

Assets returns volatility and investment horizon: The French case

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Abstract: This paper explores French assets returns predictability within a VAR setup. Using quarterly data from 1970Q4 to 2006Q4, it turns out that bonds, equities and bills returns are actually predictable. This feature implies that the investment horizon does indeed matter in the asset allocation. The VAR parameters estimates are then used to compute real returns conditional volatility across investment horizons. The results reveal the same kind of horizon effect as the one found in recent empirical studies using quarterly U.S. data. More specifically, the annualized standard deviation of French stocks returns goes down from 22% for a 1-year horizon to only 2.8% for a 25-year investment horizon. They suggest that long-horizon investors overstate the share of bonds in their portfolio choice when neglecting the horizon effect on risk of asset returns predictability.

Keywords: Asset return predictability, Investment horizon, Vector Autoregression.

JEL classification: G11 .

Introduction

The optimal management of an assets portfolio is dynamic by nature. This basic point was first made by Mossin [1968], Merton [1969] and Samuelson [1969]. However, they concluded that, under the commonly accepted assumptions of that time, the optimal assets allocation is myopic. This means that the investment horizon is irrelevant, or that the optimal dynamic allocation is equivalent to the static one. More generally, Merton [1973] has shown that variation in expected returns over time may induce horizon effects. However, closed-form solutions to Merton's intertemporal model are difficult to find except in the case where the investor has log utility of consumption, i.e., with unit constant relative risk aversion, which is a case of limited interest since it implies that Merton's model reduces to a static one. The lack of closed-form solutions for constant (but not unit) relative risk aversion coefficient explains why so few empirical work was devoted to the horizon effects until very recently. When assets returns are unpredictable, myopia becomes optimal in the more general case of constant relative risk aversion. Increasing the investment period raises both the expected final payoff and its volatility in such a way that these two effects counterbalance each others perfectly under these conditions.

This question of optimal myopia is particularly crucial for financial intermediaries in charge of the management of lifecycle saving programs. Most pension programs now take the form of Defined Contribution plans, which means that most of the financial risk is borne by the individuals contributing to the system. It implies that these financial intermediaries should manage their financial reserves by taking into account the long-term objectives of their customers, which is to accumulate enough wealth for their retirement. The European Union is however considering a new regulation of the solvency of (life) insurance companies in which their capital requirement would be based on the assets and liabilities risks measured on a 12-month basis. In this paper, we raise the question of whether this so-called "Solvency II" myopic rule is efficient from the viewpoint of the policyholders. If we accept the assumption that relative risk aversion is constant,¹ the answer to this question relies on whether assets returns are predictable or not.

Future assets returns are predictable if they are statistically related to some easily observable variables that are referred to as predictors. Fama and French [1988], Poterba and Summers [1988], Campbell [1996], Campbell, Lo and MacKinlay [1997], Barberis [2000], Cochrane [2001] and many others estimated significant predictability of US stocks returns. In particular, there is mean-reversion in stocks returns, in the sense that shocks in expected stocks returns are negatively correlated with shocks to realized stocks returns. For example, Barberis [2000] used a simple VAR setup for stocks returns with the dividend-price ratio as predictor. He showed that the implied standard deviation of ten-year U.S. stocks returns is 23.7 percent, much smaller than the 45.2 percent value implied by the standard deviation of monthly returns. Using a more sophisticated VAR analysis of bills, bonds and stocks returns with three predic-

¹Gollier and Zeckhauser [2002] examine this question when relative risk aversion is not constant.

tors, Campbell and Viceira [2002] showed that mean-reversion of U.S. stocks returns cuts their standard deviation from 18% at a one-year horizon to 14% at a 25-year horizon.

Because mean-reversion implies that stocks are safer in the long run, the intuition suggests that a long horizon agent should have a larger demand for stocks early in his investment period. Campbell and Viceira [1999] and Barberis [2000] numerically estimated the sensitiveness of the demand for stocks to the investment horizon. The hedging demand for stocks is surprisingly large. For an agent with a relative risk aversion equaling 10 and a ten-year time horizon, the optimal investment in stocks is about 40% of current wealth without predictability. It goes up to 100% when mean-reversion is taken into account. This suggests that limiting the risk measurement to 12-month as in the Solvency II reform would inefficiently bias the insurers' assets portfolios towards bonds and bills. Given the large equity premium that has been observed over the past century both in the United States and in Europe (Dimson, Marsh and Staunton [2002]), this regulatory failure could have dramatic effects on the accumulated wealth and welfare of future retirees.

One important limitation to applying these ideas to make policy recommendations is that this literature is entirely based on U.S. data. It is a priori not clear whether these findings can be applied to other countries, as long as we don't have a general theory that would explain these serial correlations in assets returns. Our aim in this paper is to see whether the above-mentioned findings made on U.S. markets can be extended to France. We use quarterly data from 1970Q4 to 2006Q4 to perform a VAR analysis of real assets returns on French financial markets. We use the following list of predictors to determine the intensity of predictability: past real returns of stocks, bonds and bills, the nominal interest rate, the dividend-price ratio and the spread of interest rates. We closely follows the analysis that was used by Campbell and Viceira [2002], and we obtain very similar conclusions. Namely, stocks returns exhibit mean-reversion, whereas the returns of portfolios of bonds held to maturity exhibit some degree of mean-aversion. The intensity of mean-reversion of French stocks returns is stronger than the one reported by Campbell and Viceira on U.S. data. More specifically, the annualized standard deviation of French stocks returns goes down from 22% for a 1-year horizon to only 2.8% for a 25-year investment horizon.

The paper is organized as follows. Section 1 shows how conditional second-order moments of asset returns, i.e. their volatility, can be derived from a VAR setup. Section 2 describes the data used for the VAR model estimation presented in Section 3. In Section 4, French assets conditional volatilities are compared across investment horizons. The robustness of these results is then checked in Section 5 while Section 6 concludes.

1 Asset returns predictability and horizon effects from a VAR setup

A vector autoregressive (VAR) dynamics for U.S. asset returns is considered in e.g. Campbell [1991], Campbell [1996], Barberis [2000] or Campbell and Viceira [2002].² Beyond asset returns predictability considerations, Campbell and Viceira [2002] analysis emphasizes how well-suited the VAR framework is in order to evaluate investment horizon effects. Following their approach, our empirical study allows an arbitrary set of traded assets and state variables. More specifically, we consider a short-term interest rate together with excess stock returns and excess bond returns. Let R_{0t} denote the ex post real short rate and $r_{0t} = \log(R_{0t})$ the log (or continuously compounded) real return on this asset that is used as a benchmark to compute excess returns on other asset classes. Then, with r_{et} and r_{bt} the log real stock return and the log real bond return, let $x_{et} = r_{et} - r_{0t}$ and $x_{bt} = r_{bt} - r_{0t}$ denote the corresponding log excess returns.

We retain the same VAR(1) system as in Campbell and Viceira [2002] and Campbell et al. [2003]³:

$$\mathbf{z}_t = \Phi_0 + \Phi_1 \mathbf{z}_{t-1} + \mathbf{v}_t, \quad (1)$$

where

$$\mathbf{z}_t = \begin{bmatrix} r_{0t} \\ \mathbf{x}_t \\ \mathbf{s}_t \end{bmatrix} \quad (2)$$

is a $m \times 1$ vector with \mathbf{x}_t , the $n \times 1$ vector of log excess returns and \mathbf{s}_t the $m - n - 1 \times 1$ vector of variables which have been identified as returns predictors in existing empirical analysis, such as the nominal short rate, the dividend-price ratio and the yield spread between long-term and short-term bonds. In equation (1), Φ_0 is the $m \times 1$ vector of intercepts and Φ_1 is the $m \times m$ matrix of slope coefficients. It is assumed that the roots of the characteristic polynomial $\Phi(z) = I_m - \Phi_1 z$ lie strictly outside the unit circle in absolute value, a condition which rules out nonstationary or explosive behavior in \mathbf{z}_t . Finally, \mathbf{v}_t is the $m \times 1$ vector of innovations in asset returns and return forecasting variables, which is assumed to be *i.i.d.* normally distributed:

$$\mathbf{v}_t \sim \mathcal{N}(0, \Sigma_v), \quad (3)$$

where Σ_v is the $m \times m$ covariance matrix.

²Under the assumption that asset returns are well described by such a VAR model, Campbell, Chan and Viceira [2003] show how to obtain approximate solutions to the multiperiod portfolio choice model they propose. In this model, the investor is infinitely-lived with Epstein-Zin utility and there are no borrowing or short-sales constraints on asset allocation.

³As emphasized by these authors, the analysis below can be easily extended to more than one lag. However, the number of parameters in a VAR model increases exponentially with the number of lags, which may significantly reduce the estimates precision.

As stressed in Campbell and Viceira [2004], the conditional k -period variance-covariance matrix obtains straightforwardly from the VAR model estimates. First, cumulative k -period log returns are obtained by adding one-period log returns over k successive periods. Then, under the assumption that Σ_v is constant over time⁴, the conditional k -period variance is given by:

$$\begin{aligned} Var_t(\mathbf{z}_{t+1} + \dots + \mathbf{z}_{t+k}) &= \Sigma_v + (I + \Phi_1)\Sigma_v(I + \Phi_1)' \\ &\quad + (I + \Phi_1 + \Phi_1\Phi_1)\Sigma_v(I + \Phi_1 + \Phi_1\Phi_1)' + \dots \\ &\quad + (I + \Phi_1 + \dots + \Phi_1^{k-1})\Sigma_v(I + \Phi_1 + \dots + \Phi_1^{k-1})' \end{aligned} \quad (4)$$

In order to extract the conditional moments of real returns from the VAR, we use the following $(n+1) \times m$ selection matrix:

$$M_r = \begin{bmatrix} 1 & \mathbf{0}_{1 \times n} & \mathbf{0}_{1 \times (m-n-1)} \\ \mathbf{0}_{n \times 1} & \mathbf{I}_{n \times n} & \mathbf{0}_{n \times (m-n-1)} \end{bmatrix}$$

in (4). Then, dividing both sides by the horizon in order to annualize, we get:

$$\frac{1}{k} Var_t \begin{bmatrix} r_{0,t+1}^{(k)} \\ r_{e,t+1}^{(k)} \\ r_{b,t+1}^{(k)} \end{bmatrix} = \frac{1}{k} M_r Var_t(\mathbf{z}_{t+1} + \dots + \mathbf{z}_{t+k}) M_r'. \quad (5)$$

This approach will be applied to the French data described in the next section.

2 The French assets return data

The short term rate is the 3-month PIBOR rate, obtained from Datastream. The end-of-quarter values from this monthly series are retained to get quarterly observations, and r_{0t} denotes the real ex post short term rate, i.e. the difference between the log return on the 3-month PIBOR and the log inflation rate. The inflation series is calculated from the monthly Consumer Price Index series (source: INSEE) as $100 \times (cpi_t - cpi_{t-12})/cpi_{t-12}$. The log yield on this 3-month PIBOR is also used as the measure of the log short-term nominal interest rates, r_{0t}^{nom} .

French data for equities prices and returns come from Morgan Stanley Capital International (MSCI) database and are available since December 1969. More precisely, quarterly stock market data are based on the monthly MSCI National Price and Gross Return Indices in local currency. From these data, a quarterly stock total return series and a quarterly dividend series

⁴Time variation in Σ_v could be incorporated in the model, but since the available empirical evidence suggests that the persistence of changes in risk is quite low, this should not affect too much the conclusions regarding long-term asset allocation.

are obtained following the methodology described in Campbell [1999]⁵. Note that we depart from Campbell’s approach by not including the tax credits on dividends which are applicable to France. Indeed, MSCI calculates returns from the perspective of US investors, so it excludes from its indices these tax credits which are available only to local investors. Campbell chooses to add back the tax credits quite roughly, by applying the 1992 rate of 33.33% to all the sample. Nevertheless, this rate hasn’t remained fixed over the sample considered here (1970Q1—2006Q4). On top of this, the way dividends are taxed has also changed during that period. We couldn’t find exact tax rate data for our sample, and guess that on average, the French tax credits system has increased the nominal stock returns by around 40%. Since applying this coefficient to all the observations would be neutral as long as volatility is concerned, we choose to work with data excluding tax credits. The quarterly log real return on the stock index is denoted r_{et} and defined as the difference between the log return on equities and the log inflation rate. The log excess return on equities is then $x_{et} = r_{et} - r_{0t}$. The log dividend-price ratio, denoted $ldmp_t$ is the log dividend less the log price index. Since the quarterly dividend series is calculated from the monthly dividend payments over the past year, this series starts in 1970Q4 only.

Regarding the bond market, the long term government bond yield is used as a proxy variable. The monthly observations come from Datastream, and an end-of-quarter yield has been computed by selecting the end-of-quarter values. Then, the long bond return is constructed from this series using the loglinear approximation technique described in Chapter 14 of Campbell et al. [1997]:

$$r_{b,n,t+1}^{nom} \approx D_{nt}y_{nt} - (D_{nt} - 1)y_{n-1,t+1}$$

where n is the bond maturity, D_{nt} is the bond duration and Y_{nt} is the bond yield from which the log bond yield obtains as $y_{nt} = \log(1 + Y_{nt})$. The duration at time t is calculated as:

$$D_{nt} \approx \frac{1 - (1 + Y_{nt})^{-n}}{1 - (1 + Y_{nt})^{-1}}$$

where n is set to 10 years. Following Campbell and Viceira [2002], we approximate $y_{n-1,t+1}$ by $y_{n,t+1}$. The log real return on bonds, denoted r_{bt} , is the difference between r_{bt}^{nom} and the log inflation. The log excess return on bonds is then $x_{bt} = r_{bt} - r_{0t}$. The log real returns on PIBOR, bonds and equities are plotted in Figure 5, see Appendix.

Finally, the yield spread (spr_t) is the difference between the 10-year Treasury bond yield from Datastream and the 3-month PIBOR rate, again using end-of-quarter observations.

Table 1 reports sample means and standard deviations in annualized percentage points, except for the dividend-price ratio. To annualize the raw quarterly data, means are multiplied by 400 while standard deviations are multiplied by 200 since the latter increase with the square root of the time interval in serially uncorrelated data. The mean log returns are adjusted by

⁵See also Campbell’s “Data Appendix for *Asset Prices, Consumption and the Business Cycle*”, March 1998, downloadable from Campbell’s homepage.

adding one-half their variance so that they reflect mean gross returns. These statistics are computed for the sample 1970Q4-2006Q4.

Table 1: Sample statistics for real asset log returns

	mean	standard deviation
r_0	2.34	2.67
x_e	6.94	23.03
x_b	0.73	3.39
r_0^{nom}	7.39	3.44
$ldmp$	-4.95	0.56
spr	1.02	1.39

Remind that the stock return here does not include tax credits. When adding back, say, a 40% tax credit rate, the stock excess return would reach more than 10% per year. By contrast the excess return of the 10-year Treasury bond is only 0.7%. Volatility is much higher for stocks than for bonds (23.03% and 3.39% resp.).

ADF unit root tests are reported in Table 2. The deterministic component includes at most a constant under the stationary alternative. The lag order of the ADF regression was selected as the smallest one succeeding in eliminating residuals autocorrelation up to order 8. The unit

Table 2: ADF Unit Root Tests

	r_0	x_e	x_b	r_0^{nom}	$ldmp$	spr
ADF stat	-2.65 (1)	-11.21 (0)	-8.19 (0)	-2.04 (1)	-1.50 (0)	-4.57 (1)
p-value	0.085	0.000	0.000	0.268	0.530	0.000

Number of autoregressive lags into parenthesis.

root null is strongly rejected for x_e , x_b and spr , whereas it is rejected only at the 8.5% level for r_0 and clearly not rejected for the dividend-price ratio and for the nominal short term rate.

3 The VAR model estimates

In the sequel, we will consider the same model as Campbell and Viceira [2002], i.e. $\mathbf{z}_t = (r_{0,t}, x_{e,t}, x_{b,t}, r_{0,t}^{nom}, ldmp_t, spr_t)$ in equation (1). The lag order of is set to one, according to both Akaike, Schwartz and Hannan-Quinn information criteria. Moreover, the null of no residuals autocorrelation up to order 8 is not rejected at the 23% level according to Box-Pierce statistics.

Due to the stock market data, our sample is 1970Q4–2006Q4, i.e. 145 observations. The VAR is thus estimated from 1971Q1 to 2006Q4. The results are reported in Table 3. The first column reports the real 3-month rate equation. The lagged 3-month rate and the lagged spread coefficients are significantly different from zero and hence help predicting the real 3-month return. The second column refers to the real stock log return equation. This variable is known to be hardly predictable, which is here confirmed by the R^2 value of 12%. Nevertheless, the lagged bond log return and the lagged dividend-price ratio coefficients are significantly different from zero. The third column shows that both lagged 3-month return, bond return and yield spread help predicting the log bond return. The R^2 of this equation is 21%. The last three columns reveal that the return forecasting variables are highly persistent, especially the nominal short rate and the dividend-price ratio. In order to check for the VAR model stability, we have computed the roots of its characteristic polynomial: it turns out that the modulus of the largest root is lesser than one, with a value of 0.97. Hence the VAR apparently satisfies the stability condition.

Table 4 reports standard deviations (multiplied by 100) of estimated residuals on the diagonal and their correlations off-diagonal.

The magnitude of real returns residuals standard deviations obtained here are somehow similar to those obtained on quarterly data by Campbell et al. [2003] and Campbell and Viceira [2005].⁶ Indeed, these authors find 0.57, 8.06 and 2.69 for r_0 , x_e and x_b respectively, to compare with our values of 0.93, 11.41 and 3.09. The residuals cross-correlations are also quite similar to those found by these authors, except for the log real short-term rate equation: contrary to their results, we find that unexpected real short-term returns are negatively correlated with log bond excess returns and yield spread innovations, and positively correlated with nominal short-term rate innovations. Unexpected log excess stock returns are highly negatively correlated with shocks to the log dividend-price ratio, and mildly positively correlated with unexpected log excess bond returns and yield spread innovations. Finally, unexpected log excess bond returns are negatively correlated with shocks to the nominal short-term rate and to a lesser extent with shocks to real short-term returns. They are weakly positively correlated to yield spread innovations.

The signs of these innovations cross-correlations may explain some results. As shown by Stambaugh [1999], the small-sample bias in such regressions has the opposite sign to the sign of the correlation between innovations in returns and innovations in predictive variables. Hence, one may suspect that for the 3-month return equation, where the spread innovations are negatively correlated with returns innovations, there is a positive small-sample bias which may in turn explain some apparent predictability. On the contrary, the positive correlation between bond return and yield spread innovations suggests that the predictability of bond returns is not overstated in our sample. Regarding the log excess stock returns equation, the

⁶Campbell et al. [2003] use quarterly US data from 1952Q2 to 1999Q4 while Campbell and Viceira [2005] extend the sample until 2002Q4.

Table 3: VAR estimation results

	$r_{0,t}$	$x_{e,t}$	$x_{b,t}$	$r_{0,t}^{nom}$	$ldmp_t$	spr_t
$r_{0,t-1}$	0.923 (0.059) [15.54]	0.828 (0.728) [1.14]	0.394 (0.197) [2.00]	-0.0434 (0.059) [-0.74]	-0.027 (0.008) [-3.29]	-0.032 (0.053) [-0.59]
$x_{e,t-1}$	0.005 (0.007) [0.71]	-0.022 (0.093) [-0.24]	-0.027 (0.025) [-1.06]	0.004 (0.007) [0.55]	-0.001 (0.001) [-0.39]	-0.002 (0.007) [-0.28]
$x_{b,t-1}$	-0.037 (0.027) [-1.38]	0.667 (0.332) [2.01]	0.274 (0.090) [3.04]	-0.085 (0.027) [-3.17]	-0.006 (0.004) [-1.57]	0.055 (0.024) [2.26]
$r_{0,t-1}^{nom}$	0.083 (0.048) [1.75]	-1.037 (0.585) [-1.77]	-0.145 (0.158) [-0.91]	1.010 (0.047) [21.41]	0.013 (0.007) [1.95]	-0.006 (0.043) [-0.14]
$ldmp_{t-1}$	-0.629 (0.331) [-1.90]	8.805 (4.068) [2.16]	1.127 (1.101) [1.02]	-0.323 (0.328) [-0.98]	0.850 (0.046) [18.36]	0.188 (0.298) [0.63]
spr_{t-1}	0.287 (0.075) [3.81]	0.844 (0.926) [0.91]	0.829 (0.251) [3.31]	0.219 (0.075) [2.94]	-0.017 (0.010) [-1.59]	0.703 (0.068) [10.36]
c	-3.822 (1.856) [-2.06]	47.317 (22.769) [2.08]	5.378 (6.163) [0.87]	-1.753 (1.836) [-0.95]	-0.757 (0.259) [-2.92]	1.304 (1.669) [0.78]
R-squared	0.88	0.12	0.21	0.93	0.95	0.65

Standard errors in () and t-statistics in [].

Table 4: Standard deviations and correlations of residuals

	r_0	x_e	x_b	r_0^{nom}	$ldmp_t$	spr_t
r_0	0.930	0.285	-0.432	0.731	0.109	-0.707
x_e	—	11.409	0.288	-0.337	-0.799	0.234
x_b	—	—	3.088	-0.698	-0.245	0.238
r_0^{nom}	—	—	—	0.920	0.248	-0.857
$ldmp$	—	—	—	—	0.130	-0.147
spr	—	—	—	—	—	0.836

predictability due to the log dividend-price ratio may be overstated due to the quite strong negative correlation between stock returns and dividend-price ratio innovations.

4 Comparing French assets risk across investment horizons

The conditional k -period variance-covariance matrix of real returns is obtained from the VAR(1) estimates according to equation (4). The conditional standard deviation of the cumulative returns over investment horizon is divided by the square root of the horizon, so as to get annualized values. The annualized percent standard deviations of real returns for investment horizons up to 100 quarters, i.e. 25 years, are plotted in Figure 1.

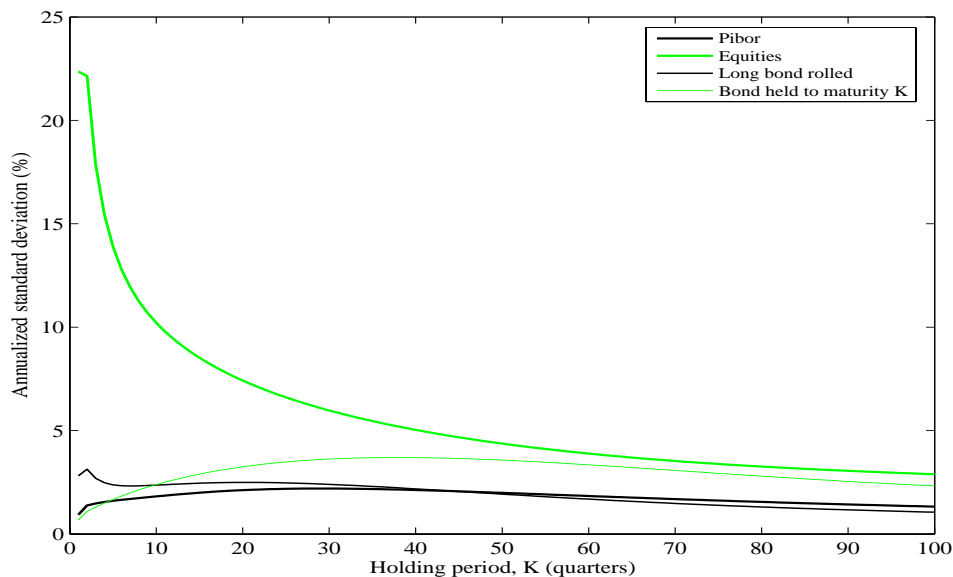


Figure 1: Annualized percent standard deviations of real returns

If returns were i.i.d., Siegel's measure of risk, i.e., the standard deviation of annualized returns, would be independent of the investment horizons. Hence, finding evidence that risk does not scale with horizon in this way would tend to confirm the predictability of returns. This is precisely the case here since the standard deviations plotted in Figure 1 depend on the investment horizon. Stock returns appear less volatile at longer horizons than at shorter ones. Hence, French stocks are *mean-reverting*. The same result is obtained for annual and quarterly US data by Campbell and Viceira [2002], Campbell et al. [2003] and Campbell and Viceira [2005]. However, French stock returns volatility decreases faster than its US analogue: here, it drops from 22% to 11% within eight quarters whereas the US stock returns volatility is divided by two after about 16 quarters. The initial steep decrease in French equities return volatility is then followed by a gradual decline, and the volatility is 2.8% at the 100-quarter horizon. By comparison, the US stock return volatility is still around 8% at this long horizon, according to Campbell and Viceira [2005] results.

This finding is a direct consequence of our VAR analysis. The mean-reversion of stocks return can be inferred from different channels. For example, we see in Table 3 that the autoregressive coefficient of the excess return of stocks is negative. But we also see that the dividend-price ratio is a good predictor of the future stocks return. Moreover, shocks to the excess return and to the dividend-price ratio are strongly negatively correlated, as seen in Table 4. It implies that a positive shock on the excess return of stocks yields a negative shock on the dividend-price ratio, which in turn yields an downwards revision of the expectation of future stocks returns.

Regarding bonds, two kinds of investment strategies are considered from the estimated model. The long bond rolled strategy is the one implicitly assumed in long-term bond returns time series: the maturity of the bond is held constant at, say, 10 years, buying a 10-year bond each period and selling it next period so as to buy a new 10-year bond. The second strategy consists in buying a bond and holding it until maturity. In this case, the standard deviation of the real return is given by the standard deviation of cumulative inflation from time t to time $t + K$, since this nominal bond held to maturity is riskless in nominal terms.

As in Campbell and Viceira [2005], we find that the long bond rolled strategy is also *mean-reverting*.⁷ Starting from about 3% at short horizons, the long bond rolled real return volatility is a little bit more than 1% at the 25-year horizon. Even though the long bond rolled return volatility is always less than the one for stocks, the gap reduces to 1.8% at the 25-year investment horizon. By contrast, the real returns on both PIBOR and the variable-maturity bond are mean-averting. The volatility of the later becomes greater than the one of the PIBOR after five quarters and than the one of the long bond rolled after ten quarters. Campbell and Viceira [2005] reach the same conclusion, but for longer investment horizons. For instance, the return on the variable-maturity bond becomes riskier than the long bond rolled from the 8-year horizon on, while it becomes riskier than the stock return for horizons greater than thirty-two

⁷Using annual data, Campbell and Viceira [2002] find a mean-averting behavior for this return.

years. According to our data, the variable-maturity bond risk never exceeds the stock risk, whatever the investment horizon considered. Finally, their study reveals that the return on short term T-bills becomes riskier than the return on long bond rolled after around forty-five years, whereas our findings from French data point to the same phenomenon for investment horizons longer than eleven years only.

Overall, the stock real return is always riskier than the other assets considered here. For horizons longer than two years and a half, the return volatility of variable-maturity bond exceeds the ones of both the constant-maturity bond and the 3-month PIBOR. From the twelve-year investment horizon on, the constant-maturity long bond becomes the less risky asset. Nevertheless, the 3-month PIBOR (resp. stock) return excess volatility at the 25-year horizon is less than 0.3% (resp. 1.84%) compared to the constant-maturity bond.

From the conditional variance-covariance matrix given in equation (5), it is straightforward to compute the correlations of the real returns at all horizons. Figure 2 shows that the correlations of stocks return and both fixed and variable-maturity long bonds return are hump-shaped at the short and medium horizons. These correlations are positive at all horizons. At the one-quarter horizon, the correlation between stock and variable-maturity bond (resp. fixed-maturity bond) returns is around 10% (resp. 21%). It then peaks at 60% (resp. 58%) at the 5-year (resp. 4-year) horizon. At the 25-year horizon, the stock—variable-maturity bond correlation is still 42%, whereas the stock—fixed-maturity bond correlation decreases to 16%. This striking hump-shaped patterns are also present in Campbell and Viceira [2005]’s study. The main discrepancy in our findings is that the correlation between stock and bond held to maturity returns remains positive at all horizons, even for the 50-year horizon, while Campbell and Viceira [2005] find that this correlation becomes negative after 45 years. Accordingly, they claim that stocks are able to hedge inflation risk in the very long term. This feature is not shared by French data. However, as stressed by these authors, the very long horizons predictions of the model must be cautiously interpreted because of the size of the sample.

5 Robustness analysis

The conclusions drawn above rely entirely on the VAR(1) model estimates reported in Tables 3 and 4. We propose to check their robustness along two dimensions. First, the stability of the coefficients estimates will be checked using the structural break analysis developed by Bai, Lumsdaine and Stock [1998]. Indeed, the Sup test developed by these authors allows to test the null of VAR parameters stability against the alternative of a structural change at unknown date. Second, since the unit root hypothesis cannot be rejected at reasonable levels for the log nominal short-term rate and the log dividend-price ratio according to the univariate Augmented Dickey-Fuller tests, these variables will be removed from the information set.

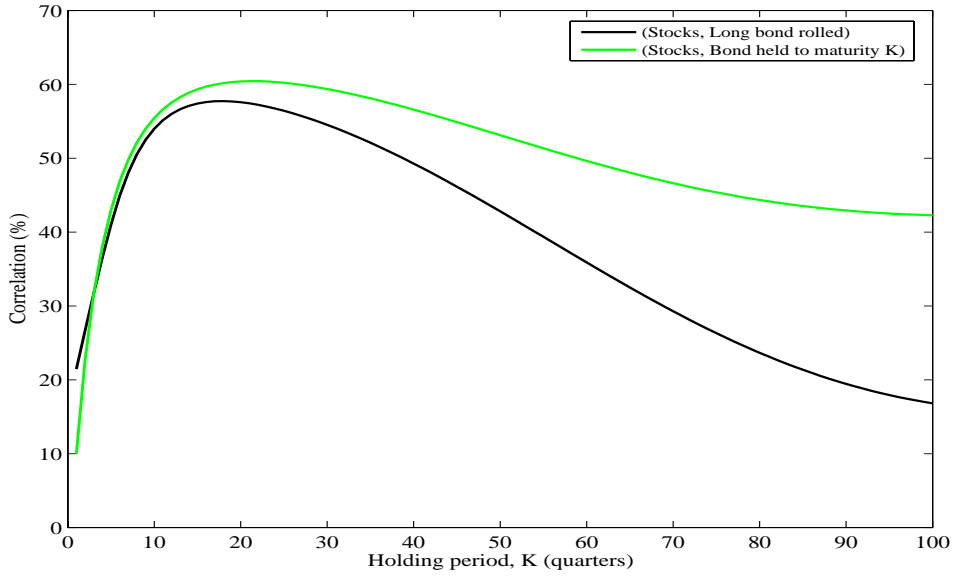


Figure 2: Correlations of real returns implied by VAR(1) estimates

5.1 Stability analysis

The liberalization of the French financial markets in the mid-eighties may have affected the returns dynamics over, say, the second half of our sample. If it were actually the case, then our conclusions would be biased since the VAR model given in equation (1) does not allow for a structural change. In order to allow for a general structural change at time $\tau = T\lambda$, with $\lambda \in (0, 1)$, the VAR model can be generalized as follows:

$$\mathbf{z}_t = \Phi_0 + \Phi_1 \mathbf{z}_{t-1} + D(t > \tau)(\Gamma_0 + \Gamma_1 \mathbf{z}_{t-1}) + \mathbf{u}_t, \quad (6)$$

where Γ_0 and Γ_1 are respectively $m \times 1$ and $m \times m$ matrices, \mathbf{u}_t is the $m \times 1$ vector of innovations assumed to be *i.i.d.* with $E(\mathbf{u}\mathbf{u}'_t) = \Sigma_u$. $D(t > \tau)$ is a dummy variable such that $D(t > \tau) = 0$ for $t \leq \tau$ and $D(t > \tau) = 1$ for $t > \tau$. Under the null hypothesis that no break occurs, model (6) reduces to the VAR given in equation (1):

$$\mathbf{z}_t = \Phi_0 + \Phi_1 \mathbf{z}_{t-1} + \mathbf{v}_t$$

with $E(\mathbf{v}_t \mathbf{v}'_t) = \Sigma_v$. Hence, the null hypothesis of no structural break at time τ corresponds to:

$$H_0 : \Gamma_0 = \mathbf{0}_{m \times 1} \text{ and } \Gamma_1 = \mathbf{0}_{m \times m}. \quad (7)$$

If the break point τ were known, then standard likelihood ratio (LR), Lagrange Multiplier (LM) or Wald (W) test statistics could be used to test H_0 . If τ is unknown, the difficulty is that there is no estimate of τ under the null hypothesis: τ is a nuisance parameter under the

null. The parameters μ_1 and B are also unidentified nuisance parameters under H_0 . With τ unknown, we will follow the common practice initiated by Davies [1987] which consists in using sup tests of the type:

$$SupLR(\tau) = \sup_{\tau \in [\tau_{inf}, \tau_{sup}]} LR_T(\tau) \quad (8)$$

where τ_{inf} corresponds to the initial fraction of the full sample T which is trimmed, in practice often set to $0.15T$ as suggested by Andrews [1993] and $\tau_{sup} = (1 - \tau_{inf})T$. In our case, since the sample length is 145 and the number of parameters is 42, the trimming parameter is chosen so as to leave 42 observations before the first break date τ_{inf} and after the last break date τ_{sup} . Moreover,

$$LR_T(\tau) = (T - n_c)(\log \det(\Sigma_v) - \log \det(\Sigma(\tau))) \quad (9)$$

where n_c denotes the number of constrained coefficients involved by assumption (7). Note that the LR statistics depends on τ through the estimate of the variance-covariance matrix of residuals under the alternative (6). Equivalently, one could define SupLM and SupW statistics. As can be shown from Andrews [1993] and Bai et al. [1998], the asymptotic null distribution of $SupLR(\lambda)$ is free of nuisance parameter. Hence, the critical values for the test statistics can be tabulated. However, as the empirical use of it will involve a finite number of observations, we will rather use a residual bootstrap method calculated by simulation. For given initial conditions, random draws are made from the residual vectors under the null. From these bootstrap residuals, one can create a simulated sample of series using model (1), and for each sample, calculate the corresponding $SupLR$ statistic. The bootstrap p -value then obtains as the percentage of simulated statistics which exceed the actual statistics.⁸

For a set of break dates ranging from 1981Q1 to 1996Q4, the SupLR statistic is 68.80, and its bootstrapped p -value — computed from 5000 replications — is 52.74%. Consequently, the null of parameters stability cannot be rejected for our VAR(1) model.

5.2 Change in the information set

As stressed earlier, the estimation results given in Tables 3 and 4 must be cautiously considered since the univariate unit root tests do not reject the null hypothesis for the log nominal short-term rate and for the log dividend-price ratio. Even though the largest root of the VAR(1) characteristic polynomial is lesser than one, it is still very close to unity with a value of 0.97. Johansen's cointegration rank test further confirms that this value may not be significantly different from unity.⁹

Therefore, we propose to check the robustness of our conclusions in two ways: *i*) by substituting the first differences to the levels of these variables, and *ii*) by excluding these two

⁸A detailed description of the method can be found in e.g. Hansen [1996] or Hansen and Seo [2002].

⁹The trace test concludes to four cointegration relations at most, at the 10% level, hence indicating two common trends.

variables from the analysis. These two alternative models lead to the same results regarding the conditional moments of the various assets across investment horizons. Indeed, the first differences of the nominal short rate and of the dividend-price ratio do not help predicting the real returns (see Table 7 in appendix). Hence, excluding them from the information set does not affect the conclusions. Consequently, we will only report the results obtained from the 4-dimensional vector $\mathbf{z}_t^* = (r_{0,t}, x_{e,t}, x_{b,t}, spr_t)$. The VAR estimates are reported in Table 5. Again, the information criteria point to a lag length of one and the Box-Pierce statistics

Table 5: VAR estimation results for \mathbf{z}_t^*

	$r_{0,t}$	$x_{e,t}$	$x_{b,t}$	spr_t
$r_{0,t-1}$	1.013 (0.035) [28.57]	-0.441 (0.437) [-1.01]	0.232 (0.117) [1.99]	-0.059 (0.032) [-1.86]
$x_{e,t-1}$	0.009 (0.007) [1.28]	-0.0791 (0.090) [-0.87]	-0.034 (0.024) [-1.41]	-0.003 (0.006) [-0.48]
$x_{b,t-1}$	-0.046 (0.026) [-1.77]	0.762 (0.324) [2.35]	0.289 (0.086) [3.34]	0.053 (0.023) [2.28]
spr_{t-1}	0.277 (0.074) [3.75]	0.897 (0.908) [0.99]	0.844 (0.243) [3.45]	0.693 (0.066) [10.55]
c	-0.279 (0.155) [-1.80]	-1.171 (1.909) [-0.61]	-0.928 (0.510) [-1.82]	0.404 (0.138) [2.93]
R-squared	0.88	0.09	0.20	0.65

Standard errors in () and t-statistics in [].

does not reject the null of no residuals autocorrelation up to order 8. This VAR model in \mathbf{z}_t^* is thus estimated using the same sample as the VAR in \mathbf{z}_t , namely 1971Q1– 2006Q4. The largest root of the characteristic polynomial is now 0.94 in absolute value and Johansen's trace test allows to conclude that the cointegration rank is four at the 7% level, hence confirming the stationarity of \mathbf{z}_t^* . The estimated equations of the four remaining variables are basically unchanged compared to the 6-variable system, except of course for the influence of the log

dividend-price ratio on the log stock real excess return which is now neglected.

Table 6 reports standard deviations (multiplied by 100) of estimated residuals on the diagonal and their correlations off-diagonal. They are very similar to the ones obtained from the 6-variable model. The only change concerns the sign of the correlation between stock and real short rate returns innovations which is now negative. This should not affect the result that much since the short-term real rate does not help predicting the excess stock return. Since

Table 6: Standard deviations and correlations of residuals

	r_0	x_e	x_b	spr_t
r_0	0.935	-0.305	-0.439	-0.703
x_e	—	11.520	0.298	0.240
x_b	—	—	3.078	0.241
spr	—	—	—	0.833

the nominal short rate has been excluded from this model, the inflation rate dynamics cannot be recovered from the system. Consequently, it is now impossible to compute the conditional moments of the variable-maturity bond strategy. The annualized standard deviations of the real returns on the three remaining assets are reported in Figure 3. The pattern of the stock real return volatility is very similar to one obtained in the previous section: the standard deviation reaches 22.5% at the one-quarter horizon, then sharply decreases and is divided by two after 10 quarters. This volatility then slowly decreases and is only 4% at the 25-year horizon. Compared to the 6-variable model, the patterns of long bond and 3-month PIBOR real returns volatilities remain largely unchanged up to the 6-year horizon. Afterwards, the log long bond return volatility stays approximately 0.5% above the log 3-month real rate and at the 25-year horizon, their respective values are 1.92% and 1.56%. So, in this 4-variable model, the three curves do not intersect each other: at all horizons, the equities return is riskier than the long bond return which in turn is riskier than the 3-month PIBOR rate. However, as can be seen from Figure 3, the volatility differentials strongly decrease with the investment horizon, as in the previous model. The most striking discrepancy between the two models appears in the conditional correlations between stocks and long bond real returns — see Figure 4. From the four-variable model, it is always increasing with the investment horizon. Starting with a correlation of around 22% at the one-quarter horizon, similarly to the six-variable model, it sharply increases to reach 50% at the 6-year horizon and then slowly raises until it stabilizes around 59% at the 25-year horizon.

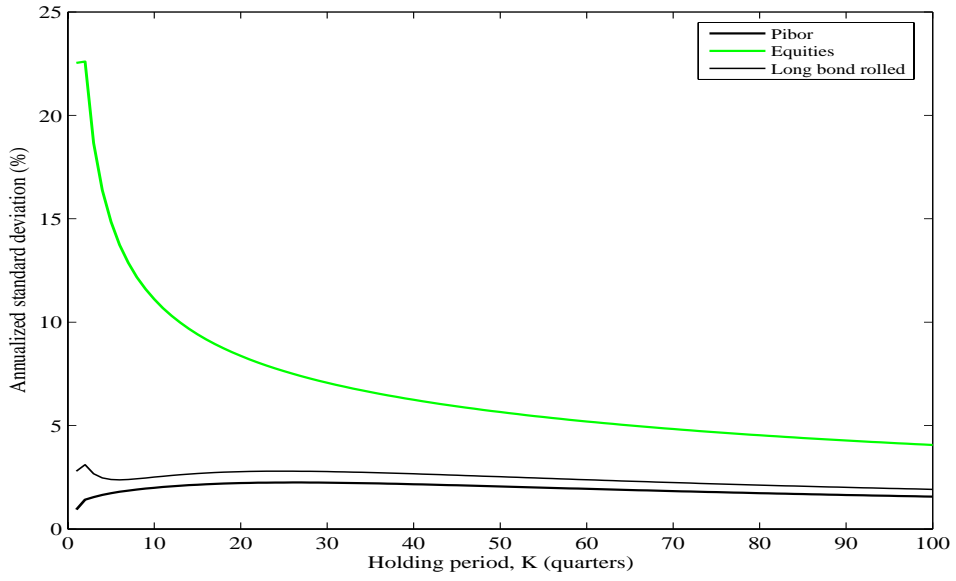


Figure 3: Annualized percent standard deviations of real returns (z_t^*)

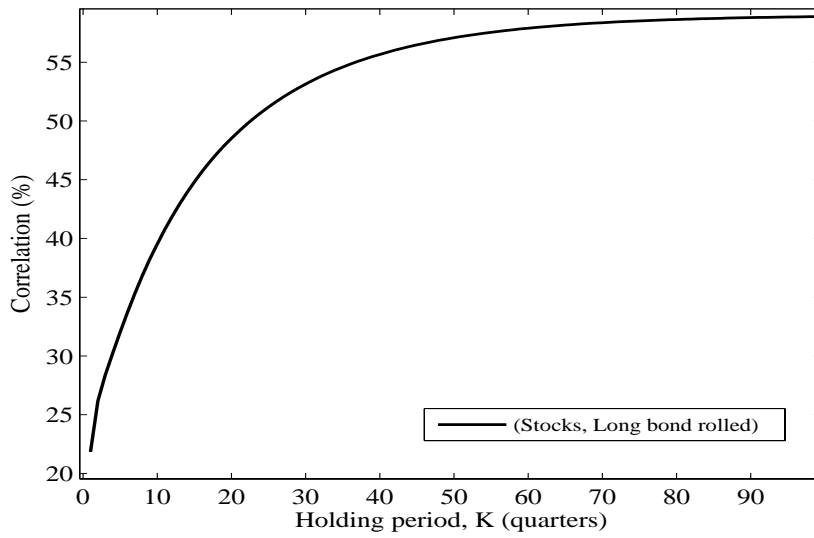


Figure 4: Correlations of real returns (z_t^*)

6 Concluding remarks

The aim of this paper was to assess French assets returns predictability within a VAR setup. Using quarterly data from 1970Q4 to 2006Q4, it turns out that bonds, equities and bills returns are actually predictable, at least to some extent. This feature has important consequences in terms of multiperiod portfolio choice. It implies that the investment horizon does indeed matter in the asset allocation. Following the approach developed by Campbell and Viceira [2002], the VAR parameters estimates are used to compute real returns conditional moments of order two. From a six-variable VAR model similar to the one estimated by these authors, we find the same kind of horizon effect for French data as they do using quarterly U.S. data.

In particular, French stock market return is mean-reverting: its volatility quickly decreases as the holding period increases, even though stocks remain the riskiest asset for all horizons. Furthermore, the relative magnitude of the four kinds of assets considered here changes with the investment horizon. At the one-quarter horizon, stocks are riskier than constant-maturity bonds, which themselves are riskier than 3-month bills, which in turn are riskier than variable-maturity bonds. For horizons longer than fourteen years, variable-maturity bonds volatility becomes very close to the one of stocks: the volatility gap between these two assets is around 0.5% only. For these long horizons, constant-maturity bonds and PIBOR volatilities are almost identical: 1.32% for the latter and 1.05% for the former, at the 25-year horizon.

Compared to Campbell and Viceira [2002] and Campbell and Viceira [2005] results, French stocks returns seem more strongly mean-reverting than their US analogues. As a result, French stocks volatility is a little bit lesser than 3% at the 25-year horizon whereas US stocks volatility is still around 8% at the same horizon. It is also worth noting that the 25-year horizon volatility of the four French assets considered here ranks from 1% to 3%. By contrast, their US analogues lie between 3% and 8%. Regarding the conditional correlation between assets across investment horizons, French data reveal the same kind of patterns as the ones found by Campbell and Viceira [2005], namely an increasing correlation at the short and medium horizons.

Finally, our main conclusions seem rather robust to changes in the information set. To sum up, they suggest that long-horizon investors may well overstate the share of bonds in their portfolio choice when neglecting the horizon effect on risk of asset returns predictability.

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Appendix

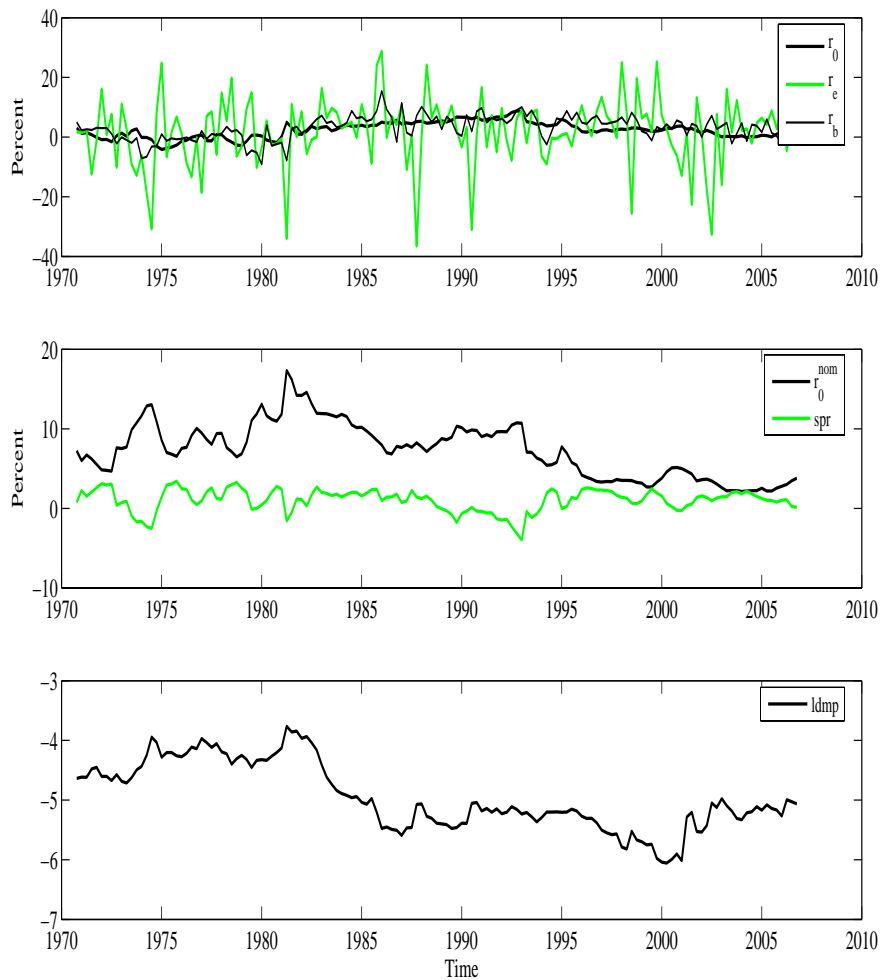


Figure 5: French data (1970Q4—2006Q4)

Table 7: VAR estimation results for z_t^Δ

	$r_{0,t}$	$x_{e,t}$	$x_{b,t}$	$\Delta r_{0,t}^{nom}$	$\Delta l d m p_t$	$s p r_t$
$r_{0,t-1}$	1.032 (0.036) [28.55]	-0.647 (0.457) [-1.42]	0.243 (0.122) [1.99]	0.000 (0.000) [0.35]	-0.006 (0.005) [-1.15]	-0.069 (0.032) [-2.14]
$x_{e,t-1}$	0.026 (0.012) [2.20]	-0.262 (0.147) [-1.78]	-0.027 (0.039) [-0.68]	0.000 (0.000) [1.31]	0.000 (0.002) [0.05]	-0.013 (0.010) [-1.21]
$x_{b,t-1}$	-0.015 (0.029) [-0.53]	0.727 (0.370) [1.96]	0.212 (0.099) [2.14]	-0.000 (0.000) [-1.42]	-0.006 (0.004) [-1.44]	0.021 (0.026) [0.80]
$\Delta r_{0,t-1}^{nom}$	18.377 (9.671) [1.90]	-4.204 (122.22) [-0.03]	-54.132 (32.60) [-1.66]	0.263 (0.09) [2.76]	0.181 (1.44) [0.12]	-20.312 (8.63) [-2.35]
$\Delta l d m p_{t-1}$	1.592 (0.974) [1.63]	-19.385 (12.306) [-1.57]	1.200 (3.283) [0.36]	0.007 (0.010) [0.75]	-0.053 (0.145) [-0.36]	-0.840 (0.868) [-0.97]
$s p r_{t-1}$	0.289 (0.072) [3.99]	0.840 (0.913) [0.92]	0.820 (0.244) [3.36]	0.002 (0.001) [3.53]	-0.014 (0.011) [-1.33]	0.682 (0.064) [10.58]
c	-0.323 (0.153) [-2.11]	-0.795 (1.936) [-0.41]	-0.878 (0.516) [-1.70]	-0.002 (0.001) [-1.67]	0.030 (0.023) [1.33]	0.439 (0.137) [3.21]
R-squared	0.89	0.10	0.22	0.17	0.05	0.67

Standard errors in () and t-statistics in [].