Innovation, Spillovers and Venture Capital Contracts

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Abstract

Innovative start-ups and venture capitalists are highly clustered, benefiting from localized spillovers: Silicon Valley is perhaps the best example. There is also substantial geographical variation in venture capital contracts: California contracts are more "incomplete". This paper proposes an economic explanation for these observations, often attributed to regional cultural differences. In the presence of significant spillovers, it becomes optimal for an innovative start-up and its financier to adopt contracts with fewer contingencies: these contracts maximize their ability to extract (part of) the surplus they generate through positive spillovers. This relaxes ex-ante financing constraints and makes it possible to induce higher innovative effort.

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1. Introduction

Venture capital firms and innovative start-ups tend to be highly clustered, benefiting from labor market pooling and localized knowledge spillovers (Chen, Gompers, Kovner and Lerner (2009)); perhaps the best example is California's highly successful Silicon Valley. The greater success of Silicon Valley, even relative to other important clusters like the Route 128 corridor in Massachusetts, has been attributed to regional cultural differences by Saxenian (1994)[1]. These cultural differences are also thought to account for observed differences in contract design (Bengtsson and Ravid (2009)): venture capital contracts in California, it turns out, contain significantly fewer contingencies linking entrepreneurs' rewards to their firm's performance (Kaplan and Stromberg (2003), Bengtsson and David (2009)). In this sense, *contracts are more "incomplete" in California.* Examples of contingencies include the achievement of financial[2] or product[3] targets, while rewards include additional equity or options for the entrepreneur, the provision of new funds, or suspension of dividend payments to venture capitalists.

This paper develops a model that endogenizes the observed "California effect" on contract design: in the presence of sufficiently large and positive *spillovers*, it becomes optimal to rely on more *incomplete* venture capital contracts. The key advantage of these incomplete contracts is that they enable the innovative start-up and its venture capital investor to *extract some of the surplus that they generate through positive spillovers for new entrants.* Ex ante, the expectation of this surplus extraction relaxes the financing constraint for the start-up, making it possible to fund *more start-ups* with the potential to generate such positive spillovers, and also making it possible to induce *higher* levels of entrepreneurial *effort.* The model therefore provides a possible rationale for the finding by Mollica and Zingales (2007) that venture capital firms increase both patents and the total number of businesses. It also shows how the interaction between localized spillovers and contract design may be at the heart of venture capital success in California.

To illustrate the mechanism at work, consider the following example. A capital-constrained entrepreneur with an innovative project seeks funding from

[1] This is by no means the only explanation put forward in the literature: see Glaeser, Kerr and Ponzetto (2009).

[2] Financial targets are based on revenues and operating profits.

[3] Examples of product targets given by Kaplan and Stromberg (2003) include: reaching a threshold number of customers who have purchased the product and given positive feedback; acquiring a technology; developing a facility.

a venture capitalist. If the project is financed, the entrepreneur chooses his effort level. This effort determines the probability that the crucial first phase of the project is successful. In the absence of any other considerations, the optimal contract agreed at the financing stage between the entrepreneur and the venture capitalist conditions on the outcome of the first phase of the project, offering a reward to the entrepreneur if, and only if, the first phase is successful. This amounts to a "complete" contingent contract: successful completion of the first phase is the performance target.

Now suppose that success in the first phase generates not only high expected future profits for the project, but also positive *spillovers* that make entry by a second entrepreneur potentially profitable. Moreover, the outcome of the first phase is observed only by those with inside knowledge of the firm: the entrepreneur (the "incumbent"), and the venture capitalist who has funded the project and interacted closely with the entrepreneur from the start ($VC1$). Without access to this information, other venture capitalists are not willing to fund the second entrepreneur. This enables $VC1$ to fund him when entry is efficient, and extract informational rents from him; i.e. *capture some of the surplus due to the positive spillovers*. *Ex ante*, when the financing contract between the incumbent and $VC1$ is agreed, the expected value of the surplus that can be subsequently extracted from the entrant is taken into account: this increases the project's pledgeable income, making it easier to satisfy the investor's participation constraint and to give more high-powered effort incentives to the entrepreneur.

All this is possible as long as information about the outcome of the first phase of the project does not become available to other venture capitalists. Otherwise, competition among venture capitalists to fund the entrant will dissipate $VC1$'s rents. This is where the "completeness" of the financing contract between the incumbent and $VC1$ becomes a disadvantage. When the incumbent is rewarded for success of the first phase of the project in a way that is publicly observable, information leakage and rent dissipation cannot be avoided. If the reward is given privately, the transfer nevertheless generates some hard evidence, available to the contracting parties (e.g. granting of new equity or options to the entrepreneur, provision of additional funding by the venture capitalist). As discussed in detail at the end of section 4, this creates scope for profitable side deals between the incumbent and other venture capitalists that also lead to information leakage and rent dissipation for $VC1$.

If the surplus that can be extracted from the entrant is sufficiently large, a more "incomplete" contract is then preferred, since it does not condition on the outcome of the first phase and links rewards only to long-term profits, ensuring there is no information leakage. In this case, the gain from avoiding rent dissipation more than offsets the loss entailed by using a less efficient reward mechanism for the entrepreneur.

This example illustrates how, in the presence of substantial positive localized spillovers, and hence high expected profitability of entrants, a preference for "incomplete" contracts can arise endogenously. Moreover, the analysis in section 4 will show that in these circumstances, reliance on optimal incomplete contracts facilitates the creation of new businesses with the potential to generate positive spillovers, and stimulates entrepreneurial effort. Our results therefore suggest that *localized spillovers and contract design are mutually reinforcing*, and are likely to be correlated in the most successful clusters - as is the case in California.

The remainder of the paper is organized as follows. I complete this section by discussing the relationship with the existing literature. Section 2 introduces the model. Section 3 briefly analyzes the benchmark case without entry (and hence without spillovers). The main analysis is presented in section 4. Empirical implications are discussed in section 5. This section also includes an extension of the model to investigate how the trade-off between complete and incomplete contracts is modified when the incumbent generates negative, rather than positive, spillovers for the potential entrant. Section 6 concludes.

1.1. Relationship to the literature

This paper is related to several important literatures:

(1) *Venture capital contracts.*

A large body of theoretical work in this area has studied the allocation of cashflow rights and control rights[4], with a particular emphasis on explaining the widespread use of convertible securities in venture capital financings. Perhaps the closest paper to ours is Cuny and Talmor (2005), which focuses instead on the choice between "milestone" and "round" financing. In the former, contracts specify that the venture capitalist will provide additional funding when specific performance milestones are met, and the terms of this additional funding. In the latter, there are no such contingencies in the contract and no pre-commitment to future funding by the venture capitalist. Milestone financing therefore corre-

[4] See, among others, Bergemann and Hege (1998), Bottazzi et al. (2005), Casamatta (2003), Cestone (2000), Cornelli and Yosha (2003), Dessí (2005), Hellmann (1998), Kaplan et al. (2003), Lerner and Schoar (2005), Repullo and Suarez (2000, 2004), and Schmidt (2003).

sponds to more complete contracts, and round financing to more incomplete ones. However, the trade-off investigated by Cuny and Talmor is very different from the one examined in this paper: in their model, the key difference is that under round financing the price of funding at each stage is set through negotiations at that stage (and different investors may provide funding at different stages), while milestone financing sets the contingent prices *ex ante* as part of a long-term contract.

(2) *Clustering and spillovers*

An extensive literature explores the importance of entrepreneurship and innovation clusters, and localized spillovers[5]. This literature has been largely separate from the literature on contract design. Our paper builds on insights from both, and provides a link between the two. In particular, it shows how spillovers and contract design can be mutually reinforcing in clusters of innovative entrepreneurial firms and venture capitalists.

(3) *Incomplete contracts*

A very large literature examines the causes and consequences of contractual incompleteness[6]. Our paper is perhaps closest in spirit to Holmstrom and Milgrom (1991). In their framework, it can be optimal to pay a fixed wage independent of measured performance when performance is easy to measure for some activities but not others. Our framework is very different, but it has a similar flavor in the sense that it can be optimal not to include some contingencies in the contract even though it would be feasible to include them. The reason is quite different though: in our setting, the "incompletenesss" of the contract makes it possible to avoid potentially damaging revelation of information. In a similar vein, Bernheim and Whinston (1998) show that when contracts cannot condition on some aspects of performance, because they are not verifiable, they may optimally leave other, verifiable, aspects unspecified, generating strategic ambiguity. The key to their

[5] See Audretsch and Feldman (2004) for a review and discussion. Glaeser and Kerr (2009) document the importance of several factors for entrepreneurship clustering: the abundant presence of small independent suppliers, labor market pooling, and knowledge spillovers. Ellison, Glaeser and Kerr (2010) study the impact of these factors on industrial agglomeration. Evidence of the importance of geographic proximity for knowledge spillovers is provided by Acs et al. (1994), Agrawal et al. (2008), Audretsch and Feldman (1996), Audretsch and Stephan (1996), Jaffe (1989), Jaffe et al. (1993), and Zucker et al. (1998). Fallick et al. (2006) and Freedman (2008) document the importance of labor market pooling.

[6] See, among others, Anderlini and Felli (1994, 1999), Battigalli and Maggi (2002), Bolton and Faure-Grimaud, Dye (1985), Grossman and Hart (1986), Hart and Moore (1990, 1999), Klein, Crawford and Alchian (1978), Segal (1999), Tirole (2008), Williamson (1975, 1985).

results is the effect that explicit contractual provisions have on the set of feasible self-enforcing implicit agreements between the parties. The present paper is also concerned with strategic incompleteness, but for a very different reason: incompleteness makes contractual execution less informative, and through this channel affects subsequent strategic interactions with other parties. Other papers that have explored the informational implications of incomplete contracts have tended to focus on the informational content of a contractual offer, as in Allen and Gale (1992) and Spier (1992).[7] Allen and Gale consider an environment in which different agents have different abilities to manipulate information about contingencies. Non-contingent contracts emerge in equilibrium because they do not create incentives to engage in such manipulation. Spier shows how, in the presence of (exogenous) transactions costs, an informed principal may prefer an incomplete contract to signal that his "type" is "good". My paper is, to my knowledge, the first to focus instead on the (hard) information generated by contractual execution, and the ways in which outside parties, as well as the contracting parties, may use strategically this information.

2. The model

The model has three dates, $t = 0, 1, 2$. At date 0, an entrepreneur may enter a new industry and invest in a project, call it project I (I for "incumbent"). At date 1, the state γ is realized (see below). At this stage another entrepreneur may enter the industry and invest in a related project, call it project E (E for "entrant"). The probability of success of project I at date 2 will depend on the state γ and on whether entry occurs. The state γ will also affect the probability of success of the second project E. Entrepreneurs possess no capital and need to raise finance from investors (venture capitalists). For simplicity, there is no discounting. All agents in the model are assumed to be risk neutral and protected by limited liability.

2.1. The incumbent

Project I requires an initial outlay of value K_I. The first entrepreneur (henceforth also called the incumbent) faces considerable uncertainty about his project's returns when he invests at $t = 0$: some of the uncertainty is resolved at $t = 1$, when the state γ is realized. For simplicity, γ is assumed to take one of two values:

[7] See also Aghion and Hermalin (1990), who study the desirability of legal restrictions on contracting to prevent inefficient signaling.

γ_G ("good" state) or γ_B ("bad" state), with $1 > \gamma_G > \gamma_B > 0$. If there is no entry, project I yields verifiable returns R at $t = 2$ with probability γ, and zero otherwise, where $R > K_I > 0$. Thus γ represents the probability of "success" (high returns) in the second period in the absence of competition. The impact of competition is considered below.

If project I is undertaken at $t = 0$, the incumbent chooses his effort level $e \in [0, e^H]$, where $0 < e^H < 1$. The cost of effort is given by $c(e) \equiv \frac{1}{2}e^2$. Entrepreneurial effort increases the probability of the good state: specifically, the good state occurs with probability e. To capture the uncertainty inherent in innovative activity, I assume that the bad state occurs with some non-trivial probability even when the incumbent exerts the maximal feasible effort. At the same time, entrepreneurial effort is crucially important; I therefore assume that, leaving aside entry considerations, the project is not worth undertaking with zero effort:

- **(A1)** $\gamma_B R < K_I$

In what follows, I denote by $\Delta\gamma = \gamma_G - \gamma_B > 0$ the difference in the probability of success between the good state and the bad state. For ease of exposition, I also assume that[8]

- **(A2)** $(\Delta\gamma + \mu)R \leqslant e^H$

2.2. The entrant

At $t = 1$, a second entrepreneur (henceforth also called the entrant) may enter the industry and invest in a related project. This project requires an initial outlay of value K_E. In the baseline version of the model, I assume that if the incumbent has been successful during the first period (i.e. $\gamma = \gamma_G$), this generates new profitable opportunities for prospective entrants. I model this as simply as possible by assuming that the entrant's expected returns are equal to π_H when $\gamma = \gamma_G$ and π_L when $\gamma = \gamma_B$, with $\pi_H > K_E > \pi_L > 0$. We can then think of $\pi_H - \pi_L$ as capturing the magnitude of the positive spillover generated by the

[8] As will become clear below, this assumption simply means that the first-best effort level is potentially feasible in most cases of interest for our analysis. The only exception is incomplete contracts in section 3, where for sufficiently high values of S the first-best effort could, in principle, exceed $(\Delta\gamma + \mu)R$. I therefore incorporate the feasibility constraint $e \leqslant e^H$ explicitly into the problem and solution for that case, described by Proposition 2.

incumbent's success. I will denote by $S \equiv \pi_H - K_E$ the expected profits (surplus) from undertaking the entrant's project in state γ_G.

In the extension studied at the end of the paper, I will modify this assumption to allow instead for the opposite case, where the incumbent's success at date 1 *reduces* the entrant's expected profits, because the existing project does not generate any positive spillovers for new projects, and the incumbent has been very successful in building up a competitive advantage (for example by forming valuable strategic alliances, developing ties with suppliers and customers, and building up a reputation that gives him a competitive advantage). I model this case by simply reversing the previous assumption, so that the entrant's expected returns are equal to π_H when $\gamma = \gamma_B$ and π_L when $\gamma = \gamma_G$. S will then denote the surplus from undertaking the entrant's project in state γ_B.

If the second entrepreneur decides to enter, he has an impact on the profitability of the incumbent. I model this by assuming that entry reduces the incumbent's success probability to $\gamma - \mu$, where $\gamma_G - \mu > \gamma_B > \mu > 0$. I further assume that it is nevertheless efficient to fund the entrant when the state is "favorable"; that is, the surplus from the entrant's project outweighs the cost of entry imposed on the incumbent:

- **(A3)** $S > \mu R$

Finally, I assume that:

- **(A4)** *It is not worth funding the entrant unless the state is known to be "favorable"*

  A4 seems a reasonable assumption when there is sufficient uncertainty *ex ante* about the incumbent's success at the intermediate stage (regardless of effort), and his success at the intermediate stage is crucial for the entrant's expected profitability - as in our baseline model[9]. In other settings, the assumption can be made for analytical convenience, since it guarantees the existence of a pure-strategy equilibrium[10].

[9] A sufficient condition for (A4) in our baseline model would be: $e^H S + (1 - e^H)(\pi_L - K_E) < 0$.

[10] Analyses of mixed-strategy equlibria when this assumption is relaxed can be found in the literature on informed lending (see Rajan (1992), Von Thadden (2004)). In these settings the informed investor is still able to exploit his informational advantage, albeit less than in the pure strategy equilibrium analyzed in this paper. The main qualitative insights of our analysis would therefore continue to hold.

2.3. Investors

Entrepreneurs seek financing from venture capitalists, who possess enough expertise and sector-specific knowledge to be able to evaluate entrepreneurs and their projects. I assume that there are N such investors, identical and competitive *ex ante*, denoted $VC1$, $VC2$... and VCN. If the first entrepreneur succeeds in obtaining funding for his project at date 0, denote by $VC1$ the venture capitalist that provides the funding. As discussed below, thanks to his involvement with the project, $VC1$ will have access to more information at date 1 than other venture capitalists.

2.4. Information

I assume that γ is only observed by the incumbent and $VC1$ at $t = 1$. The notion that firm "insiders", and in particular the firm's entrepreneur and the venture capitalist funding the firm, possess an informational advantage concerning the firm's progress and prospects seems a very reasonable assumption in the context of young, entrepreneurial firms (see, for example, Admati and Pfleiderer (1994), Dessí (2005) and Schmidt (2003)). On the other hand, as discussed in the Introduction and more fully in section 4, if the incumbent and $VC1$ have signed a contract at date 0 contingent on γ, the execution of this contract can disclose information about the realized value of γ to other parties at date 1.

2.5. Time line

$t = 0$	$t = 1$	$t = 2$
Project I undertaken? Incumbent chooses effort.	Realization of γ. Entry?	Project returns realized.

3. No entry

This section briefly presents the benchmark case where entry is ruled out *a priori*: optimal financial contracts for this case will provide a useful benchmark for comparison. In subsequent sections, I shall allow for the possibility of entry.

Suppose then that no entry can occur at date 1. In this case the only financial contract to be examined is the one agreed at date 0 to provide funding for the incumbent.

Given that γ is a sufficient statistic for effort, the most efficient way to elicit effort from the incumbent is to offer him a reward, $R_e > 0$, contingent on the realization of the "good" state at date 1 (i.e., when $\gamma = \gamma_G$), and zero otherwise (because of limited liability). $VC1$ provides the initial capital K_I at date 0 and receives the project's returns at date 2. Competition among venture capitalists at date 0 ensures that the incumbent obtains the full expected NPV of the project. The optimal financing contract, denoted by $C1$, solves the following problem, $P1$:

$$Max \qquad U = eR_e - \frac{1}{2}e^2 \tag{3.1}$$

$$e = R_e \quad (IC) \tag{3.2}$$

$$e\gamma_G R + (1-e)\gamma_B R - eR_e \geqslant K_I \quad (IR) \tag{3.3}$$

where (IC) is the entrepreneur's incentive constraint and (IR) the venture capitalist's participation constraint. It can be easily checked that the first-best effort level, which maximizes the project's expected returns net of effort costs, is given by $e^{FB} \equiv \Delta\gamma R$. To implement this would require setting $R_e = \Delta\gamma R$ (from (IC)). This would imply that the maximum income that could be pledged to $VC1$ would be equal to $\gamma_B R$. By assumption $(A1)$, this will not be sufficient to satisfy (IR). Thus (IR) will bind. To make the analysis interesting, I assume that parameter values are such that the project can be funded (see the Appendix for details). Effort will then be equal to:

$$e^N = \frac{1}{2}\Delta\gamma R + \frac{1}{2}\{[(\Delta\gamma R]^2 + 4[\gamma_B R - K_I]\}^{\frac{1}{2}} \tag{3.4}$$

Effort will be lower than the first-best level: in the absence of entry considerations, this is the only source of inefficiency. Next we consider whether allowing for the possibility of entry introduces further inefficiencies, and whether it also mitigates inefficiency in some cases. In particular, we study how this depends on whether complete or incomplete contracts are used, and the resulting trade-off.

4. Entry

I now allow for the possibility of entry at date 1. I begin by analyzing the case where the incumbent and $VC1$ at date 0 sign a contract contingent on the realization of γ, and execution of the contract at date 1 reveals γ to outside parties ("*complete contracts*"). I will then study the case where the contract is not contingent on the realization of γ, so as to avoid revealing information to outside parties ("*incomplete contracts*"). The end of the section will examine what can be achieved with *secretly-executed* complete contracts.

The timing of the game at date 1 is as follows. The state γ is realized and is observed by the incumbent and his investor ($VC1$). A second entrepreneur (the entrant) seeks financing for a related project, project E. Venture capitalists ($VC1, VC2, ..., VCN$) make simultaneous take-it-or-leave-it offers to the entrant. The entrant accepts one offer (or zero). If he accepts an offer (other than the null contract), project E is undertaken. Both projects' returns are realized at $t = 2$.

4.1. Complete contracts

The optimal *complete* contract agreed at $t = 0$ between the incumbent and $VC1$ takes the form studied above for the no-entry case: the entrepreneur receives a reward R_e if, and only if, $\gamma = \gamma_G$, while the investor receives the project's final returns. As in the no-entry case, this type of contract is optimal because it elicits effort efficiently from the incumbent. Given this form of contract between the incumbent and $VC1$, we can examine the game between the entrant and investors at date 1 and then solve backwards for the optimal date-0 contract.

4.1.1. The game at date 1

Investors and the entrant learn the realized value of γ at $t = 1$, when the incumbent is rewarded (or not). The game between them therefore takes place under symmetric information. When the realized state is unfavorable for the entrant (i.e. $\gamma = \gamma_B$), nobody is willing to fund his project. When the state is favorable, competition among investors ensures that the entrant is able to fund his project and obtain its full expected NPV. This also implies a loss for $VC1$, because entry reduces the success probability of the incumbent's project. We therefore have the following result.

Lemma 1 (date 1 game with complete contracts). When $\gamma = \gamma_B$, there is no entry. When $\gamma = \gamma_G$, entry always occurs: the entrant's expected gain from his project is equal to its full expected NPV, S, while the expected value of the incumbent's project is reduced by μR.

Proof: see Appendix.

Because he has no informational advantage at date 1, $VC1$ not only cannot extract any rents from the entrant, but he incurs a loss when $\gamma = \gamma_G$, due to the fact that other venture capitalists' funding offers to the entrant do not internalize the costs imposed by entry on the incumbent's project. We can now examine the implications for financing constraints *ex ante*.

4.1.2. The game at date 0

The optimal financial contract between the incumbent and $VC1$ at date 0 will solve the following problem:

$$Max \qquad U = eR_e - \frac{1}{2}e^2 \tag{4.1}$$

$$e = R_e \quad (IC) \tag{4.2}$$

$$e(\gamma_G - \mu)R + (1-e)\gamma_B R - eR_e \geqslant K_I \quad (IR) \tag{4.3}$$

It is straightforward to verify that the first-best effort level is now lower than in the no-entry case, and is equal to $e_C^{FB} = (\Delta\gamma - \mu)R$. This is because entry reduces the expected value of the incumbent's project, and hence the return to effort.

However, *the investor's participation constraint is harder to satisfy*, because pledgeable income is reduced by the possibility of subsequent entry. Thus once again it is not possible to implement the first-best level of effort. We therefore obtain the following result:

Proposition 1. (a) Either (i) the incumbent's project cannot be funded. This happens when K_I is "too large" (see the Appendix for precise details); or (ii) the incumbent's project is funded, and the incumbent's effort level is equal to:

$$e^C = \frac{1}{2}(\Delta\gamma - \mu)R + \frac{1}{2}\{[(\Delta\gamma - \mu)R]^2 + 4[\gamma_B R - K_I]\}^{\frac{1}{2}} \tag{4.4}$$

where $e^C < e^N$.

(b) The incumbent's expected utility when the project is funded is equal to $U = \frac{1}{2}(e^C)^2$, which is strictly lower than in the no-entry case.

Proof: see Appendix.

Thus with *complete* contracts we find that allowing for the possibility of *entry may make it impossible to undertake the incumbent's project.* Moreover, if the project is undertaken, *entrepreneurial effort on the project will be strictly lower than in the no-entry case, as will the incumbent's expected utility*. Overall then, allowing for the possibility of entry is "bad news" for the incumbent when complete contracts are used.

Can the incumbent be better off with an incomplete contract? We now turn to this question.

4.2. Incomplete contracts

In this section we examine what happens if the incumbent and $VC1$ at date 0 sign a contract that is *not* contingent on γ, and outside parties (entrant, other venture capitalists) do not have access to information about the realized value of γ at date 1.

At date 0, the contract between the incumbent and $VC1$ can only condition on the realization of final project returns. It will therefore take the form $CI = \{R_I, R_V\}$, where R_j denotes the payoff for j $(j = I, V)$ at $t = 2$ when realized final returns are equal to R. I denotes the incumbent and V the venture capitalist. The timing of the game is the same as in the case of complete contracts studied above. As before, we solve the game by backward induction, starting from date 1.

4.2.1. The game at date 1

Only $VC1$ learns the realized value of γ at date 1; the other venture capitalists and the entrant do not. The game between them therefore takes place under asymmetric information. The equilibrium is described by the following result.

Lemma 2 (date 1 game with incomplete contracts). When $\gamma = \gamma_B$, there is no entry. When $\gamma = \gamma_G$, entry always occurs: $VC1$ funds the entrant and extracts all the surplus from him, with expected value S.

Proof: see Appendix.

This result shows the main benefit from incomplete contracts in this setting: in contrast with the complete contracts case examined earlier, $VC1$ here can use his informational advantage to extract the entrant's surplus. We now explore the implications for the contract between $VC1$ and the incumbent *ex ante*, and for the incumbent's choice of effort.

4.2.2. The game at date 0

The optimal financial contract between the incumbent and $VC1$ at date 0 will solve the following problem:

$$Max \qquad U \equiv e(\gamma_G - \mu)R_I + (1-e)\gamma_B R_I - \frac{1}{2}e^2 \tag{4.5}$$

subject to the constraints:

$$e = \arg\max(U) \quad (IC) \tag{4.6}$$

$$e[(\gamma_G - \mu)R_V + S] + (1-e)\gamma_B R_V \geqslant K_I \quad (IR) \tag{4.7}$$

$$R_I + R_V = R \tag{4.8}$$

$$R_I \geqslant 0, R_V \geqslant 0 \quad (LL) \tag{4.9}$$

where the first two constraints represent, as before, the incumbent's incentive compatibility constraint and the venture capitalist's participation constraint, while the following two are the feasibility and limited liability constraints.

There are two key differences relative to the analogous problem with complete contracting. First, the venture capitalist is able to extract the entrant's surplus in the "good" state (γ_G). This makes the incumbent's effort more valuable. Moreover, the expected surplus from the entrant essentially increases pledgeable income, relaxing the investor's participation constraint. Second, the entrepreneur is now rewarded less efficiently through a stake in the project's final returns, rather than a reward directly tied to the realization of the performance signal γ. The interaction between these two effects gives the result summarized by Proposition 2. Let $\alpha \equiv \Delta\gamma - \mu$. Then:

Proposition 2. (a) Either (i) the incumbent's project cannot be funded. This happens essentially when S is "too small" (see the Appendix for precise details); or (ii) the incumbent's project is funded, and the incumbent's effort level is equal to $e^I \equiv \min[e^*, e^H]$, where

$$e^* = \frac{1}{2}[\alpha R + S - \frac{\gamma_B}{\alpha}] + \frac{1}{2}\{[\alpha R + S - \frac{\gamma_B}{\alpha}]^2 + 4[\gamma_B R - K_I]\}^{\frac{1}{2}} \tag{4.10}$$

(b) The incumbent's expected utility when the project is funded is equal to: $U = \frac{1}{2}(e^I)^2 + \frac{\gamma_B e^I}{\alpha}$.

Proof: see Appendix.

Intuitively, when the surplus that can be extracted from the entrant is too small, the inefficiency of rewarding the entrepreneur on the basis of final returns rather than intermediate performance dominates and incomplete contracts perform poorly. For higher values of the surplus, however, the venture capitalist's participation constraint is relaxed, making it possible to induce higher effort and better performance.

4.3. Complete contracts or incomplete contracts?

We can now examine the trade-off between complete and incomplete contracts. The essence of the trade-off is the following. Under complete contracting, the incumbent can be given a reward contingent on the realization of γ, which is a sufficient statistic for effort. Under incomplete contracting, his effort incentives can only be provided, less efficiently, by giving him a share of the project's final returns. This represents the disadvantage of incomplete contracting. However, incomplete contracting enables $VC1$ to extract some informational rents from the entrant when $\gamma = \gamma_G$. Moreover, the expectation of this relaxes the venture capitalist's *ex ante* participation constraint, which makes it easier to induce effort. These are the benefits of incomplete contracting.

For some parameter values, the trade-off between complete and incomplete contracts takes a particularly stark form, in the sense that the *incumbent's project can only secure funding with one type of contract.* This is easily seen by noting (see the proof of Proposition 1) that funding can only be secured under complete contracts if

$$K_I \leqslant \frac{1}{4}[\alpha R]^2 + \gamma_B R \tag{4.11}$$

while the corresponding condition under incomplete contracts (see the proof of Proposition 2) is given by:

$$K_I \leqslant \frac{1}{4}[\alpha R + S - \frac{\gamma_B}{\alpha}]^2 + \gamma_B R \quad (4.12)$$

Clearly for sufficiently small values of S, the expected value of informational rents under incomplete contracting, it may be possible to fund the incumbent under complete contracting but not under incomplete contracting. Conversely, for sufficiently large values of S it may be possible to fund the incumbent under incomplete contracting but not under complete contracting.

There is a threshold value of S such that pledgeable income is higher with incomplete contracts above the threshold, and higher with complete contracts below the threshold. This threshold is given by $\hat{S} \equiv \frac{\gamma_B}{\alpha}$. It has an intuitive interpretation: with incomplete contracts, the incumbent's expected returns in the "bad" state γ_B are equal to $\gamma_B R_I = \gamma_B(\frac{e^I}{\alpha})$, while under complete contracts they are equal to zero. Thus from an *ex-ante* perspective, incomplete contracting implies that the incumbent has to be given rents of value $\frac{e^I \gamma_B}{\alpha}$, which reduce the project income that can be pledged to the venture capitalist. On the other hand, incomplete contracting also implies that the venture capitalist expects to earn informational rents (from the entrant) of value S with probability e^I. Pledgeable income will be higher with incomplete contracts if, and only if, $e^I S > \frac{e^I \gamma_B}{\alpha}$.

Consider now the trade-off arising when the two conditions are both satisfied, and the incumbent's project can secure funding with either type of contract[11]. In this case, the incumbent's expected payoff with complete contracting is equal to the NPV of his project (since venture capitalists at date 0 are competitive, so that $VC1$ does not earn any rents from the incumbent), taking into account the effect of entry. It is given by:

$$NPV^C = e^C(\gamma_G - \mu)R + (1 - e^C)\gamma_B R - \frac{1}{2}(e^C)^2 - K_I \quad (4.13)$$

The incumbent's expected payoff under incomplete contracting is equal to the NPV of his project, taking into account the effect of entry, plus the expected value of the investor's informational rents. It is therefore given by:

$$NPV^I = e^I[(\gamma_G - \mu)R + S] + (1 - e^I)\gamma_B R - \frac{1}{2}(e^I)^2 - K_I \quad (4.14)$$

Thus, incomplete contracts will be preferred if, and only if, the incumbent's net benefit from using incomplete contracts, NBI, is positive:

[11] Obviously when neither condition is satisfied no trade-off arises because the incumbent's project cannot be undertaken with any contract.

$$NBI \equiv e^I S + (e^I - e^C)\alpha R - \frac{1}{2}[(e^I)^2 - (e^C)^2] > 0 \qquad (C^*)$$

The first term in this expression represents the expected value of the venture capitalist's informational rents: these rents are the direct benefit of incomplete contracting. The other two terms reflect the impact of any difference between the equilibrium effort levels induced under complete and incomplete contracting. Intuition might suggest that this impact should be negative, because incomplete contracts reward entrepreneurial effort less efficiently than complete contracts. If this is the case, the choice between complete and incomplete contracts will depend on the *trade-off* between the benefit of earning *informational rents* with incomplete contracts and the benefit of *inducing effort more efficiently* with complete contracts. However, effort under complete contracting may be reduced significantly below its first-best level by the need to generate sufficient pledgeable income to satisfy the investor's participation constraint. With incomplete contracts, on the other hand, the expected value of the investor's informational rents becomes part of pledgeable income, making it easier to satisfy the constraint: this is the potential indirect benefit of incomplete contracting. If this effect is sufficiently important, the sum of the last two terms in the above expression may also be positive, enhancing the net benefit of incomplete contracts.

To gain further insight into the trade-off, we can use (8.15) and (8.29) to write the net benefit of incomplete contracts as follows:

$$NBI \equiv \frac{1}{2}\{e^I S + (e^I - e^C)\alpha R + \frac{e^I \gamma_B}{\alpha}\}$$

This expression makes clear that for $e^I \geqslant e^C$ incomplete contracts will be preferred. Indeed, for complete contracts to be preferred instead, e^C has to be sufficiently greater than e^I to offset the other positive terms in the expression. It is immediately apparent from the expressions for the two efforts (given in Propositions 1 and 2) that $e^I \geqslant e^C$ if, and only if, $S \geqslant \hat{S}$. Intuitively, when this condition holds, pledgeable income is at least as high with incomplete contracting as with complete contracting, making it possible to elicit at least as much effort from the incumbent. Thus when the surplus that can be extracted from the entrant is sufficiently large, there is no longer a trade-off: incomplete contracts yield informational rents *and* induce at least as much effort as complete contracts. For lower values of S, the trade-off applies but is still favorable to incomplete contracts. Finally for sufficiently low values of S, the trade-off will switch in favor of complete contracts.

4.4. When does the trade-off apply?

In the next section we will analyze how the trade-off between complete and incomplete contracts described above is modified when the incumbent's success decreases, rather than increasing, the entrant's expected profitability. Before we do this, we need to examine carefully the basic assumption underlying the trade-off identified so far, namely the assumption that at date 1 "incomplete" contracts do not reveal information about the realized value of γ to outside parties (in particular, uninformed venture capitalists and the entrant), while "complete" contracts do.

First of all, is it the case that with incomplete contracts of the kind studied in this paper information about the realized value of γ will not be revealed to outside parties? Clearly no evidence concerning γ is generated by *execution of the contract* at date 1, because the contract is not contingent on γ. Information disclosure to outside parties could still occur at date 1 if the contracting parties had, privately and independently, access to hard evidence about the realized value of γ, irrespective of contractual execution: in this case, profitable side deals involving disclosure of the information to uninformed venture capitalists would be feasible (see more on this below). However, our focus is on settings where this is not the case, and in particular on settings where neither party has access, on his own and without resorting to the courts, to all the evidence required to establish convincingly the realized value of γ: they could only obtain such evidence through the process of *pre-trial discovery*[12].

These settings seem very relevant to the venture capital context. For example, suppose γ_G represents the development of a new product that is likely to be highly profitable, as well as generating demand for other new related products and services. The entrepreneur could present evidence that a target threshold of customers have purchased the product and given positive feedback. On the other hand, the venture capitalist who has been funding the project could be aware that there were some problems with the product. In the absence of a contract contingent on γ, neither party could, privately and independently, establish convincingly the realized value of γ. However if the parties had signed a "complete" contract (contingent on γ), and failed to agree about the realization of γ at date 1, pre-trial discovery could reveal, for example, that a number of other customers had also

[12] In U.S. law, pre-trial discovery enables each party in a lawsuit to obtain evidence from the opposing party and from non-parties through a variety of methods, including requests for answers to interrogatories, production of documents or things, admissions and depositions, and inspections.

purchased the product and given very negative feedback, or that the satisfied customers only required the product for a very specific application and that for other important applications the product performed very poorly, etc. Similar examples easily come to mind when considering other commonly used performance milestones in venture capital contracts (development of new facilities, acquisition of new technologies...). In view of these examples, it seems reasonable to assume that incomplete contracts of the kind studied in this paper will not generate credible disclosure of information to uninformed venture capitalists and the entrant.

What about complete contracts? Can information disclosure be prevented in the presence of complete contracts of the kind studied earlier? Intuition might suggest that complete contracts with secret execution could do better than any of the contracts considered so far, by combining the benefits of more efficient reward schemes for entrepreneurial effort with the benefits of not revealing information about γ to outside parties. Such "secretly-executed complete contracts" would specify that the incumbent is to be rewarded if, and only if, $\gamma = \gamma_G$, as in the complete contracting case examined earlier. However, the execution of the contract at date 1 would be kept secret; in particular, the incumbent would be rewarded secretly when $\gamma = \gamma_G$, so as not to reveal information about γ to outside parties.

My claim is that this would not work. Suppose the incumbent and $VC1$ sign an *ex-ante* (date 0) agreement to keep contractual execution secret *ex post* (date 1). When they reach date 1, the contract is executed: this requires establishing the realized value of γ and hence determining the value of the incumbent's reward (R_e or zero). If $\gamma = \gamma_G$, the incumbent receives the reward R_e from $VC1$. Large transfers typically generate *hard* evidence (e.g. bank transfers), available to both parties to the transfer. Even if $VC1$ paid the incumbent privately in cash, each party to the transaction would want to keep some hard evidence of it: entrepreneurs would want to be able to show how and why they received this large sum in cash (e.g. tax authorities, reputational benefits as successful entrepreneurs), while venture capitalists raise capital for their investments in start-up companies from limited partners in venture capital funds, and could not withdraw large amounts of money in cash which would simply disappear without trace.

Thus when $\gamma = \gamma_G$, contractual execution will generate hard evidence informative about γ, available to $VC1$ and to the incumbent. This means that in the equilibrium described by Lemma 2, in which $VC1$ extracts informational rents from the entrant, an uninformed venture capitalist will now have an incentive to "deviate" by offering the incumbent a small payment in return for seeing evidence that he has received the transfer R_e from $VC1$. The incumbent would gain by doing this secret side deal with the uninformed venture capitalist when $\gamma = \gamma_G$, and the venture capitalist would gain from becoming informed: he could then offer slightly more than $VC1$ when $\gamma = \gamma_G$, and his offer would be accepted.

It seems very difficult to rule out such behavior by including a confidentiality clause in the original contract between the incumbent and $VC1$. The "deviation" does not require the incumbent to hand over any evidence to the uninformed venture capitalist: it is enough to show it. This would make it extremely hard to prove *ex post* that the confidentiality clause had been breached. In the absence of a credible threat of punishment, information leakage would occur, undermining $VC1$'s informational advantage and associated rents. Thus secretly-executed contracts will not work.

My argument has been for contracts in which the incumbent's reward is paid at date 1 when $\gamma = \gamma_G$, but it also applies if payment of the reward is deferred until date 2: what matters is the date when the parties establish whether a reward is due or not. If they establish that the reward is due and payment is then deferred, both parties will obtain hard evidence of this: in particular, the incumbent entrepreneur will have evidence that he is due to receive the reward at date 2. This is just as informative as evidence of his obtaining the reward at date 1, so the argument goes through.

The argument would not go through if at date 0 the incumbent and $VC1$ agreed to defer until date 2 the process of establishing whether a reward is due or not. However, as discussed above, the process of establishing the realized value of γ at date 1 relies on the possibility of using pre-trial discovery if there is no agreement. If the process is delayed, pre-trial discovery will be less efficient, because as circumstances evolve over time it becomes difficult to establish what was the precise "state of the world" at a given point earlier in time (people forget information or remember it inaccurately; they move; they die; records are updated and some information is lost; products, facilities and technologies are also updated and modified; etc.). As long as this effect is sufficiently important, the argument developed above goes through, and the trade-off between complete and incomplete contracts examined in this paper continues to apply.

5. Empirical implications and robustness

When are we most likely to observe incomplete venture capital contracts in the sense of this paper (i.e. contracts that make less use of performance contingencies)? The following result provides a first answer.

Lemma 3. The net benefit from incomplete contracts, NBI, increases with S.

Proof: see Appendix.

Thus our model implies that *incomplete contracts will be more attractive when entrants are expected to be more profitable* (higher S). It also implies that when incomplete contracts are used with incumbents, *venture capitalists should be able to extract more surplus from entrants*. To my knowledge, neither of these predictions has been tested. Indeed, empirical evidence on the factors driving the degree of incompleteness of venture capital contracts is very limited. However, as discussed in the Introduction, there is substantial evidence that *venture capital contracts tend to be more incomplete in California* (Kaplan and Stromberg (2003), Bengtsson and Ravid (2009)). Bengtsson and Ravid argue that this highly significant "California effect" is hard to reconcile with most existing theoretical models of contract design, since it cannot be attributed to variations in tax or bankruptcy codes, or to differences in securities laws or legal enforcement costs. The model developed in this paper, on the other hand, is very easy to reconcile with the California effect.

As shown by Chen, Gompers, Kovner and Lerner (2009), clustering of both venture capital firms and venture capital-financed companies is very high, and California is home to arguably the most important cluster[13]. Chen et al. (2009) point to the benefits of labor market pooling and localized knowledge spillovers[14] as key factors driving clustering. In terms of our model, positive spillover effects of this kind, other things being equal, are going to increase the expected profitability of entrants, hence the value of S. This in turn increases the net benefit of incomplete contracts, providing a rationale for the California effect. Moreover, in line with the model's predictions, there is also evidence that venture capitalists are able to obtain lower valuations for their investments, and therefore extract greater surplus, in California (Hochberg, Ljungqvist and Lu (2009)).

For simplicity, the analysis so far assumed that when the incumbent is success-

[13]Chen et al. (2009) investigate the geography of venture capital firms and venture capital-backed portfolio companies. They find that in 2005 the San Jose-San Francisco area accounted for 21.6% of all venture capital firm Main Offices, the single biggest share for any location. For a sample of 28,434 venture capital investments between 1975 and 2005, they find that 29.01% were in portfolio companies located in the San Jose-San Francisco area, again the highest share for any location.

[14]For evidence on these see footnote 5.

ful at date 1, there is a (profitable) potential entrant with probability one. If we allow this probability to vary, the net benefit of incomplete contracts will be increasing in the probability, since incomplete contracts enable the venture capitalist to extract informational rents when there is a potential profitable entrant. The model therefore predicts that *incomplete contracts will be more attractive when the probability of a profitable potential entrant emerging is higher.* This probability is likely to be higher in R&D-intensive industries, with substantial investments in innovative projects whose outcomes may be complementary to the incumbent's when successful. The available evidence on this is consistent with the model: Kaplan and Stromberg (2003, 2004) find that incomplete venture capital contracts are significantly more common for firms in industries with a high R&D/sales ratio.

Overall then, the available empirical evidence seems consistent with the theoretical model developed in this paper. This yields some interesting implications. First, our model predicts that when S is sufficiently large ($S > \hat{S}$), it becomes possible to finance the incumbent with an incomplete contract even when the incumbent would be unable to obtain funding for his project with a complete contract. Second, when S is sufficiently large (again $S > \hat{S}$), the optimal incomplete contract will induce a higher level of entrepreneurial effort than the optimal complete contract. Thus when expected positive spillovers are sufficiently important (entrants' expected profitability is sufficiently high), the use of optimally incomplete venture capital contracts can increase the number of start-ups able to obtain funding, and increase innovative effort. This may help to explain the finding by Mollica and Zingales (2007) that venture capital firms increase both patents and the total number of new businesses. It may also contribute to explaining the particularly successful performance of venture capital in California.

5.1. Extension: Incumbent success reduces entrant's profitability

Our analysis in previous sections assumed that the incumbent's success at the intermediate stage would increase the potential entrant's expected profitability. We now examine the opposite case: the incumbent's success *reduces* the entrant's expected profitability. Specifically, the entrant's expected returns are now equal to π_H when $\gamma = \gamma_B$ and π_L when $\gamma = \gamma_G$. The model is otherwise unchanged. Optimal complete and incomplete contracts are derived in the Appendix. Our findings for this case may be summarized as follows.

Proposition 3. When the incumbent's success at date 1 increases the entrant's expected profitability, (a) incomplete contracts will be preferred if, and only if, the following condition holds:

$$NBI \equiv \frac{1}{2}\{(1-e^I)S + (e^I - e^C)\alpha R + \frac{e^I(\gamma_B - \mu)}{\alpha}\} > 0$$

where $\alpha \equiv \Delta\gamma + \mu$, $e^C = \frac{1}{2}\alpha R + \frac{1}{2}\{[\alpha R]^2 + 4[(\gamma_B - \mu)R - K_I]\}^{\frac{1}{2}}$ and $e^I = \frac{1}{2}[\alpha R - S - \frac{\gamma_B - \mu}{\alpha}] + \frac{1}{2}\{[\alpha R - S - \frac{\gamma_B-\mu}{\alpha}]^2 + 4[(\gamma_B - \mu)R + S - K_I]\}^{\frac{1}{2}}$.

(b) The net benefit from incomplete contracts, NBI, increases with S.

Proof: see Appendix.

This result shows that when the incumbent's success *reduces* the entrant's expected profitability, an analogous trade-off emerges between the costs and the benefits of incomplete contracts, with one important difference: the venture capitalist now extracts the entrant's surplus (S) when $\gamma = \gamma_B$, which reduces the first-best effort level. Indeed, as shown in the Appendix, it may even be possible to implement this (lower) first-best level of effort, if S is sufficiently large.

Moreover, we find once again that the net benefit of incomplete contracts increases with the magnitude of the surplus that can be extracted from the entrant, implying that the empirical predictions discussed above continue to hold.

6. Conclusions

In this paper, we have examined the interaction between innovation spillovers and contract design in venture capital. When innovative start-up firms generate sufficiently large spillovers for subsequent potential entrants, it becomes optimal to adopt more "incomplete" contracts. These contracts are incomplete in the sense of including fewer contingencies that would reveal to outside parties the private signals observed by the firm's insiders (entrepreneur and venture capitalist). Although these contracts may entail some efficiency loss (in our model, a less efficient reward scheme for the entrepreneur), this loss is more than offset by the efficiency gains. By avoiding information leakage, the contracting parties are able to extract informational rents from subsequent entrants; ex ante, the expectation of these rents relaxes financing constraints and makes it possible to provide more high-powered incentives to entrepreneurs, increasing innovative effort.

Thus spillovers and contractual design become mutually reinforcing. Our analysis can therefore explain the observed geographical correlations between contractual design and localized spillovers. We view this explanation as a valuable complement to more traditional accounts based on regional cultural differences.

7. References

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8. Appendix

The optimal contract when entry is ruled out exogenously (problem P1)

The problem is:

$$Max \qquad U = eR_e - \frac{1}{2}e^2 \tag{8.1}$$

$$e = R_e \quad (IC) \tag{8.2}$$

$$e\gamma_G R + (1-e)\gamma_B R - eR_e \geqslant K_I \quad (IR) \tag{8.3}$$

The first-best effort level maximizes the project’s expected returns net of effort costs, i.e.

$$e\gamma_G R + (1-e)\gamma_B R - \frac{1}{2}e^2 \tag{8.4}$$

and is given by $e^{FB} \equiv \Delta\gamma R$. To implement this would require setting $R_e = \Delta\gamma R$ (from (IC)). This would imply that the maximum income that could be pledged to $VC1$ would be equal to $\gamma_B R$. By assumption $(A1)$, this will not be sufficient to satisfy (IR). Thus (IR) will bind. We can write (IR) as follows:

$$e\Delta\gamma R + \gamma_B R - e^2 \geqslant K_I \tag{8.5}$$

Differentiating the LHS gives $\Delta\gamma R - 2e$, implying that the LHS increases from an initial value of $\gamma_B R$ for $e = 0$ to $\frac{1}{4}(\Delta\gamma R)^2 + \gamma_B R$ for $e = \frac{1}{2}\Delta\gamma R$, decreasing thereafter.

We assume that parameter values are such that the project can be financed:

$$\frac{1}{4}(\Delta\gamma R)^2 + \gamma_B R \geqslant K_I \qquad (A5) \tag{8.6}$$

Effort will therefore be equal to the largest root of the following equation:

$$(e\Delta\gamma + \gamma_B)R - e^2 = K_I \tag{8.7}$$

i.e.

$$e^N = \frac{1}{2}\Delta\gamma R + \frac{1}{2}\{[\Delta\gamma R]^2 + 4[\gamma_B R - K_I]\}^{\frac{1}{2}} \tag{8.8}$$

Proof of Lemma 1.

When $\gamma = \gamma_B$, by assumption it is unprofitable to fund the entrant. When $\gamma = \gamma_G$, by the same assumption it is profitable to fund the entrant. Competition among venture capitalists $VC2, ..., VCN$ ensures that they are willing to fund the entrant on terms that give them zero expected profits. Thus the entrant's expected gain from his project is equal to its full expected NPV, S. Entry reduces the expected value of the incumbent's project by μR, implying a corresponding loss for $VC1$ since he receives the final returns from the incumbent's project.

Proof of Proposition 1

(a) The problem is:

$$Max \qquad U = eR_e - \frac{1}{2}e^2 \tag{8.9}$$

$$e = R_e \quad (IC) \tag{8.10}$$

$$e(\gamma_G - \mu)R + (1-e)\gamma_B R - eR_e \geqslant K_I \quad (IR) \tag{8.11}$$

The first-best effort level, e_C^{FB}, maximizes the project's NPV, taking into account costs of entry; i.e.

$$e[\gamma_G - \mu - \gamma_B]R + \gamma_B R - K_I - \frac{1}{2}e^2 \tag{8.12}$$

and is equal to

$$e_C^{FB} = (\Delta\gamma - \mu)R \tag{8.13}$$

Implementing this effort level would require setting $R_e = (\Delta\gamma - \mu)R$, which would not satisfy (IR). Thus (IR) will bind. We can write (IR) as follows:

$$e(\Delta\gamma - \mu)R + \gamma_B R - e^2 \geqslant K_I \tag{8.14}$$

Differentiating the LHS gives $(\Delta\gamma - \mu)R - 2e$, implying that the LHS increases from an initial value of $\gamma_B R$ for $e = 0$ to $\frac{1}{4}[(\Delta\gamma - \mu)R]^2 + \gamma_B R$ for $e = \frac{1}{2}(\Delta\gamma - \mu)R$, decreasing thereafter.

Thus if $K_I > \frac{1}{4}[(\Delta\gamma - \mu)R]^2 + \gamma_B R$, (IR) cannot be satisfied and the project cannot be funded. Otherwise, the project will be funded and effort will be given by the largest root of:

$$e(\Delta\gamma - \mu)R + \gamma_B R - e^2 = K_I \tag{8.15}$$

i.e.

$$e^C = \frac{1}{2}(\Delta\gamma - \mu)R + \frac{1}{2}\{[(\Delta\gamma - \mu)R]^2 + 4[\gamma_B R - K_I]\}^{\frac{1}{2}} \tag{8.16}$$

To show that $e^C < e^N$, suppose not; i.e. $e^C \geqslant e^N$. From (8.15) we know that

$$(e^C\Delta\gamma + \gamma_B)R - (e^C)^2 = K_I + \mu R e^C > K_I \tag{8.17}$$

This means that e^C would also be feasible in the no-entry case, and indeed that a higher effort than e^C would be feasible in the no-entry case since there is some slack in the investor's participation constraint for the no-entry case evaluated for effort equal to e^C. Thus e^N could not be the solution to problem $P1$.

(b) The incumbent's expected utility with complete contracts is equal to $U = \frac{1}{2}(e^C)^2$, his expected utility in the no-entry case is $U = \frac{1}{2}(e^N)^2$, and we have just proved that $e^C < e^N$.

Proof of Lemma 2

Consider the following candidate equilibrium strategies:

(i) $VC1$. If the realized state is γ_B, never offer to fund the entrant. If the realized state is γ_G, offer to fund him on terms that extract the full surplus from his project (i.e. $VC1$ provides the initial capital K_E in return for the project's final returns).

(ii) Uninformed venture capitalists. Never offer to fund the entrant.

(iii) Entrant. Accept the best offer.

Given these strategies, if an uninformed venture capitalist deviates by offering to fund the project on more favorable terms for the entrant (i.e. he offers to provide the initial capital K_E in return for a share of the project's final returns, the share being less than one), he knows that his offer will be accepted by the entrant in both states. He therefore expects to make a loss (by assumption $(A4)$). If he offers to fund the entrant on the same terms as $VC1$ (i.e. he offers to provide the initial capital K_E in return for the project's final returns), his offer will be accepted with probability one when $\gamma = \gamma_B$, and with probability $p = \frac{1}{2}$ when $\gamma = \gamma_G$, so again he expects to make a loss[15]. Thus uninformed venture capitalists have no incentive to deviate. $VC1$ has no incentive to deviate either because his strategy yields the highest possible expected return for him in the date 1 game.

Proof of Proposition 2.

(a) The problem is:

$$Max \qquad U \equiv e(\gamma_G - \mu)R_I + (1-e)\gamma_B R_I - \frac{1}{2}e^2 \tag{8.18}$$

subject to the constraints:

$$e = \arg\max(U) \quad (IC) \tag{8.19}$$

$$e[(\gamma_G - \mu)R_V + S] + (1-e)\gamma_B R_V \geqslant K_I \quad (IR) \tag{8.20}$$

$$R_I + R_V = R \tag{8.21}$$

$$R_I \geqslant 0, R_V \geqslant 0 \quad (LL) \tag{8.22}$$

From (IC) we have

$$e = (\Delta\gamma - \mu)R_I \tag{8.23}$$

[15] Obviously this will also be true for any other value of p.

The first-best effort level, e_I^{FB}, maximizes the project's NPV, taking into account costs of entry and surplus extracted from the entrant; i.e.

$$e[(\gamma_G - \mu)R + S] + (1-e)\gamma_B R - K_I - \frac{1}{2}e^2 \tag{8.24}$$

and is equal to

$$e_I^{FB} = [\Delta\gamma - \mu]R + S \tag{8.25}$$

Implementing e_I^{FB} would require setting

$$R_I = R + \frac{S}{\Delta\gamma - \mu} \tag{8.26}$$

which would not satisfy (IR). Thus either (IR) binds or the feasibility constraint $e \leqslant e^H$ binds. Define $\alpha \equiv \Delta\gamma - \mu$. We can then write (IR) as follows

$$e[\alpha R_V + S] + \gamma_B R_V \geqslant K_I \tag{8.27}$$

and replace $R_V = R - R_I = R - \frac{e}{\alpha}$ (using (IC)) to obtain

$$e[\alpha R + S - \frac{\gamma_B}{\alpha}] - e^2 + \gamma_B R \geqslant K_I \tag{8.28}$$

Differentiating the LHS gives $\alpha R + S - \frac{\gamma_B}{\alpha} - 2e$, and differentiating again gives -2. If $\alpha R + S - \frac{\gamma_B}{\alpha} \leqslant 0$, the LHS is maximized at $e = 0$, and the project cannot be funded (by assumption $(A1)$). If $\alpha R + S - \frac{\gamma_B}{\alpha} > 0$, the LHS is maximized at $e = \frac{1}{2}[\alpha R + S - \frac{\gamma_B}{\alpha}]$. Thus if $\frac{1}{4}[\alpha R + S - \frac{\gamma_B}{\alpha}]^2 + \gamma_B R < K_I$, again the project cannot be funded. When $\frac{1}{4}[\alpha R + S - \frac{\gamma_B}{\alpha}]^2 + \gamma_B R \geqslant K_I$, we have two possibilities: either the feasibility constraint $e \leqslant e^H$ is not binding, implying that the project is funded and e is the largest root of the equation

$$e[\alpha R + S - \frac{\gamma_B}{\alpha}] - e^2 + \gamma_B R = K_I \tag{8.29}$$

i.e.

$$e^I = \frac{1}{2}[\alpha R + S - \frac{\gamma_B}{\alpha}] + \frac{1}{2}\{[\alpha R + S - \frac{\gamma_B}{\alpha}]^2 + 4[\gamma_B R - K_I]\}^{\frac{1}{2}} \tag{8.30}$$

or the feasibility constraint is binding, implying that the project is funded if, and only if, $e^H[\alpha R + S - \frac{\gamma_B}{\alpha}] - (e^H)^2 + \gamma_B R \geqslant K_I$. In this case $e^I = e^H$.

(b) The incumbent's expected utility is given by

$$U \equiv e[\gamma_G - \mu - \gamma_B]R_I + \gamma_B R_I - \frac{1}{2}e^2 = \frac{1}{2}e^2 + \frac{\gamma_B e}{\alpha} \tag{8.31}$$

Proof of Lemma 3

The net benefit of incomplete contracts is

$$NBI \equiv \frac{1}{2}\{e^I S + (e^I - e^C)\alpha R + \frac{e^I \gamma_B}{\alpha}\}$$

Hence,

$$\frac{dNBI}{dS} = \frac{1}{2}\{e^I + (S + \alpha R + \frac{\gamma_B}{\alpha})\frac{de^I}{dS}\}$$

Using (8.29):

$$e^I[\alpha R + S - \frac{\gamma_B}{\alpha}] - (e^I)^2 + \gamma_B R = K_I \tag{8.32}$$

we obtain

$$de^I[\alpha R + S - \frac{\gamma_B}{\alpha} - 2e^I] = -e^I dS \tag{8.33}$$

Thus $\frac{de^I}{dS} = \frac{-e^I}{\alpha R + S - \frac{\gamma_B}{\alpha} - 2e^I}$ and $\frac{dNBI}{dS} = \frac{1}{2}\{\frac{-2e^I(\frac{\gamma_B}{\alpha} + e^I)}{\alpha R + S - \frac{\gamma_B}{\alpha} - 2e^I}\} > 0$ for $e^I > \frac{1}{2}[\alpha R + S - \frac{\gamma_B}{\alpha}]$. When $e^I = \frac{1}{2}[\alpha R + S - \frac{\gamma_B}{\alpha}]$ it is straightforward to verify that a marginal increase in S makes it possible to increase e^I without violating the IR constraint. Thus for $e^I \geqslant \frac{1}{2}[\alpha R + S - \frac{\gamma_B}{\alpha}]$, i.e. the range of values of interest, NBI increases with S.

8.1. Extension: incumbent success reduces entrant's expected profitability

Proof of Proposition 3

(a) First note that Lemma 1 and Lemma 2 apply if we simply reverse the states: entry now occurs in state γ_B instead of state γ_G. For reasons of space I do not repeat the lemmas here.

I now derive the optimal complete contract. The derivation is analogous to the one in the proof of Proposition 1.

Optimal complete contract

The problem is:

$$Max \qquad U = eR_e - \frac{1}{2}e^2 \tag{8.34}$$

$$e = R_e \quad (IC) \tag{8.35}$$

$$e\gamma_G R + (1-e)[\gamma_B - \mu]R - eR_e \geqslant K_I \quad (IR) \tag{8.36}$$

The first-best effort level, e_C^{FB}, maximizes the project's NPV, taking into account costs of entry; i.e.

$$e[\gamma_G - (\gamma_B - \mu)]R + [\gamma_B - \mu]R - K_I - \frac{1}{2}e^2 \tag{8.37}$$

and is equal to

$$e_C^{FB} = (\Delta\gamma + \mu)R \tag{8.38}$$

Implementing this effort level would require setting $R_e = (\Delta\gamma + \mu)R$, which would not satisfy (IR). Thus (IR) will bind. We can write (IR) as follows:

$$e(\Delta\gamma + \mu)R + (\gamma_B - \mu)R - e^2 \geqslant K_I \tag{8.39}$$

Differentiating the LHS gives $(\Delta\gamma + \mu)R - 2e$, implying that the LHS increases from an initial value of $(\gamma_B - \mu)R$ for $e = 0$ to $\frac{1}{4}[(\Delta\gamma + \mu)R]^2 + (\gamma_B - \mu)R$ for $e = \frac{1}{2}(\Delta\gamma + \mu)R$, decreasing thereafter. Thus if $K_I > \frac{1}{4}[(\Delta\gamma + \mu)R]^2 + (\gamma_B - \mu)R$, (IR) cannot be satisfied and the project cannot be funded. Otherwise, the project will be funded and effort will be given by the largest root of:

$$e(\Delta\gamma + \mu)R + (\gamma_B - \mu)R - e^2 = K_I \tag{8.40}$$

i.e.

$$e^C = \frac{1}{2}(\Delta\gamma + \mu)R + \frac{1}{2}\{[(\Delta\gamma + \mu)R]^2 + 4[(\gamma_B - \mu)R - K_I]\}^{\frac{1}{2}} \tag{8.41}$$

The incumbent's expected utility with this contract is equal to $U = \dfrac{1}{2}(e^C)^2$.

Optimal incomplete contract

The problem is:

$$Max \qquad U \equiv e\gamma_G R_I + (1-e)(\gamma_B - \mu)R_I - \frac{1}{2}e^2 \tag{8.42}$$

subject to the constraints:

$$e = \arg\max(U) \quad (IC) \tag{8.43}$$

$$e\gamma_G R_V + (1-e)[(\gamma_B - \mu)R_V + S] \geqslant K_I \quad (IR) \tag{8.44}$$

$$R_I + R_V = R \tag{8.45}$$

$$R_I \geqslant 0, R_V \geqslant 0 \quad (LL) \tag{8.46}$$

From (IC) we have

$$e = [\gamma_G - (\gamma_B - \mu)]R_I \tag{8.47}$$

The first-best effort level, e_I^{FB}, maximizes the project's NPV, taking into account costs of entry and surplus extracted from the entrant; i.e.

$$e\gamma_G R + (1-e)[(\gamma_B - \mu)R + S] - K_I - \frac{1}{2}e^2 \tag{8.48}$$

and is equal to

$$e_I^{FB} = [\gamma_G - (\gamma_B - \mu)]R - S \tag{8.49}$$

Implementing e_I^{FB} would require setting

$$R_I = R - \frac{S}{\gamma_G - (\gamma_B - \mu)} \Longrightarrow R_V = \frac{S}{\gamma_G - (\gamma_B - \mu)} \tag{8.50}$$

Replacing the above expression for R_V into the LHS of (IR) gives

$$\frac{\gamma_G}{\Delta\gamma + \mu}S \tag{8.51}$$

We therefore have the following possibilities:

Case 1: $K_I \leqslant \frac{\gamma_G}{\Delta\gamma+\mu}S$. In this case the first-best effort level e_I^{FB} can be implemented.

Case 2: $K_I > \frac{\gamma_G}{\Delta\gamma+\mu}S$. In this case the first-best effort level e_I^{FB} cannot be implemented because it would violate (IR). Define $\alpha \equiv \Delta\gamma + \mu$. We can then write (IR) as follows

$$e\alpha R_V + (\gamma_B - \mu)R_V + (1-e)S \geqslant K_I \tag{8.52}$$

and replace $R_V = R - R_I = R - \frac{e}{\alpha}$ (using (IC)) to obtain

$$e[\alpha R - S - \frac{\gamma_B - \mu}{\alpha}] - e^2 + (\gamma_B - \mu)R + S \geqslant K_I \tag{8.53}$$

Differentiating the LHS gives $\alpha R - S - \frac{\gamma_B - \mu}{\alpha} - 2e$, and differentiating again gives -2. We know from $(A2)$ and $(A5)$ that $\alpha R - S - \frac{\gamma_B-\mu}{\alpha} > 0$, implying that the LHS is maximized at $e = \frac{1}{2}[\alpha R - S - \frac{\gamma_B-\mu}{\alpha}]$. Thus: either $\frac{1}{4}[\alpha R - S - \frac{\gamma_B-\mu}{\alpha}]^2 + (\gamma_B - \mu)R + S < K_I$, and the project cannot be funded; or the project is funded and e is the largest root of the equation

$$e[\alpha R - S - \frac{\gamma_B - \mu}{\alpha}] - e^2 + (\gamma_B - \mu)R + S = K_I \tag{8.54}$$

i.e.

$$e^I = \frac{1}{2}[\alpha R - S - \frac{\gamma_B - \mu}{\alpha}] + \frac{1}{2}\{[\alpha R - S - \frac{\gamma_B - \mu}{\alpha}]^2 + 4[(\gamma_B - \mu)R + S - K_I]\}^{\frac{1}{2}} \tag{8.55}$$

The incumbent's expected utility with this contract is given by

$$U \equiv e^I[\gamma_G - (\gamma_B - \mu)]R_I + (\gamma_B - \mu)R_I - \frac{1}{2}(e^I)^2 = \frac{1}{2}(e^I)^2 + \frac{(\gamma_B - \mu)e^I}{\alpha} \tag{8.56}$$

The net benefit from incomplete contracts can be derived as the difference between the incumbent's utility with the optimal incomplete contract and his utility with the optimal complete contract, obtained above:

$$NBI \equiv \frac{1}{2}[(e^I)^2 - (e^C)^2] + \frac{(\gamma_B - \mu)e^I}{\alpha}$$

Using (8.40) and (8.54), it can also be written as follows:

$$NBI \equiv \frac{1}{2}\{(1 - e^I)S + (e^I - e^C)\alpha R + \frac{e^I(\gamma_B - \mu)}{\alpha}\}$$

(b) We have:

$$\frac{dNBI}{dS} = \frac{1}{2}\{1 - e^I + (\alpha R - S + \frac{\gamma_B - \mu}{\alpha})\frac{de^I}{dS}\}$$

Using (8.54):

$$e^I[\alpha R - S - \frac{\gamma_B - \mu}{\alpha}] - (e^I)^2 + (\gamma_B - \mu)R + S = K_I \tag{8.57}$$

we obtain

$$de^I[\alpha R - S - \frac{\gamma_B - \mu}{\alpha} - 2e^I] = -(1 - e^I)dS \tag{8.58}$$

Thus $\frac{de^I}{dS} = \frac{-(1-e^I)}{\alpha R - S - \frac{\gamma_B - \mu}{\alpha} - 2e^I} > 0$ and $\frac{dNBI}{dS} > 0$ for $e^I > \frac{1}{2}[\alpha R - S - \frac{\gamma_B - \mu}{\alpha}]$. When $e^I = \frac{1}{2}[\alpha R - S - \frac{\gamma_B - \mu}{\alpha}]$ it is straightforward to verify that a marginal increase in S makes it possible to increase e^I without violating the IR constraint. Thus for $e^I \geqslant \frac{1}{2}[\alpha R - S - \frac{\gamma_B - \mu}{\alpha}]$, i.e. the range of values of interest, NBI increases with S.