Working Paper Series n° 610

February, 2010

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Optimal capture and sequestration from the carbon emission flow and from the atmospheric carbon stock with heterogeneous energy consuming sectors

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February 11, 2010

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Abstract

We characterize the optimal exploitation paths of two primary energy resources. The first one is a non-renewable polluting resource, the second one a pollution-free renewable resource. Both resources can supply the energy needs of two sectors. Sector 1 is able to reduce the potential carbon emissions generated by its non-renewable energy consumption at a reasonable cost while sector 2 cannot. Another possibility is to capture the carbon spread in the atmosphere but at a significantly higher cost. We assume that the atmospheric carbon stock cannot exceed some given ceiling and that this constraint is effective. We show that there may exist paths along which it is optimal to begin by fully capturing the sector 1's potential emission flow before the ceiling constraint begins to be effective. Also there may exist optimal paths along which both capture devices have to be activated, in which case the potential emission flow of sector 1 is first fully abated and next the society must resort to the atmospheric carbon reducing device.

Keywords: Fossil resource, Carbon stabilization cap, Heterogeneity, CCS, Air capture.

JEL classifications: Q31, Q32, Q41, Q42, Q54, Q58.

Contents

1 Introduction

As recommended by the Intergovernmental Panel on Climate Change, abatement technologies must be used to reduce the anthropogenic carbon dioxide emissions in a climate change mitigation objective (IPCC, 2007). Among all the alternatives, a particular interest should be given to the carbon capture and sequestration (CCS) (IPCC, 2005). CCS technology consists in filtering CO_2 fluxes at the source of emission, that is, in fossil energy-fueled power plants, by use of scrubbers installed near the top of chimney stacks. The carbon would be sequestered in reservoirs, such as exhausted salt and coal mines, depleted oil and gas fields or deep saline aquifers.

Even if the efficiency of such a technology as well as the potential capacities of such carbon sinks are still under assessment, current engineering estimates suggest that CCS could be a credible cost-effective approach for eliminating most of the emissions from coal and natural gas power plants (MIT, 2007). Along this line of arguments, Islegen and Reichelstein (2009) point out that CCS has considerable potential to reduce CO_2 emissions at a social cost that most economists would consider as reasonable, given the social costs of carbon emissions predicted for a business-as-usual scenario without regulation. For regulated scenarios, CCS is also intended to have a major role in limiting the effective carbon tax, that is, the market price for CO_2 emission permits under a cap-and-trade system. In that case, the crucial point is to estimate how far would the CO_2 price have to rise before the operator of power plants would find it advantageous to install CCS technology rather then buy emission permits or pay the carbon tax. McKinsey & Company (2007) estimates a break-even price, that is a carbon price from which CCS becomes an economically viable alternative, in the range of \$30-45 for coal-fire plants. This price rises to at least \$60 per tonne of CO_2 in the case of power plants running on natural gas.

However, geologic CCS presents the disadvantage to apply to the sole large point sources of pollution such as power plants or huge manufacturings. This technology is prohibitively costly to filter for instance the CO_2 emissions from transportation as far as the energy input is gasoline or kerosene[1], small residence heating or scattered agricultural activities. Hence, the ultimate device to abate carbon dioxide fluxes from any concentrated as well as diffuse sources would consist in capturing them directly from the atmosphere. Deliberately expressing a double meaning, McKay (2009) claims about this alternative that "*captur-*

[1]Note that an electric railway system escapes this constraint.

ing carbon dioxide from thin air is the last thing we should talk about" (p.240). On the one hand, the energy requirements for atmospheric carbon capture are so enormous that, according to McKay, it seems actually almost absurd to talk about it. But on the other hand, McKay argues that "*we should talk about it, contemplate how best to do it, and fund research into how to do it better, because capturing carbon from thin air may turn out to be our last line of defense, if climate change is as bad as the climate scientists say, and if humanity fails to take the cheaper and more sensible options that may still be available today*" (p.240).

Technically speaking, removing carbon from the atmosphere can be achieved in different ways. The most obvious approach consists in exploiting the process of photosynthesis. A close idea can be transposed to the oceans. To make them able to capture carbon faster then normal, phytoplankton blooms can be stimulated by fertilizing some oceanic iron-limited regions. A third way is to enhance weathering of rocks, that is to pulverize rocks that are capable of absorbing $C0_2$, and leave them in the open air. This idea can be pitched as the acceleration of a natural process. Unfortunately, as claimed by Barrett (2009), the effects of all these devices are difficult to verify, their potential is limited in any event, and there are concerns about some unknown ecological consequences.

A probably most credible way of sucking carbon from thin air is to use a chemical process. This involves a technology that brings air into contact with a chemical "sorbent" (an alkaline liquid). The sorbent absorbs CO_2 in the air, and the chemical process then separates out the CO_2 and recycles the sorbent. Finally, the captured CO_2 is stored in geologic deposits, just like the CCS from power plant described above. However, chemical air capture is expensive. Estimates of marginal cost range from \$100-200/tCO_2, which is larger than the cost of alternatives for reducing emissions such as power plant CCS. They are also larger than current estimates of the social cost of carbon, which range from about \$7-85/tCO_2. But, as concluded by Barrett (2009), bearing the cost of chemical air capture can become profitable in the future under constraining cap-and-trade scenarios. Furthermore, we may hope that the cost will decrease, thanks to R&D and learning by doing.

In the present study, we address the question of the heterogeneity of energy users regarding the way their carbon footprint can be reduced. We then consider two abatement technologies and two sectors. The first technology is a conventional emission abatement device (CCS) which is available at a marginal cost assumed to be socially acceptable. However, this abatement technology cannot apply to carbon emissions from any type of activity, but only from large point sources of emissions. The second technology directly captures carbon in the atmosphere. Its marginal cost is highly costlier than the emission capture technology, but it allows to reduce carbon from any sources since the capture process and the generation of emissions are now disconnected. The first sector, in which pollution sources are spatially concentrated, can abate its carbon emissions, but not the second one since energy users are too small and too scattered. The ultimate way for abating pollution is to directly capture carbon in the atmosphere. But since the atmosphere is a public good, this kind of pollution reduction will also benefit to sector 1. Whatever the capture process, we assume that carbon is stockpiled into reservoirs whose size is very large. Then, as in Chakravorty *et al.*, (2006-b) this suggests a generic abatement scheme of unlimited capacity[2]. Finally, energy in each sector can be supplied either by a carbon-based fossil fuel, contributing to climate change (oil, cal, gas), or by a carbon-free renewable and non biological resource such, as solar energy.

Using a standard Hotelling model for the non-renewable resource and assuming that the atmospheric carbon stock should not exceed some critical threshold, we then characterize the optimal time path of sectoral energy prices, sectoral energy consumptions, emission and atmospheric abatements. The key results of the paper are: i) Irrespective of the availability of the air capture technology, it may happen that it is optimal for the first sector to abate its carbon emissions before the atmospheric carbon concentration cap is attained. ii) Since this type of carbon capture is unable to filter the emissions from the second sector, it is also optimal for the first sector to abate the totality of its own emissions, at least at the beginning. These two first results are at variance with Chakravorty *et al.* (2006-b), Lafforgue *et al.* (2008-a) and (2008-b) who consider a single sector using energy and a single abatement technology. iii) The atmospheric carbon capture is only used when the atmospheric carbon stock reaches the ceiling, maintaining the stock at its critical level. Hence the flow of carbon captured in the atmosphere is lower than the emission flow of the second sector.

The paper is organized as follows. Section 2 presents the model. In section 3, we lay down the social planner program and we derive the optimality conditions. In section 4,

[2]The question of the size of carbon sinks and of the time profile of their filling up is addressed by Lafforgue *et al.* (2008-a, 2008-b).

we examine the restricted problem in which only the emission carbon capture device is available. In section 5, we examine how the model reacts when the atmospheric carbon capture technology is introduced. We also investigate the time profile of the optimal carbon tax as well as, for each sector, the total burden induced by the mitigation of their emissions. Finally, we briefly conclude in section 6.

2 Model and notations

Let us consider a stationary economy with two sectors, indexed by $i = 1, 2$, in which the instantaneous gross surplus derived from energy consumption are the same. For an identical energy consumption in the two sectors, $q_1 = q_2 = q$, the sectoral gross surplus $u_1(q)$ and $u_2(q)$ are such that: $u_1(q) = u_2(q) = u(q)$. We assume that this common function u satisfies the following standard assumptions. $u : \mathbb{R}_+ \to \mathbb{R}_+$ is a function of class $\mathcal{C}^2$, strictly increasing, strictly concave and verifying the Inada conditions: $\lim_{q\downarrow 0} u'(q) = +\infty$ and $\lim_{q\uparrow +\infty} u'(q) = 0$. We denote by $p(q)$ the sectoral marginal gross surplus function and by $q^d(p) = p^{-1}(p)$, the sectoral direct demand function.

In each sector, energy can be supplied by two primary natural resources: a dirty non-renewable resource (let say oil for instance) and a carbon-free renewable resource (let say solar energy).

Let us denote by X^0 the initial oil endowment of the economy, by $X(t)$ the remaining part of this initial endowment at time t, and by $x_i(t)$, $i = 1, 2$, the instantaneous consumption flow of oil in sector i at time t, so that:

$$\dot{X}(t) = -[x_1(t) + x_2(t)], \quad \text{with } X(0) \equiv X^0 \text{ and } X(t) \geq 0 \tag{1}$$

$$x_i(t) \geq 0, \ i = 1, 2. \tag{2}$$

The delivery cost of oil is the same for both sectors. We denote by c_x the corresponding average cost, assumed to be constant and hence equal to the marginal cost. The delivery cost includes the extraction cost of the resource, the cost of industrial processing (refining of the crude oil) and the transportation cost, so that the resource is ready for use by the consumer in the concerned sector. To keep matter as simple as possible, we assume that no oil is lost during the delivery process. Equivalently, the oil stock $X(t)$ may be understood as measured in ready for use units.

Let $Z(t)$ be the stock of carbon within the atmosphere at time t, and Z^0 be the initial stock, $Z^0 \equiv Z(0)$. We assume that a carbon cap policy is prescribed to prevent catastrophic damages which would be infinitely costly. This policy consists in forcing the atmospheric stock to stay under some critical level $\bar{Z}$, with $\bar{Z} > Z^0$.

The atmospheric carbon stock is fed by carbon emission flows resulting from the use of oil. Let ζ be the quantity of carbon which would be potentially released by unit of oil consumption whatever the sector in which the oil is used. Thus, the gross pollutant flow amounts to $\zeta[x_1(t) + x_2(t)]$. However, this gross emission flow can be abated before being released into the atmosphere. We assume that emissions from sector 1 can be abated, but not emissions from sector 2 (or at a prohibitive cost). Emission abatement by carbon capture and sequestration (CCS) can be achieved when burning oil is spatially concentrated, as it is the case for instance in the electricity or cement industries, which are good examples of sector 1's activities. At the other extreme of the spectrum, i.e. in sector 2, there exists some activities with prohibitively costing emission captures since users are too small or too scattered. Transportation by cars, trucks and diesel train are good examples of sector 2's industry[3].

Let $s_e(t)$ be this part of carbon emissions from sector 1 which is captured and sequestered at some average cost c_e, assumed to be constant. Thus the net pollution flow issued from sector 1 amounts to:

$$\zeta x_1(t) - s_e(t) \geq 0, \quad s_e(t) \geq 0. \tag{3}$$

In sector 2, the net pollution flow amounts to $\zeta x_2(t)$.

Carbon emission capture is not the unique way to reduce the atmospheric carbon concentration. The other process consists in capturing the carbon present in the atmosphere itself. We denote by $s_a(t)$ the instantaneous carbon flow which is abated owing to this second device, and by c_a the corresponding average cost, also assumed to be constant. Although atmospheric carbon capture seems technically feasible, it is proved to be more costly than emission capture: $c_a > c_e$.[4] The only constraint on this capture flow is:

$$s_a(t) \geq 0 \tag{4}$$

[3]Note that electric traction trains could be good examples of sector 1 users.

[4]Classical devices allowing for the carbon capture and sequestration from the atmosphere consists in increasing the forestlands and changing the agricultural processes. This is not the type of device we consider in the present paper.

Last, there is also some natural self regeneration effect of the atmospheric carbon stock. We assume that the natural proportional rate of decay, denoted by $\alpha > 0$, is constant. Taking into account all the components of the dynamics of $Z(t)$ results into:

$$\dot{Z}(t) = \zeta[x_1(t) + x_2(t)] - [s_a(t) + s_e(t)] - \alpha Z(t), \quad Z(0) \equiv Z^0 < \bar{Z} \tag{5}$$

$$\bar{Z} - Z(t) \geq 0. \tag{6}$$

When the atmospheric carbon stock reaches its critical level, i.e. when $Z(t) = \bar{Z}$, and absent any active capture policy, i.e. $s_a(t) = s_e(t) = 0$, then the total oil consumption $x(t) \equiv x_1(t)+x_2(t)$ is constrained to be at most equal to $\bar{x}$, where $\bar{x}$ is solution of $\zeta x - \alpha\bar{Z} = 0$, that is:

$$\bar{x} = \frac{\alpha}{\zeta}\bar{Z}.$$

We assume that it may be optimal to abate the pollution for delaying the date of arrival at the critical threshold and for relaxing the constraint on the oil consumption flow, that is:

$$c_x + c_a < u'(\bar{x}) \Rightarrow c_x + c_e < u'(\bar{x}).$$

The alternative energy source is supplied by the carbon-free renewable resource, the solar energy. We denote by $y_i(t)$ the solar energy consumption in sector i, $i = 1, 2$, and by c_y the average delivery cost of this alternative energy. Because c_x and c_y both include all the costs necessary to deliver a ready for use energy unit to the potential users, then both resources may be seen as perfect substitutes for the consumers, so that we may define the aggregate energy consumption of sector i as $q_i = x_i + y_i$, $i = 1, 2$, as far as the costs c_x and c_y are incurred.

The average cost c_y is assumed to be constant, the same for both sectors, and higher than $u'(\bar{x}/2)$. This last condition implies that the optimal energy consumption paths can be split into two periods: a first one during which only oil is consumed and a second one during which only solar energy is used. We also have to assume that the natural flow of available solar energy, denoted by y^n, is large enough to supply the energy needs in both sectors during the second period described above. Let $\tilde{y}$ be the sectoral energy consumption that it would be optimal to consume at the marginal cost c_y, that is $\tilde{y} = q^d(c_y)$ for which $u'(\tilde{y}) = c_y$. Then we assume that $y^n > 2\tilde{y}$. Under this assumption, no rent has ever to be imputed for using the solar energy. Thus the only constraint on $y_i(t)$ having to be taken into account along any optimal path is a non-negativity constraint:

$$y_i(t) \geq 0, \quad i = 1, 2. \tag{7}$$

Finally, the instantaneous social rate of discount, denoted by ρ, $\rho > 0$, is assumed to be constant over time.

3 Social planner problem and optimality conditions

The problem of the social planner consists in maximizing the sum of the discounted net current surplus. Let (P) be this program:

$$
\begin{aligned}
(P) \qquad \max_{s_a, s_e, x_i, y_i, i=1,2} \int_0^\infty & \{u\left[x_1(t) + y_1(t)\right] + u\left[x_2(t) + y_2(t)\right] - c_x\left[x_1(t) + x_2(t)\right] \\
& -c_y\left[y_1(t) + y_2(t)\right] - c_a s_a(t) - c_e s_e(t)\} \, e^{-\rho t} dt
\end{aligned}
$$

subject to (1)-(7).

Let us denote by λ_X the costate variable of the state variable X, by λ_Z minus the costate variable of the state variable Z, by γ's the Lagrange multipliers associated with the non-negativity constraints on the command variables, and by ν the Lagrange multiplier associated with the ceiling constraint on Z. As usually done in this kind of problem, we do not take explicitly into account the non-negativity constraint on X. Thus we may write the current value Lagrangian $\mathcal{L}$ of problem (P) as follows:

$$
\begin{aligned}
\mathcal{L}(t) \quad = \quad & u\left[x_1(t) + y_1(t)\right] + u\left[x_2(t) + y_2(t)\right] - c_x\left[x_1(t) + x_2(t)\right] - c_y\left[y_1(t) + y_2(t)\right] \\
& -c_a s_a(t) - c_e s_e(t) - \lambda_X(t)\left[x_1(t) + x_2(t)\right] \\
& -\lambda_Z(t)\left\{\zeta\left[x_1(t) + x_2(t)\right] - \left[s_a(t) + s_e(t)\right] - \alpha Z(t)\right\} \\
& +\nu(t)\left[\bar{Z} - Z(t)\right] + \sum_i \gamma_{x_i}(t) x_i(t) + \sum_i \gamma_{y_i}(t) y_i(t) \\
& +\gamma_{s_a}(t) s_a(t) + \gamma_{s_e}(t) s_e(t) + \bar{\gamma}_{s_e}(t)\left[\zeta x_1(t) - s_e(t)\right]
\end{aligned}
$$

The first-order conditions relative to the command variables are:

$$
\frac{\partial \mathcal{L}}{\partial x_1} = 0 \quad \Rightarrow \quad u'[x_1(t) + y_1(t)] = c_x + \lambda_X(t) + \zeta[\lambda_Z(t) - \bar{\gamma}_{s_e}(t)] - \gamma_{x_1}(t) \tag{8}
$$

$$
\frac{\partial \mathcal{L}}{\partial x_2} = 0 \quad \Rightarrow \quad u'[x_2(t) + y_2(t)] = c_x + \lambda_X(t) + \zeta \lambda_Z(t) - \gamma_{x_2}(t) \tag{9}
$$

$$
\frac{\partial \mathcal{L}}{\partial y_i} = 0 \quad \Rightarrow \quad u'[x_i(t) + y_i(t)] = c_y - \gamma_{y_i}(t), \quad i = 1, 2 \tag{10}
$$

$$
\frac{\partial \mathcal{L}}{\partial s_a} = 0 \quad \Rightarrow \quad c_a = \lambda_Z(t) + \gamma_{s_a}(t) \tag{11}
$$

$$
\frac{\partial \mathcal{L}}{\partial s_e} = 0 \quad \Rightarrow \quad c_e = \lambda_Z(t) - \bar{\gamma}_{s_e}(t) + \gamma_{s_e}(t) \tag{12}
$$

The associated complementary slackness conditions are:

$$\gamma_{x_i}(t) \geq 0,\ x_i(t) \geq 0 \quad \text{and} \quad \gamma_{x_i}(t)x_i(t) = 0, \quad i = 1,2 \tag{13}$$

$$\gamma_{y_i}(t) \geq 0,\ y_i(t) \geq 0 \quad \text{and} \quad \gamma_{y_i}(t)y_i(t) = 0, \quad i = 1,2 \tag{14}$$

$$\gamma_{s_a}(t) \geq 0,\ s_a(t) \geq 0 \quad \text{and} \quad \gamma_{s_a}(t)s_a(t) = 0 \tag{15}$$

$$\gamma_{s_e}(t) \geq 0,\ s_e(t) \geq 0 \quad \text{and} \quad \gamma_{s_e}(t)s_e(t) = 0 \tag{16}$$

$$\bar{\gamma}_{s_e}(t) \geq 0,\ \zeta x_1(t) - s_e(t) \geq 0 \quad \text{and} \quad \bar{\gamma}_{s_e}(t)[\zeta x_1(t) - s_e(t)] = 0 \tag{17}$$

The dynamics of the costate variables must satisfy:

$$\dot{\lambda}_X = \rho\lambda_X - \frac{\partial \mathcal{L}}{X} \quad \Rightarrow \quad \dot{\lambda}_X(t) = \rho\lambda_X(t) \tag{18}$$

$$\dot{\lambda}_Z = \rho\lambda_Z - \frac{\partial \mathcal{L}}{Z} \quad \Rightarrow \quad \dot{\lambda}_Z(t) = (\rho + \alpha)\lambda_Z(t) - \nu(t) \tag{19}$$

together with the complementary slackness condition:

$$\nu(t) \geq 0,\ \bar{Z} - Z(t) \geq 0 \quad \text{and} \quad \nu(t)[\bar{Z} - Z(t)] \geq 0. \tag{20}$$

Last, the transversality conditions take the following forms:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) \quad = \quad 0 \tag{21}$$

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) \quad = \quad 0 \tag{22}$$

Remarks:

1. The shadow marginal value of the stock of oil, or mining rent, $\lambda_X(t)$, must grow at the social rate of discount ρ. From (18), we get:

$$\lambda_X(t) = \lambda_{X_0}e^{\rho t},\ \lambda_{X_0} \equiv \lambda_X(0). \tag{23}$$

   Thus the transversality condition (21) reduces to:

$$\lambda_{X_0} \lim_{t\uparrow\infty} X(t) = 0. \tag{24}$$

   If oil is to have some value, $\lambda_{X_0} > 0$, then it must be exhausted along the optimal path.

2. Concerning the shadow marginal cost of the atmospheric carbon stock, $\lambda_Z(t)$, note that before the date $\underline{t}_Z$ at which the ceiling constraint is beginning to be active, we must have $\nu(t) = 0$ since $\bar{Z} - Z(t) > 0$. Then (19) reduces to $\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z$ so that:

$$t < \underline{t}_Z \ \Rightarrow \ \lambda_Z(t) = \lambda_{Z_0}e^{(\rho+\alpha)t},\ \lambda_{Z_0} \equiv \lambda_Z(0). \tag{25}$$

Once the ceiling constraint is no more active and forever, $\lambda_Z(t) = 0$. Thus, denoting by $\bar{t}_Z$ the latest date at which $Z(t) = \bar{Z}$, we get:

$$t > \bar{t}_Z \;\Rightarrow\; \lambda_Z(t) = 0. \tag{26}$$

In order to simplify the notations in the next sections, it is useful to define the following prices or full marginal costs and the corresponding sectoral consumption levels for which the F.O.C's (8) and (9) relative to $x_1(t)$ and to $x_2(t)$, respectively, are satisfied[5].

- Price or full marginal cost of oil and sectoral oil consumption before the ceiling and absent any abatement, whatever the sector under consideration:

$$p^1(t, \lambda_{X_0}, \lambda_{Z_0}) \equiv c_x + \lambda_{X_0} e^{\rho t} + \zeta \lambda_{Z_0} e^{(\rho+\alpha)t} \tag{27}$$

$$\tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) \equiv q^d\left(p^1(t, \lambda_{X_0}, \lambda_{Z_0})\right) \tag{28}$$

- Price or full marginal cost of oil for consumption in sector 1 given that emissions from this sector are fully or partially abated, i.e. $s_e(t) > 0$, and corresponding oil consumption of sector 1:

$$p_e^2(t, \lambda_{X_0}) \equiv c_x + \lambda_{X_0} e^{\rho t} + \zeta c_e \tag{29}$$

$$\tilde{q}_e^2(t, \lambda_{X_0}) \equiv q^d\left(p_e^2(t, \lambda_{X_0})\right) \tag{30}$$

- Price or full marginal cost of oil for consumption in sector 2 given that some part of the atmospheric carbon stock is captured, $s_a(t) > 0$, and corresponding consumption in this sector:

$$p_a^2(t, \lambda_{X_0}) \equiv c_x + \lambda_{X_0} e^{\rho t} + \zeta c_a \tag{31}$$

$$\tilde{q}_a^2(t, \lambda_{X_0}) \equiv q^d\left(p_a^2(t, \lambda_{X_0})\right) \tag{32}$$

- Price or full marginal cost of oil once the ceiling constraint $\bar{Z} - Z(t) \geq 0$ is no more active and forever, and corresponding sectoral consumptions, whatever the sector:

$$p^3(t, \lambda_{X_0}) \equiv c_x + \lambda_{X_0} e^{\rho t} \tag{33}$$

$$\tilde{q}^3(t, \lambda_{X_0}) \equiv q^d\left(p^3(t, \lambda_{X_0})\right) \tag{34}$$

This last case corresponds to a pure Hotelling regime.

[5] The upper indexes $n = 1, 2, 3$ correspond to the order in which the price p^n and the quantity $\tilde{q}^n$ are appearing along the optimal path. If both $p^n(t, ...)$ and $p^{n+m}(t', ...)$ are appearing along the same path, then it implies that $t < t'$.

Problem solving strategy:

To solve the social planner problem (P), we proceed as follows. First, we check whether the most costly device to capture the carbon has ever to be used. The test consists in solving the social planner problem assuming that the atmospheric carbon capture device is not available. This is inducing some path of atmospheric carbon shadow cost $\lambda_Z(t)$. Then:

- either this shadow cost is permanently lower than the marginal cost of atmospheric carbon capture, that is $\lambda_Z(t) < c_a$ for any $t \geq 0$, and then the atmospheric carbon capture device has never to be used because too costly;

- or there exists some time interval during which $\lambda_Z(t)$ is higher than c_a so that, in this case, the atmospheric carbon capture device must be activated since the loss in the marginal net surplus induced by not using it is higher than its marginal cost of use.

4 Optimal policy without atmospheric carbon capture device

This kind of policies have been investigated and characterized in Chakravorty *et al.* (2006-a, 2006-b) and in Lafforgue *et al.* (2008-a, 2008-b), but for economies in which any potential emissions can be captured and sequestered irrespective of the oil consumption sector. Thus, in their models, there is a single consumption sector, similar to the sector 1 of the present model.

Two important conclusions of these studies are that: i) it is never optimal to abate the potential flow of emissions before attaining the critical level $\bar{Z}$ of atmospheric carbon concentration; ii) along the phase at the ceiling during which it is optimal to abate, only some part of the potential emission flow must be abated; because abating is never optimal excepted during this phase, then it is never optimal to fully abate the potential flow of emissions along the optimal path.

As we shall show, it may happen in the present context that: i) abating the potential emissions of the sector 1 has to begin before the ceiling level $\bar{Z}$ is attained; ii) when it is optimal to begin to capture the sector 1 potential emissions, before the ceiling is attained, then it is optimal to capture its whole potential emission flow.

4.1 Restricted social planner problem

Assuming that the atmospheric carbon capture technology is not available, the social planner problem reduces to the following restricted problem $(R.P)$:

$$(R.P) \qquad \max_{s_e, x_i, y_i, i=1,2} \int_0^\infty \{u\,[x_1(t)+y_1(t)] + u\,[x_2(t)+y_2(t)] - c_x\,[x_1(t)+x_2(t)] \\ -c_y\,[y_1(t)+y_2(t)] - c_e s_e(t)\}\, e^{-\rho t} dt$$

subject to (1), (2), (3), (6), (7) and:

$$\dot{Z}(t) = \zeta[x_1(t)+x_2(t)] - s_e(t) - \alpha Z(t), \quad Z(0) = Z^0 < \bar{Z} \tag{35}$$

The current value Lagrangian of $(R.P)$, denoted by $\mathcal{L}_R$, writes now:

$$\begin{aligned} \mathcal{L}_R(t) &= u\,[x_1(t)+y_1(t)] + u\,[x_2(t)+y_2(t)] - c_x\,[x_1(t)+x_2(t)] - c_y\,[y_1(t)+y_2(t)] \\ &\quad -c_e s_e(t) - \lambda_X(t)\,[x_1(t)+x_2(t)] - \lambda_Z(t)\,\{\zeta\,[x_1(t)+x_2(t)] - s_e(t) - \alpha Z(t)\} \\ &\quad +\nu(t)\left[\bar{Z} - Z(t)\right] + \sum_i \gamma_{x_i}(t) x_i(t) + \sum_i \gamma_{y_i}(t) y_i(t) \\ &\quad +\gamma_{s_e}(t) s_e(t) + \bar{\gamma}_{s_e}(t)\,[\zeta x_1(t) - s_e(t)]\,. \end{aligned}$$

The new F.O.C's relative to the command variables, except s_a, together with the associated complementary slackness conditions, are the same then the ones of the unrestricted problem (P), namely (8)-(12) and (13)-(17). Also the equations (18) and(19) determining the dynamics of the costate variables, the complementary slackness condition (20) on ν and the associated transversality conditions (21) and (22) when t tends to infinity, must hold.

We can conclude that remarks 1 and 2 of the previous section 3 also hold in the present restricted context.

4.2 Optimal paths along which it is optimal to capture and sequester before being at the ceiling

Let us assume that the initial oil endowment is large enough to justify some period at the ceiling during which $Z(t) = \bar{Z}$, and that there exists some period during which the emissions of sector 1 are wholly or partially abated, $s_e(t) > 0$. Two kinds of such paths can be optimal, depending on whether sector 1's emissions have to be captured from the beginning of the planning horizon or later.

4.2.1 Optimal paths along which it is not optimal to abate from the beginning

Such paths are illustrated in the Figure 1 below.

[Figure 1 here]

The optimal price path depicted in Figure 1 is a seven phases path. Denoting by $p_i(t)$, for $i = 1, 2$, the price – or full marginal cost – of oil for sector i, these phases are the following:

- Phase 1, before the ceiling and without abatement: $[0, \underline{t}_e)$

During this phase, the oil price is the same for each sector and it is given by $p_1(t) = p_2(t) = p^1(t, \lambda_{X_0}, \lambda_{Z_0})$. The existence of such a phase requires that $\lambda_{Z_0} < c_e$, so $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) < p_e^2(t, \lambda_{X_0})$, that is capturing sector 1's emissions would be too costly. $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) - p_e^2(t, \lambda_{X_0}) = \zeta \left[\lambda_{Z_0} e^{(\rho+\alpha)t} - c_e\right] < 0$, is increasing so that supporting the marginal shadow cost of the atmospheric carbon stock, $\lambda_Z(t) = \lambda_{Z_0} e^{(\rho+\alpha)t}$, is less costly than abating, that is supporting the marginal cost of abating the sector 1's emissions, c_e.

The oil consumption of each sector is given by $x_1(t) = x_2(t) = \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})$.

The common oil price $p^1(t, \lambda_{X_0}, \lambda_{Z_0})$ is increasing at an instantaneous rate which is higher than the rate of growth of $p_e^2(t, \lambda_{X_0})$. At the end of the phase, denoted by $\underline{t}_e$, both prices are equated $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_e^2(t, \lambda_{X_0})$.

Note that, since $p_1(t) = p_2(t) < u'(\bar{x})$ and $Z^0 < \bar{Z}$, then during this phase both $x_1(t)$ and $x_2(t)$ are higher than $\bar{x}$ so that $Z(t)$ is increasing. However, the existence of this phase requires that, at its end, $Z(t)$ is lower than the critical level $\bar{Z}$: $Z(\underline{t}_e) < \bar{Z}$.

- Phase 2, before the ceiling with full abatement of sector 1's emissions: $[\underline{t}_e, \underline{t}_Z)$

From $\underline{t}_e$ onwards, we have $p_e^2(t, \lambda_{X_0}) < p^1(t, \lambda_{X_0}, \lambda_{Z_0})$. Thus it is now strictly less costly for sector 1 to abate than not to abate, hence $p_1(t) = p_e^2(t, \lambda_{X_0})$, implying that $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$.[6] Moreover, since the inequality is strict then the potential sector 1's emissions are fully abated: $s_e(t) = \zeta x_1(t)$.

[6]Note that during such a phase, because $s_e(t) > 0$ then $\gamma_{s_e}(t) = 0$, so that from (12) we obtain:

$$\lambda_Z(t) = c_e + \bar{\gamma}_{s_e}(t).$$

Substituting for $\lambda_Z(t)$ in (8) and taking into account that $x_1(t) > 0$, hence $\gamma_{x_1}(t) = 0$, and $y_1(t) = 0$, we get:

$$u'(x_1(t)) = c_x + \lambda_{X_0} e^{\rho t} + \zeta c_e,$$

from which we conclude that $p_1(t) = p_e^2(t, \lambda_{X_0})$ and $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$.

Sector 2 is not able to abate its emissions and it must support the carbon shadow cost $\zeta\lambda_{Z_0}e^{(\rho+\alpha)t}$ per unit of burned oil, so that $p_2(t) = p^1(t, \lambda_{X_0}, \lambda_{Z_0})$ and $x_2(t) = \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})$.

Note that, during this phase, since $Z(\underline{t}_e) < \bar{Z}$ and $p_2(t) < u'(\bar{x})$, then $x_2(t) > \bar{x}$ and the atmospheric carbon stock increases. Finally, since $p_2(t) > p_1(t)$, the first of these two prices reaching $u'(\bar{x})$ is $p_2(t)$. However, in order that sector 2's consumption begins to be blockaded at $t = \underline{t}_Z$, we must have simultaneously $p_2(t) = u'(\bar{x})$ and $Z(t) = \bar{Z}$ at the end of the phase.

- Phase 3, at the ceiling with sector 2's oil consumption blockaded and sector 1's emissions fully abated: $[\underline{t}_Z, \tilde{t})$

During this phase, the oil price in sector 2 is given by $p_2(t) = u'(\bar{x})$ and the oil consumption of this sector is set to the maximum consumption level allowed by the ceiling constraint, i.e. $x_2(t) = \bar{x}$. Note that this implies that $\lambda_Z(t) = [u'(\bar{x}) - p^3(t, \lambda_{X_0})]/\zeta$ is decreasing over time during the phase[7].

Since $p_e^2(\underline{t}_Z, \lambda_{X_0}) < u'(\bar{x})$, then $c_e < \lambda_Z(t)$ at the beginning of the phase. Then, once again, abating emissions is proved to be less costly for sector 1 than supporting the shadow cost of the atmospheric carbon stock. Consequently, the sector 1's emissions are fully captured: $s_e(t) = \zeta x_1(t)$. Since $p_1(t) = p_e^2(\underline{t}_Z, \lambda_{X_0})$, we still have $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$.

Given that sector 2's emissions are $\zeta x_2(t) = \zeta\bar{x}$, full abatement in sector 1 implies that, during this phase at the ceiling, the atmospheric carbon stock stays at its critical level: $\dot{Z}(t) = 0$ and $Z(t) = \bar{Z}$. Finally, $p_1(t) = p_e^2(t, \lambda_{X_0})$ is increasing during the phase. At the end of the phase, $p_e^2(t, \lambda_{X_0}) = u'(\bar{x})$ or, equivalently, $\lambda_Z(t) = c_a$.

- Phase 4, at the ceiling with partial abatement of sector 1's emissions: $[\tilde{t}, \bar{t}_e)$

From time $\tilde{t}$ onwards, $p_e^2(t, \lambda_{X_0})$ becomes higher than $u'(\bar{x})$. Thus, the only way to satisfy simultaneously the F.O.C's (8) and (9) on the x_i's is to set $p_1(t) = p_2(t) = p_e^2(t, \lambda_{X_0})$, which implies $x_1(t) = x_2(t) = \tilde{q}_e^2(t, \lambda_{X_0})$ together with a partial abatement of sector 1's emissions. As far as $p_e^2(t, \lambda_{X_0})$ is staying under $u'(\bar{x}/2)$, then the potential emissions amount to $2\zeta\tilde{q}_e^2(t, \lambda_{X_0}) > \zeta\bar{x} = \alpha\bar{Z}$. As far as $p_e^2(t, \lambda_{X_0})$ is now higher than $u'(\bar{x})$, then the

[7] Since the ceiling constraint is active, then $\nu(t)$ is strictly positive and sufficiently high so that $\dot{\lambda}_Z(t) = (\rho+\alpha)\lambda_Z(t) - \nu(t) < 0$.

potential emissions $2\zeta\tilde{q}_e^2(t,\lambda_{X_0})$ stays at a lower level than $2\zeta\bar{x}$, so that:

$$\bar{x} < 2\tilde{q}_e^2(t,\lambda_{X_0}) < 2\bar{x}. \tag{36}$$

In order to satisfy the atmospheric carbon constraint $Z(t) = \bar{Z}$, it is sufficient to abate this part $s_e(t)$ of the sector 1's emissions for which $\dot{Z}(t) = 0$. Thus we may have:

$$2\zeta\tilde{q}_e^2(t,\lambda_{X_0}) - s_e(t) = \zeta\bar{x}. \tag{37}$$

Conditions (36) and (37) imply that:

$$s_e(t) = \zeta\left[2\tilde{q}_e^2(t,\lambda_{X_0}) - \bar{x}\right] < \zeta\tilde{q}_e^2(t,\lambda_{X_0}) = \zeta x_1(t). \tag{38}$$

Hence, during this phase, emissions from sector 1 are only partially abated and, since $\tilde{q}_e^2(t,\lambda_{X_0})$ is decreasing through time then the instantaneous rate of capture $s_e(t)$ is also decreasing. This solution may be optimal if and only if abating and supporting the shadow marginal cost of the atmospheric carbon stock are resulting into the same full marginal cost, that is if and only if $\lambda_Z(t)$ is constant and equal to c_e. Since sector 2 cannot abate its emissions, it is supporting the marginal shadow cost of atmospheric carbon and the condition $p_1(t) = p_2(t) = p_e^2(t,\lambda_{X_0}) = c_x + \lambda_{X_0}e^{\rho t} + \zeta\lambda_Z(t)$ guarantees that $\lambda_Z(t) = c_e$ is satisfied[8].

Since $p_e^2(t,\lambda_{X_0})$ is increasing over time, there exists some date $\bar{t}_e$ at which $p_e^2(t,\lambda_{X_0}) = u'(\bar{x}/2)$. At this date, $x_1(t) = x_2(t) = \bar{x}/2$ and sector 1 ceases to capture its emissions, $s_e(t) = 0$. From $\bar{t}_e$ onwards, we have $p_e^2(t,\lambda_{X_0}) > u'(\bar{x}/2)$ so that the cost of capture of sector 1's emissions becomes prohibitive.

- Phase 5, at the ceiling and without abatement of sector 1's emissions: $[\bar{t}_e, \bar{t}_Z)$

Since abating the sector 1's emissions is now too costly, there is no more abatement and, in order to not overshoot the critical atmospheric carbon level, we must have $p_1(t) = p_2(t) = u'(\bar{x}/2)$ and $x_1(t) = x_2(t) = \bar{x}/2$, so that $\dot{Z}(t) = 0$.

During such a phase, $\lambda_Z(t) = u'(\bar{x}) - p^3(t,\lambda_{X_0})/\zeta$ is decreasing. The phase is ending at time $t = \bar{t}_Z$ when $\lambda_Z(t) = 0$, which implies that $p^3(t,\lambda_{X_0}) > u'(\bar{x}/2)$ for $t > \bar{t}_Z$.

[8] Again, because the ceiling constraint is effective then $\nu(t) > 0$ and, in order that $\dot{\lambda}_Z(t) = 0$, we have: $\nu(t) = (\rho+\alpha)\lambda_Z(t) = (\rho+\alpha)c_e$.

- Phase 6, pure Hotelling phase: $[\bar{t}_Z, t_y)$

This phase is the last one during which energy needs are supplied by oil. This is a pure Hotelling phase. The energy price is the same for the two sectors: $p_1(t) = p_2(t) = p^3(t, \lambda_{X_0}) > u'(\bar{x}/2)$, also generating an identical oil consumption in the two sectors: $x_1(t) = x_2(t) < \bar{x}/2 \Rightarrow x(t) < \bar{x}$.

Since $x(t) < \bar{x}$ and $Z(t) = \bar{Z}$ at the beginning of the phase, then $Z(t) < \bar{Z}$ for $t > \bar{t}_Z$ justifying the fact that now $\lambda_Z(t) = 0$ from $\bar{t}_Z$ onwards[9]. Then $\lambda_Z(t)Z(t) = 0$ and the transversality condition (22) is satisfied.

During the phase, the price is ever increasing and there must exist some time $t = t_y$ at which $p^3(t, \lambda_{X_0}) = c_y$. At this time, this level of oil price makes the renewable resource competitive. To be optimal, the switch from the pure Hotelling regime to a pure renewable regime requires that, at time $t = t_y$, $X(t) = 0$ so that from t_y onwards $\lambda_X(t)X(t) = 0$ warranting that the transversality condition (21) relative to X is satisfied.

- Phase 7, carbon-free renewable energy permanent regime: $[t_y, +\infty)$

From t_y onwards, the economy follows a pure renewable energy regime which is free of carbon emissions: $p_1(t) = p_2(t) = c_y$, $x_1(t) = x_2(t) = 0$ and $y_1(t) = y_2(t) = \tilde{y}$. Since $x_i(t) = 0$, $i = 1, 2$, then $\dot{Z}(t) = -\alpha Z(t)$ so that $Z(t)$ is permanently decreasing down to 0 at infinity: $Z(t) = Z(t_y)e^{-\alpha(t-t_y)}$.

Determination of the characteristics of the optimal path:

The optimal path described above is parametrized by eight variables whose values have to be determined: λ_{X_0}, λ_{Z_0}, $\underline{t}_e$, $\underline{t}_Z$, $\tilde{t}$, $\bar{t}_e$, $\bar{t}_Z$ and t_y. They are given as the solutions of the following eight equations system.

- Balance equation of non-renewable resource consumption and supply:

$$\begin{aligned} 2\int_0^{\underline{t}_e} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})dt &+ \int_{\underline{t}_e}^{\underline{t}_Z} \left[\tilde{q}_1(t, \lambda_{X_0}, \lambda_{Z_0}) + \tilde{q}_e^2(t, \lambda_{X_0})\right] dt \\ &+ \int_{\underline{t}_Z}^{\tilde{t}} \left[\tilde{q}_e^2(t, \lambda_{X_0}) + \bar{x}\right] dt + 2\int_{\tilde{t}}^{\bar{t}_e} \tilde{q}_e^2(t, \lambda_{X_0})dt \\ &+ \left[\bar{t}_Z - \bar{t}_e\right]\bar{x} + 2\int_{\bar{t}_Z}^{t_y} \tilde{q}^3(t, \lambda_{X_0})dt = X^0. \qquad (39) \end{aligned}$$

[9]However, note that $Z(t)$ is not necessarily monotonically decreasing during this phase. What is sure is that there exists some critical time interval $(\bar{t}_Z, \bar{t}_Z + \epsilon)$, with ϵ positive and small enough, during which $\dot{Z}(t) < 0$. For $t > \bar{t}_Z + \epsilon$, it may happen that $\dot{Z}(t) > 0$. But, because $x(t) < \bar{x}$, even if $\dot{Z}(t)$ were temporally increasing, it would not be able to go back to $\bar{Z}$.

- Continuity of the carbon stock at time $\underline{t}_Z$:

$$\begin{aligned} Z^0 e^{-\alpha \underline{t}_Z} + 2\zeta \int_0^{\underline{t}_e} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) e^{-\alpha(\underline{t}_Z - t)} dt & \\ + \zeta \int_{\underline{t}_e}^{\underline{t}_Z} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) e^{-\alpha(\underline{t}_Z - t)} dt &= \bar{Z}. \end{aligned} \tag{40}$$

- Price continuity equations:

$$p^1(\underline{t}_e, \lambda_{X_0}, \lambda_{Z_0}) = p_e^2(\underline{t}_e, \lambda_{X_0}) \tag{41}$$

$$p^1(\underline{t}_Z, \lambda_{X_0}, \lambda_{Z_0}) = u'(\bar{x}) \tag{42}$$

$$p_e^2(\tilde{t}, \lambda_{X_0}) = u'(\bar{x}) \tag{43}$$

$$p_e^2(\bar{t}_e, \lambda_{X_0}) = u'(\bar{x}/2) \tag{44}$$

$$p^3(\bar{t}_Z, \lambda_{X_0}) = u'(\bar{x}/2) \tag{45}$$

$$p^3(t_y, \lambda_{X_0}) = c_y. \tag{46}$$

Assuming a positive solution of system (39)-(46), then it is easy to check that all the optimality conditions of the restricted problem $(R.P)$ are satisfied. Reciprocally, it is clear that there exists values of the parameters of the system c_x, c_y, c_e, ζ, α and ρ together with values of initial endowments of oil X^0 and of atmospheric carbon stock Z^0 such that the path described above is the solution of the restricted problem $(R.P)$. However, other solutions may exist.

4.2.2 Optimal paths along which it is optimal to abate from the beginning

Consider the optimal path characterized in the previous subsection 4.2.1 and some date $\hat{t}$ between $\underline{t}_e$ and $\underline{t}_Z$ (see Figure 1). At this time $\hat{t}$:

- The stock of oil amounts to:

$$\begin{aligned} X(\hat{t}) &= X^0 - 2\int_0^{\underline{t}_e} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) dt - \int_{\underline{t}_e}^{\hat{t}} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) dt \\ &\quad - \int_{\underline{t}_e}^{\hat{t}} \tilde{q}_e^2(t, \lambda_{X_0}) dt. \end{aligned} \tag{47}$$

- The stock of atmospheric carbon is:

$$\begin{aligned} Z(\hat{t}) &= Z^0 e^{-\alpha \hat{t}} + 2\zeta \int_0^{\underline{t}_e} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) e^{-\alpha(\hat{t} - t)} dt \\ &\quad + \zeta \int_{\underline{t}_e}^{\hat{t}} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) e^{-\alpha(\hat{t} - t)} dt + \zeta \int_{\underline{t}_e}^{\hat{t}} \tilde{q}_e^2(t, \lambda_{X_0}) e^{-\alpha(\hat{t} - t)} dt \end{aligned} \tag{48}$$

Assume that the oil endowment of the economy is $X(\hat{t})$ instead of X^0 and that the initial atmospheric carbon stock is $Z(\hat{t}) < \bar{Z}$ instead of Z^0. Since in the present model the solution of $(R.P)$ is time consistent, then the optimal price path restricted by this new initial conditions (47) and (48) proves to be this part of the previous path beginning at time $t = \hat{t}$. More precisely, if $p_i(t)$, $i = 1, 2$, is the solution described in the previous subsection 4.2.1, then the solution, denoted by $\hat{p}_i(t)$, corresponding to the initial conditions (47) and (48) is given by:

$$\hat{p}_i(t) = p_i(t + \hat{t}),\ i = 1, 2. \tag{49}$$

Now, the first phase (i.e. phase 1) before the ceiling and without emission abatement is going out and sector 1's emissions must be captured from the beginning of the planning horizon, that is from $\hat{t}$.

4.3 Paths along which the oil prices path is the same for the two sectors

4.3.1 Paths along which it is optimal to abate sector 1's emissions

Example of such a path, solution of the restricted problem $(R.P)$, is illustrated in Figure 2 below.

[Figure 2 here.]

This kind of paths is characterized by the fact that, at time $t = \underline{t}_e$ at which $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_e^2(t, \lambda_{X_0})$, then the common value of these two prices is larger than $u'(\bar{x})$ while $Z(\underline{t}_e) = \bar{Z}$ simultaneously.

Because $Z^0 < \bar{Z}$ there must exist a first phase $[0, \underline{t}_e)$ during which the ceiling $\bar{Z}$ is not yet attained and $p_1(t) = p_2(t) = p^1(t, \lambda_{X_0}, \lambda_{Z_0}) < p_e^2(t, \lambda_{X_0})$, hence it is not optimal to abate sector 1's emissions. At the end of this first phase, both $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_e^2(t, \lambda_{X_0})$ and $Z(\underline{t}_e) = \bar{Z}$ so that $\underline{t}_e$ coincides with $\underline{t}_Z$.

The next phase $[\underline{t}_e, \bar{t}_e)$ is a phase at the ceiling during which $p_1(t) = p_2(t) = p_e^2(t, \lambda_{X_0})$. As in the phase 4 of the previous case – $[\tilde{t}, \bar{t}_e)$ of the path illustrated in Figure 1 – because sector 2 cannot abate its emissions, we must have $\lambda_Z(t) = c_e$ during the second phase of the present path. Also because $u'(\bar{x}) < p_e^2(t, \lambda_{X_0}) < u'(\bar{x}/2)$, then only some part of the sector 1's emissions have to be captured (cf. the above equation (38)), $s_e(t) <$

$\zeta\tilde{q}_e^2(t,\lambda_{X_0}) = \zeta x_1(t)$, and the capture intensity $s_e(t)$ diminishes. At the end of this phase, $p_e^2(t,\lambda_{X_0}) = u'(\bar{x}/2)$, $x_1(t) = x_2(t) = \bar{x}/2$ and $s_e(t) = 0$.

The third phase $[\bar{t}_e, \bar{t}_Z)$ is still a phase at the ceiling but without capture of sector 1's emissions: $p_1(t) = p_2(t) = u'(\bar{x}/2)$ and $x_1(t) = x_2(t) = \bar{x}/2$. The phase is ending when $p^3(t,\lambda_{X_0}) = u'(\bar{x}/2)$, that is when $\lambda_Z(t) = 0$. The fourth and fifth phases are respectively the standard pure Hotelling phase $[\bar{t}_Z, t_y)$ and the pure renewable energy phase $[t_y, \infty)$.

4.3.2 Paths along which it is never optimal to capture sector 1's emissions

When the abatement cost c_e is very high, capturing is proved to never be an optimal strategy. In this case, we get a four phases optimal price path as illustrated in Figure 3.

[Figure 3 here.]

In Figure 3, $p_e^2(t,\lambda_{X_0})$ is higher than $p^1(t,\lambda_{X_0},\lambda_{Z_0})$ along the whole time interval $[0, \underline{t}_Z)$ before the ceiling. Hence, it is never optimal to capture sector 1's emissions. Such optimal paths have been characterized in Chakravorty *et al.* (2006-a, 2006-b).

5 Optimal policies requiring to activate both capture devices

In this section, we first determine the conditions under which it is optimal to activate the atmospheric carbon capture device. Next we characterize the optimal paths along which both carbon capture technologies must be used. Last, we discuss about the time profile of the optimal carbon marginal shadow cost, that is the optimal unitary carbon tax, as well as the total burden induced by climate change mitigation policies in each sector, including the tax burden and the abatement cost.

5.1 Checking whether the atmospheric carbon capture is optimal

Let us consider the four kinds of optimal price paths which may solve the planner restricted problem $(R.P)$ and which have been discussed in the previous section. Clearly, since $p_a^2(t,\lambda_{X_0}) > p_e^2(t,\lambda_{X_0})$, then for the two last kinds of optimal paths illustrated in Figures 2 and 3 in the subsections 4.3.1 and 4.3.2 respectively, the price trajectory $p_a^2(t,\lambda_{X_0})$ (not depicted in these figures) is always located above the optimal price path. Hence, it is never optimal to use the atmospheric carbon capture device.

For the two first kinds of optimal paths illustrated in Figure 1, with a starting point at time $t = 0$ as studied in the subsection 4.2.1, or at time $\hat{t}$ as characterized in the subsection 4.2.2, then it may happen that using the atmospheric carbon capture technology reveals optimal. To check whether this technology is optimal or not, the test runs as follows. Consider the price path $p_a^2(t, \lambda_{X_0})$ (not depicted in Figure 1). Then at time $t = \underline{t}_Z$, either $p_a^2(t, \lambda_{X_0}) < u'(\bar{x})$ or $p_a^2(t, \lambda_{X_0}) \geq u'(\bar{x})$. In the first case, there must exist a time interval around $t = \underline{t}_Z$ such that $p_2(t) > p_a^2(t, \lambda_{X_0})$ and it would be less costly for sector 2 to bear the cost of the atmospheric capture c_a than the burden of the shadow cost of the atmospheric carbon stock $\lambda_Z(t)$. In the second case, using the atmospheric carbon capture technology could not allow to improve the welfare.

5.2 Optimal paths

Let us assume now that the atmospheric carbon capture technology has to be used. Then we may obtain two kinds of optimal paths depending on whether the least costly emission capture technology has to be activated from the beginning or not. The typical optimal path along which it is not optimal to capture the sector 1's emission flows from the start is illustrated in Figure 4 below.

[Figure 4 here.]

The path is an eight phases path and the difference with the trajectory depicted in Figure 1 is that a new phase $[\underline{t}_a, \bar{t}_a)$ – the third one in the present case – appears now during which some of the atmospheric carbon is captured. The seven other phases are similar to the ones which have been described in section 4.2.1. This new phase begins at $t = \underline{t}_a$ when $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_a^2(t, \lambda_{X_0})$, that is when $\lambda_Z(t) = c_a$. Then for $t > \underline{t}_a$, it becomes less costly for sector 2 to undertake atmospheric carbon capture rather than to pay the social cost of the carbon accumulation within the atmosphere. At the time sector 2's abatement begins, the ceiling is reached, so that $\underline{t}_a$ coincides with $\underline{t}_Z$.

During this phase $[\underline{t}_a, \bar{t}_a)$, each sector uses simultaneously its own abatement technology. We have $p_1(t) = p_e^2(t, \lambda_{X_0})$ and $p_2(t) = p_a^2(t, \lambda_{X_0})$, which implies $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$ and $x_2(t) = \tilde{q}_a^2(t, \lambda_{X_0})$. Since $c_e < c_a$, we also have $p_1(t) < p_2(t)$ and then $x_1(t) > x_2(t)$. Remember that, during this phase, as in the phase 3 of subsection 4.2.1, sector 1's emissions are fully captured: $s_e(t) = \zeta x_1(t)$. Because this is a phase at the ceiling, sector 2 has just to capture in the atmosphere the necessary amount of carbon in order to maintain the atmospheric carbon stock at its critical level. It is thus optimal for sector 2 to abate at a level which is smaller than its own carbon emissions: $s_a(t) = \zeta x_2(t) - \alpha \bar{Z} < \zeta x_2(t)$. Moreover, since $s_a(t) > 0$, we have $\zeta x_2(t) > \alpha \bar{Z}$, or equivalently, $x_2(t) > \bar{x}$, implying in turns $p_2(t) < u'(\bar{x})$. The price path $p_2(t) = p_a^2(t, \lambda_{X_0})$ being increasing through time, first the amount of abated carbon by the atmospheric device $s_a(t)$ is decreasing, second there must exist a date at which $p_2(t) = u'(\bar{x})$, that is at which $x_2(t) = \bar{x}$ and $s_a(t) = 0$. At that time, denoted by $\bar{t}_a$, since sector 1 still fully abates all its emissions, it is no more optimal for sector 2 to pursue the atmospheric carbon capture. All the efforts to maintain the carbon stabilization cap are now supported by the sole sector 1 and the economy identically behaves as in section 4.2.1 from phase 3, that if from the date $\underline{t}_Z$ as depicted in Figure 1.

To the eight variables parameterizing the optimal path in the case without atmospheric capture technology (cf. subsection 4.2.1), we must here identify two additional variables: $\underline{t}_a$ (equal to $\underline{t}_Z$) and $\bar{t}_a$. We thus obtain nine variables that must solve the following nine equations system:

- Balance equation of non-renewable resource consumption and supply:

$$
\begin{aligned}
2\int_0^{\underline{t}_e} \tilde{q}^1(t,\lambda_{X_0},\lambda_{Z_0})dt + \int_{\underline{t}_e}^{\underline{t}_a=\underline{t}_Z} \left[\tilde{q}_1(t,\lambda_{X_0},\lambda_{Z_0}) + \tilde{q}_e^2(t,\lambda_{X_0})\right] dt \\
+ \int_{\underline{t}_a=\underline{t}_Z}^{\bar{t}_a} \left[\tilde{q}_e^2(t,\lambda_{X_0}) + \tilde{q}_a^2(t,\lambda_{X_0})\right] dt + \int_{\bar{t}_a}^{\tilde{t}} \left[\tilde{q}_e^2(t,\lambda_{X_0}) + \bar{x}\right] dt \\
+2\int_{\tilde{t}}^{\bar{t}_e} \tilde{q}_e^2(t,\lambda_{X_0})dt + \left[\bar{t}_Z - \bar{t}_e\right]\bar{x} + 2\int_{\bar{t}_Z}^{t_y} \tilde{q}^3(t,\lambda_{X_0})dt \quad = \quad X^0. \qquad (50)
\end{aligned}
$$

- Continuity of the carbon stock at time $\underline{t}_Z$: identical to (40).

- Price continuity equations: identical to (41)-(46) except that (42) is now replaced by the two following equations:

$$
p^1\left(\underline{t}_a, \lambda_{X_0}, \lambda_{Z_0}\right) = p_a^2(\underline{t}_a, \lambda_{X_0}) \qquad (51)
$$

$$
p_a^2\left(\bar{t}_a, \lambda_{X_0}\right) = u'(\bar{x}) \qquad (52)
$$

5.3 Time profile of the optimal carbon tax

The trajectory of the carbon marginal shadow cost corresponding to the optimal path illustrated in Figure 4 is characterized by:

$$\lambda_Z(t) = \begin{cases} \lambda_{Z_0} e^{(\rho+\alpha)t} & , t \in [0, \underline{t}_Z) \\ c_a & , t \in [\underline{t}_Z, \bar{t}_a) \\ \left[u'(\bar{x}) - p^3(t, \lambda_{X_0}) \right] / \zeta & , t \in [\bar{t}_a, \tilde{t}) \\ c_e & , t \in [\tilde{t}, \bar{t}_e) \\ \left[u'(\bar{x}/2) - p^3(t, \lambda_{X_0}) \right] / \zeta & , t \in [\bar{t}_e, \bar{t}_Z) \\ 0 & , t \in [\bar{t}_Z, \infty) \end{cases} \tag{53}$$

This shadow cost can be interpreted as the optimal unitary tax to be levied on the net carbon emissions. Its time profile is illustrated in Figure 5 below.

[Figure 5 here.]

The unitary tax rate is first increasing but is bounded from above by the highest marginal abatement cost c_a which is attained when it becomes optimal to use this abatement device and, simultaneously, when the atmospheric carbon stock constraint begins to be active, that is at time $t = \underline{t}_a = \underline{t}_Z$. Given that it is always possible to choose to abate rather than release the carbon in the atmosphere, the maximal tax rate of carbon emissions is necessarily determined by the highest marginal cost permitting to avoid polluting carbon releases.

During the ceiling phases, from $\underline{t}_Z$ up to $\bar{t}_Z$, the carbon tax is either constant or decreasing. First, as long as sector 2 abates, that is between $\underline{t}_a$ and $\bar{t}_a$, it is sufficient to set the tax rate equal to c_a to induce an optimal atmospheric capture by sector 2, given that sector 1 fully abates its own emissions. The same applies between $\tilde{t}$ and $\bar{t}_e$ for sector 1 by setting the tax rate equal to c_e, given that sector 2 no more abates. Between these two phases, that is between $\bar{t}_a$ and $\tilde{t}$, and during the last phase at the ceiling, that is between $\bar{t}_e$ and $\bar{t}_Z$, the tax rate strictly decreases. This is due to the oil price increase and to the fact that the emission level is constrained by $\bar{x}$ for sector 2 during $[\bar{t}_a, \tilde{t})$, and by $\bar{x}/2$ for each sector during $[\bar{t}_e, \bar{t}_Z)$.

5.4 Time profile of the tax burden and of the sequestration cost

Assume now that the tax rate described above is implemented. The instantaneous induced fiscal income from each sector is defined by $\Gamma_1(t) \equiv [\zeta x_1(t) - s_e(t)]\lambda_Z(t)$ for sector 1 and by $\Gamma_2(t) \equiv [\zeta x_2(t) - s_a(t)]\lambda_Z(t)$ for sector 2. The sequestration cost in each sector simply writes as the sequestered carbon flow times the respective marginal cost of sequestration:

$S_1(t) \equiv s_e(t)c_e$ and $S_2(t) \equiv s_a(t)c_a$. Then, the total burden of carbon mitigation efforts for each sector is the sum of the fiscal burden and the sequestration cost. Denoting by $B_i(t)$ $i = 1, 2$ such a total burden, the two following tables details its components for each sector.

$\Gamma_1(t)$	$S_1(t)$	$B_1(t)$	Phases
$\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$	0	$\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$	$[0, \underline{t}_e)$
0	$\zeta\tilde{q}_e^2(t)c_e$	$\zeta\tilde{q}_e^2(t)c_e$	$[\underline{t}_e, \tilde{t})$
$\zeta\left[\bar{x} - \tilde{q}_e^2(t)\right]c_e$	$\zeta\left[2\tilde{q}_e^2(t) - \bar{x}\right]c_e$	$\zeta\tilde{q}_e^2(t)c_e$	$[\tilde{t}, \bar{t}_e)$
$(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$	0	$(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$	$[\bar{t}_e, \bar{t}_Z)$
0	0	0	$[\bar{t}_Z, \infty)$

Table 1. Decomposition of the total carbon reduction burden in sector 1.

$\Gamma_2(t)$	$S_2(t)$	$B_2(t)$	Phases
$\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$	0	$\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$	$[0, \underline{t}_a)$
$\zeta\bar{x}c_a$	$\zeta[\tilde{q}_a^2(t) - \bar{x}]c_a$	$\zeta\tilde{q}_a^2(t)c_a$	$[\underline{t}_a, \bar{t}_a)$
$\bar{x}\left[u'(\bar{x}) - p^3(t)\right]$	0	$\bar{x}\left[u'(\bar{x}) - p^3(t)\right]$	$[\bar{t}_a, \tilde{t})$
$\zeta\tilde{q}_e^2(t)c_e$	0	$\zeta\tilde{q}_e^2(t)c_e$	$[\tilde{t}, \bar{t}_e)$
$(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$	0	$(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$	$[\bar{t}_e, \bar{t}_Z)$
0	0	0	$[\bar{t}_Z, \infty)$

Table 2. Decomposition of the total carbon reduction burden in sector 2.

Their time profile are depicted by Figure 6 below.

[Figure 6 here.]

Before the ceiling phases, the shapes of the total burden trajectories may be either increasing or decreasing depending upon oil demand elasticity. Once the ceiling is reached, the total burden gradually declines down to zero at the end of the ceiling phase.

For sector 1, the total burden identifies to the sole tax burden as long as abatement is not activated, that is before $\underline{t}_e$. Between $\underline{t}_e$ and $\tilde{t}$, sector 1, fully abating its emissions, does not bear the carbon tax burden ($\Gamma_1(t) = 0$), but it bears the sequestration cost $S_1(t)$. During this phase, since sector 1's emissions decrease, so does its sequestration cost and thus its total burden. During the next phase, between $\tilde{t}$ and $\bar{t}_e$, it is no more optimal for sector 1 to fully abate its emissions and then, this sector bears a mix of tax burden and abatement cost. Even if its gross carbon emissions diminish, its sequestration flow decreases at a higher rate resulting in an increase in the net emission flow. The cost of sequestration thus decreases. Since the tax rate is constant and equal to c_e, the fiscal burden rises. The combined effect of these two evolutions results in a decline of the total carbon burden of sector 1. Over the last ceiling phase, between $\bar{t}_e$ and $\bar{t}_Z$, sector 1 no more abates and bears only the fiscal burden. Then its total burden is declining and it reaches zero when the ceiling constraint becomes no more active, that is at time $\bar{t}_Z$.

Sector 2 bears simultaneously the tax and the sequestration cost burden only during the atmospheric capture phase, that is between $\underline{t}_a$ and $\bar{t}_a$. During this phase, its fiscal burden is constant because i) the tax rate is constant and equal to c_a and ii) sector 1 fully abates its emissions and sector 2's net emissions are constrained by $\bar{x}$. Its sequestration effort decreases since gross emissions decline. After $\bar{t}_a$ and during all next phases at the ceiling, the total burden of sector 2 reduces to the sole fiscal burden and it is thus decreasing over time as discussed above.

We conclude by two remarks. First, the total fiscal income, that is $\Gamma_1(t) + \Gamma_2(t)$, jumps down twice at each time where either sector 1 or sector 2 begins to abate. Then, any redistributive environmental policy should take into account the ability of polluters to undertake abatement activities and thus to escape from the tax. Second, since sector 2 is constrained by the higher cost of its abatement technology, its fiscal contribution as well as its total burden are larger or equal than the total burden of sector 1 even if pollutive intensities and demand functions are the same for both sectors.

6 Conclusion

In a Hotelling depletion model, we have determined the optimal exploitation time paths of two energy resources, one being depletable and carbon-emitting, the other being renewable and carbon-free, by two sectors that are heterogeneous regarding their respective abatement capacities. The optimal paths have been considered along with the following features. First, sector 1 is able to abate its carbon emissions, but not sector 2. Second, to reduce pollution, sector 2 can only have recourse to the atmospheric capture technology, which is highly more expensive than the emission capture. Third, the cumulative atmospheric carbon stock is set not to exceed some critical threshold.

We have shown that the optimal path requires that emission abatement by sector 1 must be undertaken before the point of time at which the atmospheric carbon stock reaches its critical threshold and that sector 1's emissions must be wholly abated. This first result contrasts with the results established by Chakravorty *el al.* (2006-b) in a model with a single energy using sector and a single abatement technology. It can thus be explained by the assumption of heterogeneity introduced here, which constraints the potential of emission capture to be at the most equal to the sole emissions of sector 1 and then to be always smaller the total carbon emissions of fossil energy users.

Heterogeneity means that the abatement costs are not the same for all the energy users. This is the crucial assumption generating the early and full abatement of sector 1's potential emissions. Clearly in the present model we could have assumed that the sector 2 can abate its emissions at the marginal cost c_a instead of having to capture the carbon in the atmosphere. The reason is that the flow of carbon captured in the atmosphere is lower than the potential carbon emission flow of sector 2 when carbon is captured in the atmosphere. Thus would the carbon be captured from sector 2's emissions at the marginal cost c_a the result would be the same, that is the optimal price paths, optimal sectoral consumption paths and captured carbon paths by the two abatement technologies would be the same. To reinforce the heterogeneity argument, we show in a companion paper (Amigues *et al.*, 2010) that, when all the energy consumers have access to the same abatement costs, then even learning by doing in the abatement technology does not justify to begin to abate too early, that is before being at the ceiling[10].

Finally, atmospheric carbon capture by sector 2 is implemented only once the ceiling is reached and sector 2's intensity of abatement is always smaller than its real contribution to the common atmospheric carbon accumulation, which is now in accordance with the results of Chakravorty *et al.* (2006-b).

[10]However, note that the time at which $Z(t)$ reaches its critical level $\bar{Z}$ is endogenous. Thus learning by doing is not without effect on this date.

References

[1] Amigues, J.P., Lafforgue, G. and Moreaux, M. (2010). Optimal abatement policies of potential carbon emissions with learning by doing in abatement. *LERNA working paper*, Toulouse School of Economics.

[2] Barrett, S. (2009). Climate treaties with a backstop technology. *Working Paper*, Columbia University.

[3] Chakravorty, U., Magné, B. and Moreaux, M. (2006-a). Plafond de concentration en carbone et substitutions entre ressources énergétiques. *Annales d'Economie et de Statistiques*, 81, 141-168.

[4] Chakravorty, U., Magné, B. and Moreaux, M. (2006-b). A Hotelling model with a ceiling on the stock of pollution. *Journal of Economic Dynamics and Control*, 30, 2875-2904.

[5] Edmonds, J., Clarke, J., Dooley, J., Kim, S.H., Smith, S.J. (2004). Stabilization of CO2 in a B2 world: insights on the roles of carbon capture and disposal, hydrogen, and transportation technologies. *Energy Economics*, 26, 517-537.

[6] Gerlagh, R., van der Zwaan, B.C. (2006). Options and instruments for a deep Cut in CO2 emissions: carbon capture or renewable, taxes or subsidies? *Energy Journal*, 27, 25-48.

[7] Intergovernmental Panel on Climate Change (2005). Special Report on Carbon Dioxide Capture and Storage, Working Group III.

[8] Intergovernmental Panel on Climate Change (2007). Climate Change 2007, Synthesis Assessment Report, Working Group III.

[9] Islegen, O., Reichelstein, S. (2009). The economics of carbon capture. *The Economist's Voice*, December: The Berkeley Electronic Press.

[10] Lafforgue, G., Magne B., Moreaux M. (2008-a). Energy substitutions, climate change and carbon sinks. *Ecological Economics*, 67, 589-597.

[11] Lafforgue, G., Magne B., Moreaux M. (2008-b). The optimal sequestration policy with a ceiling on the stock of carbon in the atmosphere. In *The Design of Climate Policy (Chapter 14, pp. 273-304)*, R. Guesnerie and H. Tulkens eds, Boston: The MIT Press.

[12] McFarland, J.R., Herzog, H.J., Reilly, J.M., 2003. Economic modelling of the global adoption of carbon capture and sequestration technologies. In: *Proceedings of the Sixth International Conference on Greenhouse Gas Control Technologies*, Gale, J., Kaya, Y. (Eds.). Elsevier Science, Oxford.

[13] McKay, D. (2009). Sustainable energy - Without the hot air. UIT Cambridge Ltd.: Cambridge.

[14] McKinsey & Company (2007). Reducing U.S. greenhouse gas emissions: How much at what cost? Available at:
http://www.mckinsey.com/clientservice/ccsi/greenhousegas.asp.

[15] MIT (2007). The future of coal. Available at: http://web.mit.edu/coal.

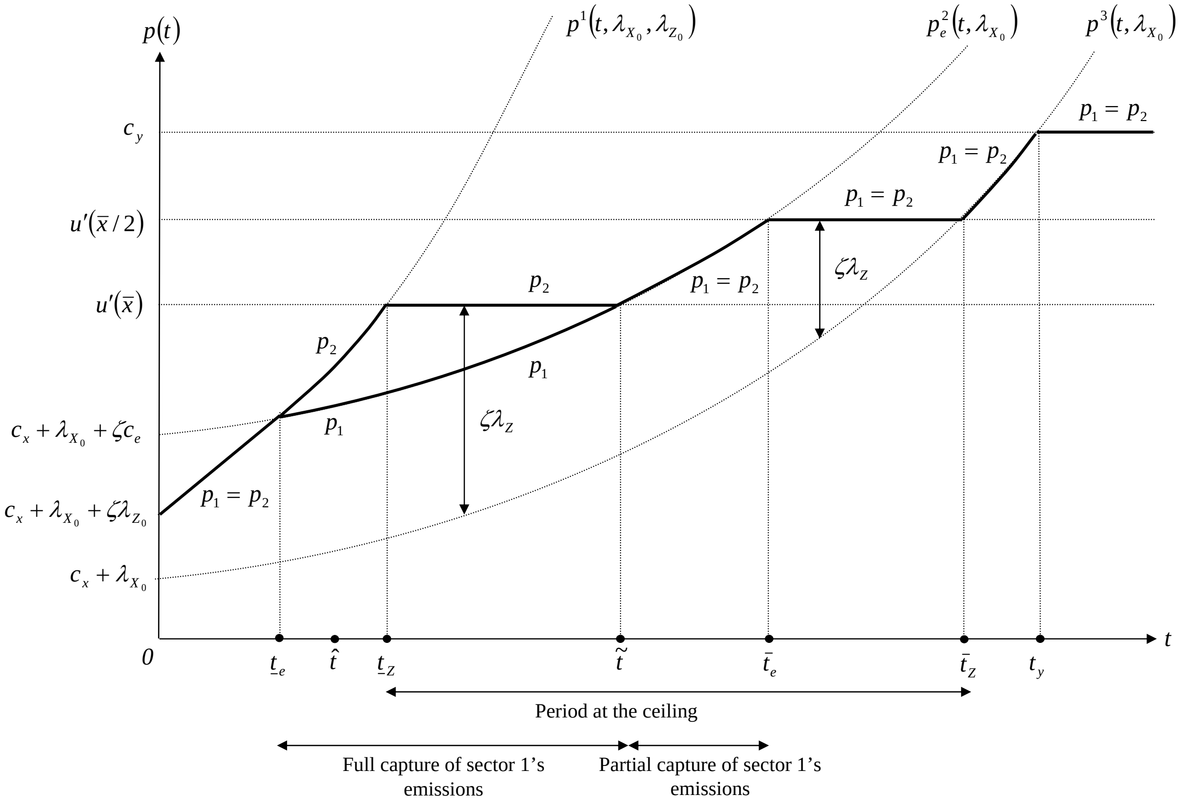

Figure 1: Optimal path along which it is optimal to abate before the ceiling, but not from the beginning of the planning horizon

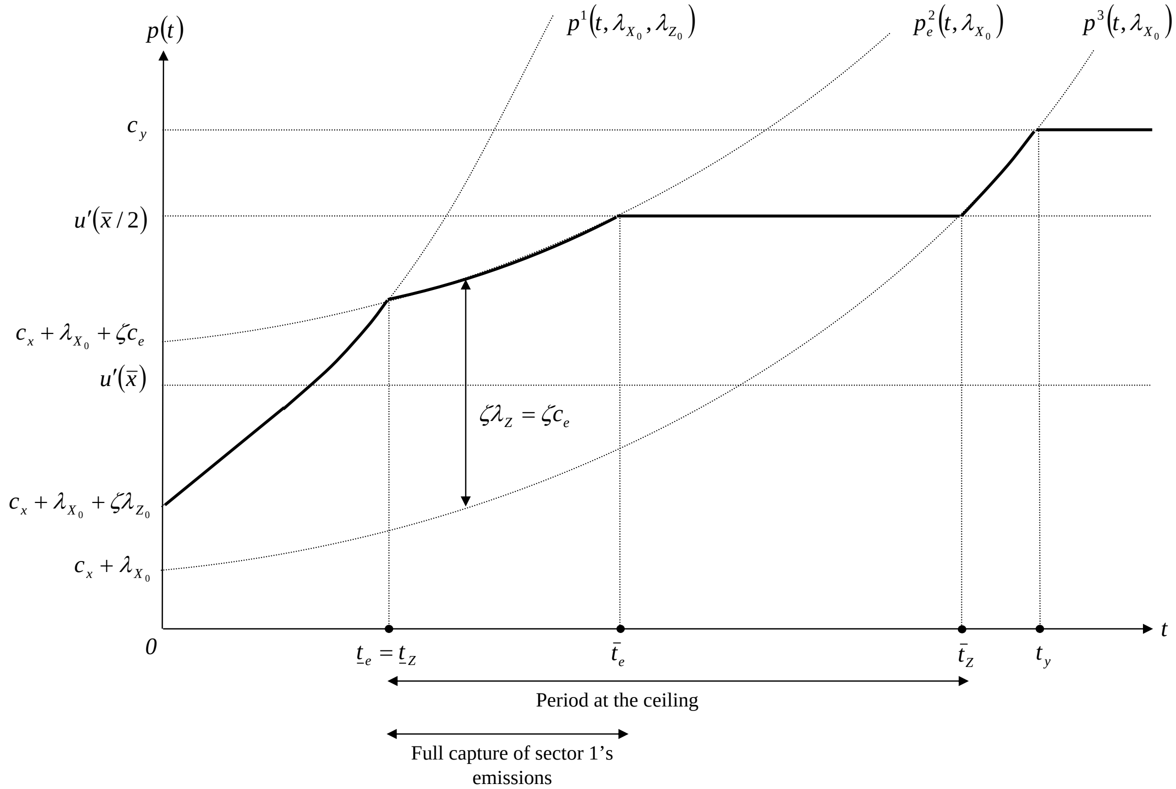

Figure 2: Optimal path along which the energy price is the same for each sector and it is optimal to abate sector 1's emissions

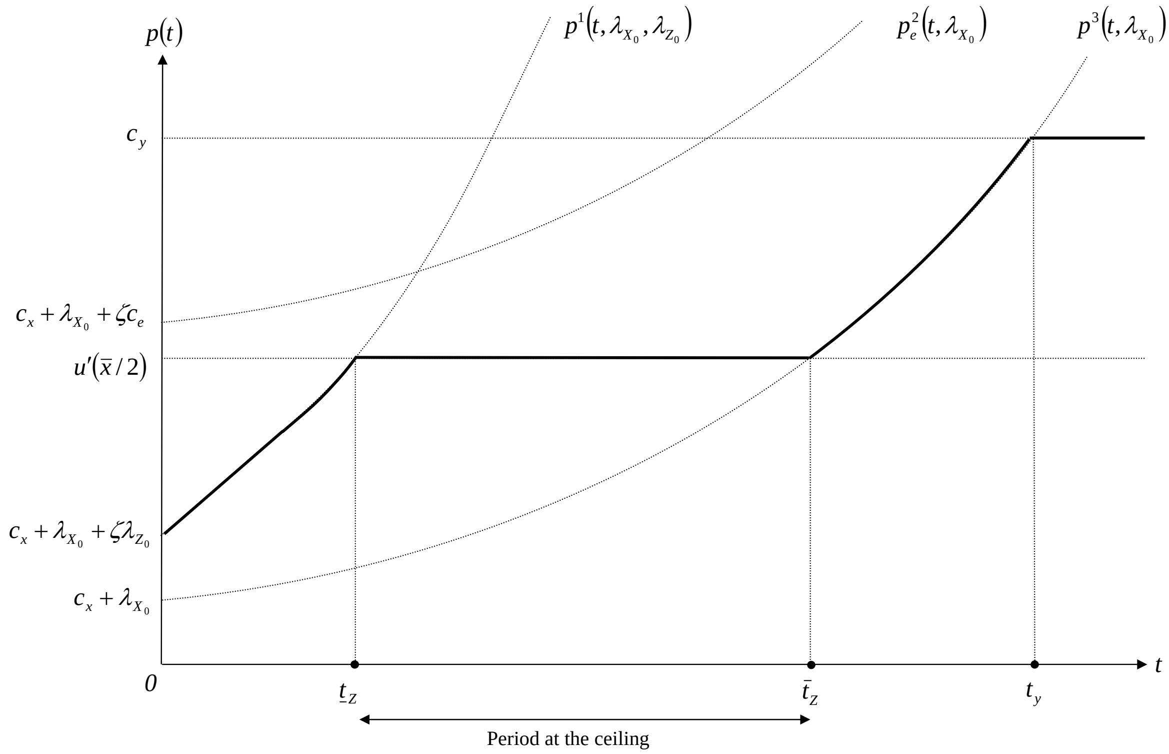

Figure 3: Optimal path along which the energy price is the same for each sector and it is not optimal to abate sector 1's emissions

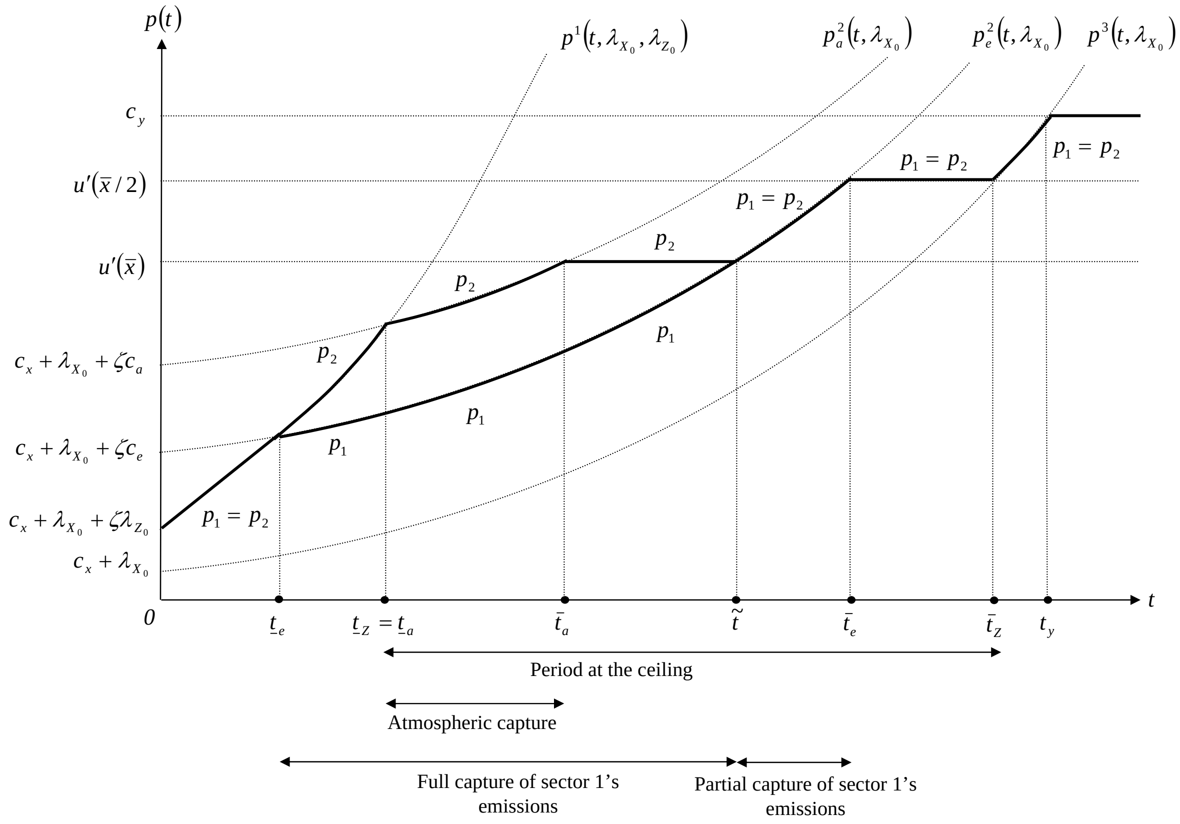

Figure 4: Optimal path requiring to activate the both carbon capture devices

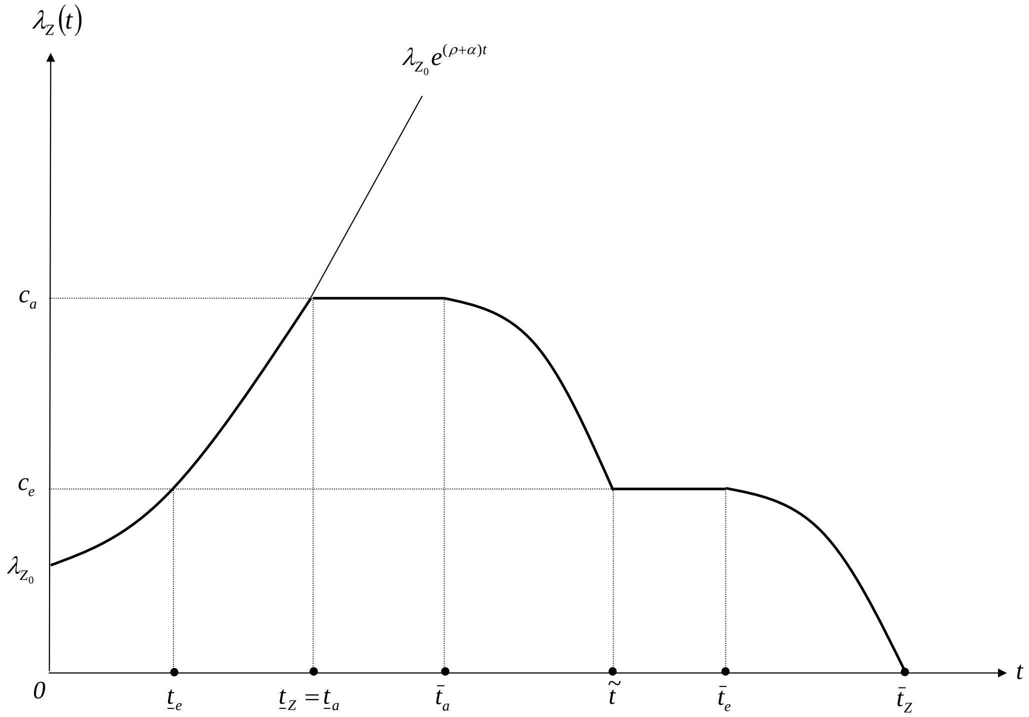

Figure 5: Time profile of the optimal unitary carbon tax

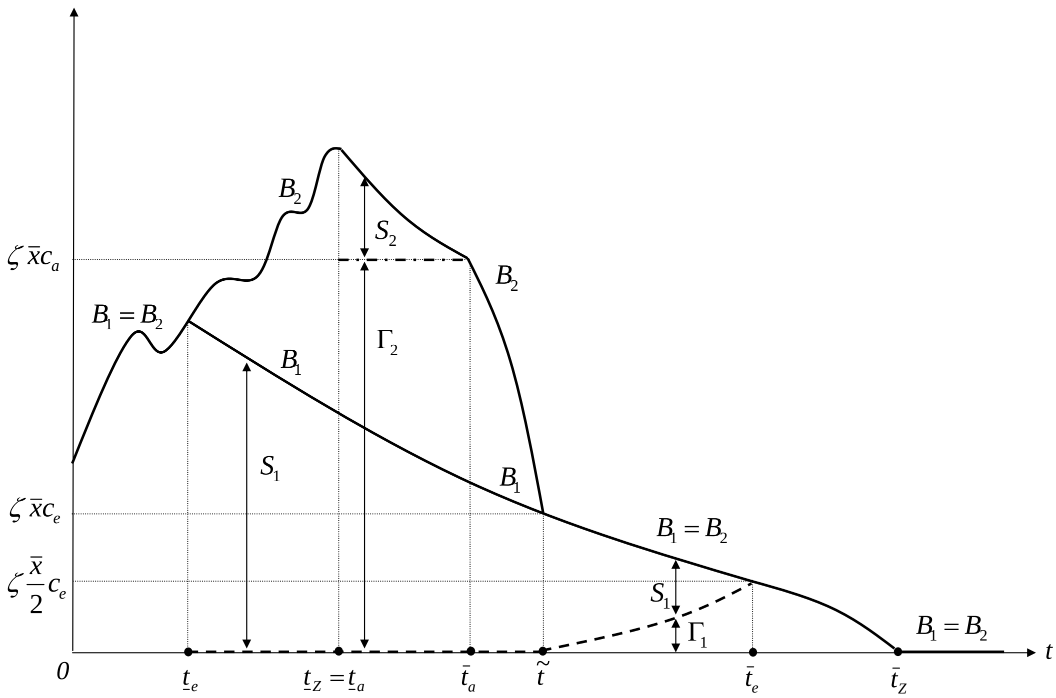

Figure 6: Total burden of carbon for each sector