Innovation markets in the policy appraisal of climate change mitigation*

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Abstract

We propose a complete framework for the assessment of climate change mitigation policies in an endogenous growth model with two dedicated R&D sectors. First, we characterize the optimum analytically. Second, we derive the equilibrium paths in a decentralized economy. Since knowledge is not embodied into intermediate goods, its price is defined in an alternative way (as a part of its social value, which is equal to the sum of its marginal profitabilities in all sectors using it). Moreover, the two types of market failures arising in our setting, i.e. the pollution from fossil resource use and the research spillovers, are corrected by two economic policy instruments: a carbon tax and a research subsidy for each R&D sector. Third, we determine the optimal policies. Finally, we illustrate the theoretical model using some calibrated functional specifications. In particular, we investigate the effects of various combinations of public policies (including the optimal ones) by determining the deviation of each corresponding equilibrium from the "laisser-faire" benchmark.

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1 Introduction

Emerging energy technologies, such as clean coal or renewable energy, are crucial for a cost-effective climate change mitigation policy. The relevant appraisal of a climate policy should thus include the appropriate incentives for R&D investments in carbon-free energies that will drive the substantial technical improvements necessary to their large scale deployment[1] (see Energy Journal, 2006, Special issue on endogenous technical change and the economics of atmospheric stabilization). In this methodological paper, we propose a complete framework for the assessment of climate change mitigation policies in an endogenous growth model with two dedicated R&D sectors. Moreover, we put emphasis on the design of the innovation markets and their pricing, underlying clean energy investments.

The strand of literature on economic growth and climate change mostly contains optimization models (see for instance Bosetti et al., 2006; Edenhofer et al., 2005 and 2006; Gerlagh 2006; Gerlagh and Van Der Zwaan 2006; Popp, 2004, 2006a and 2006b). In those models, only the optimum is generally characterized, together with the system of prices that implements this optimum and, eventually, the optimal environmental policy instruments (e.g. a carbon tax defined as the co-state variable associated with the stock of cumulative atmospheric pollution, namely CO_2 and other greenhouse gases, or as the marginal cost of an abatement process, if available). Nonetheless, the models mentioned above generally do not study the equilibrium in a decentralized economy[2]. They also lack some insights as for the channels of financing the innovations in the energy sector and the discrepancy between the private investment decision and the socially desirable amounts (Köhler et al., 2006).

The study of the decentralized economy offers one major advantage: it allows for the entire characterization of the continuum of all existing equilibria and not only the optimal one. Indeed, a particular equilibrium is associated with each feasible vector of policy instruments. The approach followed in this paper gives some insights on how the economy reacts to policy changes: when the economy faces one or several market failures, e.g. pollution, market power or research spillovers, this characterization of market equilibria reveals crucial for measuring the impacts of economic tools such as environmental taxes,

[1]In 2004, wind and solar energy represented roughly 0.4% of world energy supply (IEA, 2006).

[2]For instance, as Edenhofer et al. (2006) put it about their MIND model: "Therefore, designing a general intertemporal equilibrium version of MIND for a comparison with the social planner solution would be the natural next step". An exception is Laurent-Lucchetti and Leach (2006) that derive numerically the decentralized outcome of their model.

pollution permits or research subsidies. Because of budgetary, socioeconomic or political constraints, the enforcement of first best optimum is usually difficult to achieve for the policy-maker that would rather implement second-best solutions.

The objective of this paper is to complete the literature mentioned above by setting up a global analysis, i.e. a general equilibrium analysis, that includes explicitly both the optimal outcome and the decentralized equilibrium. We develop an endogenous growth model in which energy services can be produced from a polluting non-renewable resource as well as a clean backstop. As in Popp (2006a), we introduce two R&D sectors, the first one improving the efficiency of energy production and the second one, the efficiency of the backstop. With this respect, we have to consider two types of market failures: the pollution from fossil resource consumption and the research spillovers of each R&D sector. That is why, in the decentralized equilibrium, we introduce two kinds of economic policy instruments in accordance: a tax on the fossil fuel use[3] and a research subsidy for the energy and backstop R&D sectors. There is an equilibrium associated to each vector of instruments, which allows to study the impact of one or several policy changes on the equilibrium trajectories. Clearly, when public instruments are optimally set, the equilibrium of the decentralized economy coincides with the first best optimum.

However, the main difficulty of this exercise lies in the introduction and the characterization of a specific market for knowledge, together with its associated prices. The standard endogenous growth theory advocates the introduction of an intermediate market for each kind of innovation (Aghion and Howitt, 1998; Romer, 1990; Grossman and Helpman, 1991). But embodying knowledge into intermediate goods becomes inextricable in more general computable endogenous growth models with pollution and/or natural resources such as the ones previously mentioned. In addition, those technical difficulties are emphasized when dealing with several research sectors, i.e. when there are several types of specific knowledge, each of them being dedicated to a particular input (resource, labor, capital, backstop...) as it is proposed in Acemoglu (2002). It follows that research activity is not expected to be funded by profits of monopolies on these goods. We suppose in fact that it is directly financed by the innovation users, eventually completed by public funds.

We thus propose a method, based on Grimaud and Rougé (2005)[4], that consists in three

[3]Fischer et al. (2003) and Nordhaus (2005) analyze the relative advantage of such a carbon tax as compared to a tradable emission permit system. In an earlier paper, Pizer (2002) argues that, when uncertainties about climate change mitigation costs are accounted for, price controls are much more efficient as long as damages are not too abrupt.

[4]See also Grimaud and Tournemaine (2007).

points. First, we define the optimal price of one unit of specific knowledge (associated with the energy or backstop R&D sectors) as the sum of the marginal profitabilities of this unit in each sector using this specific knowledge. Second, by referring to several empirical studies (see for instance Jones, 1995; Jones and Williams, 1998; Popp, 2004, 2006a), we assume that, in the decentralized economy, the equilibrium price of knowledge is in fact equal to a given proportion of this optimal value, usually on the order of a quarter to a third. Hence, the market value is lower than the social one because of observability, information and excludability problems. Third, we assume that the R&D sectors can be subsidized in order to reduce the gap between these social and private values, the maximum subsidy percentage ranging from two thirds to three quarters according to the assumed level of the equilibrium price.

Solving such an endogenous growth model both in a centralized and in a decentralized economy provides three main streams of results. First, the fact that the optimal policy instruments, which have been computed analytically, implement numerically the optimal trajectories confirms the consistency of the equilibrium concept used here, in particular regarding the characterization of the knowledge market and the computation of the innovation social values for each R&D sector. Second, comparing numerically the optimum and the equilibrium situation of "laisser-faire" allows us to measure the impact of the optimal instruments on the vector of prices and quantities of all economic sectors. In addition, since the analysis is undertaken at a decentralized level, we are able to dissociate those effects according to the various sectors: energy, backstop, fossil fuel, R&D. Third, our methodology renders possible the impact study of any economic policy on all variables, prices and quantities. In particular, we isolate the effects of the environmental policy from the ones of the research policies and vice versa. For instance, we show that an increase in the carbon tax has no effect on the R&D activities. It reduces the flow of fossil fuel extraction and stimulates the backstop penetration. It also implies a rent transfer from the resource-holders to the government. Moreover, jointed research policy in each R&D sector increases the knowledge accumulation in both sectors. Simultaneously, such a policy reduces the fossil fuel extraction and increases the backstop use. However, a research policy in a given R&D sector has no effect on the level of knowledge in the other R&D sector.

The article is organized as follows. Section 2 presents the theoretical model. In section 3, we determine the optimal solutions owing to five characterizing conditions. Section 4 studies the decentralized economy. We first analyze the behavior of each agent in the economy. Next, we characterize the equilibrium solutions owing to five conditions and we compute the equilibrium prices for any policy levels. We also show in a short methodological note how to solve the decentralized equilibrium as a single maximization problem, which is necessary to solve the model numerically. In section 5, we implement the first best optimum by comparing the two corresponding sets of characterizing conditions, which allows us to determine the optimal policies. In section 6, we derive a selection of numeric results focusing on i) the simultaneous effects of all the optimal policies (i.e. comparison between the optimum and the "laisser-faire" equilibrium), and ii) the differentiated effects of one policy, the other ones being given[5]. We conclude in section 7.

2 The model

We consider an economy in which, at each time t, a quantity Q_t of a homogeneous good is produced according to the following technology:

$$Q_t = Q(K_t, E_t, L_t, A_t), \tag{1}$$

where K_t is the amount of physical capital used within the production process, E_t is the flow of energy services, L_t, $L_t \equiv L_0 e^{\int_0^t g_{L,s} ds}$, denotes labor and A_t, $A_t \equiv A_0 e^{\int_0^t g_{A,s} ds}$, is an efficiency index that measures the total productivity of factors. Growth rates $g_{L,t}$ and $g_{A,t}$ are exogenously given. Since, as we will see later, climate change affects global income and not utility, Q_t is in fact the final output that we would get without any environmental damage. Function $Q(.)$ is assumed to be increasing and concave in each of his arguments and exhibits constant return to scale.

As in Popp (2006a), production of energy services requires some specific knowledge $H_{E,t}$, fossil fuels F_t and a backstop energy source B_t:

$$E_t = E(H_{E,t}, F_t, B_t). \tag{2}$$

Production function $E(.)$ is increasing and concave in each argument and the backstop and the fossil fuel are supposed to be imperfect substitutes. $H_{E,t}$ represents technological improvements into overall energy production process, in the form of energy efficiency improvements.

[5] The functional specifications used for numerical computations are provided in the appendix.

The fossil fuel end product is obtained from some carbon-based non-renewable resource and some specific investment[6]:

$$F_t = F(Q_{F,t}, Z_t), \tag{3}$$

where $Q_{F,t}$ is the amount of final product devoted to the production of fossil fuel and Z_t, $Z_t \equiv \int_0^t F_s ds$, is the cumulative extraction of the exhaustible resource from the initial date up to t. We assume that function $F(.)$ is increasing and concave in Q_F, decreasing and convex in Z, and that the fossil fuel extraction is constrained by a ceiling $\bar{Z}$: $Z_t \leq \bar{Z}$, $\forall t \geq 0$.[7]

The backstop resource is produced from specific investment and knowledge[8]:

$$B_t = B(Q_{B,t}, H_{B,t}), \tag{4}$$

where $B(.)$ is an increasing and concave function in $Q_{B,t}$, the amount of final product that is devoted to the backstop production sector, and in $H_{B,t}$, the stock of knowledge pertaining to the backstop.

In this model, there are two stocks of knowledge, H_E and H_B, each associated with a specific R&D sector (i.e. the energy and the backstop ones). We now specify the dynamics of these two stocks. In the energy (resp. the backstop) R&D sector, we consider that each innovation is a public, indivisible and infinitely durable good which is simultaneously used by the energy (resp. backstop) production sector and by the R&D sector in question. Formally, it is a point on the segment $[0, H_{E,t}]$ (resp. $[0, H_{B,t}]$). At each time t, the stock of knowledge in sector i, $i = \{B, E\}$, evolves as follows:

$$\dot{H}_{i,t} = H^i(R_{i,t}, H_{i,t}), \tag{5}$$

where $R_{i,t}$ is the R&D investment into sector i, i.e. the amount of final output that is devoted to R&D sector i, and $H^i(.)$ an innovation function assumed to be increasing and

[6] An appreciable difference with the DICE stream of models lies in the definition of such a production function which, in fact, replaces the cost (or price) function of the fossil fuel. In Nordhaus and Boyer (2000) or in Popp (2004, 2006a), such a full cost function is equal to the full extraction cost augmented by the scarcity rent that depends on Z_t. By making this transformation, this utility/technology canonical model allows for an endogenous determination of the resource market price when solving the equilibrium (see section 4 below). However, we will analytically specify function $F(.)$ in such a way that there exists a correspondence with the cost function mentioned below and such that the calibration of the DICE model still applies.

[7] Here, the capacity constraint of the exhaustible resource is not characterized by the limited availability of initial stocks, but by the decreasing relationship between the flow of produced fossil fuel and the amount of resource that has already been extracted. Then, resource scarcity is not physically but economically captured.

[8] The same remark as the one formulated for the fossil fuel production function applies, i.e. the backstop price as defined in the ENTICE-BR model is here replaced by a production function.

concave in each argument. Then, the stock of knowledge $H_{i,t}$ increases due to increases in R&D effort and in already accumulated knowledge, but there are diminishing returns to research over time.

Pollution is generated by fossil fuel use. Let α be the unitary carbon content of fossil fuel such that, without any abatement policy, the carbon flow released into the atmosphere would be equal to αF_t. Let G_0 be the stock of carbon in the atmosphere at the beginning of the planning period, G_t the stock at time t and ζ, $\zeta > 0$, the natural rate of decay, so that[9]:

$$\dot{G}_t = \alpha F_t - \zeta G_t. \tag{6}$$

As in the DICE model (see also Farzin and Tahvonen, 1996), the atmospheric carbon concentration does not directly enter the damage function. In fact, the increase in carbon concentration drives the global mean temperature away from a given state – here the 1990 level – and the difference between this state and the present global mean temperature should be taken as an index of climate change. Let T_t denote this difference, whose dynamics is governed by the following state equation:

$$\dot{T}_t = \Phi(G_t) - mT_t, \tag{7}$$

where $\Phi(.)$ is an increasing and concave function that links the atmospheric carbon concentration to the dynamics of temperature (i.e. the radiative forcing as characterized in Nordhaus and Boyer, 2000) and m, $m > 0$, is a constant parameter[10].

We denote by $D(T_t)$ the instantaneous unitary damage. This damage affects society through the global income Y_t. Then, the final output when taking into account climate change effects is:

$$Y_t = D(T_t) \times Q_t, \tag{8}$$

[9] In the analytical treatment of the model, we assume for the sake of clarity that the emission and natural decay rates are constant, despite what the DICE model recommends. However, in the numerical simulations, we adopt the carbon cycle characterization from DICE, that represents the carbon enhances between the oceans and the atmosphere. Based on Nordhaus and Boyer (2000), Goulder and Mathai (2000) estimate parameters α and ζ that take into account the inertia of the climatic system. They state that only 64% of current emissions actually contribute to the augmentation of atmospheric CO_2 and that the portion of current CO_2 concentration in excess is removed naturally at a rate of 0.8% per year.

[10] As for the dynamics of the atmospheric carbon stock, the state equation (7) replaces in fact a more complex and general set of dynamic equations which considers two measures of temperature – the atmospheric temperature and the lower oceanic one – and the interactions between both. Kriegler and Bruckner (2004) have recourse to such simplified dynamics by using a log function for Φ and by estimating the associated parameter m. However, for numerical simulations, we keep the DICE formulation that fully describes temperature variations.

where $D(.)$ is a strictly concave inverted U-shaped function: as the global mean temperature increases, the unitary environmental damage first grows until it reaches a peak and next, it diminishes.

The final output is devoted to either aggregated consumption C_t, fossil fuel production $Q_{F,t}$, backstop production $Q_{B,t}$, investment in physical capital I_t or in the two R&D sectors $R_{E,t}$ and $R_{B,t}$:

$$Y_t = C_t + Q_{F,t} + Q_{B,t} + I_t + R_{E,t} + R_{B,t}. \tag{9}$$

The dynamic equation of the physical capital stock is:

$$\dot{K}_t = I_t - \delta K_t, \tag{10}$$

where δ, $\delta > 0$, is the capital depreciation rate.

Finally, the social welfare function W_t is defined as:

$$W_t = \int_0^t U(C_s) e^{-\int_0^s \rho_\tau d\tau} ds = \int_0^t L_s u(c_s) e^{-\int_0^s \rho_\tau d\tau} ds, \tag{11}$$

where ρ_t, $\rho_t \equiv \rho_0 e^{-g_\rho t}$, is the instantaneous social rate of time preferences, g_ρ is the constant declining rate of ρ_t, $U(C_t)$ is the instantaneous utility function from aggregated consumption, $c_t \equiv C_t/L_t$ is the per capita consumption and $u(c_t)$ is the per capita instantaneous utility function. As usual, functions $U(.)$ and $u(.)$ are increasing, concave and satisfy Inada conditions. The model is summarized in Figure 1.

[Figure 1]

To conduct numerical simulations, we assign functional specifications to the utility and technological functions so as to obtain a calibrated model. Those functional forms are listed in Appendix A1.

3 Welfare analysis

The social planner problem consists in choosing $\{C_t, Q_{F,t}, Q_{B,t}, R_{E,t}, R_{B,t}\}_{t=0}^{\infty}$ that maximizes W_∞, as defined by (11), subject to constraints (1)-(10). After eliminating the co-state variables, the first order conditions reduce to the five characteristic conditions of Proposition 1 below, which hold at each time t.

Proposition 1 *At each time t, an optimum is characterized by the following five conditions:*

$$\begin{aligned}
&\left[D(T_t)Q_E E_F - \frac{1}{F_{Q_F}}\right] U'(C_t)e^{-\int_0^t \rho_s ds} + \int_t^\infty \frac{F_Z}{F_{Q_F}} U'(C_s)e^{-\int_0^s \rho_x dx} ds \\
&+\alpha \int_t^\infty \left[\int_s^\infty D'(T_x)Q_x U'(C_x)e^{-\left[m(x-s)+\int_0^x \rho_y dy\right]}dx\right] \Phi'(G_s)e^{-\zeta(s-t)}ds = 0 \quad (12)
\end{aligned}$$

$$D(T_t)Q_E E_B B_{Q_B} = 1 \quad (13)$$

$$D(T_t)Q_K - \delta = \Psi_t \quad (14)$$

$$H^B_{H_B} + \frac{H^B_{R_B} B_{H_B}}{B_{Q_B}} - \frac{\dot{H}^B_{R_B}}{H^B_{R_B}} = \Psi_t \quad (15)$$

$$H^E_{H_E} + \frac{H^E_{R_E} E_{H_E}}{E_B B_{Q_B}} - \frac{\dot{H}^E_{R_E}}{H^E_{R_E}} = \Psi_t \quad (16)$$

where J_X stands for the partial derivative of function $J(.)$ with respect to X and $\Psi_t \equiv \rho_t - \dot{U}'(C_t)/U'(C_t)$.

Proof. See Appendix A2.

Equation (12) reads as a particular version of the Hotelling rule in this model, which takes into account the carbon accumulation in the atmosphere, the dynamics of temperatures and their effects on output. We will see later (cf. equation (35) in Proposition 2) that this equation allows for the computation of the optimal tax on the fossil fuel. Equation (13) tells that the marginal productivity of specific input $Q_{B,t}$ equals its marginal cost. The three last equations are Keynes-Ramsey conditions. Equation (14) characterizes the optimal trade-off between physical capital K_t and consumption C_t, as in more standard growth models. Equation (15) (resp. (16)) characterizes the same kind of optimal trade-off between specific investment into backstop R&D sector, $R_{B,t}$ (resp. energy R&D sector, $R_{E,t}$) and consumption.

4 Decentralized equilibrium

4.1 Behavior of agents

In the decentralized economy, we assume that all sectors are perfectly competitive. The price of output Y_t is normalized to one and $p_{F,t}$, $p_{B,t}$, $p_{E,t}$, w_t and r_t are, respectively, the prices at date t of fossil fuel, backstop, energy, labor (real wage) and the interest rate on financial market.

Recall that production of fossil fuels generates some carbon emission flow. The accumulation of carbon in the atmosphere drives the global mean temperature to increase and, in that way, it induces an environmental damage. This environmental externality should create a market failure without any corrective policy since the fossil fuel user, i.e. the energy producer, does not take into account its negative impact on social welfare. That is why we introduce an environmental policy defined as a tax τ_t on the fossil fuel consumption.

We have seen above that both R&D sectors produce innovations which are public, indivisible and infinitely durable pieces of knowledge. A basic feature of the present model is that these innovations are not embodied into private intermediate goods, as it is done for instance in the standard models of endogenous growth. Thus, we cannot assume that the research activity is funded by profits of monopolies on these goods. Here, we suppose that research is directly financed. First, in each research sector, we determine the *social* value of an innovation. Since an innovation is a public good, this social value is the sum of marginal profitabilities of this innovation in all sectors which use it. If this value is paid to the inventor, the first best optimum is implemented[11]. But we know that, in the real world, only a part of this sum is generally extracted (for instance, Jones and Williams, 1998, estimate that actual investment in research are at least four times below what would be socially optimal; on this point, see also Popp, 2006a). Thus, in a second step, we define the *market* value as a percentage of the social value. Basically, the market value is lower than the social one because the innovator faces observability, information and excludability problems. However, we can assume that the research sectors are subsidized in order to reduce the gap between the social and the private values of innovations.

To sum up, there are two types of policy tools in the model: an environmental tax on the resource and two subsidies for the backstop and energy research sectors.

4.1.1 The fossil resource sector

The program of the fossil fuel producer writes:

$$\max_{\{Q_F, t \geq 0\}} \int_0^\infty \left(p^s_{F,t} F_t - Q_{F,t}\right) e^{-\int_0^t r_s ds} dt \quad \text{s.t. (3) and } Z_t = \int_0^t F_s ds,$$

[11] This result will be proved by Proposition 3 below. In fact, what we call social value is the sum of the Lindahl prices associated with the innovations.

where $p^s_{F,t}$ denotes the selling price of the fossil resource, i.e. the price which is received by the resource-holder. Static and dynamic first order conditions are:

$$
\begin{aligned}
(p^s_{F,t}F_{Q_F}-1)e^{-\int_0^t r_s ds}+\eta_t F_{Q_F} &= 0 \qquad (17)\\
p^s_{F,t}F_Z e^{-\int_0^t r_s ds}+\eta_t F_Z &= -\dot{\eta}_t, \qquad (18)
\end{aligned}
$$

together with the transversality condition $\lim_{t\to\infty}\eta_t Z_t=0$. Replacing p_F into (18) by its expression coming from (17), it comes:

$$
\dot{\eta}_t=-\frac{F_Z}{F_{Q_F}}e^{-\int_0^t r_s ds}. \qquad (19)
$$

By integrating (19) and using (17) again, we find:

$$
\eta_t=\int_t^\infty \frac{F_Z}{F_{Q_F}}e^{-\int_0^s r_u du}ds \quad \text{and} \quad p^s_{F,t}=\frac{1}{F_{Q_F}}-\int_t^\infty \frac{F_Z}{F_{Q_F}}e^{-\int_t^s r du}ds. \qquad (20)
$$

4.1.2 The backstop sector

At each time t, the backstop producer maximizes its profit $\Pi^B_t=[p_{B,t}B_t-Q_{B,t}]$, subject to technological constraint (4). The first order condition determines, for each time t, the inverse demand function for specific investment $Q_{B,t}$:

$$
p_{B,t}=\frac{1}{B_{Q_B}}. \qquad (21)
$$

4.1.3 The energy sector

At each time t, the energy producer maximizes $\Pi^E_t=\left[p_{E,t}E_t-p^m_{F,t}F_t-p_{B,t}B_t\right]$ subject to (2), where $p^m_{F,t}$ is the fossil fuel market price, i.e. the price which is paid by the firm and which includes the environmental tax τ_t. This tariff is assumed to be additive: $p^m_{F,t}=p^s_{F,t}+\tau_t$. However, our results can easily be extended to the case of an ad-valorem tax τ^a_t: $p^m_{F,t}=p^s_{F,t}(1+\tau^a_t)$. The first order conditions write:

$$
\begin{aligned}
\frac{\partial\Pi^E_t}{\partial F_t} &= 0 \quad \Rightarrow p_{E,t}=\frac{p^m_{F,t}}{E_F}=\frac{p^s_{F,t}+\tau_t}{E_F} \qquad (22)\\
\frac{\partial\Pi^E_t}{\partial B_t} &= 0 \quad \Rightarrow p_{E,t}=\frac{p_{B,t}}{E_B}. \qquad (23)
\end{aligned}
$$

Those conditions determine respectively the inverse demand functions for fossil fuel and backstop.

4.1.4 The R&D sectors

The behaviors of the backstop and energy R&D sectors are analogous so that we will study a single problem indexing by i, $i = \{B, E\}$, the sector in question. As we have mentioned above, knowledge is not embodied into intermediate goods, which implies that it can not be financed by the sale of these goods. As in Grimaud and Rougé (2005), we suppose that it is in fact directly financed by public funds, the government paying to the innovator an amount which is equal to a part of the willingness to pay of both sectors using this type of knowledge, i.e. the R&D sector i and the energy or the backstop sectors. In other words, the government subsidizes the sectors which buy knowledge[12].

Let us consider for instance the energy R&D sector. Each innovation produced by this sector is used by the sector itself as well as by the energy sector. Thus, at each date t, the instantaneous social value of this innovation is $\bar{v}_{H_E,t} = v^E_{H_E,t} + v^{H_E}_{H_E,t}$, where $v^E_{H_E,t}$ and $v^{H_E}_{H_E,t}$ are the marginal profitabilities of this innovation in the energy production sector and in the energy R&D sector, respectively. The social value of this innovation at t is $\bar{V}_{H_E,t} = \int_t^\infty \bar{v}_{H_E,s} e^{-\int_t^s r_x dx} ds$. Assume that only a part $\gamma_{E,t}$, with $0 < \gamma_{E,t} \leq 1$, $\forall t \geq 0$, is paid to the innovator. Then, the instantaneous market value is:

$$v_{H_E,t} = \gamma_{E,t} \bar{v}_{E,t} = \gamma_{E,t} \left(v^E_{H_E,t} + v^{H_E}_{H_E,t} \right), \tag{24}$$

and the market value at date t is:

$$V_{H_E,t} = \int_t^\infty v_{H_E,s} e^{-\int_t^s r_x dx} ds. \tag{25}$$

Similarly, the instantaneous social value of an innovation in the backstop R&D sector is $\bar{v}_{H_B,t} = v^B_{H_B,t} + v^{H_B}_{H_B,t}$, where $v^B_{H_B,t}$ and $v^{H_B}_{H_B,t}$ are the marginal profitabilities of an innovation in the backstop sector and in the backstop R&D sector, respectively. Then, $\bar{V}_{H_B,t} = \int_t^\infty \bar{v}_{H_B,s} e^{-\int_t^s r_x dx} ds$ is the social value of an innovation at date t, and $V_{H_B,t} = \int_t^\infty v_{H_B,s} e^{-\int_t^s r_x dx} ds$ is the market value, in which $v_{H_B,t} = \gamma_{B,t} \bar{v}_{H_B,t}$, with $0 < \gamma_{B,t} \leq 1$, $\forall t \geq 0$. Note that differentiating (25) (and the corresponding equation for $V_{H_B,t}$) with respect to time leads to the usual arbitrage relation:

$$r_t = \frac{\dot{V}_{H_i,t}}{V_{H_i,t}} + \frac{v_{H_i,t}}{V_{H_i,t}}, \quad \forall i = \{B, E\}, \tag{26}$$

which reads as the equality between the rate of return on the financial market (left hand side) and the rate of return on the R&D sector i (right hand side).

[12] This assumption is in fact a simplification of a more general framework in which firms using knowledge as input sell their goods on imperfect competitive (e.g. Cournot) markets that allow them to get strictly positive profits to buy knowledge (see Grimaud and Tournemaine, 2007).

At each time t, the R&D sector i, $i = \{B, E\}$, supplies the flow of innovations $\dot{H}_{i,t}$ at price $V_{H_i,t}$ and demands some specific investment $R_{i,t}$ at price 1, so that the profit function to be maximized is $\Pi_t^{H_i} = \left[V_{H_i,t} H^i(R_{i,t}, H_{i,t}) - R_{i,t}\right]$. The first order condition implies:

$$\frac{\partial \Pi_t^{H_i}}{\partial R_{i,t}} = 0 \quad \Rightarrow V_{H_i,t} = \frac{1}{H_{R_i}^i}. \tag{27}$$

The marginal profitability for specific knowledge of R&D sector i is:

$$v_{H_i,t}^{H_i} = \frac{\partial \Pi_t^{H_i}}{\partial H_{i,t}} = V_{H_i,t} H_{H_i}^i = \frac{H_{H_i}^i}{H_{R_i}^i}, \quad \forall i = \{B, E\}. \tag{28}$$

In order to determine the value of an innovation in both research sectors, we need to know the marginal profitabilities of innovations in the energy and backstop production sectors. From (21) and (23), those values are given by:

$$v_{H_E,t}^{E} = \frac{\partial \Pi_t^E}{\partial H_{E,t}} = \frac{E_{H_E}}{E_B B_{Q_B}}, \tag{29}$$

$$v_{H_B,t}^{B} = \frac{\partial \Pi_t^B}{\partial H_{B,t}} = \frac{B_{H_B}}{B_{Q_B}}. \tag{30}$$

4.1.5 The final good sector

At each time t, the firm chooses $\{K_t, E_t, L_t\}_{t=0}^{\infty}$ that maximizes its profit function $\Pi_t^Q = [D(T_t)Q_t - P_{E,t}E_t - w_t L_t - (r_t + \delta)K_t]$, subject to (1). The first order conditions are:

$$\frac{\partial \Pi_t^Q}{\partial K_t} = 0 \Rightarrow r_t = D(T_t)Q_K - \delta \tag{31}$$

$$\frac{\partial \Pi_t^Q}{\partial E_t} = 0 \Rightarrow p_{E,t} = D(T_t)Q_E \tag{32}$$

$$\frac{\partial \Pi_t^Q}{\partial L_t} = 0 \Rightarrow w_t = D(T_t)Q_L. \tag{33}$$

4.1.6 The household

The representative household maximizes W_∞ subject to the following dynamic budget constraint: $\dot{M}_t = rM_t + w_t L_t + \Pi_t - C_t - T_t^a$, where M_t is the stock of bonds at time t, Π_t is the total profits gained in the economy (including the resource rent) and T_t^a is a lump-sum tax (subsidy free) that allows to balance the budget constraint of the government. This maximization leads to the following condition:

$$\rho_t - \frac{\dot{U}'(C)}{U'(C)} = r_t \Rightarrow U'(C_t) = U'(C_0) e^{\int_0^t (\rho_s - r_s) ds}. \tag{34}$$

4.1.7 The government

Assuming that the government's budget constraint holds at each time t, then it writes:

$$T_t^a + \tau_t F_t = V_{H_B,t}\dot{H}_{B,t} + V_{H_E,t}\dot{H}_{E,t},$$

where $V_{H_i,t}$ depends on the subsidy level γ_i, $i = \{B, E\}$ (see (24) and (25) above).

4.2 Characterization of the decentralized equilibrium

From the previous analysis of individual behaviors, we can now characterize an equilibrium in the decentralized economy, which is done by the following Proposition:

Proposition 2 *For a given triplet of policies $\{\gamma_{B,t}, \gamma_{E,t}, \tau_t\}_{t=0}^{\infty}$, the equilibrium conditions can be summed up as follows:*

$$\left[D(T_t)Q_E E_F - \tau_t - \frac{1}{F_{Q_F}}\right] e^{-\int_0^t r_s ds} + \int_t^{\infty} \frac{F_Z}{F_{Q_F}} e^{-\int_0^s r_x dx} ds = 0 \quad (35)$$

$$D(T_t)Q_E E_B B_{Q_B} = 1 \quad (36)$$

$$D(T_t)Q_K - \delta = \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} \quad (37)$$

$$-\frac{\dot{H}_{R_B}^B}{H_{R_B}^B} + \gamma_{B,t}\left[\frac{B_{H_B} H_{R_B}^B}{B_{Q_B}} + H_{H_B}^B\right] = \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} \quad (38)$$

$$-\frac{\dot{H}_{R_E}^E}{H_{R_E}^E} + \gamma_{E,t}\left[\frac{E_{H_E} H_{R_E}^E}{E_B B_{Q_B}} + H_{H_E}^E\right] = \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} \quad (39)$$

and the equilibrium corresponding prices are:

$$r_t^* = D(T_t)Q_K - \delta \quad (40)$$

$$w_t^* = D(T_t)Q_L \quad (41)$$

$$p_{F,t}^{s*} = \frac{1}{F_{Q_F}} - \int_t^{\infty} \frac{F_Z}{F_{Q_F}} e^{-\int_t^s r_x dx} ds \quad (42)$$

$$p_{B,t}^* = \frac{1}{B_{Q_B}} \quad (43)$$

$$p_{E,t}^* = \frac{p_{F,t}^{s*} + \tau_t}{E_F} = \frac{p_{B,t}^*}{E_B} = D(T_t)Q_E \quad (44)$$

$$v_{H_B,t}^* = \gamma_{B,t}\left[\frac{B_{H_B}}{B_{Q_B}} + \frac{H_{H_B}^B}{H_{R_B}^B}\right] \text{ and } V_{H_B,t}^* = \frac{1}{H_{R_B}^B} \quad (45)$$

$$v_{H_E,t}^* = \gamma_{E,t}\left[\frac{E_{H_E}}{E_B B_{Q_B}} + \frac{H_{H_E}^E}{H_{R_E}^E}\right] \text{ and } V_{H_E,t}^* = \frac{1}{H_{R_E}^E}. \quad (46)$$

Proof. See Appendix A3.

Equations (35)-(39) are related to the quantities $Q_{F,t}$, $Q_{B,t}$, I_t, $R_{B,t}$ and $R_{E,t}$, respectively. They have to be compared one by one to equations (12)-(16) of Proposition 1 which characterize the optimum. In particular, by analyzing condition (35) and the optimal corresponding one (12), we will be able to compute the tax that implements the first best optimum (see next section). Equation (40) gives the interest rate and equations (41)-(46), the equilibrium prices of L_t, F_t, E_t, $H_{B,t}$ and $H_{E,t}$, respectively. A particular equilibrium is associated to a given triplet of policies $\{\gamma_{B,t}, \gamma_{E,t}, \tau_t\}_{t=0}^{\infty}$ and the set of equations given by Proposition 2 allows to compute quantities and prices for this equilibrium. If the triplet of policy tools is optimal, this set of equations gives the same quantities as the ones obtained from Proposition 1; it also gives the first best prices.

Finally, in order to solve numerically the market outcome, we need to transform the decentralized problem described above into a single maximization program. Proposition 3 explains how to proceed.

Proposition 3 *Solving the following program:*

$$\max_{\{C_t, Q_{F,t}, Q_{B,t}, R_{E,t}, R_{B,t}, t\geq 0\}} \int_0^{\infty} U(C_t) e^{-\int_0^t \rho_s ds} dt \qquad \textit{subject to:}$$

$$\begin{aligned}
\dot{K}_t &= D(T_t)Q(.) - C_t - \delta K_t - R_{E,t} - R_{B,t} - Q_{F,t} - Q_{B,t} - \tau_t F(Q_{F,t}, Z_t) \\
\dot{H}_{i,t} &= \gamma_{i,t} H^i(R_{i,t}, H_{i,t}), \quad \forall i = \{B, E\} \\
Z_t &= \int_0^t F(Q_{F,s}, Z_s) ds.
\end{aligned}$$

leads to the same characterizing conditions (35)-(39) as the decentralized equilibrium.

Proof. See Appendix A4.

5 Implementation of the optimum and determination of the optimal policies

Recall that for a given set of public policies, a particular equilibrium is characterized by conditions (35)-(39) of Proposition 2. This equilibrium will be said to be optimal if it satisfies the optimum characterizing conditions (12)-(16) of Proposition 1. By analogy between these two sets of conditions, we can show that there exists a single triplet $\{\gamma_{B,t}, \gamma_{E,t}, \tau_t\}_{t=0}^{\infty}$ that implements the optimum.

Since conditions (13) and (14) have the same expressions as (36) and (37) respectively, we only have to compare the three remaining conditions of each proposition. First, by identifying (12) to (35) and using (34), the level of the additive environmental tax that implements the optimum – referred to as the optimal tax from now on – is defined by:

$$\tau_t^o = -\frac{\alpha}{U'(C_t)} \left\{ \int_t^{\infty} \Phi'(G_t) e^{-\zeta(s-t)} \left[\int_s^{\infty} D'(T_t) Q_x U'(C_x) e^{-m(x-t)-\int_t^x \rho_y dy} dx \right] ds \right\}. \quad (47)$$

The interpretation of (47) is quite standard. This expression reads as the ratio between the marginal social cost of climate change – the marginal damage in terms of utility coming from the consumption of an additional unit of final good – and the marginal utility obtained by consuming this unit, i.e. the marginal rate of substitution between pollution and consumption. Equivalently, that corresponds to the social cost of one unit of carbon in terms of final good.

Next, the correspondence between the equilibrium characterizing condition (38) (resp. (39)) and the optimum characterizing condition (15) (resp. (16)) is achieved if and only if $\gamma_{B,t}$ (resp. $\gamma_{E,t}$) is equal to one, i.e. if both sectors are fully subsidized. These results are summarized in Proposition 4 below.

Proposition 4 *The equilibrium defined in Proposition 2 is optimal if and only if the triplet of policies* $\{\gamma_{B,t}, \gamma_{E,t}, \tau_t\}_{t=0}^{\infty}$ *is such that* $\gamma_{B,t} = \gamma_{E,t} = 1$ *and* $\tau_t = \tau_t^o$*, for all* $t \geq 0$.

Using the specified model as introduced in Appendix A1, the optimal carbon tax can be illustrated by Figure 2.

[Figure 2]

This tax τ_t^o starts from some low 5 US$ per ton of carbon and follows an inverted U-shape trajectory, reaching around 90 US$ by 2100, 200$ in 250 years, before plummeting. As we will see later, this carbon policy increases the delivered price of the resource, i.e. the market price including the carbon tax (p_t^m). We will see also that this more expensive fossil energy provides strong incentives for developing alternative energy supply. The next section focuses on the analysis of those incentives.

6 Impacts of economic policies

In this empirical section, we use the analytical model developed so far to conduct numerical simulations. We appraise the impacts of environmental and research policies on all variables – prices and quantities – in the decentralized equilibrium and we emphasize their transmission channels. We proceed as follows. In sub-section 6.1, we compare the so-called "laisser-faire" case, that consists in determining the outcome in the decentralized economy without neither climate nor public policy, as well as the optimal outcome of the model. In other words, starting from the equilibrium without any public policy, we analyze the effect of the simultaneous introduction of optimal environmental and research policies. In sub-section 6.2, we analyze the impacts of carbon tax variations on equilibrium trajectories, given an optimal research policy ($\gamma_E = \gamma_B = 1$). In sub-section 6.3, given an optimal carbon tax scheme, we analyze how the trajectories evolve when i) γ_E and γ_B are simultaneously modified; ii) either γ_E or γ_B is modified while the other one is set to 0.3, i.e. when the research policy focuses on a single sector, the remaining one being not subsidized at all[13].

We adopt the following notations that will help us pointing at various facts when describing graphs. $\Delta^{\tau,\gamma}|_X$ stands for the change in variable X due to a simultaneous increase of τ from 0 to τ^o and of $\gamma_B = \gamma_E$ from 0.3 to 1 (cf. sub-section 6.1). $\Delta^{\tau}|_X$ is the change of X due to an increase in τ from 0 to τ^o, given $\gamma_B = \gamma_E = 1$ (cf. sub-section 6.2). Finally, given $\tau = \tau^o$, $\Delta^{\gamma_B,\gamma_E}|_X$ denotes the change in variable X due to a simultaneous increase of γ_B and γ_E from 0.3 to 1, and $\Delta^{\gamma_i}|_X$ the change in variable X due to an increase of γ_i from 0.3 to 1, with $\gamma_j = 0.3$, for $i, j = \{B, E\}$ and $i \neq j$ (cf. sub-section 6.3). The table 1 summarizes the findings from our sensitivity analysis conducted consequently, i.e. the signs of the Δs.

6.1 Optimum vs laisser-faire

In both R&D sectors, the implementation of optimal policies clearly translates into much faster knowledge accumulation (i.e. $\Delta^{\tau,\gamma}|_{H_B} > 0$ and $\Delta^{\tau,\gamma}|_{H_E} > 0$), as seen from Figure 3(a)[14]. Notice that R&D dedicated to energy efficiency is hardly carried out in the "laisser-faire" equilibrium. The innovation selling prices $V_{H_E,t}$ and $V_{H_B,t}$ follow diverging time-paths: $V_{H_E,t}$ decreases over time, while $V_{H_B,t}$ follows a reverse upward trend, at least for the first two centuries (see Figure 3(b)). The optimal instruments shift the price of an innovation dedicated to energy efficiency below its laisser-faire counterpart: $\Delta^{\tau,\gamma}|_{V_{H_E}} <$

[13] According to Jones (1995), we adopt the following convention: a situation without any public subsidy is equivalent to setting the value of γ_i to 0.3, for $i = \{B, E\}$.

[14] Optimal and "laisser-faire" trajectories are referred to as "opti" and "l-f" in the graphs, respectively.

	Optimum vs Laisser-faire	Environmental policy	R&D policies		
X	$\Delta^{\tau,\gamma}\|_X$	$\Delta^{\tau}\|_X$	$\Delta^{\gamma_B,\gamma_E}\|_X$	$\Delta^{\gamma_E}\|_X$	$\Delta^{\gamma_B}\|_X$
H_E	+	∼	+	+	∼
H_B	+	∼	+	∼	+
F	−	−	−	−	−
B	+	+	+	−	+
E	+	−	+	+	+
D	−	−	−	−	−
T	−	−	−	−	−
V_{H_E}	−	∼	−	−	∼
V_{H_B}	+	∼	+	−	+
p_F^m	+	+	−	−	−
p_F^s	−	−	−	−	−
p_B	−	∼	−	∼	−
p_E	−	+	−	−	−

Table 1: Summary of economic policy effects

0. Simultaneously, they shift the selling price of innovations dedicated to the backstop production above the laisser-faire level: $\Delta^{\tau,\gamma}|_{V_{H_B}} < 0$. As will be seen in sub-section 6.3, those results are essentially caused by the R&D policies.

[Figure 3]

In the fossil fuel sector, the introduction of optimal policies implies a reduction of the instantaneous flow of extraction: $\Delta^{\tau,\gamma}|_F < 0$ (see Figure 4(a)). Since this result is observed at each date, the cumulative extraction is also reduced. The resource market price increases, whereas its selling price diminishes: $\Delta^{\tau,\gamma}|_{p_F^m} > 0$ and $\Delta^{\tau,\gamma}|_{p_F^s} < 0$. This overall effect on fuel prices is essentially due to the environmental policy and will be commented in sub-section 6.2.

[Figure 4]

In the backstop and energy sectors, the price of carbon-free energy $p_{B,t}$, as well as of final energy $p_{E,t}$, is reduced: $\Delta^{\tau,\gamma}|_{p_B} < 0$ and $\Delta^{\tau,\gamma}|_{p_E} < 0$; their respective consumption B_t and E_t are intensified, overriding the fossil use reduction: $\Delta^{\tau,\gamma}|_B > 0$ and $\Delta^{\tau,\gamma}|_E > 0$ (see Figures 4(b) and 5(a)).

Finally, the optimal time-paths of temperature variation and environmental damage start diverging from the laisser-faire case by the middle of the century: $\Delta^{\tau,\gamma}|_T < 0$ and

$\Delta^{\tau,\gamma}|_D < 0$ (see Figure 5(b)). In 2100, the optimal temperature variation is 9.4% lower than the no-climate-policy case, reaching almost 5 degrees in the very long run. The resulting loss in gross world product reveals only marginally affected by the end of this century, the most prominent impacts occurring only later on[15].

[Figure 5]

6.2 Sensitivity to environmental tax

In order to quantify how sensitive are the economic variables to the environmental policy, we analyze the impacts of carbon tax variations on equilibrium trajectories, given an optimal research policy ($\gamma_E = \gamma_B = 1$). The carbon tax τ_t is assumed to develop over time according to the following patterns: the carbon tax is first assumed to be nil throughout the entire time horizon ($\tau_t = 0$ for all t); second, it is assumed to correspond to half of the optimal carbon tax ($\tau_t = 0.5 \times \tau_t^o$ for all t); third, these outcomes are compared to the optimal carbon tax implementation ($\tau_t = \tau_t^o$ for all t). We summarize our findings as follows.

In the R&D sectors, the choice of the environmental tax affects neither the innovation prices nor the knowledge levels: $\Delta^\tau|_{H_i} \approx 0$ and $\Delta^\tau|_{V_{H_i}} \approx 0$, for $i = \{B, E\}$[16].

As far as the resource market is concerned, reinforcing the carbon tax level throughout the entire time horizon is shifting the fossil fuel market price upward, and then the resource use downward, as depicted in Figure 6: $\Delta^\tau|_{p_F^m} > 0$ and $\Delta^\tau|_F < 0$. However, it is worth observing that the selling price of the fossil resource is decreasing: $\Delta^\tau|_{p_F^s} > 0$. This reduction implies a rent transfer from the resource-holder to the government. The idea that environmental taxes generally imply some redistributive effects in addition to the expected efficiency gains has already been evoked by economists (see for example Nordhaus and Boyer, 2000 and Grimaud and Rougé, 2005)[17]. Our framework provides an unambiguous characterization of those redistributive effects and allows for the assessment of their extent.

[15]The quantitative results from our simulations should not be taken as granted. They would rather exhibit some qualitative robustness of the model functional specifications. In particular, some further developments of the model will require, among others, embedding an updated damage function from the DICE model. Contrary to the current specification, the most recent calibration in Nordhaus (2007) shows some positive non-market damages from the earliest phases of climate changes. Those revised cost estimates would likely reinforce the attractiveness of a sounder climate policy.

[16]Note that we have conducted this sensitivity analysis to τ_t in the case of fully subsidized research sectors, i.e. $\gamma_E = \gamma_B = 1$. However, according to some alternative runs not documented here, this conclusion still applies when the research sectors are only partially financed ($\gamma_E = \gamma_B = 0.3$). One could extrapolate this conclusion and conjecture that those results hold for any type of research policy.

[17]Nordhaus and Boyer (2000) justify the existence of a potential rent transfer by the fact that fossil fuel availability is generally limited (i.e. resources are scarce) and their supply curves are relatively price-inelastic. As they mention p.54, "In the limited case of perfectly price-inelastic supply of carbon-energy

[Figure 6]

Concerning the backstop sector, the backstop price reveals unaffected by the environmental policies, i.e. $\Delta^\tau|_{p_B} \approx 0$, since its production cost – that partly depends on the specific level of knowledge in the backstop R&D sector H_B – remains fairly constant. However, the demand for the non-carbon backstop energy is stimulated by such policies (see Figure 8(a)): $\Delta^\tau|_B > 0$. Indeed, in the energy sector, since firms face a higher fossil fuel price, they substitute the backstop for the polluting resource. Nonetheless, this more intensive backstop use is not sufficient to maintain the laisser-faire level of energy consumption (see Figure 7(a)): $\Delta^\tau|_E < 0$. This last result comes from the increase of the energy market price when a carbon tax is levied on the fossil fuel use (Figure 7(b)): $\Delta^\tau|_{p_E} > 0$. The implementation of the optimal carbon tax causes the energy market price to grow by 15% whereas, when the carbon tax is halved, the energy price increase is limited to 6% in 2100.

[Figures 7 and 8]

Finally, the reduced carbon intensity of the global economy stemming from the carbon tax policy, slows down the increase in the temperature variation, and in turn, reduces the environmental damage to some 0.3% of GWP in 2100 as compared with the laisser-faire case, as shown in Figure 8(b). In the longer run, the discrepancies are growing such that the optimal carbon tax implementation saves 1.5% of GWP loss by the end of our time horizon.

6.3 Sensitivity to research subsidies

When implementing a research policy, the regulator can act either on the sole energy R&D sector, on the sole backstop R&D sector, or on both sectors simultaneously. Then, a complete analysis of the effect of such a policy requires the dissociation of the joint effects from the marginal ones on each sector. For this matter, we allow for γ_E and γ_B to take the following values: 0.3, 0.6 and 1 and we proceed to two kinds of comparisons. For a given optimal carbon tax scheme, i.e. $\tau_t = \tau_t^o$ for all t, we analyze how the trajectories evolve when i) γ_E and γ_B are simultaneously modified; ii) either γ_E or γ_B is modified while the

with zero extraction costs, carbon taxes may have no economic effect at all and would simply redistribute rents from the resource owners to the government". The situation described in our model is not so extreme since the resource will never be exhausted given the specified extraction technology. Nevertheless, beyond the efficiency effects, we still observe a rent effect.

other one is set to 0, i.e. when the research policy focuses on a single sector, the remaining one being not subsidized at all.

Let us begin with the R&D sectors. We have previously noted (cf. sub-section 6.2) that the environmental policy has no effect on the level of knowledge and on the innovation selling prices. Thus, the overall effect of optimal policies on R&D sectors, as mentioned in sub-section 6.1, is only due to the research policies. A simultaneous increase in γ_B and γ_E makes the knowledge accumulation in both R&D sectors faster: $\Delta^{\gamma_B,\gamma_E}|_{H_i} > 0$, for $i = \{B, E\}$. However, as shown in Figures 9(a) and 10(a), if we decompose the aggregate effect according to each sector, we observe that there is no cross-sector effects: $\Delta^{\gamma_B,\gamma_E}|_{H_i} \approx \Delta^{\gamma_i}|_{H_i}$ and $\Delta^{\gamma_j}|_{H_i} \approx 0$, for $i, j = \{B, E\}$ and $i \neq j$.

The innovation selling prices being equal to the marginal costs of innovations, apprehending how they are affected by the optimal policy proves difficult. Contrary to the stock of knowledge in each R&D sector, the innovation selling prices do not react the same way to any research policy, as depicted in Figures 9(b) and 10(b): $\Delta^{\gamma_B,\gamma_E}|_{V_{H_E}} < 0$ and $\Delta^{\gamma_B,\gamma_E}|_{V_{H_B}} > 0$. From (45) and (46), we have:

$$V_{H_i,t} = \frac{1}{H^i_{R_i}} = \frac{R_{i,t}}{b_i H^i(R_{i,t}, H_{i,t})}, \text{ for } i = \{B, E\}\,.$$

For any $i = \{B, E\}$, both the numerator and the denominator of this ratio increase. Nevertheless, when $i = E$, $R_{E,t}$ grows less than $b_E H^E(.)$, which leads to a decrease of the marginal cost of innovation in the energy R&D sector, along with its selling price $V_{H_E,t}$. Alternatively, when $i = B$, $R_{B,t}$ grows more than $b_B H^B(.)$, which leads to the opposite result. Those complex interactions stem from our general equilibrium framework.

Some further analysis of the R&D policy make the effect of each type of R&D subsidy on the innovation selling prices clearer. First, $\Delta^{\gamma_E}|_{V_{H_E}} \approx \Delta^{\gamma_B,\gamma_E}|_{V_{H_E}}$ and $\Delta^{\gamma_B}|_{V_{H_E}} \approx 0$: there is no cross-sector effects on the energy R&D sector, i.e. γ_B has no effect on $V_{H_E,t}$ (see Figure 9(b)). Second, $\Delta^{\gamma_B}|_{V_{H_B}} > \Delta^{\gamma_B,\gamma_E}|_{V_{H_B}} > 0$ and $\Delta^{\gamma_E}|_{V_{H_B}} < 0$ (see Figure 10(b)). In this case, strong cross-sector effects are occurring. When the only backstop R&D sector is subsidized, the increase in the innovation selling price in backstop R&D is higher than the increase which is observed when both R&D sectors are subsidized. Moreover, subsidizing the only energy R&D causes the backstop innovation price to move in the opposite direction, i.e. to decrease.

[Figures 9 and 10]

In turn, we examine the simultaneous effects of research policies on the fossil fuel, the backstop and the energy sectors, and we try to give some intuitions on the results. Since a simultaneous increase in γ_B and γ_E stimulates the knowledge accumulation in both sectors (i.e. H_B and H_E increase), this directly reduces the production costs of the backstop and the energy services, as well as their respective market prices p_B and p_E: $\Delta^{\gamma_B,\gamma_E}|_{p_B} < 0$ and $\Delta^{\gamma_B,\gamma_E}|_{p_E} < 0$ (Figures 11(b) and 12(b)). This implies an increase in the backstop and energy productions: $\Delta^{\gamma_B,\gamma_E}|_B > 0$ and $\Delta^{\gamma_B,\gamma_E}|_E > 0$ (Figures 11(a) and 12(a)). Since the backstop is relatively less costly than the fossil fuel (p_B/p_F decreases), then the energy producers substitute the former for the latter: $\Delta^{\gamma_B,\gamma_E}|_F < 0$ (Figure 13(a)). The demand for the fossil fuel being reduced, its price decreases: $\Delta^{\gamma_B,\gamma_E}|_{p_F} < 0$. Remark that an increase in γ_E reduces the backstop production, but leaves its market price unchanged: $\Delta^{\gamma_E}|_B < 0$ (which implies $\Delta^{\gamma_B}|_B > \Delta^{\gamma_B,\gamma_E}|_B > 0$) and $\Delta^{\gamma_E}|_{p_B} \approx 0$.

[Figures 11, 12 and 13]

Finally, the damage and the temperature changes are positively affected by a rise in energy subsidies to one or both sectors (see Figure 13(b)).

7 Conclusion

This paper establishes the template of a climate change integrated assessment model, capable of defining the decentralized outcome, i.e. the equilibrium, of a given climate policy architecture. One of the main features of the model lies in the analytical derivation of the innovation prices. In our context, those innovations are dedicated to knowledge accumulation in two sectors: the backstop energy sector and the energy efficiency sector. Since knowledge is not embodied into intermediate goods, its price is defined in an alternative way (as a part of its social value that is equal to the sum of its marginal profitabilities in all sectors using it).

Another key feature of the model lies in its ability and suitability to assess various economic policies. As the economy encompasses three market distortions, i.e. the pollution from fossil resource consumption and the two research spillovers, two types of economic policy instruments are implemented: a tax on the fossil fuel use and a research subsidy for each R&D sector. As one obtains a distinct equilibrium for each vector of instruments, we are able to test for any policy architectures, including suboptimal carbon taxes and research subsidies, contrary to Popp (2006c). This should be of particular interest for studying second best policy in the context of climate change mitigation.

We use a calibrated version of the model to simulate the socially optimum outcome and compare it to its laisser-faire counterpart in the decentralized economy. We assess the impacts on all economic and environmental variables and characterize the efficiency of the policy measures, and particularly the efficiency of the R&D funding that have to be devoted to energy technologies. The laisser-faire situation results in some additional gross world product losses of 1.6% in the long run, as compared to the socially desirable outcome. We exhibit the significant influence of R&D activities aiming at reducing the polluting fossil energy use. This setting advocates for higher subsidies dedicated to renewable energies, and, to a lower extent, for subsidies aiming at improving energy efficiency. This mainly comes from the underlying assumption on the potential improvements of energy efficiency that are limited to 20%, suggesting that improvement in energy efficiency would rather be a short term option for tackling the climate change issue, while bringing the backstop energy to the market is more beneficial in the longer term.

The natural extension of the model will consist in introducing a richer set of climate mitigation options such as the possibility of capturing and storing the carbon in geological formations. One might also introduce biofuel energy, the feedstock then encompassing the features of a renewable resource. The specificities of nuclear energy may also be incorporated in our model. The flexibility of the tool at hand allows for the modeling of specific knowledge stocks for each of the energy supply technologies.

Finally, the calibration of this model may require some further adjustment. In this respect, alternative functional forms may be experienced (See Nordhaus's comment on Stern review and the accompanying data update – Nordhaus, 2007). Moreover, as suggested by the IPCC report (IPCC, 2000), a number of plausible scenarios may arise in the future. The DICE model calibration may be revised so as to match more closely the GDP projections of other long term studies. In particular, it would be worthwhile analyzing the effects of a more sustained long term growth. An enhanced world economic growth would turn into more intensive fossil energy use, at least in the early decades where the renewable energy does not exhibit sufficient cost reduction. Besides the increased externality resulting from more rapid climate change, the modified economically recoverable resource base may, in turn, confront us to lower fossil resource availabilities in the long run. The effect on the fossil fuel prices and the incentive for increased investment in clean energy R&D deserves some further investigation.

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Appendix

A1. Analytical specification and calibration of the model

To characterize analytically our model, we use a mix of functional forms considered in the DICE and ENTICE-BR models:

$$\begin{aligned}
Q(K,E,L,A) &= AK^{\gamma}E^{\beta}L^{1-\gamma-\beta}, \quad \text{with} \quad \beta,\gamma \in (0,1) \\
g_i &= \left(\frac{g_{i0}}{d_i}\right)\left(1-e^{-d_i t}\right), \quad \text{with} \quad d_i > 0,\, \forall i = \{A, L\} \\
E(H_E,F,B) &= \left[(\alpha_H H_E)^{\rho_H} + (F^{\rho_B} + B^{\rho_B})^{\frac{\rho_H}{\rho_B}}\right]^{\frac{1}{\rho_H}}, \quad \text{with} \quad \alpha_H, \rho_H, \rho_B \in (0,1) \\
H^i(R_i,H_i) &= a_i R_i^{b_i} H_i^{\phi_i}, \quad \text{with} \quad a_i > 0, \text{ and } b_i, \phi_i \in [0,1],\, \forall i = \{E, B\} \\
D(T) &= \frac{1}{1 + a_1 T + a_2 T^2}, \quad \text{with} \quad a_1 < 0 \text{ and } a_2 > 0 \\
U(C) &= k_1 L \log\left(\frac{C}{L}\right) + k_2, \quad \text{with} \quad k_1, k_2 > 0 \\
\Phi(G) &= \epsilon_1 \frac{\log(G/\epsilon_2)}{\log 2} + O(t),
\end{aligned}$$

where $O(t) = \epsilon_3 t - \epsilon_4$ for $t < \bar{t}$, $O(t) = \epsilon_5$ otherwise, $\epsilon_i > 0$, $i = \overline{1,5}$. We also consider the following production functions:

$$\begin{aligned}
F(Q_F, Z) &= \frac{Q_F}{c_F + \alpha_F \times (Z/\bar{Z})^{\eta_F}}, \quad \text{with} \quad c_F, \alpha_F, \eta_F > 0 \\
B(Q_B, H_B) &= Q_B \times \frac{H_B^{\eta_B}}{\alpha_B}, \quad \text{with} \quad \alpha_B, \eta_B > 0.
\end{aligned}$$

For numerical computations, we use the same values of exogenous parameters as in the ENTICE-BR model [18]. Since we have transformed the cost functions of fossil fuel and backstop into production functions, we also specify the parameters of these production functions in such a way that the calibration of the ENTICE-BR model still applies to our model. Finally, we consider a finite time horizon starting at date $t_0 = 1990$ and ending at $T = t_0 + 350$.

A2. Proof of Proposition 1

Let H be the discounted value of the Hamiltonian of the optimal program (we drop time subscripts for notational convenience):

$$
\begin{aligned}
H &= U(C)e^{-\int_0^t \rho ds} + \lambda D(T)Q\left\{K, E\left[H_E, F(Q_F, Z), B(Q_B, H_B)\right]\right\} \\
&\quad -\lambda\left(C + Q_F + Q_B + \sum_i R_i + \delta K\right) + \sum_i \nu_i H^i(R_i, H_i) \\
&\quad +\mu_G(\alpha F - \zeta G) + \mu_T[\Phi(G) - mT] + \eta F.
\end{aligned}
$$

The associated first order conditions are:

$$
\frac{\partial H}{\partial C} = U'(C)e^{-\int_0^t \rho ds} - \lambda = 0 \tag{48}
$$

$$
\frac{\partial H}{\partial Q_F} = \lambda[D(T)Q_E E_F F_{Q_F} - 1] + \alpha\mu_G F_{Q_F} + \eta F_{Q_F} = 0 \tag{49}
$$

$$
\frac{\partial H}{\partial Q_B} = \lambda[D(T)Q_E E_B B_{Q_B} - 1] = 0 \tag{50}
$$

$$
\frac{\partial H}{\partial R_i} = -\lambda + \nu_i H^i_{R_i} = 0, \quad i = \{B, E\} \tag{51}
$$

$$
\frac{\partial H}{\partial K} = \lambda[D(T)Q_K - \delta] = -\dot{\lambda} \tag{52}
$$

$$
\frac{\partial H}{\partial H_i} = \lambda D(T)Q_E E_{H_i} + \nu_i H^i_{H_i} = -\dot{\nu}_i, \quad i = \{B, E\} \tag{53}
$$

$$
\frac{\partial H}{\partial G} = -\zeta\mu_G + \mu_T\Phi'(G) = -\dot{\mu}_G \tag{54}
$$

$$
\frac{\partial H}{\partial T} = \lambda D'(T)Q - m\mu_T = -\dot{\mu}_T \tag{55}
$$

$$
\frac{\partial H}{\partial Z} = \lambda D(T)Q_E E_F F_Z + \alpha\mu_G F_Z + \eta F_Z = -\dot{\eta} \tag{56}
$$

[18] For the sake of simplicity, the exogenous land use emissions have been omitted. Those emissions are likely small (see Nordhaus, 2007) and would alter neither our qualitative nor our quantitative results.

and the transversality conditions are:

$$\lim_{t\to\infty} \lambda K = 0 \tag{57}$$

$$\lim_{t\to\infty} \nu_i H_i = 0, \quad i = \{B, E\} \tag{58}$$

$$\lim_{t\to\infty} \mu_G G = 0 \tag{59}$$

$$\lim_{t\to\infty} \mu_T T = 0 \tag{60}$$

$$\lim_{t\to\infty} \eta Z = 0 \tag{61}$$

First, we show how to obtain condition (12), the less evident one. From (49), we have:

$$\alpha\mu_G + \eta = -\lambda\frac{[D(T)Q_E E_F F_{Q_F} - 1]}{F_{Q_F}},$$

where $\lambda = U'(C)e^{-\int_0^t \rho ds}$ from (48). Substituting this expression into (56) and after simplifications, we get the following differential equation:

$$\dot{\eta} = -\frac{F_Z}{F_{Q_F}}U'(C)e^{-\int_0^t \rho ds}.$$

Integrating this expression and using transversality condition (61), we obtain:

$$\eta = \int_t^\infty \frac{F_Z}{F_{Q_F}}U'(C)e^{-\int_0^s \rho du}ds. \tag{62}$$

From (48) and (55), we have:

$$\dot{\mu}_T = m\mu_T - D'(T)QU'(C)e^{-\int_0^t \rho ds}.$$

Solution of such a differential equation is given by:

$$\mu_T = e^{mt}\left[\mu_{T,0} - \int_0^t D'(T)QU'(C)e^{-\left(ms+\int_0^s \rho dx\right)}ds\right].$$

Using (60), this expression becomes:

$$\mu_T = e^{mt}\int_t^\infty D'(T)QU'(C)e^{-\left(ms+\int_0^s \rho dx\right)}ds = \int_t^\infty D'(T)QU'(C)e^{-\left[m(s-t)+\int_0^s \rho dx\right]}ds. \tag{63}$$

Now, let us consider condition (54). Using transversality condition (59), this differential solution is solved for:

$$\mu_G = e^{\zeta t}\int_t^\infty \mu_T\Phi'(G)e^{-\zeta s}ds = \int_t^\infty \mu_T\Phi'(G)e^{-\zeta(s-t)}ds, \tag{64}$$

where μ_T is defined by (63). Finally, condition (12) is equivalent to condition (49) when replacing λ, μ_G and η by their expressions coming from (48), (64) and (62) respectively, and dividing each side of the equation by F_{Q_F}.

Second, the characterizing condition (13) is directly provided by (50). To continue, remark that (48) implies:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{U}'(C)}{U'(C)} - \rho. \tag{65}$$

Then, condition (14) is simply obtained from (52) and (65).

Finally, differentiating (51) with respect to time and using (13), (53) and (65), we get the characterizing conditions (15) and (16), which concludes the proof.

A3. Proof of proposition 2

The first characterizing condition (35) is obtained by replacing η into (17) by its expression coming from (20) and by noting that $p_F^s = p_E E_F - \tau$ from (22), where $p_E = D(T)Q_E$ from (32). Second, combining (21), (23) and (32) leads to condition (36). Next, using (31) and (34), we directly get condition (37). Finally, the differentiation of (27) with respect to time leads to:

$$\frac{\dot{V}_{H_i}}{V_{H_i}} = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}}, \quad i = \{B, E\}.$$

Substituting this expression into (26) and using (24), (27) and (28), it comes:

$$r = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}} + \gamma_i H^i_{R_i}\left(v^i_{H_i} + \frac{H^i_{H_i}}{H^i_{R_i}}\right), \quad \forall i = \{B, E\}.$$

We thus obtain the two last characterizing equilibrium conditions (38) and (39) by replacing into this last equation $v^B_{H_B}$ and $v^E_{H_E}$ by their expressions coming from (30) and (29) respectively.

A4. Proof of Proposition 3

The Hamiltonian in discounted value of the program writes:

$$\begin{aligned} H &= U(C)e^{-\int_0^t \rho_s ds} + \lambda\left[D(T)Q(.) - C - \sum_i R_i - Q_F - Q_B - \delta K - \tau F(Q_F, Z)\right] \\ &\quad + \sum_i \nu_i \gamma_i H^i(R_i, H_i) + \eta F(Q_F, Z) \end{aligned}$$

The associated first order conditions are:

$$\frac{\partial H}{\partial C} = U'(C)e^{-\int_0^t \rho ds} - \lambda = 0 \Rightarrow \frac{\dot{\lambda}}{\lambda} = \frac{\dot{U}'(C)}{U'(C)} - \rho \tag{66}$$

$$\frac{\partial H}{\partial Q_F} = \lambda[D(T)Q_E E_F F_{Q_F} - 1 - \tau F_{Q_F}] + \eta F_{Q_F} = 0 \tag{67}$$

$$\frac{\partial H}{\partial Q_B} = \lambda[D(T)Q_E E_B B_{Q_B} - 1] = 0 \tag{68}$$

$$\frac{\partial H}{\partial R_i} = -\lambda + \nu_i \gamma_i H^i_{R_i} = 0, \quad i = \{B, E\} \tag{69}$$

$$\frac{\partial H}{\partial K} = \lambda[D(T)Q_K - \delta] = -\dot{\lambda} \tag{70}$$

$$\frac{\partial H}{\partial H_i} = \lambda D(T)Q_E E_{H_i} + \nu_i \gamma_i H^i_{H_i} = -\dot{\nu}_i, \quad i = \{B, E\} \tag{71}$$

$$\frac{\partial H}{\partial Z} = \lambda[D(T)Q_E E_F F_Z - \tau F_Z] + \eta F_Z = -\dot{\eta} \tag{72}$$

and the transversality conditions are:

$$\lim_{t \to \infty} \lambda K = 0 \tag{73}$$

$$\lim_{t \to \infty} \nu_i H_i = 0, \quad i = \{B, E\} \tag{74}$$

$$\lim_{t \to \infty} \eta Z = 0 \tag{75}$$

As in Appendix A.2, we use (67), (72) and (75) to determine η. It is the same expression than (62). Next, replacing into (67) λ and η by their expressions (66) and (62) respectively, we have the first characterizing condition (35). Condition (36) is a direct consequence of (68). Condition (37) is obtained from (66) and (70). Finally, differentiating (69) with respect to time and using (66) and (71) imply the characterizing conditions (38) and (39).

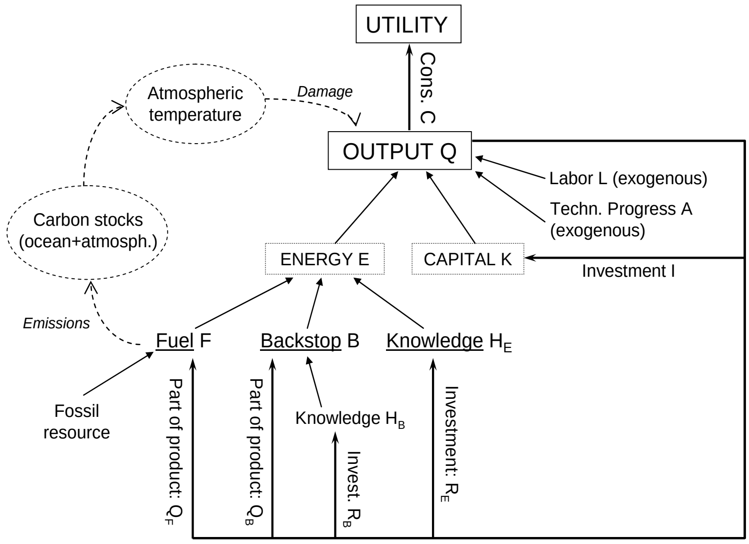

Figure 1: Description of the model

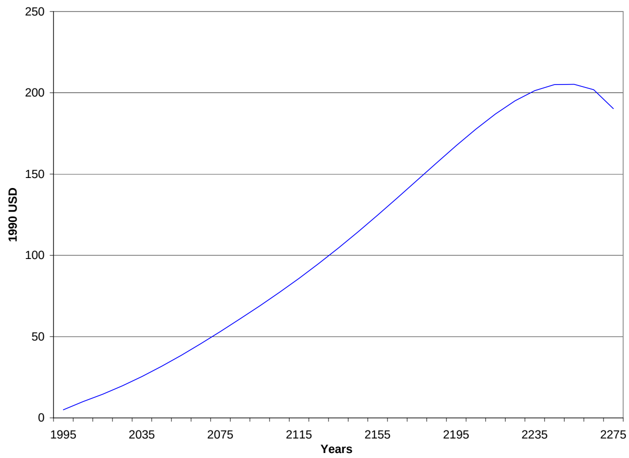

Figure 2: Optimal carbon tax τ^o

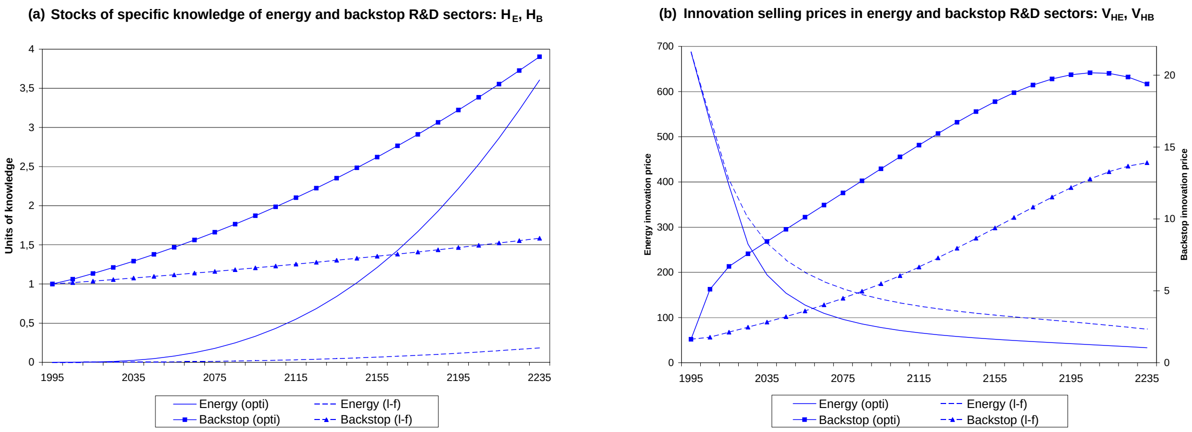

Figure 3: Effect of optimal instruments on H_E and H_B (a) – V_{H_E} and V_{H_B} (b)

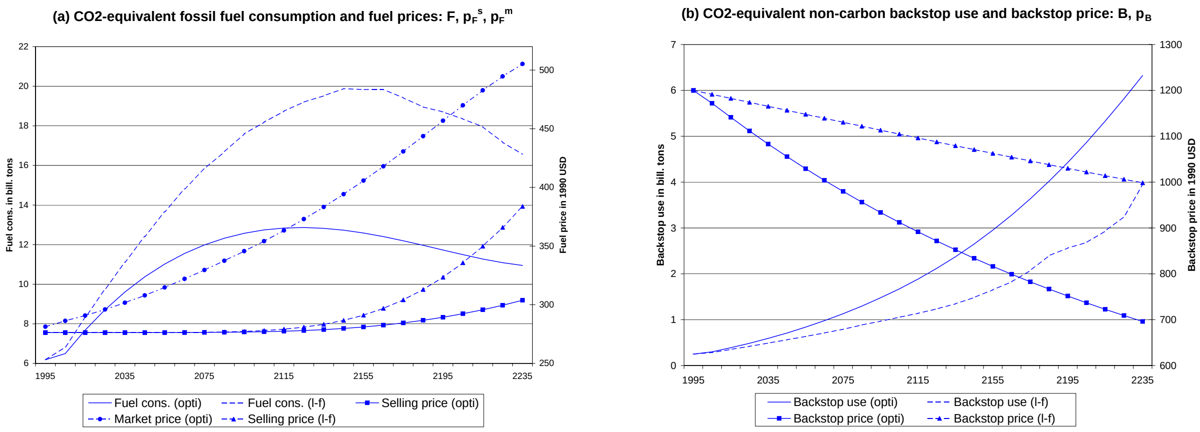

Figure 4: Effect of optimal instruments on F, p_F^s and p_F^m (a) – B and p_B (b)

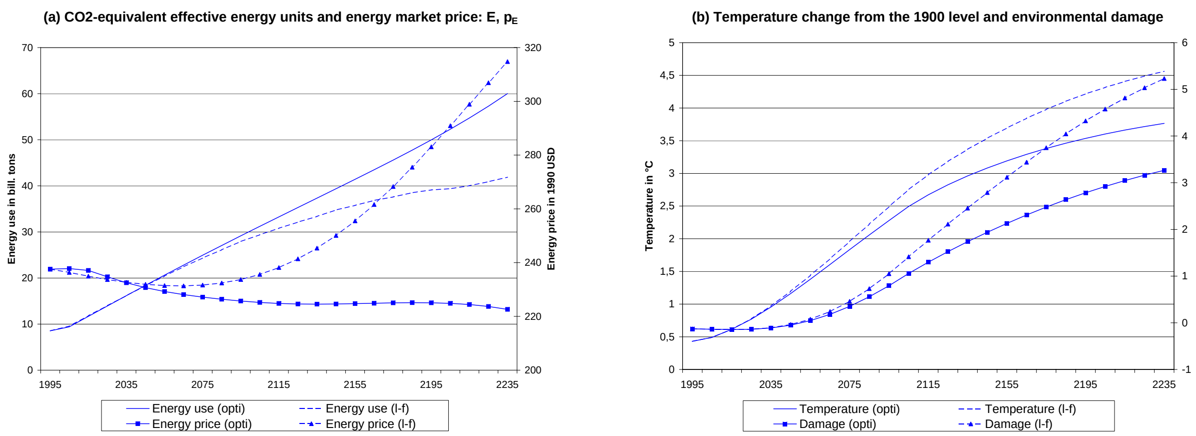

Figure 5: Effect of optimal instruments on E and p_E (a) – T and $D(T)$ (b)

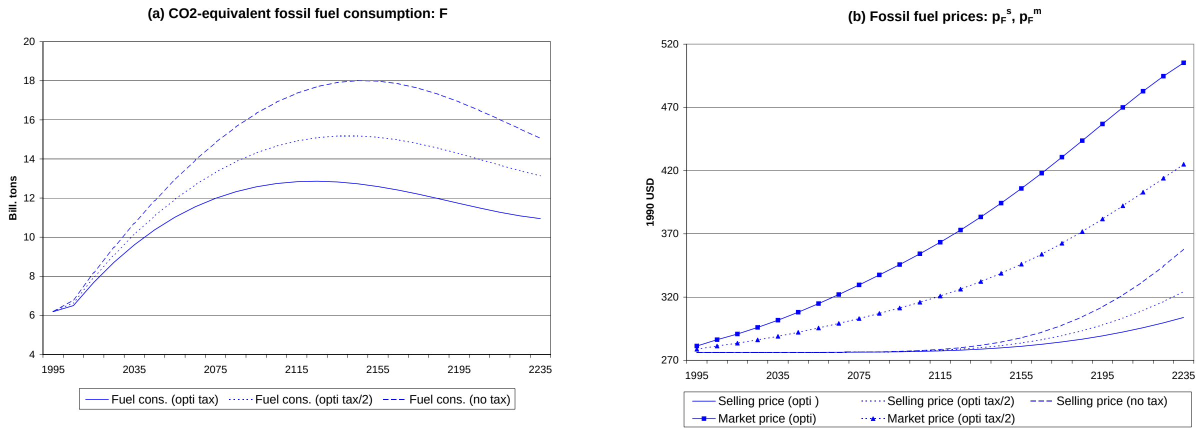

Figure 6: Effect of the environmental tax on F (a), p_F^s and p_F^m (b)

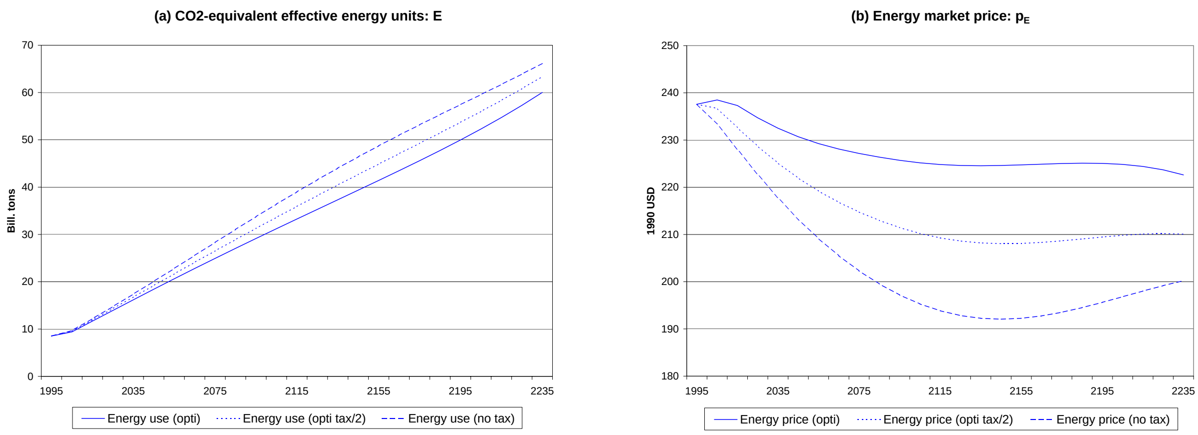

Figure 7: Effect of the environmental tax on E (a) and p_E (b)

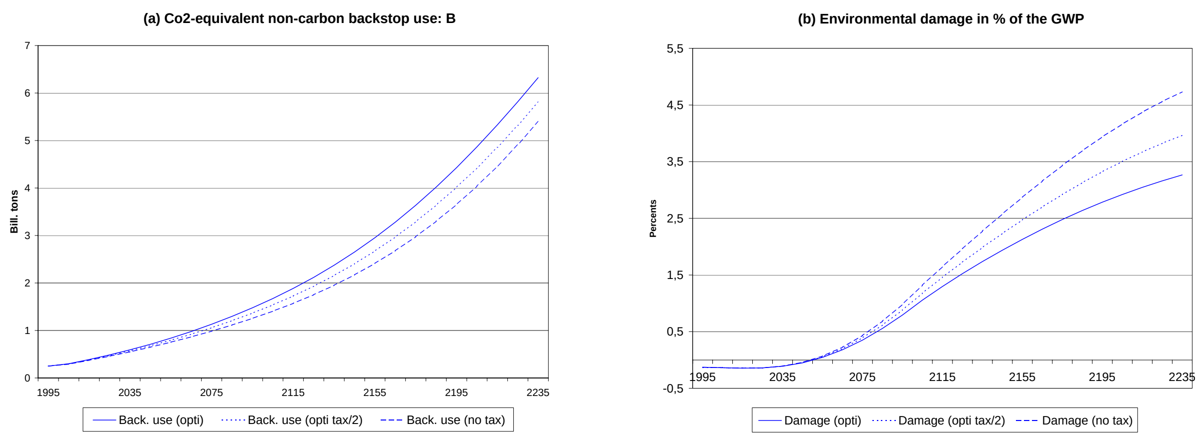

Figure 8: Effect of of the environmental tax on B (a) and $D(T)$ (b)

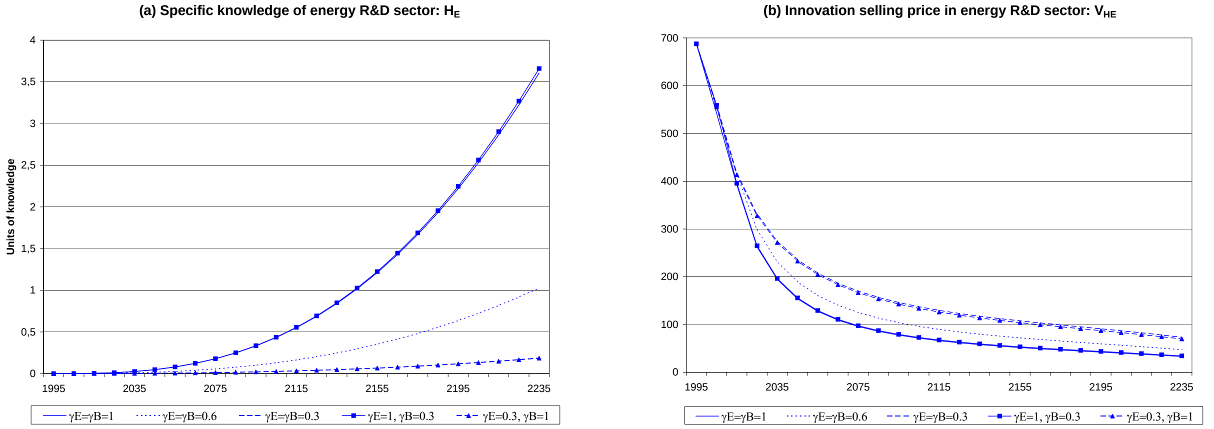

Figure 9: Effect of the research policy on H_E (a) and V_{H_E} (b)

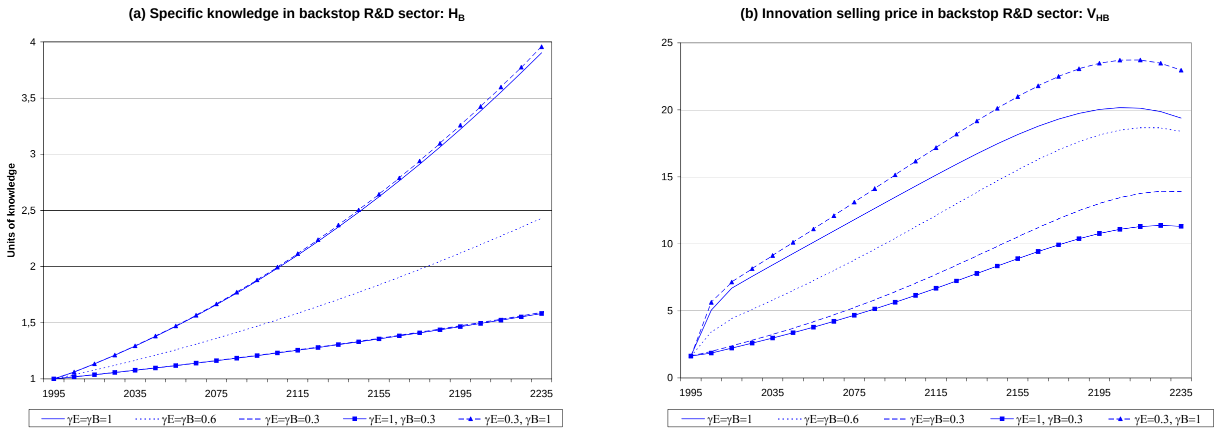

Figure 10: Effect of the research policy on H_B (a) and V_{H_B} (b)

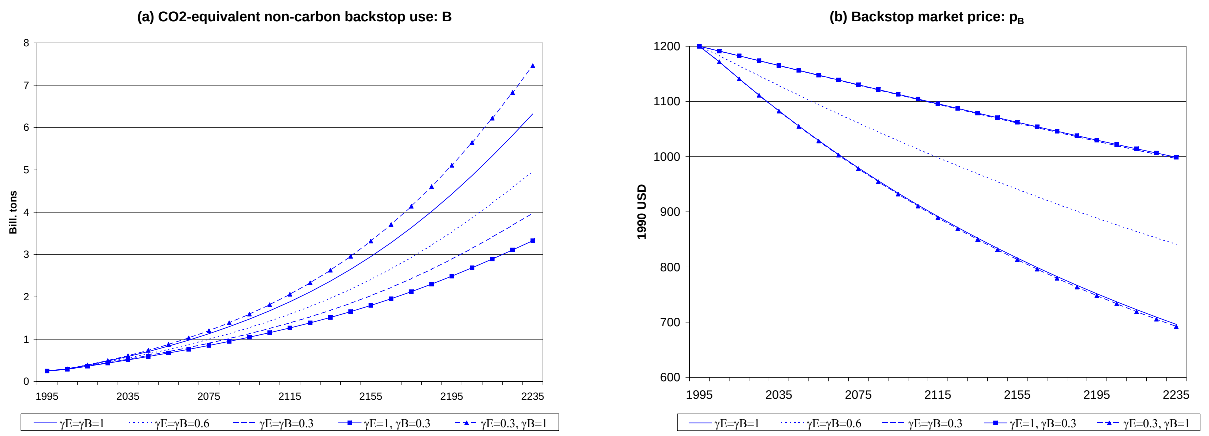

Figure 11: Effect of the research policy on B (a) and p_B (b)

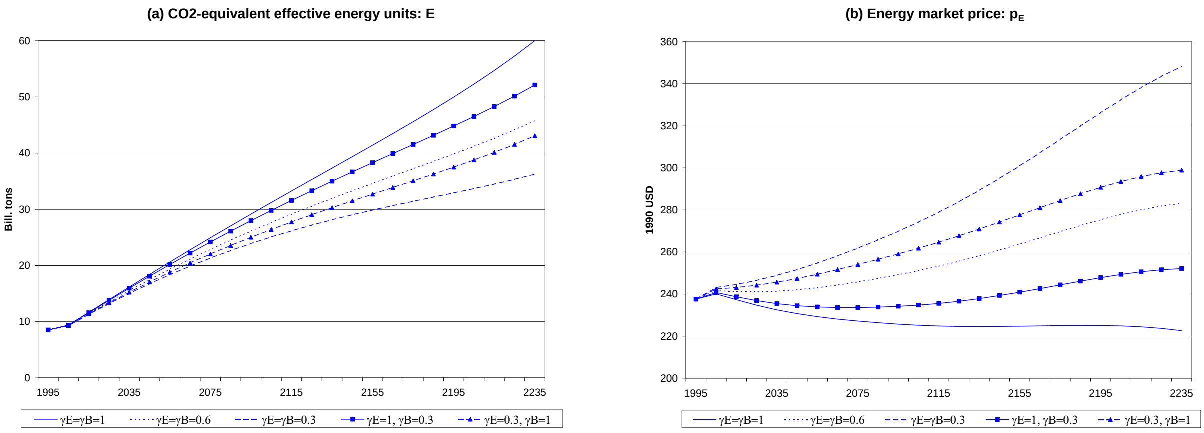

Figure 12: Effect of the research policy on E (a) and p_E (b)

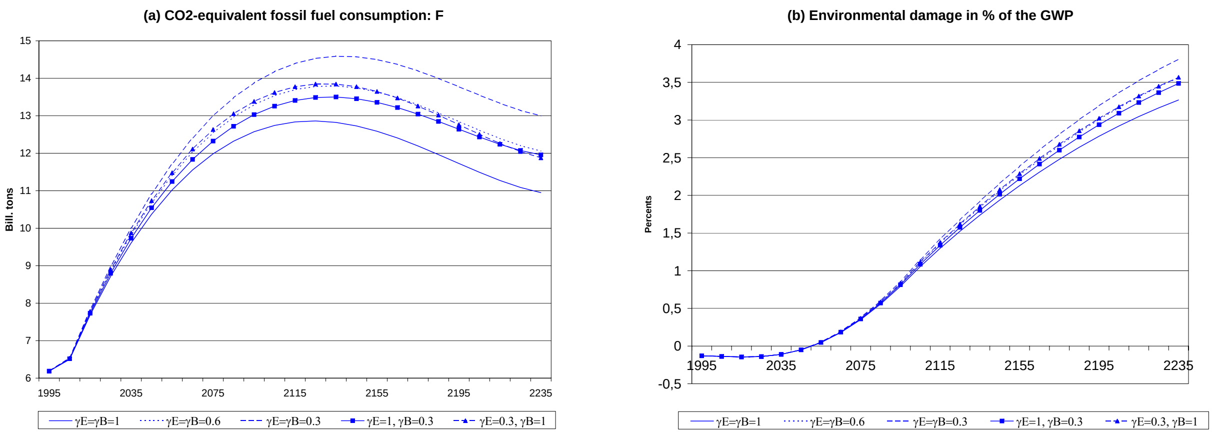

Figure 13: Effect of the research policy on F (a) and $D(T)$ (b)