Hidden limit orders and liquidity in order driven markets[1]

Sophie Moinas

Toulouse School of Economics (CRM, Toulouse University and IDEI)[2]

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[2] Université de Toulouse 1, 2 rue du Doyen-Gabriel-Marty F-31042 Toulouse, e.mail: Sophie.Moinas@univ-tlse1.fr, Tel. : (33) (0)5.61.12.85.89, Fax : (33) (0)5.61.12.86.37

Abstract

This paper analyzes the rationale for the submission of hidden limit orders, and compares opaque and transparent limit order books. In my sequential model, the limit order trader may be informed with some probability. Both informed and large uninformed liquidity suppliers submit hidden orders in order to decrease the informational impact of their large orders, while ensuring a large trading volume. As they cannot adopt such a strategy in the transparent market, I find that pre-trade opacity improves market liquidity, and the welfare of the participants. My model further yields empirical predictions on the use and revelation of hidden orders in opaque markets.

Keywords: Limit order, hidden order, iceberg order, market transparency, informed trading.

JEL Classification:G10, G14, G18

"The Turquoise MTF's integrated market combines dark and visible orders to increase execution and price improvement for small orders, whilst minimising information leakage and market impact for larger, institutional-size orders."(www.tradeturquoise.com)

1 Introduction

This paper analyzes the rationale for the submission of hidden limit orders, and compares opaque limit order books to transparent ones.

Pre-trade transparency seems to be a necessary condition for brokers to ensure best execution in fragmented markets. In contrast though, regulations like the MIF directive in Europe and the Reg NMS in the U.S., both applied in 2007, have favored the emergence of dark pools, which are now successfully competing with traditional exchanges.[1] According to the Securities and Exchange Commission, which records approximately 30 dark pools in the U.S., 8.5% of the U.S. trading volume occurred in dark pools in 2009. In Europe, the opaque platforms represented 4.1% of the trading volume in 2009, according to Tabb.[2]

Even before the emergence of dark pools, pre-trade transparency was not fully implemented in regulated exchanges. In limit order books, traders either supply liquidity by posting non-marketable limit orders that specify a price and a total order size, or demand liquidity by submitting market orders or marketable limit orders, that yield immediate partial or full execution. Some markets like Euronext allow agents to submit hidden limit orders (also called iceberg or undisclosed orders), that specify prices, total order sizes, and displayed sizes (also called peak sizes). When the quantity initially displayed has been fully executed, the system automatically refreshes and displays another peak, until the full execution of the order. In some opaque limit order books, traders may even either fully hide the quantity of their limit order and disclose the price only (as in the Australian Stock Exchange), or hide the price and the quantity of their order (as in the dark pool Turquoise). Various empirical studies have highlighted the importance of hidden depth in limit order markets. On a sample of 100 stocks traded on Euronext Paris, Bessembinder et al. (2009) find that hidden orders represent approximately 44% of order volume and

[1] Dark pools are private alternative trading systems or platforms that provide investors with non-displayed liquidity sources. According to Jeremy Grant, "Pre-trade prices - the price at which shares are offered for sale - are not visible to anyone, even the participants in them, and the price at which shares change hands is only revealed after the trade is done" (The Trading Room, Financial Times).

[2] Mizen Miranda, European Equity Trading 2009: Counterparties, Capital and Control, April 2009, available on http://www.tabbgroup.com.

that 18% of the incoming orders include some hidden size. Besides, conditional on the presence of hidden orders at the five best quotes, 80% of the total depth is hidden.[3] Finally, the recent development of algorithms, among which some replicate the behavior of hidden limit orders, further confirms that limit order traders would want some opacity.

These elements raise two intriguing issues: why would limit order traders need to hide, and why would transparency harm market quality? Both questions matter for market regulators, for designers of trading systems, as well as for practitioners. In this paper, I compare various measures of market quality under different transparency regimes, and I derive new empirical predictions on the use and on the detection of hidden orders.

I propose a sequential model of trading in a limit order book. The limit order book is initially endowed with buy and sell orders. A new liquidity supplier enters the market, and submits either a small or a large limit order, that undercuts the current quotes. Then a liquidity demander observes the visible depth in the limit order book and chooses whether to submit no order, a small or a large market order, or a large marketable limit order. In the transparent market, the total depth in the limit order book is fully displayed. In the opaque market, the liquidity suppliers can submit a large hidden limit order. In that case, the total depth in the limit order book is actually larger than the visible depth.

The model builds on three key assumptions. First, the liquidity supplier may be informed on the fundamental value of the security with some probability. Due to the presence of an informed liquidity supplier ("she"), the uninformed liquidity demander ("he") updates his beliefs on the value of the security conditional on the visible depth he observes. When the probability of informed trading is low, the magnitude of his update in beliefs is limited. Therefore, he trades a large volume when he observes a large depth. In contrast, when the probability of informed trading is large, his gains from trade can be offset by his adverse selection risk. Consequently, he prefers not to match a large limit order.

Second, I assume that the uninformed liquidity supplier ("it") may be restricted in the size of its limit order with some probability. The presence of a small uninformed liquidity supplier obscures the informational content of the observation of a small displayed order, with respect to that of a large displayed order. For high levels of asymmetric information, the liquidity demander will trade if the visible

[3]Accordingly, D'Hondt and De Winne (2007) report that the hidden depth accounts for 45% of the total depth available at the five best quotes on average, for the stocks which constitute the CAC 40 index in Euronext Paris. In other trading platforms in which hidden orders are provided, Hasbrouck and Saar (2002) report that hidden orders account for 12% of all order executions in Island. In the Nasdaq, Tuttle (2006) finds that the hidden liquidity represents 20% of the inside depth in Nasdaq 100 stocks. In Xetra, Frey and Sandas (2009) show that iceberg orders represent 9% of all non-marketable limit orders.

depth is small, not if the visible depth is large. In this case, both the informed and the large uninformed liquidity suppliers must display a small order in order to get at least partial execution. In particular, even uninformed, liquidity suppliers must reduce the visible size of their limit orders to avoid being taken for an informed agent. This holds both in the transparent and in the opaque market. By doing so in the transparent market, liquidity suppliers restrict their trading volume: they cannot trade a larger size than their order size. This may not be the case in the opaque market, as hidden orders introduce a discrepancy between the visible size of the order, and its total size. Liquidity suppliers are not compelled to submit a small order, they only need to display a small order.

Third, I assume that the limit order book is initially filled with buy and sell orders behind the best quotes, such that the liquidity demander would be willing to trade at those prices in the absence of adverse selection, provided that his gains from trade are sufficiently large. The difference in the total size of a small and a hidden order only matters for liquidity suppliers when the liquidity demander submits a larger order than the visible depth. Otherwise, both orders would be perfectly equivalent in their informational impact and in their execution conditions. If the liquidity demander does so and discovers hidden depth, the informational content of such a revelation is similar to that of the observation of a large order. Therefore he is equally reluctant to trade as when large limit orders are visible. Here is where my third key assumption kicks in. Given this assumption, if the informed liquidity supplier only submits large hidden orders, then small limit orders have no informational content. Consequently, the liquidity demander would benefit from trading in the absence of hidden depth. I show that the gains realized when his order walks up or down the book in the absence of hidden depth offset the losses the liquidity demander bears if there is a hidden order, provided that his gains from trade and the proportion of small uninformed liquidity suppliers are sufficiently high. Accordingly, he submits a large market order, which enables the liquidity suppliers to execute a larger volume than the quantity that they display.

Finally, I show that the uninformed and the informed liquidity suppliers hide their large order in the opaque market in order to decrease its informational impact. As they cannot adopt such a strategy in the transparent market, they are compelled to submit a small limit order. Market depth is therefore larger in the opaque market. Besides, submitting a small limit order in the transparent market instead of a hidden limit order in the opaque market decreases trading volume. Therefore, I find that pre-trade opacity improves the welfare of the incoming liquidity suppliers and demanders.

My paper proposes an explanation for the extensive use of hidden orders observed in practice, that provides an alternative to the rationale traditionally put forward by researchers. Unlike Harris (1997) and Buti and Rindi (2009), I model the placement of hidden orders by informed traders. Thus, I show i.

that both liquidity-motivated and information-motivated traders use hidden orders, and ii. that liquidity-motivated traders do not use hidden orders to manage their exposure but to soften the informational impact of their large limit orders.[4] My theoretical results on the use and on the detection of hidden orders in opaque markets are consistent with the empirical findings of Aitken et al. (2001), Fleming and Mizrach (2009), Anand and Weaver 2004).

The paper is organized as follows. Section 2 introduces the theoretical model. In Section 3, I describe the equilibrium's strategies. Section 4 offers a comparative static analysis generating new empirical predictions, and analyzes the impact of transparency on market quality. Section 5 concludes. The proofs that do not appear in the text are collected in the appendix.

2 The model

2.1 Timing and participants

I consider a sequential model of trading in a limit order book for a single risky security, with three periods. The final value of the security, $\tilde{v}$, is realized at date 3:

$$\tilde{v} = v_0 + \tilde{\varepsilon}, \tag{1}$$

where $\tilde{\varepsilon}$ is equal to $+\sigma$ or $-\sigma$, with probability $\frac{1}{2}$.

Since I aim at understanding the process of liquidity provision in two different trading systems, namely a transparent and an opaque market, I fix the prices, and I analyze the quantity submitted and displayed at these prices. Limit prices are constrained by a price grid, and I denote by Δ the minimum tick size between two consecutive prices. I define $A_k = v_0 + k\Delta$ (respectively $B_k = v_0 - k\Delta$), where $k = 1, 2...N$, as the k^{th} best ask price above (respectively bid price below) the unconditional value of the asset, v_0.

[4]A recent experiment designed by Gozluklu (2009) to disentangle these two motivations for submitting hidden orders confirms that the two views are not mutually exclusive, although the strategies implemented by the informed and uninformed agents with respect to the choice of prices and quantities differ. See also the theoretical analysis of Esser and Moench (2007), which is based on the assumption that large orders would have such an adverse informational impact that the execution risk of limit orders would increase with their displayed size. In a continuous setting, they then determine the limit price and the optimal size of a hidden order for a static liquidation strategy. In this model as in mine, hidden orders would be conceptually similar to the automation of splitting orders' strategies.

I assume that the limit order book is symmetrically filled with sell orders at price A_2, and with buy orders at price B_2 at date 0. Because price and time priority is enforced, only new limit orders undercutting the current quotes by (at least) one tick have a non-zero probability of being executed. The initial symmetry in the limit order book enables me to focus on the case where $\tilde{v} = v_0 - \sigma$, and to analyze only the ask side of the limit order book. I discuss the case where the limit order book is not initially symmetric in Section 3.

Order submission occurs sequentially. First, at date 1, a liquidity supplier submits a limit order at price A_1. Next, at date 2, a liquidity demander observes the limit order book and potentially trades by submitting a market or a marketable limit order. I analyze the order submission's behavior of both traders in two different trading systems: (i) the transparent limit order market, and (ii) the opaque limit order market. In the transparent market, agents are able to observe the *total* depth D^t supplied at the best quotes in the limit order book. In the opaque limit order market however, liquidity suppliers have the opportunity to hide a fraction of the total quantity of their limit order. Thus, before submitting a market or a marketable limit order, the liquidity demander is only able to observe the *visible* depth at the best quotes D^v, but not the *hidden* depth D^h. Notice that by definition, $D^t = D^v + D^h$.

The liquidity demander ("he") entering the market at date 2 may trade a maximum of two units of the asset. He is a buyer or a seller with equal probabilities. I assume that he is strategic and Bayesian, and that he is ready to pay higher transaction costs in order to trade immediately. To represent his impatience, I assume that he has a marginal private value from trading β, that may either be due to an inventory holding cost, or to a difference in the valuation of the security (*cf.* Foucault, 1999 or Parlour, 1998).[5] If the private value β is positive, then the liquidity demander wants to buy the security. His utility if he trades a quantity Q at price P writes:

$$U_D(Q) = Q \times (\tilde{v} - P + \beta), \tag{2}$$

In most market structures, impatient traders may either submit a market order, or a marketable limit order. Both are immediately executed against the opposite order(s) standing in the limit order book. A market order guarantees a full execution, but may have a price impact by walking up or down the limit order book. Conversely, a marketable limit order has no price impact, but if its size exceeds the total depth available at the best limit price, then its non-executed part automatically becomes a limit order at the opposite side of the book. To account for both order types, I consider that the liquidity demander may submit any order $M \in \{0, 1, 2^{MLO}, 2^{MO}\}$, where $M = 2^{MLO}$ stands for a large marketable limit order,

[5] Introducing speculators willing to trade against the limit order book would obscure the presentation of the results without adding new insights to the model.

and $M = 2^{MO}$ for a large market order.

My model builds on the assumption that the liquidity supplier entering the market at date 1 may possess private information on the value of the security. Liquidity suppliers may therefore have different information sets, and I denote by I_j the information set of agent j. With probability π, the liquidity supplier is *informed* ("she") on the realized value of the security before trading occurs. She perfectly observes $\tilde{v}$, that is, in this case, $\tilde{v} = v_0 - \sigma$. Her profit if she sells a quantity Q at price P writes:

$$U_I(Q) = Q \times (P - (v_0 - \sigma)), \tag{3}$$

With probability $1 - \pi$, the strategic liquidity supplier is *uninformed* ("it"), and it is a net seller or buyer with identical probabilities. Another key assumption is the existence of *small* uninformed liquidity suppliers, who can trade a maximum of 1 unit of the security. We denote by θ the probability of entry of a *small* uninformed liquidity supplier, and by $(1 - \theta)$ that of a *large* uninformed liquidity supplier, who can trade up to 2 units of the security. Its profit if it sells a quantity Q at price P writes:

$$U_N(Q) = Q \times (P - \tilde{v}), \tag{4}$$

Given the absence of any informed liquidity demander in my model, the uninformed liquidity supplier is not exposed to adverse selection. Consequently, it is always profitable for it to trade, and it is straightforward to show that it will submit a limit order of at least one unit at price A_1. Besides, equation (4) implies that undercutting by more than one tick would not be profitable for the uninformed liquidity supplier. It can be shown that consequently, the informed limit order trader also submits a limit order at price A_1, for a maximum quantity of 2 units. This enables me to focus on the quantity that limit order traders supply, and display to the market at price A_1. Liquidity suppliers have the choice between the three following offers at price A_1: (a) Offer L: submit and display a large limit order $D^t = D^v = 2$ and $D^h = 0$, (b) Offer S: submit a small limit order $D^t = D^v = 1$ and $D^h = 0$, or (c) Offer H: in the opaque market, submit a large limit order $D^t = 2$, disclose a fraction of it, $D^h = 1$, and display one unit to the market, so that $D^v = 1$.

Offers L and H are not available to the small liquidity supplier. Consequently, since it is profitable for it to trade at price A_1, it submits a small limit order. In what follows, we will consider this strategy as given, although it is not exogenous but the outcome of its strategic behavior.

The agents' decision planning on the ask side of the limit order book is depicted in Figure 1.

Insert Figure 1 about here.

2.2 Adverse selection, market structure and parametrization of the model

To better understand the key features of my model, I analyze here two benchmark cases where there is not asymmetric information. First, when there is no private information (that is, π=0), the liquidity demander expects to receive $(v_0 + \beta)$ for each unit traded. In particular, when he observes an offer D^v, his expected utility from trading a quantity $M \in \{0, 1, 2^{MLO}, 2^{MO}\}$ units of the security, provided that $M \leq D^v$, simply writes:

$$EU_D^{benchmark}(M|D^v) = \gamma \times M, \tag{5}$$

where $\gamma \equiv v_0 + \beta - A_1 = \beta - \Delta$.

I refer to this parameter γ as to his "gains from trade" at price A_1, as he obtains a surplus γ for each unit traded at this price (in the absence of adverse selection). If this parameter was negative, he would never submit a market order, whatever the size of the depth available at price A_1. I therefore assume that $\gamma \geq 0$.

Now, when he observes a small offer, the liquidity demander may be willing to submit a larger order than the visible depth. The parameter γ determines the aggressiveness of his order. If $\gamma > \Delta$, the liquidity demander is ready to trade at price A_2. Consequently, when there are orders at price A_2, he would submit a large market order whatever the quantity supplied in the limit order book at price A_1. His expected utility from submitting a large market order, while observing a limit order of size $D^v = 1$, is indeed:

$$\begin{aligned} &EU_D^{benchmark}(M = 2^{MO}|D^v = 1) = EU_D(M = 1|D^v = 1) \\ &+ Pr(D^t = 2|D^v = 1) \times \gamma + Pr(D^t = 1|D^v = 1) \times (\gamma - \Delta) \end{aligned} \tag{6}$$

The initial presence of limit orders standing in the limit order book at price A_2 is crucial to induce the liquidity demander to submit a large order in the absence of adverse selection, when he is characterized by $\gamma > \Delta$. Accordingly, it will also be key to our more general results derived in the next Section.

Conversely, when $\gamma \leq \Delta$, the liquidity demander is willing to trade, but not at any price. In particular, equation (6) shows that in a transparent market, where the total depth cannot be larger than the visible depth, he would not submit a large market order. Still, he would be ready to submit a large marketable limit order, yielding the following expected profit:

$$\begin{aligned} &EU_D^{benchmark}(M = 2^{MLO}|D^v = 1) \\ &= EU_D^{benchmark}(M = 1|D^v = 1) + Pr(D^t = 2|D^v = 1) \times \gamma \end{aligned} \tag{7}$$

I show that this one-to-one relationship between the value of γ with respect to the tick size Δ, and the type of large order which would be preferred by the uninformed liquidity demander, also holds in the presence of adverse selection in equilibrium.

Second, if all traders were symmetrically informed that the value of the security is $\tilde{v} = v_0 - \sigma$, the liquidity demander would expect to earn a net profit of $(\gamma - \sigma)$ for each unit traded. I therefore focus on the case where $\gamma \leq \sigma$, so that he would not be ready to buy the asset in this case. To summarize, the parameter γ is such that:

$$0 \leq \gamma \leq \sigma. \tag{8}$$

Finally, I restrict my analysis to the case where adverse selection does not cause a market breakdown, namely:

$$\pi \leq \frac{\gamma}{\sigma}. \tag{9}$$

For $\pi > \frac{\gamma}{\sigma}$, it can indeed been shown that adverse selection deters the liquidity demander from trading, whatever the strategies implemented by the liquidity suppliers.

2.3 Notations and definition of an equilibrium

In the presence of asymmetric information, this model is actually a reverse version of Glosten (1994). His model is based on the paradigm that is traditionally used to model adverse selection in a financial market: the liquidity supplier is uninformed while the *liquidity demander* is potentially informed. The former faces an adverse selection risk, and infers information based on the expected transaction size. In contrast, in my model, I assume that the *liquidity supplier* is potentially informed, while the liquidity demander is uninformed. In this case, the later faces an adverse selection risk, and tries to infer information from his observation of the visible depth in the limit order book. Visible depth may indeed signal the presence of an informed liquidity supplier.[6]

Accordingly, my model represents a signaling game between a liquidity supplier whose type is “informed” with an *ex ante* probability π or “uninformed” with the complementary probability, moving first, and an uninformed liquidity demander, moving second. In this dynamic game with incomplete information, I look for a perfect Bayesian equilibrium. Such an equilibrium is characterized by a Bayesian

[6]Various recent studies, either empirical (Kaniel and Liu, 2006, Aitken et al., 2001, Anand et al., 2005 or Cao, Hansch and Wang, 2009), experimental (Bloomfield and O’Hara, 1999), or theoretical (Goettler, Parlour, and Rajan, 2005, or Rosu, 2009) show that speculators, who possess some private information on the true value of the security, do not only submit market orders that would be executed against the best limit orders standing in the limit order book, but may themselves supply liquidity by submitting limit orders. According to Kaniel and Liu (2006), such a behavior may indeed increase their expected profits thanks to a better execution price, when their information is long-lived.

update of the beliefs of the uninformed liquidity demander at date 2, conditional on his observation of the limit order book. For clarity, I will however not report the equilibrium beliefs of the uninformed liquidity demander in the Propositions in the next Section.

I look for the equilibria in pure strategies. To show their existence and derive the comparative statics in Section 4, I however do not exclude mixed strategies. In equilibrium, if agents are indifferent between different types of orders, then their strategies' set is not the set including their possible orders, but a set of probability vectors, which describe for each possible order the probability with which the agent chooses this order. To characterize this perfect Bayesian equilibrium in mixed strategies, I adopt the following notations. On the one hand, let $\mu_{M|D^v}$ be the probability with which the liquidity demander submits an order $M \in \{0, 1, 2^{MLO}, 2^{MO}\}$ when he observes the depth D^v. On the other hand, the liquidity supplier chooses a large unhidden order L with probability l, a large hidden order H with probability h, or a small order S with probability s, all probabilities subscripted by i if she is informed, or by u if it is large and uninformed.

3 The equilibria

In this section, I solve for the Perfect Bayesian equilibria by backward induction. I first analyze how the uninformed liquidity demander updates his beliefs on the value of the security and on the total depth in the book at date 2, conditional on the visible depth he observes. I then find his optimal reaction, for each possible state of the visible depth. I finally derive the optimal strategies of the informed and uninformed liquidity suppliers at date 1.

3.1 Beliefs of the uninformed liquidity demander

Due to the presence of an informed liquidity supplier, visible depth may reveal information on the value of the security. Esser and Moench (2007) exemplify empirically that current variations in the visible order book imbalance of XETRA are positively correlated with future returns. Consistently and following Bayes' rule:

$$E(\tilde{v}|D^v = 1) = v_0 - \sigma \frac{\pi(s_i + h_i)}{\pi(s_i + h_i) + (1-\pi)(\theta + (1-\theta)(s_u + h_u))}, \tag{10}$$

$$E(\tilde{v}|D^v = 2) = v_0 - \sigma \frac{\pi(l_i)}{\pi(l_i) + (1-\pi)(1-\theta)l_u}. \tag{11}$$

As expected, the magnitude of the beliefs' update increases with the probability of informed trading π and with the asset volatility σ, that both characterize the adverse selection risk. But it also depends on the strategies played by the informed liquidity supplier. Suppose for instance that she always submits small orders, that is, $s_i = 1$. Then, for the uninformed liquidity demander, observing a large order $D^v = 2$ is not informative about the value of the security. Suppose now that the uninformed liquidity supplier never submits large orders, that is, $l_u = 0$. Then observing a large order $D^v = 2$ perfectly reveals the presence of the informed liquidity supplier, and her private information that the value of the security is low. The conditional expectation of the value of the security becomes in this case $E(\tilde{v}|D^v = 2) = v_0 - \sigma$. The magnitude of the beliefs' update therefore relies on the informed agent's strategy, and on the trading "noise" created by the presence of the small uninformed liquidity supplier on the one hand, and by the behavior of the large uninformed liquidity supplier on the other hand.

But the liquidity demander may also submit an order which size exceeds the visible depth. He may indeed submit a large market order or a large marketable order when the visible depth is small, that is, when $D^v = 1$. If there is hidden depth at price A_1, then both types of orders are fully executed at this favorable price. If not, then the non-executed part of a buy marketable limit order $M = 2^{MLO}$ would become a limit order at the bid price $B = A_1$. Given our assumptions, a market order $M = 2^{MO}$ would walk up the book and trade a second unit at price A_2. In any case, submitting a larger order than the visible depth reveals the presence (or the absence) of hidden depth at the best quote. As a consequence, the liquidity demander does not only update his beliefs on the value of the security by observing the visible depth, but also conditional on the execution of his large order. Following Bayes' rule:

$$E(\tilde{v}|D^v = 1 \cap D^h = 1) = v_0 - \sigma \frac{\pi h_i}{\pi h_i + (1-\pi)(1-\theta)h_u}, \tag{12}$$

$$E(\tilde{v}|D^v = 1 \cap D^h = 0) = v_0 - \sigma \frac{\pi s_i}{\pi s_i + (1-\pi)(\theta + (1-\theta)s_u)}. \tag{13}$$

The revelation of hidden depth impacts both the trading costs and the beliefs of the liquidity demander. Hopefully, observing a small limit order $D^v = 1$ is a noisy signal of the total depth supplied at price A_1, D^t. The probability with which there is hidden depth at price A_1 is indeed:

$$\Pr(D^h = 1|D^v = 1) = \frac{\pi h_i + (1-\pi)(1-\theta)h_u}{\pi(h_i + s_i) + (1-\pi)(\theta + (1-\theta)(h_u + s_u))}. \tag{14}$$

This probability may be interpreted in terms of correlation between the total depth and the visible depth. Bessembinder et al. (2009) accordingly find that market participants can detect hidden size to a substantial extent, as indicated by the high out-of-sample forecast power of their tests.[7]

[7] The authors also document that it remains difficult to predict the magnitude of the hidden part. This goes beyond the analysis here, as I assume that the size of the hidden depth, if any, is normalized to one.

3.2 The determinants of the liquidity demander's strategy

The uninformed liquidity demander's optimal reaction is conditional on the visible depth, which is informative both on the value of the security, and on his trading conditions.

Observing a large limit order at price A_1 allows the liquidity demander to perfectly know his transaction price, namely A_1, and the fact that a large order would be fully executed. He is thus induced to submit a large order. However, he faces an adverse selection risk, which could prevent him from trading. Each unit that the liquidity demander trades against the book when he observes a visible depth $D^v = 2$ leads to an expected gain of $E(\tilde{v}|D^v) + \beta$, which yields:

$$EU_D(M|D^v = 2) = (E(\tilde{v}|D^v = 2) + \beta - A_1) \times M \text{ for } M \in \{0, 1, 2^{MLO}, 2^{MO}\}. \tag{15}$$

His optimal reaction when he observes $D^v = 2$ thus results from a trade-off between his gain from trade and his adverse selection risk. Let us define:

$$\Phi^l = \frac{\gamma}{\sigma - \gamma} \frac{1-\pi}{\pi}(1-\theta)l_u - l_i. \tag{16}$$

Equation (15) yields:

$$EU_D(M|D^v = 2) = \Phi^l \times \frac{(\sigma - \gamma)\pi}{\pi l_i + (1-\pi)(1-\theta)l_u} \times M \text{ for } M \in \{0, 1, 2^{MLO}, 2^{MO}\}. \tag{17}$$

Lemma 1 directly follows the comparison of the liquidity demander's expected utility for his different possible actions, as described in equation (17).

Lemma 1 *The uninformed liquidity demander's optimal strategy when he observes $D^v = 2$ is as follows:*

- *If $\Phi^l > 0$, he submits a large market or marketable limit order $M = 2^{MO}$ or $M = 2^{MLO}$.*
- *If $\Phi^l < 0$, he submits no order $M = 0$.*
- *If $\Phi^l = 0$, he is indifferent between any order size.*

The incentives of the liquidity demander to submit large orders thus increase with Φ^l. This value summarizes the trade-off faced by the uninformed liquidity demander at date 2, when $D^v = 2$. On the one hand, he gains from trade: Φ^l increases with γ. On the other hand, he faces adverse selection. The larger the probability with which the informed liquidity supplier submits $D^v = 2$, that is, the higher l_i, or the larger π, the larger the belief's update of uninformed liquidity demander (see Equation (11)), the less he is willing to trade. Accordingly, Φ^l decreases with l_i and π.

The cut-off that delimits the regions in which the liquidity demander submits a large order or no order is such that $\Phi^l = 0$. At this point, the relative value of l_i with respect to l_u are such that gains from trade are perfectly offset by the adverse selection risk.

Similarly when $D^v = 1$, the liquidity demander faces a trade-off between his gains from trade, γ, and his adverse selection risk. If he submits an order that matches the visible depth, his expected utility is indeed:

$$EU_D(M|D^v = 1) = (E(\tilde{v}|D^v = 1) + \beta - A_1) \times M \text{ for } M \in \{0, 1\}. \tag{18}$$

The case where $D^v = 1$ is however more complex than the previous case where $D^v = 2$. Even if the *visible* depth in the limit order book is small, the *total* depth may be large due to the potential presence of a hidden order. This may induce the liquidity demander to submit a larger order than the visible depth. Such a strategy is linked to two types of risks, namely an adverse selection risk, and an execution risk, as the trading conditions of a large order are uncertain when $D^v = 1$.

On the one hand, revealing a hidden order enables the liquidity demander to trade a large volume at a low price, namely A_1. On the other hand, revealing a hidden order may lead the uninformed liquidity demander to update his beliefs more strongly than if there is no hidden depth, as shown in the following equivalence:

$$\begin{aligned} &|E(\tilde{v}|D^v = 1 \cap D^h = 1) - v_0| \geq |E(\tilde{v}|D^v = 1 \cap D^h = 0) - v_0| \\ &\quad \Leftrightarrow h_i(\theta + (1-\theta)s_u) \geq (1-\theta)h_u s_i. \end{aligned} \tag{19}$$

Conversely, the absence of hidden depth may signal the low informational content of the small limit order, but yields the liquidity demander either to restrict his trading volume if he submits a marketable limit order, or to trade his second unit at a higher price if he submits a large market order.

Since the revelation of hidden depth is not only informative on his execution conditions, but also on the value of the security, the liquidity demander needs to form beliefs on the probability with which there is hidden depth at the best quotes, using the existing correlation between the visible depth and the total depth (see Equation (14)). I shall now distinguish large orders depending on their type. Submitting a large marketable limit order guarantees a transaction price A_1, but there is uncertainty on the transaction size.

$$\begin{aligned} &EU_D(M = 2^{MLO}|D^v = 1) \\ &\quad = (E(\tilde{v}|D^v = 1 \cap D^h = 0) + \beta - A_1) \times \Pr(D^v = 1 \cap D^h = 0) \\ &\quad +2 \times (E(\tilde{v}|D^v = 1 \cap D^h = 1) + \beta - A_1) \times \Pr(D^v = 1 \cap D^h = 1). \end{aligned} \tag{20}$$

In the same situation, a large market order ($M = 2^{MO}$) would be fully executed, but it may have a price impact if there is no hidden depth at the best quotes.

$$EU_D(M = 2^{MO}|D^v = 1) = (2 \times (E(\tilde{v}|D^v = 1 \cap D^h = 0) + \beta) - A_1 - A_2)$$

$$\times \Pr(D^v = 1 \cap D^h = 0)$$
$$+2 \times (E(\tilde{v}|D^v = 1 \cap D^h = 1) + \beta - A_1) \times \Pr(D^v = 1 \cap D^h = 1) \tag{21}$$

At equilibrium, whether the liquidity demander would rather submit a large market order or a large marketable limit order proves to depend only on his gains from trade. As in the case where there is no informed liquidity supplier, which is described in the previous section, he is better off submitting a large marketable limit order than a large market order at equilibrium if and only if If $\gamma \leq \Delta$.

Let us define:

$$\Phi^h = \frac{\gamma}{\sigma - \gamma} \frac{1-\pi}{\pi}(1-\theta)h_u - h_i, \tag{22}$$

$$\Phi^s = \frac{\gamma}{\sigma - \gamma} \frac{1-\pi}{\pi}(\theta + (1-\theta)s_u) - s_i. \tag{23}$$

Lemma 2 follows. [8]

Lemma 2 *Let us define:*

$\Phi_1^h = \Phi^h + \mathbb{1}_{\Phi^s > \Delta(\frac{\frac{\gamma}{\sigma-\gamma}\frac{(1-\pi)}{\pi}(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma})} \times \left(\Phi^s - \frac{\Delta}{\sigma-\gamma}(\frac{1-\pi}{\pi}(\theta + (1-\theta)s_u) + s_i\right)$*, and*
$\Phi_2^h = \Phi^h + \Phi^s$,

The uninformed liquidity demander's optimal strategy when he observes $D^v = 1$ is as follows:

- *If $\Phi^s > 0$, then by definition $\Phi_2^h > \Phi_1^h$, and:*
  - *If $\Phi_1^h > 0$, he submits a large order.*

    More precisely, he submits a large market order if γ is sufficiently large and if $s_i < \frac{\gamma-\Delta}{\sigma-\gamma+\Delta}\frac{(1-\pi)}{\pi}(\theta + (1-\theta)s_u)$, and a large marketable limit order otherwise.
  - *If $\Phi_1^h = 0$, then he is indifferent between a large order and a small order.*
  - *If $\Phi_1^h < 0 < \Phi_2^h$, then he submits a small order.*
  - *If $\Phi_2^h = 0$, then he is indifferent between a small order and no order.*
  - *If $\Phi_1^h < \Phi_2^h < 0$, then he submits no order.*
- *If $\Phi^s \leq 0$ then:*
  - *If $\Phi^h + \frac{1}{2}\Phi^s > 0$, then he submits a large marketable limit order.*
  - *If $\Phi^h + \frac{1}{2}\Phi^s = 0$, then he is indifferent between a large marketable limit order and no order.*

[8] I introduce the dummy function $\mathbb{1}_X$, that takes the value 1 if the condition $\{X\}$ holds, and 0 otherwise.

– *If* $\Phi^h + \frac{1}{2}\Phi^s < 0$*, then he submits no order.*

The condition on Φ^l which determines the trading volume when $D^v = 2$ in Lemma 1 here translates into two conditions, on Φ^s and on Φ^h, when he observes $D^v = 1$. Two offers indeed lead to the observation of $D^v = 1$, namely a small limit order (S) and a hidden order (H), which adds one degree of freedom. The liquidity demander first needs to trade off the informational impact of a small visible depth with his gains from trade if he submits a small market order. This trade-off is captured in Φ^s. Then he trades off the informational impact of a revealed hidden order with his gains from trade if he submits a large market or marketable limit order, which is captured in Φ^h. Finally, he compares the expected profits of these strategies, which depend on the relative values of Φ^s and Φ^h. Submitting a small market order or a large order yield different informational impact and execution conditions.

For instance, if Φ^s is negative, the liquidity demander is not induced to trade at all, due to the large informational content of a small visible order. Still, he accounts for the presence of hidden depth. If the informed liquidity supplier submits a hidden order with a probability h_i that is sufficiently low, so that Φ^h is large, then revealing a hidden order has low informational content, therefore the liquidity demander would be ready to submit a larger order than the depth displayed at price A_1. Conversely, even if Φ^s is positive, so that $D_v = 1$ has a low informational content, the liquidity demander might fear the informational content of a hidden order that would be revealed by the submission of a large market or marketable limit order. Consequently, if Φ^h is sufficiently negative, he would not trade.

3.3 The determinants of the liquidity suppliers' strategies

The expected profits of the liquidity suppliers rely on their net profit from selling at price A_1 an asset which is worth $E(\tilde{v}|I)$, but only in the case where they actually trade. Execution depends on the presence of a buyer as liquidity demander, which occurs with probability $\frac{1}{2}$, and on the liquidity demander's reaction, which is conditional on the visible depth, as described above in Lemma 1 and 2. In turn, the liquidity suppliers' limit order submission strategy plays a crucial role, as shown in their expected utility:

$$\begin{aligned} & EU_j(l_j, h_j, s_j) \\ & = (A_1 - E(\tilde{v}|I_j)) \times \frac{1}{2} \\ & \times \begin{pmatrix} l_j \times (2 \times (\mu_{2^{MLO}|D^v=2} + \mu_{2^{MO}|D^v=2}) + \mu_{1|D^v=2}) \\ +h_j \times (2 \times (\mu_{2^{MLO}|D^v=1} + \mu_{2^{MO}|D^v=1}) + \mu_{1|D^v=1}) \\ +s_j \times (\mu_{2^{MLO}|D^v=1} + \mu_{2^{MO}|D^v=1} + \mu_{1|D^v=1}) \end{pmatrix} \end{aligned} \tag{24}$$

Though the informed and the large uninformed liquidity suppliers are induced to submit large limit orders to increase their trading volume, they need to take into account the impact of such a strategy on the liquidity demander's beliefs, thus on the probability of execution of their limit order.

3.3.1 The transparent market

In the transparent market, participants are able to observe the full quantity supplied at the best quotes in the limit order book. Consequently, the liquidity demander perfectly anticipates his trading conditions. Depth only acts as a potential signal on the value of the security. His demand is therefore highly sensitive to the probability of informed trading, π, as shown in the following Proposition.

Proposition 1 (Transparent market) *In the transparent market, there exists a unique equilibrium in pure strategies such that:*

1. *If $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$, the large uninformed and the informed liquidity suppliers submit a large limit order, that is, $l_u^* = l_i^* = 1$. The liquidity demander submits a large order when $D^v = 2$. If $D^v = 1$, he submits a small order if $\gamma \leq \Delta$, and a large market order if $\gamma > \Delta$.*

2. *If $\pi \geq \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$, all liquidity suppliers submit a small order, that is, $s_u^* = s_i^* = 1$. When $D^v = 1$, the liquidity demander submits a small order if $\pi \geq \frac{\gamma-\Delta}{\sigma}$, and a large market order otherwise.*

If $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$, whatever the strategy used by the informed liquidity supplier, the liquidity demander's adverse selection risk is sufficiently low to be compensated by his gains from trade. Consequently, the informed liquidity supplier does not fear to reveal her presence and she is able to always display a large limit order without decreasing its probability of execution. This equilibrium is characterized by a large trading volume, namely $M = 2$.

If $\pi \geq \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$ however, such a strategy cannot be sustained in equilibrium without deterring the liquidity demander. The increase in the adverse selection risk faced by the liquidity demander induces him to have a closer look at the informational content of the limit order book, and his reaction is conditional on the strategies of all liquidity suppliers. Consequently, the informed liquidity supplier needs to submit small orders to avoid being detected. Unfortunately, so does also the large uninformed liquidity supplier. This equilibrium is characterized by a small trading volume, namely $M = 1$.

3.3.2 The opaque market

In the opaque market, I extend the strategies' space of the liquidity suppliers to enable them to submit hidden orders. For the liquidity demander, the visible depth is still informative on the value of the security, and it becomes informative on the trading conditions. Therefore, his reaction is not only sensitive to the probability of informed trading, π, but also to his gains from trade, γ, as shown in Proposition 2.

Proposition 2 (Opaque market) *In the opaque market, the set of equilibria in pure strategies is characterized as follows:*

1. *If $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$, there exist two equilibria in pure strategies:*

   - *Either the large uninformed and the informed liquidity suppliers submit a large limit order, that is, $l_u^* = l_i^* = 1$. The liquidity demander submits a large order when $D^v = 2$, and when $D^v = 1$, he submits a large market order if $\gamma > \Delta$, and he is indifferent between submitting a small or a large marketable limit order otherwise.*

   - *Or the large uninformed and the informed liquidity suppliers submit a large hidden order, that is, $h_u^* = h_i^* = 1$, and the liquidity demander submits a large market or marketable limit order when $D^v = 1$.*

2. *If $\pi \geq max(\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta})$, there exist three equilibria in pure strategies, characterized as follows.*

   - *Either the large uninformed and the informed liquidity suppliers submit a small order, that is, $s_u^* = s_i^* = 1$.*

   - *Or the large uninformed and the informed liquidity suppliers submit a large hidden order, that is, $h_u^* = h_i^* = 1$.*

   - *Or the large uninformed liquidity supplier submits a small order, that is, $s_u^* = 1$, the informed liquidity supplier submits a large hidden order, that is, $h_i^* = 1$.*

   In any case, the liquidity demander submits a small order when $D^v = 1$.

3. *If $\gamma > \Delta$, then:*

   - *There exists an interval $[\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}]$ such that when π belongs to this interval, there exists a unique equilibrium in pure strategies characterized as follows: the large uninformed and the informed liquidity suppliers submit a large hidden order, that is, $h_u^* = h_i^* = 1$, and the liquidity demander submits a large market order when $D^v = 1$.*

   - *If $\pi < \frac{\gamma\theta-\Delta\theta}{\sigma-\gamma+\gamma\theta-\Delta\theta}$, there also exists an equilibrium in pure strategies such that the large uninformed liquidity supplier submits a large order, that is, $l_u^* = 1$, the informed liquidity supplier submits a hidden order, that is, $h_i^* = 1$, and the liquidity demander submits a large market order whatever the visible depth.*

If $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$, as in the transparent market, adverse selection is sufficiently low to induce the liquidity demander to trade a large quantity. Still, the informed liquidity supplier needs to mimic the strategy of the large uninformed liquidity supplier, so as to hide in the trading noise. Both liquidity suppliers indeed must have similar strategies in equilibrium, that is, either they both display a large order, or they both submit a hidden order. Conversely, if $\pi \geq max(\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta})$, then adverse selection is so high that even the potential use of hidden orders cannot induce the liquidity demander to trade a larger quantity than $M = 1$.

Let us now focus on the case where $\gamma > \Delta$ and $\pi \in [\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}]$, which is the key of the main results of the model. Adverse selection prevents the large uninformed liquidity supplier to submit a large order. It would indeed be followed in this strategy by the informed liquidity supplier, deterring the liquidity demander from trading when he observes $D^v = 2$. However, observing $D^v = 1$ is a more noisy signal on the value of the security, due to the presence of a small uninformed liquidity supplier in proportion θ. As in the transparent market, this induces the liquidity demander to trade at least one unit when he observes $D^v = 1$, while he would not trade if he observed $D^v = 2$. Proposition 2 shows that if he has large gains from trade, and if adverse selection keeps sufficiently low, he is even induced to submit a large market order.

This seems surprising, because if there is hidden depth, then the informational content of such a revelation is identical to the informational content of a large visible order. Consequently, his expected utility from trading each unit at price A_1 is negative, as it would have been if he had observed a large limit order. However, if there is no hidden depth, it must be the case that the order was submitted by a small uninformed liquidity supplier. Therefore, it bears no informational content. In Section 2, we have seen that when $\gamma > \Delta$, the liquidity demander would be ready to trade at price A_2 in the absence of adverse selection. Due to the initial presence of limit orders at this price, the liquidity demander who is characterized by large gains from trade would therefore profitably trade in the absence of hidden depth. I find that the gains realized when he trades in the later case offset the losses he would bear if there was a hidden order, provided that $\pi < \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}$.

In this case, there exists a unique equilibrium in pure strategies, in which the large uninformed liquidity supplier must submit a hidden order to have the opportunity to trade a large volume. My model therefore suggests a rationale for the use of hidden orders by uninformed limit order traders which differs from the arguments usually put forward. First, the uninformed liquidity supplier does not fear adverse selection that would be due to the presence of an informed liquidity demander. Neither does it want to avoid the risk of being picked off, described by Buti and Rindi (2009). Besides, such arguments may not hold in my model, as I allow the liquidity demander to submit marketable limit orders. My model suggests that in the presence of informed liquidity suppliers, liquidity suppliers must hide their large orders in order to decrease their informational impact.

My results therefore emphasize a rational for the use of hidden orders that differs from the existing literature. Following Harris (1997), hidden orders would attract large limit order traders by enabling them to reduce their exposure. First, limit orders are exposed to the risk of being picked off. This refers to a situation where fast traders (namely scalpers) react to the arrival of public information before the limit order trader cancels his mispriced order. Quote-matchers may even first undercut the limit order to gain price priority. If need be, they would rapidly undo their position by scalping the undercut limit order. Second, limit order traders face an adverse selection risk, as some market order traders may be privately better informed on the fundamental value of the security. The model proposed by Buti and Rindi (2009) analyses the behavior of limit order traders who are exposed to the risk of being picked off by fast traders. Their model allows traders to take the simultaneous three dimensional strategic decisions of price, quantity and exposure. Their numerical results show that traders use hidden orders both to compete for the provision of liquidity, as subsequent traders may not feel the need to undercut a small visible order while they would undercut a large order, and to avoid the risk of being picked off, as the supply of hidden depth complicates the detection of large mispriced orders. Accordingly, the authors find complementarities between price aggressiveness and order exposure.

The literature argues that liquidity-motivated traders may use hidden orders to lower the costs of managing an exposure to the risks linked to the option feature of limit orders. In contrast, my model suggests that both liquidity-motivated and information-motivated traders submit hidden orders to decrease the informational impact of their trades. This complements our understanding of opaque markets.

3.4 Discussion

I now discuss the assumption that the limit order book is initially symmetrically filled in with buy and sell orders at prices B_2 and A_2, which simplifies the resolution of the model.

Assuming a symmetry in the limit order book at date 0 enables me to study one side of the book only. I argue here that relaxing this assumption would not change the main findings, while obscuring the presentation of the results.

Assume first that there already is a sell limit order at price A_1 at date 0. Whatever the size of this order, a seller would not be able to trade more than one unit. Consequently, there is no question whether to submit or not a hidden order.

Assume now that there already is a buy limit order at price B_1. Let D^v_{ask} be the visible size of a sell limit order at price A_1, and D^v_{bid} be the visible size of the buy limit order at price B_1. Following Bayes' rule:

$$E(\tilde{v}|D^v_{ask} = 1, D^v_{bid} = 1) = E(\tilde{v}|D^v_{ask} = 2, D^v_{bid} = 2) = v_0, \tag{25}$$

$$E(\tilde{v}|D^v_{ask}=1, D^v_{bid}=2) = v_0 - \sigma \frac{\pi(s_i+h_i)}{\pi(s_i+h_i)+(1-\pi)(\theta+(1-\theta)(s_u+h_u))}$$
$$+\sigma \frac{\pi(l_i)}{\pi(l_i)+(1-\pi)(1-\theta)l_u}, \tag{26}$$

$$E(\tilde{v}|D^v_{ask}=2, D^v_{bid}=1) = v_0 - \sigma \frac{\pi(l_i)}{\pi(l_i)+(1-\pi)(1-\theta)l_u}$$
$$+\sigma \frac{\pi(s_i+h_i)}{\pi(s_i+h_i)+(1-\pi)(\theta+(1-\theta)(s_u+h_u))}, \tag{27}$$

The initial presence of a limit order at the bid side of the limit order book softens the adverse selection risk faced by the liquidity demander. This may induce him to trade a larger volume for a given probability of informed trading, reducing the incentives of liquidity suppliers to use hidden orders. The exposition of this case would therefore not deliver additional insights on the use of hidden orders.

4 Theoretical implications and empirical predictions

In this section, comparative statics enable me to analyze the impact of transparency on market quality, and to derive new empirical predictions. I focus on the case where $\gamma > \Delta$. In the opposite case indeed, even if there exists equilibria in which the liquidity suppliers submit hidden orders in the opaque market, these equilibria co-exists with equilibria in which they display the total size of their limit order. The liquidity demander would indeed never submit a larger marketable limit order than the visible depth. My model being characterized by a multiplicity of equilibria in pure and/or in mixed strategies, I only report here the comparative statics that hold whatever the equilibrium.

4.1 Adverse selection and the visible depth

My model confirms a well-known result in the literature (see Glosten,1994 for example): adverse selection harms market liquidity .

Corollary 1 (Depth and adverse selection) *In the opaque and in the transparent market, the visible and the total depth decrease with: i. the probability of informed trading, π, and ii. the asset volatility, σ.*

In my model though, adverse selection impacts the behavior of the uninformed liquidity supplier even if it does not directly face an adverse selection risk, as the liquidity demander is assumed to be uninformed. Liquidity drops when the level of asymmetric information increases because liquidity suppliers anticipate the increased informational impact of their large orders.

4.2 Transparency and market quality

I now analyze the impact of market transparency on market liquidity, and the welfare of the agents at equilibrium. Since prices are fixed in this model, market liquidity is measured by the size of the limit order submitted and displayed at price A_1, that is, by the depth at this price. Actually, there exist two measures of this depth, namely the visible depth, D^v, and the total depth, D^t. The move to an opaque market has different effects on both measures.

Corollary 2 (Transparency and liquidity) *If $\gamma > \Delta$, the visible depth D^v may decrease in the move to the opaque market, but the total depth D^t increases. The trading volume is larger in the opaque market. In particular, whatever the equilibria in both markets, the total depth and the trading volume are larger in the opaque market if $\pi \in [\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}]$.*

This prediction is in line with the empirical findings of natural experiments. Anand and Weaver (2004) analyze the reintroduction of hidden orders in the Toronto Stock Exchange in 2002, and show that it increased the total depth of the limit order book. Similarly, Aitken et al. (2001) study two successive increases in the level of pre-trade transparency in the Australian Stock Exchange in October 1994 and in October 1996, characterized by a mandatory increase in the proportion of the displayed size with respect to the total size of the order. They find significant volume reductions. I show in Corollary 3 that this increase both in market depth and in trading activity improves the welfare of the participants.

Corollary 3 (Transparency and welfare) *If $\gamma > \Delta$, the ex ante expected utility of all the traders, that is, the informed and uninformed liquidity suppliers and the liquidity demander, are higher in the opaque than in the transparent market. In particular, they are strictly better off if $\pi \in [\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}]$.*

On the one hand, the liquidity demander with large gains from trades is induced to submit a larger order than visible depth, as he would be ready to trade even at price A_2 in the absence of adverse selection. If there is a hidden order at price A_1, he looses on average, but this loss is balanced by an increase in utility in the absence of hidden depth. On the other hand, the welfare of the liquidity suppliers depends on the equilibrium trading volume. Hidden orders sometimes enable them to decrease the informational impact of their large orders. Consequently, both the informed and the large uninformed liquidity suppliers are better off in the opaque market. Consistently, Bessembinder et al. (2009) analyze the various costs and benefits of order exposure, and find that the option to hide order size is valuable to many market participants, as hidden orders seem to be characterized by lower opportunity costs, despite a decreased likelihood of full execution.

4.3 Empirical predictions

My model shows that under some conditions, liquidity suppliers are obliged to submit hidden orders. This enables me to derive some empirical predictions on the frequency and on the informational impact of hidden orders in opaque limit order books. I first analyze the probability to observe hidden orders.

Corollary 4 (Hidden depth) *Whatever the equilibrium in the opaque market, when $\gamma > \Delta$, the frequency with which hidden orders are submitted is strictly positive if $\pi \in [\frac{\gamma-\gamma\theta}{\Delta-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}]$. This frequency decreases with the tick size, Δ, while the impact of the volatility σ is ambiguous.*

A couple of empirical results seems to confirm that the submission of hidden orders depends on the level of information asymmetries. First, analyzing the intraday patterns of depth proportions, Fleming and Mizrach (2009) show that hidden depth proportions are higher outside of New York trading hours for the U.S. BrokerTec Platform. Second, Bessembinder et al. (2009) find that the usage of hidden orders is more prevalent for less liquid firms in Euronext Paris, and that traders choose to hide more of their orders when the bid-ask spread is wide. Finally, Chakrabarty and Shaw (2008) report a significant increase in the hidden order trade volume around earnings announcements.

However, the impact of an increase in volatility on the probability to observe hidden orders is ambiguous. When the asset volatility σ increases, liquidity suppliers need to hide their large orders for lower probabilities of informed trading. Indeed, $\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$ decreases. However, simultaneously, $\frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}$ decreases as well: the liquidity suppliers need not to hide as the liquidity demander would never submit a larger order above this threshold. Accordingly, Aitken et al. (2001) and Fleming and Mizrach (2009) empirically find that volatility increases hidden order proportions, while Bessembinder et al. (2009) find that a greater return volatility is associated with a lower likelihood of hidden order usage.

Two effects go along with a decrease in the tick size, and reinforce each other. First, there may be a move from the case where $\gamma \leq \Delta$ to the case where $\gamma > \Delta$. Second, the threshold $\frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}$ decreases with Δ. Intuitively, when Δ decreases, the liquidity demander may be more likely to submit a large market order, as the cost of walking up the book in the absence of hidden depth decrease. In turn, this encourages the submission of hidden orders by liquidity suppliers. This result is in line with the findings of Aitken et al. (2001). Notice that if the liquidity suppliers also use hidden orders to avoid quote-matching, the impact of a decrease in the tick size on hidden depth would be similar, as this would favor parasitic behavior. My model proposes an alternative explanation.

As compared to a small limit order that offers the same visible depth, a hidden order may only be useful for liquidity suppliers if the liquidity demander submits larger orders than the visible depth. By convention in this case, the large market or marketable limit order is said to "test" the presence of hidden depth, and this hidden depth is then “revealed”. My model also enables to draw new empirical predictions on the conditions under which the liquidity demander tests the presence of hidden depth.

Corollary 5 (Revelation of hidden depth) *Whatever the equilibrium in the opaque market, when $\gamma > \Delta$, the frequency with which the uninformed liquidity demander submits a larger order than the visible depth, that is,* $\Pr(M = 2|D^v = 1)$*, decreases with: i. the probability of informed trading π, and ii. the volatility of the asset σ.*

A question that naturally arises is then: does the revelation of a hidden order provide information? Researchers seem to debate on this point.

On the one hand, the hidden depth seems to have an informational content, as documented by Anand and Weaver (2004), Tuttle (2006), Fleming and Mizrach (2009) or Frey and Sandas (2009). These results are coherent with Corollary 6.

Corollary 6 (Informational impact of a revealed hidden order) *Whatever the equilibrium in the opaque market, the revelation of a hidden order has an informational impact, that is, $\left|E\left(\tilde{v}|D^h_{rev} = 1\right) - v_0\right| > 0$, where $E\left(\tilde{v}|D^h_{rev} = 1\right)$ stands for the expected value of the security, conditional on a hidden order to have been submitted and revealed.*

Proposition 2 indeed shows that in equilibrium, the probability with which the informed liquidity supplier submits a hidden order, h_i, is always greater or equal to the probability with which the large uninformed liquidity supplier submits a hidden order, h_u.

But on the other hand, various empirical studies question the informational content of hidden orders. According to Pardo and Pascual (2007), revealing a hidden order does not have a larger permanent price impact than an equally sized and matched ordinary trade in the Spanish Stock Exchange. Consistently, De Winne and D’Hondt, 2007 or Bessembinder et al., 2009 for Euronext Paris, Frey and Sandas, 2009 for Xetra report no significant effect of hiding size on price impact, conditional on execution. My model however shows that this result does not contradict the previous one.

Corollary 7 (Informational impact of large orders) *There exist equilibria in the opaque market where a revealed hidden order has a lower informational impact than a non-hidden order of the same size.*

For instance, if $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$ and $\gamma > \Delta$, there exists an equilibrium in pure strategies such that: i. the informed liquidity supplier submits a large visible order, $l_i^* = 1$, ii. the large uninformed liquidity supplier submits a large order with probability $l_u^* = \frac{1}{\Gamma(1-\theta)} + \varepsilon$ and a hidden order with the complementary probability, and iii. the liquidity demander submits a large market order whatever the depth he observes.

In equilibrium, revealing a hidden size has no informational impact, while observing a large limit order yields:

$$\left|E\left(\tilde{v}|D^t = 2 \cap D^h = 0\right) - v_0\right| = \frac{\sigma\pi}{\pi + \frac{1-\pi}{\Gamma}} + \varepsilon. \tag{28}$$

In any case, the informed liquidity supplier often needs to mimic the behavior of the large uninformed liquidity supplier, in order to avoid being detected.

5 Conclusion

I analyze the strategy of orders' submission in a limit order book characterized by asymmetric information. I show that hidden orders may be used to soften the informational impact of large orders, while allowing liquidity suppliers to trade a large volume. My model yields empirical predictions, which I confront to the existing empirical literature. Recent empirical findings highlight both the predictability of the presence of hidden orders, and the unpredictability of the magnitude of the hidden part (Bessembinder et al., 2009 or Frey and Sandas, 2009). In my model, I fix the size of the hidden part, but it would be interesting to extend the decision set of limit order traders to multiple units.

Besides, I show that opacity improves market liquidity in a centralized limit order book, which supports the success of hidden orders and of dark pools. It would however be interesting to extend this analysis to fragmented markets. Opacity would indeed increase search costs in such an environment, which may offset its beneficial effects.

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6 Appendix: Proofs

6.1 Proof of Lemma 2

Proof 1 (Lemma 2) *Let us define* $\Gamma = \frac{\gamma}{\sigma-\gamma}\frac{(1-\pi)}{\pi}$*. Comparing the expected utility of the liquidity demander for each* $M \in \{0, 1, 2^{MLO}, 2^{MO}\}$*, conditional on* $D^v = 1$*, yields the following system of 6 inequalities.*

$$M = 2^{MLO} \succ M = 0 \Leftrightarrow h_i < \Gamma(1-\theta)h_u + \frac{1}{2}(\Gamma(\theta+(1-\theta)s_u) - s_i) \text{ (S1)}$$

$$M = 2^{MLO} \succ M = 1 \Leftrightarrow h_i < \Gamma(1-\theta)h_u \text{ (S2)}$$

$$M = 2^{MLO} \succ M = 2^{MO} \Leftrightarrow s_i > \frac{\gamma-\Delta}{\sigma-\gamma+\Delta} \times \frac{(1-\pi)}{\pi}(\theta+(1-\theta)s_u) \text{ (S3)}$$

$$M = 2^{MO} \succ M = 1$$

$$\Leftrightarrow h_i < \Gamma(1-\theta)h_u + \Gamma(\theta+(1-\theta)s_u) - s_i - \Delta(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}) \text{ (S4)}$$

$$M = 2^{MO} \succ M = 0$$

$$\Leftrightarrow h_i < \Gamma(1-\theta)h_u + \Gamma(\theta+(1-\theta)s_u) - s_i - \frac{\Delta}{2}(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}) \text{ (S5)}$$

$$M = 1 \succ M = 0 \Leftrightarrow h_i < \Gamma(1-\theta)h_u + \Gamma(\theta+(1-\theta)s_u) - s_i \text{ (S6)}$$

With our notations, the system becomes:

$$M = 2^{MLO} \succ M = 0 \Leftrightarrow \Phi^h + \frac{1}{2}\Phi^s > 0 \text{ (S1)}$$

$$M = 2^{MLO} \succ M = 1 \Leftrightarrow \Phi^h > 0 \text{ (S2)}$$

$$M = 2^{MLO} \succ M = 2^{MO} \Leftrightarrow \Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right) < 0 \text{ (S3)}$$

$$M = 2^{MO} \succ M = 1 \Leftrightarrow \Phi^h + \Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right) > 0 \text{ (S4)}$$

$$M = 2^{MO} \succ M = 0 \Leftrightarrow \Phi^h + \Phi^s - \frac{\Delta}{2}(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}) > 0 \text{ (S5)}$$

$$M = 1 \succ M = 0 \Leftrightarrow \Phi^h + \Phi^s > 0 \text{ (S6)}$$

1. *Let us first assume that* $\Phi^s > 0$.

    - *Assume that* $\Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right) > 0$*. In this case, given the inequality (S3), the submission of a large market order is preferred to the submission of a large marketable limit order. Besides, we have:*

$$\Phi^h < \min\left(\Phi^h + \frac{1}{2}\Phi^s, \Phi^h + \Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right)\right)$$

$$< \max\left(\Phi^h + \frac{1}{2}\Phi^s, \Phi^h + \Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right)\right)$$
$$< \Phi^h + \Phi^s - \frac{\Delta}{2}\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right) < \Phi^h + \Phi^s$$

Consequently, the liquidity demander submits a large market order as long as $\Phi^h + \Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right) > 0$*, that is, as long as* $\Phi^h_1 > 0$*, no order if* $\Phi^h + \Phi^s < 0$*, that is, if* $\Phi^h_2 < 0$*, and a small order in-between.*

- *Assume that* $\Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right) \leq 0$*. In this case, given the inequality (S3), the submission of a large marketable limit order is preferred to the submission of a large market order. Besides, we have:*

$$\Phi^h + \Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right)$$
$$\leq \min\left(\Phi^h, \Phi^h + \Phi^s - \frac{\Delta}{2}\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right)\right)$$
$$< \max\left(\Phi^h, \Phi^h + \Phi^s - \frac{\Delta}{2}\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right)\right)$$
$$< \Phi^h + \frac{1}{2}\Phi^s < \Phi^h + \Phi^s$$

Consequently, the liquidity demander submits a large marketable limit order as long as $\Phi^h > 0$*, which is equivalent in this case to* $\Phi^h_1 > 0$*, no order if* $\Phi^h + \Phi^s < 0$*, that is, if* $\Phi^h_2 < 0$*, and a small order in-between.*

2. *Let us now assume that* $\Phi^s \leq 0$*. It must then be the case that* $\Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right) < 0$*, and that the following series of inequalities holds:*

$$\Phi^h + \Phi^s - \Delta\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right) < \Phi^h + \Phi^s - \frac{\Delta}{2}\left(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}\right)$$
$$< \Phi^h + \Phi^s \leq \Phi^h + \frac{1}{2}\Phi^s \leq \Phi^h$$

Consequently, the liquidity demander submits a marketable limit order if $\Phi^h + \frac{1}{2}\Phi^s > 0$*, and no order if* $\Phi^h + \frac{1}{2}\Phi^s < 0$*. If* $\Phi^h + \frac{1}{2}\Phi^s = 0$*, then he is indifferent between these two strategies.*

6.2 Proof of Proposition 1

Proof 2 (Proposition 1 Part 1) *For* $\pi < \frac{\gamma(1-\theta)l_u}{\sigma-\gamma+\gamma(1-\theta)l_u}$*, whatever the strategies of the informed liquidity supplier* l_i*, and of the large uninformed liquidity supplier* l_u*, the condition* $\Phi^l > 0$ *holds since the former inequality is equivalent to* $\Gamma(1-\theta)l_u > 1$*. Consequently, according to Lemma 1, the liquidity demander* *always submit a large order when he observes $D^v = 2$. This induces both the informed and the large uninformed liquidity supplier to display a large order. For $l_u = 1$, the threshold of π that sustains such strategies in equilibrium is $\pi < \frac{\gamma - \gamma\theta}{\sigma - \gamma\theta}$.*

Proof 3 (Proposition 1 Part 2) *For $\pi \geq \frac{\gamma - \gamma\theta}{\sigma - \gamma\theta}$, then even if the large uninformed liquidity supplier displays a large order with probability $l_u = 1$, adverse selection deters the liquidity demander from trading, except if l_i is sufficiently low relative to l_u to ensure that $\Phi^l > 0$. But if this was the case, submitting a small order with some positive probability would not be sustainable for the informed agent, as she would earn strictly more by submitting a large order than by submitting a small order. It must therefore be the case that the informed liquidity supplier is at least indifferent between submitting a small or a large order (that is, that she either is perfectly indifferent, or strictly better off submitting a small order), and so does the large uninformed liquidity supplier.*

Given that there cannot exist an equilibrium in pure strategies where the large uninformed and the informed liquidity demander submit and trade a large quantity, we now look for an equilibrium in pure strategies, where the liquidity suppliers submit a small limit order. If $s_u = s_i = 1$, then $E(\tilde{v}|D^v = 1) = v_0 - \pi\sigma$. Provided that $\pi < \frac{\gamma}{\sigma}$, the liquidity demander submits a small order if $E(\tilde{v}|D^v = 1) \leq \Delta$, and a large market order otherwise. This proves Part 2 of Proposition 1.

To complete this proof, we now show that there also exist equilibria in mixed strategies. In an equilibrium in mixed strategies, the informed liquidity supplier must be indifferent between submitting a small or a large order, and so does the large uninformed liquidity supplier. This may only be the case if the liquidity demander submits a small order when $D^v = 2$ and when $D^v = 1$. According to Lemma 1, this imposes that $l_i = \Gamma(1-\theta)l_u$. Under the later condition,

$$\begin{aligned}
\Phi^s &= \Gamma(\theta + (1-\theta)s_u) - s_i \\
&= \Gamma(1 - (1-\theta)l_u) - (1 - l_i) \\
&= \Gamma - \Gamma(1-\theta)l_u - 1 + \Gamma(1-\theta)l_u \\
&= \Gamma - 1
\end{aligned} \tag{29}$$

Our assumption that $\gamma < \sigma$ implies that $\Phi^s > 0$. If $\Phi^s > \Delta(\frac{\Gamma(\theta+(1-\theta)s_u}{\gamma} + \frac{s_i}{\sigma-\gamma})$, Lemma 2 shows that the liquidity demander submits a large order, otherwise, he submits a small order when $D^v = 1$. Consequently, there exists a multiplicity of equilibria in mixed strategies, which are characterized as follows:

- *The informed liquidity supplier submits a large order with probability $l_i^* = \Gamma(1-\theta)l_u^*$ and a small order with probability $s_i^* = 1 - l_i^*$,*

- *The large uninformed liquidity supplier submits a large order with probability* $0 \leq l_u^* \leq 1$ *and a small order with probability* $s_u^* = 1 - l_u^*$,
- *If* $\Phi^s > \Delta(\frac{\Gamma(\theta+(1-\theta)s_u}{\gamma} + \frac{s_i}{\sigma-\gamma})$, *the liquidity demander submits a large order, otherwise, he submits a small order when* $D^v = 1$. *He submits an order of size 1 if* $D^v = 2$.

The probability l_i^* *is such that the informed liquidity supplier perfectly "mimics" the uninformed liquidity supplier's strategy, so as to "hide in the trading noise".*

6.3 Proof of Proposition 2

Proof 4 (Proposition 2 Parts 1 and 3) *We first look for all the equilibria in pure strategies which are characterized by a large trading volume, that is, such that the liquidity demander submits a large market or marketable limit order. There are two possible cases.*

1. *Either* $l_i^* = 1$. *According to Lemma 1, such a strategy is sustainable in equilibrium for the informed liquidity supplier provided that* $\pi < \frac{\gamma(1-\theta)l_u}{\sigma-\gamma+\gamma(1-\theta)l_u}$. *In this case, two pure strategies may yield large profits for the large uninformed liquidity demander. Either* $h_u^* = 1$, *but in this case, the former condition becomes* $\pi < 0$ *so there can be no equilibrium such that* $l_i^* = 1$ *and* $h_u^* = 1$. *Or* $l_u^* = 1$. *It yields that this equilibrium exists for* $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$. *Now, given that the informed liquidity supplier never displays small orders,* $\Phi^s > 0$ *and* $\Phi_2^h > 0$. *Thus when* $D^v = 1$, *according to Lemma 2, the liquidity demander submits a large market order if* $\gamma > \Delta$, *and he is indifferent between submitting a small or a large marketable limit order otherwise.*

2. *Or* $h_i^* = 1$. *Given that in this case* $\Phi^s > 0$, *Lemma 2 shows that we need* $\Phi_1^h > 0$ *to ensure that the liquidity demander submits a large order when the visible depth is small. Re-arranging terms after accounting for* $s_i^* = 0$, *we have:*

$$\Phi_1^h > 0 \Leftrightarrow \Gamma(1-\theta)h_u - h_i + \mathbb{1}_{\gamma>\Delta} \times \frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} > 0 \tag{30}$$

   - *If* $\gamma \leq \Delta$, *the inequality (30) becomes* $h_i < \Gamma(1-\theta)h_u$. *The same reasoning as in case 1. for* $l_i^* = 1$ *applies and yields to the conclusion that the only equilibrium in pure strategies such that* $h_i^* = 1$ *is characterized by* $h_u^* = 1$, *and therefore exists as long as* $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$. *Now, given that only the small uninformed liquidity supplier submits small orders,* $\Phi^s = \Gamma\theta$ *which is positive, while* $\Phi^h = \Gamma(1-\theta)$. *This implies that* $\Phi_1^h = \Gamma(1-\theta)$ *and* $\Phi_2^h = \Gamma$, *so both parameters are strictly positive. Thus when* $D^v = 1$ *and for* $\gamma \leq \Delta$, *according to Lemma 2, the liquidity demander submits a large marketable limit order.*

- *If $\gamma > \Delta$, the inequality (30) becomes:*

$$h_i < \Gamma(1-\theta)h_u + \frac{\Gamma(\theta+(1-\theta)s_u}{\gamma} \tag{31}$$

After some manipulations, we show that for $h_i^* = 1$, this inequality only holds if the following inequality holds:

$$\pi < \frac{\gamma - \gamma(1-\theta)l_u - \Delta(\theta+(1-\theta)s_u)}{\sigma - \gamma(1-\theta)l_u - \Delta(\theta+(1-\theta)s_u)} \tag{32}$$

If the uninformed liquidity demander submits a large order when $D^v = 1$, submitting a small limit order is a strictly dominated strategy for the large uninformed liquidity supplier. Therefore $s_u^* = 0$. Now, given that the liquidity demander submits a large order whatever the visible depth if $h_i^* = 1$ and if the inequality (32) holds, the large uninformed liquidity supplier could either submit a large or a hidden order. If $l_u^* = 1$, then this equilibrium where $h_i^* = 1$ only exists for $\pi < \frac{\gamma-\gamma(1-\theta)-\Delta\theta}{\sigma-\gamma(1-\theta)-\Delta\theta}$. If $h_u^* = 1$, then this equilibrium where $h_i^* = 1$ exists for $\pi < \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}$.

Now, given that $\gamma > \Delta$, we have $\frac{\gamma-\Delta\theta}{\sigma-\Delta\theta} > \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$. This further proves Part 3. of Proposition 2.

3. *More generally for $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$,*

- *When $\gamma \leq \Delta$, any set of strategies characterized by $s_u^* = 0$ for the large uninformed liquidity supplier and $s_i^* = 0$ for the informed liquidity supplier can sustain an equilibrium in which the liquidity demander submits a large order, as long as the two following inequalities hold:*

$$l_i^* < \Gamma(1-\theta)l_u^* \tag{33}$$

$$h_i^* < \Gamma(1-\theta)h_u^* \tag{34}$$

These inequalities imply that on the one hand $\Phi^l > 0$, and that on the other hand that $\Phi_1^h > 0$ since $\Phi^s = \Gamma\theta$ when $s_u^* = s_i^* = 0$, which is strictly positive. In this case, Lemma 1 and 2 respectively show that the liquidity demander submits a large market or marketable limit order whatever the visible depth. A necessary condition for both inequalities to hold is obtained by summing both inequalities for $s_u^* = s_i^* = 0$, which yields $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$.

- *Similarly when $\gamma > \Delta$, any set of strategies characterized by $s_u^* = 0$ for the large uninformed liquidity supplier and $s_i^* = 0$ for the informed liquidity supplier can sustain an equilibrium in which the liquidity demander submits a large order, as long as the two following inequalities hold:*

$$l_i^* < \Gamma(1-\theta)l_u^* \tag{35}$$

$$h_i^* < \Gamma(1-\theta)h_u^* + (\gamma-\Delta)\frac{\Gamma\theta}{\gamma} \tag{36}$$

A necessary condition for both inequalities to hold is obtained by summing both inequalities for $s_u^* = s_i^* = 0$, which yields $\pi < \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}$.

Proof 5 (Proposition 2 Part 2) *When $\pi \geq max(\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta})$, Proof 4 shows that there cannot be any equilibrium, either in pure or in mixed strategies, such that the liquidity suppliers are able to trade a large volume $M = 2$. We therefore first look for all the equilibria in pure strategies which are characterized by a small trading volume, that is, such that the liquidity demander submits a small market order. Given that the liquidity demander submits a small order, a small and a hidden order not only represent the same signal $D^v = 1$ on the value of the security, but they also have the same execution conditions. Consequently, both strategies are perfectly equivalent for the informed and the large uninformed liquidity supplier. We therefore simply check whether the conditions required for the liquidity demander to submit a small order when $D^v = 1$ are fulfilled when $l_u^* = l_i^* = 0$ and $\pi \geq max(\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta})$. There are four possible cases.*

1. *Either $s_u^* = s_i^* = 1$, in which case $\Phi^s = \Gamma - 1$ and $\Phi^h = 0$. Given our assumption $\pi < \frac{\gamma}{\sigma}$, $\Gamma > 1$, and the two thresholds of Lemma 2 become:*

   $\Phi_1^h = \mathbb{1}_{\Gamma-1>\Delta(\frac{\Gamma}{\gamma}+\frac{1}{\sigma-\gamma})} \times (\Gamma - 1 - \Delta(\frac{\Gamma}{\gamma} + \frac{1}{\sigma-\gamma})$, *and*

   $\Phi_2^h = \Gamma - 1.$

   Given that $\Gamma - 1 \leq \Delta(\frac{\Gamma}{\gamma} + \frac{1}{\sigma-\gamma})$ for $\pi \geq max(\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta})$, we indeed have $\Phi_1^h = 0 < \Phi_2^h$. According to Lemma 2, the liquidity demander in this case is indifferent between a large marketable limit order and a small order.

2. *Or $h_u^* = h_i^* = 1$, in which case $\Phi^s = \Gamma\theta$ which is strictly positive, and $\Phi^h = \Gamma(1-\theta) - 1$. So the two thresholds of Lemma 2 become:*

   $\Phi_1^h = \Gamma(1-\theta) - 1 + \mathbb{1}_{\gamma>\Delta} \times \Gamma\theta(1 - \frac{\Delta}{\gamma})$, *and*

   $\Phi_2^h = \Gamma - 1.$

   Again, $\Phi_2^h > 0$. For $\pi \geq max(\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta})$, we indeed have $\Phi_1^h < 0$. According to Lemma 2, the liquidity demander submits a small order.

3. *Or $s_u^* = 1$ and $h_i^* = 1$, in which case $\Phi^s = \Gamma$ which is strictly positive, and $\Phi^h = -1$. So the two thresholds of Lemma 2 become:*

   $\Phi_1^h = -1 + \mathbb{1}_{\gamma>\Delta} \times \Gamma(1 - \frac{\Delta}{\gamma})$, *and*

   $\Phi_2^h = -1 + \Gamma.$

Similarly, $\Phi_2^h > 0$, and for $\pi \geq max(\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta})$, we indeed have $\Phi_1^h < 0$. According to Lemma 2, the liquidity demander submits a small order.

4. *Or $h_u^* = 1$ and $s_i^* = 1$, in which case $\Phi^s = \Gamma\theta - 1$ and $\Phi^h = \Gamma(1-\theta)$. Therefore, $\Phi^s > 0$ if and only if $\pi < \frac{\gamma\theta}{\sigma-\gamma+\gamma\theta}$. But for $\theta < \frac{1}{2}$, we have $\frac{\gamma\theta}{\sigma-\gamma+\gamma\theta} < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$, so we must have $\Phi^s \leq 0$. In this case, depending on the parameters' values, if $\Phi^h + \frac{1}{2}\Phi^s > 0$, the liquidity demander would submit a large marketable limit order, so the informed agent would be induced to submit a large hidden order, therefore there can be no equilibrium such that $h_i = 0$. Or $\Phi^h + \frac{1}{2}\Phi^s > 0$, in which case the liquidity demander submits no order, consequently, deviations of the liquidity suppliers can be expected. Finally, if $\Phi^h + \frac{1}{2}\Phi^s = 0$, that is, if $\pi = \frac{\gamma+\gamma(1-\theta)}{\sigma+\gamma(1-\theta)}$, then the liquidity demander is indifferent between submitting no order and a large marketable limit order. But in this case again, the informed liquidity demander would be induced to submit a large hidden order, so $h_i = 0$ cannot hold in equilibrium.*

5. *More generally, we now look for all equilibria in mixed strategies which are characterized by a small trading volume. On the one hand, to ensure that the liquidity demander trades an average quantity of 1 unit when $D^v = 2$, Lemma 1 shows that we must have : $\Phi^l = 0$. On the other hand, to ensure that the liquidity demander trades an average quantity of 1 unit when $D^v = 1$, Lemma 2 shows that we must have: $\Phi^s > 0$ and $\Phi_1^h \leq 0 \leq \Phi_2^h$. This yields the following set of conditions:*

$$l_i^* = \Gamma(1-\theta)l_u^* \tag{37}$$

$$\begin{aligned}
&s_i^* < \Gamma(\theta + (1-\theta)s_u^*) \\
&s_i^* + h_i^* \leq \Gamma(\theta + (1-\theta)(s_u^* + h_u^*)) \\
&h_i^* \leq \Gamma(1-\theta)h_u^* + \mathbb{1}_{s_i < \frac{\gamma-\Delta}{\sigma-\gamma+1}\frac{1-\pi}{\pi}} \\
&\times(\Gamma(\theta + (1-\theta)s_u) - s_i - \Delta(\frac{\Gamma(\theta+(1-\theta)s_u)}{\gamma} + \frac{s_i}{\sigma-\gamma}))
\end{aligned} \tag{38}$$

Given that $\Gamma > 1$ for $\pi < \frac{\gamma}{\sigma}$, there are multiple equilibria in mixed strategies.

6.4 Proof of the corollaries

Proof 6 (Proof of Corollary 1) *The proofs of Propositions 1 and 2 show that whatever the equilibrium, the large and the uninformed liquidity suppliers do not submit small limit orders in equilibrium either if $\pi < \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}$ when $\gamma > \Delta$ in the opaque market, or if $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$ otherwise. Both thresholds decrease in π, in σ and in θ. Besides, the equilibria in mixed strategies such that the liquidity demander submits a* *small order whatever the book he observes are all characterized by $l_i^* = \Gamma(1-\theta)l_u^*$, which is decreasing in the same set of variables.*

Proof 7 (Proof of Corollary 2) *If $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$ or $\pi > \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}$, the total depth available at the best quotes for a fixed level of adverse selection π does not depend on market transparency. In the opposite case though, market opacity provides the liquidity suppliers with the opportunity to trade a large trading volume, which they could not do in the transparent market for the same level of the parameter π. Accordingly, they submit larger orders in this case, which increases the total depth. However, these are hidden orders, so the visible depth does not increase.*

Proof 8 (Proof of Corollary 3) *If he submits a large market order, the ex ante expected utility of the liquidity demander characterized by $\gamma > \Delta$ writes:*

$$EU_D = 2(\gamma - \pi\sigma) - \Delta(\pi s_i + (1-\pi)[(1-\theta)s_u + \theta) \tag{39}$$

If he submits a small order, his ex ante expected profit writes: $EU_D = (\gamma - \pi\sigma)$

1. *Whatever the equilibrium in the opaque market, we therefore have:*

   - *If $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$, then:*

$$\begin{aligned} EU_D^t = EU_D^o &= 2(\gamma - \pi\sigma) - (1-\pi)\theta \times min(\gamma, \Delta) \\ EU_U^t = EU_U^o &= \Delta \\ EU_I^t = EU_I^o &= \Delta + \sigma \end{aligned} \tag{40}$$

   - *If $\pi \geq max(\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta})$, then:*

$$\begin{aligned} EU_D^t = EU_D^o &= \gamma - \pi\sigma \\ EU_U^t = EU_U^o &= \frac{1}{2}\Delta \\ EU_I^t = EU_I^o &= \frac{1}{2}(\Delta + \sigma) \end{aligned} \tag{41}$$

2. *If $\gamma > \Delta$, then for $\pi \in [\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}]$, we have in the transparent market:*

$$\begin{aligned} EU_D^t &= \gamma - \pi\sigma \\ EU_U^t &= \frac{1}{2}\Delta \\ EU_I^t &= \frac{1}{2}(\Delta + \sigma) \end{aligned} \tag{42}$$

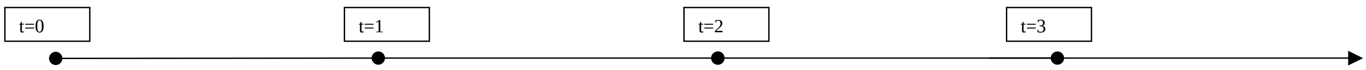
And in the opaque market:

$$
\begin{aligned}
EU_D^o &= 2(\gamma - \pi\sigma) - \Delta\theta(1-\pi) \\
EU_U^o &= \Delta \\
EU_I^o &= \Delta + \sigma
\end{aligned} \tag{43}
$$

Proof 9 (Proof of Corollary 4) *In the opaque market, when $\gamma > \Delta$, if π is uniformly distributed in $[0,1]$, then whatever the equilibrium played in the opaque market when $\pi < \frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}$ or when $\pi > \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}$, the probability of observe hidden orders increases with the size of the interval $[\frac{\gamma-\gamma\theta}{\sigma-\gamma\theta}, \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}]$. In this interval indeed, there exists a unique equilibrium which is characterized by the use of hidden orders by the large uninformed and the informed liquidity suppliers. Straightforward computations show that: i. both thresholds of the interval decrease in σ, and ii. the size of this interval decreases with σ.*

Proof 10 (Proof of Corollary 5) *The proof is straightforward, as the liquidity demander submits a larger order than the visible depth as long as $\pi \leq \frac{\gamma-\Delta\theta}{\sigma-\Delta\theta}$.*

FIGURE 1 : TRADING PROCESS



t=0

- The limit order book is filled in with buy and sell limit orders.
- The best quotes are (A_2,B_2).

t=1

- Agents observe the best quotes (A_2,B_2).
- One agent submits a limit order that undercuts the best quotes by one tick (at price A_1 for a sell order or B_1 for a buy order)

t=2

- Agents observe the depth of the limit order book at price (A_1,B_1).
- One liquidity demander enters the market and potentially submits a market or a marketable limit order to buy or sell the asset.

t=3

- The final value of the asset is revealed.