Partial Regulation
in Vertically Differentiated Industries*

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Abstract

In this paper we provide theoretical foundations for a price-and-quality cap regulation of recently liberalized utilities. We model a partially regulated oligopoly where vertically differentiated services are provided by a regulated incumbent and an unregulated entrant competing in price and quality. The model may equally well represent competition within or across industries. We establish that weights in the cap need to depend also on the market served by the entrant, despite the latter is not directly concerned by regulation. This calls for the possibility that regulators use information about the whole industries, rather than on the sole incumbent. In the unit demand case, however, regulation can be conducted as if the regulated firm were still a monopoly.

Keywords: Price-and-Quality Cap; Partial Regulation; Vertical Differentiation

J.E.L. Classification Numbers: L11, L13, L51, L9

*This paper has been presented at the XVIII SIEP Meeting, Università di Pavia, at the 5th Conference on Applied Infrastructure Research, Technische Universität Berlin, at the 4th Annual Conference on Railroad Industry Structure, Competition and Investment, Universidad Carlos III de Madrid, and at the 47th SIE Meeting, Università di Verona. We would like to thank seminar participants for their comments and, in particular, Carlo Fiorio and Ingo Vogelsang. All remaining errors are ours. The paper was completed while the second author was visiting Université de Montreal whose hospitality is gratefully acknowledged. The third author also whish to thank the Florence School of Regulation, at the EUI that she visited in quality of Jean Monnet Fellow.

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"Price cap regulation rates as one of the success stories of applied economic theory. (...) it strikes a very good compromise between the theoretically rigorous foundation of the theory of optimal regulation for multiproduct firms (...) and the practitioner’s requirement of the simple, easy-to-understand, easy-to-apply rule." (De Fraja and Iozzi [17], p.1)

1 Introduction

In a seminal work, Laffont and Tirole [23] show that, capping the prices of a multiproduct monopolist by means of a constraint in which any such price is attributed a weight equal (or proportional) to the correctly forecasted optimal quantity leads to Ramsey pricing. The second-best optimum under monopoly thus entails. This result "restates" Brennan [11]’s finding that price cap emerges as the solution to a problem of welfare-constrained profit maximization, in which quantities are appropriate weights to be attached to allowable deviations from socially desirable prices.

Those aforementioned are definitely not the sole papers about the so-called ideal price cap. Indeed, numerous other authors have worked on the subject. Some generically look at industries providing services of general interest. One may recall, for instance, De Fraja and Iozzi [16] and [17] as well as Iozzi, Poritz and Valentini [22]. Others focus on particular sectors, such as Billette de Villemeur, Cremer, Roy and Toledano [8], who propose a price-cap scheme that fits postal sector specificities.

Under price-cap regulation, firms have incentives to cut costs, which likely translates into quality under-provision. Rovizzi and Thompson [28] point out that noticeable quality reduction was registered in British Telecom’s services as soon as the privatized company went subject to price-cap regulation. In consideration of this drawback, De Fraja and Iozzi [16] lay down theoretical foundations for joint regulation of monopoly price and quality. Integrating quality dimensions into standard price cap, they find that appropriate price weights are still the optimal quantities, whereas quality weights should equal consumer marginal surplus evaluated at the optimal prices and qualities. Similar results emerge in Billette de Villemeur [7] as for the airline industry.

The contributions recalled above all refer to monopoly regulation. This circumstance is easily explained. After the second World War, the provision of services of general interest was typically concentrated in State controlled-and-owned vertically integrated monopolies. Over time, this organizational model has lost momentum.

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1 The list of papers about price cap regulation we mention is far from exhaustive. There also exists a full class of works which focus on the practical application of price cap regulation, either in a static or in a dynamic version. Some such papers are Vogelsang and Finsinger [30], Littlechild [21] and Foreman [19]. However, we do not strictly hinge on the literature background about implementation, to which those studies belong. For the same reason, Brennan [11] and De Fraja and Iozzi [16] are solely referred to as for the theoretical foundation of price cap and price-and-quality cap respectively, not as for the implementation schemes they propose. More generally, see Vogelsang [29] for a survey of the literature about incentive regulation in general and its implementation over the last two decades. See Armstrong and Sappington [2] as well.
Network industries, such as telecommunications, water, energy and transportation, have undergone a major privatization process. Thus statutory monopolists have often become regulated commercial players\(^2\). With such real-world scenarios in mind, economists have long focused on monopoly regulation, with and without quality adjustments.

However, at more recent stage, the segments of network industries that are not concerned by sub-additive technologies have been opened up to competition. Partly regulated oligopolies thus arise, where incumbents are subject to regulatory obligations, while entrants are allowed to operate uncontrolled. On one side, such policies are meant to promote access and (some) competition. On the other side, they aim at guaranteeing reliable service supply at affordable prices. This calls for making incumbents less market powerful, though still financially viable\(^3\).

A crucial issue arises. Consolidated monopoly regulation does not need to suit imperfectly competitive partially regulated sectors. Biglaiser and Ma [6] show that regulatory programs targeted to sole incumbents are yet sensitive to the presence of competitors endowed with market power. New constraints and trade-offs add up in the regulatory process, leading to third-best outcomes.

Observe in particular that price-and-quality-cap regulation lacks any normative basis with reference to these (newly relevant) situations. As for pure price cap, Brennan [11] proves that, if an incumbent competes with a passive fringe whose profits are taken to contribute to social welfare, then relevant quantities are the optimal ones of the sole regulated firm. Yet, he acknowledges, when competitors are not price takers, a different recipe is required, which he does not identify though\(^4\).

The ultimate goal of our paper is to provide theoretical foundations to price-and-quality cap regulation of oligopolies where a regulated dominant firm (a Stackelberg leader) competes in price and quality with one (or more) strategic follower(s). To capture the importance of quality provision in nearly all utilities, we focus on a market where vertically differentiated services are supplied to consumers exhibiting heterogeneous quality valuations.

The interpretation of our model is twofold. Firstly, it represents competition between asymmetrically regulated operators within some given industry. To fix ideas, one may think about a regulated dominant train operator competing with one or more unregulated rivals, whose services display different qualities (such as high-

\(^2\)See Martimort [25] for a discussion about the costs and benefits that are associated with privatization in an incentive theory perspective.

\(^3\)Biglaiser and Ma [6] provide the example of AT&T competing as a regulated dominant firm with the unregulated MCI and Spring in the long-distance telecommunications market. See also Helm and Jenkinson [20], who report that regimes of partial regulation apply to railways and truck freight transport in Argentina and in the USA, as well as to the natural gas and oil sectors in Germany, Finland and Hong Kong. Other forms of partial regulation emerge when former monopolies are now engaged in both regulated and unregulated activities in different market segments.

\(^4\)See footnote 6 at page 144 and footnote 10 at page 145 in Brennan [11]. The latter also points out that, if fringe profits were not to be included in social welfare, the optimal price-cap would not differ from the monopoly case (i.e. price weights in the price-cap formula are proportional to optimal quantities). By contrast, we will consider that unregulated firm’s profits do contribute to social welfare.
speed and low-speed services). Secondly, our model stylizes competition between regulated and unregulated industries. Insisting on transportation, one may consider inter-modal competition between regulated train operators and deregulated air carriers, whose services differ as for speed, frequency, scheduling reliability or comfort.

Albeit transportation is particularly well placed to illustrate the flexibility of our model, the latter does apply to most utilities. Indeed, as further examples of relevant quality dimensions, one may recall continuity and constancy of Internet service supply as well as reaction lags in electricity generation and provision5.

We begin by characterizing the optimal (partial) regulatory policy. In the presence of strategic rivals, such a policy is no more the second-best monopoly policy. In particular, Ramsey pricing no longer yields (constrained) efficiency. Relevant benchmark for the regulator is now the policy that arises when a welfare-maximizing firm acts as a Stackelberg leader with respect to one (or more) profit-maximizing rival(s), facing the requirement that profits be non-negative. In this perspective, our work is reminiscent of mixed oligopoly models where the public firm is taken to be a first mover vis-à-vis the private operator(s)6.

At a later stage, we demonstrate how the optimal policy can be decentralized by means of a price-and-quality cap targeted to the sole dominant firm. Decentralization requires that weights in the cap depend not only on the market served by the incumbent, but also on the market covered by the unregulated competitor(s). It follows that regulatory bodies of liberalized industries should not be restricted to access and use information about the sole regulated firms. They should rather be allowed to extract and utilize information about the overall industry. However, in the particular case of unit demand, the optimal policy can yet be decentralized by means of a “standard” price-and-quality cap, which uniquely refers to the regulated firm. This may come out as an interesting result for those sectors (e.g., transportation) where relevant markets are difficult to define.

The remainder of the paper is organized as follows. Section 2 presents the framework. Section 3 illustrates the impact of incumbent’s actions on entrant’s decisions and characterizes the optimal (partial) regulatory policy in a Stackelberg oligopoly. Section 4 evidences how the policy target can be decentralized by means of an appropriate price-and-quality cap. The latter is further investigated for the unit demand case, which returns especially useful and intuitive insights. Section 5 concludes.

5 Crampes and Moreaux [12] stress that this quality aspect of energy provision introduces a heterogeneity dimension in generated electricity, which is otherwise an homogeneous product.

6 In particular, addressing the issue of which pricing policy a public firm should pursue if it faces constraints (namely, the requirement of operating at zero profits), Bös [10] stresses that the firm should opt for a modified Ramsey-pricing rule. Beato and Mas Colell [4] show in turn that the solution to a homogeneous-product Stackelberg game (where the public firm moves first) corresponds to average-cost pricing for the public firm. Indeed, since the latter selects its output before the private competitor, it chooses so that, at equilibrium, it obtains zero profits. See Nett [26] for a survey about the homogeneous-good mixed oligopoly literature.
2 The Model

Consider an industry where two firms provide vertically differentiated products. The two providers play a Stackelberg game. One firm, denoted $L$ (for leader), is subject to regulation. The other, denoted $F$ (for follower), is not. Strategic variables are prices ($p_L$ and $p_F$) and qualities ($q_L$ and $q_F$). They are both assumed to be observable and verifiable. Although this assumption excludes many environments, it appears yet to be a natural and realistic hypothesis in a variety of contexts. Transportation, where length of trip or frequency of services may play the role of quality, provides a good example of such a context. We will often refer to it all along the paper.

Except for quality differences, the goods provided by the two firms turn out to be perfect substitutes.

Consumers differ only in their valuation for quality, as represented by means of a parameter $\theta$. More precisely, we adopt a quasi-linear framework. The net surplus that a consumer of characteristic $\theta$ derives from the consumption of $x$ units of quality $q$ bought at unit price $p$ writes

$$v_\theta (x, p, q) = u (x) - (p - \theta q) x.$$  \hfill (1)

The parameter $\theta$ is distributed over the interval $[\theta, \bar{\theta}]$ according to a continuous density function $f(\theta)$. The associated cumulative distribution function is denoted $F(\theta)$. Given her quality valuation, a $\theta$-consumer patronizing firm $i \in \{L, F\}$ faces the so-called generalized price $\bar{p}(\theta) \equiv p - \theta q$, which is equal to the unit price $p$ net of the benefits $\theta q$, associated with product quality. A $\theta$-consumer prefers to purchase the good from firm $i$ rather than from firm $j$, whenever she bears a smaller generalized price by doing so, i.e. whenever $\bar{p}_i (\theta) < \bar{p}_j (\theta)$. Observe that, by construction, no consumer finds it profitable to patronize both firms.

The marginal consumer, who is indifferent between the two operators, is characterized by the parameter value

$$\theta_m \equiv \frac{p_i - p_j}{q_i - q_j},$$  \hfill (2)

where without any loss of generality $p_i \geq p_j$ and $q_i > q_j$. Individuals whose $\theta$ exceeds $\theta_m$ patronize firm $i$, whereas individuals whose $\theta$ is smaller than $\theta_m$ patronize firm $j$.

2.1 Consumer Valuation of Quality, Demand and Surplus

The consumption of a $\theta$-consumer is pinned down by maximizing (1) with respect to $x$, which yields

$$\frac{\partial u}{\partial x} = \bar{p} (\theta),$$  \hfill (3)

where we have $\bar{p}(\theta) = \arg \min \{\bar{p}_L (\theta), \bar{p}_F (\theta)\}$. This says that individual consumption $x_i (p_i, q_i; \theta)$ appears to be a function of the sole generalized price of the consumed
commodity \( p_i(\theta) \). Since \( p(\theta) \) decreases with \( \theta \) and \( p_i(\theta_m) = p_j(\theta_m) \), the ranking of consumers in terms of quality valuation conveys when individual consumptions \( x_\theta \) are considered. Formally \( x_{\theta_1} \leq x_{\theta_2} \) whenever \( \theta_1 \leq \theta_2 \).

Relying upon (3), it is possible to establish the relationship between the impacts on consumption of marginal changes in price and quality. To see this, observe first that (3) holds for any \( p_i \) and \( q_i \). Differentiating both sides with respect to \( p_i \) and to \( q_i \) and combining the two equations, we obtain

\[
\frac{\partial x_i}{\partial q_i} \frac{\partial q_i}{\partial p_i} = \theta, \quad i = L, F. \tag{4}
\]

This evidences that, for \( \theta \)-consumer demand to remain unchanged as price \( p_i \) is increased by one unit, quality \( q_i \) should be augmented by an amount equal to individual marginal valuation for quality, namely \( \theta \). It also follows that a consumer with a strictly higher quality valuation that patronizes the same firm would consider an increase in \((p_i, q_i)\) that leaves a \( \theta \)-consumer indifferent as strictly beneficial. Conversely a consumer with a strictly lower valuation would find it detrimental. Opposite appreciations would arise if a decrease in \((p_i, q_i)\) that leaves a \( \theta \)-consumer indifferent were considered.

Aggregate demands are immediately obtained by summing over the relevant ranges of \( \theta \)'s. They are given by

\[
X_j(p, q) = \int_{\theta_m}^{\theta} x_j(p_j, q_j; \theta) f(\theta) d\theta \tag{5a}
\]

\[
X_i(p, q) = \int_{\theta_m}^{\theta} x_i(p_i, q_i; \theta) f(\theta) d\theta \tag{5b}
\]

where \( \theta_m = (p_i - p_j) / (q_i - q_j) \) while \( p \) and \( q \) stand for the vector of prices and quality respectively. The properties they display are rather standard. We hereafter briefly recall them. For any \( i, j \in \{L, F\} \):

1. \( (\partial X_i / \partial p_i) < 0 \) : firm \( i \)'s demand decreases in its own price \( p_i \);
2. \( (\partial X_i / \partial q_i) > 0 \) : firm \( i \)'s demand increases in its own quality \( q_i \);
3. \( (\partial X_i / \partial p_j) > 0 \) : firm \( i \)'s demand increases in the rival price \( p_j \);
4. \( (\partial X_i / \partial q_j) < 0 \) : firm \( i \)'s demand decreases in the rival quality \( q_j \).

It is also straightforward to obtain aggregate consumer surplus as a function of prices and qualities. To this aim, plug individual demands pinned down by (3) into the surplus function (1) and sum over the relevant ranges of \( \theta \)'s. This ultimately returns

\[
V(p, q) = \int_{\theta_m}^{\theta} v_\theta(x_\theta, p_j, q_j) f(\theta) d\theta + \int_{\theta_m}^{\theta} v_\theta(x_\theta, p_i, q_i) f(\theta) d\theta. \tag{6}
\]
2.2 Technologies and Profits

Let $C_i(X_i, q_i)$ be firm $i$'s cost function. This function is assumed to be continuous and increasing in both production level and quality. Formally $(\partial C_i/\partial X_i) > 0$ and $(\partial C_i/\partial q_i) > 0$.

We further assume that $C_i(\cdot, +\infty) = +\infty$. This says that high quality products are so costly to improve that perfect products ($q_i = +\infty$) are never actually provided on markets. We finally assume that firms never find it profitable to decrease the quality of their products down to zero. Taken together, these hypotheses ensure an interior solution to the definition of quality. Formally, there exists a finite $\overline{q}$ such that $0 < q_i < \overline{q}$, for $i = L, F$.

Finally, defining firm $i$'s revenues as $R_i \equiv p_i X_i$, the profit function is written

$$\pi_i(p, q) = R_i - C_i(X_i, q_i), \quad i = L, F. \quad (7)$$

3 Optimal Partial Regulation

This section is devoted to the characterization of optimal (partial) regulatory policy. By the latter we mean the policy which would materialize in a mixed duopoly where a welfare-maximizing (public) firm were to play the market game as a Stackelberg leader vis-à-vis a profit-maximizing (private) competitor, under a non-negative profit constraint.

As follower’s reaction to incumbent’s policy is taken into account, analysis is performed backward. We therefore start by investigating entrant’s behaviour and look subsequently for leader’s price-and-quality bundle.

3.1 Price-and-Quality Policy of Firm F

Firm $F$ behaves as a follower vis-à-vis the incumbent. Therefore, it takes firm $L$'s price and quality as given and optimizes accordingly.

The first-order condition that derives from the maximization of (7) with respect to price $p_F$ gives rise to the standard formula

$$\frac{p_F - (\partial C_F/\partial X_F)}{p_F} = \frac{1}{\varepsilon_F}, \quad (8)$$

where $\varepsilon_F \equiv (p_F/X_F)(-\partial X_F/\partial p_F)$ is (the absolute value of) the demand elasticity with respect to price. Equation (8) is nothing but the Lerner formula, evidencing that firm $F$ acts as a monopolist vis-à-vis residual demand.

The first-order condition that derives from the maximization of (7) with respect to quality $q_F$ is given by

$$p_F \frac{\partial X_F}{\partial q_F} = \frac{\partial C_F}{\partial q_F} + \frac{\partial C_F}{\partial X_F} \frac{\partial X_F}{\partial q_F}. \quad (9)$$

As revealed by equation (9), quality $q_F$ is chosen so that marginal benefits from
quality improvements (the left-hand side) equate their marginal costs (the right-hand side), as expressed by the sum of direct costs of quality (the first term) and indirect costs (as reflected by the second term), that follow from demand increases resulting from quality increases. The latter equation can be rearranged and combined with the first-order condition with respect to \( p_F \) to obtain

\[
\frac{\partial X_F}{\partial q_F} - \frac{\partial X_F}{\partial p_F} = \frac{1}{X_F} \frac{\partial C_F}{\partial q_F}.
\]

By analogy with equation (4), the ratio on the left-hand side of (10) can be interpreted as the aggregate marginal valuation of quality by firm \( F \)'s clients. The right-hand side of (10) is nothing but the average cost of a marginal increase in quality for this same firm. Although the firm is profit maximizing, no distortion is introduced by the firm’s choices in terms of quality. Given consumer valuation, a further quality improvement would not appear to be worth its costs.\(^7\)

### 3.2 Optimal Regulatory Policy

We now characterize the socially optimal policy. Remind that firm \( L \), the regulated firm is assumed to act as a Stackelberg leader vis-à-vis firm \( F \), the (profit-maximizing) follower. Optimal policy is thus pinned down by maximizing the unweighted social welfare function

\[
W (p, q) = V (p, q) + \pi_L (p, q) + \pi_F (p, q)
\]

with respect to \( p_L \) and \( q_L \), while \( p_F \) and \( q_F \) are assumed to obey the rules (8) and (9) identified above. The set of policy considered is also restricted to allow firm \( L \) to break-even. Formally, the pair \((p_L, q_L)\) should be such that \( \pi_L (p, q) \geq 0 \).

Let \( \lambda \) be the Lagrange multiplier associated with firm \( L \)'s budget constraint. The first-order condition that follows from the constrained maximization of (11) with respect to price \( p_L \) is given by

\[
\frac{d\pi_L}{dp_L} = \left( \frac{-1}{1 + \lambda} \right) \left( \frac{dV}{dp_L} + \frac{d\pi_F}{dp_L} \right).
\]

The left-hand side of (12) is the total derivative of firm \( L \)'s profits with respect to \( p_L \). In other words, it also accounts for the indirect effects on \( \pi_L \) of a variation in \( p_L \). The latter indeed result in induced adjustments of rival’s price and quality, \( p_F \) and \( q_F \). Formally, we have:

\[
\frac{d\pi_L}{dp_L} = \frac{\partial \pi_L}{\partial p_L} + \frac{\partial \pi_L}{\partial p_F} \frac{dp_F}{dp_L} + \frac{\partial \pi_L}{\partial q_F} \frac{dq_F}{dp_L}
\]

\(^7\)This does not mean that consumer and firm’s objectives are perfectly aligned, even when attention is restricted to quality issues. In fact, were the price be lower, the demand would be larger. As a result, the average cost of quality would be lower, calling for a strict improvement in terms of quality.
Similarly the impact on consumer surplus of a change in price $p_L$ can be decomposed as
\[
\frac{dV}{dp_L} = \frac{\partial V}{\partial p_L} + \frac{\partial V}{\partial p_F} \frac{dp_F}{dp_L} + \frac{\partial V}{\partial q_F} \frac{dq_F}{dp_L}. \tag{13}
\]

By contrast, since $(\partial \pi_F/\partial p_F) = 0$ and $(\partial \pi_F/\partial q_F) = 0$, the derivative of firm $F$’s profits with respect to $p_L$ simplifies to
\[
\frac{d\pi_F}{dp_L} = \frac{\partial \pi_F}{\partial p_L} = \left( p_F - \frac{\partial C_F}{\partial X_F} \right) \frac{\partial X_F}{\partial p_L} = X_F \left( \frac{\partial X_F/\partial p_L}{-\partial X_F/\partial p_F} \right). \tag{14}
\]

Assume that the regulated firm provides higher quality services than firm $F$. In other words, assume that firm $L$ serves consumers with quality valuation in $[\theta_m, \overline{\theta}]$ and firm $F$ consumers with quality valuation in $[\underline{\theta}, \theta_m)$. This is a natural assumption in a variety of contexts. In particular, it happens to be often the case when firm $L$ is the incumbent of a previously fully regulated industry while firm $F$ is a frail newcomer, that entered market after liberalization. Observe however that, *mutatis mutandis*, the analysis would be identical if the unregulated follower would have served the high market segment $[\theta_m, \overline{\theta}]$. The assumption is thus to be considered as an explanatory device introduced to avoid redundancy. It does not put any restriction upon the analysis.

From Roy’s identity, we have
\[
\frac{\partial V}{\partial p_L} = -\int_{\theta_m}^{\overline{\theta}} x_L(p_L, q_L; \theta) f(\theta) \, d\theta = -X_L, \tag{15}
\]
\[
\frac{\partial V}{\partial p_F} = -\int_{\underline{\theta}}^{\theta_m} x_F(p_F, q_F; \theta) f(\theta) \, d\theta = -X_F, \tag{16}
\]
\[
\frac{\partial V}{\partial q_L} = \int_{\theta_m}^{\overline{\theta}} x_L(p_L, q_L; \theta) \theta f(\theta) \, d\theta = \tilde{\theta}_L X_L, \tag{17}
\]
\[
\frac{\partial V}{\partial q_F} = \int_{\underline{\theta}}^{\theta_m} x_F(p_F, q_F; \theta) \theta f(\theta) \, d\theta = \tilde{\theta}_F X_F, \tag{18}
\]

where $\tilde{\theta}_L$ and $\tilde{\theta}_F$ as defined by
\[
\tilde{\theta}_L = \int_{\theta_m}^{\overline{\theta}} \frac{x_L(p_L, q_L; \theta)}{X_L} \theta f(\theta) \, d\theta,
\]
\[
\tilde{\theta}_F = \int_{\underline{\theta}}^{\theta_m} \frac{x_F(p_F, q_F; \theta)}{X_F} \theta f(\theta) \, d\theta,
\]
denote the average valuation of quality by each firm clients, as weighed by their relative demand.

Plugging (15), (16) and (18) into (13) and then (14) and (13) into (12), we
ultimately obtain
\[
\frac{d\pi_L}{dp_L} = \left( \frac{1}{1 + \lambda} \right) \left[ X_L - \left( \frac{\partial X_F/\partial p_L}{-\partial X_F/\partial p_F} \right) X_F + X_F \frac{dp_F}{dp_L} - \tilde{\theta}_F X_F \frac{dq_F}{dq_L} \right].
\] (19)

Turning now to the second relevant dimension, the first-order condition for a constrained maximum of (11) with respect to quality \(q_L\) is given by
\[
\frac{d\pi_L}{dq_L} = \left( \frac{-1}{1 + \lambda} \right) \left( \frac{dV}{dq_L} + \frac{\partial \pi_F}{\partial q_L} \right).
\] (20)

A similar analysis yields a decomposition of consumer surplus variation that writes
\[
\frac{dV}{dq_L} = \tilde{\theta}_L X_L - X_F \frac{dp_F}{dq_L} + \tilde{\theta}_F X_F \frac{dq_F}{dq_L}.
\] (21)

Again, relying upon the first-order conditions of firm \(F\)'s profit-maximization, we can write
\[
\frac{\partial \pi_F}{\partial q_L} = X_F \left( \frac{\partial X_F/\partial q_L}{-\partial X_F/\partial p_F} \right).
\] (22)

Replacing (21) and (22) into (20), we ultimately obtain
\[
\frac{d\pi_L}{dq_L} = -\left( \frac{1}{1 + \lambda} \right) \left[ \tilde{\theta}_L X_L + \left( \frac{\partial X_F/\partial q_L}{-\partial X_F/\partial p_F} \right) X_F - X_F \frac{dp_F}{dq_L} + \tilde{\theta}_F X_F \frac{dq_F}{dq_L} \right].
\] (23)

To sum up, the optimal partial regulatory policy under the incumbent’s budget constraint is given by the price-and-quality combination \((p_L, q_L)\) which simultaneously satisfies the pair of conditions (19) and (23). Observe that both equations contain terms that reflect strategic interactions across firms (like \((dp_F/dp_L)\) and \((dp_F/dq_L)\)) and cross-price effects (like \(\partial X_F/\partial p_L\)). Some may thus consider the definition of optimal policy as a purely theoretical exercise, with no practical value. If optimal policy does not find an explicit expression, exact implementation is indeed likely to be beyond reach. This makes more striking the results we present hereafter.

4 Decentralization through an Extended Price Cap

In this Section, we propose an extended version of the price cap scheme and evidence that it allows to decentralize the optimal allocation identified above.

Assume that the incumbent is left free to choose both price and quality provided that a “price-and-quality” cap is satisfied. Formally, let
\[
\left\{ \begin{array}{l}
\max_{\{p_L, q_L\}} \pi_L \equiv p_L X_L - C_L (X_L, q_L), \\
s.t. \quad \alpha p_L - \beta q_L \leq P + \gamma p_F - \delta q_F,
\end{array} \right.
\] (24)

be the regulated firm’s program and denote \(\mu\) the multiplier associated with the price-and-quality constraint. The first-order conditions for a constrained maximum
of \( \pi_L \) with respect to \( p_L \) and \( q_L \) are respectively given by

\[
\frac{d\pi_L}{dp_L} = \mu \left[ \alpha - \gamma \frac{dp_F}{dp_L} + \delta \frac{dq_F}{dp_L} \right], \quad (25)
\]

\[
\frac{d\pi_L}{dq_L} = -\mu \left[ \beta + \gamma \frac{dp_F}{dq_L} - \delta \frac{dq_F}{dq_L} \right]. \quad (26)
\]

With \( \alpha, \beta > 0 \), the regulatory constraint is tightened by an increase in price \( p_L \), relaxed by an increase in quality \( q_L \). These effects are mitigated or enhanced through their impact on firm \( F \)'s decisions, which are explicitly considered in the extended price cap.

For the proposed extended price cap to ultimately implement the partial regulatory policy \((p_L, q_L)\) previously characterized, it is sufficient (i) to set coefficients \((\alpha, \beta)\) to

\[
\alpha = X_L^{PR} - X_F^{PR} \left( \frac{\partial X_F^{PR}/\partial p_L}{-\partial X_F^{PR}/\partial p_F} \right), \quad (27a)
\]

\[
\beta = \tilde{\theta}_L^{PR} X_L^{PR} + X_F^{PR} \left( \frac{\partial X_F^{PR}/\partial q_L}{-\partial X_F^{PR}/\partial p_F} \right), \quad (27b)
\]

(ii) to set coefficients \((\gamma, \delta)\) to

\[
\gamma = -X_F^{PR}, \quad \delta = -\tilde{\theta}_F^{PR} X_F^{PR}, \quad (27c)
\]

and (iii) to decrease \( P \) enough as to wash out firm \( L \)'s profits so that

\[
\mu = \frac{1}{1 + X^{PR}}. \quad (28a)
\]

The presence of the superscript \( PR \) indicates that exact values are those obtained at the Partial Regulatory optimum.

Observe that both derivatives \((\partial X_F^{PR}/\partial p_L)\) and \((\partial X_F^{PR}/\partial q_L)\) in the right-hand sides of (27a) and (27b) reflect only marginal variations, as firm \( F \)'s infra-marginal customers are not concerned by changes in firm \( L \)'s price and quality. More precisely, we have

\[
\frac{\partial X_F^{PR}}{\partial p_L} = x_m^{PR} f (\theta_m^{PR}) \frac{\partial \theta_m^{PR}}{\partial p_L}, \quad (29a)
\]

\[
\frac{\partial X_F^{PR}}{\partial q_L} = x_m^{PR} f (\theta_m^{PR}) \frac{\partial \theta_m^{PR}}{\partial q_L} = -\theta_m^{PR} \frac{\partial X_F^{PR}}{\partial p_L}, \quad (29b)
\]

where \( x_m^{PR} f (\theta_m^{PR}) \) measures consumption by marginal clients, \( i.e. \) consumers characterized by quality valuation

\[
\theta_m^{PR} \equiv \frac{p_L^{PR} - p_F^{PR}}{q_L^{PR} - q_F^{PR}}.
\]
Plugging (29a) and (29b) into (27a) and (27b) ultimately returns more interpretable expressions for the regulatory price and quality weights

\[
\begin{align*}
\alpha &= X_{L}^{PR} - \nu X_{L}^{PR}, \\
\beta &= \tilde{\theta}_L X_{L}^{PR} - \theta_m^{PR} \nu X_{L}^{PR},
\end{align*}
\]  

(30a)

(30b)

where

\[
\nu = \frac{x_m^{PR} f (\theta_m^{PR}) \frac{\partial \theta_m^{PR}}{\partial p_L}}{\int_\mathcal{Q} \frac{\partial x_m^{PR}}{\partial p_F} f (\theta) d\theta + x_m^{PR} f (\theta_m^{PR}) \frac{\partial \theta_m^{PR}}{\partial p_F}}.
\]  

(31)

The coefficient \(\nu\) as defined by (31) displays, at the numerator, the marginal variation in \(X_{F}^{PR}\) induced by an increase in \(p_L\); at the denominator, the (absolute value of the) overall (marginal and infra-marginal) variation in the same quantity \(X_{F}^{PR}\) as induced by an increase in \(p_F\). The (cross) effect of price \(p_L\) on the follower’s demand is likely to be smaller than the (own) effect of price \(p_F\). Therefore, the ratio under scrutiny can be assumed to be smaller than one.

According to (30a), the appropriate price weight in the cap is given by the difference between two terms. The first term is the regulated firm’s quantity evaluated at \((p_F^{PR}, q_F^{PR})\), namely \(X_{L}^{PR}\). The second term consists in firm \(F\)'s quantity evaluated at \((p_F^{PR}, q_F^{PR})\), namely \(X_{F}^{PR}\), as multiplied by the coefficient \(\nu\). Since \(\nu \leq 1\), firm \(L\)'s output is given a larger relevance than firm \(F\)'s output in the composition of the price weight. In other words, \(\alpha\) is obtained by subtracting from the regulated firm’s quantity \(X_{L}^{PR}\) (the “standard” weight in cap formula) a fraction of its (unregulated) competitor’s one, \(\nu X_{F}^{PR}\), where the downsizing of \(X_{F}^{PR}\) reflects the difference between cross and own-price effects.

Similarly, the quality weight \(\beta\) defined by (30b) is also given by the difference between two terms. The first term, \(\tilde{\theta}_F^{PR} X_{F}^{PR}\), is an aggregate measure of the quality appreciation by firm \(L\)'s consumers. The second term is linked to the appreciation of quality by firm \(F\)'s consumers. However, since the sole marginal clients of firm \(F\) are concerned by changes in \(q_L\), the quality appreciation refers to \(\theta_m^{PR}\) and not to \(\tilde{\theta}_F\). Interestingly enough, the marginal quality valuation is to be multiplied by \(\nu X_{F}^{PR}\) (the exact same part of \(\alpha\) that refers to firm \(F\)), i.e. the coefficient is to be calculated by using the whole demand for firm \(F\)'s products, \(X_{F}^{PR}\) (which is eventually observable), and not the consumption by firm \(F\)'s marginal clients (which is not).

A clear message emerges from (30a) and (30b). First, proper regulation of firm \(L\) cannot spare reference to its competitor, firm \(F\). Second, the larger the market share of the unregulated firm, the lower the required regulatory pressure on the regulated firm. In fact, if \(X_{PR}\) denotes the market size at \((p_F^{PR}, q_F^{PR})\), the optimal weight (30a) attached to price \(p\) in the price-cap formula rewrites \(\alpha = X_{PR} - (1 + \nu) X_{F}^{PR}\). That is to say, competition has a bigger impact on markets than what appears when considering the sole market share of unregulated firms \(X_{F}^{PR}\). Third, the magnitude of this competitive effect depends on the coefficient \(\nu\), as defined by (31), which reflects firms’ differentiation. Fourth, the higher \(\tilde{\theta}_m^{PR}\), the lower the rewards for
regulated firm product quality.

We now turn to the unit demand case that yields particularly simple results, i.e. an almost “ready-for-use” scheme to regulators.

4.1 The Unit Demand Case

We hereafter focus on the case where each customer allocates a single unit of consumption to her preferred operator. Despite the restrictions attached to this case, it does fit numerous real world situations where regulation is actually performed. Consider, for instance, (passenger) transportation markets. Demand is naturally modelled as a discrete choice across available alternatives. The unit demand assumption corresponds to adopting trips as consumption units and considering that alternatives are attached to the various ways to reach the envisioned (single) destination. Note also that a quality attribute like travel time is both observable and verifiable. The latter may thus be used to regulate, say, the operator attached to one of two competing modes.

In the unit demand case, neither changes in prices nor in quality impact infra-marginal consumer decisions. Thus changes affect marginal consumers only. Clearly, any lost consumer is a gain for the competitor. It follows that

\[
\frac{\partial X_F}{\partial p_F} = f (\theta_m) \frac{\partial \theta_m}{\partial p_F} = -f (\theta_m) \frac{\partial \theta_m}{\partial p_L} = -\frac{\partial X_F}{\partial p_L}
\]

hence

\[
\nu = \frac{\partial X_{PR}^F / \partial p_L}{-\partial X_{PR}^F / \partial p_F} = 1,
\]

which yields

\[
\begin{align*}
\alpha^U &= X_{PR}^L - X_{PR}^F, \\
\beta^U &= \tilde{\theta}_L^{PR} X_{PR}^L - \theta_m^{PR} X_{PR}^F.
\end{align*}
\]

(32a)  

(32b)

Summing up, the generalized price-and-quality cap writes as

\[
(X_{PR}^L - X_{PR}^F) p_L - (\tilde{\theta}_L^{PR} X_{PR}^L - \theta_m^{PR} X_{PR}^F) q_L \leq P - (p_F - \tilde{\theta}_F^{PR} q_F) X_{PR}^F.
\]

(33)

This says that the regulated firm is compelled to choose \( p_L \) and \( q_L \) so that its average generalized price \( \tilde{p}_L \) verifies

\[
\tilde{p}_L X_{PR}^L = (p_L - \tilde{\theta}_L^{PR} q_L) X_{PR}^L \leq P + \left[ (p_L - \theta_m^{PR} q_L) - (p_F - \tilde{\theta}_F^{PR} q_F) \right] X_{PR}^F.
\]

\[\text{Recall that, for each individual, the preferred operator is the one which ensures the lower generalized price, given her personal valuation for quality.}\]

\[\text{This does not mean however that regulation can be drawn according to the sole characteristics of marginal consumers. See appendix.}\]
Recall that, by definition of the marginal consumer,

\[ p_L - \theta_m^{PR} q_L = p_F - \theta_m^{PR} q_F. \]

Thus, the regulatory constraint can finally be rewritten

\[ \tilde{p}_L X_L^{PR} \leq P - \left( \theta_m^{PR} - \tilde{\theta}_F^{PR} \right) q_F X_F^{PR}. \] (34)

The latter formula deserves a few comments. First, strategic interactions across firms do not yield any change in the fundamental structure of the incentives to be given to the firm. Indeed, ultimately, what matters is \( \tilde{p}_L \), the average generalized price associated to the regulated firm services. Second, this average generalized price \( \tilde{p}_L = p_L - \tilde{\theta}_L^{PR} q_L \) involves an estimate of the marginal value of quality. This estimate \( \tilde{\theta}_L^{PR} \) is nothing but the average quality valuation by firm \( L \)'s consumers. Third, at optimum, the cap \( P \) is adjusted by the regulator so as to wash out firm \( L \)'s profits, according to (28a). This does mean that the other terms on the right-hand side of (34) can be safely removed.\(^{10}\) As a result, in the unit demand case, optimal regulation follow from imposing a “standard” quality adjusted price-cap:

\[ p_L - \tilde{\theta}_L^{PR} q_L \leq \mathfrak{p}. \] (35)

The proposed regulatory scheme hinges upon a single parameter, \textit{exogenously} set by the regulator. If quality is to be taken into account by the regulator, this parameter is the simplest information one can think of. This is indeed an \textit{average} marginal valuation of quality by consumers of the regulated firm, upon which the regulator may legitimately and more easily collect data.

De Fraja and Iozzi [16] evidence a difficulty attached to the convergence of their quality adjusted price-caps. For this reason, they introduced an additional constraint that further limits the regulated firm choices. Observe however that this difficulty streams directly from their ambitious approach. Indeed, in their regulatory scheme, quality valuation is \textit{endogenously} determined by computing at each step consumer marginal surplus \( \partial V \partial q \). By contrast, and along the actual practice, we assume that social valuation of quality is a policy attribute chosen and made public by the regulator. Of course, with our mechanism, a bias in quality valuation \( \tilde{\theta}_L^{PR} \) would ultimately result in a sub-optimal allocation. However, the scheme is more transparent, hence more likely to get public support. Moreover, with a fixed coefficient for quality valuation it is less prone to manipulations.

Interestingly, although the regulatory policy accounts for its impacts on the whole industry, the optimal allocation can be characterised by the means of an extended price-cap that looks at the sole regulated firm. This may come out to be of importance when handling with industries where relevant markets are difficult to define.

\(^{10}\)This holds true at optimum only. In terms of incentives (\textit{i.e.} in order to guarantee the convergence of the adjustment process), this second term may appear to be useful. In any case, observe that the price \( p_F \) of the competitor does not play any role in the scheme; by contrast, a higher quality \( q_F \) and a larger supply \( X_F \) appear to call for a greater regulatory pressure.
5 Concluding Remarks

In this paper, we provide theoretical foundations to a price-and-quality cap regulation of recently liberalized industries. For this purpose, we stylize a partially regulated oligopoly where vertically differentiated services are provided by a regulated incumbent (the Stackelberg leader) and a strategic unregulated operator (the follower), competing in price and quality. The model may equally well represent competition across asymmetrically regulated industries.

Within the context just described, we first characterize the optimal (partial) regulatory policy. In the environment under scrutiny, this is the policy that arises when a welfare-maximizing firm acts as a Stackelberg leader vis-à-vis one (or more) profit-maximizing rival(s), under the constraint firms must break-even. We subsequently propose a regulatory scheme able to decentralize properly this policy target. The latter is part of the broad class of “extended” price(-and-quality) caps. Appropriate weights depend on (optimal) quantities provided by both the regulated incumbent and the follower, despite the latter is not directly concerned by regulation. In the particular case of unit demand, optimal policy is however characterised by a “standard” price-and-quality cap, that refers to the sole regulated firm.

There are essentially two insights to be drawn from our analysis. First, in a partially regulated industry, there is need for the regulatory agency to be able to hinge upon information on the whole industry; information about the sole regulated firm does not appear to allow an efficient regulation, in general. Second, in the particular case where consumers would typically consume only one unit of the patronized good, the optimal policy is however characterised by the sole regulated firm settings; regulators can thus legitimately focus on one firm and regulate the latter as if it were a monopolist. The present contribution also presents several limits; it is far from exhausting the research questions attached to the topic. In particular, it would be of importance to consider the issue of convergence in the line of the recent contribution by Earle, Schmedders and Tatur [18]. This is left however for further research.

References


Appendix

Consider the unit demand case and assume that firm L produces the high quality good while firm F produces the low quality one. This says that firm L serves consumers with a marginal valuation of quality \( \theta \) in \([\theta_m, \theta]\), while firm F serves consumers with marginal variation in \( [ \theta, \theta_m] \). Remind that the threshold valuation \( \theta_m \) is given by the ratio \( \theta_m = (p_I - p_F) / (q_I - q_F) \).

By definition, in the unit demand case price and quality changes have only an impact on the marginal consumer. As a result, changes in \( p_F \) and \( q_F \) have the following marginal impact on demand \( X_F \):

\[
\frac{\partial X_F}{\partial p_F} = f(\theta_m) \frac{\partial \theta_m}{\partial p_F} = -\frac{f(\theta_m)}{q_I - q_F},
\]

\[
\frac{\partial X_F}{\partial q_F} = f(\theta_m) \frac{\partial \theta_m}{\partial q_F} = \theta_m \frac{f(\theta_m)}{q_I - q_F}.
\]

It follows that

\[
\frac{\partial X_F}{\partial q_F} = -\theta_m \frac{\partial X_F}{\partial p_F},
\]

that is to say,

\[
\frac{\partial \theta_m}{\partial q_F} = -\theta_m \frac{\partial \theta_m}{\partial p_F}.
\]

We hereafter check whether and, if so, under which conditions, a similar relationship holds true when changes in \( p_L \) and \( q_L \) are considered.

Observe that in the considered framework, changes made by firm L (the leader) have an impact on firm F (the follower) choices. This explains why the impact of changes in \( p_L \) and \( q_L \) on the marginal consumer are not straightforward. A standard decomposition leads to:

\[
\frac{d\theta_m}{dp_L} = \frac{\partial \theta_m}{\partial p_L} + \frac{\partial \theta_m}{\partial p_F} \frac{\partial p_F}{\partial p_L} + \frac{\partial \theta_m}{\partial q_F} \frac{\partial q_F}{\partial p_L},
\]

\[
= \frac{1}{q_L - q_F} \left( 1 - \frac{\partial p_F}{\partial p_L} + \theta_m \frac{\partial q_F}{\partial p_L} \right),
\]

\[
\frac{d\theta_m}{dq_L} = \frac{\partial \theta_m}{\partial q_L} + \frac{\partial \theta_m}{\partial p_F} \frac{\partial p_F}{\partial q_L} + \frac{\partial \theta_m}{\partial q_F} \frac{\partial q_F}{\partial q_L},
\]

\[
= \frac{1}{q_L - q_F} \left( -\theta_m - \frac{\partial p_F}{\partial q_L} + \theta_m \frac{\partial q_F}{\partial q_L} \right). \tag{39b}
\]

If, as assumed, firm F profit maximization gives rise to an interior solution, \( p_F \) and \( q_F \) are determined by the system of first-order conditions

\[
\left( p_F - \frac{\partial C_F}{\partial X_F} \right) \frac{f(\theta_m)}{q_I - q_F} = F(\theta_m),
\]

\[
\left( p_F - \frac{\partial C_F}{\partial X_F} \right) \frac{f(\theta_m)}{q_I - q_F} = 1 \frac{\partial C_F}{\partial q_F}. \tag{38}
\]
This yields

\[ \theta_m F(\theta_m) = \frac{\partial C_F}{\partial q_F}. \]  

(40)

Differentiating both sides of (40) with respect to \( p_L \) and \( q_L \) yields the following pair of equalities:

\[ \frac{d\theta_m}{dp_L} F(\theta_m) + \theta_m f(\theta_m) \frac{d\theta_m}{dp_L} = \frac{\partial^2 C_F}{\partial q_F^2} \frac{dq_F}{dp_L} + \frac{\partial^2 C_F}{\partial q_F \partial X_F} \left( \frac{\partial X_F}{\partial p_F} dq_F + \frac{\partial X_F}{\partial q_F} dp_L \right), \]

(41)

\[ \frac{d\theta_m}{dq_L} F(\theta_m) + \theta_m f(\theta_m) \frac{d\theta_m}{dq_L} = \frac{\partial^2 C_F}{\partial q_F^2} \frac{dq_F}{dq_L} + \frac{\partial^2 C_F}{\partial q_F \partial X_F} \left( \frac{\partial X_F}{\partial p_F} dq_F + \frac{\partial X_F}{\partial q_F} dp_L \right). \]

(42)

Making use of both (36) and (37), they can be rewritten respectively as

\[ \frac{d\theta_m}{dp_L} = \frac{\partial^2 C_F}{\partial q_F^2} \frac{dq_F}{dp_L} - \frac{\partial^2 C_F}{\partial q_F \partial X_F} f(\theta_m) \left( \frac{dq_F}{dp_L} - \theta_m \frac{dq_F}{dp_L} \right), \]

(43)

\[ \frac{d\theta_m}{dq_L} = \frac{\partial^2 C_F}{\partial q_F^2} \frac{dq_F}{dq_L} - \frac{\partial^2 C_F}{\partial q_F \partial X_F} f(\theta_m) \left( \frac{dq_F}{dq_L} - \theta_m \frac{dq_F}{dq_L} \right), \]

(44)

Remind that both left-hand sides were defined in (39a) and (39b). This allows to rewrite both equations as a function of the sole follower reactions to leader decision changes, i.e. \( (dp_F/dp_L) \), \( (dq_F/dp_L) \), \( (dp_F/dq_L) \) and \( (dq_F/dq_L) \). More precisely, we obtain:

\[ \frac{dp_F}{dp_L} - \theta_m \frac{dq_F}{dp_L} = \frac{F(\theta_m) + \theta_m f(\theta_m) - (q_L - q_F) \frac{\partial^2 C_F}{\partial q_F^2} \frac{dq_F}{dp_L}}{F(\theta_m) + \theta_m f(\theta_m) - f(\theta_m) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}, \]

(43)

\[ \frac{dp_F}{dq_L} - \theta_m \frac{dq_F}{dq_L} = \frac{-\theta_m [F(\theta_m) + \theta_m f(\theta_m)] - (q_L - q_F) \frac{\partial^2 C_F}{\partial q_F^2} \frac{dq_F}{dq_L}}{F(\theta_m) + \theta_m f(\theta_m) - f(\theta_m) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}. \]

(44)

If \( C_F(\cdot) \) is linear in \( q_F \) so that \( (\partial^2 C_F/\partial q_F^2) \equiv 0 \), it follows immediately that

\[ \left( \frac{dp_F}{dq_L} - \theta_m \frac{dq_F}{dq_L} \right) = \theta_m \left( \frac{dp_F}{dp_L} - \theta_m \frac{dq_F}{dp_L} \right). \]

As a result, (41) and (42) yields to

\[ \frac{d\theta_m}{dq_L} = -\theta_m \frac{d\theta_m}{dp_L}, \]

(45)

an equality that exactly mirrors the relationship (38) obtained for the follower.
More generally, we know from (40) that
\[
\frac{\partial^2 C_F}{\partial q_F^2} = \frac{\partial}{\partial q_F} [\theta_m F'(\theta_m)]
= \frac{\theta_m}{q_L - q_F} [F'(\theta_m) + \theta_m f'(\theta_m)].
\]
From the unit-demand assumption, we have \(X_F = F(\theta_m)\) so that (40) also yields
\[
\frac{\partial^2 C_F}{\partial q_F \partial X_F} = \frac{\partial}{\partial X_F} [\theta_m F'(\theta_m)] = \theta_m.
\]
By plugging this last two results into (41) and (42), one obtains:
\[
\frac{d\theta_m}{dp_L} = \frac{\theta_m}{q_F} \left( \frac{dq_F}{dp_L} - \frac{f'(\theta_m)}{F'(\theta_m) + \theta_m f'(\theta_m)} \left( \frac{dp_F}{dp_L} - \theta_m \frac{dq_F}{dp_L} \right) \right), \quad (46)
\]
\[
\frac{d\theta_m}{dq_L} = \frac{\theta_m}{q_F} \left( \frac{dq_F}{dq_L} - \frac{f'(\theta_m)}{F'(\theta_m) + \theta_m f'(\theta_m)} \left( \frac{dp_F}{dq_L} - \theta_m \frac{dq_F}{dq_L} \right) \right), \quad (47)
\]
while substituting these same results into (43) and (44) yields
\[
\frac{dp_F}{dp_L} - \theta_m \frac{dq_F}{dp_L} = \left( 1 + \frac{\theta_m f'(\theta_m)}{F'(\theta_m)} \right) \left( 1 - \theta_m \frac{dq_F}{dp_L} \right), \quad (48)
\]
\[
\frac{dp_F}{dq_L} - \theta_m \frac{dq_F}{dq_L} = -\theta_m \left( 1 + \frac{\theta_m f'(\theta_m)}{F'(\theta_m)} \right) \left( 1 + \frac{dq_F}{dq_L} \right). \quad (49)
\]
Combining (46) with (48) and (47) with (49) we obtain:
\[
\frac{d\theta_m}{dp_L} = \frac{\theta_m}{q_F} \left( \left( 1 + \frac{\theta_m f'(\theta_m)}{F'(\theta_m)} \right) \frac{dq_F}{dp_L} - \frac{f'(\theta_m)}{F'(\theta_m)} \right),
\]
\[
\frac{d\theta_m}{dq_L} = \frac{\theta_m}{q_F} \left( \left( 1 + \frac{\theta_m f'(\theta_m)}{F'(\theta_m)} \right) \frac{dq_F}{dq_L} + \frac{\theta_m f'(\theta_m)}{F'(\theta_m)} \right).
\]
Thus:
\[
\left( \frac{d\theta_m}{dq_L} + \theta_m \frac{d\theta_m}{dp_L} \right) = \frac{\theta_m}{q_F} \left( 1 + \frac{\theta_m f'(\theta_m)}{F'(\theta_m)} \right) \left( \frac{dq_F}{dq_L} + \theta_m \frac{dq_F}{dp_L} \right), \quad (50)
\]
that is \((d\theta_m/dq_L) = -\theta_m (d\theta_m/dp_L)\) if and only if
\[
\theta_m = -\frac{(dq_F/dq_L)}{(dp_F/dp_L)}.
\]
In words, even in the unit demand case, an increase of one unit in \(q_L\) is not equivalent to a decrease of \(\theta_m\) units in \(p_L\), in general. Unless costs can be assumed to be linear in quality, the impact of price and quality changes are not related to each other through marginal consumers quality valuation \(\theta_m\).