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Generalized Disappointment Aversion, Long-Run Volatility Risk and Asset Prices *

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Abstract

We propose an asset pricing model where preferences display generalized disappointment aversion (Routledge and Zin, 2009) and the endowment process involves long-run volatility risk. These preferences, which are embedded in the Epstein and Zin (1989) recursive utility framework, overweight disappointing results as compared to expected utility, and display relatively larger risk aversion for small gambles. With a Markov switching model for the endowment process, we derive closed-form solutions for all returns moments and predictability regressions. The model produces first and second moments of price-dividend ratios and asset returns and return predictability patterns in line with the data. Compared to Bansal and Yaron (2004), we generate: i) more predictability of excess returns by price-dividend ratios; ii) less predictability of consumption growth rates by price-dividend ratios. Differently from the Bansal and Yaron model, our results do not depend on a value of the elasticity of intertemporal substitution greater than one.

Keywords: Equilibrium Asset Pricing, Disappointment Aversion, Long-run Risks, Volatility Risk, Equity Premium, Risk-free Rate Puzzle, Predictability of returns

JEL Classification: G1, G12, G11, C1, C5

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1 Introduction

The Consumption-based Capital Asset Pricing Model (CCAPM) has recently been revived by models of long-run risks (LRR) in mean and volatility[1]. Bansal and Yaron (2004) explain several asset market stylized facts by a model with a small long-run predictable component driving consumption and dividend growth and persistent economic uncertainty measured by consumption volatility, together with recursive preferences that separate risk aversion from intertemporal substitution (Epstein and Zin, 1989; Kreps and Porteus, 1978). These preferences play a crucial role in the long-run risks model. In a canonical expected utility risk, only short-run risks are compensated, while long-run risks do not carry separate risk premia. With Kreps-Porteus preferences, long-run risks earn a positive risk premium as long as investors prefer early resolution of uncertainty. Routledge and Zin (2009) introduced recently preferences that exhibit generalized disappointment aversion (GDA) and showed that they can generate a large equity premium and countercyclical risk aversion. Compared with expected utility, GDA overweights outcomes below a threshold set at a fraction of the certainty equivalent of future utility. Disappointment aversion (Gul, 1991) sets the threshold at the certainty equivalent.

Despite the economic appeal to link expected consumption growth to asset prices, the existence of a long-run risk component in expected consumption growth is a source of debate. If a very persistent predictable component exists in consumption growth, as proposed by Bansal and Yaron (2004), it is certainly hard to detect it as consumption appears very much as a random walk in the data[2]. Moreover, this slow mean-reverting component has the counterfactual implication of making consumption growth predictable by the price-dividend ratio. There is less controversy about the persistence in consumption growth volatility. Bansal and Yaron (2004) show that the variance ratios of the absolute value of residuals from regressing current annual consumption growth on five lags increase gradually up to 10 years, suggesting a slow-moving predictable variation in this measure of consumption growth volatility. Calvet and Fisher (2007) find empirical evidence of volatility shocks of much longer duration than in Bansal and Yaron (2004), creating the potential of a more important contribution of volatility risk in explaining asset pricing stylized facts.

[1]Another featured approach is the rare disaster model of Barro (2006). The extensive literature about the equity premium puzzle and other puzzling features of asset markets are reviewed in a collection of essays in Mehra (2008). See also Campbell (2000, 2003), Cochrane and Hansen (1992), Kocherlakota (1996), and Mehra and Prescott (2003).

[2]See in particular Campbell (2003). Bansal (2007) cites several studies that provide empirical support for the existence of a long-run component in consumption. Bansal, Gallant and Tauchen (2007) and Bansal, Kiku and Yaron (2007a) test the LRR in mean and volatility model using the efficient and generalized method of moments, respectively. Hansen, Heaton and Li (2008) and Bansal, Kiku and Yaron (2007a, 2009) present evidence for a long-run component in consumption growth using multivariate analysis.

In this paper, we revisit the LRR model with GDA preferences. In Bansal, Kiku and Yaron (2007b), the presence of a slow mean-reverting long-run component in the mean of consumption and dividend growth series, coupled with Kreps-Porteus preferences, is essential to achieve an equity premium commensurate with historical data[3]. Given the debate about the nature of the consumption process, we start by restricting the LRR model to a random walk model with LRR in volatility to investigate whether persistent fluctuations in economic uncertainty are sufficient, with GDA preferences, to explain observed asset pricing stylized facts.

This benchmark model reproduces asset pricing stylized facts and predictability patterns put forward in the previous literature. The equity premium and the risk-free rate are very closely matched, as well as the volatility of the price-dividend ratio and of returns. The price-dividend ratio predicts excess returns at various horizons even though consumption and dividend growth rates are assumed to be unpredictable.

The intuition becomes clear from the simplest representation of GDA preferences, where the only source of risk aversion is disappointment aversion (the utility function is otherwise linear with a zero curvature parameter and an infinite elasticity of intertemporal substitution). With these simple preferences, the stochastic discount factor has only two values in each state of the economy at time t. The SDF for disappointing outcomes is φ times the SDF for non-disappointing outcomes, where $\varphi - 1 > 0$ is the extra weight given by disappointment averse preferences to disappointing outcomes. This could give rise to a sizable negative covariance between the pricing kernel and the return on a risky asset, making the risk premium substantial.

More generally, the SDF has an infinite number of outcomes with a kink at the point where future utility is equal to a given fraction of the certainty equivalent. When volatility of consumption growth is persistent, an increase in volatility increases the volatility of future utility. A more volatile future utility increases the probability of disappointing outcomes, making the SDF more volatile. Since both consumption and dividends share the same stochastic volatility process, an increase in volatility will increase the negative covariance between the SDF and the equity return, implying a substantial increase in the stock risk premium.

If volatility is persistent, as it is the case in the long-run volatility risk model we assume, this will result in persistent and predictable conditional expected returns. As argued by Fama and French (1988), such a process for expected returns generates mean reversion in asset prices. Therefore, the price-dividend ratio today should be a good predictor of returns

[3]Although a persistent volatility would also increase the equity premium with Kreps-Porteus preferences, it would do it only in the presence of this first source of LRR.

over several future periods.

Bansal and Yaron (2004), as most recent models, rely on parameter calibration for consumption and dividend processes as well as preferences to derive asset pricing implications from the model. The solution technique to solve for asset valuation ratios is based on loglinear approximations. Since the GDA utility is non-differentiable at the kink where disappointment sets in, one cannot rely on the same approximation techniques to solve the model. In this paper, we propose a methodology that provides an analytical solution to the LRR in mean and volatility model with GDA preferences and a fortiori with Kreps-Porteus preferences, yielding formulas for the asset valuation ratios in equilibrium. The key to this analytical solution is to use a Markov switching process for consumption and dividends that matches the LRR specifications. In addition, we report analytical formulas for the population moments of equity premia as well as for the coefficients and R^2 of predictability regressions that have been used to assess the ability of asset pricing models to reproduce stylized facts.

Thanks to our analytical formulas, we are able to conduct a thorough comparative analysis between models by varying extensively the preference and endowment parameters. We produce graphs that exhibit the sensitivity of asset pricing statistics or predictability regressions R^2 to key parameters such as the persistence in volatility or expected consumption growth. This provides a very useful tool to measure the robustness of model implications.

We consider in particular the value of the elasticity of intertemporal substitution, which has been a source of lively debate. Bansal, Kiku and Yaron (2009) report empirical evidence in favor of a value greater than 1[4], but Beeler and Campbell (2009), as well as Hall (1988) and Campbell (1999), estimate an elasticity of intertemporal substitution below 1[5]. One important aspect of our model is that the elasticity of intertemporal substitution value of one is not pivotal for reproducing asset pricing stylized facts. Moment fitting and predictability results remain intact with values of the elasticity of intertemporal substitution below 1. The main effect of setting the elasticity of intertemporal substitution below 1 is, of course, an increase in the level and volatility of the risk-free rate, but these moments remain in line with the data.

We also conduct a sensitivity analysis with respect to the specification of risk preferences. We investigate the simplest specification among disappointment averse preferences. We

[4]They cite Hansen and Singleton (1982) and Attanasio and Weber (1989), Bansal, Kiku and Yaron (2007a), and Hansen, Heaton, Lee and Roussanov (2007), among others.

[5]Bansal and Yaron (2004) also argue that in the presence of time-varying volatility, there is a severe downward bias in the point estimates of the elasticity of intertemporal substitution. While the argument is correct in principle, Beeler and Campbell (2009) simulate the Bansal and Yaron (2004) model and report no bias if the riskless interest rate is used as an instrument. They confirm the presence of a bias (negative estimate of the elasticity of intertemporal substitution) when the equity return is used and attribute it to a weak instrument problem.

set the threshold at the certainty equivalent, as in the original disappointment aversion model of Gul (1991), and we do not allow for any curvature in the stochastic discount factor, except for the disappointment kink. In other words, if disappointment aversion were not present, the stochastic discount factor would be equal to the constant time discount parameter. This pure disappointment aversion model reproduces rather well predictability of returns. Routledge and Zin (2009) stress the importance of GDA for obtaining a counter-cyclical price of risk in their Mehra and Prescott (1985) economy. Since we have a richer endowment process, there is not such a stark contrast between DA and GDA preferences on this implication of the model.

The results obtained with a random walk consumption (and LRR in volatility) combined with GDA preferences are maintained when we introduce a long-run risk in expected consumption growth. We verify that all the statistics reproduced for the GDA preferences are very close to what we obtained with the random walk model. This is in contrast to the results obtained using Kreps-Porteus preferences, where the role played by the small long-run predictable component in expected consumption growth is essential.

Disappointment aversion preferences were introduced by Gul (1991) to be consistent with the Allais Paradox. They are endogenously state-dependent through the certainty equivalent threshold and, therefore, are apt to produce counter-cyclical risk aversion. Investors may become more averse in recessions if the probability of disappointing outcomes is higher than in booms. Bernartzi and Thaler (1995) also feature asymmetric preferences over good and bad outcomes to match the equity premium, but they start from preferences defined over one-period returns based on Kahneman and Tversky (1979)'s prospect theory of choice. By defining preferences directly over returns, they avoid the challenge of reconciling the behavior of asset returns with aggregate consumption.

Models with exogenous reference levels, such as Campbell and Cochrane (1999) and Barberis, Huang and Santos (2001), generate counter-cyclical risk aversion and link it to return predictability. Investors will be willing to pay a lower price in bad states of the world, implying higher future returns. In Lettau and Van Nieuwerburgh (2008), predictability empirical patterns can be explained by changes in the steady-state mean of the financial ratios. These changes can be rationalized by a LRR model with GDA preferences.

Recently, Ju and Miao (2009) have embedded a model of smooth ambiguity aversion in a recursive utility framework. While ambiguity aversion implies attaching more weight to bad states, as in disappointment aversion, the mechanism is very different. An ambiguity averse decision maker will prefer consumption that is more robust to possible variations in probabilities. They fear stocks because they build pessimistic views about consumption

growth realizations.[6]

In this paper we match the heteroscedastic autoregressive models for consumption and dividend growth rates in Bansal and Yaron (2004) with a four-state Markov switching model. Markov switching models have been used in the consumption-based asset pricing literature to capture the dynamics of the endowment process. While Cechetti, Lam and Mark (1990) and Bonomo and Garcia (1994) estimate univariate models for either consumption or dividend growth, Cechetti, Lam and Mark (1993) estimate a homoscedastic bivariate process for consumption and dividend growth rates, and Bonomo and Garcia (1993, 1996) a heteroscedastic one. Recently, Lettau, Ludvigson and Wachter (2008), Bhamra, Kuehn, and Strebulaev (2010), and Ju and Miao (2009) have also estimated such processes. Calvet and Fisher (2007) estimate multifractal processes with Markov switching for a large number of states in a consumption-based asset pricing model. Apart from capturing changes in regimes, another distinct advantage of Markov switching models is to provide a flexible statistical tool to match other stochastic processes such as autoregressive processes as in Tauchen (1986). Recently, Chen (2008) has approximated the dynamics of consumption growth in the Bansal and Yaron (2004) model using a discrete-time Markov and the quadrature method of Tauchen and Hussey (1991) in a model to explain credit spreads.

This paper extends considerably the closed-form pricing formulas provided in Bonomo and Garcia (1994) and Cecchetti, Lam and Mark (1990) for the Lucas (1978) and Breeden (1979) CCAPM model. Bonomo and Garcia (1993) have studied disappointment aversion in a bivariate Markov switching model for consumption and dividend growth rates and solved numerically the Euler equations for the asset valuation ratios. For recursive preferences, solutions to the Euler equations have been mostly found either numerically or after a log linear approximate transformation. However, Chen (2008) and Bhamra, Kuehn, and Strebulaev (2010) use a Markov chain structure for consumption growth to solve analytically for equity and corporate debt prices in an equilibrium setting with Kreps-Porteus preferences, while Calvet and Fisher (2007) focused on the equity premium[7]. Other papers have developed analytical formulas for asset pricing models[8].

The rest of the paper is organized as follows. Section 2 sets up the preferences and endowment processes. Generalized disappointment averse preferences and the Bansal and Yaron (2004) long-run risks model for consumption and dividend growth are presented. In

[6]Ambiguity aversion increases the conditional equity premium when there is uncertainty about the current state of the economy (and its future prospects). However, various versions of the ambiguity model have difficulty reproducing predictability patterns and magnitudes.

[7]The paper by Calvet and Fisher (2007) has been developed contemporaneously and independently from the first version of the current paper titled "An Analytical Framework for Assessing Asset pricing Models and Predictability", presented in May 2006 at the CIREQ and CIRANO Conference in Financial Econometrics in Montreal.

[8]See in particular Abel (1992, 2008), Eraker (2008), and Gabaix (2008).

section 3, we describe a moment-matching procedure for the LRR in mean and volatility model based on a Markov switching process, solve for asset prices and derive formulas for predictive regressions. Section 4 explains how endowment and preference parameters are chosen for the benchmark random walk model of consumption and dividends. We also explore the asset pricing and predictability implications of the model. A thorough sensitivity analysis is conducted in Section 5 for preference parameters and persistence in consumption volatility. Section 6 provides a comparison with the LRR model of Bansal and Yaron (2004). Section 7 concludes.

2 An Asset Pricing Model with GDA Preferences and LRR Fundamentals

Our primary goal in this section is to formulate a model that includes both a long-run risk specification for consumption and dividends, and recursive preferences. In Bansal and Yaron (2004), where a long-run risk asset pricing model is developed, the Kreps-Porteus recursive preferences have an expected utility certainty equivalent that disentangles risk aversion from intertemporal substitution. In this paper the certainty equivalent is extended to represent generalized disappointment aversion preferences (GDA) recently introduced by Routledge and Zin (2009). These preferences generalize the former disappointment aversion specification of the recursive utility family introduced by Epstein and Zin (1989) and studied empirically by Bonomo and Garcia (1993) and Bekaert, Hodrick and Marshall (1997) in the context of asset pricing, and Ang, Bekaert and Liu (2005) for portfolio allocation.

GDA preferences distort the probability weights of expected utility by overweighting outcomes below a threshold determined as a fraction of the certainty equivalent. Two parameters are added with respect to the Kreps-Porteus specification, one that determines the threshold at which the investor gets disappointed as a percentage of the certainty equivalent, and another one that sets the magnitude of disappointment incurred by the investor below this threshold. GDA preferences admits both Kreps-Porteus and simple disappointment aversion as particular cases. In the latter case, the threshold is set right at the certainty equivalent. In a simple Mehra-Prescott economy, Routledge and Zin (2009) show that recursive utility with GDA risk preferences generates effective risk aversion that is countercyclical, where effective risk aversion refers to the risk aversion of an expected utility agent that will price risk in the same way as a disappointment-averse agent. The economic mechanism at play is an endogenous variation in the probability of disappointment in the representative investor's intertemporal consumption-saving problem that underlies the asset-pricing model. We extend their investigation by combining GDA preferences with a more complex long-run risk model for consumption and dividends.

2.1 Generalized Disappointment Aversion

Routledge and Zin (2009) generalized Gul's (1991) disappointment aversion preferences and embedded them in the recursive utility framework of Epstein and Zin (1989). Formally, let V_t be the recursive intertemporal utility functional:

$$V_t = \left\{(1-\delta)\, C_t^{1-\frac{1}{\psi}} + \delta\left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi \neq 1 \tag{2.1}$$

$$= C_t^{1-\delta}\left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{\delta} \quad \text{if } \psi = 1, \tag{2.2}$$

where C_t is the current consumption, δ (between 0 and 1) is the time preference discount factor, ψ (greater than 0) is the elasticity of intertemporal substitution and $\mathcal{R}_t\left(V_{t+1}\right)$ is the certainty equivalent of random future utility conditional on time t information.

With GDA preferences the certainty equivalent function $\mathcal{R}\left(.\right)$ is implicity defined by:

$$\frac{\mathcal{R}^{1-\gamma}}{1-\gamma} = \int_{(-\infty,\infty)} \frac{V^{1-\gamma}}{1-\gamma} dF\left(V\right) - \left(\alpha^{-1}-1\right)\int_{(-\infty,\kappa\mathcal{R})}\left(\frac{\left(\kappa\mathcal{R}\right)^{1-\gamma}}{1-\gamma} - \frac{V^{1-\gamma}}{1-\gamma}\right) dF\left(V\right) \quad , \tag{2.3}$$

where $0 < \alpha \leq 1$ and $0 < \kappa \leq 1$. When α is equal to one, $\mathcal{R}$ becomes the certainty equivalent corresponding to expected utility while V_t represents the Kreps-Porteus preferences. When $\alpha < 1$, outcomes lower than $\kappa\mathcal{R}$ receive an extra weight $(\alpha^{-1}-1)$, decreasing the certainty equivalent. Thus, α is interpreted as a measure of disappointment aversion, while the parameter κ is the percentage of the certainty equivalent $\mathcal{R}$ such that outcomes below it are considered disappointing[9]. Formula (2.3) makes clear that the probabilities to compute the certainty equivalent are redistributed when disappointment sets in, and that the threshold determining disappointment is changing over time.

With Kreps-Porteus preferences, Hansen, Heaton and Li (2008) derive the stochastic discount factor in terms of the continuation value of utility of consumption, as follows:

$$M_{t,t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{\frac{1}{\psi}-\gamma}. \tag{2.4}$$

If $\gamma = 1/\psi$, equation (2.4) corresponds to the stochastic discount factor of an investor with time-separable utility with constant relative risk aversion, where the powered consumption growth values short-run risk as usually understood. When $1/\psi < \gamma$, the ratio of future utility V_{t+1} to the certainty equivalent of this future utility $R_t\left(V_{t+1}\right)$ will add a premium for long-run risk. If consumption growth is persistent, a shock will cause a variation in

[9]Notice that the certainty equivalent, besides being decreasing in γ, is also increasing in α and decreasing in κ (for $\kappa \leq 1$). Thus α and κ are also measures of risk aversion, but of a different type than γ.

$V_{t+1}/R_t(V_{t+1})$, which will have an important impact on the SDF whenever the γ exceeds substantially $\frac{1}{\psi}$.

For GDA preferences, long-run risk enters in an additional term capturing disappointment aversion[10], as follows:

$$M_{t,t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right)^{\frac{1}{\psi}-\gamma} \left(\frac{1+(\alpha^{-1}-1)\, I\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} < \kappa\right)}{1+\kappa^{1-\gamma}(\alpha^{-1}-1)\, E_t\left[I\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} < \kappa\right)\right]}\right), \tag{2.5}$$

where $I(.)$ is an indicator function that takes the value 1 if the condition is met and 0 otherwise.

Generalized disappointment aversion kicks in whenever the ratio of future utility to its certainty equivalent is less than the threshold κ. For disappointment aversion, this threshold is one.

A persistent increase in the volatility of consumption will make future utility more volatile, enhancing the volatility of the third term in (2.5) because of a higher probability of disappointing outcomes. Therefore, the impact on the SDF volatility of a more volatile future utility will be more substantial for GDA than for Kreps-Porteus preferences. A more persistent consumption growth can also increase the volatility of future utility and of the GDA stochastic discount factor, but the effect is indirect. As we will see, this effect will be much smaller in magnitude.

Persistent volatility, as in the long-run volatility risk model we propose, will result in persistent and predictable conditional expected returns. As argued by Fama and French (1988), such a process for expected returns generates mean reversion in asset prices. Therefore, the price-dividend ratio today should be a good predictor of returns over several future periods.

Notice that the new multiplicative term that appears in the SDF when there is disappointment aversion does not depend on the relation between the IES and γ. For this reason, long-run volatility risk may make the SDF volatile even when $1/\psi$ is greater than γ. As a consequence, it is possible to generate realistic asset pricing outcomes even when the IES is smaller than one, as we show in our sensitivity analysis. Whenever the difference between γ and $1/\psi$ is small, the persistence of consumption growth will have little impact on our GDA SDF, as the effect on the second term of (2.5) becomes of small magnitude.

[10]Although Routledge and Zin (2009) do not model long-run risk, they discuss how its presence could interact with GDA preferences in determining the marginal rate of substitution.

2.2 A Long-Run Volatility Risk Benchmark Model for Consumption and Dividends

In the long-run risk model of Bansal and Yaron (2004), the consumption and dividend growth processes are evolving dynamically as follows:

$$\begin{aligned} \Delta c_{t+1} &= x_t + \sigma_t \epsilon_{c,t+1} \\ \Delta d_{t+1} &= (1-\phi_d)\,\mu_x + \phi_d x_t + \nu_d \sigma_t \epsilon_{d,t+1} \\ x_{t+1} &= (1-\phi_x)\,\mu_x + \phi_x x_t + \nu_x \sigma_t \epsilon_{x,t+1} \\ \sigma^2_{t+1} &= (1-\phi_\sigma)\,\mu_\sigma + \phi_\sigma \sigma^2_t + \nu_\sigma \epsilon_{\sigma,t+1}, \end{aligned} \tag{2.6}$$

where c_t is the logarithm of real consumption and d_t is the logarithm of real dividends. In this characterization, x_t, the conditional expectation of consumption growth, is modeled as a slowly reverting AR(1) process (ϕ_x smaller but close to one). Notice that $\phi_d x_t$ also governs the conditional expectation of dividend growth, and ϕ_d is assumed to be greater than one - the leverage ratio on consumption growth. The volatility of consumption growth σ_t is also assumed to be a very persistent process (ϕ_σ smaller but close to one) with unconditional mean μ_σ. The innovations in the expected growth processes and in the volatility process are assumed to be independent.

In this LRR in mean and volatility model, two key mechanisms are at play to determine asset prices. The first one relates to expected growth: both consumption and dividend growth rates contain a small long-run component in the mean. Shocks today have a very persistent effect on expected consumption growth far in the future. The second channel reflects time-varying economic uncertainty, and is captured by fluctuating conditional volatility of consumption. As Bansal, Kiku and Yaron (2009) show clearly, the first channel is essential with Kreps-Porteus preferences to achieve an equity premium commensurate with historical data. By choosing a random walk benchmark model with LRR in volatility, we want to show that fluctuations in economic uncertainty are sufficient with generalized disappointment averse investors to generate a similar equity premium as well as most stylized facts looked at in the literature.

Campbell and Cochrane (1999) use a random walk model for consumption and a heteroscedastic slowly mean-reverting surplus that is dynamically driven by consumption growth innovations that feeds into habit persistent preferences. More recently, Calvet and Fisher (2007) have proposed a model where consumption growth is i.i.d. and where the log dividend follows a random walk with state-dependent drift and volatility. They also extend the model to allow consumption growth to exhibit regime shifts in drift and volatility.

The model that we propose differs from these previous specifications in the sense that both drifts of consumption and dividend growth are constant while volatilities are time-varying; likewise, we also depart from the LRR model of Bansal and Yaron (2004) by

allowing a correlation ρ between innovations in consumption growth and in dividend growth, as in Bansal, Kiku and Yaron (2007b):

$$\begin{aligned} \Delta c_{t+1} &= \mu_x + \sigma_t \varepsilon_{c,t+1} \\ \Delta d_{t+1} &= \mu_x + \nu_d \sigma_t \varepsilon_{d,t+1} \\ \sigma^2_{t+1} &= (1-\phi_\sigma)\mu_\sigma + \phi_\sigma \sigma^2_t + \nu_\sigma \varepsilon_{\sigma,t+1}. \end{aligned} \tag{2.7}$$

As we will see in the next section combining GDA preferences with models (2.7) or (2.6) for fundamentals necessitates a solution technique that departs from the usual approximate solutions based on log linearization.

3 Solving a Long-Run Risks Model with GDA Preferences

To solve the LRR model with Kreps-Porteus preferences, Bansal and Yaron (2004) use Campbell and Shiller (1988) approximations and obtain analytical expressions that are useful for understanding the main mechanisms at work, but when it comes to generate numerical results they appeal to numerical simulations of the original model. Following Kogan and Uppal (2002), Hansen, Heaton and Li (2008) proposed a second type of approximation around a unitary value for the elasticity of intertemporal substitution ψ.

Since the GDA utility is non-differentiable at the kink where disappointment sets in, one cannot rely on the same approximation techniques to obtain analytical solutions of the model. In this paper we propose a methodology that solves analytically the LRR in mean and volatility model with GDA preferences and a fortiori with Kreps-Porteus preferences, yielding formulas for the asset valuation ratios in equilibrium[11]. The key to this analytical solution is to use a Markov switching process for consumption and dividends that matches the LRR specifications. In addition, we report analytical formulas for the population moments of equity premia as well as for the coefficients and R^2 of predictability regressions that have been used to assess the ability of asset pricing models to reproduce stylized facts.

3.1 A Matching-Moment Procedure for the Long-Run Risk Model

We will describe the matching procedure for the general LRR in mean and volatility model in (2.6) since it will apply equally to the restricted version (2.7) that we set as our benchmark model. Let s_t be a Markov state process at time t. We postulate that the consumption and

[11] Based on these formulas, a previous version of this paper (SSRN working paper no. 1109080) compared the respective accuracy of the Campbell-Shiller and the Hansen-Heaton-Li approximations for several sets of parameter values of KP preferences.

dividend growth processes evolve dynamically as a function of s_t as follows[12]:

$$\begin{array}{rcl} \Delta c_{t+1} & = & \mu_c(s_t) + (\omega_c(s_t))^{1/2}\varepsilon_{c,t+1} \\ \Delta d_{t+1} & = & \mu_d(s_t) + (\omega_d(s_t))^{1/2}\varepsilon_{d,t+1}, \end{array} \tag{3.1}$$

where $\varepsilon_{c,t+1}$ and $\varepsilon_{d,t+1}$ follow a bivariate standard normal process with mean zero and correlation ρ.

The following are the main features of the (2.6) process to be matched:

1. The expected means of the consumption and dividend growth rates are a linear function of the same autoregressive process of order one denoted x_t;

2. The conditional variances of the consumption and dividend growth rates are a linear function of the same autoregressive process of order one denoted σ_t^2;

3. The variables x_{t+1} and σ_{t+1}^2 are independent conditionally to their past;

4. The innovations of the consumption and dividend growth rates are correlated.

In the Markov switching case, the first characteristic of the LRR model implies that one has to assume that the expected means of the consumption and dividend growth rates are a linear function of the same Markov chain with two states given that a two-state Markov chain is an AR(1) process. Likewise, the second feature implies that the conditional variances of the consumption and dividend growth rates are a linear function of the same two-state Markov chain. According to the third feature, the two Markov chains should be independent. Consequently, we shall assume that the Markov chain has 4 states, two states for the conditional mean and two states for the conditional variance and that the transition matrix P is restricted such as the conditional means and variances are independent. Finally, the last feature is captured by the correlation parameter ρ. By combining the two states - high and low - in mean and in volatility we obtain four states, $s_t \in \{\mu_L\sigma_L, \mu_L\sigma_H, \mu_H\sigma_L, \mu_H\sigma_H\}$. The states evolve according to a 4 by 4 transition probability matrix P.

The details of the matching procedure are given in a technical appendix to this paper[13]. We apply this matching procedure first to the restricted random walk version of the general LRR model defined in (2.7). Then, in Section 6, we apply it to the general LRR in mean and volatility model in (2.6).

While the matching procedure concerns unconditional moments of the consumption and dividend processes, we verify that the fit of the Markov switching model is also adequate in

[12]Bonomo and Garcia (1996) proposed and estimated specification (3.1) for the joint consumption-dividends process with a three-state Markov switching process to investigate if an equilibrium asset pricing model with different types of preferences could reproduce various features of the real and excess return series.

[13]This appendix can be downloaded from $http://gremaq.univ-tlse1.fr/perso/meddahi$.

finite samples. To assess the fit, we simulate 10,000 samples of the size of the original data for both the autoregressive consumption and dividend processes and the matched Markov switching process, and compute empirical quantiles of several moments of the consumption and dividend processes. For space consideration, the results are reported in the precited technical appendix. The percentile values are very close between the two processes.[14].

3.2 Solving for Asset-Valuation Ratios

Solving the model means finding explicit expressions for the price-consumption ratio $P_{c,t}/C_t$ (where $P_{c,t}$ is the price of the unobservable portfolio that pays off consumption), the price-dividend ratio $P_{d,t}/D_t$ (where $P_{d,t}$ is the price of an asset that pays off the aggregate dividend), and finally the price $P_{f,t}/1$ of a single-period risk-free bond that pays for sure one unit of consumption. To obtain these three valuation ratios, we need expressions for $\mathcal{R}_t\left(V_{t+1}\right)/C_t$, the ratio of the certainty equivalent of future lifetime utility to current consumption, and for V_t/C_t, the ratio of lifetime utility to current consumption.

The Markov property of the model is crucial for deriving analytical formulas for these expressions. In general, the Markov state s_t in (3.1) will arbitrarily have N possible values, say $s_t \in \{1, 2, .., N\}$, although four values as described in the previous section are sufficient to provide a good approximation of the Bansal and Yaron (2004) LRR model. Let $\zeta_t \in \mathbb{R}^N$ be the vector Markov chain equivalent to s_t and such that:

$$\zeta_t = \begin{cases} e_1 = (1, 0, 0, .., 0)^\top & \text{if } s_t = 1 \\ e_2 = (0, 1, 0, .., 0)^\top & \text{if } s_t = 2 \\ ... \\ e_N = (0, 0, .., 0, 1)^\top & \text{if } s_t = N, \end{cases}$$

where e_i is the $N \times 1$ column vector with zeroes everywhere except in the i^{th} position which has the value one, and $\top$ denotes the transpose operator for vectors and matrices.

We show in the appendix that the variables $\mathcal{R}_t\left(V_{t+1}\right)/C_t$, V_t/C_t, $P_{d,t}/D_t$, $P_{c,t}/C_t$ and $P_{f,t}/1$ are (nonlinear) functions of the state variable s_t. However, since the state variable s_t takes a finite number of values, any real non-linear function $g(\cdot)$ of s_t is a linear function of ζ_t, that is a vector in $\mathbb{R}^N$. This property will allow us to characterize analytically the price-payoff ratios while other data generating processes need either linear approximations or numerical methods to solve the model. The structure of the endowment process implies that there will be one such payoff-price ratio per regime and this will help in computing

[14]In fact, the mean and median volatilities for consumption and dividend growth produced by the Markov switching model are closer to the mean and median volatility values computed with the data than the original autoregressive processes of consumption and dividend growth.

closed-form analytical formulas. For these valuation ratios, we adopt the following notation:

$$\frac{\mathcal{R}_t\left(V_{t+1}\right)}{C_t} = \lambda_{1z}^{\top}\zeta_t, \quad \frac{V_t}{C_t} = \lambda_{1v}^{\top}\zeta_t, \quad \frac{P_{d,t}}{D_t} = \lambda_{1d}^{\top}\zeta_t, \quad \frac{P_{c,t}}{C_t} = \lambda_{1c}^{\top}\zeta_t, \text{ and } \frac{P_{f,t}}{1} = \lambda_{1f}^{\top}\zeta_t. \tag{3.2}$$

Solving the GDA model amounts to characterize the vectors λ_{1z}, λ_{1v}, λ_{1d}, λ_{1c} and λ_{1f} as functions of the parameters of the consumption and dividend growth dynamics and of the recursive utility function defined above. In Appendix B, we provide expressions for these ratios.

3.3 Analytical Formulas for Expected Returns, Variance of Returns and Predictability Regressions

Since the seminal paper of Mehra and Prescott (1985), reproducing the equity premium and the risk-free rate has become an acid test for all consumption-based asset pricing models. Follow-up papers added the volatilities of both excess returns and the risk-free rate, as well as predictability regressions where the predictor is most often the price-dividend ratio and the predicted variables are equity returns or excess returns or consumption and dividend growth rates.

Bansal and Yaron (2004) use a number of these stylized facts to assess the adequacy of their LRR model and Beeler and Campbell (2009) provide a thorough critical analysis of the Bansal and Yaron (2004) model for a comprehensive set of stylized facts. The methodology used in Beeler and Campbell (2009) to produce population moments from the model rests on solving a loglinear approximate solution to the model and on a single simulation run over 1.2 million months (100,000 years). This simulation has to be run for each configuration of preference parameters considered. Typically, as in most empirical assessments of consumption-based asset pricing models, they consider a limited set of values for preference parameters and fix the parameters of the LRR in mean and volatility model at the values chosen by Bansal and Yaron (2004) or Bansal, Kiku and Yaron (2007b). Therefore, it appears very useful to provide analytical formulas for statistics used to characterize stylized facts in the literature.

Given expressions for the asset valuation ratios, it is easy to develop formulas for expected (excess) returns and unconditional moments of (excess) returns, formulas for predictability of (excess) returns and consumption and dividend growth rates by the dividend-price ratio, and formulas for variance ratios of (excess) returns. These analytical formulas, given in the appendix, will allow us to assess the sensitivity of the results to wide ranges of the parameters of the LRR model and to several sets of preference parameter values.

4 The Benchmark Model of Random Walk Consumption and Dividends and GDA Preferences: Calibration and Asset Pricing Implications

In this section we explain in detail how we choose the parameters for the fundamentals and for the preferences. Then, based on these calibrated values, we look at the asset pricing implications in terms of matching moments and predictability. We conclude the section by interpreting the results through an SDF analysis.

4.1 Choosing Parameters for Consumption and Dividends Risks

To calibrate this process at the monthly frequency, we start with the parameters of the long-run risk model (2.6) chosen by Bansal, Kiku and Yaron (2007b), that is $\mu_x = 0.0015$, $\phi_d = 2.5$, $\nu_d = 6.5$, $\phi_x = 0.975$, $\nu_x = 0.038$, $\sqrt{\mu_\sigma} = 0.0072$, $\nu_\sigma = 0.28 \times 10^{-5}$ and $\rho = 0.39985$, except that we set ϕ_σ at a less persistent value of 0.995 instead of 0.999[15]. The later value implies a half-life of close to 58 years. The value 0.995 corresponds to the value estimated by Lettau, Ludvigson and Wachter (2008)[16]. It implies a more reasonable half-life of 11.5 years.

From this long-run risk model, we set $\phi_x = 0$ and $\nu_x = 0$ to obtain the random walk model and we adjust the other parameters when necessary such that consumption and dividend growth means, variances and covariance remain unchanged from the original model. The random walk model is thus calibrated with $\mu_x = 0.0015$, $\nu_d = 6.42322$, $\sqrt{\mu_\sigma} = 0.0073$, $\phi_\sigma = 0.995$, $\nu_\sigma = 0.28 \times 10^{-5}$ and $\rho = 0.40434$.

We then apply the matching procedure described in Section 2.2 to recover the parameters of the corresponding Markov switching process with two states in volatility. The calibrated Markov switching random walk parameters are reported in Panel A of Table 1. The unconditional probability of being in the low volatility state is close to 80%. The volatilities of consumption and dividend are roughly multiplied by three in the high volatility state compared with the low volatility state.

For comparison purposes, we also matched the LRR in mean and volatility model calibrated in Bansal, Kiku, and Yaron (2007b), except for the persistence of volatility. The calibrated Markov switching LRR parameters are reported in Panel B of Table 1. We have now four states, two for the means and two for the volatilities, as explained in Section 3.1. We observe that introducing two mean states does not alter much the values of parameters associated with the volatility states in the random walk specification. This LRR extended

[15]The calibration with $\phi_\sigma = 0.999$ is currently the reference model in the long-run risk literature; two recent papers by Beeler and Campbell (2009) and Bansal, Kiku and Yaron (2009) use it. We will look at its implications with GDA preferences in the robustness section.

[16]They estimate a two-state Markov switching process for quarterly consumption growth and found transition probabilities of 0.991 and 0.994 for the high and low states respectively. The equivalent persistence parameter is 0.991+0.994-1=0.9850 for quarterly frequency, or 0.995 for monthly frequency.

set of Markov switching parameters will be used in Section 6 to compare the model of Bansal and Yaron (2004) with Kreps-Porteus preferences to a model with the same endowment process and with GDA preferences.

4.2 Choosing parameters for GDA Preferences

We need to choose values for the five preference parameters δ, ψ, γ, α and κ. For the time preference parameter δ we follow Bansal, Kiku and Yaron (2007b) and use 0.9989 for a monthly frequency, which corresponds to 0.9869 at an annual frequency or a marginal rate of time preference of 1.32%. Observe that Lettau, Ludvigson and Wachter (2008) and Routledge and Zin (2009) use a value of 0.970 or a marginal rate of 3%.

The value of the elasticity of intertemporal substitution is a source of debate. In the literature on long-run risk, Bansal and Yaron (2004) and Lettau, Ludvigson and Wachter (2008) adopt a value of 1.5. In their models, ψ must be greater than 1 for a decline in volatility to raise asset prices. Empirically, some researchers have found that the elasticity of intertemporal substitution is relatively small and often statistically not different from zero; see among others Campbell and Mankiw (1989) and Campbell (2003). Others like Attanasio and Weber (1993) and Vissing-Jorgensen and Attanasio (2003) have found higher values of ψ using cohort or household level data. Bansal and Yaron (2004) also argue that in the presence of time-varying volatility, there is a severe downward bias in the point estimates of the elasticity of intertemporal substitution. Beeler and Campbell (2009) simulate the Bansal and Yaron (2004) model and report no bias if the riskfree interest rate is used as an instrument. In this benchmark model we follow the literature and keep a value of 1.5 for ψ. However, we will look at the sensitivity of results to values of ψ lower than one in Section 5.

The remaining parameters all act on effective risk aversion. The parameter γ representing risk aversion in Epstein-Zin utility function is set at 10 in Bansal and Yaron (2004) and at a very high value of 30 in Lettau, Ludvigson and Wachter (2008). Since the disappointment aversion parameters α and κ interact with γ to determine the level of effective risk aversion of investors, we certainly need to lower γ. To guide our choice for γ and α together we rely on Epstein and Zin (1991). In this paper, they estimate a disappointment aversion model (κ=1) by GMM with two measures of consumption. The values estimated for γ and α are 1.98 and 0.38 for nondurables consumption, and 7.47 and 0.29 for nondurables and services. With these estimated parameters they cannot reject the disappointment aversion model according to the Hansen J-statistic of over-identifying restrictions at conventional levels of confidence. We choose an intermediate set of parameters, that is $\gamma = 2.5$ and $\alpha = 0.3$. Finally, we have to choose the parameter κ that sets the disappointment cutoff.

In our random walk process with LRR in volatility, we have a consumption volatility risk that triggers a precautionary savings motive. Moving κ below one reduces this motive and drives the equilibrium interest rate upwards. We finally choose $\kappa = 0.989$ for matching the stylized facts [17].

Another way to assess the level of risk aversion implied by these parameter values is to draw indifference curves for the same gamble for an expected utility model and a disappointment aversion model. Figure 1 plots indifference curves for a hypothetical gamble with two equiprobable outcomes, where we compare GDA preferences calibrated as described above to expected utility preferences with two values (5 and 10) for the coefficient of relative risk aversion. While GDA preferences exhibit higher risk aversion than both expected utility cases for small gambles, the same is not true for larger gambles. When the size of the gamble is about 20%, the GDA indifference curve crosses the expected utility indifference curve with risk aversion equal to 10, becoming less risk averse for larger gambles. For higher gamble sizes it approaches the expected utility with relative risk aversion equal to 5.

4.3 Asset Pricing Implications

We look at a set of moments for returns and price-dividend ratios, namely the expected value and the standard deviation of the equity premium, the risk-free rate and the price-dividend ratio. The moments are population moments and are computed with the analytical formulas discussed in Section 3.3 and reported in the appendix.

We also report the median of the finite-sample distribution and the p-value of the statistics computed with the data with respect to the finite-sample distribution. To generate the distribution in finite sample, we choose a sample size of 938 months as in the data sample we used to reproduce the stylized facts. We then simulate the random walk model 10,000 times and report the percentile of the cross-sectional distribution of the model's finite-sample statistics that corresponds to the value of this statistic in the data. This percentile can be interpreted as a p-value for a one-sided test of the model based on the data statistic.

We also consider several predictability regressions by the price-dividend ratio, for excess returns, consumption growth and dividend growth. We compute the R^2 and the regression coefficients analytically with the formulas reported in the previous section.

There is also an active debate about the predictability of returns by the dividend yield. Econometric and economic arguments fuel the controversy about the empirical estimates of R^2 in predictive regressions of returns or excess returns over several horizons on the current

[17] Routledge and Zin (2009) discuss the value of this parameter in connection with the autocorrelation of consumption growth in a simple two-state Markov chain. In order to generate counter-cyclical risk aversion, they state that a value less than one for κ is needed when there is a negative autocorrelation of consumption growth and a value greater than one when the autocorrelation is positive. The economic mechanism behind this link is the substitution effect.

dividend yield. Some claim that the apparent predictability is a feature of biases inherent to such regressions with persistent regressors, others that it is not spurious since if returns were not predictable, dividend growth should, by accounting necessity, be predictable, which is not the case in the data[18]. Therefore, providing evidence that a consumption-based asset pricing model is able to reproduce these predictability patterns based on data certainly clarifies the debate.

We compare these model-produced statistics to the corresponding empirical quantities computed with a dataset of quarterly consumption, dividends and returns for the US economy. We use a sample starting in 1930 as in Bansal, Kiku and Yaron (2007a, 2009) and Beeler and Campbell (2009) and extend it until 2007. The empirical first and second moments of asset prices and the empirical predictability results are reported first in the second column of Table 2 and then repeated for convenience of comparison in all relevant tables. The reported statistics are annualized moments based on quarterly data estimation. The computed values are close to the usual values found for these statistics with an equity premium mean of 7% and a volatility of roughly 20%. The real interest rate is close to 1% and its volatility is in order of 4%. Finally, the mean of the price-dividend ratio is close to 30 and the volatility of the dividend yield is about 1.5%.

4.3.1 Matching the Moments

The asset pricing results for the benchmark RW process are reported in Panel A of Table 2. We consider a set of moments, namely the expected value and the standard deviation of the equity premium, the real risk-free rate and the price-dividend ratio.

The population values produced by the benchmark model with the random walk model described in Section 4.1 and the preference parameters set in Section 4.2 are reported in the second column of Table 2. Except for the volatility of the real interest rate, which is about half the value computed in the data, and the somewhat low level of the expected price-dividend ratio relative to the data[19], all other population moments are very much in line with the data statistics. Given the random walk process with LRR in volatility for consumption in the benchmark model, it means that for an investor with GDA preferences, it is the macroeconomic uncertainty that solely explains the high equity premium and a low risk-free rate. In the high volatility state, which happens about 20% of the time in the benchmark case, the required premium is much higher than in the low volatility state. It is also the variation of the price-dividend ratio over the two states of volatility that gives

[18]See in particular Valkanov (2003), Stambaugh (1999), Cochrane (2008) and the 2008 special issue of the Review of Financial Studies about the topic of predictability of returns.

[19]However, the level we obtain with our calibration is not out of line with values found in the sample until the year 2000 where it reached a peak of close to 90 and stayed relatively high afterwards. The more robust median estimate of 24.95 is closer to our population mean of 23.30.

enough variability to the dividend yield to match what is observed in the data.

In finite samples, the model is rejected for the standard deviation of the risk-free rate, which is much too low compared to the data. As we will see in the robustness section, it is due in part to the higher-than-one value of the elasticity of intertemporal substitution. A value greater than one implies that the investor perceives consumption at two different times as substitutes and does not want to borrow from the future to smooth out volatile consumption. This results in a low and less volatile riskless interest rate. The p-value for the expected price-dividend ratio confirms that the value of 30.57 is out of the realm of the finite-sample distribution produced by the model. The median of the finite-sample distribution is indeed close to the population moment and to the median of the price-dividend ratio in the data. For the four other moments, the p-values indicate that the empirical values are quite close to the center of the finite-sample distribution.

4.3.2 Predictability

The predictability results for the benchmark RW process are reported in Panels B, C and D of Table 2. In panel B, we reproduce the predictability of excess returns[20]. For both the R^2 statistics and the slopes of the regression of excess returns on the dividend-price ratio, we reproduce the increasing pattern over horizons of one to five years. In terms of magnitude we are a bit over the data values. However, our small-sample R^2 medians are very close to the data with p-values of about 50%.

We included the predictability of consumption growth and dividend growth even though in the random walk model there is no predictability in population. Data shows little predictability and the p-values of the finite-sample statistics confirm that the model is not rejected.

Observe that both Beeler and Campbell (2009) and Bansal, Kiku and Yaron (2009) report evidence on the relation between asset prices and volatility of returns in the data and in the LRR model with Kreps-Porteus preferences. We derived analytical formulas for the coefficients and R^2 of similar regressions of these volatility measures on the dividend price ratio. Predictability of return volatility by the dividend-price ratio is weak in the data. The GDA model produces R^2s that match quite closely the data, with a maximum R^2 of 15% at a 5-year horizon[21].

[20]Stylized facts show a strong predictability of (excess) returns by the dividend-price ratio, which increases with the horizon. Although a vast literature discusses whether this predictability is actually present or not because of several statistical issues, we will sidestep the various corrections suggested since we are looking for a model that rationalizes the estimated stylized facts.

[21]For space considerations, we do not report detailed results on predictability of return volatility. They are available upon request from the authors.

4.4 Understanding the results through the SDF

To better understand why the generalized disappointment benchmark model explains well the stylized facts, we have a closer look at the underlying stochastic discount factor. As we showed before in the description of the endowment matching, the Markov switching endowment process we are using has two states in volatility σ_L and σ_H. Panel A of Table 1 reports the transition probability matrix between the states. Both variance states are very persistent, with the transitions from the high state to the low state occurring more frequently than the reverse.

Table 3 reports the moments of the SDF of the GDA benchmark specification in the two states. These are the mean and the variance of the state-conditional distributions of the SDF. The state with low variance has a higher probability mass associated with a non-disappointing outcome. Therefore, the mean SDF in this state is low (0.99820) and not very variable (0.11481), resulting in a low risk premium. The state with high variance is the one with a more variable SDF (0.66929), a higher SDF mean (1.00304) and a corresponding higher risk premium. The switching between the low and high persistent variance states produces slow-moving state-dependent risk aversion, which is essential for predictability.

Another way to understand our results is to see how they change when we vary either the preferences or the stochastic processes of the fundamentals. This is the object of the next section.

5 Sensitivity Analysis

We start by looking at a set of specific preferences in the family of disappointment aversion preferences and at the Kreps-Porteus preferences used in Bansal and Yaron (2004) and other ensuing papers. We then measure the sensitivity of results to the persistence in consumption volatility, a key parameter of the benchmark model. In this sensitivity analysis, we report results obtained with specific values of the parameters but we also illustrate with graphs the sensitivity of results to variations in the parameters.

5.1 Sensitivity to Preference Specifications

In this section we show the implications of different calibrations for the preference parameters. First, we reproduce tables similar to Table 2 for three specific configurations of interest, namely a similar GDA than the benchmark case but with an elasticity of intertemporal substitution lower than 1 (that we will call GDA1), another with $\kappa = 1$, a pure disappointment aversion model, with linear preferences ($\gamma = 0$) and infinite elasticity of intertemporal substitution ($\psi = \infty$) (called DA0), which will isolate the role of disappointment aversion

alone, and finally the Kreps-Porteus preferences ($\alpha = 1$), which have been associated with the long-run risk model. Second, we produce graphs showing the sensitivity of asset pricing and predictability implications to continuous variations of preference parameters over large sets of values.

5.1.1 Specific Configurations of Preferences

Table 4 reports the population and finite sample p-values for moments and predictability associated with the three specifications GDA1, DA0, and Kreps-Porteus (KP).

(*a*) Elasticity of intertemporal substitution lower than one - GDA1 As already mentioned, the value of the elasticity of intertemporal substitution ψ is a matter of debate. Bansal and Yaron (2004) argue for a value larger than 1 for this parameter since it is critical for reproducing the asset pricing stylized facts.

Given this debate over the value of the elasticity of substitution ψ, we set it at 0.75. We maintain for the other parameters the same values as in the benchmark model. It can be seen in the second column of Table 4 that the random walk model with this GDA1 configuration of preferences can reproduce almost as well the asset pricing stylized facts. Therefore, we see that the elasticity of intertemporal substitution is not pivotal for the results. It does affect however the level and volatility of the riskless interest rate. Since the investor perceives consumption at two different dates as complementary, he wants to borrow from the future to smooth out volatile consumption. This implies a higher (1.97% instead of 0.93% with GDA) and a more volatile (3.25% instead of 2.34%) interest rate. The higher interest-rate mean is reflected in Table 3 by the fact that the mean of the SDF spread in the most frequent low-volatility state is smaller for GDA1 than for GDA. A wider spread between the conditional means of the SDFs for GDA1 than for GDA explains the higher volatility of the interest rate.

One dimension over which GDA1 performs less well than GDA is the volatility of the dividend-price ratio, which falls to 1.04 from 1.38. This translates into higher coefficients in the return predictability regressions but the patterns and the finite sample values are very similar to the ones obtained with GDA. The finite sample results for consumption and dividend growth predictability are the same as with GDA.

Generalized disappointment aversion preferences shed a new light on the debate about the value of the intertemporal elasticity of substitution in long-run risks models. As argued in section 2.1, the main mechanism at play with GDA preferences does not depend on the value of ψ. The need for an elasticity higher than one to match asset pricing moments is a specific feature of the Kreps-Porteus preferences.

(*b*) Pure Disappointment Aversion - DA0 The specification denoted DA0 is the simplest one among disappointment averse preferences. First, as $\kappa = 1$, the threshold is the certainty equivalent. Furthermore, other than the kink, the stochastic discount factor has no curvature, as $\gamma = 0$ and $\psi = \infty$. In other words, if disappointment aversion were not present ($\alpha = 1$), the stochastic discount factor would be equal to the constant time discount factor δ. This simplistic specification of the GDA preferences will allow us to gain intuition about the potential for such a pure disappointment aversion model, that does not use the curvature engendered by the other preference parameters, to replicate the asset pricing and predictability stylized facts we analyzed with GDA.

The results reported in Table 4 show that DA0 reproduces rather well predictability of returns but not so much moments. With respect to GDA1, the average price-dividend ratio is too low and the volatility of the dividend-price ratio too high. The equity premium is also higher than in the data. These deteriorating statistics are brought about by an enlarged set of disappointing outcomes when κ is increased from 0.989 to one. The other drawback of such simplistic preferences is a constant risk-free rate. Indeed, with $\kappa = 1$, the conditional expectation of the SDF is equal to δ, the time-discount parameter. However, for predictability of excess returns, the patterns obtained for population statistics are maintained in finite sample and the p-values associated with the R^2 and the coefficients of the return predictability regressions are close to the median.

Routledge and Zin (2009) stress the importance of generalized disappointment aversion for obtaining counter-cyclical price of risk in a Mehra-Prescott economy. In their setting, disappointment aversion alone cannot generate enough variation in the distribution of the stochastic discount factor, leading to a similar conditional equity premium in both states. Since they have two possible outcomes, one is necessarily above the certainty equivalent and the other is below. Then, for each state there is always one disappointing outcome. With generalized disappointment aversion it is possible to carefully calibrate the kink at a fraction of the certainty equivalent such that for one of the states both results are non-disappointing. Then, there is disappointment only in the bad state, engendering a counter-cyclical equity premium.

Since we have a richer endowment process, with an infinite number of possible outcomes, there is not such a stark contrast between DA and GDA preferences in our model. For each state there will always be a very large number of disappointing outcomes for both types of preferences. The probability of disappointment may change with the state even with DA preferences, generating predictable time-variation in returns. When DA is combined with $\gamma = 0$ and $\psi = \infty$, the risk-free rate becomes constant and equal to $r_f = \ln R_f = -\ln \delta$ as mentioned above. This does not imply a constant risk premium, since the conditional

covariance between the SDF and the equity return is state-dependent.

(c) Kreps-Porteus Preferences The Kreps-Porteus preferences are a key ingredient in the long-run risks model of Bansal and Yaron (2004). Recall that in the latter a small persistent component adds risk in expected consumption growth. Here we evaluate whether volatility risk alone is enough to replicate the stylized facts. We use the preference parameter values used in Bansal and Yaron (2004). It is clear that volatility risk alone is not sufficient to generate statistics in line with the data. The equity premium is very small, 1.42% compared to 7.25% in the data, the expected price-dividend ratio is much too high and the volatility of the dividend price ratio is practically zero. The last two facts translate into very high and negative slope coefficients and low R^2 in the predictability regressions of excess returns. Beeler and Campbell (2009) argue that high persistence in volatility is essential to reproduce the results. Bansal, Kiku and Yaron (2009) show that persistence in expected consumption growth is necessary for the volatility risk to play a role. Here, we see clearly that Kreps-Porteus preferences with a heteroskedastic random walk consumption is not enough to reproduce the moments and explain predictability.

5.1.2 Sensitivity to Preference Parameter Values

We gauge the sensitivity of the statistics to changes in preference parameters through graphs. In Figure 2, we keep the value of the risk aversion parameter γ to 2.5 and vary the disappointment aversion parameter α, the elasticity of intertemporal substitution ψ and the kink parameter κ. We choose three values for α (0.3, 0.35 and 0.40), three values for ψ (0.75, 1 and 1.5), while we vary continuously κ between 0.980 and 0.990. We produced three horizontal panels for expected excess returns, the risk-free rate and the price-dividend ratio respectively.

The equity premium increases with κ and decreases with α. Increasing α makes the agent less averse to disappointment and therefore prices will be higher and risky returns lower. The parameter κ acts in the opposite direction. When it gets closer to 1, there are more outcomes that makes the investor disappointed. As the elasticity of intertemporal substitution increases, it produces a rather small increase in the level of the equity premium.

The risk-free rate goes down as aversion to disappointment and the set of disappointing outcomes increase, that is when α decreases and κ increases. The effect of κ is much more pronounced since the curves fan out as we lower κ, especially for $\psi = 1.5$. The effect of ψ on the risk-free rate is important since it affects directly intertemporal trade-offs in terms of consumption. Below the value of 1 the investor sees consumption at two different times as complementary, resulting in a higher level of the risk-free rate, while with a value above 1

consumption today and tomorrow are perceived as substitutes and the equilibrium risk-free rate is lower.

Finally, the expected price-dividend ratio decreases with disappointment aversion, with the main factor being κ, since the curves bunch up as κ gets closer to 1. Decreasing ψ lowers the level of the expected price-dividend ratio and makes it less sensitive to changes in α.

In Figure 3, we apply a similar sensitivity analysis, with identical changes in the parameters, to the predictability of excess returns at one, three and five-year horizons. The main conclusion is that predictability increases when both the intensity of disappointment aversion and the set of disappointing outcomes increase (lower α and higher κ). Changing ψ does not affect much predictability since both the levels and the slopes are identical across graphs. These features apply to all horizons, but effects are amplified as the horizon lengthens.

5.2 Sensitivity to Persistence in Consumption Volatility

A key parameter in our benchmark model is the persistence of consumption volatility. Bansal, Kiku and Yaron (2007b) chose an extreme value of 0.999 while we reduced it to 0.995 based on a more reasonable value for the half-life of a shock to volatility. In Figures 4 and 5, we plot the sensitivity of the asset pricing statistics and predictability statistics, respectively, to variations in the persistence parameter of consumption volatility ϕ_σ for all preference specifications (GDA, GDA1, DA0, and KP). In Figure 4 we observe that all asset pricing statistics for Kreps-Porteus preferences, while out of line with the data, remain roughly insensitive to variations of ϕ_σ from 0.9 to 1. This is not surprising since Bansal, Kiku and Yaron (2009) showed that the sensitivity to the persistence in consumption volatility depends on the expected consumption growth persistence. For GDA, the patterns are similar across the three specifications. The biggest changes occur in the volatility of the dividend yield that goes towards zero as we approach 0.9. Otherwise, the other statistics remain pretty much the same as we vary ϕ_σ from 0.9 to 1. In Figure 5, the patterns in R^2 for all preference specifications are similar. Their values decrease steeply as ϕ_σ approaches 0.9. As we mentioned before, Kreps-Porteus preferences show some predictability but the values of the slopes become unrealistically large in magnitude and negative so they do not appear in the graphs. One can see that the magnitude of predictability for GDA specifications depends very much on the value of ϕ_σ, but that some predictability remains for a sizable range of values. It should be stressed that the curves for GDA and GDA1 are very similar both in terms of asset pricing moments and predictability statistics, except for the volatility of the risk-free rate, which is higher for GDA1 as mentioned before. What the graph tells us in this case is that the difference remains uniform across the values of ϕ_σ between 0.9

and 1.

6 Comparison with the Long-Run Risks Model of Bansal-Yaron (2004): Risks in Both Expected Consumption Growth and Consumption Volatility

The long-run risks model introduced by Bansal and Yaron (2004) features two main sources of risk, a risk in expected consumption growth and a risk in volatility of consumption. We saw that our benchmark model, featuring only the second risk, could explain the stylized facts when combined with GDA preferences but not with the Kreps-Porteus preferences chosen by Bansal and Yaron (2004). An important question is to establish whether the results obtained with the random walk consumption (with LRR in volatility) and GDA preferences are affected by the introduction of a long-run risk in expected consumption growth. For comparison purposes, we will also study the asset pricing implications in population of the Bansal and Yaron (2004) model with Kreps-Porteus preferences. In the LRR in mean and volatility model, the persistence of expected consumption growth is the key parameter. Therefore, we will assess the sensitivity of results to variations in this parameter.

For calibration, we keep the parameter values chosen in Bansal, Kiku and Yaron (2007b) and used also by Beeler and Campbell (2009): $\mu_x = 0.0015$, $\phi_d = 2.5$, $\nu_d = 6.5$, $\phi_x = 0.975$, $\nu_x = 0.038$, $\phi_\sigma = 0.995$, $\sqrt{\mu_\sigma} = 0.0072$, $\nu_\sigma = 0.28 \times 10^{-5}$ and $\rho = 0.39985$. The main difference with respect to our benchmark random walk process is the presence of a persistent component in the mean of the consumption and dividend growth processes. Note again that in this calibration the volatility persistence parameter is lower (0.995) than in the LRR calibration of Bansal, Kiku and Yaron (2007b)(0.999). We apply to this calibrated set of parameters the matching procedure described in Section 3.1 to obtain the equivalent set of parameters for the Markov switching model in (3.1). The Markov switching matching parameters are reported in Panel B of Table 1. We have two states for the means (μ_L and μ_H) and two states for the volatility (σ_L and σ_H), that combine into four states: $\{\mu_L\sigma_L, \mu_L\sigma_H, \mu_H\sigma_L, \mu_H\sigma_H\}$. In the low state both consumption and dividend growth means are negative, while they are positive and between 2.5 % and 3 % annually in the high state. The estimated volatilities are close to what we obtained in the random walk model. Overall, we are in the high mean-low variance 70% of the time and 19% of the time in the high mean-high variance state. The low mean state occurs about 10% of the time, mostly with the low volatility state.

6.1 Asset Pricing Implications

In Table 5, we report moments and predictability statistics for the benchmark GDA model and the three specifications GDA1, DA0 and Kreps-Porteus analyzed with the benchmark random walk model with LRR in volatility. Two main conclusions can be drawn. First, all the statistics reproduced for the GDA or DA preferences are very close to what we obtained with the random walk model. This confirms that volatility risk is the main economic mechanism behind the asset pricing results. Adding a risk in the expected consumption growth does not affect much the GDA investor, given our choice for γ and ψ. Recall that Bansal and Yaron (2004) rely on the the second term in the SDF equation (2.5) to generate their results. As $1/\psi - \gamma$ is not very large in magnitude in the disappointment aversion preference configurations, expected consumption growth risk does not have an important effect.

Second, the results are changing for Kreps-Porteus preferences in several dimensions. Since $1/\psi - \gamma$ is negative and large in magnitude for the BY preference configuration, the moments are now closer to the data, except still for the volatility of the riskless interest rate and of the dividend-price ratio. This confirms the essential role played by the small long-run predictable component in expected consumption growth in the Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007) models. For excess return predictability we arrive at a surprising result. While the random walk model with LRR in volatility generated some predictability in population, the LRR in mean and volatility model does not produce any predictability at all. Also, in finite sample, we can reject the model in this dimension at a 10% level of confidence.

For consumption growth, the LRR in mean and volatility model with Kreps-Porteus preferences overpredicts strongly in population, with R^2 in the order of 20%, but the distribution in finite sample is such that we cannot reject the model at the 5% level. The p-values for the R^2 are 0.07, 0.08 and 0.14 respectively at the one, three and five-year horizon. It should be stressed that the GDA and DA models give statistics and p-values that do not differ too much from the Kreps-Porteus model. It is therefore hard to differentiate between the models in finite sample. In population, the difference is clear and the Kreps-Porteus model produces too much predictability in consumption growth.

For dividend growth, the LRR in mean and volatility model with Kreps-Porteus preferences overpredicts a bit compared to the three disappointment specifications but again it is hard to distinguish between the models based on finite-sample p-values.

6.2 Sensitivity to Persistence of Expected Consumption Growth

We illustrate through graphs the sensitivity of the asset pricing and predictability statistics to large variations in the persistence of expected consumption growth (ϕ_x) in Figures 6 and 7 respectively. We start with the robustness of asset pricing moments in Figure 6. We exhibit 6 graphs, one for each moment. All the curves associated with GDA are almost parallel straight lines to the horizontal axis showing that the computed moments are insensitive to the expected growth persistence parameter. For DA0, the patterns are a bit different for values of ϕ_x close to 1 but settle to straight lines as we reduce ϕ_x. For Kreps-Porteus preferences, as already mentioned, the parameter ϕ_x is key. All results obtain for values close to 1, emphasizing the essential role of a very persistent component in expected consumption growth. The pattern of the expected price dividend ratio for Kreps-Porteus preferences is particularly striking, increasing steeply from a low value of 20 for the benchmark Bansal and Yaron (2004) value of 0.975 to values greater than 100 as we just move away from it.

In Figure 7 we explore the implications for predictability of variations in ϕ_x. We show two sets of six graphs, that is three horizons and two statistics (R^2 and slope) for the prediction of excess returns and consumption growth. In each graph, we plot the three specifications of disappointment averse preferences and Kreps-Porteus preferences. All three disappointment-averse specifications exhibit predictability patterns of excess returns consistent with what is observed in the data, while it is not the case for Kreps-Porteus preferences. Predictability stays close to zero over the whole set of values of ϕ_x for Kreps-Porteus model, increasing a bit when the value of the persistence parameter decreases but we know that the moments are no longer matched for these values. For consumption and dividend growth, the benchmark ϕ_x produces too much predictability when it gets close to 1. Otherwise it is flat at zero. Here again, we cannot reproduce the low predictability of consumption and dividend growth and the moments at the same time.

We can conclude from this sensitivity analysis that the source of long-run risk, whether in the mean or the volatility of consumption growth, needs to be persistent for the agent's preferences to operate in a way consistent with the observed data. For Kreps-Porteus preferences in the Bansal and Yaron (2004) model, we see a strong tension as ϕ_x, the persistence of expected consumption growth, moves away from 1. The ability to reproduce asset pricing moments deteriorates quickly while the predictability statistics improve. For the GDA preferences that we advocate in this paper, the persistence in the volatility of consumption growth ϕ_σ is key for reproducing the predictability stylized facts but results are not as sensitive to this persistence as they are with Kreps-Porteus preferences for the persistence of expected consumption growth. The means of the equity and risk-free returns are pretty insensitive to ϕ_σ, while their volatilities decrease but not drastically as ϕ_σ moves

away from one. It is really for the volatility of the dividend-price ratio that the persistence of volatility is very important, since it is decreasing fast as the value of ϕ_σ is approaching 0.9.

7 Conclusion

We have examined an asset pricing model with long-run risk where preferences display generalized disappointment aversion (Routledge and Zin, 2009). Our benchmark endowment process had only one of the two sources of long-run risks proposed by Bansal and Yaron (2004): the volatility risk. The persistent volatility of consumption growth strongly interacts with disappointment aversion to generate moments and predictability patterns in line with the data. Differently from the Bansal and Yaron (2004) model, our results do not depend on a value of the elasticity of intertemporal substitution greater than one: similar results may be obtained with values lower than one.

Disappointment aversion preferences introduce a kink in the utility function, raising a challenge to solve the asset pricing model. We propose a matching procedure that allows to solve analytically the model and to obtain closed-form formulas for asset-valuation ratios, asset-return moments and predictability regression coefficients and R^2, making it easy to assess the sensitivity of results to variations in the parameters of the model.

While we have focused in this paper on the time-series implications of our generalized disappointment aversion model with long-run volatility risk, it will be fruitful to investigate whether this model can rationalize the evidence put forward by Tédongap (2010) about consumption volatility and the cross-section of stock returns. He shows that growth stocks have a lower volatility risk than value stocks and that, for most investment horizons, consumption volatility risk is more correlated with multiperiod returns on the Fama-French size and book-to-market sorted portfolios than consumption level risk.

Appendix

Appendix A. In what follows, we will use the following notation. The transition probability matrix P of the Markov chain is given by

$$P^\top = [p_{ij}]_{1\leq i,j\leq N}, \quad p_{ij} = P(\zeta_{t+1} = e_j \mid \zeta_t = e_i). \tag{A.1}$$

We assume that the Markov chain is stationary with ergodic distribution and second moments given by:

$$E[\zeta_t] = \Pi \in \mathbb{R}^N_+, \quad E[\zeta_t \zeta_t^\top] = Diag(\Pi_1, .., \Pi_N) \text{ and } Var[\zeta_t] = Diag(\Pi_1, .., \Pi_N) - \Pi\Pi^\top,$$

where $Diag\,(u_1, .., u_N)$ is the $N \times N$ diagonal matrix whose diagonal elements are u_1,..,u_N. The time-varying variables $\mu_c(s_t)$, $\mu_d(s_t)$, $\omega_c(s_t)$, $\omega_d(s_t)$, and $\rho(s_t)$ defined in (3.1) are given by

$$\mu_c(s_t) = \mu_c^\top \zeta_t, \ \mu_d(s_t) = \mu_d^\top \zeta_t, \ \omega_c(s_t) = \omega_c^\top \zeta_t, \ \omega_d(s_t) = \omega_c^\top \zeta_t, \ \rho(s_t) = \rho^\top \zeta_t.$$

We define the vectors μ_{cd}, ω_{cd}, μ_{cc}, and ω_{cc} by

$$\mu_{cd} = -\gamma\mu_c + \mu_d, \ \omega_{cd} = \omega_c + \omega_d - 2\gamma\rho \odot \omega_c^{1/2} \odot \omega_d^{1/2}, \ \mu_{cc} = (1-\gamma)\,\mu_c, \omega_{cc} = (1-\gamma)^2\,\omega_c, \tag{A.2}$$

where the vector operator $\odot$ denotes the element-by-element multiplication. The vector ι denotes the $N \times 1$ vector with all components equal to one. Likewise, Id is the $N \times N$ identity matrix.

Appendix B. This Appendix provides the formulas of the vectors λ that appear in (3.2). These vectors are computed in two steps. In the first one, we characterize the ratio of the certainty equivalent of future lifetime utility to current consumption and the ratio of lifetime utility to consumption. In the second step, we characterize the price-consumption ratio, the equity price-dividend ratio and the single-period risk-free rate. These characterizations are done by solving the Euler equation for different assets. One has

$$\frac{\mathcal{R}_t\,(V_{t+1})}{C_t} = \lambda_{1z}^\top \zeta_t \text{ and } \frac{V_t}{C_t} = \lambda_{1v}^\top \zeta_t,$$

where the components of the vectors λ_{1z} and λ_{1v} are given by:

$$\lambda_{1z,i} = \exp\left(\mu_{c,i} + \frac{1-\gamma}{2}\omega_{c,i}\right)\left(\sum_{j=1}^N p_{ij}^* \lambda_{1v,j}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{B.1}$$

$$\lambda_{1v,i} = \left\{(1-\delta) + \delta\lambda_{1z,i}^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \text{ if } \psi \neq 1 \text{ and } \lambda_{1v,i} = \lambda_{1z,i}^{\delta} \text{ if } \psi = 1, \tag{B.2}$$

while the matrix $P^{*\top} = \left[p^*_{ij}\right]_{1\leq i,j\leq N}$ is defined by

$$p^*_{ij} = p_{ij}\frac{1+(\alpha^{-1}-1)\,\Phi\left(\dfrac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}-(1-\gamma)\,\omega_{c,i}^{1/2}\right)}{1+(\alpha^{-1}-1)\,\kappa^{1-\gamma}\sum_{j=1}^{N}p_{ij}\Phi\left(\dfrac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}\right)}. \tag{B.3}$$

The second step leads to

$$\frac{P_{d,t}}{D_t} = \lambda_{1d}^\top\zeta_t,\quad \frac{P_{c,t}}{C_t} = \lambda_{1c}^\top\zeta_t \;\text{ and }\; \frac{1}{R_{f,t+1}} = \lambda_{1f}^\top\zeta_t,$$

where the components of the vectors λ_{1d}, λ_{1c}, and λ_{1f} are given by

$$\lambda_{1d,i} = \delta\left(\frac{1}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma}\exp\left(\mu_{cd,i}+\frac{\omega_{cd,i}}{2}\right)\left(\lambda_{1v}^{\frac{1}{\psi}-\gamma}\right)^\top P^{**}\left(Id-\delta A^{**}\left(\mu_{cd}+\frac{\omega_{cd}}{2}\right)\right)^{-1}e_i, \tag{B.4}$$

$$\lambda_{1c,i} = \delta\left(\frac{1}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma}\exp\left(\mu_{cc,i}+\frac{\omega_{cc,i}}{2}\right)\left(\lambda_{1v}^{\frac{1}{\psi}-\gamma}\right)^\top P^{*}\left(Id-\delta A^{*}\left(\mu_{cc}+\frac{\omega_{cc}}{2}\right)\right)^{-1}e_i, \tag{B.5}$$

$$\lambda_{1f,i} = \delta\exp\left(-\gamma\mu_{c,i}+\frac{\gamma^2}{2}\omega_{c,i}\right)\sum_{j=1}^{N}\tilde{p}^*_{ij}\left(\frac{\lambda_{1v,j}}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma}, \tag{B.6}$$

where the matrices $P^{**\top} = \left[p^{**}_{ij}\right]_{1\leq i,j\leq N}$, $\tilde{P}^{*\top} = \left[\tilde{p}^*_{ij}\right]_{1\leq i,j\leq N}$ are given by

$$p^{**}_{ij} = p_{ij}\frac{1+(\alpha^{-1}-1)\,\Phi\left(\dfrac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}-\left(\rho_i\omega_{d,i}^{1/2}-\gamma\omega_{c,i}^{1/2}\right)\right)}{1+(\alpha^{-1}-1)\,\kappa^{1-\gamma}\sum_{j=1}^{N}p_{ij}\Phi\left(\dfrac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}\right)}, \tag{B.7}$$

$$\tilde{p}^*_{ij} = p_{ij}\frac{1+(\alpha^{-1}-1)\,\Phi\left(\dfrac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}+\gamma\omega_{c,i}^{1/2}\right)}{1+(\alpha^{-1}-1)\,\kappa^{1-\gamma}\sum_{j=1}^{N}p_{ij}\Phi\left(\dfrac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}\right)}, \tag{B.8}$$

while, for $u \in \mathbb{R}^N$, the matrix functions $A^*(u)$ and $A^{**}(u)$ are given by

$$A^*(u) = Diag\left(\left(\frac{\lambda_{1v,1}}{\lambda_{1z,1}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_1), ..., \left(\frac{\lambda_{1v,N}}{\lambda_{1z,N}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_N)\right)P^*, \tag{B.9}$$

(B.10)

$$A^{**}(u) = Diag\left(\left(\frac{\lambda_{1v,1}}{\lambda_{1z,1}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_1), ..., \left(\frac{\lambda_{1v,N}}{\lambda_{1z,N}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_N)\right)P^{**}. \tag{B.11}$$

Appendix C. This Appendix provides the formulas of the expected returns and some of their properties. We define the return process, R_{t+1}, and aggregate returns over h periods, $R_{t+1:t+h}$, by

$$R_{t+1} = \frac{P_{d,t+1}+D_{t+1}}{P_{d,t}} = \left(\lambda_{2d}^\top\zeta_t\right)\left(\lambda_{3d}^\top\zeta_{t+1}\right)\exp(\Delta d_{t+1}) \quad \text{and} \quad R_{t+1:t+h} = \sum_{j=1}^{h} R_{t+j}, \tag{C.1}$$

with $\lambda_{2d} = 1/\lambda_{1d}$ and $\lambda_{3d} = \lambda_{1d} + \iota$. We also define the excess returns R^e_{t+1} and aggregate excess returns $R^e_{t+1:t+h}$, i.e., $R^e_{t+1} = R_{t+1} - R_{f,t+1}$ and $R^e_{t+1:t+h} = R_{t+1:t+h} - R_{f,t+1:t+h}$. One has

$$E[R_{t+j} \mid J_t] = \psi_d^\top P^{j-1}\zeta_t \text{ and } E\left[R^e_{t+j} \mid J_t\right] = (\psi_d - \lambda_{2f})^\top P^{j-1}\zeta_t, \quad \forall j \geq 2, \tag{C.2}$$

$$E[R_{t+1:t+h} \mid J_t] = \psi_{h,d}^\top\zeta_t \quad \text{and} \quad E\left[R^e_{t+1:t+h} \mid J_t\right] = (\psi_{h,d} - \lambda_{h,2f})^\top\zeta_t, \tag{C.3}$$

where $\lambda_{2f} = 1/\lambda_{1f}$ and

$$\psi_{d,i} = \lambda_{2d,i}\exp(\mu_{d,i} + \omega_{d,i}/2)\lambda_{3d}^\top P e_i, \quad i = 1, ..., N, \tag{C.4}$$

$$\psi_{h,d} = \left(\sum_{j=1}^{h} P^{j-1}\right)^\top \psi_d \quad \text{and} \quad \lambda_{h,2f} = \left(\sum_{j=1}^{h} P^{j-1}\right)^\top \lambda_{2f}. \tag{C.5}$$

The variance of returns over h periods is given by:

$$\begin{aligned} Var[R_{t+1:t+h}] &= h\theta_2^\top E\left[\zeta_t\zeta_t^\top\right]P^\top\theta_3. \\ &+ h(\theta_1 \odot \theta_1)^\top E\left[\zeta_t\zeta_t^\top\right]P^\top(\lambda_{3d} \odot \lambda_{3d}) - h^2\left(\theta_1^\top E\left[\zeta_t\zeta_t^\top\right]P^\top\lambda_{3d}\right)^2 \\ &+ 2\sum_{j=2}^{h}(h-j+1)\theta_1^\top E\left[\zeta_t\zeta_t^\top\right]P^\top\left(\lambda_{3d} \odot \left(\left(P^{j-2}\right)^\top\left(\theta_1 \odot \left(P^\top\lambda_{3d}\right)\right)\right)\right), \end{aligned} \tag{C.6}$$

where

$$\theta_1 = \lambda_{2d} \odot (\exp(\mu_{d,1} + \omega_{d,1}/2), ..., \exp(\mu_{d,N} + \omega_{d,N}/2))^\top, \tag{C.7}$$

$$\theta_2 = (\theta_1 \odot \theta_1 \odot (\exp(\omega_{d,1}), ..., \exp(\omega_{d,N}))^\top) - (\theta_1 \odot \theta_1), \tag{C.8}$$

$$\theta_3 = \lambda_{3d} \odot \lambda_{3d}. \tag{C.9}$$

One can get similar formulas for excess returns.

Appendix D. This section deals with predictive regressions. When one runs a predictive regression, i.e., one regresses a variable $y_{t+1:t+h}$ onto a variable x_t and a constant, one gets

$$y_{t+1:t+h} = a(h) + b(h) x_t + \eta_{y,1,t+h}(h), \tag{D.1}$$

$$\text{with} \qquad b(h) = \frac{Cov(y_{t+1:t+h}, x_t)}{Var[x_t]} \text{ and } R^2(h) = \frac{(Cov(y_{t+1:t+h}, x_t))^2}{Var[y_{t+1:t+h}] Var[x_t]}, \tag{D.2}$$

where $R^2(h)$ is the corresponding population coefficient of determination. Consequently, the characterization of the predictive ability of the dividend-price ratio for future expected returns requires the variance of payoff-price ratios, covariances of payoff-price ratios with aggregate returns and variance of aggregate returns. We show that

$$Var\left[\frac{D_t}{P_{d,t}}\right] = \lambda_{2d}^{\top} Var[\zeta_t] \lambda_{2d} \text{ and } Cov\left(R_{t+1:t+h}, \frac{D_t}{P_{d,t}}\right) = \psi_{h,d}^{\top} Var[\zeta_t] \lambda_{2d}, \tag{D.3}$$

and the variance of aggregate returns is given by (C.6). One gets similar formulas for excess returns, consumption and dividend growth processes.

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Table 1: **Parameters of the Random Walk and the Long-Run Risks Markov Switching Models.**
The long-run risks model defined in (2.6) is calibrated with $\mu_x = 0.0015$, $\phi_d = 2.5$, $\nu_d = 6.5$, $\phi_x = 0.975$, $\nu_x = 0.038$, $\sqrt{\mu_\sigma} = 0.0072$, $\phi_\sigma = 0.995$, $\nu_\sigma = 0.62547 \times 10^{-5}$ and $\rho_1 = 0.39985$. In Panel A, we report the parameters of the two-state monthly Markov switching model of the form (3.1) such that $\mu_{c,1} = \mu_{c,2}$ and $\mu_{d,1} = \mu_{d,2}$. From the LRR in mean and volatility model, we set $\phi_x = 0$ and $\nu_x = 0$ to obtain a random walk model, and we adjust the other parameters when necessary such that consumption and dividend growth means, variances and covariance remain unchanged from the original model. The random walk model is then calibrated with $\mu_x = 0.0015$, $\nu_d = 6.42322$, $\sqrt{\mu_\sigma} = 0.0073$, $\phi_\sigma = 0.995$, $\nu_\sigma = 0.62547 \times 10^{-5}$ and $\rho_1 = 0.40434$. In Panel A, we report the parameters of the four-state monthly Markov switching model of the form (3.1) that matches the full long-run risk model of Bansal, Kiku and Yaron (2007). In both panels, μ_c and μ_d are conditional means of consumption and dividend growths, ω_c and ω_d are conditional variances of consumption and dividend growths and ρ is the conditional correlation between consumption and dividend growths. $P^\top$ is the transition matrix across different regimes and Π is the vector of unconditional probabilities of regimes. Means and standard deviations are in percent.

Panel A	σ_L	σ_H
$\mu_c^\top$	0.15	0.15
$\mu_d^\top$	0.15	0.15
$\left(\omega_c^\top\right)^{1/2}$	0.46	1.32
$\left(\omega_d^\top\right)^{1/2}$	2.94	8.48
$\rho^\top$	0.40434	0.40434
	$P^\top$	
σ_L	0.99894	0.00106
σ_H	0.00394	0.99606
$\Pi^\top$	0.78868	0.21132

Panel B	$\mu_L\sigma_L$	$\mu_L\sigma_H$	$\mu_H\sigma_L$	$\mu_H\sigma_H$
$\mu_c^\top$	-0.19513	-0.19513	0.19393	0.19393
$\mu_d^\top$	-0.71283	-0.71283	0.25982	0.25982
$\left(\omega_c^\top\right)^{1/2}$	0.44071	1.31462	0.44071	1.31462
$\left(\omega_d^\top\right)^{1/2}$	2.86569	8.54824	2.86569	8.54824
$\rho^\top$	0.39985	0.39985	0.39985	0.39985
		$P^\top$		
$\mu_L\sigma_L$	0.97679	0.00103	0.02215	0.00002
$\mu_L\sigma_H$	0.00386	0.97397	0.00009	0.02209
$\mu_H\sigma_L$	0.00282	0.00000	0.99612	0.00105
$\mu_H\sigma_H$	0.00001	0.00281	0.00393	0.99325
$\Pi^\top$	0.08905	0.02386	0.69963	0.18746

Table 2: **(RW) Asset Prices and Predictability: Benckmark**

The entries of Panel A are model population values of asset prices. The expressions $E[R-R_f]$, $E[R_f]-1$ and $E[P/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma[R]$, $\sigma[R_f]$ and $\sigma[D/P]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B, C and D show the R^2 and the slope of the regression $y_{t+1:t+12h} = a(h) + b(h)\left(\frac{D}{P}\right)_{t-11:t} + \eta_{t+12h}(h)$, where y stands for excess returns, consumption growth and dividend growth respectively.

	Data	GDA	50%	PV
δ		0.9989		
γ		2.5		
ψ		1.5		
α		0.3		
κ		0.989		
Panel A. Asset Pricing Implications				
$E[R-R_f]$	7.25	7.21	6.14	0.61
$\sigma[R]$	19.52	19.33	16.90	0.45
$E[R_f]-1$	1.21	0.93	1.39	0.62
$\sigma[R_f]$	4.10	2.34	1.84	1.00
$E[P/D]$	30.57	23.30	24.20	1.00
$\sigma[D/P]$	1.52	1.38	1.07	0.79
Panel B. Predictability of Excess Returns				
$R^2(1)$	7.00	12.04	7.44	0.48
$[b(1)]$	3.12	5.05	6.25	0.20
$R^2(3)$	14.67	28.35	17.27	0.46
$[b(3)]$	7.05	14.30	16.91	0.18
$R^2(5)$	27.26	38.00	22.47	0.56
$[b(5)]$	12.34	22.49	23.14	0.25
Panel C. Predictability of Consumption Growth				
$R^2(1)$	0.06	0	0.76	0.16
$[b(1)]$	-0.02	0	0.02	0.47
$R^2(3)$	0.09	0	1.67	0.13
$[b(3)]$	-0.05	0	0.07	0.46
$R^2(5)$	0.24	0	2.23	0.18
$[b(5)]$	-0.11	0	0.04	0.47
Panel D. Predictability of Dividend Growth				
$R^2(1)$	0.00	0	0.71	0.00
$[b(1)]$	0.04	0	0.11	0.49
$R^2(3)$	0.20	0	1.44	0.21
$[b(3)]$	-0.48	0	0.17	0.46
$R^2(5)$	0.08	0	1.75	0.14
$[b(5)]$	-0.37	0	-0.48	0.51

Table 3: **(RW) Conditional Moments of the Stochastic Discount Factor**
The entries of the table are the mean and the volatility of the stochastic discount factor in each state of the economy (i.e. low volatility and high volatility of aggregate consumption growth). The benchmark Random Walk dynamics is calibrated, with $\mu_x = 0.0015$, $\nu_d = 6.42322$, $\sqrt{\mu_\sigma} = 0.0073$, $\phi_\sigma = 0.995$, $\nu_\sigma = 0.62547 \times 10^{-5}$ and $\rho = 0.40434$.

	$\mu(M \mid \sigma_L)$	$\sigma(M \mid \sigma_L)$	$\mu(M \mid \sigma_H)$	$\sigma(M \mid \sigma_H)$
GDA	0.99820	0.11481	1.00304	0.66929
GDA1	0.99695	0.11394	1.00365	0.67033
DA0	0.99890	0.63412	0.99890	0.61151
KP	0.99814	0.21202	0.99935	0.13928

Table 4: **(RW) Asset Prices and Predictability: Robustness to Preference Parameters**

The entries of Panel A are model population values of asset prices. The expressions $E[R-R_f]$, $E[R_f]-1$ and $E[P/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma[R]$, $\sigma[R_f]$ and $\sigma[D/P]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B, C and D show the R^2 and the slope of the regression $y_{t+1:t+12h} = a(h) + b(h)\left(\frac{D}{P}\right)_{t-11:t} + \eta_{t+12h}(h)$, where y stands for excess returns, consumption growth and dividend growth respectively.

	Data	GDA1	50%	PV	DA0	50%	PV	KP	50%	PV
δ		0.9989			0.9989			0.9989		
γ		2.5			0			10		
ψ		0.75			∞			1.5		
α		0.3			0.3			1		
κ		0.989			1			1		
Panel A. Asset Pricing Implications										
$E[R-R_f]$	7.25	6.12	5.00	0.69	10.32	9.56	0.12	1.42	1.16	0.98
$\sigma[R]$	19.52	18.04	15.75	0.27	19.14	16.94	0.00	16.38	13.96	0.05
$E[R_f]-1$	1.21	1.97	2.60	0.68	1.32	1.32	0.61	1.93	2.04	0.75
$\sigma[R_f]$	4.10	3.25	2.55	1.00		0.00	1.00	0.59	0.46	1.00
$E[P/D]$	30.57	22.05	22.74	1.00	13.10	13.59	1.00	470.66	467.82	0.00
$\sigma[D/P]$	1.52	1.04	0.81	1.00	2.32	1.80	0.44	0.01	0.00	1.00
Panel B. Predictability of Excess Returns										
$R^2(1)$	7.00	13.53	7.78	0.47	8.00	4.54	0.61	1.29	0.87	0.88
$[b(1)]$	3.12	6.70	7.98	0.18	2.38	3.08	0.51	-294.98	-253.63	0.65
$R^2(3)$	14.67	30.54	17.33	0.46	19.88	11.10	0.57	3.33	1.69	0.87
$[b(3)]$	7.05	18.94	20.94	0.18	6.73	8.43	0.42	-834.28	-712.09	0.65
$R^2(5)$	27.26	39.72	21.94	0.56	27.78	14.63	0.67	4.81	2.26	0.92
$[b(5)]$	12.34	29.79	28.89	0.24	10.58	11.35	0.54	-1312.45	-807.13	0.61
Panel C. Predictability of Consumption Growth										
$R^2(1)$	0.06	0	0.76	0.16	0	0.76	0.16	0	0.75	0.17
$[b(1)]$	-0.02	0	0.02	0.47	0	0.01	0.45	0	-3.39	0.52
$R^2(3)$	0.09	0	1.68	0.13	0	1.66	0.13	0	1.65	0.13
$[b(3)]$	-0.05	0	0.09	0.47	0	0.04	0.45	0	-14.53	0.52
$R^2(5)$	0.24	0	2.23	0.18	0	2.23	0.18	0	2.24	0.18
$[b(5)]$	-0.11	0	0.05	0.48	0	0.02	0.46	0	-9.98	0.51
Panel D. Predictability of Dividend Growth										
$R^2(1)$	0.00	0	0.72	0.00	0	0.71	0.00	0	0.70	0.00
$[b(1)]$	0.04	0	0.14	0.49	0	0.07	0.50	0	-24.52	0.52
$R^2(3)$	0.20	0	1.44	0.21	0	1.44	0.21	0	1.44	0.21
$[b(3)]$	-0.48	0	0.23	0.47	0	0.10	0.44	0	-47.28	0.51
$R^2(5)$	0.08	0	1.75	0.14	0	1.75	0.14	0	1.76	0.14
$[b(5)]$	-0.37	0	-0.63	0.51	0	-0.28	0.50	0	107.27	0.48

Table 5: **(LRR) Asset Prices and Predictability: Bansal and Yaron (2004) Endowment Process**

The entries of Panel A are model population values of asset prices. The expressions $E[R-R_f]$, $E[R_f]-1$ and $E[P/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma[R]$, $\sigma[R_f]$ and $\sigma[D/P]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B, C and D show the R^2 and the slope of the regression $y_{t+1:t+12h} = a(h) + b(h)\left(\frac{D}{P}\right)_{t-11:t} + \eta_{t+12h}(h)$, where y stands for excess returns, consumption growth and dividend growth respectively.

	Data	GDA	50%	PV	GDA1	50%	PV	DA0	50%	PV	KP	50%	PV
δ		0.9989			0.9989			0.9989			0.9989		
γ		2.5			2.5			0			10		
ψ		1.5			0.75			∞			1.5		
α		0.3			0.3			0.3			1		
κ		0.989			0.989			1			1		
Panel A. Asset Pricing Implications													
$E[R-R_f]$	7.25	8.60	7.54	0.46	6.92	5.83	0.62	11.47	10.68	0.06	6.69	6.33	0.65
$\sigma[R]$	19.52	19.35	17.91	0.56	18.04	16.84	0.62	20.78	19.01	0.52	18.11	16.22	0.65
$E[R_f]-1$	1.21	0.96	1.33	0.47	2.19	2.67	0.27	1.32	1.32	0.00	1.21	1.28	0.44
$\sigma[R_f]$	4.10	2.48	1.95	1.00	3.70	2.85	0.96	0.00	0.00	1.00	1.05	0.78	1.00
$E[P/D]$	30.57	17.70	18.23	1.00	18.06	18.48	1.00	11.93	12.28	1.00	22.50	22.56	1.00
$\sigma[D/P]$	1.52	1.56	1.18	0.68	1.11	0.86	1.00	2.59	1.95	0.42	0.48	0.29	1.00
Panel B. Predictability of Excess Returns													
$R^2(1)$	7.00	10.30	6.49	0.52	11.90	7.06	0.50	6.38	4.75	0.62	0.05	0.83	0.94
$[b(1)]$	3.12	4.13	4.72	0.30	5.86	6.37	0.23	2.05	2.48	0.61	0.86	2.45	0.53
$R^2(3)$	14.67	24.07	14.31	0.50	26.81	14.89	0.50	15.84	11.14	0.58	0.05	1.98	0.94
$[b(3)]$	7.05	11.62	12.55	0.25	16.52	17.15	0.21	5.71	7.00	0.50	1.54	6.05	0.52
$R^2(5)$	27.26	32.07	18.72	0.61	34.80	18.00	0.61	22.08	15.11	0.71	0.03	2.64	0.99
$[b(5)]$	12.34	18.21	18.75	0.29	25.92	25.01	0.24	8.88	10.43	0.63	1.41	8.26	0.56
Panel C. Predictability of Consumption Growth													
$R^2(1)$	0.06	1.68	2.70	0.10	1.30	2.68	0.10	2.96	2.87	0.09	16.39	5.49	0.07
$[b(1)]$	-0.02	-0.24	-0.28	0.68	-0.29	-0.37	0.68	-0.19	-0.19	0.70	-2.42	-2.00	0.81
$R^2(3)$	0.09	2.14	4.15	0.09	1.66	4.08	0.09	3.77	4.35	0.09	20.91	4.86	0.08
$[b(3)]$	-0.05	-0.54	-0.65	0.65	-0.67	-0.84	0.65	-0.43	-0.42	0.66	-5.52	-3.58	0.76
$R^2(5)$	0.24	1.85	4.87	0.14	1.44	4.90	0.14	3.27	4.95	0.14	18.09	4.31	0.14
$[b(5)]$	-0.11	-0.71	-0.77	0.60	-0.88	-1.09	0.61	-0.57	-0.49	0.61	-7.21	-3.65	0.71
Panel D. Predictability of Dividend Growth													
$R^2(1)$	0.00	0.31	1.66	0.00	0.24	1.64	0.00	0.55	1.63	0.00	3.05	1.52	0.00
$[b(1)]$	0.04	-0.60	-0.99	0.63	-0.73	-1.37	0.63	-0.48	-0.62	0.66	-6.05	-5.38	0.76
$R^2(3)$	0.20	0.51	3.31	0.14	0.40	3.22	0.14	0.90	3.31	0.14	4.96	2.63	0.17
$[b(3)]$	-0.48	-1.36	-2.15	0.59	-1.68	-2.92	0.59	-1.09	-1.41	0.58	-13.80	-9.90	0.71
$R^2(5)$	0.08	0.50	3.79	0.07	0.39	3.77	0.08	0.88	3.59	0.07	4.87	2.70	0.09
$[b(5)]$	-0.37	-1.78	-2.85	0.57	-2.19	-3.87	0.57	-1.42	-1.70	0.57	-18.03	-10.24	0.65

Figure 1: **Indifference Curves for GDA Preferences**
Indifference curves over two outcomes x and y with the fixed probability $p = Prob\,(x) = 1/2$.

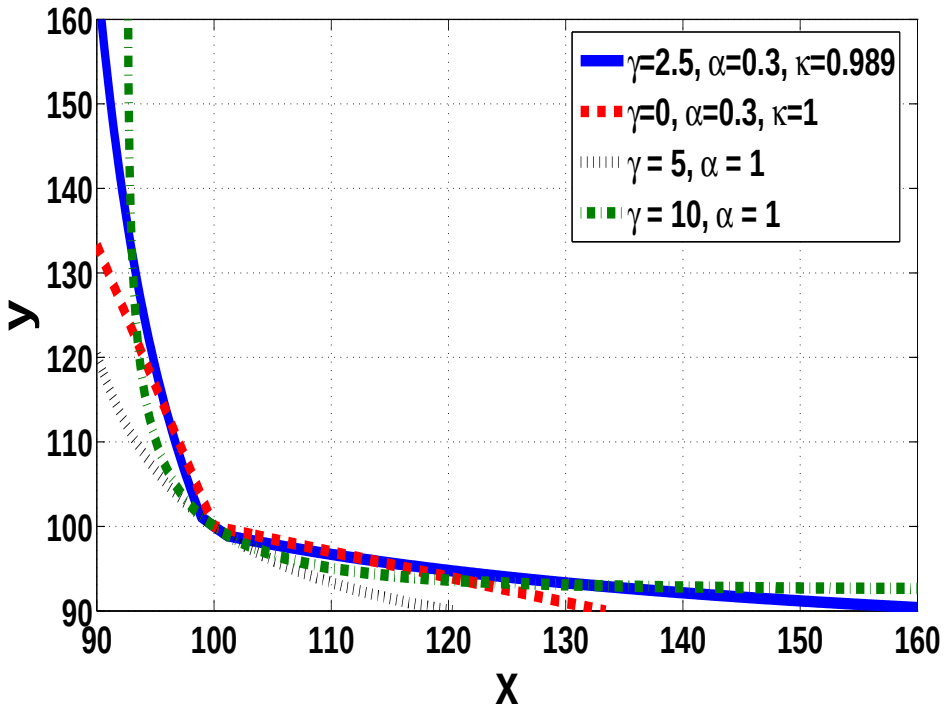

Figure 2: **(RW) Equity Premium, Risk-Free Rate and Valuation Ratio, GDA**
The figure displays population values of asset prices. The expressions $E[R - R_f]$, $E[R_f] - 1$ and $E[P/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The parameter of risk aversion is set to $\gamma = 2.5$.

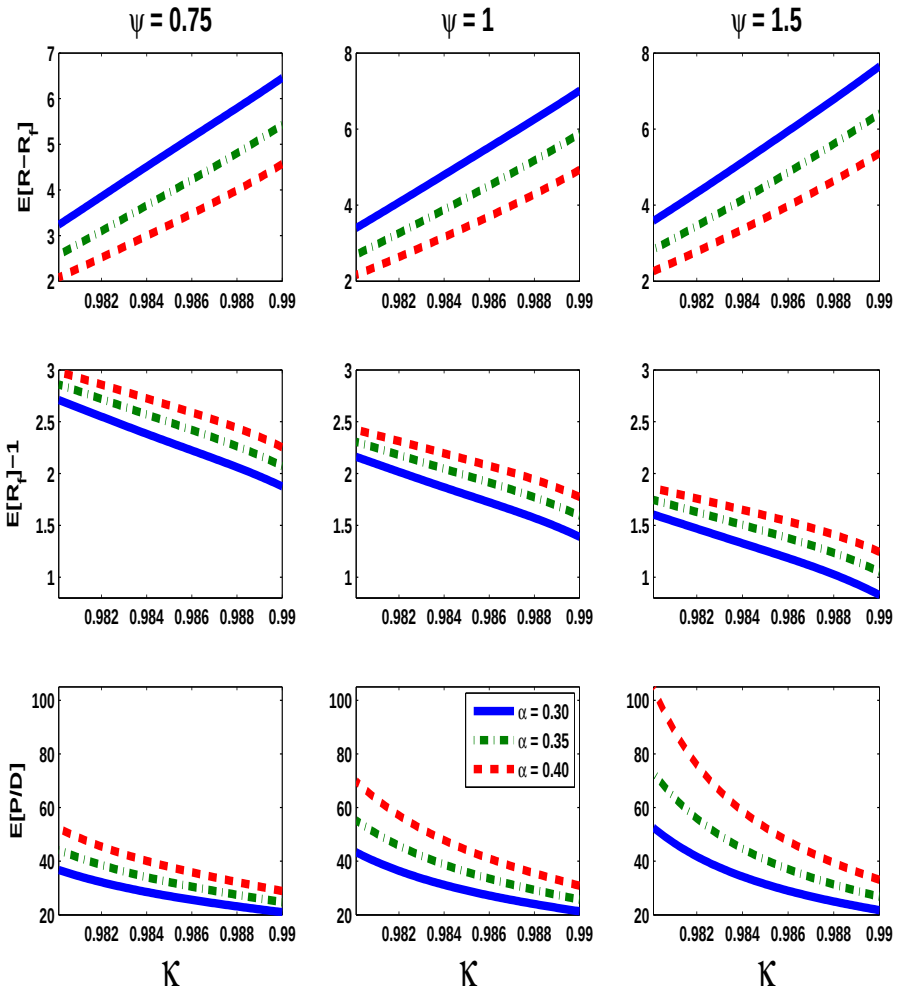

Figure 3: **(RW) Predictability of Excess Returns (R^2), GDA**
The figure shows the population R^2 of the monthly regression $R^e_{t+1:t+h} = a(h) + b(h)\frac{D_t}{P_{d,t}} + \eta_{t+h}(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The parameter of risk aversion is set to $\gamma = 2.5$.

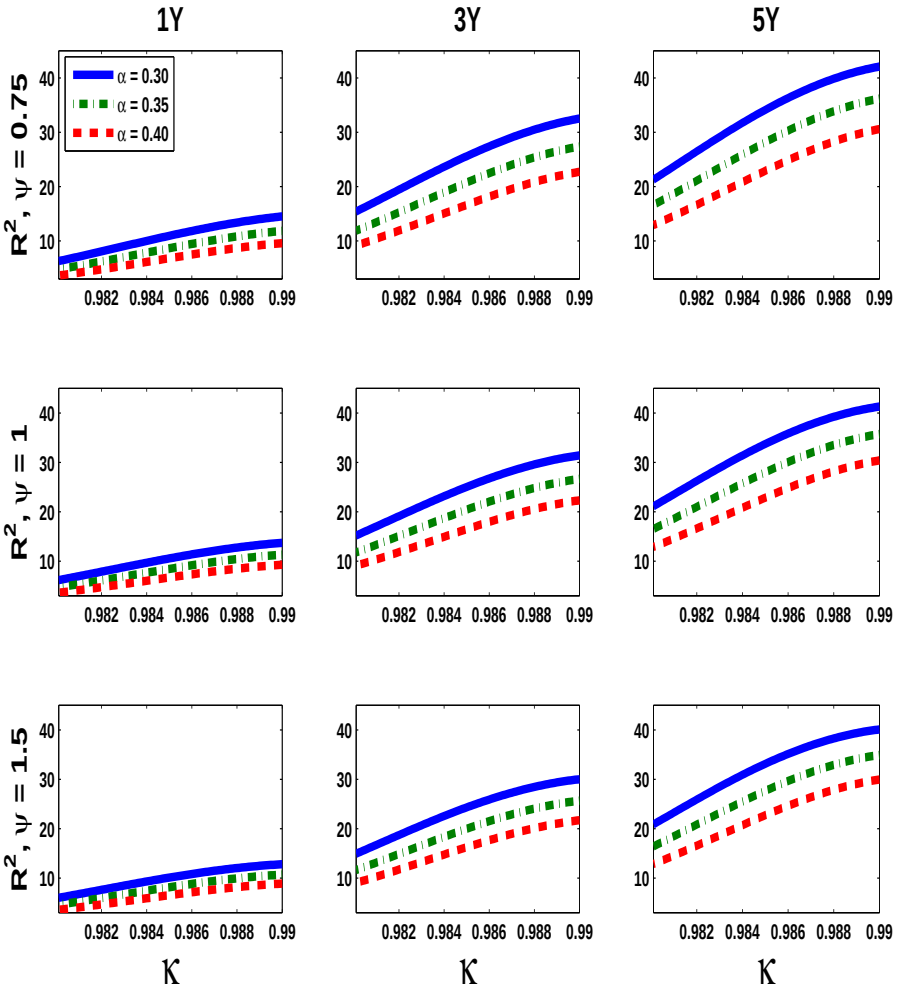

Figure 4: **(RW) Sensitivity of Asset Prices to the Persistence of Consumption Volatility: KP and GDA.**
The figure displays population values of asset prices as functions of the persistence of consumption volatility. The expressions $E[R - R_f]$ and $E[P_d/D]$ are respectively the annualized equity premium and mean price-dividend ratio. The expressions $\sigma[R - R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of the equity excess return and the equity dividend-price ratio.

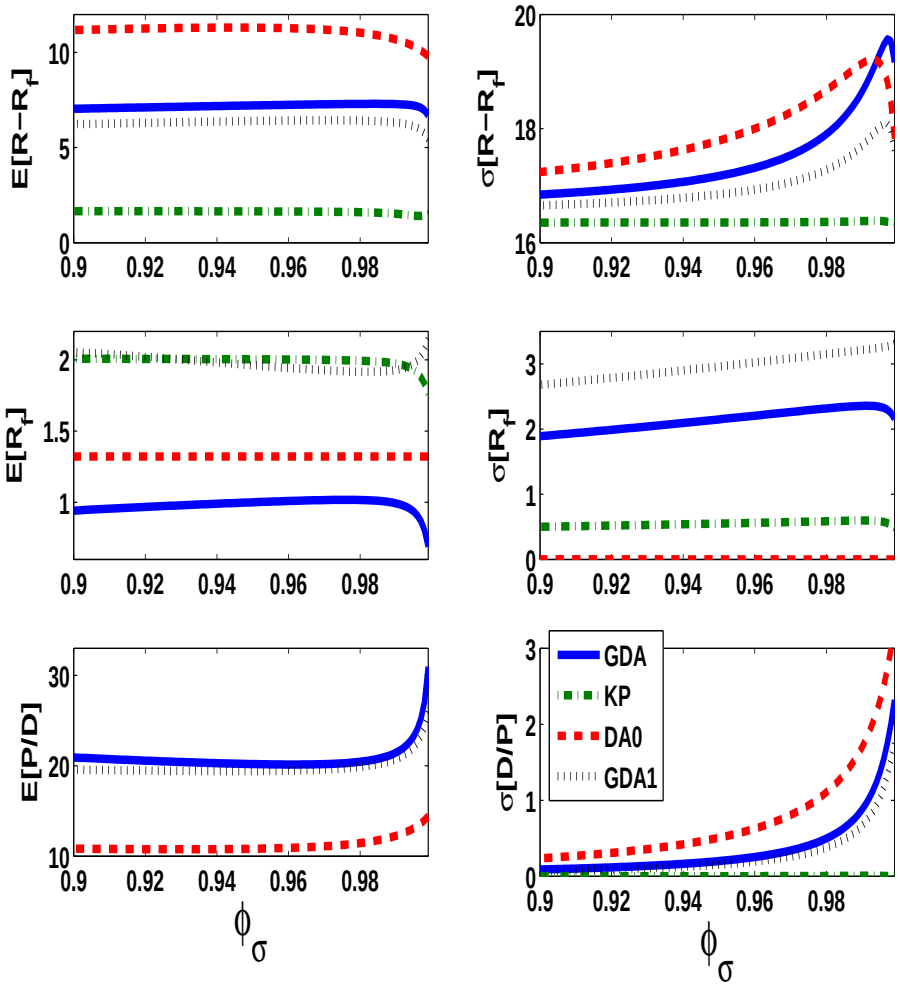

Figure 5: **(RW) Sensitivity of Excess Return Predictability to the Persistence of Consumption Volatility.**

The figure shows the population R^2 of the monthly regression $y_{t+1:t+h} = a(h) + b(h)\left(\frac{D}{P_d}\right)_{t-11:t} + \eta_{t+h}(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The variable y stands for excess returns $R - R_f$. The R^2 is plotted as a function of the persistence of consumption volatility.

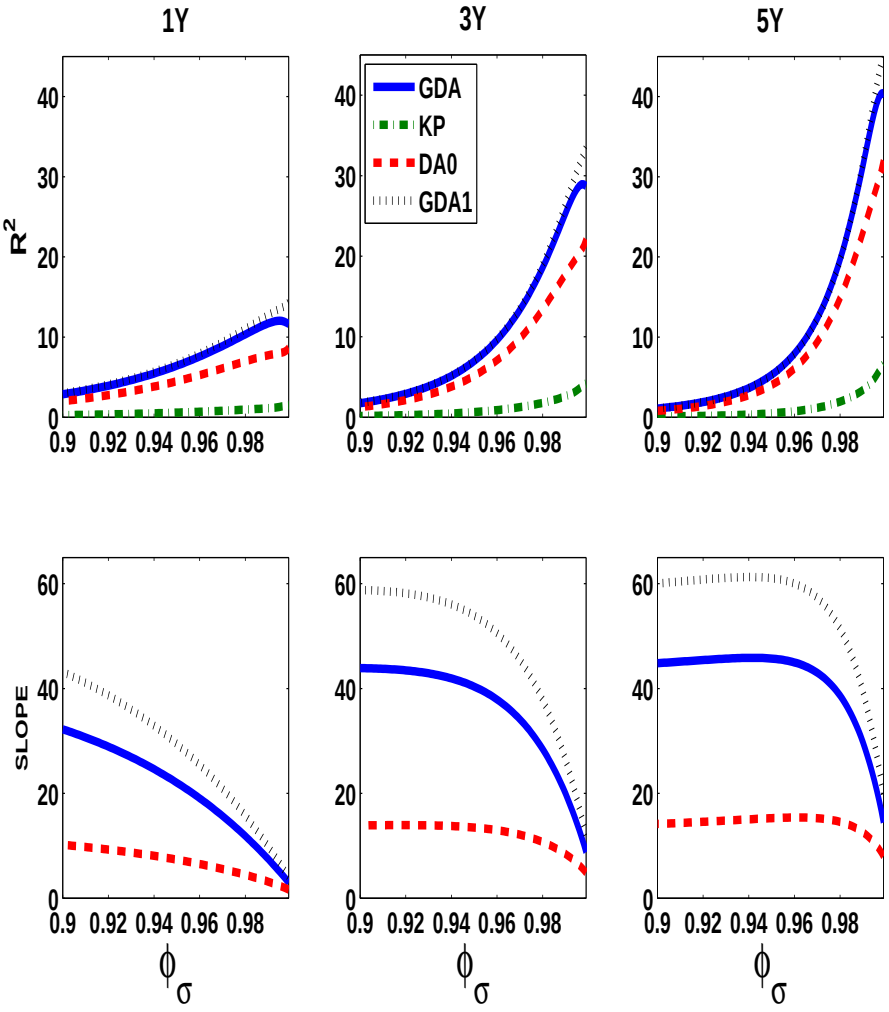

Figure 6: **(LRR) Sensitivity of Asset prices to the Persistence of Expected Consumption Growth: KP and GDA.**
The figure displays population values of asset prices as functions of the persistence of expected consumption growth. The expressions $E[R - R_f]$ and $E[P_d/D]$ are respectively the annualized equity premium and mean price-dividend ratio. The expressions $\sigma[R - R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of the equity excess return and the equity dividend-price ratio.

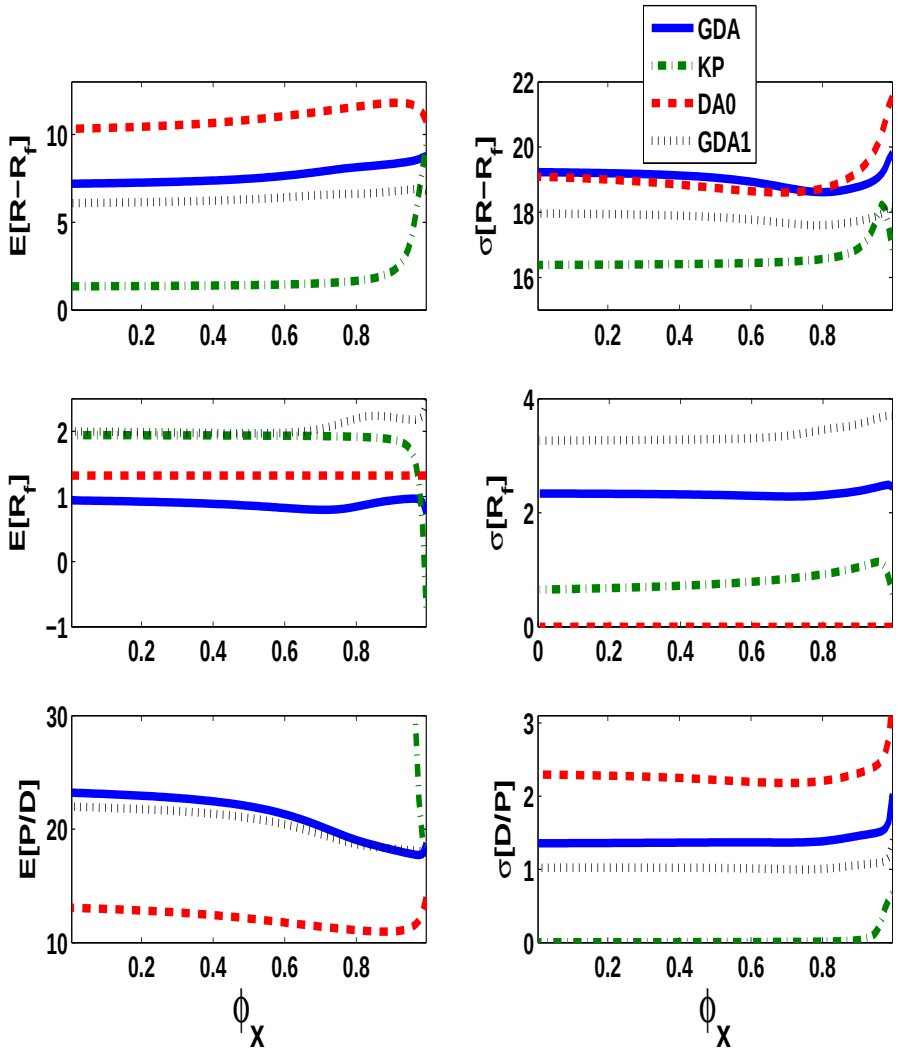

Figure 7: **(LRR) Sensitivity of Excess Return and Growth Rates Predictability to the Persistence of Expected Consumption Growth: KP and GDA.**
The figure shows the population R^2 of the monthly regression $y_{t+1:t+h} = a(h) + b(h)\left(\frac{D}{P_d}\right)_{t-11:t} + \eta_{t+h}(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The variable y stands for excess returns $R - R_f$ and consumption growth Δc. The R^2 is plotted as a function of the persistence of expected consumption growth.