Monetary Policy Inertia or Persistent Shocks: A DSGE Analysis

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Abstract

In this paper, we propose a simple econometric framework to disentangle the respective roles of monetary policy inertia and persistent shocks in interest rate rules. We exploit the restrictions of a DSGE model which is confronted to a monetary SVAR. We show that, provided enough informative variables are included in the formal test, the data favor a monetary policy representation with modest inertia and highly serially correlated monetary shocks. To the contrary, when the procedure is based solely on the dynamic behavior of the nominal interest rate, no clear-cut conclusion can be reached about the correct representation of monetary policy.

Keywords: Taylor rule, Monetary policy identification, Policy Inertia

JEL Class.: C52, E31, E32, E52

1 Introduction

The purpose of this paper is to investigate whether the dynamics of the nominal interest rate are better described as featuring monetary policy inertia or as characterized by highly persistent factors or shocks. This paper deals with this long debated issue by reconciling some of the earlier, inconclusive results based on single-equation estimates. Essentially, we show that a multivariate

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system brings useful information to answer this question. Using such an approach, we show that the data favor a representation of monetary policy with serially correlated shocks and modest policy inertia.

Over the recent years, there has been a renewed interest in modeling monetary policymaking in terms of simple rules. In this context, the Taylor rule has become the workhorse description of central bank behavior. Nevertheless, Taylor (1993) pointed out that one should not expect that policymakers “follow policy rules mechanically.” Instead, one should consider the Taylor rule as a “hypothetical but representative” description summarizing the complex process of monetary policy.

Applied monetary economists have followed this general guideline by specifying and estimating extended Taylor rules to better approximate the central bank policy. An important result obtained in the literature, as exemplified by Clarida et al. (2000), is that the lagged interest rate is highly significant in the estimated policy rule, suggesting that the nominal interest rate exhibits a sizable degree of inertia. It has been argued that this apparent inertia might result from deliberate policy inertia from the central bank, the latter enforcing a partial adjustment process on its instrument. However, Rudebusch (2002, 2006) claimed that, if the central bank actually smooths its policy, then future adjustments in the interest rate should be largely predictable. Unfortunately, Rudebusch (2002) showed that this is not supported by financial data (see also Söderlind et al., 2003).

An alternative explanation to the persistence of monetary policy is the presence of serially correlated shocks in the realizations of the interest rate. These shocks represent a set of special factors that cannot be systematically modeled by a simple, parsimonious interest rate rule such as an augmented Taylor rule. If these factors are persistent, the interest rate will display inertia.

While these two competing views entail very different conclusions about the behavior of central banks, aggregate data have been fairly silent as to which is the correct representation of actual policy. For instance, Rudebusch (2002) cannot distinguish the two competing specifications in an estimated interest rate rule. English et al. (2003) find that there is supportive evidence for both representations to be significant components of the Federal Reserve behavior. Castelnuovo (2003) suggests that both views of monetary policy are equally important to describe the central bank decisions. This lack of clear-cut conclusions may be due to a well known

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1 See also Taylor (1999) for different possible interpretations of this monetary policy rule.
2 This result was also found by Amato and Laubach (2003), Kozicki (1999), Levin, Wieland and Williams (1999), and Sack and Wieland (2000), among others.
3 Gerlach-Kristen (2004) and Apel and Jansson (2005) find similar results using Kalman filtering to account for omitted unobserved factors in the interest rate rule.
problem of identification and multiple optima typically arising in models of partial adjustment with serially correlated shocks. The latter calls into question the use of a single equation, i.e. a Taylor rule taken in isolation, as a proper way to discriminate between the two competing views of monetary policy.

To eschew this identification problem, we propose to resort to a Dynamic Stochastic General Equilibrium (DSGE) model to interpret the data and disentangle these two alternative views about monetary policy. An important and celebrated virtue of such models is that they can generate very different aggregate dynamics when subjected to different policy rules, and this does not just hold for the nominal interest rate but also for a broader set of macroeconomic variables. We build on this property to assess which of the two views generates aggregate dynamics in accordance with the data.

In order to implement these ideas, we resort to a limited information approach that allows us to exclusively focus on that portion of aggregate fluctuations due to monetary shocks in US data. We first estimate a structural vector autoregression (SVAR) with short-run restrictions to identify monetary policy shocks and the implied Impulse Response Functions (IRFs) of a set of aggregate variables. Second, we estimate the DSGE parameters that govern policy inertia and the amount of serial correlation in monetary shocks. These parameters are pinned down so that the DSGE model matches as well as possible the IRFs drawn from the SVAR.

When we consider the IRFs of output, inflation, wage inflation, the Fed funds rate, and money growth, we are able to unambiguously discriminate between the two different representations. Our results suggest that the dynamics of these variables are better fitted by a scheme with moderate policy inertia and a high degree of serial correlation. Thus, the smoothness in the interest rate is mainly explained by persistent factors beyond the target level of the policy rule. In contrast, when we consider only the responses of the Fed funds rate, we find that there is not enough information to discriminate between the two views. Therefore, we insist that in order to disentangle the relative importance of each regime, one should take into account informative features of the data. In our case, this role is devoted to the responses of inflation and wage inflation. We also investigate two practical differences between these two alternative views about monetary policy: (i) in terms of the responsiveness of the nominal interest rate to inflation and the output gap; (ii) in terms of how the economy responds under the two rules during a specific episode, namely the Volcker disinflation. In both cases, our findings confirm that the persistent shock view provides a more accurate approximation of actual monetary policy.

The remainder is as follows. Section 2 presents the monetary policy rule and discusses
identification problems when partial adjustment and serial correlation are included. Section 3 describes the main ingredients of the DSGE model used in our empirical exercise. Section 4 presents the econometric approach employed. Section 5 discusses the main estimation results. Practical differences between policy inertia and persistent shocks are reviewed in section 6. Finally, the last section offers some concluding comments.

2 Identification Problems with Monetary Policy Rules

Following recent studies that have estimated models of central bank behavior, we postulate a monetary policy rule of the form

\[ \hat{i}_t^* = a_\pi \hat{\pi}_t + a_y \hat{y}_t, \]  
\[ \hat{i}_t = \rho_i \hat{i}_{t-1} + (1 - \rho_i) \hat{i}_t^* + e_t, \]  
\[ e_t = \rho_e e_{t-1} + \nu_t, \quad \nu_t \sim \text{iid}(0, \sigma^2_e). \]

Equation (1) specifies how the target level \( \hat{i}_t^* \) evolves in response to current inflation \( \hat{\pi}_t \) and output \( \hat{y}_t \). More precisely, \( a_\pi \) and \( a_y \) govern the sensitivity of the desired level of the nominal interest rate to the log deviations of inflation and output, respectively.\(^4\) If the actual nominal interest rate \( \hat{i}_t \) were equal to \( \hat{i}_t^* \), this would correspond to the policy rule proposed by Taylor (1993). Instead, equation (2) allows for a partial adjustment of the nominal interest rate to its target level at rate \( \rho_i \). In addition, the rule is hit by monetary shocks \( e_t \). If the latter were iid, this would correspond to a standard specification for an augmented Taylor rule. Instead, equation (3) specifies a parametric model of serial correlation in \( e_t \) that can potentially account for part of the actual persistence found in \( \hat{i}_t \). These shocks may represent any contingent event the central bank faces when deciding the interest rate, such as credit crunches or financial crises (see Taylor, 1993, or Rudebusch, 2002). Moreover, the use of real-time data could also reinforce the apparent degree of serial correlation in policy shocks (Orphanides, 2004). In addition, a persistent change of the inflation target can be interpreted as a serially correlated shock to monetary policy (see Smets and Wouters, 2003, 2005).

The empirical literature on Taylor rules has had trouble reaching clear-cut conclusion about the correct representation of monetary policy. Although there is no evidence that the partial adjustment hypothesis is fully responsible for the significance of the lagged interest rate term,

\(^4\)Without loss of generality, we omit a constant term. Notice that, here, we assume that the Taylor rule penalizes the log deviations of output rather than those of the output gap. In the DSGE model presented in the next section, it turns out that this distinction is irrelevant because the implied natural level of output is insensitive to monetary shocks, which are the only shocks considered in the analysis. Thus, in this framework, the output gap exactly coincides with output. See Woodford (2003, chap. 6, p. 420).
there is also no evidence supporting the total rejection of monetary policy inertia.\textsuperscript{5} The absence of clear-cut conclusion is in part due to a well-known problem of identification and multiple optima in the partial adjustment model with serially correlated shocks (see, e.g. Griliches, 1967, Blinder, 1986, Harvey, 1990, McManus et al. 1994). Rational expectation econometrics suffer from the same problems, especially when the framework conveys little information, as in Sargent (1978) or Kennan (1988).

To see this problem, let us consider our simple representation of monetary policy (1)–(3)

$$\hat{\dot{i}}_t = (\rho_i + \rho e)\hat{\dot{i}}_{t-1} - \rho_i \rho e \hat{\dot{i}}_{t-2} + (1 - \rho_i) (\hat{\dot{i}}^*_t - \rho e \hat{\dot{i}}^*_{t-1}) + \nu_t,$$

where the target $\hat{\dot{i}}^*_t$ is a linear function of shocks that hit the economy. We assume for simplicity a single shock, namely the monetary shock $\nu_t$:

$$\hat{\dot{i}}^*_t = \sum_{k=0}^{\infty} \eta_k \nu_{t-k},$$

where $\eta_k$ is a complicated nonlinear function of the policy rule parameters, as well as other deep parameters. Suppose that $\eta_k$ for $k = 0, ..., \infty$ are small and not sensitive to $\rho_i$ and $\rho e$. In this case, $\hat{\dot{i}}^*_t$ is essentially zero with a very small amount of variance. The policy function accordingly rewrites

$$\hat{\dot{i}}_t \approx (\rho_i + \rho e)\hat{\dot{i}}_{t-1} - \rho_i \rho e \hat{\dot{i}}_{t-2} + \nu_t.$$

In this case, the parameters $\rho_i$ and $\rho e$ are not identified in general. To see this, consider the reduced form associated to the approximate monetary policy

$$\hat{\dot{i}}_t = \beta_1 \hat{\dot{i}}_{t-1} + \beta_2 \hat{\dot{i}}_{t-2} + \nu_t.$$

Provided that $\rho_i \neq \rho e$, there does not exist a unique solution for $\rho_i$ and $\rho e$ as a function of the reduced form parameters $\beta_1$ and $\beta_2$. Indeed, as long as $\beta_2 \neq 0$, the solutions for $\rho_i$ and $\rho e$ are given by $\rho_i = (\beta_1 \pm (\beta_1^2 + 4\beta_2)^{1/2})/2$ and $\rho e = \beta_1 - \rho_i$, where $\beta_1^2 + 4\beta_2 = (\rho_i - \rho e)^2 \geq 0$. This means that two sets of values for $\rho_i$ and $\rho e$ are observationally equivalent. The first solution is associated to the monetary policy inertia view ($\rho_i$ large and $\rho e$ small) whereas the second is related to the persistent shocks view ($\rho_i$ small and $\rho e$ large). When $\hat{\dot{i}}^*_t \approx 0$, we cannot distinguish between a highly inertial monetary policy with transitory shocks and a monetary policy with small partial adjustment and highly serially correlated shocks. In contrast, if $\rho_i = \rho e$, the parameters are identified, but this configuration is inconclusive since it assigns the same weights to both views about monetary policy.

When $i_t^*$ is responsive to shocks, and thus is more volatile, this multiple optima problem can potentially disappear, provided that $\eta_k$ is highly sensitive to perturbations in $\rho_i$ and $\rho_e$. However, nothing guarantees this in practice, so that estimating $\rho_i$ and $\rho_e$ by focusing on the nominal interest rate only might fail to reveal the correct information about monetary policy.

A way to eschew this problem is to consider additional variables and equations in the estimation stage. We argue that moving from a single equation setup to a system of equations is likely to aid in discriminating between the two views of policy dynamics (see Rudebusch and Wu, 2004, for a similar approach). Our strategy to identify the policy parameters rests on the restrictions imposed by a DSGE model. When the policy rule parameters have strong effects on aggregate dynamics, this give us an opportunity to properly identify $\rho_i$ and $\rho_e$ and to deliver clear-cut conclusions. The next section gives a brief overview of the model used in our empirical analysis.

3 The DSGE Model

We consider a standard New Keynesian model with price and wage stickiness, along the lines of Giannoni and Woodford (2004) and Galí and Rabanal (2005). Since, later on, we will seek to compare this model with a monetary SVAR à la Christiano et al. (1996, 1999), it is important to make sure that they both embed the same timing restrictions. To achieve this, we assume that output, inflation, and wage inflation are decided prior to observing the monetary shock, as in Rotemberg and Woodford (1997, 1999).

The first equation is the New Keynesian Phillips curve:

$$\tilde{\pi}_t - \gamma_p \tilde{\pi}_{t-1} = E_{t-1} \left\{ \frac{(1 - \alpha_p)(1 - \beta \alpha_p)}{\alpha_p((1 - \mu_p s_q)^{-1}(1 + \theta_p \epsilon_{\mu_p} + \theta_{\mu_p \epsilon_{\mu_p}}))(\tilde{w}_t + \omega_p \tilde{y}_t) + \beta(\tilde{\pi}_t + 1 - \gamma_p \tilde{\pi}_t) \right\},$$

where $E_{t-1}$ is the expectation operator conditional on information available to the firm when reoptimizing its price. $\tilde{\pi}_t$, $\tilde{y}_t$, and $\tilde{w}_t$ are the logdeviations of inflation, output, and real wage, respectively. The parameter $\beta \in (0, 1)$ is the subjective discount factor, $\gamma_p \in [0, 1]$ is the degree of indexation of prices to the most recently available inflation measure, $\alpha_p \in [0, 1]$ is the degree of nominal rigidity, $s_q \in (0, 1)$ represents the share of material goods, $\theta_p > 0$ is the steady state price elasticity of demand, $\mu_p > 1$ is the steady state markup factor, $\epsilon_{\mu_p}$ is the steady state elasticity of the parmkup factor, and $\omega_p$ is the real marginal cost elasticity with respect to the level of production.

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6 See section A for more details about the model.
A second set of equations defines the IS and LM curves:

\[ E_{t-1}\{\beta b(\hat{y}_{t+1} - b\hat{y}_t) - (\hat{y}_t - b\hat{y}_{t-1}) + \sigma\chi(\hat{m}_t - \beta b\hat{m}_{t+1}) - \varphi^{-1}\hat{\lambda}_t\} = 0, \quad (5) \]

\[ \hat{\lambda}_t = i_t + E_t\{\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\}, \quad (6) \]

\[ \hat{m}_t = \eta_y(\hat{y}_t - b\hat{y}_{t-1}) - \eta_i\hat{i}_t. \quad (7) \]

where \( \hat{m}_t, \hat{i}_t, \) and \( \hat{\lambda}_t \) are the logdeviations of real balances, the nominal interest rate, and the representative household’s marginal utility of wealth, respectively. The parameter \( b \in [0, 1) \) represents the degree of habit formation. The additional parameters \( \sigma, \chi, \eta_y \) and \( \eta_i \) are deduced from the utility function. Notice that we enforce the implied constraints on these parameters when we calibrate the model. Equation (5) illustrates the role played by habits in consumption, which reinforces the backward dimension of the IS curve. Provided \( \sigma\chi > 0 \), this equation includes a real balance effect. Equation (6) is the standard Euler equation on bond holdings. Finally, equation (7) is the money demand function. The difference in the information sets in equations (5) and (6) reflects the timing of decisions. Prior to observing the monetary policy shock, the households decides how much to consume and sets its nominal wage. The shock is then realized, and bond and money holdings decisions are taken.

The wage setting equation is given by:

\[ \hat{\pi}_w - \gamma w\hat{\pi}_{t-1} = E_{t-1}\left\{ \frac{(1 - \alpha_w)(1 - \beta\alpha_w)}{\alpha_w(1 + \omega_w\theta_w)}(\omega_w\phi\hat{y}_t - \hat{\lambda}_t - \hat{w}_t) + \beta(\hat{\pi}_{t+1} - \gamma w\hat{\pi}_t) \right\} \quad (8) \]

where \( \hat{\pi}_w \) is the logdeviation of wage inflation. The parameter \( \gamma_w \in [0, 1] \) is the degree of wages indexation to the most recently available inflation measure, \( \alpha_w \in [0, 1) \) is the degree of nominal wage rigidity, \( \theta_w > 0 \) is the wage elasticity of labor demand, \( \omega_w > 0 \) is the elasticity of the marginal disutility of labor, and \( \phi > 1 \) is the inverse elasticity of output with respect to the labor input. Finally, \( \hat{\pi}_t \) and \( \hat{\pi}_w \) are linked together through the relation

\[ \hat{\pi}_w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t, \quad (9) \]

The model is closed by postulating the monetary policy rule (1)–(3).

### 4 Econometric Approach

This section details our monetary SVAR, the implied IRFs used to estimate the DSGE model, and presents the MDE approach.
4.1 The Monetary SVAR

We start our analysis by characterizing the actual economy’s response to a monetary policy shock. As is now standard, this is done by estimating a monetary SVAR in the lines of Christiano et al. (1996, 1999) so as to identify monetary policy shocks.\footnote{See also Christiano et al. (1997, 2005), and Rotemberg and Woodford (1997,1999) for other examples of this identifying strategy.} We consider a structural VAR of the form

\[ A_0 Z_t = A_1 Z_{t-1} + \cdots + A_\ell Z_{t-\ell} + \eta_t, \]

where the data vector \( Z_t \) can be decomposed according to \( Z_t = (Z'_1, t, i_t, Z'_2, t)' \). \( Z_1, t \) is a \( n_1 \times 1 \) vector composed of variables whose current and past realizations are included in the information set available to the policymaker at \( t \) and that are assumed to be predetermined with respect to the monetary shock \( \epsilon_t \). \( Z_2, t \) is a \( n_2 \times 1 \) vector containing variables that are allowed to respond contemporaneously to \( \epsilon_t \) but whose value is unknown to monetary policy authorities at \( t \). The lag length \( \ell \) is determined by minimizing the Hannan-Quinn information criterion. In our empirical analysis, we found that \( \ell = 4 \).

4.2 Minimum Distance Estimation

Let \( \psi \) denote the whole set of model parameters. Let \( \psi_2 = (\rho_i, \rho_e, \sigma_v)' \) and let \( \psi_1 \) denote the vector collecting all the remaining parameters, so that \( \psi = (\psi_1', \psi_2')' \). To implement our approach, it is important that \( \psi_1 \) be fixed, so that variations in the empirical performance of the DSGE model result only from changes in \( \psi_2 \), thus revealing information about the relevant specification of the monetary policy rule.

The policy parameters \( \psi_2 \) are estimated by minimizing a measure of the distance between the empirical responses of key aggregate variables and their model counterparts.\footnote{See Rotemberg and Woodford (1997), Altig et al. (2004), Amato and Laubach (2003), Boivin and Giannoni(2006), Christiano et al. (2005), and Giannoni and Woodford (2004). Following these studies, we implicitly assume that the SVAR is able to identify the structural monetary policy reaction function which can differ from the reaction function in the DSGE model (see Rudebusch, 1998).} More precisely, we focus our attention on the responses of the vector \( X_t \) regrouping the actual data which we are interested in. Here, \( X_t \) is a subset of \( Z_t \). We define \( \theta_j \) the vector of responses of the variables in \( X_t \) to a monetary shock at horizon \( j \geq 0 \), as implied by the above SVAR.

Then, the object which we seek to match is \( \theta = \text{vec}([\theta_0, \theta_1, \ldots, \theta_\ell])' \) where \( \ell \) is the selected horizon.\footnote{Notice that we have to exclude from \( \theta_0 \) the responses corresponding to the elements in \( X_t \) that belong to the information set available to the policy maker at \( t \). It is important to emphasize that the DSGE model previously expounded embeds the same exclusion restrictions as the monetary SVAR.} Then let \( h(\cdot) \) denote the mapping from the structural parameters \( \psi_2 = (\rho_i, \rho_e, \sigma_v)' \) to
the DSGE counterpart of $\theta$. Our estimate of $\psi_2$ is obtained by minimizing

$$J_T = (h(\psi_2) - \hat{\theta}_T)V_T(h(\psi_2) - \hat{\theta}_T)'.$$ 

where $\hat{\theta}_T$ is an estimate of $\theta$, $T$ is the sample size, and $V_T$ is a weighting matrix which we assume is the inverse of a matrix containing the asymptotic variances of each element of $\theta$ along its diagonal and zeros elsewhere. We make this particular choice for the weighting matrix to avoid singularity problems of the covariance matrix of IRFs. In addition, as suggested by Christiano et al. (2005), this choice of weighting matrix ensures that the model-based IRFs lie as much as possible inside the confidence interval of the SVAR-based IRFs. Under the null hypothesis that the DSGE model is true, $J_T$ is asymptotically distributed as a chi-squared with $\text{dim}(\hat{\theta}_T) - \text{dim}(\psi_2)$ degrees of freedom. We will use the statistic $J_T$ as a discriminating criterion between the two representations of monetary policy. Additionally, we decompose $J_T$ into components pertaining to each element of $X_t$. This decomposition provides a simple diagnostic tool that allows us to locate on which dimension the model succeeds or fails to replicate the IRFs implied by the SVAR.

5 Empirical Results

In this section, we first present our data and results drawn from our SVAR analysis. Second, we discuss the calibration of the model’s parameters. Third, we present our estimation results. Finally, we provide a sensitivity analysis to calibration.

5.1 Data and SVAR

In addition to the Fed Funds rate, we use data from the Non Farm Business (NFB) sector over the sample period 1960(1)-2002(4).\(^{10}\) The variables used for estimation are the linearly detrended logarithm of per capita GDP, $\hat{y}_t$, the growth rate of GDP’s implicit price deflator, $\hat{\pi}_t$, and the growth rate of nominal hourly compensation, $\hat{\pi}_t^w$.\(^{11}\) We also include two “information” variables in the SVAR model. First, though not formally justified by the theoretical model,

\(^{10}\)Arguably, this sample period might be characterized by significant changes in monetary policy. As a consequence, assuming that monetary policy can be represented by a single Taylor rule is rather strong. Unfortunately, the estimated IRFs from the SVAR in the period 1985(1)-2002(4) exhibit a number of pathologies. For example, output persistently rises after a contractionary monetary policy shock. In addition, the estimated IRF are not precisely estimated, implying that estimating DSGE parameters so as to replicate these responses is meaningless. This is reminiscent of the point raised by Sims (1998) that SVARs estimated on short time series can produce very erratic IRFs. Thus we follow Christiano et al. (1996, 1999, 2005) and adopt a longer sample. In addition, Sims and Zha (2006) found more evidence in favor of stable dynamics with unstable disturbance variances than of clear changes in model dynamics. See also Leeper and Roush (2003) and Rudebusch and Wu (2004).

\(^{11}\)The civilian non-institutional population over 16 is used as our measure of population. We also experimented with quadratically detrended or first-differenced output, without quantitatively altering our conclusions.
the growth rate of the logarithm of the Commodity Research Bureau price index of sensitive commodities, \( \hat{\pi}_c^t \), is included to mitigate the so-called price puzzle (see Sims, 1992, Eichenbaum, 1992, Christiano et al., 1996, 1999). Second, the growth rate of M2, \( \hat{\xi}_t \), is included to exploit information included in money growth.\(^\text{12}\) To implement the identification strategy outlined above, we set \( Z_{1,t} = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_w^t, \hat{\pi}_c^t) \)' and \( Z_{2,t} = (\hat{\xi}_t) \). In addition, the variables of interest are \( X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_w^t, \hat{\xi}_t) \)' . The empirical responses of \( X_t \) are reported on figure 1, with \( k = 30 \). The plain line is our point estimates of the empirical responses of \( X_t \) and the shaded areas indicate the asymptotic 95% confidence interval about the point estimates.

Though we focus on a different dataset and a different sample period, our findings echo previous results reported by Christiano et al. (1996, 1997, 1999, 2005).\(^\text{13}\) Output initially responds very little, and then sharply drops, with an inverted hump pattern. Notice that the latter is precisely estimated. The response of inflation displays a persistent U-shaped profile, with a narrow confidence interval. Inflation’s lowest response is reached several quarters (more than three years) after output has reached its trough. Moreover, inflation does not present a significant price puzzle in the very short-run. The response of wage inflation is qualitatively similar, with a trough response slightly lagging that of inflation. As discussed in Woodford (2003), the delayed response of inflation is a key stylized fact that any monetary DSGE model should accurately mimic. The Federal Funds rate instantaneously increases, and then gradually declines, up to the point where it will eventually cross the x axis before reverting back to its steady state value. Finally, nominal money growth drops sharply and rapidly reaches back its steady state level.

5.2 Calibration

As explained above, parameters other than \( \psi_2 \) are calibrated prior to estimation. The rationale for doing this is that we want to make sure that the model’s IRFs depend only on the particular specification of monetary policy. The calibration is reported in table 1.

Preferences. First, we set \( \beta = 0.99 \), implying a steady state annualized real interest rate of 4%. The habit persistence parameter \( b \) is set to 0.75, lying in the range of available estimates based on aggregate data (see Boivin and Giannoni, 2006 and Christiano et al., 2005). We then set \( \sigma = 1 - b \), which implies intertemporal complementarities in consumption decisions (see

\(^\text{12}\)The data are extracted from the Bureau of Labor Statistics website, except for the Fed Funds rate and M2 which are obtained from the FREDII database.

\(^\text{13}\)See also Rotemberg and Woodford (1997, 1999), Bernanke and Mihov (1998), and Leeper and Roush (2003) for similar IRFs profiles.
As in Christiano et al. (2005) and Altig et al. (2004), the elasticity of marginal labor disutility, \( \omega_w \), is set to 1. The money demand function implied by our model is of the form (7). The parameter \( \eta_y \) governs the elasticity of real money demand to output. Following Woodford (2003), the latter is normalized to 1.

Calibrating the semi elasticity \( \eta_i \) raises specific issues, especially so if the model has to reproduce the short-run behavior of money demand, as explained by Christiano et al. (2005). To pin down the value of \( \eta_i \), we follow a different approach from theirs, yielding very similar results. From the SVAR and identified monetary shocks, we construct data series for real balances \( \tilde{m}_t \), real output \( \tilde{y}_t \), and the nominal interest rate \( \tilde{i}_t \) when only monetary shocks hit the SVAR. We then estimate a linear money demand function using OLS. The estimated money demand takes the form

\[
\tilde{m}_t = 0.8571\tilde{m}_{t-1} + 0.1429\tilde{y}_t - 0.1072\tilde{y}_{t-1} - 1.1846\tilde{i}_t + \vartheta_t.
\]

We use the estimated short-run semi-elasticity of money demand to the nominal interest rate (1.1846) to calibrate \( \eta_i \). Notice that in the course of estimation, we imposed \( \eta_y = 1 \) and took account of the calibrated value of \( b \). The implied long-run semi elasticity is slightly above 8, which is the value obtained by Lucas (1988), Chari et al. (2000), and Mankiw and Summers (1986). Consequently, our calibration of \( \eta_i \) must be interpreted as a way to account for the short-run response of money growth, as in Christiano et al. (2005).

Recall that the parameter \( \chi \) governs the extent to which a real balance effect is present in our model. Under our calibration, we use the restriction \( \chi = (1 - \beta b)\eta_y/\eta_i \). We calibrate the money velocity from actual data, and obtain \( \bar{\nu} = 1.36 \). From these calibrated values, we obtain \( \chi = 0.138 \), implying a non negligible real balance effect.

**Technology.** Here \( \phi \) is the inverse of the elasticity of value added to labor input. We set \( \phi = 1.333 \), which corresponds to a steady state share of labor income of 62.5%, after correcting for the markup. Assuming further that the production function is Cobb-Douglas, direct calculations yield \( \omega_p = \phi - 1 \). The share of material goods in gross output, \( s_q \), is set to 50%, as in Rotemberg and Woodford (1995) and Basu (1995). Following Rotemberg and Woodford (1997) and Christiano et al. (2005), we set the markup on prices to 20%, i.e. \( \mu_p = 1.20 \). This implies an elasticity of demand for goods \( \theta_p = 6 \). The markup elasticity to relative demand, \( \epsilon_\mu \), is set

\[\text{An important limit of our approach is that it assumes that OLS consistently estimate } \eta_i. \text{ However, our estimate is not far from previous estimations. Moreover, we conduct a sensitivity analysis of our results to } \eta_i \text{ (see Section 5.4). Our findings are not qualitatively affected.} \]
to 1, as in Bergin and Feenstra (2000) and Woodford (2003). Finally, we set $\theta_w$ to 21 as in Christiano et al. (2005), implying a wage markup of 5%.

**Price/Wage Setting.** Following Rotemberg and Woodford (1997), we set $\alpha_p$ to 0.66, implying an average spell of no price reoptimization of 2.5 quarters. This value is consistent with microeconomic evidence, e.g. Bils and Klenow (2004). We set $\gamma_p = 1$, as in Christiano et al. (2005). This value allows us to reinforce the backward dimension of inflation. Following Amato and Laubach (2003), we symmetrically set $\alpha_w = 0.66$. As in Christiano et al. (1995), we also set $\gamma_w = 1$.

**Nominal Interest Rate Target Level.** Following Taylor (1993), we set $a_\pi = 1.5$ and $a_y = 0.5/4$, since we focus on quarterly measures of $y_t$, $\pi_t$ and $i_t$. These values are approximately the same as those considered by Christiano et al. (2005) in their sensitivity analysis.

### 5.3 Estimation Results

The estimation results are reported in table 2, for different $X_t$ and different restrictions on the policy rule parameters. In each case, we set the IRFs horizon $k$ to 30. The table is organized as follows: the left panel reports parameters estimates when $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_w^w, \hat{i}_t, \hat{\xi}_t)'$, i.e. when $\psi_2$ is selected so as to reproduce the responses of output, inflation, wage inflation, the Fed Funds rate, and money growth to a monetary policy shock; the right panel corresponds to the case where $X_t = \hat{i}_t$, i.e. when we exclusively focus on the Fed Funds rate’s behavior. In each panel, we consider five cases, depending on the minimum value of $J_T$ reached at convergence and on restrictions on $\rho_i$ or $\rho_e$. More precisely, column (1) corresponds to the minimum value of $J_T$ reached when using as an initial condition a large $\rho_e$ and a small $\rho_i$. Conversely, column (2) corresponds to the case with a large $\rho_i$ and a small $\rho_e$. Column (3) corresponds to the restriction $\rho_i = 0$, i.e. to a model with only serially correlated shocks and no policy inertia. Column (4) corresponds to the restriction $\rho_e = 0$, i.e. to a model with nominal interest rate inertia and iid shocks to monetary policy. Finally, column (5) reports the estimation outcome when imposing the constraint $\rho_i = \rho_e$, thus granting the same weight on both alternative views about monetary policy. The point estimates of $\psi_2$ are reported together with their standard errors, in parentheses. The table also reports the value of $J_T$ at convergence, together with the associated $P$-value in brackets. Finally, with our choice of weighting matrix, we can further decompose the $J_T$ statistic into various components pertaining to each element of $X_t$.

Let us first consider the case with $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_w^w, \hat{i}_t, \hat{\xi}_t)'$. In this context, we obtain a
global minimum associated to the model with serially correlated monetary shocks. Additionally, the model successfully passes the over-identification test (see column 1). In contrast, the local minimum associated to a model with monetary policy inertia is blatantly rejected by the data (see column 2). The global minimum distance estimator yields $\rho_i = 0.30$ and $\rho_e = 0.87$. This suggests that the correct representation of monetary policy is a mix of serially correlated shocks and a modest degree of policy inertia, in the line of Rudebusch (2002, 2006). Notice that these two parameters are found to be significant. In addition, the data do not reject a model version imposing $\rho_i = 0$ while they reject the restriction $\rho_e = 0$ (see columns 3 and 4 in table 2). Notice that a quasi-likelihood ratio test would however reject the restriction of no monetary policy inertia (see columns 1 and 3).

To understand why the data reject the model with high monetary policy inertia, it is instructive to consider the decomposition of $J_T$ according to the components of $X_t$. When comparing columns (1) and (2), we see that the two representations of monetary policy deliver very similar results when it comes to output, the nominal interest rate, and money growth. In other words, these three variables are weakly informative about the relevant form of monetary policy. What turns out to be really discriminating is the behavior of inflation and wage inflation. In this case, the DSGE model with policy inertia proves unable to mimic the delayed and persistent responses of these variables.

This failure is illustrated by comparing IRFs in figure 1. The lines marked with circles correspond to the DSGE point estimates with monetary policy inertia, whereas the lines marked with stars correspond to the model with persistent shocks. The dynamic responses of output, the Fed Funds rate, and money growth do not appear to be qualitatively affected by the specification of monetary policy. To the contrary, the model’s IRFs of inflation and wage inflation sharply differ. The model with persistent shocks and moderate interest rate inertia successfully matches the essential features of the data. This is no longer the case when we consider a model with a large degree of interest rate inertia, especially so when it comes to inflation and wage inflation.

Column (5) shows that the restriction $\rho_i = \rho_e$ is not supported by the data. Indeed, such a restriction deteriorates the model fit on virtually all dimensions, except maybe for money growth. Thus, a specification of monetary policy which grants the same weights to policy inertia and persistent shocks provides a fit which is substantially worse than the one with highly persistent shocks and moderate policy inertia.

Second, let us consider the case with $X_t = \hat{\mathbf{i}}_t$. The latter is investigated as a simple way of illustrating the lack of information resulting from a quantitative assessment of our model.
based on a single variable. In some sense, this problem is reminiscent of the absence of clear-cut conclusions obtained in the literature focussing on a single policy rule equation, see Rudebusch (2002, 2006). Now, we face the “multiple optima” problem, since the two representations of monetary policy deliver very close objective functions at convergence. In addition, none are rejected by the data, so that they appear to be “observationally equivalent” in terms of the $J_T$ statistic (see columns (1)–(5) in the right panel of table 2). This experiment illustrates that focusing only on the nominal interest rate does not yield a clear conclusion as to the relevant representation of monetary policy. What really matters is the aggregate dynamics (especially the dynamics of inflation and wage inflation) implied by the alternative specifications of monetary policy.

The previous results are obtained for an horizon $k = 30$. Under this assumption, we were able to discriminate between the two competing representations of monetary policy because a model with large interest rate inertia fails to mimic the delayed U-shaped responses of inflation and wage inflation. To further illustrate the information contained in these hump-shaped patterns, we now vary the horizon $k$ between 10 and 40. Figure 2 reports the $J_T$ statistic as well as its decomposition according to the elements of $X_t$. In this exercise, we select $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_w^t, \hat{\xi}_t)'$ and reestimate the policy parameters for each selected horizon. In each panel, the plain lines correspond to the value of the objective function $J_T$ as well as its decomposition in the case of monetary policy inertia while the dashed lines correspond to the case with persistent shocks. Let us first focus on the global test, i.e. the $J_T$ statistic, in the upper-left panel. We see that for relatively short horizons $(k = 10,...,15)$, the two representations of monetary policy yield comparable results. Clearly, focussing only on short-run responses does not allow us to discriminate between the two specifications. However, as soon as $k$ is sufficiently large to include the delayed hump patterns of inflation and wage inflation (see the third and fourth panels), the performances of the two competing versions start to dramatically diverge. In particular, the monetary policy inertia specification faces more and more troubles reproducing the data.

5.4 Sensitivity to Calibration

We check whether the previous findings crucially depend on our particular calibration. A simple way to assess the importance of our calibration is to redo our analysis perturbing some key model parameters. Table 3 reports the outcome of this sensitivity analysis. We identify key parameters governing the dynamic behavior of our model relating to preferences, technology, price/wage setting, and the nominal interest rate target level. For each alternative parameter value, we
reestimate the model and recompute the $J_T$ statistic at convergence with $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_w, \hat{i}_t, \hat{\xi}_t)'$ or $X_t = (\hat{i}_t)$.

Preferences. The “Preferences” panel of table 3 reports the effect of shutting habit formation down (i.e. $b = 0$). When $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_w, \hat{i}_t, \hat{\xi}_t)'$, this has the obvious effect of dramatically worsening the model’s performance. Notice that in this case, the two representations are unambiguously rejected by the data. Following Giannoni and Woodford (2004), we drastically decrease the elasticity of labor supply, setting $\omega_w = 10$. In this case, the model’s performances are always improved, but the model with policy inertia is still rejected. Finally, we increase the sensitivity of money demand to the nominal interest rate, i.e. $\eta_i = 3$. The model’s performances with persistent shocks are affected but it still passes the over-identification test. In contrast, when we focus exclusively on the nominal interest rate ($X_t = (\hat{i}_t)$), none of the alternative representations can be rejected. More importantly, we cannot discriminate between these two policies based on the $J_T$ statistic. This means that while the estimated models cannot generically mimic the dynamic responses of inflation and wage inflation, focusing exclusively on $\hat{i}_t$ would lead us to incorrectly fail to reject any model versions. This is a further illustration of the need for considering the dynamic behavior of alternative variables to properly discriminate between the competing monetary policies.

Technology. In the “technology” panel of table 3, we investigate the sensitivity of our results to perturbations on technology parameters. Following Galí and Rabanal (2005), we assume constant returns to scale in labor input, thus imposing $\phi = 1$. The model’s performances are improved for both specifications of monetary policy. However, the policy inertia is again rejected. We also modify the markups on prices without affecting our results. To the contrary, when we increase the degree of market power on the labor market, we substantially reduce the model’s ability to reproduce the IRFs of $X_t$. Under this assumption, both versions are rejected by the data. Once again, when we focus on $X_t = (\hat{i}_t)$, we fail to reject any of the two competing representations of monetary policy.

Price/Wage Setting. In the “Price/Wage Setting” panel of table 3, we experiment with altering the details of the price and wage setting side of the model. We first shut down the indexation to past inflation in either the price or wage equations ($\gamma_p = 0$ or $\gamma_w = 0$). In both cases, this dramatically worsens the model’s fit, especially so when it comes to inflation and wage inflation. Recall that these two variables were crucial in helping us sort out which specification of
monetary policy was supported by the data. Not surprisingly, in the present case, both versions are rejected. Second, we assume perfect flexibility of either prices or wages ($\alpha_p = 0$ or $\alpha_w = 0$). In both cases, the model is rejected. Contrary to the previous experiment, when we focus on $X_t = (\hat{i}_t)$, we cannot reject any of the two competing representations of monetary policy, which prove almost completely insensitive to such parameters perturbations. This illustrates once more the need for further information.

**Nominal Interest Rate Target Level.** Finally, in the “Target Level” panel of table 3, we experiment with the parameters governing the target level of the nominal interest rate, namely $a_{\pi}$ and $a_y$. We set $a_{\pi}$ to a larger value than considered by Taylor (1993), $a_{\pi} = 3$. When $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_w, \hat{\pi}_w, \hat{\xi}_t)'$, the discrepancy between the two alternative specifications of monetary policy widens, especially so when it comes to inflation and wage inflation. This results from the fact that increasing $a_{\pi}$ increases the amount of information in the target level of the nominal interest rate. When it comes to $a_y$, the quantitative findings are left unaffected. Conversely, when we focus on $X_t = (\hat{i}_t)$, we fail to reject any of the two competing representations of monetary policy. This is more troubling than one would have expected. Indeed, increasing the volatility of the target can potentially eliminate the identification problem. This is not the case in practice. When we focus only on $\hat{i}_t$, the discriminating power of inflation and wage inflation is shut down, which keeps us away from reaching a clear-cut conclusion.

6 Practical Differences between policy inertia and persistent shocks

This section presents two illustrations of the practical differences between policy inertia and persistent shocks. We first investigate their quantitative implications for policy rule estimation. Second, we perform forecasting exercises using the Volcker disinflation a a case study that can potentially reveal striking differences between the two views.

6.1 Implications for Monetary Policy

The previous exercise has allowed us to discriminate between two alternative representations of monetary policy. However, since all the parameters were calibrated, including the responsiveness of monetary policy to inflation and output, this exercise is necessarily silent on the consequences of a monetary policy misspecification. The question which we ask now is the following: Would we get different estimates of the responsiveness of monetary policy to inflation and output in
the case of monetary policy inertia and in the case of persistent shocks?

So as to answer this question, we reestimate our model under the two alternative representations and allow \( a_\pi \) and \( a_y \) to be freely estimated. Table 4 reports the estimation results. The first column reports results obtained with persistent shocks while the second corresponds to policy inertia. As before, the global minimum is obtained with highly serially correlated monetary shocks and a small degree of interest rate smoothing. Once again, the responses of inflation and wage inflation allow us to discriminate between the two competing views.

Our results also suggest is that the two alternative views yield very contrasted findings relative to the reaction of monetary authorities to inflation and output. In the case of policy inertia, the latter is almost passive regarding inflation while highly reactive to output fluctuations. Under persistent shocks, we obtain a reverse configuration suggesting a very aggressive monetary policy in response to inflation and a zero concern for output fluctuations. The global minimum thus corresponds to a policy rule enforcing the Taylor principle.

6.2 Inspecting the Monetary Policy Rules Through the Lenses of the Volcker Disinflation

In this section, we compare the performances of the two alternative representations of monetary policy using the Volcker disinflation as an episode that can potentially reveal striking differences between these policy rules. This episode corresponds to what can be a priori viewed as a period of large contractionary monetary policy shocks. Thus, comparing the model under the two rules with what actually happened during this episode constitutes a legitimate experiment.

Since our limited information approach exclusively relies on monetary policy shocks, we start by reconstructing historical data from the SVAR after having shut down all other shocks than monetary shocks. We then feed the identified monetary shocks in our DSGE model using either rules and compute artificial data. To compare the performances of the two rules on output and inflation, we focus on the sample period preceding and succeeding to the Volcker disinflation, 1970(1)–1990(4). To make things comparable, we use the same initial conditions in 1970 either for the SVAR for the model. The outcome of these comparisons is reported in figures 3 and 4 for output and inflation, respectively. In each case, the plain line corresponds to the SVAR-based historical data while the lines marked with circles and stars correspond to the DSGE model with policy inertia and serially correlated shocks, respectively.\(^\text{15}\)

Figure 3 confirms our previous findings: the two alternative policy rules have similar im-

\(^{15}\)Notice that this exercise contains the same information as our IRF-based assessment of the model's performance. What is interesting here is that it allows us to focus on a specific episode within our sample.
lications when it comes to output dynamics. As is clear from the picture, in both cases, the simulated samples are very similar, either in terms of persistence and volatility. To the contrary, as shown in figure 4, the two alternative rules have very different implications in terms of inflation dynamics. Under the rule with policy inertia, inflation dynamics exhibit a smaller variance when compared to its SVAR-based counterpart. In addition, inflation drops too slowly during the Volcker disinflation and rises much too fast after 1983. Overall, this gives an inaccurate description of actual inflation dynamics during this particular episode. In contrast, when the rule features serially correlated shocks and a modest degree of policy inertia, the model is better suited to capture the large inflation peak of 1978 as well as the sharp decline following the disinflation. Additionally, the model does not predict a rapid rise in inflation after 1983, consistent with what the SVAR-based path suggests.

This exercise provides a confirmation that inflation dynamics contain more useful pieces of information than the dynamics of output for the purpose of disentangling the two alternative representations of monetary policy. The Volcker disinflation, taken as a case study, favors the persistent shock view as a practical approximation of actual monetary policy, as was to be expected from our previous quantitative investigation.

7 Conclusion

In this paper, we proposed a simple econometric framework to discriminate between two alternative representations of monetary policy. This approach draws heavily from the restrictions contained in the monetary DSGE model used in our empirical analysis. More precisely, thanks to these restrictions, different monetary policies can have radically different implications in terms of aggregate dynamics. Building on this well known property of DSGE models, we are able to identify which policy rule best fits the data.

Our results are twofold. First, when the framework contains enough information, a policy rule with modest interest rate inertia and highly serially correlated shocks, which contrasts with most current implementations of monetary policy rules, satisfactorily matches the data. In particular, we found that the dynamics of inflation and wage inflation are particularly helpful for inferring the correct specification of monetary policy. However, output, the nominal interest rate, and the money growth rate do not contain very discriminating information. In addition, the inverted hump patterns displayed by the impulse responses of inflation and wage inflation are found to be particularly relevant for this purpose. Second, when the framework is not informative enough, i.e. when we focus on the sole dynamics of the Fed funds rate, we are unable to discriminate between
the two alternative monetary policy rules. These results highlight the low discriminating power of single equation approaches. Overall, our results suggest that using extra macroeconomic information can help reach clear-cut conclusions as to the correct empirical representation of monetary policy rules. These two main findings are confirmed when we investigate practical differences between these two alternative monetary policy representations.
References


A Model Details

A.1 Production Side

A large number of competitive firms produce a homogeneous good that can be either consumed ($y_t$) or used as a material input in production ($q_t$). The overall aggregate demand is $d_t \equiv y_t + q_t$, and $P_t$ is the associated nominal price. Following Kimball (1995) and Woodford (2003), the production function is of the form

$$\int_0^1 G \left( \frac{d_t(\varsigma)}{d_t} \right) d\varsigma = 1,$$

where $d_t(\varsigma)$ denotes the input of intermediate good $\varsigma \in [0, 1]$, and the function $G$ is increasing, strictly concave, and satisfies the normalization $G(1) = 1$. The representative final good producer chooses $\{d_t(\varsigma), \varsigma \in [0, 1]\}$ and $p_t$ in order to maximize profits

$$\max_{\{d_t(\varsigma)\}} P_t d_t - \int_0^1 P_t(\varsigma)d_t(\varsigma)d\varsigma,$$

subject to (A.1), where $P_t(\varsigma)$ is the nominal price of intermediate good $\varsigma$. Monopolistic firms produce the intermediate goods $\varsigma \in [0, 1]$. Each firm $\varsigma$ is the sole producer of intermediate good $\varsigma$. Following Woodford (2003), we assume that monopolist $\varsigma$ produces good $\varsigma$ with the inputs of aggregate labor $n_t(\varsigma)$ and material goods $q_t(\varsigma)$ according to the following production possibilities

$$\min \left\{ \frac{F(n_t(\varsigma))}{1 - s_q}, \frac{q_t(\varsigma)}{s_q} \right\} \geq d_t(\varsigma),$$

where $F(\cdot)$ is an increasing and concave production function and $s_q$ is the share of material goods in gross output. Let $\theta_p(z)$ denote the elasticity of demand for a producer of intermediate good facing the relative demand $z = d_t(\varsigma)/d_t$. According to our specification, $\theta_p(z) \equiv -G'(z)/(zG''(z))$. This illustrates that intermediate good firms face a varying elasticity of demand for their output, implying a varying markup, which is denoted by $\mu_p(z) \equiv \theta_p(z)/(\theta_p(z) - 1)$.

Following Calvo (1983), we assume that in each period of time and prior to observing the monetary policy shock, a monopolistic firm can reoptimize its price with probability $1 - \alpha_p$, irrespective of the elapsed time since it last revised its price. As in Woodford (2003), if the firm cannot reoptimize its price, the latter is rescaled according to the simple revision rule $P_T(\varsigma) = (1 + \delta^p_{t,T})P_t(\varsigma)$, where

$$1 + \delta^p_{t,T} = \begin{cases} \prod_{j=t}^{T-1} (1 + \pi)^{1-\gamma_p}(1 + \pi_j)^\gamma_p & \text{if } T > t \\ 1 & \text{otherwise} \end{cases},$$

where $\pi_t = P_t/P_{t-1} - 1$ represents the inflation rate, $\pi$ is the steady state inflation rate, and $\gamma_p \in [0, 1]$ measures the degree of indexation to the most recently available inflation measure. Let $P_T^*(\varsigma)$ denote the price chosen in period $t$ by monopolist $\varsigma$ if drawn to reoptimize. Then, firm $\varsigma$ chooses $P_T^*(\varsigma)$ in order to maximize

$$E_{t-1} \sum_{T=t}^\infty (\beta \alpha_p)^{T-t} \lambda_T \left\{ \frac{(1 + \delta^p_{t,T})P_T(\varsigma)}{P_T}d_{T,T}(\varsigma) - S(d_{T,T}(\varsigma)) \right\},$$

where $\lambda_T$ is the representative household’s marginal utility of wealth in period $T$, $E_{t-1}\{\cdot\}$ is the expectation operator conditional on information available when the firm sets its price, $S(d_t(\varsigma))$
is the real cost of producing \( d_t(\zeta) \) units good of \( \zeta \), and \( d^*_{t,T}(\zeta) \), the demand for good \( \zeta \) at \( T \) if firm \( \zeta \) last reoptimized its price at \( t \), obeys

\[
G' \left( \frac{d^*_{t,T}(\zeta)}{d_T} \right) = \left( \frac{1 + \delta^*_t(T) P_t^*(\zeta)}{P_T} \right) \int_0^1 \frac{d_t(u)}{d_t} G' \left( \frac{d_t(u)}{d_t} \right) du.
\]

Standard manipulations yield the loglinear New Keynesian Phillips curve

\[
\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \mathbb{E}_{t-1} \{ \kappa_p (\hat{w}_t + \omega_p \hat{y}_t) + \beta (\hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t) \},
\]

(A.2)

with

\[
\kappa_p \equiv \frac{(1 - \alpha_p)(1 - \beta \alpha_p)}{\alpha_p}, \quad \zeta \equiv \frac{1}{(1 - \mu_p s_q)^{-1} (1 + \theta_p \epsilon_{\mu}) + \theta_p \omega_p}.
\]

In equation (A.2), \( \hat{\pi}_t \) is the logdeviation of \( 1 + \pi_t \), \( \hat{y}_t \) and \( \hat{w}_t \) are the logdeviations of \( y_t \) and \( w_t \) (real wage), respectively, \( \theta_p \equiv \theta_p(1) \) is the steady state elasticity of demand for a producer of intermediate good, \( \mu_p \equiv \mu_p(1) \) is the steady state markup factor, and

\[
\omega_p \equiv -\frac{F''(n) n}{F'(n) F(n)}.
\]

Here, \( F(n), F'(n), \) and \( F''(n) \) denote the value of \( F \) and its first and second derivatives, evaluated at the steady state value of \( n \). Following Woodford (2003), we let \( \epsilon_{\mu} \) denote the elasticity of \( \mu_p(z) \) in the neighborhood of \( z = 1 \), i.e. \( \epsilon_{\mu} = \mu'_p(1)/\mu_p(1) \).

**A.2 Aggregate Labor Index and Households**

Following Erceg et al. (2000), we assume for convenience that a set of differentiated labor inputs, indexed by \( \nu \in [0,1] \), are aggregated into a single labor index \( h_t \) by competitive firms, which will be referred to as labor intermediaries. They produce the aggregate labor input according to the following Constant Elasticity of Substitution technology

\[
h_t = \left( \int_0^1 h_t(\nu)(\theta_w-1)/\theta_w \, d\nu \right)^{\theta_w/(\theta_w-1)},
\]

where \( \theta_w > 1 \) is the elasticity of substitution between any two labor types. The associated aggregate nominal wage obeys

\[
W_t = \left( \int_0^1 W_t(\nu)^{1-\theta_w} \, d\nu \right)^{1/(1-\theta_w)},
\]

where \( W_t(\nu) \) denotes the nominal wage rate paid to type-\( \nu \) labor. The economy is inhabited by a continuum of differentiated households, indexed by \( \nu \in [0,1] \). A typical household, say household \( \nu \), must select a sequence of consumptions and nominal money and bond holdings, as well as a nominal wage. The timing of events is as follows. Prior to observing the monetary policy shock, the household decides how much to consume and sets its nominal wage. The shock is then realized, and bond and money holdings decisions are taken. Household \( \nu \)'s goal in life is to maximize

\[
\mathbb{E}_\Phi_t \sum_{T=t}^{\infty} \beta^{T-t}[U(c_T - bc_{T-1}, m_T) - V(h_T(\nu))]
\]

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where $\beta \in (0, 1)$ is the subjective discount factor, $b \in (0, 1)$ is the habit parameter, $c_t$ is consumption, $m_t \equiv M_t/P_t$ denotes real cash balances at the end of the period, where $M_t$ denotes nominal cash balances; $h_t(v)$ denotes household $v$’s labor supply at period $t$. Here, $E_{\Phi_t}$ is a conditional expectation operator reflecting the particular information sets at the household’s disposal when taking their decisions. Household $v$ maximizes his intertemporal utility subject to the sequence of constraints

$$P_t \text{tax}_t + P_t c_t + M_t + \frac{B_t}{1 + i_t} \leq W_t(v) h_t(v) + B_{t-1} + M_{t-1} + P_t \text{div}_t,$$

where $\text{div}_t$ denotes real profits redistributed by monopolistic firms; $B_t$ denotes the nominal bonds acquired in period $t$ and maturing in period $t + 1$; $i_t$ denotes the gross nominal interest rate; $\text{tax}_t$ is a lump-sum tax levied by the government. As in Woodford (2003), we assume that there is a satiation level $m^*$ for real balances such that $U_m = 0$ for $m \geq m^*$. Thus, when $m_t$ reaches $m^*$ from below, the transaction services of real cash balances yield lower and lower marginal utility. Let $\lambda_t$ denote the Lagrange multiplier associated with the household’s budget constraint. According to the timing of decisions embedded in $\Phi_t$, the loglinearization of the first order conditions associated with $c_t$, $B_t$, and $M_t$ yields

$$E_{t-1} \{ \beta b (\hat{c}_{t+1} - b \hat{c}_t) - (\hat{c}_t - b \hat{c}_{t-1}) + \sigma \lambda (\hat{m}_t - \beta b \hat{m}_{t+1}) - \varphi^{-1} \hat{\lambda}_t \} = 0, \quad (A.3)$$

$$\hat{\lambda}_t = \hat{i}_t + E_t \{ \hat{\lambda}_{t+1} - \hat{\pi}_{t+1} \}, \quad (A.4)$$

$$\hat{m}_t = \eta_b (\hat{c}_t - b \hat{c}_{t-1}) - \eta_i \hat{i}_t. \quad (A.5)$$

where $\hat{c}_t$, $\hat{m}_t$, $\hat{i}_t$, and $\hat{\lambda}_t$ are the logdeviations of $c_t$, $m_t$, $1 + i_t$, and $\lambda_t$, respectively, and where we defined the auxiliary parameters $\sigma^{-1} = -U_{cc}c/U_c$, $\chi = U_{cm}m/U_c$, $\varphi^{-1} = (1 - \beta b) \sigma$, $\eta_b = -U_{mc}/(U_{mm}m)$, $\eta_i = -1 - (1 - \beta b) U_c/(U_{mm}m)$. Notice that $\chi = (1 - \beta b) \eta_b / (\bar{v} \eta_i)$, where $\bar{v}$ is the steady state value of $c_t/m_t$.

A typical household acts as a monopolistic supplier of type-$v$ labor. It is assumed that at each point in time, and prior to observing the monetary policy shock, only a fraction $1 - \alpha_w$ of the households can set a new wage, which will remain fixed until the next time period the household is drawn to reset its wage. The remaining households simply revise their wages according to the simple rule $W_T(v) = (1 + \delta_w W_{t,T}) W_t(v)$, where

$$1 + \delta_w = \begin{cases} \prod_{j=t}^{T-1} (1 + \pi)^{1 - \gamma_w} (1 + \pi_j)^{\gamma_w}, & \text{if } T > t \\ 1, & \text{otherwise} \end{cases},$$

where $\gamma_w \in [0, 1]$ measures the degree of indexation to the most recently available inflation measure.

Let us now consider the wage setting decision confronting a household drawn to reoptimize its nominal wage rate in period $t$, say household $v$. Let us define wage inflation $\pi^w_t \equiv W_t/W_{t-1} - 1$. Now, let $W_t^*(v)$ denote the wage rate chosen in date $t$, and $h^*_t(v)$ denote hours worked in period $T$ if household $v$ last reoptimized its wage in period $t$, which obey the relationship

$$h^*_t(v) = (\frac{1 + \delta_w W_t^*(v)}{P_t})^{-\sigma_w} h_T.$$

$W_t^*(v)$ is then selected so as to maximize

$$E_{t-1} \sum_{T=t}^{\infty} (\beta \alpha_w)^{T-t} \left\{ \lambda T (1 + \delta_w W_t^*(v)) h^*_t(v) - V(h^*_t(v)) \right\}$$

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Loglinearizing the associated first order condition yields

\[ \hat{\pi}_t^w - \gamma \hat{\pi}_{t-1} = E_{t-1} \{ \kappa_w (\omega_w \hat{h}_t - \hat{\lambda}_t - \hat{\omega}_t) + \beta (\hat{\pi}_{t+1}^w - \gamma \hat{\pi}_t) \} \]  

(A.6)

where \( \hat{\pi}_t^w \) is the logdeviation of \( 1 + \pi_t^w \) and where we defined the composite parameters

\[ \kappa_w = \frac{(1 - \alpha_w)(1 - \beta \alpha_w)}{\alpha_w (1 + \omega_w \theta_w)}, \quad \omega_w = \frac{V_{hh} h}{V_h}. \]

Finally, \( \hat{\pi}_t \) and \( \hat{\pi}_t^w \) are linked together through the relation

\[ \hat{\pi}_t^w = \hat{\omega}_t - \hat{\omega}_{t-1} + \hat{\pi}_t, \]  

(A.7)

The model is closed by specifying the policy rule (1)–(3).

In equilibrium, it must be the case that \( y_t = c_t \) and \( h_t = n_t \). Furthermore, from the aggregate production function, it must also be the case that \( \bar{n}_t = \phi \bar{y}_t \), where \( \phi^{-1} = F'(n) n / F(n) \). Substituting these relations in the system composed of (A.2)–(A.7), augmented with eqs. (1)–(3), we obtain a rational expectations system of linear equations which we solve using standard methods.
Figure 1: IRFs to a Monetary Policy Shock (SVAR and DSGE Models)

\[ \hat{\pi}_t \]

\[ \hat{\pi}_\text{w} \]

\[ \hat{\xi}_t \]

<table>
<thead>
<tr>
<th>Periods after shock</th>
<th>Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>-0.05</td>
</tr>
<tr>
<td>20</td>
<td>-0.1</td>
</tr>
<tr>
<td>30</td>
<td>-0.15</td>
</tr>
</tbody>
</table>

- SVAR Model: persistent shocks
- Model: monetary policy inertia
Figure 2: Decomposition of the $J_T$ statistic as a function of the time horizon

$J$

- *Large $\rho_i$ and small $\rho_c$*
- *Large $\rho_c$ and small $\rho_i$*

$J_y$

$J_\pi$

$J_i$

$J_\xi$
Figure 3: Historical Simulation of Output

Note: plain line: SVAR, line marked with circles: DSGE model with policy inertia, line marked with stars: DSGE model with persistent shocks. All the data are demeaned prior to simulation.
Figure 4: Historical Simulation of Inflation

Note: plain line: SVAR, line marked with circles: DSGE model with policy inertia, line marked with stars: DSGE model with persistent shocks. All the data are demeaned prior to simulation.
Table 1. Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$b$</td>
<td>Habit persistence</td>
<td>0.75</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Intertemporal elasticity of substitution ($= 1 - b$)</td>
<td>0.25</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>Elasticity of marginal labor disutility</td>
<td>1.00</td>
</tr>
<tr>
<td>$\bar{v}$</td>
<td>Steady state money velocity</td>
<td>1.36</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>Money demand elasticity wrt $\hat{y}_t$</td>
<td>1.00</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>Money demand elasticity wrt $\hat{i}_t$</td>
<td>1.18</td>
</tr>
<tr>
<td>Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse of the elasticity of $\hat{y}_t$ wrt $\hat{n}_t$</td>
<td>1.33</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>$\phi - 1$</td>
<td>0.33</td>
</tr>
<tr>
<td>$s_q$</td>
<td>Share of material goods</td>
<td>0.50</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Elasticity of demand for goods</td>
<td>6.00</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>Markup ($= \theta_p/(\theta_p - 1)$)</td>
<td>1.20</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Markup elasticity</td>
<td>1.00</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Elasticity of demand for labor</td>
<td>21.00</td>
</tr>
<tr>
<td>$\mu_w$</td>
<td>Markup ($= \theta_w/(\theta_w - 1)$)</td>
<td>1.05</td>
</tr>
<tr>
<td>Price/Wage Setting</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>Price indexation</td>
<td>1.00</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>Wage indexation</td>
<td>1.00</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>Prob. of no price adj.</td>
<td>0.66</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>Prob. of no wage adj.</td>
<td>0.66</td>
</tr>
<tr>
<td>Nominal Interest Rate Target Level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_\pi$</td>
<td>Monetary policy reaction to $\hat{\pi}_t$</td>
<td>1.500</td>
</tr>
<tr>
<td>$a_y$</td>
<td>Monetary policy reaction to $\hat{y}_t$</td>
<td>0.125</td>
</tr>
</tbody>
</table>
Table 2. Estimation Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Based on $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_w, \hat{\xi}_t)'$</th>
<th>Based on $X_t = (\hat{\xi}_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.2976</td>
<td>0.8986</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>0.8740</td>
<td>0.0676</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.092)</td>
</tr>
<tr>
<td>$\sigma_\nu$</td>
<td>0.1691</td>
<td>0.1720</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>$\mathcal{J}$</td>
<td>145.66</td>
<td>245.54</td>
</tr>
<tr>
<td></td>
<td>[62.94]</td>
<td>[0.90]</td>
</tr>
<tr>
<td>$\mathcal{J}_y$</td>
<td>40.22</td>
<td>40.76</td>
</tr>
<tr>
<td>$\mathcal{J}_\pi$</td>
<td>34.22</td>
<td>86.25</td>
</tr>
<tr>
<td>$\mathcal{J}_{\pi_w}$</td>
<td>17.03</td>
<td>75.55</td>
</tr>
<tr>
<td>$\mathcal{J}_\xi$</td>
<td>30.70</td>
<td>28.48</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, $P$-value in brackets. (1): initialization with $\rho_i$ small and $\rho_e$ large; (2): initialization with $\rho_e$ small and $\rho_i$ large; (3) constrained case $\rho_i = 0$; (4) constrained case $\rho_e = 0$; (5) constrained case $\rho_e = \rho_i$. In columns (3) and (4), a star denotes a standard error not available.
Table 3. Sensitivity to Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>$J_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.00</td>
<td>396</td>
</tr>
<tr>
<td>$\omega_w$</td>
<td>10.00</td>
<td>120</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>0.00</td>
<td>181</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.00</td>
<td>103</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>11.00</td>
<td>110</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>11.00</td>
<td>183</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.00</td>
<td>235</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>0.00</td>
<td>266</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>0.00</td>
<td>356</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>0.00</td>
<td>481</td>
</tr>
<tr>
<td>$a_\pi$</td>
<td>3.00</td>
<td>133</td>
</tr>
<tr>
<td>$a_y$</td>
<td>0.50</td>
<td>147</td>
</tr>
</tbody>
</table>

Notes: the label $\rho_e \gg \rho_i$ refers to an initialization of the estimation with $\rho_e$ larger than $\rho_i$. Symmetrically, the label $\rho_i \gg \rho_e$ refers to an initialization of the estimation with $\rho_i$ larger than $\rho_e$. $P$-value in brackets. A # in the "$\rho_i \gg \rho_e$" panel refers to an initialization of the estimation with $\rho_i$ larger than $\rho_e$. $P$-value in brackets. A # in the "$\rho_e \gg \rho_i$" panel refers to the corresponding figure in the "$\rho_e \gg \rho_i$" panel.
Table 4. Estimation Results for the Complete Taylor Rule

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\rho_c \gg \rho_i$</th>
<th>$\rho_i \gg \rho_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_i$</td>
<td>0.2773 (0.187)</td>
<td>0.8832 (0.233)</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>0.9501 (0.013)</td>
<td>0.4169 (0.223)</td>
</tr>
<tr>
<td>$a_\pi$</td>
<td>3.0815 (0.568)</td>
<td>1.0509 (0.809)</td>
</tr>
<tr>
<td>$a_y$</td>
<td>0.0000 (---)</td>
<td>0.7065 (2.152)</td>
</tr>
<tr>
<td>$\sigma_\nu$</td>
<td>0.1670 (0.010)</td>
<td>0.1594 (0.010)</td>
</tr>
</tbody>
</table>

| $J$       | 129.43 [88.64]      | 173.64 [9.06]       |
| $J_y$     | 46.28               | 42.32               |
| $J_\pi$   | 26.56               | 51.13               |
| $J_{\pi^w}$ | 10.90               | 34.73               |
| $J_i$     | 20.87               | 24.43               |
| $J_\xi$   | 24.82               | 21.03               |

Notes: the label $\rho_c \gg \rho_i$ refers to an initialization of the estimation with $\rho_c$ larger than $\rho_i$. Symmetrically, the label $\rho_i \gg \rho_c$ refers to an initialization of the estimation with $\rho_i$ larger than $\rho_c$. Standard errors in parentheses, $P$-value in brackets.