"Private, social and self insurance for long-term care: A political economy analysis"

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Abstract

We analyze the determinants of the demand for social, private and self-insurance for long-term care in an environment where agents differ in income, probability of becoming dependent and of receiving family help. Uniform social benefits are financed with a proportional income tax and are thus redistributive, while private insurance is actuarially fair. We obtain a rich pattern of insights, depending on whether private insurance is available or not, on its loading factor, and on the correlation between, on the one hand, income and risk, and, on the other hand, income and family help. Although the availability of private insurance decreases the demand for social insurance, it only affects a minority of agents so that the majority-chosen social insurance level remains unaffected. Family support crowds out the demand for both private and social insurance, and may even suppress any demand for private insurance. Family help crowds out self-insurance only for agents whose demand for both social and private insurance is nil. A general increase in the probability of becoming dependent need not increase the demand for social insurance, since it decreases its return.

1 Introduction

While health care services aim at changing a health condition (from unwell to well), long-term care (hereafter LTC) merely aims at making the current condition (unwell) more bearable. Individuals need LTC due to disability, chronic condition, trauma, or illness, which limit their ability to carry out basic self-care or personal tasks that must be performed every day. Such activities are defined as activities of daily living (eating, dressing, bathing, getting in and out of bed, toileting and continence) or instrumental activities of daily living (preparing own meals, cleaning, laundry, taking medication, getting to places beyond walking distance, shopping, managing money affairs and using the telephone/Internet). A person is dependent if he or she has limitations in either type.

In most welfare states, one observes a puzzling gap between the financing of health care and of LTC. The size of current public LTC spending is limited compared with health care. Moreover, health care insurance coverage is often universal while LTC is covered only for the most needy and quite partially through local means-tested programs. This lack of universal social insurance has been criticized on the grounds of social equity and because it leads a number of elderly people to some strategic impoverishment required to become eligible. To be fair, we have to note that LTC spending is heavily concentrated among the elderly while health care covers the entire population.

Table 1: Health and LTC expenditures, % of GDP

	Health care		Long term care	
	Public	(Total)	Public	(Total)
Australia	5.8	(8.5)	0.8	(1)
France	8.7	(11.2)	1.4	(1.6)
Germany	8.1	(10.5)	0.9	(1.3)
Japan	6.7	(8.1)	1.4	(1.6)
Spain	6.6	(9)	0.6	(0.8)
US	6.9	(16)	0.6	(1)

Source: OECD (2010)

Table 1 provides some figures on the relative importance of health and long-term care expenditures in a sample of OECD countries. Public LTC expenditures range from 0.6% of GDP to 1.4% while public health care expenditures vary from 5.8% of GDP to 8.1%. Beyond the heterogeneity in the size of public programs, countries also differ in how they intervene. In most countries, public authorities provide financing at the local level through social assistance programs. Only a tiny number of countries have a formal social insurance for LTC: Germany, the

Netherlands and France.[1] Another source of LTC financing takes place via the market, either through self-insurance or private insurance. Except for a handful of countries (such as the US and France), private insurance's role is even smaller than that of the State. Finally, the family provides the most important part of LTC in an informal way, so that its interventions are not measured in Table 1.

Faced with such a large heterogeneity in how LTC expenditures are financed across countries, we study in this paper the determinants of the individual demand (and political support) for social, private and self-insurance (i.e., saving) in an environment where people differ in income, risk and availability of family help. As stated above, the availability of family help is of first importance for LTC, and distinguishes our approach from the literature studying the political support for other kinds of social insurance programs, such as health or social security.[2]

We start with a setting where only social and self-insurance are available, with social insurance providing a uniform benefit to any dependent person, financed by a proportional payroll tax. We obtain that the demand for social insurance decreases with income (because of its redistributiveness across income levels), with family help and increases with the probability of becoming dependent, when income, risk and family support are independent from each other. Agents with a large income or a very low risk prefer self-insurance (saving) to social insurance. One important characteristic of LTC is that, unlike for health care, the correlation between income and risk is positive, since average life expectancy increases with income, with dependency being a condition associated overwhelmingly with old age. We provide numerical examples to illustrate that, with such a positive correlation, the relationship between income and most-preferred social insurance level can go both ways. Income may also be correlated with the probability of receiving family help, but the sign of the correlation is far from clear. Using macro data in Europe, one observes a negative correlation between income and family support, with richer Northern European countries providing less family help, on average, than poorer Southern countries (the so-called "North-South gradient").[3] Focusing on micro data, Bonsang (2009) finds a positive correlation between income and family help. With a positive correlation, we obtain unambiguously that richer people prefer less social insurance, while the relationship between income and most-preferred social insurance can go both ways with a negative correlation between income and family support. Finally, we show the existence of a majority chosen social insurance level (although the characteristics of the decisive voter are

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[1] In EC (2009), one finds that public spending for LTC in the Netherlands amounts to 3.4% of GDP relative to a European average of 1.3% (EU15).

[2] There does not exist good estimates of the importance of family in LTC. One however knows that over 80% of dependent elderly live in their home or with their children, and for these people most of the care is informal. See Stone (2000).

[3] On the North-South gradient, see SHARE (2005).

difficult to pinpoint with our three-dimensional traits' space) and show that it decreases with the availability of family help in the economy.

We then introduce an actuarially fair private insurance into the picture. We first show that, if agents differ only in income, then the introduction of private insurance, although it decreases the support for social insurance by some voters, does not affect its majority chosen level. The intuition is that private insurance induces all above-average-income agents to switch their support in favor of private (rather than social) insurance, but does not affect the preferences of below-average-income agents. With a positively skewed income distribution, the majority chosen social insurance level (the one most-preferred by the individual with the median income) is then not affected by the availability of private insurance. We then introduce a loading factor on private insurance and show that, as this loading factor increases so that private insurance's return decreases away from actuarial fairness, the demand for private insurance (among the richer agents) decreases both at the extensive and intensive margin, up to the point where it becomes nil and where rich agents prefer to exclusively self-insure. We finally reintroduce heterogeneity in both income and risk, and perform some comparative statics analysis with respect to family help (assumed to be the same for all agents). We obtain that agents with low income-to-risk ratio prefer social insurance while other agents prefer private insurance. Family help crowds out both social insurance (for low income-to-risk ratio) and private insurance (for large ratios) at the intensive margin, up to the point where private insurance demand becomes nil. From that point on, any increase in family help crowds out the demand for social insurance both at the intensive and at the extensive margins. The reason why social insurance resists better than private insurance to increases in family help is due to the redistribution performed by the former but not by the latter. Also, self insurance is not affected by family help as long as agents most prefer either social or private insurance, while it decreases with family help for agents who rely on neither private or social insurance. In other words, as it increases family help first crowds out private insurance and then self-insurance.

There is surprisingly little literature on the determination of the (socially or individually) optimal level of social LTC insurance, especially when compared with the related issues of health care, social security and annuities.[4] On the normative side, Cremer and Pestieau (2011) use a model close to the one of this paper; they show that the case for social LTC insurance can only be defended when tax redistribution is restricted. On the positive side, Nuscheler and Roeder (2010) study how the heterogeneity in individual income and risk affects the preferences for redistributive income taxation vs public financing of LTC. Their model allows LTC to be provided by informal help received from the family, or through family

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[4]The economic literature on various other aspects of LTC is surveyed in Cremer *et al.* (2009).

transfers in cash and government's transfers. Insurance (whether social, private or self-insurance in the form of saving) is not available since voters know whether the elderly in the family is dependent or not when taking their decisions. There is also no room for the correlation between income and risk, since the proportion of dependent elderly is the same in the two income classes considered. Their main result is the prediction of a negative correlation between income inequality and public LTC spending.

Our paper is organized as follows. Section 2 presents the model when private insurance is not available.[5] Section 3 solves this model for the individually most-preferred social insurance and saving levels, and for the majority-chosen social LTC insurance contribution rate. Section 4 introduces private insurance and studies how its availability affects the preferences for social and self-insurance. Section 5 concludes.

2   The Model without private insurance

We consider a continuum of individuals living two periods. When young, they earn a wage, pay income taxes, save and make a transfer to their parents conditional on the parents needing LTC. When old, they live out of their saving, plus the social transfer if they need LTC, plus a transfer from the family if they have caring children and they need LTC. There are three sources of heterogeneity among individuals i: their exogenous income, denoted by w_i, their probability of needing LTC (Π_i) and their probability of having (caring and close)[6] children when needing LTC (p_i). An agent of type i is thus characterized by the triplet (w_i, Π_i, p_i).

Young individual i's lifetime utility function is given by

$$
\begin{aligned}
U_i \;=\; & w_i(1-\tau) - s_i - l_i d \\
& + (1-\Pi_i)u(s_i) + \Pi_i p_i H(c_c) + \Pi_i(1-p_i)H(c_n).
\end{aligned} \tag{1}
$$

The first three terms of (1) measure the instantaneous utility of individual i when young, while the remaining terms measure his utility when old (for simplicity, we assume away any discounting of future utility). The first term measures disposable income when young, with τ the (proportional) contribution rate on labor income. The second term s_i is private saving, while d is the private transfer made by

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[5]We start by assuming away private insuranc, which is unusual in public economics. The reason is twofold. First, it makes the presentation simpler and second, it fits the reality of most countries where private LTC insurance is missing.

[6]There are many reasons why some parents cannot count on any assistance from their offspring: (i) they do not have children or their children prematurely died; (ii) their children are not altruistic; (iii) they migrated far away from each other; (iv) parents and children do not get along.

children towards parents in need of LTC (we assume that d is exogenously set, for instance by a social norm[7]), and l_i is a binary variable that takes the value 1 if individual i's parents are in need of care (when individual i is young) and 0 otherwise. Observe that this term will play no role in the analysis, since first period utility is linear in consumption (i.e., this payment does not affect the marginal utility of income when young).

We now move to the second part of (1), namely the utility obtained in the second period of life. We distinguish the utility function when in good health, denoted by $u(.)$, from the utility when needing LTC, denoted by $H(.)$. Both $u(.)$ and $H(.)$ are increasing and concave functions of consumption. We assume both satisfy the condition of infinite marginal utility for zero consumption levels. We also assume that $u(c) > H(c)$ for any consumption level c, but that $u'(c) < H'(c)$ for all c: people are happier if not in need of LTC, but "need more money" (i.e., have a higher marginal utility of consumption) if in LTC.[8] Observe that one family of functions satisfying these assumptions is $H(c) = u(c - z)$ where $0 < z < c$: in that case, becoming dependent is equivalent to suffering a monetary loss.

With probability $1 - \Pi_i$, the individual is not dependent and enjoys his saving (without loss of generality we posit a zero interest rate on savings). If the individual becomes dependent (with probability Π_i), his consumption level depends on whether he receives help from his family. He does not receive such help with probability $1 - p_i$, in which case his consumption level is given by

$$c_n = s_i + b,$$

where b is the lump sum public transfer paid to all people in LTC. This transfer is financed by a linear tax on young people's income at a rate τ. If the dependent individual receives a transfer d from his family, his consumption level is given by[9]

$$c_c = s_i + b + d.$$

For simplicity, we assume away demographic (and economic) growth, so that the social insurance program's budget constraint is given by

$$\tau(1 - \delta\tau)\bar{w} = b\bar{\Pi}, \tag{2}$$

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[7]Admittedly this is a strong assumption. In general, one assumes some substituability between family support and either type of insurance.

[8]This assumption is reasonable up to a certain consumption level. One can make the point that the marginal utility from consumption drops close to zero beyond a cerain threshold when dependent. We implicitly assume in this paper that this consumption threshold is not reached.

[9]Observe that there is no altruism in this model but simply a social norm imposing to children to transfer d to needy parents. All young people do have parents, but some older people may not have children, at least caring children.

where $\bar{w}$ is the average income, $\bar{\Pi}$ is the average probability of needing LTC and $\delta \geq 0$ is a "black box" parameter measuring the distortions created by the financing of the public transfer. It will play a role analogous to that of the cost of public funds in a normative setting.

The timing of the model runs as follows. Individuals first choose the value of τ by majority voting. We assume that only young agents vote (observe that, in the absence of altruism, old agents would be in favor of the value of τ which maximizes the transfer b if they need LTC, and would be indifferent as to the value of τ if not dependent) and that they vote as if the result of the vote would continue to hold in the next period (this assumption is standard in the positive literature on pensions)[10]. They then observe the result of the vote, and decide privately how much to save. They learn at the beginning of their second period of life whether they need LTC or not and whether they receive a transfer from their children or not.

3 Solving the Model

People vote over τ anticipating the private decisions (here, savings decisions) of everyone. We thus look first at these private decisions, and then look at preferences over τ.

3.1 Private choice of saving

The first-order condition for saving of individual i is given by[11]

$$\frac{\partial U_i}{\partial s_i} = -1 + (1 - \Pi_i)u'(s_i) + \Pi_i E H'_i = 0, \tag{3}$$

where

$$EH'_i = p_i H'(c_c) + (1 - p_i)H'(c_n)$$

is the expected marginal utility in case of LTC of an agent of type i, and where τ is set at an exogenous level. It is clear from (3) that the optimal saving s_i depends upon Π_i and p_i, but not upon w_i. We denote the optimal saving of an agent of type i as s_i^*.

We obtain the following comparative statics result.

Proposition 1 *For any given τ, the individually optimal amount of saving, s_i^* is independent of income w_i, increases with the inefficiency of the taxation system*

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[10]See Casamatta *et al.* (2000)

[11]Observe that the condition of infinite marginal utility when consumption tends toward zero guarantees positive saving and that (3) holds with equality.

(δ), and decreases with the probability of having caring kids (p_i) and with the transfer d. It also increases with the probability of becoming dependent (Π_i), provided that $u'(s_i) < EH'_i$. Finally, saving decreases with τ provided that $\tau < 1/2\delta$.

Proof: Straightforward differentiation of the FOC (3). ∎

Intuitively, the need for saving increases with the probability of higher future needs as measured by Π_i (provided that additional resources obtained if dependent are not so large that marginal utility is lower if dependent than if not) and with the inefficiency of the tax system (δ), while it decreases with family help (measured by either p_i or d). With linear utility in the first period, saving does not depend on income (since marginal utility of first period consumption is one whatever the consumption level). There is substitution between private saving and the social insurance contribution rate τ provided that τ is low enough that we are on the upward sloping side of the Laffer curve (i.e., that raising τ increases the transfer b).

This analysis is valid for any given value of τ. We now turn to the political determination of the tax level–i.e., we move to the first stage choice of τ.

3.2   Most-preferred tax and saving bundle

The first-order condition for the most-preferred tax rate τ of individual i is given by

$$\frac{\partial U_i}{\partial \tau} = -w_i + \Pi_i EH'_i \frac{\partial b}{\partial \tau} = 0. \tag{4}$$

We denote this optimal tax rate of type i as τ_i^*.

We now perform the comparative statics analysis of the most-preferred bundle (s_i^*, τ_i^*) with respect to individual characteristics w_i, p_i and Π_i.

Proposition 2 *Comparative statics of the most-preferred (interior) bundle (s_i^*, τ_i^*):*
(i) a larger income w_i increases s_i^* and decreases τ_i^*
(ii) a larger family help (either p_i or d) decreases both s_i^* and τ_i^*.
(iii) a larger risk Π_i increases τ_i^* but may not always decrease s_i^* .

Proof: Straightforward application of the implicit function theorem on the system given by the FOCs (3) and (4). ∎

Intuitively, the most-preferred value of τ decreases with income, because social insurance redistributes across income levels (with a lump sum transfer financed with a proportional tax on income). Although richer people do not save more than poorer people when τ is exogenously set (as per Proposition 1), richer people save more when they obtain their most-preferred value of τ, since they substitute

private saving to the less appealing social insurance scheme. Second period needs decrease with family help (measured by either p_i or d), and thus the need for both saving[12] and social insurance also decreases. A larger risk Π_i also calls for more social insurance (a transfer which is targeted to states of nature where the individual is dependent) but may not call for more saving (which is a non targeted transfer received also when not dependent).

Proposition 2 has assumed that the most-preferred tax and saving bundle is interior. This is always the case for saving[13], and close observation of the FOC for τ reveals the following result.

Proposition 3 *(i) Richer people want no social insurance: whatever their other characteristics, there always exists a threshold value of w above which individuals prefer $\tau_i^* = 0$.*
(ii) People non likely to turn dependent want no social insurance: whatever their other characteristics, there always exists a threshold Π_i below which $\tau_i^* = 0$.
(iii) No such thresholds always exist, whatever their other characteristics, for family help (p_i, d).

Part (ii) is intuitive: people who face no risk of becoming dependent have no demand for social insurance, whatever their other characteristics, and by continuity people with a very low value of Π_i also prefer self-insurance to social insurance. Part (iii) is also intuitive: even though family help decreases the demand for social insurance, a very large risk Π_i coupled with a very low income w_i may generate a positive support for some social insurance. Part (i) is may be more surprising: even though social insurance is a transfer targeted to the states of nature where marginal utility is especially high, very rich agents prefer to self insure rather than favoring even a very small amount of social insurance.

The comparative statics results of Propositions 2 and 3 assume that individual characteristics are modified one at a time (i.e., independently from one another). We have argued in the introduction that individual characteristics are correlated. Observe that, if richer people tend to live longer and hence to have a larger probability of needing LTC (i.e., $cov(w, \Pi) > 0$), then the net impact of a higher w_i coupled with a higher Π_i on s_i^* and τ_i^* is ambiguous. Whether one impact is larger than the other one is essentially an empirical matter of both the intensity of the correlation and the amount of variance in the two characteristics. For instance, if (as we surmise), the variance in income levels is larger than the variance in the

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[12]As shown in Proposition 10, this result requires that $\delta > 0$. If $\delta = 0$, saving does not depend on p_i or d, as explained after Proposition 10 and in footnote 20.

[13]As stated above, when marginal utility tends towards infinity for zero consumption level, everyone saves a positive amount.

risk levels, then richer people will favor lower social insurance contribution rate (even though they may be riskier than poorer people).

Throughout the paper, we illustrate our analytical results with a numerical example. We assume that agents's utility in case of non dependency is logarithmic, with $H(x) = u(x-2)$. Individual income is distributed uniformly over the interval [1,10], so that average and median income are both equal to 5.5.[14] With $\delta = 0.8$, the transfer b is maximized for $\tau = 0.6$. In Figure 1, we assume that $p_i = 0.5$ for all i with $d = 1$ and we report the most-preferred bundle (s_i^*, τ_i^*) when w_i varies continuously from 1 to 10. The green curve corresponds to the case where $\Pi_i = 0.5$ for all, so that $cov(w, \Pi) = 0$. In that case, as income increases, individuals substitute private saving to public insurance, so that τ^* decreases and s^* increases, as shown in Proposition 2. Agents with an income larger than 8.5 have the same most-preferred bundle, with a zero most-preferred tax rate and the same amount of saving (due to the linearity of utility in the first period). The other two curves correspond to cases where correlation is perfect between w and Π, but differ in sign. The blue curve corresponds to the case where the risk Π decreases linearly with income, from 0.9 for the lowest income level to 0.1 for the highest income level. In that case, τ^* decreases with income, with low income workers exhibiting a larger τ^* than when they differ only in income, while the opposite relationship holds for high income workers. (By construction, all three curves intersect for the median and average income level, because average risk and income are the same in all three cases while the risk level of the average income individual is also the same in all three cases). The picture is more complex for saving: s^* increases with w as long as w is low enough that the agent wants a positive tax rate, but then decreases with income above this threshold. The intuition for the latter result is to be found in Proposition 1: for an exogenous value of τ (zero here), saving is not affected by income but decreases when the risk Π_i decreases. Finally, the red curve illustrates the more empirically relevant positive correlation between income and LTC risk, with the risk increasing linearly with income, from 0.1 for the lowest income agent to 0.9 for the highest income. We obtain that the income effect trumps the risk effect, in the sense that τ^* decreases with income while s^* increases with income, as we surmised above.[15] The counter impact of a larger Π when w increases can be seen in the fact that both τ^* and s^* vary less with income

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[14]We have chosen the simpler uniform distribution over the positively-skewed income distributions observed empirically because our objective is not to replicate empirical results but to give the simplest numerical illustration of our analytical results.

[15]We provide another numerical example after Proposition 9 in section 4.2., based on the same functional forms, but where Π_i increases with w_i in such a way that their ratio remains a constant (rather than having w_i/Π_i increasing in w_i as here). In that case, a larger income (and risk) is associated with a larger τ_i^* and a smaller s_i^*. This confirms that whether the income effect trumps the risk effect is essentially an empirical question.

than in the other cases. Also, this is the only case among the three depicted here where even the highest income individual has a strictly positive most-preferred value of τ.

Insert Figure 1 around here

As for the correlation between income and family help (measured by p_i), the sign of the correlation depends on the type of data used, as mentioned in the Introduction. With macro data, the correlation seems to be negative (the so-called "South-North gradient"), and the impact of a larger income/lower family help on τ^* is ambiguous. On the opposite, with micro data (see Bonsang (2009)), the correlation between income and family help seems to be positive, and richer people should favor less social insurance (the impact on saving being ambiguous).

Figure 2 illustrates the impact of the correlation between income and family help. We proceed in the same way as for Figure 1. We assume that $\Pi_i = 0.5$ for all i. The green curve corresponds to the case where $p_i = 0.5$ for all i (no correlation) and is identical to the green curve in Figure 1. The red curve represents positive and perfect correlation, with the probability of family help increasing linearly with income from 0.1 to 0.9. The most-preferred tax rate decreases with income, and is larger than without correlation for lower-than-average income agents, and lower above this threshold. Symmetrically, saving is lower than without correlation for low income agents, and larger above. Saving increases with income as long as τ^* is positive, and then decreases with income (i.e., with p_i) above this threshold. Finally, the blue curve illustrates the perfect negative correlation between income and family help. Here also, the income effect trumps the family help effect. Optimal saving increases with income over the whole range of income levels (since even agents with a zero value for τ^* increase their saving level when p_i decreases).

Insert Figure 2 around here

We now move to the determination, by majority voting, of τ

3.3 Majority voting over τ

Preferences over τ are locally concave around τ_i^* since we obtain, using the envelope theorem, that

$$\frac{\partial^2 U_i}{\partial \tau^2} < 0.$$

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We can then apply the median voter theorem, so that the majority chosen value of τ, denoted by τ^V, corresponds to the median most-preferred value of τ_i^*.

The identity of the decisive agent is difficult to pinpoint, because agents differ in three dimensions. One way to depict the preferences over τ of individuals of different types consists in drawing "iso-τ*" curves in the (w_i, p_i) plane, the (w_i, Π_i) plane and the (Π_i, p_i) plane.

In the (w_i, p_i) plane, using the implicit function theorem for a system of two FOCs together with Cramer's rule, the slope of the "iso-τ*" curves is given by

$$\frac{\partial w_i}{\partial p_i} = -\frac{\partial \tau_i^*/\partial p_i}{\partial \tau_i^*/\partial w_i} < 0. \tag{5}$$

We then have that agents with low w_i and low p_i prefer a large value of τ, while rich people with a large probability of having children prefer a low (possibly nil) value of τ.

In the (w_i, Π_i) plane, the slope of the "iso-τ*" curves is given by

$$\frac{\partial w_i}{\partial \Pi_i} = -\frac{\partial \tau_i^*/\partial \Pi_i}{\partial \tau_i^*/\partial w_i} > 0.$$

We then have that agents with low w_i and large Π_i prefer a large value of τ, while rich people with a small probability of having LTC needs prefer a low (possibly nil) value of τ.

In the (Π_i, p_i) plane, the slope of the "iso-τ*" curves is given by

$$\frac{\partial \Pi_i}{\partial p_i} = -\frac{\partial \tau_i^*/\partial p_i}{\partial \tau_i^*/\partial \Pi_i} > 0.$$

We then have that agents with low p_i and large Π_i prefer a large value of τ, while people with a small probability of having LTC needs and a larger probability of having children prefer a low (possibly nil) value of τ.

As for the majority-voting value of τ, it corresponds to the value of τ^* that is such that the corresponding iso-τ^* surface divides the three-dimensional space of types such that half of voters are below this plane and half above.

Formally, we have that

Proposition 4 *Assume that the distribution of types is given by the distribution function $F(w_i, p_i, \Pi_i)$ with frequency $f(w_i, p_i, \Pi_i)$. Observe that the first-order condition (4) implicitly defines, for any level of τ°, of p_i and of Π_i, a value of w_i such that the individual with (w_i, p_i, Π_i) most-prefers τ°. Let us denote this value by $w_i^*(\tau^\circ, p_i, \Pi_i)$. The majority-voting value of τ, denoted by τ^V, is the value τ° which satisfies*

$$\int_0^1 \int_0^1 \int_0^{w_i^*(\tau^\circ, p_i, \Pi_i)} f(w_i, p_i, \Pi_i) dw_i dp_i d\Pi_i = 1/2 \tag{6}$$

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This formula is obviously difficult to manipulate analytically without specifying functional forms and the distribution of types. There is one comparative statics result that can be obtained very easily:

Proposition 5 *If society becomes less family centered (i.e., if p_i or d decreases for all agents), then τ^V increases.*

Proof: Immediate result since τ_i^* decreases with p_i and with d. ∎

Figure 3 illustrates (s^*, τ^*) as a function of w (same functional forms and distribution assumptions as above) for three different values of d, when $p_i = \Pi_i = 0.5$.

Insert Figure 3 around here

Unfortunately, we cannot proceed in the same way for variations of w_i and Π_i (although the impact of their variations on τ_i^* is the same for all agents) because modifying the value of w_i and Π_i for all agents would affect $\bar{w}$ and $\bar{\Pi}$ which play a role in the government's budget balance equation (2). Moreover, the individual (direct) impact of, say, increasing w_i on τ_i^* is negative, while the increase in all w_i's would raise $\bar{w}$, which would increase τ_i^* since it would increase the return of the social insurance scheme. Likewise, the individual (direct) impact of increasing Π_i on τ_i^* is positive, while the increase in all Π_i's would raise $\bar{\Pi}$, which would decrease τ_i^* since it would decrease the return of the social insurance scheme.

We now introduce private insurance into the picture.

4   Introducing private insurance

We model a private insurance scheme which is actuarially fair: the premium does not depend on income but is based on the individual risk Π_i (which is assumed to be observable by the insurer). Since LTC need is binary, there is no place for ex post moral hazard, and no distinction between lump sum payment and payment increasing with the LTC expenses. Also, we assume that insurers do not condition the payment on the transfer made by children (for instance because they cannot observe it).

Individuals can choose the quantity of private insurance that they buy, as measured by the insurance premium a_i paid in the first period of life. In case they need LTC, they then receive an actuarially fair amount

$$x_i(a_i) = \frac{a_i}{\Pi_i}.$$

12

The utility of individual i is then given by

$$\begin{aligned}
U_i &= w_i(1 - \tau) - s_i - l_i d - a_i \\
&+ (1 - \Pi_i)u(s_i) \\
&+ \Pi_i p_i H(c_c) + \Pi_i(1 - p_i)H(c_n),
\end{aligned}$$

with

$$\begin{aligned}
c_c &= s_i + b + d + x_i, \\
c_n &= s_i + b + x_i, \\
b &= \tau(1 - \delta\tau)\frac{\bar{w}}{\bar{\bar{\Pi}}}, \\
x_i &= \frac{a_i}{\Pi_i}.
\end{aligned}$$

The FOCs with respect to contribution rate, saving and private insurance coverage are

$$FOC\tau_i \quad : \quad -w_i + \Pi_i EH'_i \frac{\partial b}{\partial \tau} \le 0 \qquad (7)$$

$$FOCs_i \quad : \quad -1 + (1 - \Pi_i)u'(s_i) + \Pi_i EH'_i = 0, \qquad (8)$$

$$FOCa_i \quad : \quad -1 + EH'_i \le 0. \qquad (9)$$

Observe that the formulation of the FOC for saving is not affected by the presence of private insurance, so that we know from the assumption that $\lim_{c \to 0} u'(c) = \infty$ that it holds with equality (i.e., everyone saves a positive amount at his optimum, although the specific amount may of course depend on whether private insurance is available or not).

In order to better understand the driving forces of this model, we simplify the traits space by first assuming away family help, tax distortions and risk heterogeneity.

4.1 Individuals differ in income only

In this section, we assume that there is no family help (because either $p_i = 0$ or $d = 0$), no tax distortion ($\delta = 0$) and that all individuals have the same risk level, $\Pi_i = \Pi = \bar{\Pi}$. Individuals then differ only in their income levels, w_i. We first show that the optimal bundle of individuals consists in either social or private insurance, but not both simultaneously, together with private saving.

Proposition 6 *Assume that $d = 0$ or $p_i = 0$, $\delta = 0$ and $\Pi_i = \Pi = \bar{\Pi}$, for all i. Agents with $w_i < \bar{w}$ most-prefer social insurance together with no private insurance ($a_i^* = 0$, $\tau_i^* > 0$) while agents with $w_i > \bar{w}$ have the opposite preferences ($a_i^* > 0$, $\tau_i^* = 0$).*

Proof: See appendix. ∎

The intuition for this result is that both forms of insurance transfer income from first period to the same second period state of nature, so that individuals choose the cheaper among the two forms, which depends upon how their income compares with the average income given the redistributiveness embedded in the social insurance program.[16]

We then show that the introduction of private insurance, although it decreases the support for social insurance, does not affect its majority chosen level.

Proposition 7 *Assume that $d = 0$ or $p_i = 0$, $\delta = 0$ and $\Pi_i = \Pi = \bar{\Pi}$, for all i, and that the median income level is smaller than the average. Although allowing for private insurance decreases the political support for social insurance of above-average-income agents, it does not modify the politically chosen level of social insurance, which remains positive and determined by the median income agent.*

Proof: See Appendix ∎

The intuition for this result is simply that the preferences of all agents with lower-than-average income (including the decisive voter with median income) for social insurance are not affected by the availability of private insurance, while all agents with larger-than-average income support private insurance when available. This is illustrated on Figure 4, where the red curve depicts $\tau^*(w)$ in the absence of private insurance and where $\Pi = 0.5$. When private insurance is introduced, agents up to the average income of 5.5 prefer the same τ^* together with $a^* = 0$, while above-average income agents prefer $\tau^* = 0$ together with $a^* > 0$ (the blue curve).

Insert Figure 4 around here

We now introduce a loading factor on private insurance, assuming that the payment received from the private insurer is less than actuarially fair (due to administrative and/or marketing costs), so that a premium of a_i by individual i generates a payment of $\lambda a_i / \Pi_i$ (with $\lambda < 1$) in case of dependency. We then obtain the following result.

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[16]The fact that agents prefer either private or social insurance hinges on the absence of distortion in social insurance. With $\delta > 0$, low income agents would favor some social insurance, then top up with private insurance once the distortions associated with social insurance become large enough. This point is developed (in the context of social security) by Casamatta *et al.* (2000).

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Proposition 8 *Assume that $d = 0$ or $p_i = 0$, $\delta = 0$ and $\Pi_i = \Pi = \bar{\bar{\Pi}}$, for all i. Introducing a loading factor $1/\lambda > 1$ on private insurance has the following impact compared to no loading factor ($\lambda = 1$): (i) if λ is large enough, the minimum income compatible with $\tau_i^* = 0$ increases to $\bar{w}/\lambda$, with agents below this income threshold favoring $\tau_i^* > 0$ and $a_i^* = 0$ while agents above the threshold most prefer $\tau_i^* = 0$ and $a_i^* > 0$. Moreover, agents above this income threshold substitute private saving to private insurance as λ decreases; (ii) if λ is small enough, no one wishes to buy private insurance so that preferences are the same as when private insurance is not available.*

Proof: See Appendix ∎

The green curve in Figure 4 illustrates part (i) of Proposition 8 where we assume a loading factor such that $\lambda = 0.8$ on private insurance. The income threshold above which agents switch their support from social to private insurance increases compared to the blue curve (private insurance with $\lambda = 1$). As λ further decreases, this threshold increases, up to the point where it reaches the income threshold when private insurance is not available (given by the intersection of the red curve with the horizontal axis in Figure 4). For this and lower values of λ ($\lambda \leq 0.62$ in our numerical simulations), private insurance becomes so unattractive that no one wishes to buy it, and preferences are identical to the case where private insurance is not available at all.

In other words, the total demand for private insurance (measured at the optimum policy bundles of individuals) decreases for two reasons when private insurance becomes less actuarially fair (i.e., when λ decreases): first, the threshold income level above which agents wish to buy private insurance increases; second, the optimal amount of private insurance bought by these agents decreases.

We now reintroduce heterogeneity in risk as well as in income, but revert to an actuarially fair private insurance.

4.2 Individuals differ in income and risk

In this section, individuals are characterized by the pair (w_i, Π_i). We first show that there exists an equivalent to Proposition 6, where the threshold determining whether an agent most prefers social or private insurance is expressed in terms of the ratio of income to risk.

Proposition 9 *Assume that $d = 0$ or $p_i = 0$ for all i, and that $\delta = 0$ and $\lambda = 1$. Agents with $w_i/\Pi_i < \bar{w}/\bar{\bar{\Pi}}$ most prefer social insurance together with no private insurance ($a_i^* = 0$, $\tau_i^* > 0$) while agents with $w_i/\Pi_i > \bar{w}/\bar{\bar{\Pi}}$ have the opposite*

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preferences ($a_i^* > 0$, $\tau_i^* = 0$). Moreover, all agents with $w_i/\Pi_i > \bar{w}/\bar{\Pi}$ have the same most-preferred saving level s_i^* and private insurance transfer a_i^*/Π_i (so that a_i^* increases with Π_i).

Proof: See Appendix. ∎

The intuition for this result closely mirrors the intuition for Proposition 6, except that what determines the return on social insurance for agent i is now the ratio of income to risk (observe that neither income nor risk affects the return of private insurance, by assumption). This is *not* to say that this ratio determines the most-preferred *level* of social insurance for agents with $w_i/\Pi_i < \bar{w}/\bar{\Pi}$.[17] There are two consequences on the determination of the majority chosen contribution rate. First, the majority chosen tax rate is positive provided that a majority of agents have a ratio of income to risk that is lower than the ratio of average income to average risk.[18] Second, it is not possible at this level of generality to determine who the decisive voter is, since preferences for τ are *not* determined only by the ratio of income to risk. For instance, with our numerical simulations, we obtain that τ_i^* increases while s_i^* decreases as both w_i and Π_i are increased simultaneously such that w_i/Π_i remains constant: unlike in section 3.2, the risk effect trumps the income effect when income and risk are positively correlated in that way![19] Finally, once agents have an income-to-risk ratio large enough to prefer private to social insurance, they all save the same amount (such that marginal utility when not dependent is one) and buy an amount of private insurance which is increasing in their own risk Π_i (because all such agents buy just enough private insurance to equalize marginal utility across states of nature, while the return of private insurance decreases with individual risk).

The introduction of a loading factor on private insurance has the same qualitative impact as in the previous section, and is left to the reader.

Finally, we reintroduce family help, assuming that everyone shares the same probability p of receiving the transfer $d > 0$ if dependent. We obtain the following result.

Proposition 10 *Assume that $d > 0$, $\delta = 0$, $\lambda = 1$, $p_i = p$ for all i. (i) If p and d are low enough, then (a) the same results as in Proposition 9 hold, so that agents with $w_i/\Pi_i < \bar{w}/\bar{\Pi}$ most prefer social insurance together with no private insurance*

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[17]Observe for instance that w_i does not impact the FOC for saving.

[18]Observe that the relevant threshold is *not* the average ratio of income to risk: average income determines fiscal proceeds for a given τ while average risk determines the amount of the lump sum transfer for given tax proceeds.

[19]Note that, in Figure 1, w_i/Π_i is increasing in w_i for the 3 cases reported.

($a_i^* = 0$, $\tau_i^* > 0$) while agents with $w_i/\Pi_i > \bar{w}/\bar{\Pi}$ have the opposite preferences ($a_i^* > 0$, $\tau_i^* = 0$) and share the same most-preferred bundle ($a_i^*/\Pi_i > 0$, $s_i^* > 0$). (b) The optimal saving level (of any agent) is not affected by either d or p, while (c) the optimal tax rate τ_i^* (resp., private insurance amount a_i^*) decreases as both d and p increase for agents with $w_i/\Pi_i < \bar{w}/\bar{\Pi}$ (resp., $w_i/\Pi_i > \bar{w}/\bar{\Pi}$).

(ii) If both p and d are large enough, then (a) no one wishes to buy private insurance: agents with w_i/Π_i lower than a threshold x prefer social insurance only ($a_i^* = 0$, $\tau_i^* > 0$) while people with w_i/Π_i larger than x prefer neither social nor private insurance ($a_i^* = \tau_i^* = 0$) but rely only on (the same amount of) self insurance. (b) The threshold value x is lower than $\bar{w}/\bar{\Pi}$ and decreases as both d and p increase. (c) The optimal saving level is not affected by either d or p for agents with a positive τ_i^*, but decreases with both p and d when $\tau_i^* = 0$. (d) For agents with w_i/Π_i larger than x, saving is independent of income but decreases with Π_i. (e) The most-preferred tax rate decreases as both d and p decrease for w_i/Π_i lower than x.

Proof: See Appendix ∎

The intuition for Proposition 10 runs as follows. When family help (as measured by d and p) is small enough, the results from Proposition 9 still hold, so that agents with low income and large risk most prefer social insurance while high income/low risk agents most prefer private insurance. Any increase in family help then crowds out the demand for social insurance (for low w_i/Π_i) and for private insurance (for large w_i/Π_i). Observe that the agents with $w_i/\Pi_i = \bar{w}/\bar{\Pi}$ are indifferent between social and private insurance, provided that their most-preferred amount of insurance (whatever its source) is provided, i.e. we have $\tau_i^* w_i = a_i^*$. This amount a_i^* (which is the same for all agents with $w_i/\Pi_i > \bar{w}/\bar{\Pi}$) decreases with p and d, up to the point where it reaches zero. Once this point is reached, no one wishes to buy private insurance, and any further increase in p or d crowds out social insurance along two margins: at the intensive margin (as low w_i/Π_i agents decrease their most-preferred value of τ_i^*) and at the extensive margin (with a decrease in the threshold value of w_i/Π_i above which people resort exclusively to self insurance).

The behavior of private saving is also interesting, since it is not affected by family help (d or p) as soon as agents most prefer some social or private insurance. When they most prefer private insurance, agents save and buy insurance in order to equalize marginal utility to one both if they are dependent and if they are not. Hence, the optimal private insurance amount is affected by the size and probability of family help, but saving is not. Similarly, if agents most prefer social insurance, they choose a combination of saving and tax rate such that the expected utility in case of dependence remains equal to a constant (now smaller than one) which

depends only on factors influencing social insurance's individual return, namely income and risk, but not family help. Hence, the optimal tax rate is affected by family help, but saving is not.[20] Finally, optimal saving decreases with both p_i and d for agents who exclusively self insure.

To summarize, family help crowds out the demand for both private and social insurance, but in different ways. When p and d are low enough, they crowd out the demand for social insurance by low w_i/Π_i types and for private insurance by large w_i/Π_i types. The crowding out is exclusively at the intensive margin, since the threshold value of w_i/Π_i (equal to $\bar{w}/\bar{\Pi}$) which determines whether agents prefer social or private insurance is not affected by d or p. When family help is large enough that it drives to zero the demand for private insurance of the agents with $w_i/\Pi_i = \bar{w}/\bar{\Pi}$, demand for private insurance disappears. From that point on, any increase in family help crowds out the demand for social insurance both at the intensive and at the extensive margins.

Finally, the result mentioned in point (ii) (d) that saving decreases with risk for agents who rely exclusively on self insurance when family help is large enough is due to the fact that marginal utility is lower when dependent than when not (thanks to the large family help), so that a higher probability of being dependent (and hence of receiving the large family transfer) decreases the expected marginal utility when old, leading to a lower saving level.

Figure 5 illustrates the most-preferred level of the social insurance contribution rate as a function of income for three values of d (when $\Pi_i = p_i = 0.5$ for all agents). With no family help ($d = 0$), all agents with $w < \bar{w}$ prefer social insurance, with τ_i^* decreasing in w_i. Observe that agents with average income are indifferent between any combination of social and private insurance which give them their most-preferred insurance level a_i^*. Agents with larger-than-average income most prefer private to social insurance. Increases in family help crowd out the demand for social insurance, up to the point (reached when $d = 0.75$) where the average income agent prefers to self insure (i.e., $\tau_i^* = a_i^* = 0$). From that level on, any further increase in family help d crowds out social insurance both at the intensive and extensive margins, as can be seen by comparing the green and red curves.

Insert Figure 5 around here

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[20]The fact that saving is independent of p and d depends crucially on the assumption that there is no distortion associated to the funding of social insurance. If $\delta > 0$, the individual return from social insurance decreases with the tax rate, so that the expected utility in case of dependency is not a constant anymore. Indeed, Proposition 2 shows that optimal saving decreases with family help if $\delta > 0$.

Figure 6 illustrates the optimal saving and private insurance bundle of larger-than-average-income individuals as a function of family help d. For low values of d, the demand for private insurance is positive and decreasing in d, while the optimal saving amount is a constant. When d becomes large enough, the demand for private insurance becomes nil, and any further increase in family help crowds out saving. In a nutshell, family help first crowds out private insurance, and then saving.

Insert Figure 6 around here

5   Conclusion

This paper has studied the determinants of the demand for private, social and self-insurance for LTC in an environment where individuals differ in earnings, family support and dependence risk. We obtain an interesting pattern of insights, depending on whether private insurance is available or not, on its loading factor, and on the correlation between, on the one hand, income and risk, and, on the other hand, income and family help.

We can use the results of our analysis to try and shed light on the future development of the three types of insurance for LTC. The two main changes expected to affect LTC in the near future are (i) the doubling in the number of dependent individuals in the next twenty years within the OECD, associated with the rapid increase of very old (75+) people in the population, and (ii) the decline in family solidarity due to increased participation of women in the labor market, increased mobility and changing family values. The first effect can be modelled in our setting as an increase in the risk of becoming dependent of all agents. This higher risk will undoubtedly increase the needs when old, but we obtain that it does not necessarily imply an increase in the demand for social insurance, because a larger average risk of becoming dependent decreases the return of the social LTC insurance. Observe that the return of the (actuarially fair) private insurance decreases with the individual risk, while self-insurance return is not affected. The impact of a larger aggregate risk on the demand for social insurance thus depends on its distribution across people, and especially on its correlation with income. The impact of a diminishing family support is easier to ascertain: as we show, it unambiguously increases the demand for social insurance among agents with a low-income-to-risk ratio. As for individuals with a high ratio, a decrease in family help will first increase their self-insurance level, and then increase their demand for private insurance. These results are obtained under the assumption that agents share the same probability of receiving family support. In case they

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differ in this dimension as well, the correlation between income and family help will play an important role.

6 Appendix

Proof of Proposition 6

Observe first that the three FOCs become

$$
\begin{aligned}
FOC\tau_i &: \quad \bar{w}\left[-\frac{w_i}{\bar{w}} + EH'_i\right] \leq 0, \\
FOCa_i &: \quad -1 + EH'_i \leq 0, \\
FOCs_i &: \quad -1 + (1 - \Pi_i)u'(s_i) + \Pi_i EH'_i = 0,
\end{aligned}
$$

with

$$
EH'_i = H'(s_i + b + \frac{a_i}{\Pi_i}).
$$

Comparing the FOCs for τ and a, one sees that

$$
\begin{aligned}
FOC\tau_i &< FOCa_i \text{ for all } w_i > \bar{w}, \\
FOC\tau_i &= FOCa_i \text{ for all } w_i = \bar{w}, \\
FOC\tau_i &> FOCa_i \text{ for all } w_i < \bar{w}.
\end{aligned}
$$

We then show that people buy either private or social insurance—i.e., that $a_i^* = \tau_i^* = 0$ is impossible. If it were the case, we would obtain that $EH'_i > u(s_i)$, so that, by the FOC for saving, we would have $EH'_i > 1 > u'(s_i)$, which in turn would imply that $FOCa_i > 0$, a contradiction with $a_i^* = 0$.

Observe that $a_i^* > 0$ implies that $EH'_i = 1$. This in turn implies that $FOC\tau_i > 0$ if $w_i < \bar{w}$, an impossibility, and that $FOC\tau_i < 0$ if $w_i > \bar{w}$, so that $\tau_i^* = 0$. In the latter case, the FOC for saving implies that $EH'_i > u(s_i) = 1$, which is compatible with the starting assumption that $a_i^* > 0$.

Finally, observe that $\tau_i^* > 0$ implies that $EH'_i = w_i/\bar{w}$. If $w_i > \bar{w}$, we then obtain that $EH'_i > 1$ and so that $FOCa_i > 0$, a contradiction. On the other hand, if $w_i < \bar{w}$, we have that $EH'_i < 1$ and that $FOCa_i < 0$, implying that $a_i^* = 0$. Finally, it is obvious that $w_i = \bar{w}$ is indifferent between a and τ, provided that $EH'_i = 1$. ∎

Proof of Proposition 7

A) In the absence of private insurance, we have already shown in section 3.2. that τ_i^* is decreasing in w_i, and that there exists a threshold (denote it by $\hat{w}$) above which τ_i^* is zero. We now show that $\hat{w} > \bar{w}$ when $d = 0$ or $p_i = 0$, $\delta = 0$ and $\Pi_i = \Pi = \bar{\Pi}$, for all i. Assume that $\tau_i^* = 0$. Then, by the FOC for saving, we

have that $EH_i' > 1 > u(s_i)$. Looking at $FOC\tau_i$, we then see that the minimum wage level compatible with $\tau_i^* = 0$ is $\hat{w} = \bar{w}EH_i' > \bar{w}$.

B) When we introduce private insurance, we know from Proposition 6 that agents with $w_i < \bar{w}$ do not wish to buy private insurance, so that their most-preferred level of social insurance is unaffected by the availability of private insurance. Agents with $w_i > \bar{w}$ wish to buy private insurance but no social insurance. ■

Proof of Proposition 8

Introducing the loading factor on private insurance modifies the FOC for private insurance to become

$$FOCa_i : -1 + \lambda EH_i' \leq 0.$$

Comparing the FOCs for private and social insurance, it is easy to see that agents prefer social to private insurance if $w_i < \tilde{w} = \bar{w}/\lambda$, and private to social if the opposite relationship holds. As in the proof of Proposition 7, we denote by $\hat{w}$ the threshold income level above which agents prefer $\tau^* = 0$ in the absence of private insurance. Two cases may occur, depending on the value of λ:

(i) Assume that λ is large enough that $\tilde{w} < \hat{w}$. We then have that $\tau_i^* > 0$ and $a_i^* = 0$ for $w_i < \tilde{w}$. For agents with $\tilde{w} < w_i < \hat{w}$, private insurance is a better deal than social insurance while the most-preferred tax rate is strictly positive when private insurance is not available. We then have that $\tau_i^* = 0$ while $a_i^* > 0$ for these individuals. Since neither the FOC for private insurance nor for saving depends on income, all agents with $w_i > \tilde{w}$ have the same most-preferred bundle (a_i^*, s_i^*). The most-preferred bundle (τ_i^*, s_i^*) of agents with $w_i < \tilde{w}$ is not affected by λ (which plays no role in the FOCs for social insurance and for private saving). Finally, straightforward exploitation of the FOCs for a_i and s_i shows that $\partial s_i^*/\partial\lambda < 0$, $\partial a_i^*/\partial\lambda > 0$ while $\partial(s_i^* + \lambda a_i^*/\Pi)/\partial\lambda > 0$ (since $H'(s_i^* + \lambda a_i^*/\Pi) = 1/\lambda$ when $a_i^* > 0$).

(ii) Assume that λ is low enough that $\tilde{w} > \hat{w}$. We obtain that $\tau_i^* > 0$ and $a_i^* = 0$ for $w_i < \hat{w}$, since private insurance performs worse than social insurance for these individuals, while they prefer a strictly positive amount of social insurance in the absence of private insurance. For agents with $\hat{w} < w_i < \tilde{w}$, social insurance remains a better deal than private insurance, but income is large enough that agents prefer to self insure in the absence of private insurance: we then have that $\tau_i^* = 0$ while $a_i^* = 0$ for these individuals. Since the FOCs for saving and private insurance are not affected by income, we know that all agents above $\hat{w}$ share this same most-preferred policy bundle. ■

Proof of Proposition 9

Observe first that the three FOCs become

$$FOC\tau_i \quad : \quad \bar{w}\frac{\Pi_i}{\bar{\Pi}}\left[-\frac{w_i}{\Pi_i}\frac{\bar{\Pi}}{\bar{w}} + EH'_i\right] \le 0,$$

$$FOCa_i \quad : \quad -1 + EH'_i \le 0,$$

$$FOCs_i \quad : \quad -1 + (1 - \Pi_i)u'(s_i) + \Pi_i EH'_i = 0,$$

with

$$EH'_i = H'(s_i + b + \frac{a_i}{\Pi_i}).$$

Observe that the FOC for τ_i^* is the only one whose formula is affected by the heterogeneity in risks. The rest of the proof of the first statement follows the same path as the proof of Proposition 6. Also, for all agents who prefer private to social insurance, we obtain from the FOCs for a_i and s_i that $u'(s_i) = 1 = EH'_i = H'(s_i^* + a_i^*/\Pi_i) = 1$ which in turn imply that they all save the same amount s_i^* and obtain the same transfer from private insurance, a_i^*/Π_i. ∎

Proof of Proposition 10

Observe first that the three FOCs are

$$FOC\tau_i \quad : \quad \bar{w}\frac{\Pi_i}{\bar{\Pi}}\left[-\frac{w_i}{\Pi_i}\frac{\bar{\Pi}}{\bar{w}} + EH'_i\right] \le 0,$$

$$FOCa_i \quad : \quad -1 + EH'_i \le 0,$$

$$FOCs_i \quad : \quad -1 + (1 - \Pi_i)u'(s_i) + \Pi_i EH'_i = 0,$$

with

$$EH'_i = (1 - p)H'(s_i + b + \frac{a_i}{\Pi_i}) + pH'(s_i + b + \frac{a_i}{\Pi_i} + d).$$

(i) We first show that people buy either private or social insurance—i.e., that $a_i^* = \tau_i^* = 0$ is impossible– if d and p are low enough. More precisely, if p and d are low enough that $EH'_i > u'(s_i)$ when $a_i^* = \tau_i^* = 0$, i.e. that

$$(1 - p)H'(s_i) + pH'(s_i + d) > u'(s_i),$$

then, by the FOC for saving, we would have $EH'_i > 1 > u(s_i)$, which in turn would imply that $FOCa_i > 0$, a contradiction with $a_i^* = 0$.

The rest of the proof of (i) (a) is the same as in the previous proposition (except that $EH'_i = (1-p)H'(s_i^* + a_i^*/\Pi_i) + pH'(s_i^* + a_i^*/\Pi_i + d)$ for $w_i/\Pi_i > \bar{w}/\bar{\Pi}$, which does not affect the conclusion).

(i) (b) From $FOC\tau_i = 0$, we obtain that

$$\Pi_i EH'_i = \frac{w_i}{\bar{w}}\bar{\Pi},$$

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which we plug into $FOCs_i = 0$ to obtain that

$$1 - (1 - \Pi_i)u'(s_i) = \frac{w_i}{\bar{w}}\bar{\Pi},$$

so that s_i^* is independent of both p_i and d for all agents with $\tau_i^* > 0$. Agents with $a_i^* > 0$ are such that $EH_i' = 1 = u'(s_i^*)$ so that s_i^* is also independent of both p_i and d.

(i) (c) Straightforward differentiation of the system of equations ($FOC\tau_i = 0, FOCs_i = 0$) for $w_i/\Pi_i < \bar{w}/\bar{\Pi}$ and of ($FOCa_i = 0, FOCs_i = 0$) for $w_i/\Pi_i > \bar{w}/\bar{\Pi}$ with respect to p and d shows that both τ_i^* (for $w_i/\Pi_i < \bar{w}/\bar{\Pi}$) and a_i^* (for $w_i/\Pi_i > \bar{w}/\bar{\Pi}$) decrease with both p and d.

(ii) (a) If p and d are large enough that $EH_i' \le u'(s_i)$ when $a_i^* = \tau_i^* = 0$, i.e. that

$$(1 - p)H'(s_i) + pH'(s_i + d) \le u'(s_i), \tag{10}$$

then, by the FOC for saving, we have $EH_i' \le 1 \le u'(s_i)$, which in turn implies that $FOCa_i \le 0$, consistent with $a_i^* = 0$. We then have that $FOC\tau_i \le 0$ for $\tau_i = 0$ provided that $w_i/\Pi_i \ge x = (\bar{w}/\bar{\Pi})EH_i'$, with $x \le \bar{w}/\bar{\Pi}$ since $EH_i' \le 1$.

In that case, nobody wishes to have $a_i^* > 0$. This would imply by $FOCa_i$ that $EH_i' = 1$, but this is incompatible with the assumption above that $EH_i' \le 1 \le u(s_i)$ when $a_i^* = 0$, since increasing EH_i' decreases in a_i.

The argument that $\tau_i^* > 0$ is compatible with $a_i^* = 0$ for individuals with $w_i/\Pi_i \le x \le \bar{w}/\bar{\Pi}$ is the same, mutatis mutandis, as in Proposition 6.

(b) We have just shown that $x = EH_i'(\bar{w}/\bar{\Pi})$ with $EH_i' = (1 - p)H'(s_i^*) + pH'(s_i^* + d) \le 1$. We then have that

$$\begin{aligned}
\frac{dx}{dd} &= \frac{\bar{w}}{\bar{\Pi}}\left(\frac{\partial EH_i'}{\partial d} + \frac{\partial EH_i'}{\partial s_i}\frac{\partial s_i^*}{\partial d}\right) \\
&= \frac{\bar{w}}{\bar{\Pi}}\left(\frac{\partial EH_i'}{\partial d}\left[1 - \Pi_i\frac{\partial EH_i'/\partial s_i}{(1 - \Pi_i)u''(s_i) + \Pi_i EH_i''}\right]\right) < 0
\end{aligned}$$

by the concavity of u and EH_i together with the fact that $EH_i' \le 1 \le u'(s_i)$. A similar argument shows that $dx/dp < 0$.

(c) The proof that s_i^* is independent of both p_i and d for agents with $\tau_i^* > 0$ is the same as in (i) (b) above. As for agents with $\tau_i^* = 0$, differentiation of the FOC for saving shows that

$$\begin{aligned}
\frac{\partial s_i^*}{\partial d} &= -\frac{\Pi_i p H''(s_i + d)}{(1 - \Pi_i)u''(s_i) + \Pi_i EH_i''} < 0, \\
\frac{\partial s_i^*}{\partial d} &= -\frac{\Pi_i\left[H'(s_i + d) - H'(s_i)\right]}{(1 - \Pi_i)u''(s_i) + \Pi_i EH_i''} < 0.
\end{aligned}$$

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(d) It is straightforward to see that the FOC for saving does not depend on income. Applying the implicit function theorem to the FOC for saving shows that s_i^* decreases with Π_i.

(e) Already shown in Proposition 2 (ii) (the fact that we modify the probability of family help of all agents simultaneously here and not in Proposition 2 (ii) has no consequence since there is no government budget balance involved). ■

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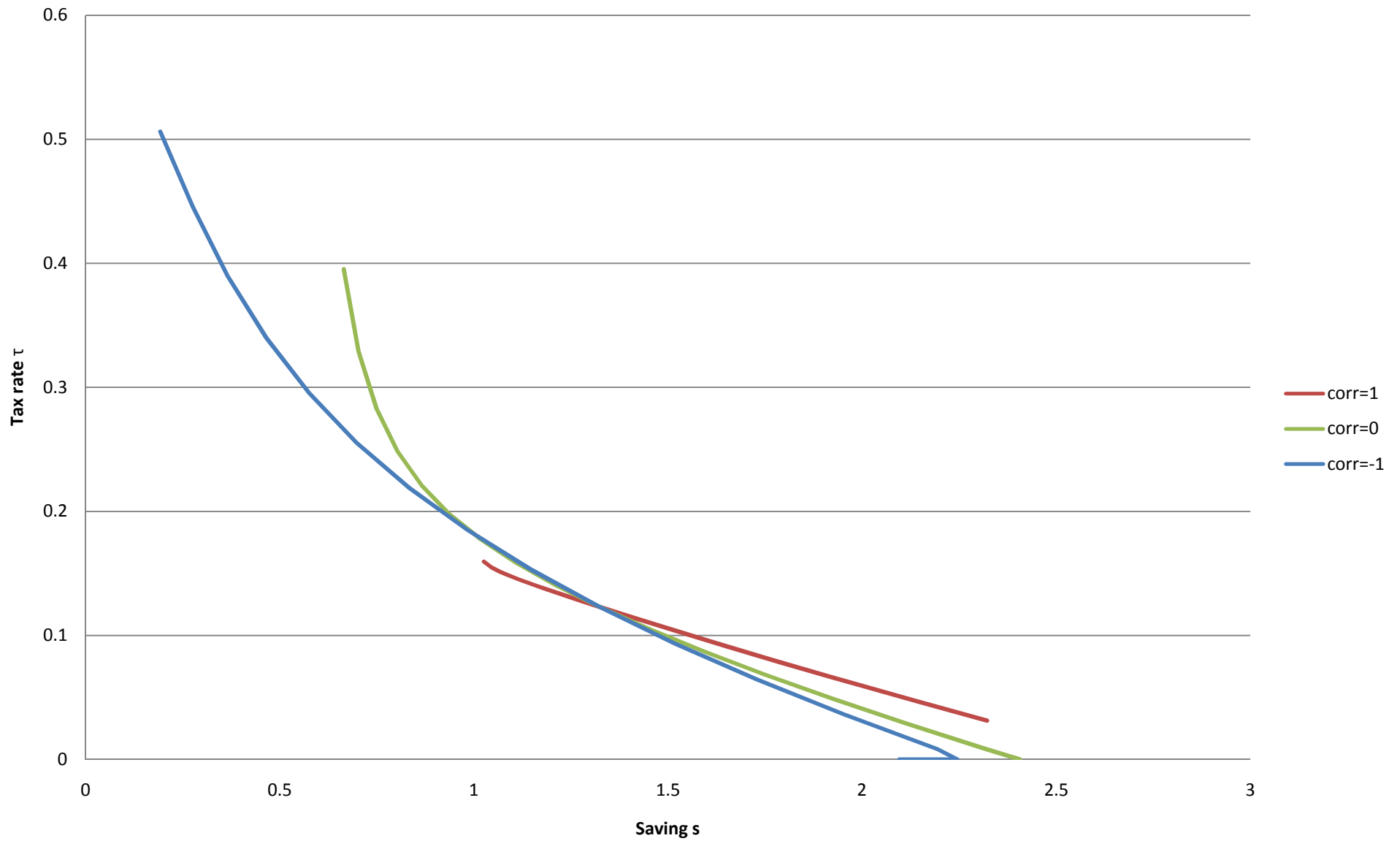
Figure 1: (s*,τ*) as a function of corr(w,Π)
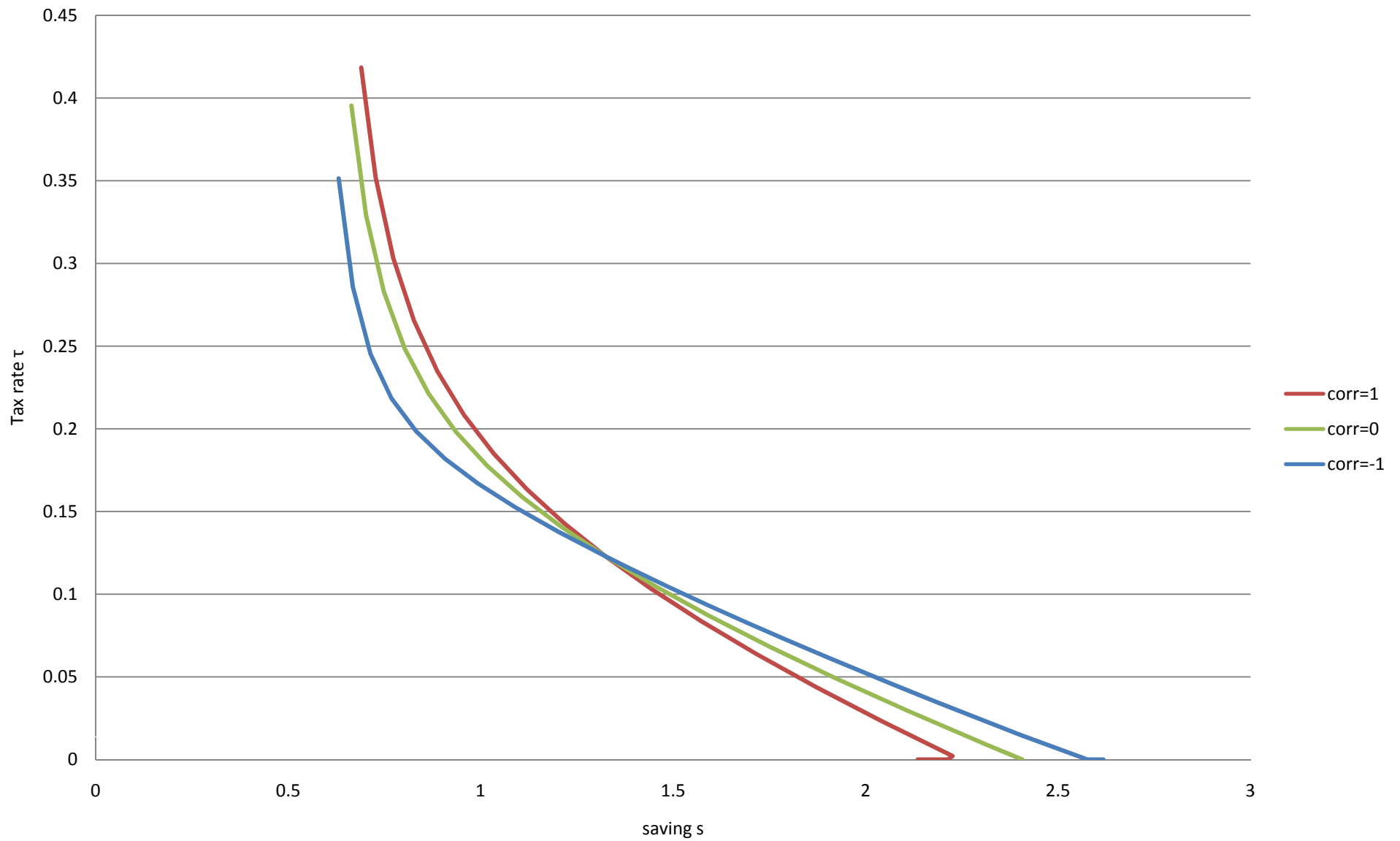
Figure 2: (s*,τ*) as a function of corr(w,p)
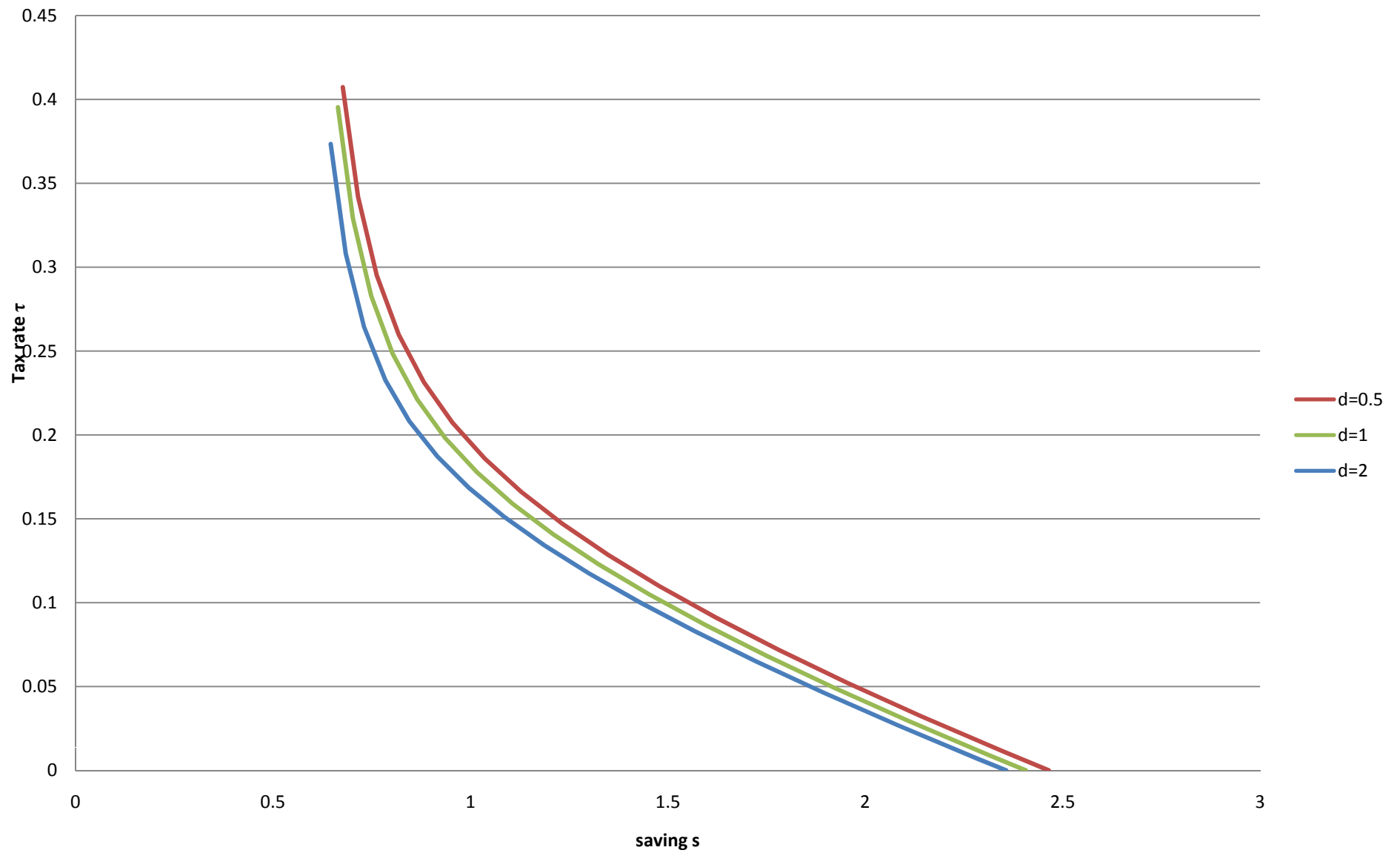
Figure 3: (s*,τ*) as a function of w
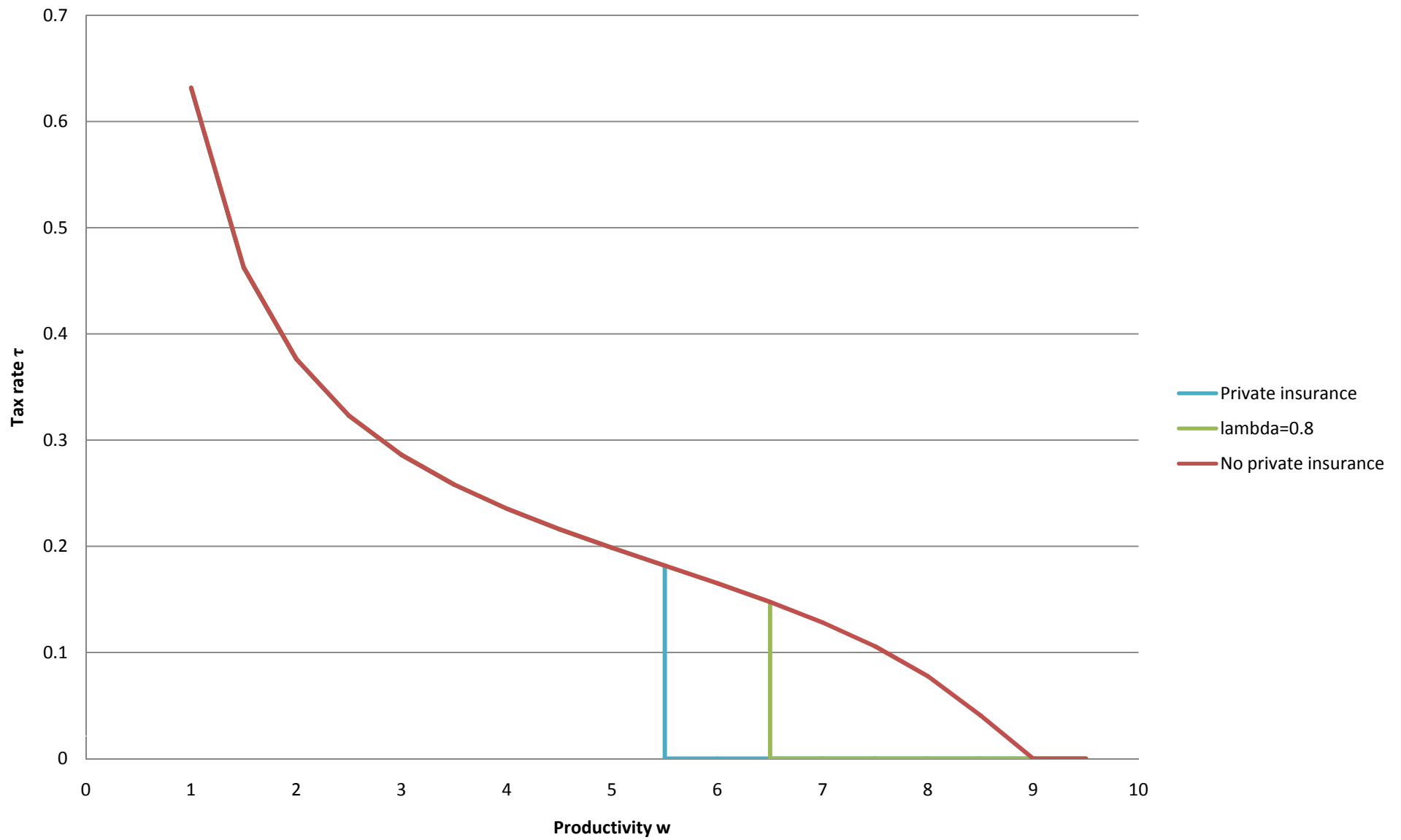
Figure 4: τ* as a function of w

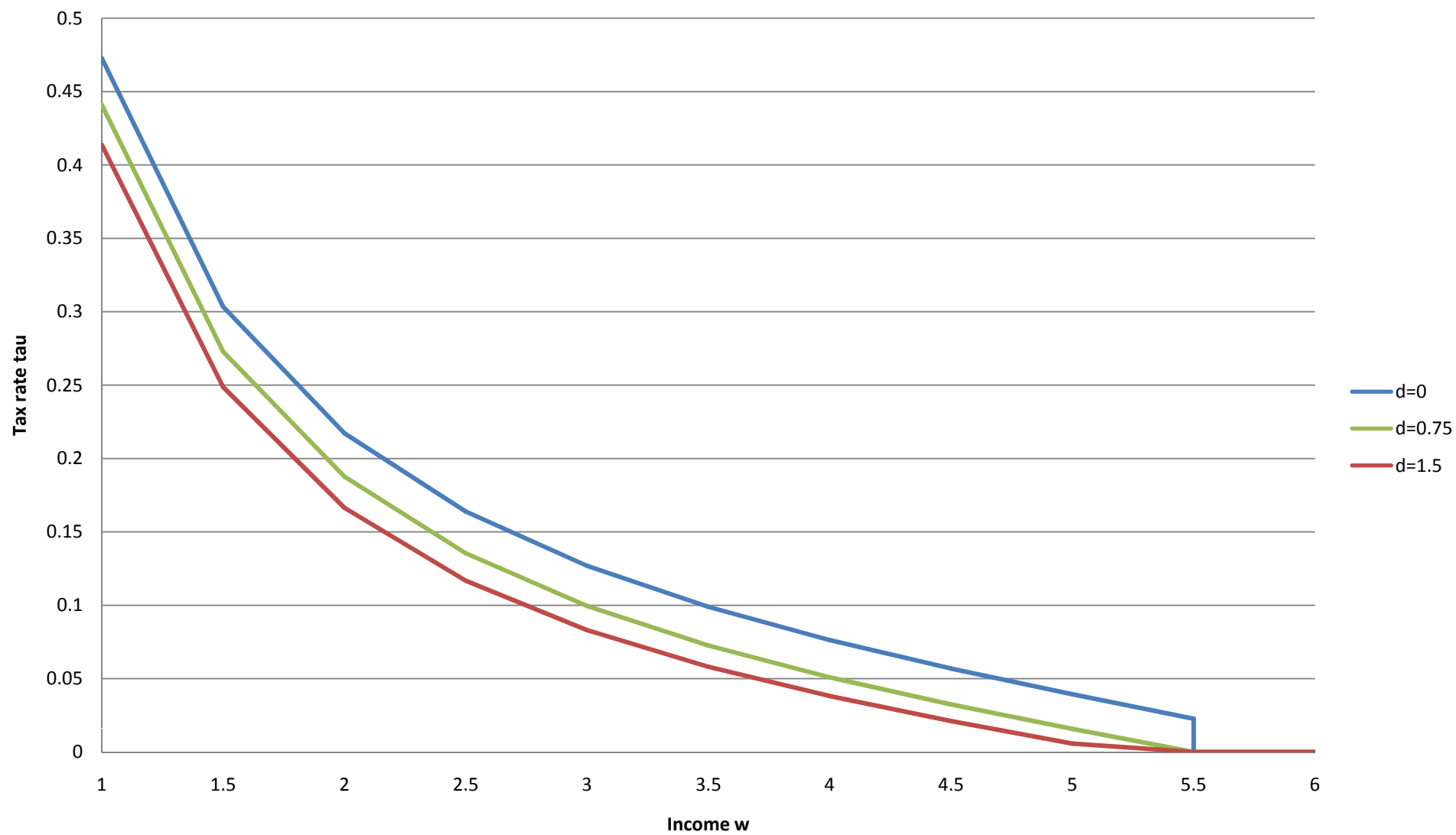Figure 5: τ^* as a function of income and of family help

Figure 6: (s*,a*) as a function of d for larger-than-average income levels