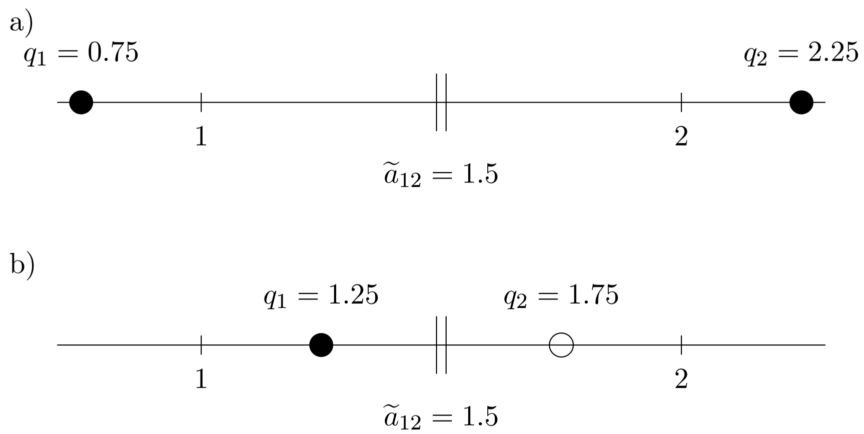

Figure 2: Two schools without peer group effects: a) private oligopoly and b) mixed oligopoly. Black dots represent private school's and empty dots represent public school's; the double lines (||) represent the marginal students.

	Efficient	Private olig.	Mixed oligopoly	
			One public i=2	Two public i=1,2
a_{12}	0.3333	0.2708	0.3750	0.3333
a_{23}	0.6667	0.7292	0.6250	0.6667
q_1	1.1667	1.1250	1.2500	1.1667
q_2	1.5000	1.5000	1.5000	1.5000
q_3	1.8333	1.8750	1.7500	1.8333
t_1	-	0.7344	0.8750	0.7917
t_2	-	1.2109	1.2188	1.2361
t_3	-	1.8594	1.6250	1.7917
π_1	-	0.0275	0.0352	0.0370
π_2	-	0.0394	0.0234	0.0370
π_3	-	0.0275	0.0352	0.0370
sw	1.1620	1.1610	1.1602	1.1620

Table 2: Three schools without peer group effect ($\alpha = 0$)

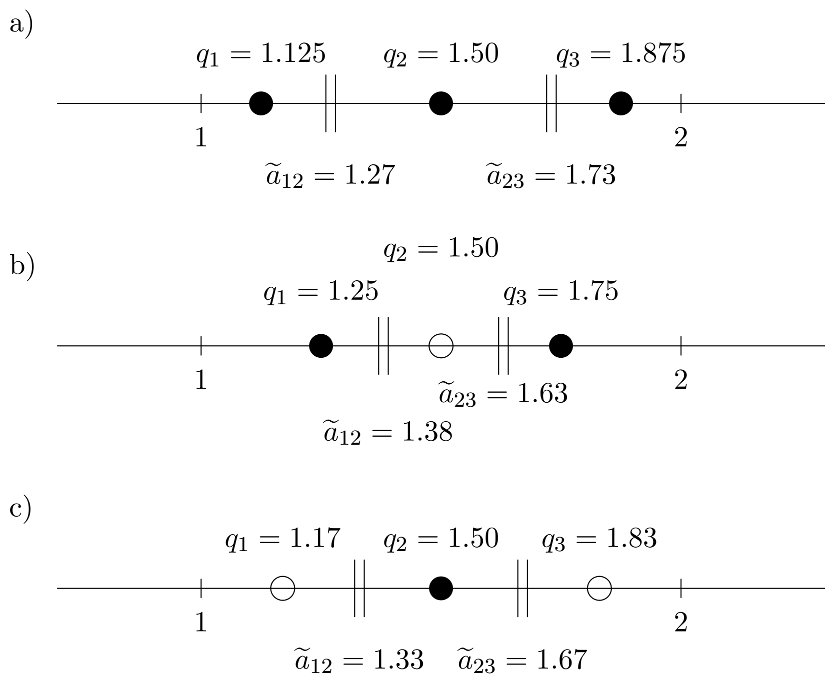

Figure 3: Three schools without peer group effects: a) private oligopoly, b) mixed oligopoly with one public school and c) mixed oligopoly with two public schools. Black dots represent private school's and empty dots represent public school's; the double lines (||) represent the marginal students.

Proposition 2 *Assume that there are no peer-group-effects ($\alpha = 0$) and normalize $\underline{a} = 0$ and $\overline{a} = 1$. We have:*

a) with two schools ($S = 2$),

(i) the private duopoly with profit maximizing schools is inefficient and given by $(q_1^{ep}, q_2^{ep}) = (3/4, 9/4)$,

(ii)the mixed duopoly with one private and public school is efficient whatever the relative position of the public school (high or low quality).

b) with three schools ($S = 3$),

(i) the private oligopoly with profit maximizing schools is inefficient and given by $(q_1^{ep}, q_2^{ep}, q_2^{ep}) = (5/4, 6/4, 7/4)$,

(ii) the mixed oligopoly with one private and two public schools is efficient whatever the relative position of the public schools (high or low or intermediate quality)

(iii) a mixed equilibria (with positive market shares for all schools) with two private and one public schools exist only when the public school offers the intermediate quality. This equilibrium is not efficient and yields lower welfare than the private oligopoly.

4 School competition with peer group effects

We now introduce peer group effects and suppose that $\alpha > 0$. Not surprisingly this makes the analytical resolution of the model even more tiresome. There are however some important conclusions that can be drawn from the best-reply functions in the tuition fee game with two schools. We shall derive and discuss these expressions and then turn to the numerical results obtained in the two and three school cases.

4.1 The tuition subgame with two schools

With two schools and our specification of peer group effects, from (2) we have $g_1 = a_{12}$ and $g_2 = 1$ and equation (1) specifying the marginal student simplifies to.

$$a_{12} = \frac{(t_2 - t_1) - (q_2 - q_1) - \alpha}{q_2 - q_1 - \alpha}. \tag{15}$$

Not surprisingly, a_{12} decreases with α; the stronger the PGE, the larger will be *ceteris paribus* the market share of the high quality school. As before the marginal student determines the demand of each school.

Private schools

A private school that offers lowest quality (q_1) will have profits

$$\pi_1 = \left[\frac{(t_2 - t_1) - (q_2 - q_1) - \alpha}{q_2 - q_1 - \alpha}\right]\left[t_1 - \frac{q_1^2}{2}\right].$$

Maximizing with respect to t_1 yields the following best-reply

$$t_1 = \frac{1}{2}\left[\frac{q_1^2}{2} + t_2 - (q_2 - q_1) - \alpha\right] \tag{16}$$

When the private school offers the highest quality we have

$$\begin{aligned} \pi_2 &= \left[1 - \frac{(t_2 - t_1) - (q_2 - q_1) - \alpha}{q_2 - q_1 - \alpha}\right]\left[t_2 - \frac{q_2^2}{2}\right] & (17) \\ &= \left[\frac{2(q_2 - q_1) - (t_2 - t_1)}{q_2 - q_1 - \alpha}\right]\left[t_2 - \frac{q_2^2}{2}\right] & (18) \end{aligned}$$

and the best-reply function is given by

$$t_2 = \frac{1}{2}\left[\frac{q_2^2}{2} + t_1 + 2(q_2 - q_1)\right] \tag{19}$$

Interestingly, the best-reply of the high quality school does not depend on α. This property arises because the market share (and hence the profit) of the high quality school is proportional to $1/(q_2 - q_1 - \alpha)$, which is a constant in the pricing game; see equation (18). Consequently, the best-reply is the same as in the absence of PGE. The low quality school sets a tuition which decreases with α. This does not come as a surprise. The PGE reduces the market power of the low quality schools. Consequently, it has to set a lower market than in the absence of PGE.

Public schools

When there are only two schools welfare is given by sw_2 as defined by (3). Substituting for a_{12} from (15), differentiating with respect to t_i's and solving yields the following best-reply functions:

$$t_1 = t_2 - \frac{1}{2}(q_2^2 - q_1^2) - \frac{\alpha\left[(q_2 - q_1) + (\frac{1}{2}q_2^2 - \frac{1}{2}q_1^2) - 3\alpha\right]}{[q_2 - q_1 - 2\alpha]} \tag{20}$$

for a low-quality public school and

$$t_2 = t_1 + \frac{1}{2}(q_2^2 - q_1^2) + \frac{\alpha\left[(q_2 - q_1) + (\frac{1}{2}q_2^2 - \frac{1}{2}q_1^2) - 3\alpha\right]}{[q_2 - q_1 - 2\alpha]} \tag{21}$$

for a high-quality public school.[8]

In the case of public schools, both reaction functions depend on the intensity of the PGE α. When $\alpha = 0$ we return to the expressions obtained without PGE which imply that the public school sets the same markup as its private neighbor. When $\alpha > 0$ public schools no longer set the same markup as their private competitor; instead, they adjust it to account for the PGE (whatever their relative position in the spectrum of qualities). The sign of the second term in (20) and (21) is not unambiguous. However, when α is sufficiently close to zero it is obviously positive. For larger levels of α this remains true as long as $q_2 - q_1 > 2\alpha$. Consequently, the PGE induces the high-quality school to increase its markup and the low-quality school to decrease it. Recall that the prices which implement the optimal allocation satisfy the same property, see expression (5). Since the public school maximizes welfare, it will set prices to achieve the best possible allocation of students between schools, considering qualities, costs and PGE. If the high quality public school sets the same markup as its neighbor the marginal consumer will be too small because no incentive is given to account for the PGE.

4.2 Numerical Results

Tables 3 and 4 and Figures 4 and 5 present the numerical results with two and three school for $\alpha = 0.04$ (a level of PGE for which it is efficient to have 3 schools).[9]

With two schools gives extreme and inefficient quality differentiation like in the case without PGE. However, the solution is no longer symmetric and both qualities are lower than in the case without PGE. In other words, there appears to be substitution between a school achieves "for free" (because of the PGE) and the conventional quality which is costly. The introduction of a public school improves welfare but does not restore

[8] Footnote about existence, restriction on α etc. We know that we must have $q_2 - q_1 > 2\alpha$ so that the welfare maximization problem be convex. We can show that $\left[(q_2 - q_1) + (\frac{1}{2}q_2^2 - \frac{1}{2}q_1^2) - 3\alpha\right] > 0$.

[9] In some cases we found multiple equilibria; however one of the equilibrium was always Pareto dominated, we report only the Pareto dominant equilibrium.

	Efficient	Private olig.	Mixed oligopoly	
			i=1	i=2
a_{12}	0.5000	0.4906	0.4628	0.5372
q_1	1.2500	0.7359	1.2314	1.2933
q_2	1.7500	2.2359	1.7067	1.7686
t_1	-	0.9871	0.9735	1.0702
t_2	-	3.2434	1.6903	1.8193
π_1	-	0.3514	0.0997	0.1256
π_2	-	0.3788	0.1256	0.1182
sw	1.2260	1.1013	1.2260	1.2260

Table 3: Two schools with peer group effect

efficiency. The two mixed equilibria (school 1 or school 2 public) are different but "symmetric" with respect to 3/2 and they yield the same level **K:of** welfare.[10]

In the three school case we obtain also that the presence of PGE lowers the quality levels of all schools in the private equilibrium. The mixed equilibrium no longer restores efficiency with two public schools. However, the equilibrium with two public schools yields a higher level of welfare than the private oligopoly and the best outcome is achieved when the public schools provides the extreme quality levels (index 1 and 3). The mixed equilibria with a single public school give lower welfare than the private oligopoly. Welfare depends on the position of the public school and the best (least harmful) solution is when it provides the lowest quality. There exists no equilibrium (yielding positive market shares for all schools) in which a high quality public school competes with two private of lower quality.

The main results of this section are summarized in the following proposition.

Proposition 3 *Assume that there are peer-group-effects ($\alpha > 0$) whose intensity is determined by the most able student attending a given school. Normalize $\underline{a} = 0$ and $\overline{a} = 1$. We have:*

a) with two schools ($S = 2$),

(i) the private duopoly with profit maximizing schools is inefficient and given by $(q_1^{ep}, q_2^{ep}) =$

[10] "Symmetry" refers to the property that the private school "locates" either at $1.5 + 0.21$ (when is is of high quality) or at $1.5 - 0.21$ (when it is of low quality). Similarly, the public school locates either at $1.5 - 0.27$ or at $1.5 + 0.27$.

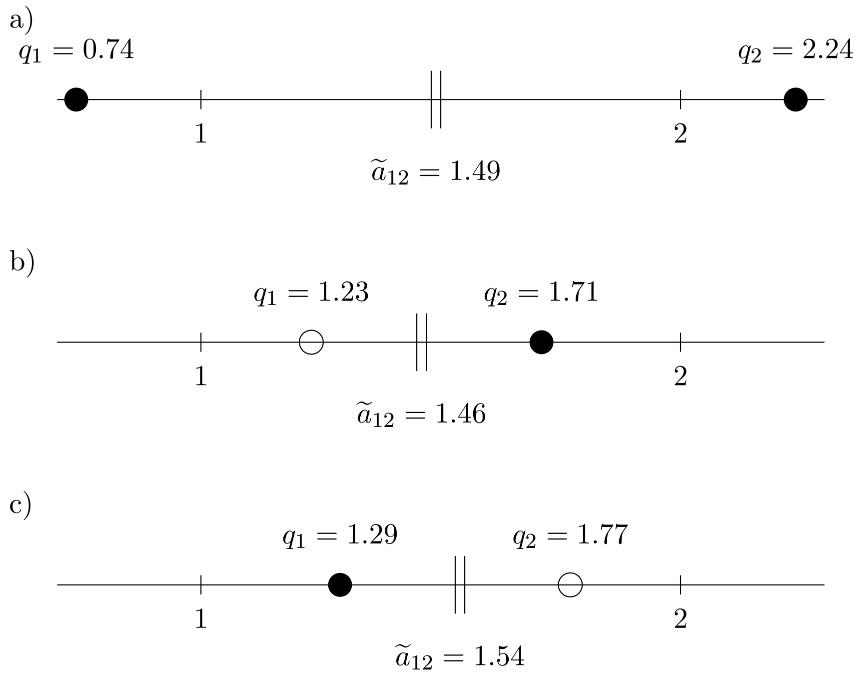

Figure 4: Two schools with peer group effects: a) private oligopoly, b) mixed oligopoly, school 1 is public and c) mixed oligopoly, school 2 is public. Black dots represent private school's and empty dots represent public school's; the double lines (||) represent the marginal students.

	Efficient	Private olig.	Mixed olig.				
			One public		Two public		
			i=1	i=2	i=1,2	i=1,3	i=2,3
a_{12}	0.3333	0.2053	0.5300	0.0499	0.2846	0.3956	1.2462
a_{23}	0.6667	0.6826	0.7299	0.8078	0.5693	0.6985	1.6201
q_1	1.1667	1.0424	1.2494	1.0216	1.1423	1.1877	1.1056
q_2	1.5000	1.4067	1.3013	1.4300	1.4270	1.5848	1.4721
q_3	1.8333	1.8233	1.7290	1.4780	1.7480	1.8750	.8339
t_1	-	0.6099	0.7614	0.5403	0.7507	0.7313	0.6915
t_2	-	1.0681	0.8488	0.9994	1.1278	1.2977	1.1632
t_3	-	1.7818	1.5994	1.0937	1.6488	1.8025	1.7646
π_1	-	0.0137	-0.0101	0.0009	0.0280	0.0103	0.0198
π_2	-	0.0376	0.0004	-0.0175	0.0312	0.0127	0.0298
π_3	-	0.0379	0.0283	0.0003	0.0521	0.0135	0.0315
sw	1,2287	1.2271	1.2138	1.2031	1.2281	1.2282	1.2281

Table 4: Three schools with peer group effect

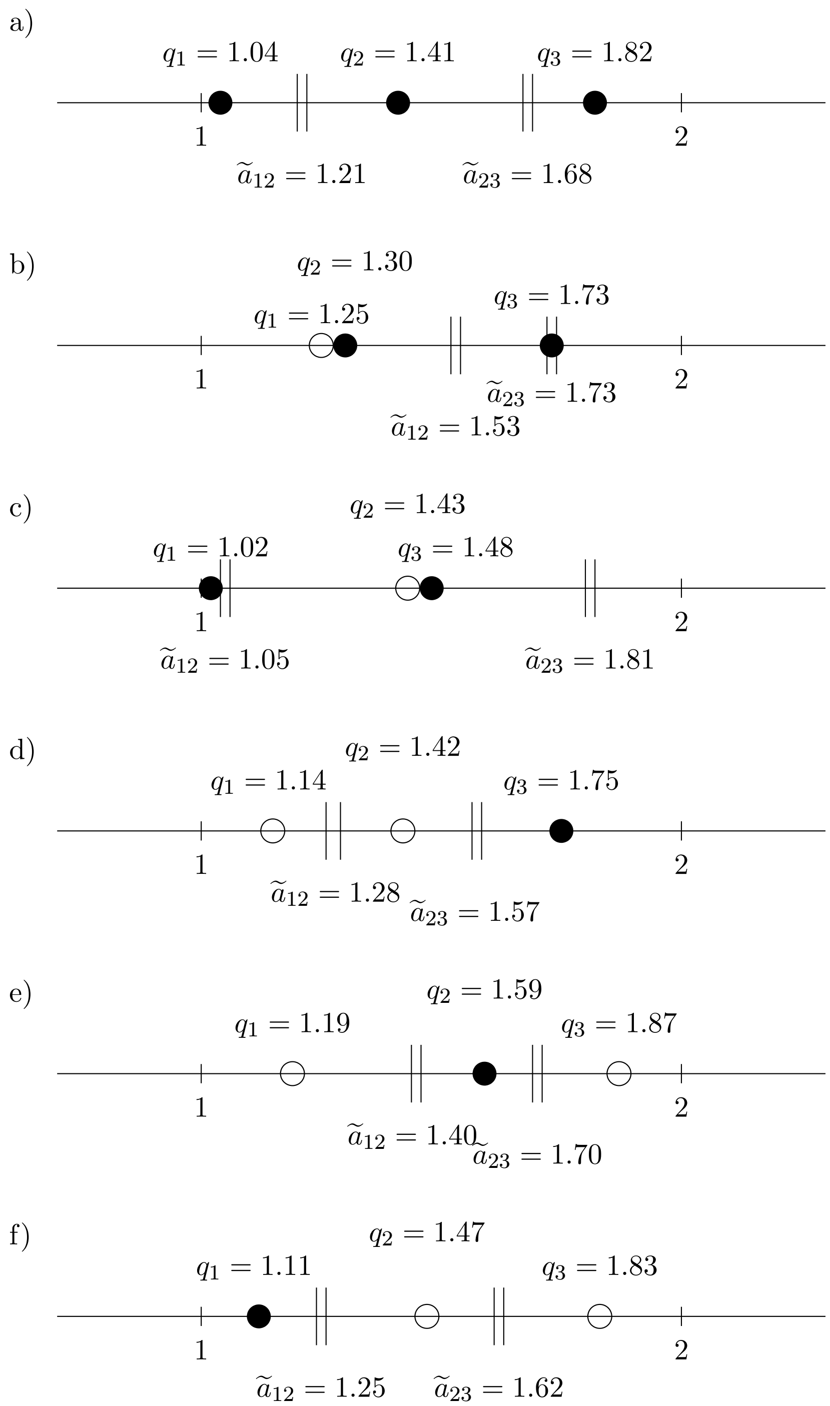

Figure 5: Three schools and peer group effects: a) private oligopoly, b) mixed oligopoly, school 1 is public, c) mixed oligopoly, school 2 is public, d) mixed oligopoly, schools 1 and 2 are public, e) mixed oligopoly, schools 1 and 3 are public and f) mixed oligopoly, schools 2 and 3 are public. Black dots represent private school's and empty dots represent public school's; the double lines (||) represent the marginal students.

$(0.74, 2.24)$; *both qualities are lower than in the absence of PGE*
(ii) the mixed duopoly with one private and public school is no longer efficient. Equilibrium qualities depend on the relative position of the public school (high or low quality) but welfare does not.

b) with three schools ($S = 3$),
(i) the private oligopoly with profit maximizing schools is inefficient and given by $(q_1^{ep}, q_2^{ep}, q_2^{ep}) = (1.04, 1.41, 1.82)$ *and all qualities are lower than in the absence of PGE.*
(ii) the mixed oligopoly with one private and two public schools is no longer efficient. However, welfare is higher than in the private equilibrium and is highest when the public school provides an extreme quality level (index 1 or 3).
(iii) a mixed equilibria (with positive market shares for all schools) with two private and one public schools exist only when the public school offers the low or the intermediate quality (index 1 or 2). This equilibrium is not efficient and yields lower welfare than the private oligopoly.

Finally, the comparison of Propositions 2 and 3 suggests that the presence of PGE reduces the effectiveness of public schools as regulatory tool in an otherwise private education sector. This may come as a surprise because PGE generate externalities which could be expected to enhance the case for public intervention. However, an effective public intervention has to rely on the appropriate instrument(s). And the lesson that emerges from our result is that the mere presence of public (welfare maximizing) schools may not be sufficient. As a matter of fact, in the absence of commitment, they may effectively be detrimental to social welfare. While this phenomenon can also occur in the absence of PGE, it appears to be exacerbated by the PGE.

5 Summary and concluding remarks

This paper has studied mixed oligopoly equilibria with private and public schools. We have examined how the equilibrium allocation (qualities, tuition fees and welfare) is affected by the presence of public schools and by their relative position in the quality range. We have studied these questions without and with PGE. The peer group effect for any given school is determined by its student with the highest ability. PGE were shown to have a significant effect on the various equilibria we consider. When there are no peer group effects, efficiency is achieved when (at least) all but one school are public. In particular, in the two school case, the impact of a public school is spectacular as we go from a setting of extreme differentiation to an efficient allocation. However, in the three school case, a single public school will lower welfare compared to the private equilibrium. More complex results have emerged with PGE. In that case the mixed equilibrium is never efficient. However, welfare continues to be improved if all but one school are public. Overall, the presence of PGE reduces the effectiveness of public schools as regulatory tool in an otherwise private education sector. As a matter of fact, in the absence of commitment, they may effectively be detrimental to social welfare. While this phenomenon can also occur in the absence of PGE, it appears to be exacerbated by the PGE. This may come as a surprise because PGE generate externalities which could be expected to enhance the case for public intervention. However, an effective public intervention has to rely on the appropriate instrument(s). And the lesson that has emerged from our result is that the mere presence of public schools may not be sufficient. The analysis is of great practical relevance since in many countries with mixed education markets, regulatory tools are limited to the introduction of public institutions. To achieve a more efficient outcome, additional tools would be required to correct for the inefficiencies brought about by the externalities.

Our analysis has ignored a number of important features of education markets. Probably the most important is wealth heterogeneity. If this aspect were introduced the assumption that the entire market is covered in any scenario would also become rather unacceptable. We leave this issue for future research. Note, however, that this feature would not render the current paper redundant. It would require an admittedly nontrivial generalization, though. Either way, our finding that the simple presence of public schools (even when they are welfare maximizing) may be counterproductive and is in any event not sufficient to eradicate the inefficiencies that arise in a decentralized education market with PGE can only be reinforced. Additional instruments are needed and the assessment of their respective roles will be an even more challenging issue under heterogeneity.

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