Vertical Integration and Regulation in the Securities Settlement Industry

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1 Introduction and summary

The European Securities Trading and Post-trading (Clearing and Settlement) Industries have recently experienced a vast movement of consolidation. This was encouraged by the European Commission, seeking to establish a single market for the trade of financial securities across the European Union, and reduce the cost of cross-border transactions. Somehow, this industry may face a competition problem similar to that encountered in the telecom and transport sectors. For the regulator, the issue as a whole hinges on the need to allow for the achievement of economies of scale, while avoiding the possibility that an upstream monopolistic position might thwart competition in the downstream market, i.e. banking activities or securities trading activities.

After a long discussion with the industry, the European Commission considered that there was no need for regulation, provided that platforms adopted and implemented a "Code of Conduct" supposed to prevent such anti-competitive behaviors. The objective

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of this article is to challenge this view by showing that such a "Code of Conduct" may not be enough.

Several recent academic studies confirm the importance of these questions and conclude that regulation plays a crucial role in preventing any abuse of a dominant position. They recommend avoiding vertical integration among infrastructures and services, be it in securities trading activities or banking services. However these theoretical models do not try to figure out what should be the optimal regulation when complete separation of activities cannot be avoided. They rather provides different insights on how far the clearing and settlement industries differ from more traditional industries.

In order to address directly the issue of regulating such industries, we consider a model where custodians are competing in order to provide both custody services and banking services to investors. We assume that there is only one Central Security Deposit (CSD) where all the net orders must be in the end settled. We assume therefore that each custodian have an omnibus account in the CSD, for which they pay a fixed fee, and must pay an additional fee for each transaction they cannot not internalize. Besides, we assume also that the CSD is directly competing with other custodians for provision of custody and banking services. This may result from a vertical integration between a CSD and one custodian, or more simply by the CSD getting a banking licence. Such an entity may be more efficient than custodians (due to technological improvement and/or internalisation of some trades), but has the ability to exclude its rivals through prohibitive access prices.

A crucial assumption of our model is that it would be socially inefficient to let several CSDs compete for the provision of depository and settlement services for a given security (natural monopoly property). This comes from the property of increasing returns to scale that seems to be confirmed by empirical analysis which are discussed in the next section, but furthermore from the fact that we are in a competitive bottleneck situation (see for example Bergman, 2003). Even though several CSDs compete, it would be total uneconomical to have the same security deposited in several CSDs. Therefore when final investors want to trade in a given security, they are obliged to access (directly or indirectly) the (unique) CSD that provide notary services for this particular security. This is analogous to the local loop in the telecom industry.

Different types of regulator may be considered, existing ones such as the European commission, or new ones close to the regulating agencies which are already in place in other industries (telecom, energy). While the former may only impose external constraints on actors such as non-discriminatory access pricing or forbidding of integration, the latter may resort to a combination of monitoring and incentive regulation in order to extract information and impose a cost-based regulation. Although such a regulation may be very difficult and costly, we do not restrict ourselves in the followings, and consider both types
of regulation.

We first analyse the impact of vertical integration when the CSD is not for profit (section 3), then determine what should be the optimal access price when the CSD is for profit (section 4). In order to do that, we explicitly take into account the ability of both the CSD and the custodians to internalize part of their transactions. We show that cost-based regulation might restore social efficiency, but might also be difficult to implement. Finally, we study the effect of a ”code of conduct” (section 5) and show that vertical integration of platforms with banks, even with a code imposing non-discriminatory access pricing and accounting separation, will typically not lead to a socially efficient industry structure.

2 A Review of the academic literature

Let’s start by reviewing two empirical papers, among the very few available on the subject. Schmeidel et al.(2004) investigates the existence and extent of economies of scale in depository and settlement systems. Their data set consists in the balance sheets and income statements of 16 settlement institutions (including the two (I)CSDs) in Europe, North America and Asia over the period 1993-2000. The authors estimate different regressions (loglinear or translog) of total operating costs of these institutions over two outputs variables (the number of settlement instructions and the value of securities deposited), one input price variable, proxied by per capita GDP, a time trend and an (I)CSD dummy. The results are not very conclusive, since the coefficients of the output variables are not significant. However, the authors conclude that economies of scale are present, but that they differ a lot by size and region of the settlement institutions (they are smaller for large institutions and in the US). The (I)CSD dummy is positive and significant and the time trend indicates improved cost effectiveness over the sample period.

Van Cayseele and Wuyts (2005) estimate a translog cost function for settlement and safekeeping services (custody). They find that economies of scale are quickly exhausted, while economies of scope are present. However, they don’t have any (I)CSD in their data set, and they do not include the activities of custodian banks in their analysis. For these two reasons, they are not able to answer the fundamental question, namely is an (I)CSD more efficient (for providing jointly the services usually provided by CSDs and custodian banks) than the CSD and the custodian bank taken separately?

Among theoretical papers the most relevant to our analysis is Holthausen and Tapking (2004). This paper models competition between a CSD and a custodian bank for servicing small banks, which can either become members of the CSD or else use the custodian bank as an intermediary. The CSD and the custodian bank thus compete for offering
conservation services, but the CSD is a monopoly on the settlement activity. The CSD sets two prices: one for opening a securities account (depository function), the other for trade settlement. The custodian bank reacts by setting its own prices for custody and settlement. A relatively complex model is constructed to generate network externalities and gains from netting trades between investors. However some investors also have needs for additional services only provided by the custodian bank. As a result of the interaction between network externalities and these exogenous preferences, the socially optimal allocations are such that a majority of the investors chooses one of the providers (i.e. they are not symmetric. But in the absence of regulation, the CSD is able to leverage its monopoly power on settlement for attracting a majority of investors. This is because the CSD is able to raise the cost of its competitor for providing services to final investors. However, due to network externalities, the proportion of final users who hold direct accounts with the CSD is not necessarily excessive, from a social welfare perspective. Hence there is no clear case for regulating CSDs.

A number of other papers are also interesting but less directly relevant to our analysis. For example two papers by Kauko (2002, 2003) study the strategic use of links between CSDs as a way to commit not to charge too high fees for secondary market services. This allows CSDs to increase their revenues from primary markets services. These papers, although quite complex, are very interesting because they are the only ones (to date) to model explicitly the two-sided nature of the industry, by taking into account the security issuers as well as the investors. They show in particular that platforms are likely to subsidize issuers and tax investors. Similarly, Tapking and Yang (2004) analyse different forms of industry structures in a two country model with exchanges and CSDs: complete separation (CS), vertical integration (VI) within each country, horizontal integration of the two CSDs, either at the purely "legal" level (LHI) or also at the technical level (THI). They show that this last system dominates all the other and that VI is better than CS. Due to the complementarily of the services provided by the exchanges and the CSDs, horizontal merger is in fact pro-competitive. Koppl and Monnet (2003) also analyse the impact of integration between CSDs an exchanges and arrive at very different conclusions. They show that vertical silos can prevent efficiency gains from horizontal consolidation. They claim that their results may explain the failure of the merger between DBorse and the LSE.

Finally let’s mention three articles that do not contain any formal analysis but contribute to the debate about the industrial organisation of the security industry. Knieps (2004) tries to put forward the view that end custodians can be put in competition (he claims that "clearing and settlement are competitive services") as long as the technical regulatory function (i.e. the notary function)- is provided in a non discriminatory fashion. Similarly Serifsoy and Weiss (2003) try to assess the pros and cons of different post mar-
ket architectures and argue that contestable monopolies are the relevant paradigm. Van Cayseele (2004) discusses possible anti-trust concerns and opposes two radical views: a contestable quasi monopoly "view and a "regulated pan-European monopoly". He argues in favour of the first.

3 Vertical integration

We consider a model in the spirit of the one developed by Holthausen and Tapking (2003). There is a unique CSD, two competing custodian banks labelled $i = 1, 2$, and a large number of investors. The target of this chapter is to analyze the impact of vertical integration when the CSD is not for profit and uses a cost-based fee structure. Two stylized models of the clearing and settlement industry will be described in such a way, depending on whether the CSD is integrated with one custodian or not.

The two custodian banks compete to provide differentiated services tailored to satisfy the needs of final investors. This product differentiation dimension is captured by a Hotelling model (a standard modelling tool in Industrial Organization, see for example Tirole (1988)) where investors are located on a unit interval $[0,1]$, and incur a transportation cost $t$ per unit of distance when they choose to patronize one of the custodian banks, located at the extremes of the unit interval. Each investor has a non increasing and convex demand function $q = D(p)$, and an associated (net) surplus function $S(p) = \int_p^\infty D(s)ds$. The parameter $t$ reflects the intensity of competition between custodian banks: the higher $t$, the more differentiated the services offered by the two banks, and thus the less competitive the downstream market.

First, assume that the CSD is independent of the two custodians and contracts with each of them for the provision of settlement services. The cost structure is as follows: the CSD incurs a set-up cost $F$ and a marginal cost $C$ for providing depository and settlement services to custodian banks. Each custodian bank incurs a fixed cost $f$ per investor account, and a marginal cost $c$ for providing complementary services to each of his final consumers. We note also $p_i$ and $a_i$ respectively the per transaction fee and the fixed fee charged by custodian bank $i$ ($i = 1, 2$). This "custodians model" is represented in figure 1.
In this case, competition leads to a price structure that maximizes welfare:

**Proposition 1** In the CSD model, if transportation costs are small enough so that all the final investors are served, then:

- Per-transaction fees charged by custodian banks equal their total marginal cost \( p_1 = p_2 = c + C \);
- Fixed fees charged by the custodian banks equals their fixed cost plus the differentiation cost parameter \( a_1 = a_2 = f + t \);
- This price structure achieves maximum social welfare;
- The market is equally shared between the two custodians who make a profit equal to \( \frac{t}{2} - F \), while the average consumer surplus is equal to \( S(c + C) - f - \frac{5t}{4} \).

All final investors are served as long as custodian’s profit is greater than the fixed cost of installment \( (\frac{t}{2} > F) \), and that the gross surplus of each final investor exceeds the sum of fixed costs for maintaining the investor’s account with the bank, the margin of the custodian bank and the maximal transportation cost incurred among investors \( (S(c + C) > f + \frac{3t}{2}) \).

Secondly, consider an integrated model where the first custodian is replaced by the CSD, either through a vertical integration as depicted below or because the CSD got
a banking licence. We assume that the integration is accompanied by a decrease in total marginal cost, due to the internalisation of some trades and/or technological improvements: the CSD is more efficient than the second custodian and incurs $\gamma < c + C$ marginal cost, while the second custodian still incurs the marginal cost $C$ for access to the CSD. This ”CSD vs custodian model” is represented in figure 2.

![Figure 2: CSD vs custodian model.](image)

In this case, competition leads to a suboptimal price structure that induces redistributive effects between different type of investors:

**Proposition 2** In the CSD-custodian model, if transportation costs are small enough so that all the final investors are served (see appendix for explicit conditions), then

- **Per transaction prices charged by the CSD is lower than for bank 2**, which keeps the same as in the Custodians model. This favours brokers and dealers;

- **The fixed fee charged by the CSD is higher than for bank 2**, which is reduced in comparison with the Custodians model. On average, this penalizes retail investors;

- **Social welfare increases in comparison with the Custodians model, due to technical improvement induced by vertical integration, but the price structure is suboptimal.**

The price structure is suboptimal because the CSD raises its fixed fees in order to increase its profit, at the expense of its market share which becomes suboptimal (since
the CSD is more cost-efficient than the custodian). This result, as well as the two previous propositions, are a consequence of a more specific property whose proof is given in appendix 7.1. This property, which is provided in the following lemma, applies to a more schematic model where two custodians are competing while incurring the same fixed cost \( f \) but possibly different marginal costs \( c_1 \leq c_2 \). The difference between \( c_1 \) and \( c_2 \) reflects the way the custodians access the CSD \((c_2 = c + C)\) but, in the "custodians model", \( c_1 = c_2 \) while \( c_1 = \gamma < c_2 \) in the "CSD vs custodian" model).

**Lemma 1** Both at social optimum and at competitive equilibrium, per transaction fees equal marginal costs \( p_1 = c_1 \) and \( p_2 = c_2 \). However, as soon as custodians do not face the same marginal costs \((c_1 \neq c_2)\), the equilibrium market share \( x_c \) of the most efficient custodian is suboptimal at the competitive equilibrium:

\[
x_c = \frac{1}{2} + \frac{\Delta S}{6t} < x^* = \frac{1}{2} + \frac{\Delta S}{2t}
\]

where \( \Delta S = S(c_1) - S(c_2) \), reflecting the fact the most efficient custodian prefers to choose higher fixed fees while a social planner would prefer to impose equal fixed fees \( a_1 = a_2 \) (at the competitive equilibrium \( a_i = f + t + (-1)^{i+1} \frac{\Delta S}{3} \)).

4 **Access Price’s Regulation**

If the CSD is for profit, as is well known from other industries, vertical integration may lead to anticompetitive behavior including foreclosure. Indeed, if we assume that the CSD can choose first its access price \( p_a \) and then compete with the other custodian for service delivery to final investors as represented in figure 3, it can be shown that the CSD will raise significantly its access price in order to increase its rival’s cost, or even deny access to the custodian if the elasticity of demand of final investors low enough (cf. appendix 7.3 for explicit computations).
Once the second custodian is excluded, which is represented in figure 4, the CSD will raise its fixed fees in order to profit from its monopoly power. He may thus not serve all the downstream market. In comparison with the custodians’ model discussed in the previous section, the impact of integration on social welfare in this case is unclear since the increase in technical efficiency due to the merger may still dominates the decrease in competition. Anyway, regulators may be tempted to intervene in order to avoid the exclusion of a fraction of final users.

Figure 3: CSD internalization model.
One way to do that is to keep an open access to the CSD and impose a "reasonable" access charge. Even if no information is hidden to the regulator (we will question this assumption in the next section), the optimal access price is not straightforward because the cost structure is endogeneous, due to the ability of the CSD to internalize a greater fraction of its transactions.

For that reason, and in order to find out what should be the optimal access price, we extend the previous model in order to take into account internalization. We first assume that all custodian’s transactions need to be settled through the CSD: their total marginal cost is therefore $c + C$ (while CSD’s total marginal cost is equal to $\gamma < c + C$). This last assumption will be relaxed at the end of this section.

We assume thereafter that it is not optimal for the social planner to exclude the second custodian. This requires that the customer’s valuation for the differentiation of services provided by the custodian banks is high enough\(^1\) and that installment cost $F$ is low enough. We then have the following result:

**Proposition 3** Cost-based access price is optimal, i.e. $p_a = C$ maximizes social welfare.

This result is based on the assumption that no custodian can internalize part of its transaction. For robustness, we analyze also what would be the optimal access-price

\[^1\text{the exact condition is } S(\gamma) - S(c + C) \leq 1.8t, \text{ see appendix 7.2 for explicit computation.}\]
regulation when we depart from this assumption. In order to do that, we assume that the volume of such transactions remains supposedly relatively small. This hypothesis reflects the fact that multiple custodians may be facing the CSD, and that only transactions settled between two investors belonging to the same custodian would be internalized by this custodian.

More precisely, we assume thereafter that the CSD is facing \( n \) custodians (labeled \( i = 1, \ldots, n \)) that must settle the major part of their transactions through the CSD. The market is divided into \( n \) unit intervals \([0,1]\) with the CSD on one side, and one custodian on the other side. By symmetry, at the equilibrium, the market share \( x_i \) and the tariffs \((a_i, p_i)\) of each custodian will be the same, and we can thereafter omit the subscripts \( i \).

The total market share of the CSD is denoted by \( nx_0 \) where \( x_0 \) is its market share on one unit interval (hence \( x = x_i = 1 - x_0 \) for all \( i \in \{1, \ldots, n\} \)). For simplicity, we assume also that all CSD’s transactions are internalized with no additional cost \((\gamma = c)\).

![Diagram](image)

**Figure 5:** custodian’s internalisation model.

We denote by \( \alpha \) the fraction of custodian’s transactions that must be settled through the CSD. The remaining part \( 1 - \alpha \) of transactions, when securities are transferred between two accounts of the same custodian bank, are internalized by the custodian. When one custodian must settle a transaction, the other end of the transaction may be either with
one of its clients, a CSD’s client or a client of one of the \(n-1\) other custodians. Therefore, on average, \(\alpha = \frac{n-(1-x_0)}{n}\).

When \(n\) is large enough, it can be shown that the optimal access price with respect to social welfare is higher than the cost :

**Proposition 4** In a CSD model with regulated access-prices and custodians’ internalization, when the number of custodians is large enough, then the optimal regulated access price is higher than the cost \((p^*_a > C)\)

The intuition behind this result is as follows. When choosing \(p_a \neq C\), there is a tradeoff between two effects: a distortion on the demand of final investors \((p_a = C\) is less distorsive), and a change of the market share of the more cost-efficient CSD \((p_a > C\) is more efficient). When the custodians do not internalize, the distorsion effect dominates the efficiency effect, and \(p^*_a = C\). When they internalize part of their transactions, their market share is more sensitive to the access price (a raise of \(p_a\) decreases their market share, and thus lower their ability to internalize, which increases their cost, and so on). The efficiency effect then dominates the distorsion effect near \(p_a = C\) and the optimal access price is greater than \(C\).

## 5 Code of conduct

A cost-based regulation is likely to be difficult to implement, due to the incentives of the ICSD to hide (or manipulate) the information about its costs. Besides, in any case, due to indirect externalities between different categories of users (namely with issuers), cost based regulation is not even desirable. Lastly, a competition authority such as the DG COMP, is not supposed to regulate prices. Let us consider the case where a competition authority imposes a ”Code of conduct”. We follow here what was agreed and presented to the EU Commissioner McCreevy on the 7th November 2006 by the European Association of Central Counterparty Clearing Houses and the European Central Securities Depositories Association. In our framework, this agreement can be summarized in the three following constraints :

- non discriminatory access to the CSD;
- accounting separation between CSD and custodian bank 1;
- independent pricing decisions of the CSD and custodian bank 1.
The following proposition shows that the code of conduct achieve one of its target, that is to avoid redistribution between different types of final investors, but lead to high monopoly access price and inefficiency:

**Proposition 5** The equilibrium under the "code of conduct" leads to:

- **equal prices for final investors**
  \[ p_1 = p_2 = p_a + c \quad a_1 = a_2 = t + f \]

- **insufficient market share for the CSD**

- **monopoly access price**
  \[ \frac{(p_a + c) - \frac{1}{2} [\gamma + c + C]}{p_a + c} = \frac{1}{\eta} \]

**6 conclusion**

This paper is a first attempt to analyze the welfare effects of vertical integration between a CSD and a custodian bank in the Clearing and Settlement Industry.

It shows that a move from the CSD model to the ICSD model is likely to entail a change in the nature and the costs of settlement services (leading to a possible decrease in per transaction fees) accompanied with a decrease in competition for providing services to final users (leading, on average, to an increase in the fixed fees charged to investors).
If vertical integration is allowed, it would be necessary to regulate access pricing, but this would introduce new inefficiencies, due to the incentives of the ICSD to hide cost information. The design of an optimal regulation would be even more complex when indirect externalities are taken into account, since socially optimal prices then depend on externalities between the different categories of users, and not only on costs.

Having several CSDs competing for issuers on one side (single homing) and offering access to custodian banks on the other (multi-homing) could be a reasonable alternative, since it would limit the need for regulation only on the investors side. However a formal analysis remains to be done.

7 proof

7.1 vertical integration

Let us first consider the competitive equilibrium’s property described in lemma 1. The market share of custodian $i$ is:

$$x_i = \frac{1}{2} + \frac{1}{2t}(S(p_i) - S(p_j) + a_j - a_i),$$

while its profit is given by:

$$\pi_i = x_i a_i - f + (p_i - c_i)D(p_i) - F.$$

Social welfare is equal to

$$W = x_1(S(p_1) + (p_1 - c_1)D(p_1)) + (1 - x_1)(S(p_2) + (p_2 - c_2)D(p_2)) - \frac{t}{2}(x_1^2 + (1 - x_1)^2) - f - 2F$$

Under competition, custodian’s profit maximization program can be solved either by choosing the tariffs $(p_i, a_i)$ or by choosing $p_i$ and the market share $x_i$, since $x_i$ is equal, up to additive and multiplicative constants, to $S(p_i) - a_i$. The partial derivative of the profit $\pi_i$ with respect to $p_i$, while maintaining $x_i$ constant, is equal to $x_i(p_i - c_i)D'(p_i)$. Therefore, at a Nash equilibrium, the variable fees must be equal to the marginal costs ($p_i = c_i$).

The profit of custodian $i$ is maximized when $\frac{\partial \pi_i}{\partial a_i} = x_i - \frac{1}{2t}(a_i - f) = 0$. Simple computations give $a_i = \frac{1}{2}(t + f + a_j + (-1)^{i+1}\Delta S)$ where $\Delta S = S(c_1) - S(c_2)$, and then $a_i = t + f + (-1)^{i+1}\frac{1}{2}\Delta S$. At the competitive equilibrium, the market share of the first custodian is thus

$$x_1 = \frac{1}{2} + \frac{\Delta S}{6t}$$

Social welfare is then equal to

$$W_c = \Delta S\left(\frac{1}{2} + \frac{\Delta S}{6t}\right) + S(c_2) - \frac{t}{2}\left(\frac{1}{2} + \frac{\Delta S^2}{18t^2}\right) - f - 2F$$

or

$$W_c = S(c_1) - \frac{\Delta S}{2} + \frac{5\Delta S^2}{36t} - f - 2F - \frac{t}{4}.$$
Those results are correct as long as the net surplus of each investor is positive, i.e. \( S(p_i) \geq a_i + x_i t \), and as long as custodians’ profit are positive, i.e. \( x_i(a_i - f) > F \). Those two relations are respectively equivalent to

\[
\frac{S(c_1) + S(c_2)}{2} > \frac{3}{2} t + f
\]

and

\[
1 - \frac{\Delta S}{3 t} \geq \sqrt{\frac{2F}{t}}.
\]

Let us now consider the social optimum. Using the same previous arguments, variable fees must be equal to marginal costs. Besides, fixed fees must be chosen in order to have \( x_1 \in \arg \max (x_1 S(c_1) + (1 - x_1) S(c_2) - \frac{t}{2} (x_1^2 + (1 - x_1)^2)) \). Simple computation gives the optimal market share for the first custodian:

\[
\pi_1^* = \frac{1}{2} + \Delta \frac{S}{2 t}
\]

which is obtained by choosing \( a_1 = a_2 \). Social welfare is then equal to

\[
W^* = \Delta S \left( \frac{1}{2} + \frac{\Delta S}{2 t} \right) + S(c_2) - \frac{t}{2} \left( \frac{1}{2} + \frac{\Delta S^2}{2 t^2} \right) - f - 2 F
\]

or

\[
W^* = S(c_1) - \Delta \frac{S}{2} + \frac{\Delta S^2}{4 t} - f - 2 F - \frac{t}{4} = W_c + \frac{\Delta S^2}{9 t}
\]

### 7.2 Access price’s regulation

When a social planner sets \( p_a \), profits are respectively given by the following equations

\[
\begin{align*}
\pi_1 &= x_1(a_1 - f + (p_1 - \gamma) D(p_1)) + (1 - x_1)(p_a - C) D(p_2) - F \\
\pi_2 &= (1 - x_1)(a_2 - f + (p_2 - c - p_a) D(p_2)) - F
\end{align*}
\]

Social welfare is equal to

\[
W = x_1 (S(p_1) + (p_1 - \gamma) D(p_1)) + (1 - x_1) (S(p_2) + (p_2 - c - C) D(p_2)) - \frac{t}{2} (x_1^2 + (1 - x_1)^2) - 2 F
\]

According to the previous section, social optimum is achieved when the market share of the first custodian is equal to \( x^* = \frac{1}{2} + \frac{\Delta S}{2 t} \) with \( \Delta S = S(\gamma) - S(c + C) \), and when per-transaction fees to the investors are equal to marginal costs \( p_1 = \gamma \) and \( p_2 = c + C \).

Under competition, using the same previous argument, the variable fees will be equal to the marginal costs at a Nash equilibrium: \( p_1 = \gamma \) and \( p_2 = c + p_a \). The fixed costs must verify the following equations

\[
\begin{align*}
\frac{\partial \pi_1}{\partial a_1} &= x_1 - \frac{1}{2 t} (a_1 - f - (p_a - C) D(c + p_a)) = 0 \\
\frac{\partial \pi_2}{\partial a_2} &= (1 - x_1) - \frac{1}{2 t} (a_2 - f) = 0
\end{align*}
\]

therefore

\[
\begin{align*}
a_1 &= \frac{1}{2} (f + t + a_2 + S(\gamma) - S(c + p_a) + (p_a - C) D(c + p_a)) \\
a_2 &= \frac{1}{2} (f + t + a_1 + S(c + p_a) - S(\gamma))
\end{align*}
\]
so

\[
\begin{align*}
& a_1 = f + t + \frac{S(c) - S(c+p_a)}{3} + \frac{2}{3}(p_a - C)(D(c+p_a)) \\
& a_2 = f + t - \frac{S(c) - S(c+p_a)}{3} + \frac{1}{3}(p_a - C)(D(c+p_a))
\end{align*}
\]

and the equilibrium market share of custodian 1 is

\[
x_{pa} = \frac{1}{2} + \frac{S(c) - S(c+p_a)}{6t} - \frac{1}{6t}(p_a - C)(D(c+p_a))
\]

The derivative of the market share with respect to \( p_a \) is

\[
\frac{\partial x_{pa}}{\partial p_a} = -\frac{1}{6t}(p_a - C)D'(c + p_a)
\]

thus the market share is minimum when \( p_a = C \), and it is then equal to

\[
x_{pa=c} = \frac{1}{2} + \frac{\Delta S}{6t}
\]

where \( \Delta S = S(c) - S(c+C) \).

Let us know find the optimal choice of \( p_a \) for the social planner. The welfare can be seen as a function of the three variables \( p_1, p_2 \) and \( x_1 \). The derivative of the social welfare with respect to \( x_1 \), when \( p_1 = \gamma, p_2 = c + p_a \) and \( x_1 = x_{pa} \) is

\[
\frac{\partial W(\gamma, c + p_a, x_{pa})}{\partial x_1} = t - 2tx + S(\gamma) - S(c + p_a) - (p_a - C)(D(c + p_a) = 4tx_{pa} - 2t \geq 0
\]

thus, the derivative of the social welfare with respect to \( p_a \) is

\[
\begin{align*}
\frac{\partial W(\gamma, c + p_a, x_{pa})}{\partial p_a} &= (p_a - C)D'(c + p_a)(1 - x_{pa} - \frac{4tx_{pa} - 2t}{6t}) \\
&= (p_a - c)D'(c + p_a)(\frac{4}{3} - \frac{5}{3}x_{pa})
\end{align*}
\]

When the transportation costs are high enough, or the custodian’s inefficiency low enough, so that \( x_{pa=c} \) is lower than 80% (which is equivalent to \( \Delta S < 1.8t \)), then there is a local maximum at \( p_a = c \). On the other hand, when \( \Delta S > 1.8t \), then \( x_{pa} \) is always higher than 80%, and it is more optimal to choose \( p_a \) different than \( c \), in order to exclude in the end the custodian. Even when \( \Delta S < 1.8t \), the point \( p_a = c \) may not be a global maximum, which would also mean that the exclusion of the custodian is optimal.

### 7.3 exclusion of the other custodians

Let’s first assume that the integrated CSD is for profit and non regulated. We consider a mix Nash and Stackelberg equilibrium : the CSD first chooses \( p_a \), then the CSD and the other custodians compete together by providing tariffs \( p_1 \) and \( p_2 \) to the investors, and move toward a Nash equilibrium. Once a tariff \( p_a \) is chosen by the CSD, the results of the previous section 7.2 apply. CSD’s profit is given by the following equation

\[
\pi_1 = x_1(a_1 - f) + (1 - x_1)(p_a - c)(D(c + p_a))
\]

with

\[
\begin{align*}
x_1 &= \frac{1}{2} + \frac{S(c) - S(c+p_a)}{6t} - \frac{1}{6t}(p_a - c)(D(c + p_a)) \\
a_1 &= f + t + \frac{S(c) - S(c+p_a)}{3} + \frac{2}{3}(p_a - c)(D(c + p_a))
\end{align*}
\]

\[16\]
so

\[ \pi_1 = x_1(a_1 - f - (p_a - c)D(c + p_a)) + (p_a - c)D(c + p_a) = 2tx_1^2 + (p_a - c)D(c + p_a) \]

and simple computation gives the following relation at the optimum

\[ (p_a - c)D'(c + p_a)(\frac{2}{3}xp_a - 1) = D(c + p_a) \]

or

\[ \frac{p_a - c}{c + p_a} = \frac{-D(c + p_a)}{(c + p_a)D'(c + p_a)} \times \frac{1}{1 - \frac{2}{3}xp_a} \]

The left term of this equation is an increasing function of \( p_a \) which takes value 0 when \( p_a = c \) and tends to 1. The right term is equal to the inverse of the elasticity of quasi-demands time a factor that is greater than \( \frac{3}{2} \) since \( xp_a > \frac{1}{2} \). Thus, CSD will choose a high access price, and even exclude the custodian as soon as the elasticity of demand is lower than 1.5.

If the other custodians is excluded, then the market share of the monopolistic CSD is \( x_1 = \frac{S(p_1) - a_1}{t} \) and its profit is maximized when \( p_1 = c_1 \) and

\[ a_1 = \arg\max \left( \frac{S(c_1) - a_1(a_1 - f)}{t} \right) = \frac{S(c_1) + f}{2} \]

Its market share is therefore equal to \( x_1 = \frac{S(c_1) - f}{2t} \) as long as this value belongs to the interval \([0, 1]\), and social welfare is equal, when fixed costs are included, to

\[ W_{ICSD} = x_1(S(c_1) - f) - \frac{t}{2}x_1^2 - F = \frac{3}{8t}(S(c_1) - f)^2 - F \]

if \( x_1 < 1 \) and to

\[ W_{ICSD} = S(c_1) - f - F - \frac{t}{2} \]

if the CSD serve all the downstream market.

7.4 custodian internalization

Under these assumptions, the market share of CSD on each unit interval is

\[ x_0 = \frac{1}{2} + \frac{1}{2t}(S(p_0) - S(p) + a - a_0) \]

If we neglect set-up costs, including those related to setting omnibus accounts for each custodian inside the CSD, the profits of the CSD and of the custodians are respectively given by:

\[ \begin{align*}
\pi_0 &= nx_0(a_0 - f + (p_0 - c)D(p_0)) + n\alpha(1 - x_0)(\gamma - c)D(p) \\
\pi &= (1 - x_0)(a - f + (p - c - \alpha\gamma)D(p))
\end{align*} \]

where \( \gamma \) is the variable fee charged by the CSD to each custodian for settling a transaction through the CSD, and \( \alpha \) is the fraction of custodian’s transactions that must be settled through the CSD. The case \( \alpha = 1 \) corresponds to the model analyzed in the previous section.

The profits of the CSD and of the custodians are respectively given by:

\[ \begin{align*}
\pi_0 &= nx_0(a_0 - f + (p_0 - c)D(p_0)) + n\alpha(1 - x_0)(\gamma - c)D(p) \\
\pi &= (1 - x_0)(a - f + (p - c - \alpha\gamma)D(p))
\end{align*} \]
Each custodian will choose its fixed tariff in order to maximize its profit, which leads to the following system of equation:

\[
\begin{align*}
\frac{\partial n_0}{\partial m_0} &= x_0 - \frac{1}{2t} (a_0 - f + (p_0 - c)D(p_0)) + (\gamma - c)D(p)(1 - \frac{1 - x_0}{6n}) = 0 \\
\frac{\partial m}{\partial m_0} &= (1 - x_0)(1 - \frac{\gamma D'(c + \alpha \gamma)}{2nt}) - \frac{1}{2t} (a - f + (p - c - \alpha \gamma)D(p)) = 0
\end{align*}
\]

since \( \frac{\partial n}{\partial x_0} = \frac{1}{n} \), \( \frac{\partial m}{\partial n_0} = -\frac{1}{2t} \) and \( \frac{\partial m(1-x_0)}{\partial m_0} = -\frac{1-x_0}{2nt} + \frac{a}{2t} = \frac{1}{2t} - \frac{1-x_0}{2nt} \). Using the same argument as in the previous section, at a Nash equilibrium, the variable fees must once again be equal to the marginal costs i.e. \( p_0 = c \) and \( p = c + \alpha \gamma \), and this system of equation becomes

\[
\begin{align*}
\frac{\partial n_0}{\partial m_0} &= x_0 - \frac{1}{2t} (a_0 - f) + (\gamma - c)D(c + \alpha \gamma)(1 - \frac{1 - x_0}{6n}) = 0 \\
\frac{\partial m}{\partial m_0} &= (1 - x_0)(1 - \frac{\gamma D'(c + \alpha \gamma)}{2nt}) - \frac{1}{2t} (a - f) = 0
\end{align*}
\]

Those two equations can be combined in order to obtain a third equation

\[
x_0 = \frac{1}{2} + \frac{S(c) - S(c + \alpha \gamma)}{6t} - \frac{1}{6t} (\gamma - c)D(c + \alpha \gamma) - \frac{1 - x_0}{3nt} (c - \gamma)D'(c + \alpha \gamma)
\]

which defines implicitly the equilibrium market share \( x_0 \) as a function of \( \gamma \). Differentiating this equation, we get

\[
\frac{\partial x_0}{\partial \gamma} \times (1 - \frac{\gamma}{6nt}D(c + \alpha \gamma) + \frac{\gamma (\gamma - c)}{6nt} D'(c + \alpha \gamma) - \frac{c - \gamma}{3nt} D'(c + \alpha \gamma)) + \frac{1 - x_0}{3nt^2} (c - \gamma) \gamma D'(c + \alpha \gamma))
\]

\[
= -\frac{1}{6t} (\gamma - c) \alpha D'(c + \alpha \gamma) + \frac{1 - x_0}{6nt} D(c + \alpha \gamma) - \frac{1 - x_0}{3nt} (c - \gamma) \alpha D'(c + \alpha \gamma)
\]

and it is easy to check that, when \( n \) is large enough, this derivative at \( \gamma = c \) is strictly positive.

Let us now find whether the optimal value for \( \gamma \) is equal to \( c \) or not. Social welfare is equal to

\[
W = n \{ x_0 S(c) + (1 - x_0) (S(c + \alpha \gamma) + \alpha (\gamma - c) D(c + \alpha \gamma)) - \frac{t}{2} (x_0^2 + (1 - x_0)^2) \} - 2F
\]

and the maximization of this value depends on the derivative

\[
\frac{\partial W}{n \partial \gamma} = (1 - x_0) \alpha^2 (\gamma - c) D'(c + \alpha \gamma) + \frac{\partial x_0}{\partial \gamma} \frac{\partial W}{n \partial x_0} |_{\gamma = \text{cte}}
\]

When \( n \) tends to infinity, the last term of this equation

\[
\frac{\partial W}{n \partial x_0} = S(c) - S(c + \alpha \gamma) - \alpha (\gamma - c) D(c + \alpha \gamma) - (1 - x_0) \frac{c}{n} D(c + \alpha \gamma) + (1 - x_0) \frac{\alpha \gamma}{n} (\gamma - c) D'(c + \alpha \gamma) + t - 2tx_0
\]

tends uniformly towards the value obtained when \( \alpha = 1 \) in the previous section, i.e. \( \left. \frac{\partial W}{n \partial x_0} \right|_{\gamma = c} = 4tx_0 - 2t > 0 \). Consequently, the derivative of the social welfare with respect to \( \gamma \) is strictly positive at \( \gamma = c \) when \( n \) is large enough.

8 bibliography


