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Competition in two-sided markets with common network externalities[1]

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Abstract

We study competition in two sided markets with *common network externality* rather than with the standard inter-group effects. This type of externality occurs when *both groups benefit*, possibly with different intensities, from an *increase* in the size of one group and from a *decrease* in the size of the other. We explain why common externality is relevant for the health and education sectors. We consider different remuneration schemes on the provider side: pure salary, fee-for service or mixed payments. We show that when the externality itself satisfies an homogeneity condition then platforms' profits and price structure have some specific properties. Our results reveal how the rents coming from network externalities are shifted by platforms from one side to other, according to the homogeneity degree. In the specific but realistic case where the common network externality is homogeneous of degree zero, platforms' profit do not depend on the intensity of the (common) network externality. Prices are affected but in such a way that platforms only transfer rents from consumers to providers. We show that a fee-for service leads to a lower remuneration for providers than a flat salary scheme. This, in turn, results in a lower prices level so that profits are the same under both regimes. When platforms combine both remuneration schemes on the providers' side, there exists a continuum of symmetric equilibria, parametrized by one of the three prices. Nevertheless, platforms' profits are the same in of all these equilibria.

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1 Introduction

The theory of two-sided markets has been developed in recent years to investigate market structures in which two groups of agents interact via platforms; see for instance Rochet and Tirole (2006). The central theme of this literature is the presence of network externalities, which occur when the benefit from joining a platform for individuals of a given group depends on the size of membership (and/or usage) of the other group. Prominent examples of sectors in which such inter-group externalities occur range from credit cards, media and software to dating clubs.

We consider a two-sided market with an externality of a different nature. We shall refer to it as a "common network externality". It occurs when externalities can be summed up by a (quality) index that positively affects the utility of both groups. More precisely, *both groups benefit*, possibly with different intensities, from an *increase* in the size of one group and from a *decrease* in the size of the other. Such externalities are relevant in a number of two-sided markets. For instance, in the health care sector, hospitals compete for patients on one side and for providers on the other side (see Pezzino and Pignatoro, 2008). It is a conventional assumption that the quality of health care depends on the providers' "workload". This is documented, for instance, by Tarnow-Mordi *et al.* (2000) who use UK data to show that variations in mortality can be explained in part by excess workload in the intensive care unit. Accordingly, health care quality is frequently related to the provider/patient ratio; see Mc Gillis Hall (2004). In other words, it increases when the number of health care professionals increases (for a given number of patients) but decreases when the number of patients increases (for a given number of providers). Both sides benefit from a higher quality albeit for different reasons and possibly with different intensity. This is quite obvious on the patients' side, where one can expect a higher quality to translate into a improvement in patients' health state (or at the very least into a reduction in waiting lines for appointments, *etc...*). Physicians benefit from a higher quality through a reduction in their workload,[1]

[1] See for instance Fergusson-Paré (2004) for the nursing workload. Griffin and Swan (2006) also find a strong relationship between nurses' workload and quality of health care.

or indirectly, through their altruism (or simply job satisfaction).[2]

Similar issues may arise in the education sector. Colleges or universities compete for students on one side and for professors on the other side. The quality of education depends on the pupil/teacher ratio and one can expect both sides to benefit from a higher quality. This is confirmed by surveys in which parents and teachers declare that they prefer a smaller class size (Mueller *et al.*, 1988). Furthermore, lower pupil/teacher ratios are associated with higher test scores for the children (see for instance Angrist and Lavy, 1999) and a smaller class size tends to increase average future earnings (Card and Krueger, 1992). On the other side, teachers enjoy an improved job satisfaction and a lower workload as the pupil/teacher ratio decreases.[3]

In this paper, we revisit the Armstrong's framework with a *common network externality* rather than with the standard inter-group effects. Two for-profit platforms compete in prices for singlehoming agents, *i.e.* customers and providers, located on two distinct Hotelling's lines.[4] The common externality enters the preferences of both groups as a quality parameter. Each group values the common externality with (possibly) different intensities but the underlying notion of quality that matters (the functional form that specifies quality) is the same for both groups. We consider two alternative schemes for the payment of providers: salary on the one hand, and fee-for-service on the other hand. We focus on the symmetric equilibrium and show that when the externality is specified by an homogeneous function, price structure and platforms' profit present some special features. We show that when providers receive a salary, network externalities affect prices in a cumulative way: the price on one side of the market depends on the sum of the externality terms on both sides of the market. Second, the effect on one side's price is, partially or entirely, shifted to the other side of the market. The extend of this shifting depends on the sign of the homogeneity degree of the common network exter-

[2] See, *e.g.*, Choné and Ma (2010).

[3] Buckingham (2003) finds that a reduction of class size slightly increases achievment, but also increases teachers work conditions by lightening their workload and easing classroom managment.

[4] In the education and health sectors, for-profit firms may also compete with non-profit organizations. To understand the implications of our common network externalities within the simplest possible setting, we focus on symmetric equilibria with for-profit firms. The study of mixed markets is a naturel extension of our analysis.

nality (or more precisely, the degree of homogeneity of the function that relates quality to the membership on both sides). Third, under both payment schemes, competition intensity is also affected by this homogeneity degree. Specifically, the homogeneity degree determines the impact of the common network externality on the platforms' profits.

Our results have particularly strong implication for the education and health sectors, where quality is known to mainly depend on consumer/provider ratio *i.e.,* the common externality is homogeneous of degree zero. In this case, platforms' profits do not depend on the intensity of the (common) network externalities. This property is in sharp contrast to the results obtained so far in the two-sided market literature. One of the major findings which has been reiterated in many settings is that the presence of network externalities in a two-sided market structure increases the intensity of competition when the externality is positive (and decreases it when the externality is negative).[5] We show that in a context of common network externality of degree zero, this is not the case. Under this assumption, prices are affected by the externality but in such a way that platforms only transfer rents from one group to the other. Roughly speaking, some rents due to the common network externality are extracted from the "consumers' side" and transferred to "providers". Furthermore, we show that for nonzero degrees of homogeneity, the conventional result can be generalized in an intuitively appealing way. When the degree of homogeneity of the common network externality is positive, platforms' profits decrease in the externality parameters. We can think about this case as that where the global impact of the externality is positive. A negative degree of homogeneity yields exactly the opposite result.

While these results hold under both types of payment schemes, price remuneration levels differ. Specifically, prices and payment per provider are lower under fee-for-service than under salary schemes. As far as profits are concerned these two effects exactly cancel out so that profits are the same for the two payment schemes. These results are particularly instructive for the health care sector where both schemes are used in practice. Most of the literature on the design of payment schemes in the health sector focuses on incentives for cost reduction within a principal-agent framework. These

[5] See, *e.g.,* Armstrong (2006).

studies typically advocate that flat payment schemes give better incentives to providers than fee-for-service.[6] We show that when competitive strategies are accounted for, fee-for-service schemes may appear in a more positive light. In particular, when hospitals compete in a two-sided environment, *i.e.* for patients and providers, a fee-for-service scheme allows to reduce the patients' bill.

Finally, we analyze the case where platforms combine both remuneration schemes on the providers' side. As expected, and in line with Armstrong's findings, we show that there exists a continuum of symmetric equilibria, parametrized by one of the three prices. More precisely, the price charged on the customers' side and the salary paid to providers are reduced according to the fee-for-service received by providers. Consequently, for a given level of quality obtained in (symmetric) equilibrium, patients' welfare increases as a larger share of the providers' remuneration is paid through a fee-for service schemes. Platforms' profits, on the other hand, are the same in all symmetric equilibria. An interpretation of this result is that the intensity of the competition is independent of the weights put on both types of payments within the providers' remuneration scheme.

Before proceeding, let us have a closer look at the relationship of our paper to the existing literature. As pointed out by Rochet and Tirole (2003), the two-sided literature is at the intersection between multi-product pricing and network theories. The main focus of this paper lies on the second aspect. Several types of network externalities have been analyzed in the two-sided markets literature. The standard one is the *inter-group* network externality which we have mentioned above. It has also been pointed out that negative *intra-group* network "externality" can occur in equilibrium. This may be the case when members of a given group compete with each other. An additional member on one side then not only creates a positive inter-group externality but, at the same time, it can adversely affect welfare of the other members of the considered group.[7] For instance, in Bardey and Rochet (2010), health plans compete for policyholders on one side and for physicians on the other side. When a health plan enlists more physicians, this directly increases welfare of its policyholders. However, at the same time, it may tend

[6] For instance, Gaynor and Pauly (1990) show this result within a theoretical model. Their structural estimations confirm that incentives affect the quantity of medical services that are delivered.

[7] Most of time, this effect occurs because it increases the number of competitors.

to attract riskier policyholders who place a higher value on the diversity of physicians. The induced adverse selection problem can be seen as a negative intra-group network "externality" that occurs on the policyholders' side.

These intra-group effects are of course strictly speaking not externalities as they operate through the price system. However, some recent papers have also considered proper negative intra-group network externalities. Belleflamme and Toulemonde (2007) develop a model where agents value positively the presence of members of the other group, but may value negatively members of their own group. For instance, both advertisers and consumers benefit from a large representation of the other group (positive inter-brand externality) but advertisers are in competition for eyeballs (negative intra-brand externality). Kurucu (2008) analyses a matching problem in which agents on one side prefers more agents on the other side but less on their own side. Such a configuration of externalities can occur for matrimonial or job matching agencies.

Our paper is inspired by Belleflamme and Toulemonde (2007) and Kurucu (2008) from whom we borrow the presence of negative intra- and positive inter-groups network externalities. However, we combine the same ingredients in a different way. In our framework, an additional consumer generates a negative intra-group and a positive inter-group network externality. Roughly speaking, the utility of a consumer is increasing in the number of providers and is decreasing in the number of the other consumers affiliated with the same platform. On the providers' side, network externalities work on the opposite direction. In other words, the utility of a provider is increasing in the number of providers affiliated to the same platform (positive intra-group network externality), while it is decreasing in the number of consumers present on the other side (negative inter-group network externality). The combination of these two characteristics leads to our concept of *common network externalities*: both groups benefit, possibly with different intensities, from an increase in the size of one group and from a decrease in the size of the other group.

The paper is organized as follows. Section 2 lays out the model. In Section 3, we determine the equilibrium and study its properties when providers are remunerated via salaries. We extend the equilibrium analysis to the case in which providers are remunerated through a fee-for-service in Section 4, while both remuneration schemes are combined in Section 5. Finally, some illustrations are provided in Section 6.

2 The model

Consider two platforms $j = \{1, 2\}$ located at both endpoints of the Hotelling's segment. They compete for two groups of agents $i = \{A, B\}$ of mass 1 (group A) and m (group B) respectively. Agents of each group are uniformly distributed over an interval of length 1. The utilities of both groups exhibit quadratic transportation costs with parameters t_A and t_B respectively. For the sake of simplicity, we shall refer to members of group A as "customers" while group B individuals are considered as "providers". We shall return to this interpretation later.

The utility of a group A individual (a customer), located at z, who patronizes platform j (consumes one unit of its product) is given by

$$V = \overline{V} + \gamma q_j - P_j - t_A (z - x_j)^2,$$

where P_j denotes platform j's price, while γ measures the preference intensity for a quality q_j. An individual of group B (a provider), located at y, who works (a given number of hours) for platform j has utility

$$U = \overline{U} + \theta q_j + T_j - t_B (y - x_j)^2,$$

where θ is the preference for quality q_j, while T_j denotes the transfer paid by platform j to its providers. Without loss of generality, reservation utilities are equal to zero. Consequently, the constants $\overline{V}$ and $\overline{U}$ denote the gross utility on sides A and B; they are assumed to be sufficiently large to ensure full coverage on both sides of the market. Platforms maximize profits and simultaneously set their price/transfer vectors (P_j, T_j), $j = 1, 2$.

2.1 Network externalities

Let n_j^i denote the *share* of type $i = A, B$ individuals affiliated with platform $j = 1, 2$, while N_j^i denotes the *number* of affiliates. With our normalizations we have $N_j^A = n_j^A$

and $N_j^B = mn_j^B$. The quality offered by platform j depends on its number of affiliates in both groups and is determined by

$$q_j = f(N_j^A, N_j^B) = f\left(n_j^A, mn_j^B\right).$$

This function specifies what we refer to as a "*common network externality*" and which is defined as follows.

Definition 1 *A common network externality, described by the function $q_j = f(N_j^A, N_j^B)$, occurs when both sides value, possibly with different intensities, the same network externality.*

An important feature of this definition is that the functional form f is the same on both sides. In other words, customers and providers agree on the ranking of quality levels. However, the taste for quality (measured by γ and θ) can differ between customers and providers. Since A refers to the consumer side, while index B is used for the provider side we assume $\partial f/\partial N_i^A < 0$ and $\partial f/\partial N_i^B > 0$.[8]

Prominent examples of such a common externality can be found in the health care and education markets. In the hospital sector, for instance, on can think of n_j^A as representing the number of patients while mn_j^B stands for the number of physicians. Alternatively, n_j^A can be interpreted as the number of students while mn_j^B stands for the number of teachers. In both of these cases, one would expect quality to increase with mn_j^B and to decrease with n_j^A. A formulation often used in the literature on education and health is given by $q_j = \left(lmn_j^B/n_J^A\right)^\delta$, where l and δ are positive constants. With this specification the quality offered by a hospital or a university depends upon provider/patient or teacher/student ratio, and the function f is homogenous of degree 0.[9] More generally, one can assume that the function specifying the quality is

[8] While this assumption reflects the spirit of our definition of the common network externality, it is of no relevance to our formal analysis. Specifically, Propositions 1, 2 and 3 do not rely on this assumption. However, the assumption is important for the interpretation of our results (and their economic content). It is also key to understanding the difference between our framework and Kurucu (2008) and Belleflamme and Toulemonde (2007). In our setting, the utility functions on both sides, $V^A(N_A, N_B)$ and $U^B(N_A, N_B)$ are both decreasing in N_A and increasing in N_B. In their framework (but with our notations), $V^A(N_A, N_B)$ is increasing in N_B but decreasing in N_A while $U^B(N_A, N_B)$ is increasing in N_A but decreasing in N_B.

[9] Krueger (2003) provides a cost-benefit analysis of class size reduction. He shows that the internal rate of return of a class size reduction from 22 to 15 students is around 6%.

homogenous of degree k, which may or may not be positive. For instance when quality is specified by

$$q_j = \left(N_j^B\right)^\beta / \left(N_J^A\right)^\alpha, \tag{1}$$

f is homogenous of degree $\beta - \alpha$.

Using subscripts to denote the derivatives of f with respect to its first and second arguments (N_j^A and N_j^B respectively) and applying Euler's law yields the following property.

Property 1 *When a common network externality is homogenous of degree k then $N_j^A f_A(N_j^A, N_j^B) + N_j^B f_B(N_j^A, N_j^B) = k f(N_j^A, N_j^B)$.*

We do *not* impose this assumption when determining the equilibrium in the next section. However, it will turn out that the equilibrium has specific properties when the common externality is homogenous of degree k. We shall focus more particularly on the realistic case $k = 0$ which has some strong implications.

2.2 Demand functions

On group A's side, the marginal consumer indifferent between two platforms is determined by

$$\tilde{z} = \frac{1}{2} + \frac{1}{2t_A}\left[\gamma\left(q_1 - q_2\right) - \left(P_1 - P_2\right)\right],$$

while in group B, the marginal provider is given by

$$\tilde{y} = \frac{1}{2} + \frac{1}{2t_B}\left[\theta\left(q_1 - q_2\right) + \left(T_1 - T_2\right)\right].$$

As both sides are fully covered, demand levels are equivalent to market shares. On side A, we have $n_1^A = \tilde{z}$ and $n_2^A = 1 - \tilde{z}$, while on side B, $n_1^B = \tilde{y}$ and $n_2^B = (1 - \tilde{y})$. Defining the quality differential between platforms as

$$g\left(n_1^A, mn_1^B\right) = f\left(n_1^A, mn_1^B\right) - f\left(1 - n_1^A, m(1 - n_1^B)\right) = q_1 - q_2,$$

the demand functions are determined by the following system of implicit equations

$$n_1^A = \frac{1}{2} + \frac{1}{2t_A}\left[\gamma g\left(n_1^A, mn_1^B\right) - \left(P_1 - P_2\right)\right], \tag{2}$$

$$n_1^B = \frac{1}{2} + \frac{1}{2t_B}\left[\theta g\left(n_1^A, mn_1^B\right) + \left(T_1 - T_2\right)\right]. \tag{3}$$

In practice, the transfer T_j can take different forms. In the education sector, schools usually pay a salary to their teachers. In the health care sector, fee-for-service schemes were traditionally predominant. However, more recently salary contracts have become increasingly popular; see Cutler and Zeckhauser (2000). Formally, the remuneration of a provider affiliated with platform j is an affine function of N_j^A/N_j^B (the number of patients per provider) where w_j represents the fixed remuneration (or salary), while c_j denotes the fee-for-service[10]

$$T_j = w_j + c_j \frac{N_j^A}{N_j^B} = w_j + c_j \frac{n_j^A}{mn_j^B}. \tag{4}$$

We successively consider the two types of remuneration systems, namely a pure wage scheme $(w_j > 0$ and $c_j = 0)$ in Section (3) and a fee-for-service plan $(w_j = 0$ and $c_j > 0)$ in Section (4). Both schemes are combined in section (5).

Let $\phi = (\gamma, \theta, t_A, t_B, m)$ denote the vector of exogenous parameters. Equations (2)–(3) define the demand levels of platform 1,

$$n_1^A (P_1, P_2, w_1, w_2, c_1, c_2, \phi) \qquad \text{and} \qquad n_1^B (P_1, P_2, w_1, w_2, c_1, c_2, \phi),$$

as functions of both platforms' price/wage/fee-for-service vectors and of the exogenous variables.[11] With full market coverage on both sides, demand levels of platform 2 are then also fully determined and given by $n_2^A = 1 - n_1^A$ and $n_2^B = 1 - n_1^B$.

Substituting (4) in (2) and (3), totally differentiating the demand expressions, rear-

[10]The fee-for-service payment differs from the "usage" or "transaction" fee usually considered in the two-sided market literature. In the traditional specification, the price paid by one group is solely an affine function of the number of agents in the other group (see *e.g.,* Rochet and Tirole, 2003 and Armstrong, 2006).

[11]We assume throughout the paper that demands are well defined and unique for any price levels. When $q_j = mn_j^B - n_j^A$, it is straightforward that demands are uniquely defined. Appendix A shows that it is also the case when $q_j = \left(mn_j^B/n_J^A\right)$.

ranging and solving yields

$$\frac{dn_1^A}{dP_1} = \frac{1}{|\Psi|}\left[-\frac{1}{2t_A}\left(1-\frac{1}{2t_B}\left(\theta m g_B - \frac{4c_1}{m}\right)\right)\right], \tag{5}$$

$$\frac{dn_1^B}{dP_1} = \frac{1}{|\Psi|}\left[-\frac{1}{4t_Bt_A}\left(\theta g_A + \frac{4c_1}{m}\right)\right], \tag{6}$$

$$\frac{dn_1^A}{dw_1} = \frac{1}{|\Psi|}\left[\frac{\gamma m g_B}{4t_Bt_A}\right], \tag{7}$$

$$\frac{dn_1^B}{dw_1} = \frac{1}{|\Psi|}\left[\frac{1}{2t_B}\left(1-\frac{\gamma g_A}{2t_A}\right)\right], \tag{8}$$

$$\frac{dn_1^A}{dc_1} = \frac{1}{|\Psi|}\left[\frac{\gamma g_B}{4t_Bt_A}\right] = \frac{1}{m}\frac{dn_1^A}{dw_1}, \tag{9}$$

$$\frac{dn_1^B}{dc_1} = \frac{1}{|\Psi|}\left[\frac{1}{2mt_B}\left(1-\frac{\gamma g_A}{2t_A}\right)\right] = \frac{1}{m}\frac{dn_1^B}{dw_1} \tag{10}$$

where $g_A = \partial g/\partial N_1^A$, $g_B = \partial g/\partial N_1^B$ and

$$\Psi = 1 - \frac{\theta}{2t_B}mg_B - \frac{\gamma}{2t_A}g_A + \frac{2c}{mt_B}\left[1-\gamma\frac{(g_A+mg_B)}{2t_A}\right].$$

These properties are used in the next sections to determine the market equilibrium.

3 Market equilibrium with salary schemes

In this section, we assume that the transfers paid by platform j to its providers is a salary w_j (which does not depend on the number of patients) and set $c_j = 0$. We first determine the price equilibrium and then study the properties of the corresponding allocation. Platform 1 maximizes its profit with respect to P_1 and w_1 and solves

$$\max_{P_1,w_1} \Pi_1 = P_1 n_1^A(P_1,P_2,w_1,w_2,\phi) - m w_1 n_1^B(P_1,P_2,w_1,w_2,\phi).$$

The first-order conditions are given by[12]

$$\frac{\partial \Pi_1}{\partial P_1} = n_1^A + P_1 \frac{\partial n_1^A}{\partial P_1} - m w_1 \frac{\partial n_1^B}{\partial P_1} = 0, \tag{11}$$

$$\frac{\partial \Pi_1}{\partial w_1} = -m w_1 \frac{\partial n_1^B}{\partial w_1} - m n_1^B + P_1 \frac{\partial n_1^A}{\partial w_1} = 0. \tag{12}$$

The first two terms of equations (11) and (12) represent the traditional marginal income tradeoff, while the third terms capture the two-sided market feature. Specifically, an increase in the price charged on one side of the market also affects the demand on the other side. Equations (11) and (12) determine platform 1's best-reply functions: $P_1 = \widetilde{P}_1 (P_2, w_2, \phi)$ and $w_1 = \widetilde{w}_1 (P_2, w_2, \phi)$. Platform 2's best-reply functions $P_2 = \widetilde{P}_2 (P_1, w_1, \phi)$ and $w_2 = \widetilde{w}_2 (P_1, w_1, \phi)$ can be determined in a similar way by the maximization of Π_2. Solving these best-reply functions yields the Nash equilibrium $[(P_1^*, w_1^*), (P_2^*, w_2^*)]$.

In the remainder of the paper, we concentrate on symmetric equilibria in which both platforms charge the same prices, pay the same wages and equally split the market on both sides ($n_1^A = n_2^A = 1/2$ and $n_1^B = n_2^B = 1/2$) so that quality levels are also identical ($g = 0$). To determine the symmetric equilibrium we solve (11) and (12). The derivatives of n_1^A and n_1^B that appear in these expressions are given by equations (5)–(8); with $n_1^A = 1/2$, $n_1^B = 1/2$ and $c_1 = c_2 = 0$, they are all well determined and the problem reduces to the solution of a system of linear equations.[13]

Using the Cramer's rule, we obtain

$$P_1 = \frac{\frac{1}{2}\left[\frac{\partial n_1^B}{\partial w_1} + m \frac{\partial n_1^B}{\partial P_1}\right]}{D}, \qquad w_1 = \frac{\frac{1}{2}\left[m \frac{\partial n_1^A}{\partial P_1} + \frac{\partial n_1^A}{\partial w_1}\right]}{mD}, \tag{13}$$

[12] In two-sided market models, second-order condition are usually quite complex. With our general specification of externality this problem can only be reinforced. To establish a link with the conditions stated in Armstrong (2006), let us consider the linear case where quality is given by $q_j = m n_j^B - n_j^A$. The second-order conditions for this specification are derived in Appendix B. They are shown to be satisfied iff

$$(2t_B - \theta m)(2t_A + \gamma) \geq \frac{m}{4} [\gamma - \theta]^2 .$$

As in Armstrong (2006), this condition implies that transportation costs are sufficiently high with respect to the externality parameters. In Section 6, we present a numerical example for the case where $q_j = m n_j^B / n_j^A$ and show that the second-order conditions are satisfied.

[13] The derivates depend on n_1^A and n_1^B (which are by definition set at 1/2) but not directly on P_1 and w_1. The underlying reason for this simplification is that for the determination of demands only *differences* in prices and wages matter; see expressions (2)–(3).

where

$$D = \left[-\frac{\partial n_1^A}{\partial P_1} \frac{\partial n_1^B}{\partial w_1} + \frac{\partial n_1^A}{\partial w_1} \frac{\partial n_1^B}{\partial P_1} \right]. \tag{14}$$

Substituting (5)–(8) and rearranging yields

$$\frac{1}{2} \left[\frac{\partial n_1^B}{\partial w_1} + m \frac{\partial n_1^B}{\partial P_1} \right] = \frac{1}{2} \left[\frac{2t_A - \gamma g_A - \theta m g_A}{4 t_A t_B \Phi} \right], \tag{15}$$

$$\frac{1}{2} \left[m \frac{\partial n_1^A}{\partial P_1} + \frac{\partial n_1^A}{\partial w_1} \right] = \frac{1}{2} \left[\frac{-2t_B + \theta m g_B + \gamma g_B}{4 t_A t_B \Phi} \right], \tag{16}$$

$$D = \frac{1}{4 t_A t_B \left(1 - \frac{\theta}{2t_B} m g_B - \frac{\gamma}{2t_A} g_A \right)}, \tag{17}$$

where g_A and g_B are evaluated at $(1/2, m/2)$. Substituting (15)–(17) into (13), simplifying and defining $g_A^* = g_A(1/2, m/2)$ and $g_B^* = g_B(1/2, m/2)$ establishes the following proposition:

Proposition 1 *When providers receive a salary, symmetric equilibrium prices are given by*

$$P_j^* = t_A - \frac{1}{2} (\gamma + m\theta) g_A^*, \tag{18}$$

$$w_j^* = -t_B + \frac{1}{2} (\gamma + m\theta) g_B^*, \ \forall j = 1, 2. \tag{19}$$

Observe that this proposition provides a closed form solution with explicit expression for the equilibrium prices. To interpret these expressions recall that per our assumption on f, we have $g_A^* < 0$ and $g_B^* > 0$. As usual, on both sides, platforms take advantage of transportation costs to increase their mark-up. Network externalities, on the other hand, affect prices in a more interesting way. Because of their specific nature *i.e.* common externality, their impact is "cumulative", as is reflected by the factor $(\gamma + m\theta)$ in the second term. In other words, network externalities are charged on the consumers' side, which generates the negative component of the CNE, and are transferred to providers, who generate the positive element of the CNE.

Using (18) and (19) we can now express equilibrium profits as

$$\begin{aligned} \Pi_j^* &= \frac{1}{2} \left(P_j^* - m w_j^* \right) \\ &= \frac{1}{2} (t_A + m t_B) - \frac{1}{4} (\gamma + m\theta) (m g_B^* + g_A^*). \end{aligned} \tag{20}$$

Differentiation with respect to the network externalities parameters gives

$$\frac{\partial \Pi_j^*}{\partial \theta} = m \frac{\partial \Pi_j^*}{\partial \gamma} = -\frac{1}{4} \left(m g_B^* + g_A^* \right). \tag{21}$$

Equation (21) establishes the following proposition.

Proposition 2 *When providers receive a salary, the impact of individual valuations of quality γ and θ on (symmetric) equilibrium profits is described by*

$$m\frac{\partial \Pi_j^*}{\partial \gamma} = \frac{\partial \Pi_j^*}{\partial \theta} \lesseqqgtr 0 \qquad \text{if and only if} \qquad -g_A^* \lesseqqgtr m g_B^*, \; \forall j = 1, 2.$$

Proposition 2 shows that the impact of the externality (or, more precisely of the relevant preference parameters) on profits depend on the relative strength of the externalities created by the membership on the two sides. To interpret this proposition, we shall concentrate on the case where the common externality is homogenous of degree k. According to Property 1 we then have $N_1^B f_A + N_1^A f_A = kf$. Moreover, at a symmetric equilibrium, we have:

$$\begin{aligned} g_A &= 2f_A, \\ g_B &= 2f_B, \end{aligned}$$

so that

$$m g_B^* + g_A^* = 4kf\left(\frac{1}{2}, \frac{m}{2}\right).$$

Proposition 3 *Assume that providers receive a salary. When f is homogenous of degree k the symmetric equilibrium implies $m g_B^* + g_A^* = 4kf\left(\frac{1}{2}, \frac{m}{2}\right)$, so that*

$$sign\left(\frac{\partial \Pi_j}{\partial \gamma}\right) = sign\left(\frac{\partial \Pi_j}{\partial \theta}\right) = -sign\left(k\right), \qquad j = 1, 2. \tag{22}$$

Proposition 1 has shown that network externalities have a cumulative effect on prices. Proposition 3 shows how profits and competition intensity are affected. When the degree of homogeneity of the common network externality is positive, platforms' profit decrease in the externality's parameters. In other words, the common network externality increases the competition intensity between platforms. This outcome occurs because in this case, platforms have to pay a higher relative price w_j^*/P_j^* on providers' side. Recall that we have a positive externality created by one side and a negative externality generated by the other side. We can think about the case of $k > 0$ as that where the global impact of the network externalities is positive. In other words, if we increase membership on both sides in the same proportion, f (and thus quality) increases. From that perspective we can think of our finding as a generalization of the conventional result in the literature (relating profits and intensity of competition to the sign of the externality).

When $k < 0$, on the other hand, we have a negative global externality which brings about extra profits for the platforms. The wage paid to providers continues to increase in the network externality parameters. However, this increased cost is now more than fully shifted to the consumers. This is because quality is more sensitive to the number of consumers–and recall that quality decreases with the number of consumers. Consequently, the common network externality tends to reduce the intensity of competition on the consumers' side. The firms are then able to extract more extra rents from the consumers than they have to concede to the providers.

Finally, let us consider the special case in which the homogeneity degree is equal to 0 *i.e.* $k = 0$.

Corollary 1 *For a symmetric equilibrium under salary schemes,* $\forall j = 1, 2$ *with* f *homogenous of degree* 0 *we have* $mg_B^* + g_A^* = 0$*, so that*

$$\frac{\partial \Pi_j}{\partial \gamma} = \frac{\partial \Pi_j}{\partial \theta} = 0.$$

In that case, the intensity of preferences for quality (and thus the intensity of the externality) has no impact on equilibrium profits. Specifically, profit levels are the same when the externality does not matter at all (in which case $\gamma = \theta = 0$) as when one or both of these parameters are positive. In other words, a common network externality that is homogenous of degree zero, has no impact on the intensity of competition which is in stark contrast with conventional results obtained in the two-sided market literature (for alternative forms of externalities). The expressions for the prices (18) and (19) make it clear why this result emerges. Assuming $g_B^* > 0$ (providers produce a positive externality) the externality in itself (or an increase in its valuation on either side) increases "rents" on the providers' side: wages are increased. However, this increase in wages has no impact on profits because it is entirely shifted to consumers: the price increase exactly matches the increase in wages.

4 Market equilibrium with fee-for-service schemes

We now consider the same market structure except that providers receive a fee-for-service. Each provider of platform $j = 1, 2$ now receives a payment that is proportional to his number of patients. Setting $w_j = 0$ for $j = 1, 2$, platform 1 maximizes its profit with respect to P_1 and c_1 and solves

$$\begin{aligned} \max_{P_1, c_1} \Pi_1 &= P_1 n_1^A (P_1, P_2, c_1, c_2, \phi) - m c_1 n_1^B (P_1, P_2, c_1, c_2, \phi) \frac{n_1^A}{m n_1^B} \\ &= n_1^A (P_1, P_2, c_1, c_2, \phi) (P_1 - c_1). \end{aligned} \tag{23}$$

The first-order conditions are given by[14]

$$\frac{\partial \Pi_1}{\partial P_1} = n_1^A + \frac{\partial n_1^A}{\partial P_1} (P_1 - c_1) = 0, \tag{24}$$

$$\frac{\partial \Pi_1}{\partial c_1} = -n_1^A + \frac{\partial n_1^A}{\partial c_1} (P_1 - c_1) = 0. \tag{25}$$

Substituting (5) and (9) where $w_1 = w_2 = 0$ into the first-order conditions, rearranging and solving establishes the following proposition.

Proposition 4 *When providers are remunerated via fee-for service, symmetric equilibrium prices are given by*

$$P_j^{**} = \frac{m t_B}{2} + t_A - \left(\frac{2 g_A + m g_B}{4} \right) (\gamma + \theta m), \tag{26}$$

$$c_j^{**} = -\frac{m t_B}{2} + \frac{m g_B (\theta m + \gamma)}{4}. \tag{27}$$

It is worth noticing that c_j^{**} is exactly equal to $w_j^* / (2m)$, that is one half of the fee-for-service implied by the equilibrium level of w in the salary game. In other words,

[14] Second order conditions are much more complex in this case and are left in appendix B2.

when platforms compete in fee-for-service levels rather than in salaries their providers' equilibrium compensation is cut in half. To understand why fee-for-service leads to lower compensations, let us start from the equilibrium salary w^*. By definition, this salary level is such that no platform can gain by decreasing its salary given the salary offered by the other platform. Now, when fee-for-service is the strategic variable, a decrease in a say c_1 induces (for a given level of c_2) a reduction in compensation offered by platform 2 (because some providers move to platform 2). This implies that a reduction in c_1 (given c_2) is beneficial, even though a reduction in w_1 (given w_2) is not. Interestingly, the price level is also smaller with the fee-for-service scheme. To see this, combining (26) and (18) to obtain $P_j^{**} = P_j^* - (m/2)w^*$. Intuitively, we can once again start from the equilibrium under salary schemes. By definition, platform 1 cannot gain by decreasing its price given P_2 and w_2. In the fee-for-service case, a reduction in P_1 brings about a reduction in the compensation (per provider) paid by platform 2 (because some patients more from platform 2 to platform 1). This in turn mitigates the negative effects of a decrease in the price and implies that a unilateral price decrease is beneficial when c_2 is held constant even though it was not beneficial when w_2 was constant. Theses results are summarized in the following proposition.

Proposition 5 *We have $P_j^{**} < P_j^*$ and $mc_j^{**} < w_j^*$. Consequently, customers pay a lower price and providers are less remunerated under a fee-for-service scheme than under a salary scheme.*

The heath economics literature has extensively dealt with the relative merits of payment schemes and specifically their incentive properties. A point that is often made is that flat payment schemes have the advantage of providing stronger incentives for cost reduction.[15] In our setting, cost reduction incentives are ignored and we focus instead on strategic aspects. Interestingly, we find that the flat compensation scheme (salary) now implies higher payments to provider and thus higher costs.

While providers' compensation and prices decrease the impact on profits is not *a priori* clear. Substituting (26) and (27) into (23) and using (20) yield

[15] See, for instance, Gosden *et al.* (1999) for a review of the literature on the remuneration of health care providers.

$$\Pi_j^{**} = \Pi_j^{*} = \frac{mt_B + t_A}{2} - \frac{(\gamma + \theta m)(g_A + mg_B)}{4}, \forall j = 1, 2.$$

In words, equilibrium profits are the same under fee-for-service as under salary schemes! This establishes the following proposition.

Proposition 6 *We have $\Pi_j^{**} = \Pi_j^{*}$ so that equilibrium profits are the same under fee-for-service as under salary schemes. Consequently, Propositions 2 and 3 as well as Corollary 1 obtained with salary schemes continue to hold under fee-for-service.*

To sum-up, while prices and provider payments depend on the type of remuneration scheme, our results pertaining to the impact of the common network externality on profits continue to hold. The next section shows that this result remains true when the two remuneration schemes are combined.

5 Mixed remuneration schemes for providers

In this section, we consider that platforms can combine salary and fee-for-service. This situation is particularly interesting to analyze the hospitals' competition where it is common that they use both schemes on the providers' side. Platforms maximize profits and simultaneously set their price/remuneration vectors (P_j, w_j, c_j), $j = 1, 2$. Without loss of generality, we still concentrate on platform 1's program which is stated as

$$\max_{P_1, w_1, c_1} \Pi_1 = n_1^A (P_1, P_2, w_1, w_2, c_1, c_2, \phi)(P_1 - c_1) - mw_1 n_1^B (P_1, P_2, w_1, w_2, c_1, c_2, \phi). \tag{28}$$

The first-order conditions are given by

$$\frac{\partial \Pi_1}{\partial P_1} = n_1^A + \frac{\partial n_1^A}{\partial P_1}(P_1 - c_1) - mw_1 \frac{\partial n_1^B}{\partial P_1} = 0, \tag{29}$$

$$\frac{\partial \Pi_1}{\partial w_1} = -mw_1 \frac{\partial n_1^B}{\partial w_1} - mn_1^B + (P_1 - c_1)\frac{\partial n_1^A}{\partial w_1} = 0, \tag{30}$$

$$\frac{\partial \Pi_1}{\partial c_1} = -n_1^A + \frac{\partial n_1^A}{\partial c_1}(P_1 - c_1) - mw_1 \frac{\partial n_1^B}{\partial c_1} = 0. \tag{31}$$

From these expressions we obtain the following proposition:

Proposition 7 *(i) When platforms compete on the providers' side in affine remuneration schemes, as defined by (4), the system of first-order conditions has an infinity of symmetric solutions.*

(ii) At a symmetric equilibrium, the price structure can be written as

$$w_1^{***} = -t_B + \left(\frac{(\theta m + \gamma)\, g_B}{2} - \frac{2c_1^{***}}{m} \right), \tag{32}$$

$$P_1^{***} = t_A - \frac{(\theta m + \gamma)\, g_A}{2} - c_1^{***}, \tag{33}$$

where c_1^{***} *is set arbitrarily.*

Proof. At a symmetric solution, $n_1^A = n_1^B = 1/2$. Substituting (7)–(10) into (30) and (31) and rearranging yields

$$\frac{\partial \Pi_1}{\partial w_1} = m \frac{\partial \Pi_1}{\partial c_1},$$

so that the system of first-order conditions is redundant. This establishes part *(i)* as long as the system has at least one solution. Such a solution is given in part *(ii).* It is obtained by combining (29) and (30), using the properties of the demand functions, (5)–(8), and solving the system of linear equations so obtained. ■

Observe that since g_A and g_B are evaluated at $n_1^A = n_1^B = 1/2$, equations (32) and (33) give a closed form solution for salaries and prices. Part *(i)* of Proposition 7 is similar to a result obtained by Armstrong (2006), albeit for a different specification of externalities and tariffs. He shows that when platforms use two-part tariffs on one side, there is a continuum of symmetric equilibria. Part *(ii)* of the proposition expresses the providers' salary and the price charged on the consumers' side as functions of the fee-for-service. This is possible because with a redundant system of first-order conditions, we can arbitrarily set one of the variables. Equations (32) and (33) show that the fee-for-service paid to providers reduces both the price charged to consumers and the fixed salary paid to providers. Indeed, the fee-for-service allows platforms to internalize the positive component generated by providers in the formation of CNE and therefore to reduce the rents the providers get in equilibrium.

Substituting the equilibrium prices into the platforms' profit defined in (28) and rearranging establishes the following proposition.

Proposition 8 *In a symmetric equilibrium, platforms' profits are given by*

$$\begin{aligned}\Pi_j &= \frac{1}{2}\left(P_j^{***} - c_j^{***} - mw_j^{***}\right), \\ &= \frac{1}{2}\left(t_A + mt_B - \frac{(\theta m + \gamma)(g_A + mg_B)}{2}\right). \end{aligned} \tag{34}$$

Consequently platforms' profits do not depend on the level of c_j^{***} on which the equilibrium prices are conditioned.

This property is in stark contrast to Armstrong's (2006) findings. Recall that in Armstrong's setting, platforms' profits are *increasing* ("competition intensity" is decreasing) in the usage fee. This is because platforms use usage fee to reduce the cross-group network effects. In the current model, externalities and tariffs are of a different nature and it turns out that with our CNE, an increase in the fee-for-service does *not* reduce competition intensity. Another consequence of this finding is that when one of the two remuneration schemes is regulated, platforms' profits do not depend on the regulated level. This result qualifies Wright's (2004) assessment that any regulatory policy will affect competition intensity in a two-sided framework (see fallacy 8, p.51).

Corollary 2 *The level of a regulated fee-for-service (resp. wages) does not affect competition intensity when the platforms strategically set their level of wages (resp. fee-for-service).*

6 Examples and illustrations

To illustrate the results and provide some additional insight, we shall now present the full analytical solutions (under the two types of payment schemes) for three special cases and give a numerical illustration for one of them. First, we consider the case where the externality simply depends on the ratio between membership on both sides (so that f is homogenous of degree zero). Then we consider a setting with different degrees of homogeneity. Finally, we provide an example for the non homogenous case.

When $q_j = \left(mn_j^B/n_j^A\right)^\delta$, the common network externality is homogeneous of degree zero. When providers receive a salary, Proposition 1 implies that equilibrium prices are given by

$$P_j^* = t_A + 2(\gamma + m\theta)\,\delta m^{\delta},$$
$$w_j^* = -t_B + 2(\gamma + m\theta)\,\delta m^{\delta-1}.$$

With this price structure, it is clear that platforms only transfer rents from "consumers" to "providers" and have equilibrium profits independent of the network externalities. To confirm this, note that with this specification (20) reduces to

$$\Pi_j^* = \frac{1}{2}(t_A + m t_B), \qquad j = 1, 2,$$

which does not depend on γ or θ. It is worth noticing that (as discussed in the introduction) this functional form (with quality depending on the ratio), has interesting applications for education and heath care sectors.[16]

To illustrate this case numerically, we consider the following parameters' values: $t_A = 0.3, t_B = 0.4, \gamma = 0.3, \theta = 0.5, m = 0.05$ and $\delta = 1$. The symmetric equilibrium leads to $P_i = 0.3325$ and $w_i = 0.25$ with a level of profit $\Pi_i = 0.16$. The following figure represents the profit function of firm 1 as a function of admissible[17] P_1 and w_1 evaluated at symmetric equilibrium prices $P_2 = 0.3325$ and $w_2 = 0.25$. One can see that the symmetric equilibrium constitutes effectively a global best reply for platform 1 (and thus by symmetry for platform 2). Though of limited scope this observation is interesting. As usual in two-sided market models, second-order conditions are quite complex in our setting. The example shows that at least for a simple nonlinear specification of f, the existence of the equilibrium is not a problem.

We now turn to the more general case where f is specified by (1) so that the CNE is homogenous of degree $k = \beta - \alpha$. The nice feature about this specification is that it shows that negative levels of k do not have to be ruled out. Under a salary scheme,

[16] Under a fee-for-service scheme, we obtain

$$c_j^{**} = -\frac{m t_B}{2} + \delta m^{\delta}(\theta m + \gamma),$$
$$P_j^{**} = \frac{m t_B}{2} + t_A + (\gamma + \theta m)\,\delta m^{\delta},$$

which in accordance with the general results yields a profit of $\Pi_j^{**} = \Pi_j^*$.

[17] By admissible, we mean that the demands lie between 0 and 1 and that profits are positive.

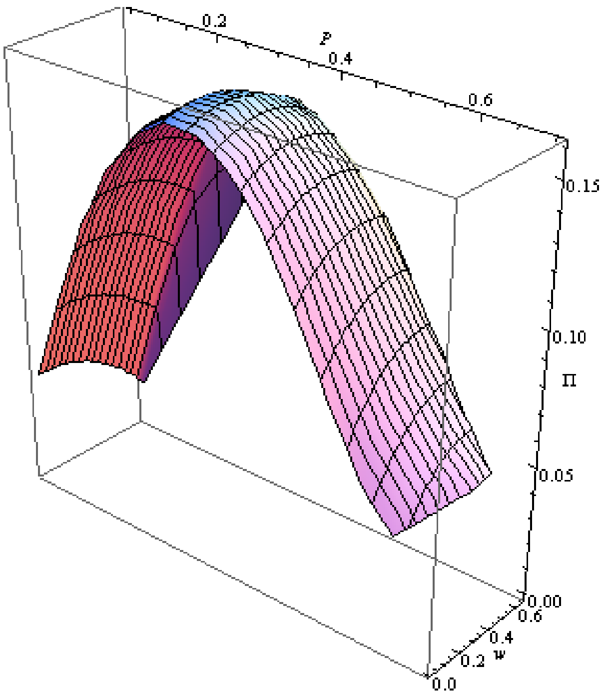

Figure 1: Profit function of platform 1 (when prices of platform 2 are set at their equilibrium levels).

equilibrium prices and profit levels are given by $(j = 1, 2)$

$$P_j^* = t_A + \left(\frac{1}{2}\right)^{\beta-\alpha-1} \alpha m^\beta (\gamma + m\theta), \tag{35}$$

$$w_j^* = -t_B - \left(\frac{1}{2}\right)^{\beta-\alpha-1} \beta m^{\beta-1} (\gamma + m\theta), \tag{36}$$

$$\Pi_j^* = \frac{1}{2}(t_A + m t_B) - \left(\frac{1}{2}\right)^{\beta-\alpha} m^\beta (\gamma + m\theta)(\beta - \alpha). \tag{37}$$

Not surprisingly, network externalities affect profits according to the sign of $k = \beta - \alpha$. When $\alpha > \beta$, quality offered by the platforms are relatively more sensitive to the number of consumers than to the number of providers. Therefore, platforms can charge a higher relative price on the consumers' side for the quality provided without transferring all the network externalities rents to the providers (negative global externality).

Finally, let us consider a case where the homogeneity property does not hold at all. For instance, think about a case where quality depends positively on the ratio $\left(m n_j^B / n_j^A\right)$ but also depends positively on the "volume" of consumers treated by the provider[18]. In such a case, we have $q_j = \left(m n_j^B / n_j^A\right) + d n_j^A$ with $d > 0$ small enough to ensure that we continue to have a negative intra-group externality on the consumers' side. Equilibrium prices, wages and profits are now given by

$$\begin{aligned} P_j^* &= t_A + 2(\gamma + m\theta)\left(m - \frac{d}{2}\right), \\ w_j^* &= -t_B + 2(\gamma + m\theta)(m), \\ \Pi_j^* &= \frac{1}{2}(t_A + m t_B) - (\gamma + m\theta) d. \end{aligned}$$

Recall that because of the parameter d (which reduces the intensity of the negative intra-group externality on the consumers' side) the common network externality does not satisfy the homogeneity property. Consequently, the impact on prices and profits is more complex. With the considered specification, platforms' profit are reduced because they charge a lower price on the consumers' side. This lower price is *not* outweighed by a lower wage paid on the providers' side.

[18] This assumption makes sense for a hospital. The quality of care can depend on the volume of patients treated.

7 Conclusion

This paper has examined a market in which two platforms compete for consumers and for providers in the presence of a common network externality. For a given price structure, consumers and providers value the same index of quality (albeit possibly with different intensities). This index depends positively on the number of providers, but negatively on the number of consumers. We have shown that the symmetric equilibrium has some specific properties. First, the common network externality has a cumulative effect on prices: its effect on one side's price is, partially or entirely, shifted to the other side of the market. Second, when the quality index is specified by a homogenous function, the sign of the homogeneity's degree determines the net impact of the externality, *(i),* on prices on both sides of the market and, *(ii),* on competition intensity. In the specific, but empirically appealing case where the homogeneity's degree is equal to zero, the presence of the common network externality has no impact on equilibrium profits; the price increase on one side of the market is totally shifted to the other side. We show that a fee-for-service leads to lower remuneration for providers than a flat salary scheme. This, in turn results in a lower price level so that profits are the same under both regimes. Finally, we have allowed platforms to combine both remuneration schemes. As expected, there exists a continuum of symmetric equilibria, parametrized by one of the prices. However, surprisingly, platforms' equilibrium profits do not depend on this price. In other words, competition intensity does not depend on the way the providers remuneration schemes mix salaries and fee-for-service. This is a rather strong and surprising result which is at odds with conclusions obtained for other types of prices and externalities. Admittedly, the robustness of this strong result must be confronted to a number of variations and generalizations of the model. In particular, providers may decide the number of procedures so that both quality and quantity matter. These features appear to be especially relevant in the health care sector (see for instance Quest *et al.*, 2008 and Fortin *et al.*, 2008).

Our analysis could also be extended in at least two different ways. First, it would be interesting to consider the case where the market is not fully covered (on one or two sides). When the severity of their illness is not high enough, some of the potential patients may decide not to consume medical (hospital) services at all. On the providers' side, some of the physicians may prefer to remain self-employed. A second possible extension concerns the type of hospitals that compete in the market for patients. For instance, for-profit providers could coexist with not-for-profit or physician's-owned hospitals. This would admittedly not be a trivial extension because in the case of mixed oligopolies one can no longer concentrate on symmetric equilibria. These issues are on our research agenda.

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Appendix

A Uniqueness of demand functions

In the ratio case we have $q_1 = mn_1^B/n_1^A$. From (3), one obtains

$$g\left(n_1^A, mn_1^B\right) = \frac{2t_B}{\theta}\left[n_1^B - \frac{1}{2} - \frac{\Delta w}{2t_B}\right],$$

where $\Delta w = w_1 - w_2$.

Substituting this expression into (2) yields

$$n_1^A = \frac{1}{2} + \frac{\gamma}{t_A}\frac{t_B}{\theta}\left[n_1^B - \frac{1}{2} - \frac{\Delta w}{2t_B}\right] - \frac{\Delta P}{2t_A}, \tag{38}$$

where $\Delta P = P_1 - P_2$. It is worth noticing that if n_1^B is unique, then n_1^A is also unique.

From (3), one has

$$n_1^B\left[1 - \frac{\theta m}{2t_B}\left(\frac{1}{n_1^A} + \frac{1}{1-n_1^A}\right)\right] = \frac{1}{2} + \frac{1}{2t_B}\left(-\frac{\theta m}{1-n_A^1} + \Delta w\right).$$

Multiplying both sides of this equality by $n_1^A\left(1-n_1^A\right)$ yields

$$n_1^B\left[n_1^A\left(1-n_1^A\right) - \frac{\theta m}{2t_B}\right] = \frac{1}{2}\left[n_1^A\left(1-n_1^A\right)\left[1 + \frac{\Delta w}{t_B}\right] - \frac{\theta}{t_B}n_1^A\right].$$

Substituting (38) from gives n_1^B as a solution to a third degree polynomial equation. Some additional tedious computations show that only one of the three solutions is a real numbers.[19]

B Second-order conditions

We provide second order conditions for the case where $q_j = mn_j^B - n_j^A$. We first deal successively with the salary and the fee-for-service payment schemes.

B.1 Salary scheme

When platforms pay a salary to providers, the symmetric equilibrium is a global maximum when the matrix H defined by

$$H = \begin{bmatrix} \frac{\partial^2 \Pi_1}{\partial P_1^2} & \frac{\partial^2 \Pi_1}{\partial P_1 \partial w_1} \\ \frac{\partial^2 \Pi_1}{\partial P_1 \partial w_1} & \frac{\partial^2 \Pi_1}{\partial w_1^2} \end{bmatrix}$$

[19] The market share belongs to $[0,1]$. If the solution of the polynomial is negative or superior to 1, it means that we are in presence of a corner solution for the demands levels.

is semi definite negative *i.e.* $\partial^2\Pi_1/\partial P_1^2 \leq 0$, $\partial^2\Pi_1/\partial w_1^2 \leq 0$ and $\det H \geq 0$. Differentiating the LHS of (11) and (12) with respect to P_1 and w_1, the second order conditions are thus satisfied if:

$$\frac{\partial^2\Pi_1}{\partial P_1^2} = 2\frac{\partial n_1^A}{\partial P_1} \leq 0, \tag{39}$$

$$\frac{\partial^2\Pi_1}{\partial w_1^2} = -2m\frac{\partial n_1^B}{\partial w_1} \leq 0, \tag{40}$$

$$\det H = \frac{\partial^2\Pi_1}{\partial P_1^2}\frac{\partial^2\Pi_1}{\partial w_1^2} - \left(\frac{\partial n_1^A}{\partial w_1} - m\frac{\partial n_1^B}{\partial P_1}\right)^2 \geq 0. \tag{41}$$

According to (5) and (8), (39) and (40) are satisfied. Substituting (5)–(8) into the LHS of (41) yields

$$\begin{aligned}\det H &= -4m\frac{\partial n_1^A}{\partial P_1}\frac{\partial n_1^B}{\partial w_1} - \left(\frac{\partial n_1^A}{\partial w_1} - m\frac{\partial n_1^B}{\partial P_1}\right)^2 \\ &= \frac{4\left(2t_B - \theta m\right)\left(2t_A + \gamma\right) - m\left[\gamma - \theta\right]^2}{\left(4t_A t_B \Psi\right)^2},\end{aligned} \tag{42}$$

so that (41) is satisfied if

$$\left(2t_B - \theta m\right)\left(2t_A + \gamma\right) \geq \frac{m}{4}\left[\gamma - \theta\right]^2.$$

B.2 Fee-for-service scheme

When platforms use a fee for service scheme, the symmetric equilibrium is a global maximum when the matrix Z defined by

$$Z = \begin{bmatrix} \frac{\partial^2\Pi_1}{\partial P_1^2} & \frac{\partial^2\Pi_1}{\partial P_1\partial c_1} \\ \frac{\partial^2\Pi_1}{\partial P_1\partial c_1} & \frac{\partial^2\Pi_1}{\partial c_1^2} \end{bmatrix}$$

is definite negative *i.e.* $\partial^2\Pi_1/\partial P_1^2 \leq 0$, $\partial^2\Pi_1/\partial c_1^2 \leq 0$ and $\det Z \geq 0$. Differentiating the LHS of (24) and (25) with respect to P_1 and c_1 show that the second-order conditions are satisfied if

$$\frac{\partial^2\Pi_1}{\partial P_1^2} = 2\frac{\partial n_1^A}{\partial P_1} \leq 0, \tag{43}$$

$$\frac{\partial^2\Pi_1}{\partial c_1^2} = -2\frac{\partial n_1^A}{\partial c_1} + \frac{\partial^2 n_1^A}{\partial c_1^2}\left(P_1 - c_1\right) \leq 0, \tag{44}$$

$$\frac{\partial^2\Pi_1}{\partial P_1\partial c_1} = \frac{\partial^2\Pi_1}{\partial P_1^2}\frac{\partial^2\Pi_1}{\partial c_1^2} - \left(\frac{\partial n_1^A}{\partial c_1} - \frac{\partial n_1^A}{\partial P_1}\right)^2 \geq 0. \tag{45}$$

Differentiating (9) with respect to c_1 yields

$$\frac{\partial^2 n_1^A}{\partial c_1^2} = -\frac{\gamma}{4m\left(t_A t_B\right)^2 \Psi^2}\left[2t_A + \gamma\left(1-m\right)\right].$$

According to (5) and (8), (43) and (44) are fulfilled provided that $2t_A + \gamma\left(1-m\right) > 0$ and that profits are positive. Introducing (9) and (5) in (44), simplifying and rearranging allows us to observe that (45) is fulfilled if and only if

$$\frac{\gamma\left(2t_B - \theta m + \frac{4c}{m}\right)\left(2t_A + \gamma\left(1-m\right)\right)\left(P_1 - c_1\right)}{m t_A t_B \Psi} \geq \frac{1}{2}\left(\gamma - \theta m - 2t_B + \frac{4c}{m}\right)^2.$$