The Mixed Strategy Nash Equilibrium of the Television News Scheduling Game

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Abstract

We characterize the unique mixed-strategy equilibrium of an extension of the "television news sheduling game" of Cancian, Bergström and Bills (1995) where viewers want to watch the first newscast broacast after they return home. A fraction of the viewers record randomly one of the newscast to watch them in case they are too late. At equilibrium, neither of the two stations broadcasts its evening news in the first part of the evening and the density function is strictly decreasing.

1 Introduction

Cancian, Bergström and Bills (1995) have analyzed the problem of scheduling evening television newscasts as an "Hotelling location problem with directional constraints". In this simple but ingenious framework, stations have to decide the time at which they are going to broadcast their evening news, given that viewers, who go back at different times in the evening, watch the first newscast broadcast after they return home. They showed that this specific game has no pure strategy Nash equilibrium.

Obviously what characterizes an "Hotelling location problem with directional constraints" is that consumers care not only about how far they are from the different firms but also about on which side the firms are located. Cancian, Bills and Bergström point out that the scheduling of airline departures is another example of this type of games: a business traveller will take the latest flight which allows him/her to arrive at his/her destination at a specific time but no flight which arrives after this time. Another possible example ("voters leaning towards the left") is in the field of political economy : each voter is characterized by his/her ideal policy t and votes for the candidate who is the closest *on the left* of this ideal point but never for a candidate on the right of t. Introducing directional constraints of this kind changes in a substantial way the outcome of the location game when the players maximize their "audience" (or the number of voters). Indeed, in the usual Hotelling model without directional constraints, it is well-known that the stations would broadcast their news at the *median time* among the ideal times of the population of viewers.

When one looks at the times at which the evening news[1] are broadcast in four European countries, one finds a rather important dispersion, except may be in France where the range is rather narrow: from 19.30 (FR3) to 19H 54 (M6) and 20H (TF1 and A2) It is much larger in RFA: 18.45 (RTL), 20.00 (ARD), 21.45 (ZDF) to name only the main Channels. In Great Britain the broadcasting times are rather late: 20.00 (BBC 4), 22.00 (BBC 1), 22.30 (BBC 2). Finally in Italy, they are as follows : 18.30 (Studio Aperto), 18.55 (TG 4), 19.00 (TG 3), 20.00 (TG 1 and TG 5), 20.30 (TG2). If the stations are indeed audience-maximizers[2], this dispersion is an argument which seems to plead against the usual minimum-differentiation result of the usual Hotelling model without directional constraints and to suggest a possible mixed-strategy equilibrium.

In this paper we show that there is unique mixed-strategy Nash equilibrium of the following extension of the CBB game. We assume that an exogenous fraction γ of the viewers record randomly one of the two evening news : these viewers watch the recorded news in case where they reach their home too late to watch one of them on live; the CBB game corresponds to the case where $\gamma = 0$. We characterize this equilibrium when the viewers are uniformly distributed on the interval $[0, 1]$. At equilibrium neither of the two stations broadcasts its evening news in the first part of the evening; depending upon the value of γ, it varies between $\frac{1}{4}$and $\frac{1}{e}$. The density is shown to be decreasing. Further, for all time T the probability of broadcasting news before T increases with γ.

2 The Non-Existence of a Pure Strategy Equilibrium

Suppose that there are viewers who are uniformly distributed on an interval $[0, T]$ It is supposed that a given type $t-$viewer comes back home at time t and watches *the first newscast* which is broadcast after his/her return if he/she can do so while a fraction $\gamma \in [0, 1]$ of those missing the last news will watch them as they have been wise enough to record one of them which they randomly recorded before leaving home There are two stations which broadcast the evening news and each station i $(= 1, 2)$ has to choose at which time $T_i \in [0, T]$ it will broadcast its evening news in order to maximize its audience. Let $T = 1$ for the sake of simplicity but without any loss of generality. The payoffs of the two stations are simply defined as

[1] In some cases it is difficult to have a clearcut ditinction between and evening and night news.

[2] If audience-maximizing is a debatable assumption for characterizing free-to-air Channels' behavior in general we think that it is a rather sensible "intermediate assumption" regarding more specifically the evening news which may be preceded and followed but not interrupted by advertising slots Having more people watching the news generally means more people watching the advertising slots before and after the news.

$$
\begin{aligned}
A_1 &= T_1 + \frac{\gamma}{2}(1-T_2), \;\; A_2 = T_2 - T_1 + \frac{\gamma}{2}(1-T_2), \text{ if } T_1 < T_2 \qquad (1) \\
A_1 &= T_1 - T_2 + \frac{\gamma}{2}(1-T_1), \;\; A_2 = T_2 + \frac{\gamma}{2}(1-T_1), \text{ if } T_1 > T_2 \\
A_1 &= A_2 = \frac{\gamma + (1-\gamma)T}{2} \;\; \text{if } T_1 = T_2 = T
\end{aligned}
$$

This game has no pure strategy Nash equilibrium. If its competitor j broadcasts its evening news before T_m the best reply of station i is to broadcast its own news at $T_i = 1$, i.e. as late as possible. If j broadcasts its news after T_m the best strategy for i is to broadcast its evening news just before j.

3 The Mixed-Strategy Equilibrium

We first characterize the unique symmetrical equilibrium with absolutely continuous distribution functions. Then, we show that this is in fact the unique mixed-strategy equilibrium of this game. In what follows, we denote by F the cumulative and by f the density.

Proposition 1 *If both medias choose absolutely continuous distribution functions with a connected support, the broadcasting news game has a unique,symmetrical, mixed strategy equilibrium where (i) both firms select* $F(T_i) = 2 - \frac{\frac{2}{\gamma}-1}{\sqrt{T_i^{\gamma}}}$ *for all* $T_i \in \left[(1-\frac{\gamma}{2})^{\frac{2}{\gamma}}, 1\right]$ *and* $F(T_i) = 0$ *for all* $T_i \in \left[0, (1-\frac{\gamma}{2})^{\frac{2}{\gamma}}\right]$ *when* $\gamma \neq 0$ *and (ii) both firms select* $F(T_i) = \ln(T_i) + 1$ *for all* $T_i \in \left[\frac{1}{e}, 1\right]$ *and* $F(T_i) = 0$ *for all* $T_i \in \left[0, \frac{1}{e}\right]$ *when* $\gamma = 0$.

Proof : At any such mixed strategy Nash equilibrium, given the strategy $f(T_j)$ of station j station i must have the same expected pay-off whatever its broadcasting time T_i provided it belongs to the interval $[\alpha, \beta]$:

$$
\int_{\alpha}^{T_i} (T_i(1-\frac{\gamma}{2}) - T_j + \frac{\gamma}{2}) f(T_j) dT_j + \int_{T_i}^{\beta} \left(T_i + \frac{\gamma}{2} - \frac{\gamma\, T_j}{2} \right) f(T_j) dT_j = C
$$

where C is the constant expected audience. Differentiating with respect to T_i we obtain

$$
\frac{\gamma}{2}(1-T_i) f(T_i) + (1-\frac{\gamma}{2}) \int_{\alpha}^{T_i} f(T_j) dT_j - \left(\frac{\gamma}{2}(1-T_i) + T_i \right) f(T_i) + \int_{T_i}^{\beta} f(T_j) dT_j = 0
$$

i.e. after simplifications

$$-T_i f(T_i) + 1 - \frac{\gamma}{2} F(T_i)) = 0$$

so that the equilibrium mixed strategy F satisfy the first order linear differential equation:

$$\frac{dF}{dx} = \frac{1}{x} - \frac{\gamma F(x)}{2x}$$

which admits as a solution

$$F(x) = \frac{2}{\gamma} - \frac{\lambda}{\sqrt{x^\gamma}} \text{ when } \gamma \neq 0 \tag{2}$$

or

$$F(x) = \ln \lambda x \text{ when } \gamma = 0$$

Obviously α must be strictly positive. On the other hand, we must have $\beta = 1$ as if this did not hold true, by choosing a broadcasting time equal to 1 a firm i would trivially have a larger expected audience.

Now $\beta = 1$ implies $\lambda = \frac{2}{\gamma} - 1$ when $\gamma \neq 0$ and $\lambda = e$ when $\gamma = 0$. and therefore :

$$F(\alpha) = \frac{2}{\gamma} - (\frac{2}{\gamma} - 1)\alpha^{-\frac{\gamma}{2}} = 0$$

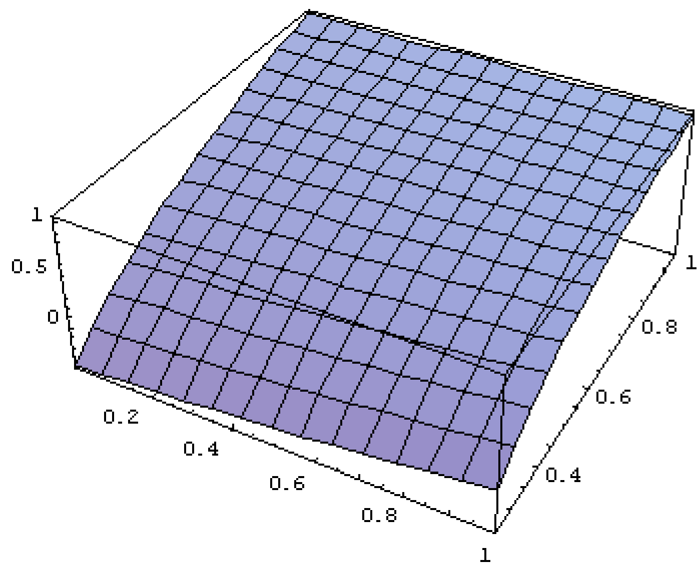
leading to $\alpha = \left(1 - \frac{\gamma}{2}\right)^{\frac{2}{\gamma}}$.

We note that $\alpha = \left(1 - \frac{\gamma}{2}\right)^{\frac{2}{\gamma}}$ is decreasing with respect to γ. *This means that the broadcasting of news is likely to shift later in the evening if the proportion of households holding and using a recorder decreases.* Note that α converges to $\frac{1}{e}$ when γ tends to 0.We plot below the probability that a station broadcasts its evening news before time t when $\gamma = 0$. The median time at which a station broadcasts its news is approximately 0.606531

Figure 1

We now show that the absolutely continuous equilibrium described in proposition 1 is unique as there are no equilibria with atoms and (or) disconnected supports.

Lemma 2 *A station i's equilibrium distribution function cannot have an atom at some $x < 1$*



Proof. Suppose that the contrary is true. Then we show that there exists $\delta > 0$ such that $\Pr\{T_j \in [x, x+\delta]\} = 0$. Indeed there always exists an arbitrarily small but strictly positive ε such that the expected pay-off of j when playing $x - \varepsilon$ is larger than when playing $x + \varepsilon$: there is indeed a first-order gain from discontinuously reducing the probability that station i is going to broadcast its news before station j which straightforwardly outweighs a second-order loss from reaching a slightly lower audience by broadcasting a bit earlier. Now station i would clearly gain by choosing a slightly later broadcasting time than x (the probability that $T_i > T_j$ would be unchanged while the audience would be increased) and so the equilibrium distribution function of i cannot have an atom at x. ■

Lemma 3 *The supports of the equilibrium distribution functions are connected.*

Proof. Suppose to the contrary that station i's equilibrium distribution function has a "hole" between x and y where $y > x$. It is straightforward to show that then, there exists some ε such that station j would be better off by broadcasting at y rather than at some $t \in [x - \varepsilon, y)$: the discontinuous increase in audience compensates (and outweights for $t > x - \varepsilon$) the increase in the probability that $T_j > T_i$. Thus j's equilibrium distribution must have a hole between $x - \varepsilon$ and y. Applying the same argument, we can now show that station i's equilibrium distribution function must have a hole beginning from some $x - \delta$, where $\delta > 0$, to y, hence a contradiction. ■

It remains to show that the connected equilibrium distribution functions cannot have an atom at 1.

Lemma 4 *The equilibrium distribution functions are atomless.*

Proof. From Lemmas 1 and 2, the equilibrium distribution functions are connected and cannot have atoms at some $t < 1$. Suppose that the distribution function of player i has an atom at 1, i.e. station i broadcasts its news at $T_i = 1$ with a operability $1 > p > 0$. From Lemma 2 the support of its distribution function must be an interval $[\alpha_i, 1]$ with $\int_{\alpha_i}^1 f(T_i)dT_i = 1 - p$. On the other hand the support of j's distribution function must be a semi-open interval $[\alpha_j, 1)$ (given that i's distribution has an atom at 1, station j will never broadcast at $T_j = 1$). . Since j has the same equilibrium expected pay-off from broadcasting at any time in $[\alpha_i, 1)$ it must be[3] that $f_i(T_i) = \frac{1}{T_T} \frac{1-\frac{\gamma}{2}}{\sqrt{T_i^\gamma}}$ so that $\int_{\alpha_i}^1 f_i(T_i)dT_i = 1 - p$ entails that $\alpha_i = \left(\frac{2-\gamma}{2-p\gamma}\right)^{\frac{2}{\gamma}}$. Applying the same reasoning to station j $\int_{\alpha_j}^1 f_j(T_j)dT_j = 1$ entails $\alpha_j = \left(1 - \frac{\gamma}{2}\right)^{\frac{2}{\gamma}} < \alpha_i$. However this is impossible in equilibrium since station j would earn a greater pay-off by broadcasting its new at $T_j = \alpha_i$ rather than at any $\alpha_j \leq T_j < \alpha_i$[4]. ■

Our last result describes the comparative statics of the equilibrium with respect to γ.

- **Proposition 2** For all $\gamma \in [0, 1]$, let $F_\gamma(x) = \frac{2}{\gamma} - \frac{\frac{2}{\gamma}-1}{\sqrt{x^\gamma}}$ over the interval $\left[\left(1 - \frac{\gamma}{2}\right)^{\frac{2}{\gamma}}, 1\right]$. Then if $\gamma < \gamma'$, F_γ dominates $F_{\gamma'}$ according to first degree stochastic dominance.

Proof Let $t \equiv \frac{2}{\gamma}$. Under this change of variable, the function F_γ, now denoted Φ, is defined as follows :

$$\Phi(t) = t - \frac{t-1}{x^{\frac{1}{t}}} \text{ with } t \in [2, +\infty[$$

We now prove that :

$$\Psi(t) \equiv \Phi'(t) \leq 0$$

Straightforward calculations lead to :

$$\Psi(t) = 1 - \frac{1 + \frac{(t-1)\ln x}{t^2}}{e^{\frac{\ln x}{t}}}$$

Therefore $\Psi(t) \leq 0$ iff $a(t) \leq b(t)$ where :

$$a(t) \equiv e^{\frac{\ln x}{t}} \text{ and } b(t) \equiv 1 + \frac{(t-1)\ln x}{t^2}$$

Note that :

$$a'(t) = -\frac{\ln x}{t^2} e^{\frac{\ln x}{t}} > 0 \text{ since } x \leq 1$$

[3] See above equation (2) and use $\lambda = \frac{2}{\gamma} - 1$.

[4] Of course a similar argument holds when $\gamma = 0$.

and

$$b'(t) = \frac{t-2}{t^3} \ln x < 0 \text{ since } t \geq 2$$

This implies that $\Psi(s) \leq 0$ for all $s \in [2, +\infty[$ iff $\lim_{t\to\infty} a(t) \leq \lim_{t\to\infty} b(t)$. Since
:

$$\lim_{t\to\infty} a(t) = \lim_{t\to\infty} b(t) = 1$$

the conclusion follows and the proof is completed.

4 Conclusion

We have shown that the "television news scheduling game" has a unique strategy equilibrium and we have characterized this equilibrium. The interest of this game goes beyond the simple understanding of evening news scheduling by TV-Channels. The model introduced by Cancian, Bergström and Bills is an extreme case of Hotelling location problems with directional constraints when moving towards the left or towards the right is completely impossible. A more general analysis of Hotelling location games with directional constraints where moving towards the right is more costly than moving towards the left, or the reverse, would certainly be interesting. with possible applications in political economy. This is on our research agenda.

References

[1] Cancian M., Bergström T. and A. Bills, 1995, Hotelling Location Problems with Directional Constraints: An Application to Television News Scheduling, Journal of Industrial Economics, 43, 121-124.