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1 Introduction

The extent to which developing countries should enforce intellectual property rights is a hotly debated topic. It is often heard that not doing so would allow a cheap access to important products such as drugs. Some authors also point out that in the nineteenth century, the US itself did not recognize foreign patents.

While the argument that a given small country can be made better-off by free-riding on intellectual property in the rest of the world is straightforward, empirically things are not so clear-cut: While Gould and Gruben (1996) find that lower enforcement of Intellectual property reduces growth in a cross-section of countries, a number of authors find adverse effects on foreign direct investment, exports and licensing by US high-tech firms (See Lee and Mansfield (1996), Smarczynska (2002), Smith, P. (1999), and Yang, G. and K. Maskus (1998)).

This paper develops a model for analyzing the costs and benefits of IP enforcement in LDCs. It is an endogenous growth model in the fashion of Grossman and Helpman (1991), with two countries, North and South. Innovation only takes place in the North. The North is more productive than the South. There are two types of goods, and each bloc has a comparative advantage in
producing a specific type of good. If the South does not enforce IP (piracy), then it is able to manufacture goods invented in the North without paying royalties. These goods are sold at marginal cost instead of monopoly price, which benefits world consumers from a static viewpoint. If comparative advantage is strong enough, even under piracy there are goods that the South will not produce. Piracy will then lead to a reallocation of innovative activity in favor of these goods. That may harm consumers (including consumers in the South) to the extent that these goods have smaller dynamic learning externalities than the other goods, and that their share in consumption is small. Thus, whether or not piracy is in the interest of the South depends on how important are the goods for which it has a comparative advantage to its consumers, and what the growth potential of these goods is. While, all else equal, the North tends to lose more (or gain less) from piracy than the South, because monopoly profits eventually accrue to the North, the South may lose more than the North if there is a strong enough home bias in favor of the goods for which it has a comparative advantage.

Our results are reminiscent of two related papers: Diwan and Rodrik (1991) consider the benefits of IPR in the South when there exists a range of products that it specifically consumes. Greater specificity of these products make it more costly for the South to opt for piracy, as it will get less innovation for the products that it needs. Their paper, however, is static and does not consider endogenous growth or comparative advantage. Thoenig and Verdier (2003) argue that the threat of piracy will induce the North to invest in more complex, skill-intensive technologies that are harder to imitate; the argument rests on a direct link between complexity and imitation, while the effects discussed here are relative price effects.¹

¹Other related works include Goh and Olivier (2003) and Grossman and Lai (2001).
2 The model

There are two countries, denoted by A and B, and two types of goods, denoted by 1 and 2. Within each type there potentially is a continuum of goods. Each good is produced with a linear technology which uses labor only. At each date \( t \) there is a continuum of goods, with a mass \( N_i \) of goods of type \( i \). Goods of either type are indexed by \( k \in [0, N_i] \). Goods of type 1 differ from goods of type 2 in that the relative productivity of country B is not the same. Specifically, country A produces all goods with a unit productivity. Country B produces goods of type 1 with productivity \( b \) and goods of type 2 with productivity \( a \). We assume that country B is less developed than country A, and that it has a comparative advantage in producing 1-goods. That is,

\[
a < b < 1
\]

Each country has a representative consumer. Utility is allowed to differ between the two countries, embodying the possibility of home bias:

\[
U_A = \alpha \ln C_A^1 + (1 - \alpha) \ln C_A^2 - (\alpha \ln \alpha + (1 - \alpha) \ln(1 - \alpha)); \\
U_B = \beta \ln C_B^1 + (1 - \beta) \ln C_B^2 - (\beta \ln \beta + (1 - \beta) \ln(1 - \beta)).
\]

where \( C_i^j, i \in \{1, 2\}, j \in \{A, B\}, \) is the aggregate consumption index of goods of type \( i \) in country \( j \), given by

\[
C_i^j = \left( \int_0^{N_{it}} c_{ijt}(k)^{\sigma-1} dk \right)^{\frac{\sigma}{\sigma-1}},
\]

where \( c_{ijt}(k) \) is consumer \( j \)'s consumption of the \( i \)-good indexed by \( k \) at date \( t \).

The demand function for good \( k \) of type \( i \) coming from country \( j \) is

\[
c_{ijt}(k) = s_{ij} \frac{Y_{jt}}{p_{it}} \left( \frac{p_t(k)}{p_{it}} \right)^{-\sigma},
\]

where \( Y_{jt} \) is aggregate nominal income in country \( j \) at \( t \); \( p_t(k) \) is the price of the good considered; \( s_{ij} \) the appropriate income share \((s_{1A} = \alpha; s_{2A} = 1 - \alpha; s_{1B} = \cdot)\).
\( \beta; s_{2B} = 1 - \beta \); and \( \bar{p}_{it} \) the appropriate price index for \( i \)-goods:

\[
\bar{p}_{it} = \left( \int_{0}^{N_{it}} p_{it}(k)^{1-\sigma} dk \right)^{\frac{1}{1-\sigma}}.
\]

Country \( j \) is endowed with \( L_j \) units of production labor. Furthermore, country \( A \) is endowed with a stock \( L_R \) of researchers, who produce new blueprints of either type. Thus, R & D can only take place in country \( A \). The cost of producing a new good is the same irrespective of the type of the good. Once the good is invented, the inventor holds a monopoly right forever on the good. Researchers decide ex-ante whether to try and invent a 1-good or a 2-good. Thus, they all specialize in the good that yields the largest present discounted value, unless the two PDVs are equal, in which case they are indifferent. To guarantee sustained long-run growth, the cost of inventing new blueprints is assumed (in a standard way) to be proportional to the total number of goods of the same type \( N_{it} \). Thus, if \( \theta_t \) is the proportion of researchers working on 1-goods, we have

\[
\begin{align*}
\dot{N}_{1t} &= \gamma_1 \theta_t N_{1t}; \\
\dot{N}_{2t} &= \gamma_2 (1 - \theta_t) N_{2t}.
\end{align*}
\]

Note that the \( \gamma \)s are allowed to differ between the two types of goods: one type of good may have stronger dynamic learning externalities than the other.

We will characterize equilibrium in two different cases:

1. Country B does not enforce any intellectual property by country A, and can export the products it copies to country A. In such a case any good \( i \) can be produced by the patent holder in country A plus a fringe of perfect competitors in country B.

2. Country B fully enforces intellectual property rights. In such a case only the patent holder can produce a good. Patent holders can be located in either country\(^2\), but all monopoly profits accrue to country A.

\(^2\) Or, equivalently, extract all the rents by licensing to a monopoly based in country B.
We now describe the main properties of the solution under different regimes, relegateing proofs to the Appendix. We first characterize the allocation of resources and prices at a given date, and then study the determinants of innovation and the evolution of the number of goods. We shall express prices in terms of labor in country A. Its wage $w_A$ is therefore normalized to $w_A = 1$.

### 3 Full piracy

Let us first analyze equilibrium in the case of full piracy. One can show that there are 4 possible cases:

1. If
   \[ a < \frac{\alpha L_A}{(1 - \beta)L_B} < \frac{b}{\mu}, \]
   then in equilibrium (Regime P1):
   
   (i) Country A only produces 2-goods.
   
   (ii) Country B only produces 1-goods
   
   (iii) Prices are given by
   
   \[ w_A = 1; \]
   \[ w_B = \frac{\alpha \mu L_A}{(1 - \beta)L_B} < b; \]
   \[ p_1 = \frac{w_B}{b}; \]
   \[ p_2 = \mu. \]

   (iv) Profits in country A are strictly positive for producers of 2-goods, and would be strictly negative for producers of 1-goods. As a result, researchers in country A only produce 2-goods.

   (v) National income in country A is equal to
   
   \[ Y_A = \mu L_A, \]
   
   while in country B it is equal to
   
   \[ Y_B = \frac{\alpha \mu L_A}{(1 - \beta)}. \]
Note that under piracy, GDP is equal to GNP as profit repatriation is the only source of foreign income in the model.

2. If

\[ a < \frac{b}{\mu} < \frac{\alpha L_A}{(1 - \beta)L_B}, \]

then in equilibrium (Regime P2)

(i) Country A produces both types of goods.

(ii) Country B produces only 1-goods

(iii) Prices are given by

\[ w_A = 1; \]
\[ w_B = b; \]
\[ p_1 = 1; \]
\[ p_2 = \mu. \]

(iv) Employment in 2-goods in country A is given by

\[ L_{2A} = \frac{(1 - \alpha) L_A + b(1 - \beta)L_B}{\alpha \mu + 1 - \alpha}. \]

(v) Profits are strictly positive for 2-goods and equal to zero for 1-goods.

Therefore, country 2’s researchers only invent 2-goods.

(vi) National incomes are given by

\[ Y_A = (\mu - 1)L_{2A} + L_A \]

and

\[ Y_B = bL_B \]

3. If

\[ \frac{b}{\mu} < a < \frac{\alpha L_A}{(1 - \beta)L_B}. \]
then in equilibrium (regime P3)

(i) Country A produces both types of goods.

(ii) Country B produces only 1-goods

(iii) Prices are given by

\[
\begin{align*}
  w_A &= 1; \\
  w_B &= b; \\
  p_1 &= 1; \\
  p_2 &= b/a.
\end{align*}
\]

(iv) Employment in 2-goods in country A is given by

\[
L_{2A} = \frac{(1 - \alpha)L_A + b(1 - \beta)L_B}{\alpha b/a + 1 - \alpha}
\]

(v) Profits are strictly positive for 2-goods and equal to zero for 1-goods. Therefore, country 2’s researchers only invent 2-goods.

(vi) National incomes are given by

\[
Y_A = (b/a - 1)L_{2A} + L_A
\]

and

\[
Y_B = bL_B.
\]

In terms of specialization, this regime is the same as regime P2. In terms of pricing, however, it implies that producers of 2-goods in country A charge a limit price equal to the cost of these goods in country B, rather than a monopoly price.

4. If

\[
\frac{\alpha L_A}{(1 - \beta)L_B} < a,
\]

then in equilibrium (Regime P4)
(i) Country A only produces 2-goods

(ii) Country B produces both types of goods

(iii) Prices are given by

\[ \begin{align*}
    w_A &= 1; \\
    w_B &= a; \\
    p_1 &= a/b; \\
    p_2 &= 1.
\end{align*} \]

(iv) Employment in 1-goods in country B is given by

\[ L_{1B} = \frac{\alpha L_A + a\beta L_B}{a} \]

(v) Profits in country A would be negative for 1-goods, and are zero for 2-goods. Consequently, no innovation takes place and the wage of researchers falls to zero.

(vi) We have

\[ Y_A = L_A \]

and

\[ Y_B = aL_B. \]

Note that regimes 2 and 3 entail the same pattern of specialization, but that regime 3 has limit pricing instead of monopoly pricing. The phase diagram corresponding to the four regimes is depicted in Figure 1. The conditions tell us that if productivity in country B for 2-goods is small compared to the relative size of the workforce in country A, then country B will only produce 1-goods for which it has a comparative advantage. Depending on its size, country A will either produce 2-goods or both types of goods. In both cases, only 2-goods are making positive profits (in the first case a 1-good would make a negative profit, in the second case a 1-good would make a zero profit). Furthermore,
if $a < b/\mu$, 2-goods producers in country A can charge monopoly prices. If $a > b/\mu$, 2-goods producers are compelled to charge a limit price $b/a$ pinned down by competition from country B. If productivity in country B for 2-goods is large compared to the relative size of country A, then country B produces both types of goods and country A only produces 2-goods. Its producers can’t make any profits, otherwise they could lower their price by $\varepsilon$ and increase their profits by getting the whole markets, so it must be that $p_2 = 1$.

4 Full enforcement

We now turn to the second case, where intellectual property is fully enforced worldwide. There are three possible cases.

1. If

$$a < \frac{\alpha \mu L_A}{[(1-\beta) + (1-\alpha)(\mu-1)]L_B} < b,$$

then (regime E1), all 1-goods are produced in country B, and all 2-goods are produced in country A. Prices are given by

$$w_A = 1,$$

$$p_2 = \mu,$$

$$w_B = \frac{\alpha \mu L_A}{(1-\beta)L_B + (1-\alpha)(\mu-1)L_B},$$

and

$$p_1 = \mu w_B/b.$$ 

Profits for a 2-good produced in country A are given by

$$\pi_2 = (\mu - 1)\frac{L_A}{N_2};$$

(4)

profits for a 1-good produced in country B are given by
\[ \pi_1 = (\mu - 1) \frac{L_B}{N_1} w_B \]
\[ = (\mu - 1) \frac{L_B}{N_1} \frac{\alpha \mu L_A}{(1 - \beta) L_B + (1 - \alpha)(\mu - 1)L_B} \] (5)

National incomes are given by

\[ Y_A = \mu L_A + (\mu - 1)L_B w_B; \]
\[ Y_B = w_B L_B. \]

If we compare to case P1 above, it is clear that enforcement redistributes income from country B to country A, in the form of monopoly profits for patent holders in country A.

2. If

\[ \frac{\alpha \mu L_A}{[(1 - \beta) + (1 - \alpha)(\mu - 1)]L_B} < a, \]

then (regime E4) country B produces both 1-goods and 2-goods, and country A only produces 2-goods. Prices are given by

\[ w_A = 1, \]
\[ w_B = a, \]
\[ p_2 = \mu, \]
\[ p_1 = \mu a / b. \]

Employment of workers in 2-goods in country B is given by

\[ L_{2B} = \frac{(1 - \beta) a L_B + (1 - \alpha)(\mu - 1) a L_B - \alpha \mu L_A}{\mu a}. \]

Profits for producers of 1-goods are equal to

\[ \pi_1 = (\mu - 1)a \frac{L_B - L_{2B}}{N_1}; \]
and for producers of 2-goods they are equal to

$$\pi_2 = (\mu - 1) \frac{L_A + aL_{2B}}{N_2}.$$ 

National incomes are given by

$$Y_A = \mu L_A + (\mu - 1)aL_B;$$
$$Y_B = aL_B.$$

3. If

$$\frac{\alpha \mu L_A}{[(1 - \beta) + (1 - \alpha)(\mu - 1)]L_B} > b,$$

then (regime E2) country A produces both types of goods, and country B produces 1-goods. Prices are given by

$$w_a = 1,$$
$$w_B = b,$$
$$p_1 = p_2 = \mu.$$

Employment in 2-goods in country A is given by

$$L_{2A} = \frac{\alpha \mu L_A}{(1 - \beta) + (1 - \alpha)(\mu - 1)}$$

Profits are now given by

$$\pi_1 = (\mu - 1) \frac{bL_B + L_A - L_{2A}}{N_1}$$

$$\pi_2 = (\mu - 1) \frac{L_{2A}}{N_2}$$

Incomes are

$$Y_A = \mu L_A + (\mu - 1)bL_B;$$
$$Y_B = bL_B$$
Figure 2 depicts which regime prevails as a function of $L_A/L_B$ and $\mu$. The
zones are substantially different from the enforcement regime. Note that profits
can be written $\pi_i = q_i/N_i$, where $q_i$ is a constant which only depends on the
regime being considered.

5 Innovation

We now analyze the determinants of innovation.

The R and D sector is competitive. All agents can borrow and lend at a fixed
nominal rate $r$. If $V_i$ denotes the value of a patent associated with an $i$-good, then

$$rV_i = \pi_i + \dot{V}_i \quad (6)$$

In equilibrium, researchers allocate themselves between the two types of
innovation. The labor market for researchers is perfectly competitive, so that
they earn the same wage in each sector.

Let us first consider the full enforcement case. If $\theta$ is interior, and we confine
the analysis to this case, the expected value from one unit of research must be
the same in both sectors, implying

$$\gamma_1 N_1 V_1 = \gamma_2 N_2 V_2. \quad (7)$$

Substituting (1) into (6), integrating forward and using (7), we get the equi-
librium value of $\theta$:

$$\theta = \frac{(r\gamma_1 + \gamma_1\gamma_2)q_1 - r\gamma_2 q_2}{\gamma_1 \gamma_2 (q_1 + q_2)}. \quad (8)$$

Note that $\theta$ does not depend on $N_1$ and $N_2$, and is constant through time. It
is increasing in the productivity of research for 1-goods $\gamma_1$ and the profitability
of these goods $q_1$, and decreasing in $\gamma_2$ and $q_2$. Finally for $\theta$ to be interior, we
need that

$$\frac{r \gamma_1 q_1}{r \gamma_2 + \gamma_1 \gamma_2} \leq q_2 \leq \frac{(r \gamma_1 + \gamma_1 \gamma_2) q_1}{r \gamma_2}$$

(8)

We can then compute the growth rate of the mass of goods for each variety

$$\dot{N}_1 = \frac{(r \gamma_1 + \gamma_1 \gamma_2) q_1 - r \gamma_2 q_2}{\gamma_2 (q_1 + q_2)} = g_1$$

$$\dot{N}_2 = \frac{(r \gamma_2 + \gamma_1 \gamma_2) q_2 - r \gamma_1 q_1}{\gamma_1 (q_1 + q_2)} = g_2$$

In the no enforcement case, $\pi_1 \leq 0$ always. If $\pi_2 > 0$, then $\theta = 0$ and $g_2 = \gamma_2$, while $g_1 = 0$. If $\pi_2 = 0$, then researchers are ‘unemployed’ and $g_1 = g_2 = \pi_1 = \pi_2 = 0$.

6 Welfare

The regimes characterized above are valid at a point in time, but note that the conditions do not depend on the values of $N_1$ and $N_2$ (That is because preferences are Cobb-Douglas between the two aggregates). Therefore, if an economy is in a given regime at $t = 0$, it will stay in that regime. If from time $t = 0$ on, $N_1$ and $N_2$ grow at constant rates $g_1$ and $g_2$, it is then easy to express the welfare of both countries as a function of their nominal national income, the price levels, the initial number of goods and their growth rates. We get

$$U_{At} = \frac{\alpha}{\sigma - 1} \ln N_{1t} + \frac{1 - \alpha}{\sigma - 1} \ln N_{2t} + \ln Y_A - \alpha \ln p_1 - (1 - \alpha) \ln p_2,$$

where time indices have been dropped from $p_t$ and $Y_A$, because they are constant throughout if one remains in the same regime. Similarly,

$$U_{Bt} = \frac{\beta}{\sigma - 1} \ln N_{1t} + \frac{1 - \beta}{\sigma - 1} \ln N_{2t} + \ln Y_B - \beta \ln p_1 - (1 - \beta) \ln p_2.$$
The present discounted values, defined after having dropped the constant terms, are given by

\[ V_A = \int_0^{+\infty} U_A e^{-\rho t} dt = \frac{\alpha}{\sigma-1} \left( \frac{\ln N_{10}}{\rho} + \frac{q_1}{\rho^2} \right) + \frac{1 - \alpha}{\sigma-1} \left( \frac{\ln N_{20}}{\rho} + \frac{q_2}{\rho^2} \right) + \frac{\ln Y_A - \alpha \ln p_1 - (1 - \alpha) \ln p_2}{\rho}. \]

\[ V_B = \frac{\beta}{\sigma-1} \left( \frac{\ln N_{10}}{\rho} + \frac{q_1}{\rho^2} \right) + \frac{1 - \beta}{\sigma-1} \left( \frac{\ln N_{20}}{\rho} + \frac{q_2}{\rho^2} \right) + \frac{\ln Y_A - \beta \ln p_1 - (1 - \beta) \ln p_2}{\rho}. \]

To facilitate comparison, I will assume that the world economy is initially in a balanced growth path with enforcement, and compare each country’s welfare if enforcement continues with its value if country B stops enforcing IPRs. If so, then under both alternatives \( N_i \) grows at a constant rate and the above formulas may be applied. We assume full specialization of each country in the enforcement case, i.e. that \( (3) \) holds; and that \( \frac{\alpha (L_A - L_R)}{(1 - \beta) L_B} < \frac{b}{\mu} \), which, given \( (3) \), implies that \( (2) \) also holds. Therefore, we focus the discussion on the case where the economy is in regime P1 (resp. E1) under piracy (resp. enforcement). We thus have from \( (5) \) and \( (4) \):

\[ q_1 = (\mu - 1) \frac{\alpha \mu L_A L_B}{(1 - \beta) L_B + (1 - \alpha)(\mu - 1)L_B} \]

and

\[ q_2 = (\mu - 1)L_A. \]

The following Table then summarizes the shifts in the variables of interest if the economy moves from an enforcement steady state to a no enforcement situation.
\[ \Delta \ln w_B = \ln(1 + \frac{(1-\alpha)(\mu-1)}{1-\beta}) \]
\[ \Delta \ln w_A = 0 \]
\[ \Delta \ln p_2 = 0 \]
\[ \Delta \ln p_1 = -\ln \mu + \ln(1 + \frac{(1-\alpha)(\mu-1)}{1-\beta}) \]
\[ \Delta g_1 = \frac{(r \gamma_1 + \gamma_1 \gamma_2) q_1 - r \gamma_2 q_2}{\gamma_1 \gamma_2 (q_1 + q_2)} < 0 \]
\[ \Delta g_2 = \frac{(r \gamma_1 + \gamma_1 \gamma_2) q_1 - r \gamma_2 q_2}{\gamma_1 (q_1 + q_2) \gamma_2 (q_1 + q_2)} > 0 \]
\[ \Delta \ln Y_B = \ln(1 + \frac{(1-\alpha)(\mu-1)}{1-\beta}) \]
\[ \Delta \ln Y_A = \ln(1 - \alpha \frac{\mu-1}{\mu-\beta}) \]

Table 1 – Changes in relevant variables when piracy is introduced.

When prices are expressed in units of labor in country A, we see that GNP falls in country A while it goes up in country B. Country A’s patent holders lose profit income from country B\(^3\), while at the same time the elimination of markups from country B’s products drives its wage up.

The relative price of 1-goods may either go up or down: while the elimination of the markup pushes it down, the higher wage in country 1 pushes it up. If \(\alpha = \beta\), then \(p_1\) is unchanged. If \(\alpha > \beta\), \(p_1\) falls. That is because the fall in the demand for 1-goods from consumers of country A (whose GNP falls) has a stronger effect than the rise in demand from consumers of country B (whose GNP goes up).

Intratemporal utility of consumers in country B goes up, given that the change in \(p_1\) never exceeds the change in \(w_B\). As for consumers in country A, their intratemporal utility may either go up or down, depending on whether or not the fall in \(p_1\) is strong enough to outweigh the foregone profits from 1-goods.

What about intertemporal welfare? Using Table 1 and noting that \(\mu = \sigma/(\sigma - 1)\), we get that

\[ \Delta V_A = \frac{\mu - 1}{\rho^2} \frac{(r \gamma_1 + \gamma_1 \gamma_2) q_1 - r \gamma_2 q_2}{\gamma_1 \gamma_2 (q_1 + q_2)} \gamma_2 - \alpha (\gamma_1 + \gamma_2) \]
\[ + \frac{1}{\rho} \left[ \ln(1 - \alpha \frac{\mu - 1}{\mu - \beta}) - \alpha \ln(1 + \frac{(1-\alpha)(\mu-1)}{1-\beta}) + \alpha \ln \mu \right]. \]

\(^3\)These are the direct redistributive effects discussed, for example, by MacCalman (2000).
\[ \Delta V_B = \frac{\mu - 1}{\rho^2} \left( r\gamma_1 + \gamma_1 \gamma_2 \right) q_1 - r\gamma_2 q_2 \left( \gamma_2 - \beta \left( \gamma_1 + \gamma_2 \right) \right) \]
\[ + \frac{1}{\rho} \left[ (1 - \beta) \ln \left( 1 + \frac{(1 - \alpha)(\mu - 1)}{1 - \beta} \right) + \beta \ln \mu \right] \]

To proceed, note that intertemporal optimization by consumers implies that
\[ r = \rho \cdot 1 \]

We can rewrite the preceding equations as

\[ \Delta V_A = \frac{\mu - 1}{\rho^2} \left( \rho\gamma_1 + \gamma_1 \gamma_2 \right) q_1 - \rho\gamma_2 q_2 \left( \gamma_2 - \alpha \left( \gamma_1 + \gamma_2 \right) \right) \]
\[ + \frac{1}{\rho} \left[ \ln \left( 1 - \alpha \frac{\mu - 1}{\mu - \beta} \right) - \alpha \ln \left( 1 + \frac{(1 - \alpha)(\mu - 1)}{1 - \beta} \right) + \alpha \ln \mu \right] \]

\[ \Delta V_B = \frac{\mu - 1}{\rho^2} \left( \rho\gamma_1 + \gamma_1 \gamma_2 \right) q_1 - \rho\gamma_2 q_2 \left( \gamma_2 - \beta \left( \gamma_1 + \gamma_2 \right) \right) \]
\[ + \frac{1}{\rho} \left[ (1 - \beta) \ln \left( 1 + \frac{(1 - \alpha)(\mu - 1)}{1 - \beta} \right) + \beta \ln \mu \right] \]

The first term of (9) and (10) represents the contribution to welfare of the dynamic effects of piracy on the evolution of \( N_i \): innovation now only takes place in 2-goods. Given condition (8), this contribution is negative if and only if

\[ \frac{\gamma_2}{\gamma_1 + \gamma_2} < \alpha \]

Hence, the dynamic effects of piracy are more likely to be negative, (i) the greater the shares of 1-goods in consumption \( \alpha \) and \( \beta \), and (ii) the greater the level of the dynamic externality in 1-goods, \( \gamma_1 \), relative to 2-goods, \( \gamma_2 \). The intuition is straightforward: The welfare loss from piracy is greater if 1-goods have a greater potential to grow, and if they are more important to consumers.
Note also that this dynamic component is proportional to $\mu - 1$. This captures the fact that new goods are more valued, the less substitutes they are to existing goods, in which case the equilibrium markup is also higher. Therefore, all gains and losses from piracy are commensurate with $\mu - 1$: the markup gives us the order of magnitude of both the level of monopoly distortions and the hedonic value of greater product diversity. While it is possible to disentangle the two effects by adding some power of $N$ to the utility function (Bénassy, 1996), the view that the elasticity of demand is lower when new goods are more valuable is intuitively appealing.

For country B, a similar condition holds for dynamic gains to be negative:

$$\frac{\gamma_2}{\gamma_1 + \gamma_2} < \beta. \quad (12)$$

The second terms of (9) and (10) are the static gains from piracy. For country B, they are always positive, since they result entirely from lower markups. Consequently, if (12) is violated, country B unambiguously gains from piracy. What if (12) holds. Static gains are positive, while dynamic gains are negative. One can check that the RHS of (12) is positive and then negative as $\rho$ falls. Thus there exists a critical level of the rate of time preference below which piracy harms country B, and above which if benefits country B.

Turning now to country A, it is more difficult to compute the sign of the static gains. One can however note that for $\beta \geq \alpha$ ("home bias"), $1 + \frac{(1-\alpha)(\mu-1)}{\beta} \geq \mu$, so that country A unambiguously loses from piracy. In such a case, it will always oppose piracy if (11) holds. If (11) does not hold, country A will oppose piracy if the discount rate is high but support it if the discount rate is low, as piracy then reshuffles research into the goods with more favorable learning externalities given its preferences. If $\beta < \alpha$, the third term in the static gains for country A dominates the second, implying that it would gain from piracy if $\mu$ is very large. However, for $\mu$ close enough to one, the static gains are again negative,
since a first-order Taylor approximation yields \( \frac{\alpha - \beta}{1 - \beta} - \frac{1}{1 - \beta} < 0 \).

To summarize, the preceding discussion suggests that the long-run gains from IP enforcement to each country depend on how the expenditure share of 1-goods compare to the relative growth potential of 2-goods. For an equal growth potential, long-run gains from IP enforcement are positive if the expenditure share on 1-good exceeds 50% (or, more generally, the expenditure share on 2-goods).

If \( \alpha = \beta \), then country A is always less likely to gain/more likely to lose from piracy. However, the assumption \( \beta > \alpha \), i.e. that goods in which a country has a comparative advantage are also goods that its consumers like better relative to other consumers in the rest of the world ("home bias"), is also intuitive. Such a property could arise if economies were once closed and if productivity levels were determined by learning by doing, or tastes by habit formation.

If 1-goods are more important to consumers in country B than to consumers in country A, and if at the same time it has a comparative advantage in producing these goods, then it may actually lose more from piracy than country A: While static gains are negative for country A and positive for country B, condition (12) is more likely to hold than (11), so that country B is more likely to suffer dynamic losses from piracy; that is because it cares more about the goods for which innovation has fallen.

Finally, one can just check that computing dynamic gains amount to computing the change in growth rates for the aggregate consumption index. The growth rate of the aggregate consumption index is

\[
g_A = \frac{\alpha g_1 + (1 - \alpha) g_2}{\sigma - 1},
\]

in country A and

\[
g_B = \frac{\beta g_1 + (1 - \beta) g_2}{\sigma - 1},
\]
in country B. Using the above formulae, we see that piracy reduces growth in country A if and only if

\[
\alpha > \frac{\gamma_2}{\gamma_1 + \gamma_2},
\]

which is a sufficient condition for country A to always lose in welfare terms. Similarly, country B’s growth is reduced iff:

\[
\beta > \frac{\gamma_2}{\gamma_1 + \gamma_2},
\]

which is a necessary condition for welfare to fall in country B. Finally note that if \( \beta > \alpha \), country B grows less fast than country A in the piracy regime. However, that is in terms of the relevant basket of goods consumed by its residents. Both countries would grow at the same rate in PPP terms.

6.1 The role of the markup

While the preceding approximation is fairly tractable, it cannot be used to study the role of the markup \( \mu \). As already explained, this parameter plays a twin role, as it goes up, so does the static loss from monopoly pricing. But, at the same time, complementarity between goods goes up, and people value innovation more.

I report the results of numerical simulations where country B may lose from IP enforcement. In the first simulation, (figures 3-5), the parameters are \( L_A = L_B = 1 \), \( \alpha = \beta = 0.3 \), \( a = 0.2 \), \( b = 0.9 \), \( \gamma_1 = 0.02 \), \( \gamma_2 = 0.006 \), \( \rho = 0.03 \).

Here dynamic losses come from the fact that 2-goods involve smaller learning externalities (\( \gamma_2 < \gamma_1 \)).\(^5\) Figure 3 plots the evolution of the static and dynamic gains for country A. Both are negative: country A loses both from slower growth and foregone profits. Furthermore, losses go up with the markup. As for

\(^5\)One may check that for these parameters, the economy is in regimes P1 and E1 for the values of \( \mu \) that are used.
country B (Fig. 4), it has static gains from the elimination of monopoly pricing, and dynamic losses from lower innovation in 1-goods. Both gains and losses are increasing with the markup. Whether the country gains or loses on net depends on the discount factor. Figure 5 plots the value of $\rho$ below which piracy does not pay for country B. A striking property is that it is increasing with the markup; that suggests that as goods become more complementary, the value of having more 1-goods becomes stronger relative to the greater static losses from monopoly pricing. Equivalently, given $\rho$, a greater markup makes it more likely that piracy is not in the interest of country B. Note however that with these parameter values the critical $\rho$ is very small.

In the second simulation, (figures 6-9), parameters are $L_A = L_B = 1$, $\alpha = 0.2$, $\beta = 0.6$, $a = 0.2$, $b = 0.9$, $\gamma_1 = \gamma_2 = 0.02$, $\rho = 0.01$. Dynamic losses to country B now come from the fact that it puts a high weight on 1-goods, contrary to country A. As Figure 6 shows, country A has static losses from piracy, but positive dynamic gains as innovation is reallocated to the goods on which it spends a high share. Thus it will prefer piracy if patient enough. Figure 7 plots the critical discount rate of country A beyond which it prefers piracy. It is again increasing with the markup, meaning that the higher the monopoly rent, the more likely it is that the innovative country A prefers piracy! This paradoxical result comes from the fact that the structure of innovation is inefficient from country A’s perspective, because too much effort is devoted to producing the 1-goods that its consumers value little. Given that country B is not productive enough to compete with country A in the production of 2-goods, letting it engage in piracy triggers a reallocation of research in favor of 2-goods which is quite welcome by consumers in country A, provided they are patient enough to be willing to trade foregone royalties against future increments in the number of varieties it likes. Greater complementarities between goods increases the foregone monopoly rents as well as the utility gain from having more 2-goods in

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6 The value of $\rho$ was lowered so as to prevent $\theta$ from being negative.
the future, and the simulations tells us that the latter effect dominates. Turning now to country B, net gains are reported on Figure 8. The pattern is similar to the preceding simulation. Again, greater monopoly power makes it more likely that country B loses from piracy, as the critical discount rate is again increasing with $\mu$. Here again, the critical discount rates are quite low: static effects seem to dominate for plausible values of the discount rate.

7 Summary and conclusion

The model yields a number of insights with respect to the costs and benefits of piracy in an international trade context. First, piracy triggers a reallocation of R and D in favor of the goods (2-goods) in which the richer country has a comparative advantage. Second, this reallocation tends to reduce welfare if (i) dynamic learning externalities are high in 1-goods relative to 2-goods, and if (ii) the share of 1-goods in consumption is high. Third, if consumers across the world spend the same fraction of their income on each type of goods, the richer country is more likely to incur a net loss than the poorer, because of the foregone royalties associated with its intellectual property. However, if there is a "home bias" in that consumers spend more on the goods in which their country has a comparative advantage than foreign consumers, then the poorer country may lose, and it will grow less fast, in utility terms, under the piracy regime than the richer country. Finally, a higher monopoly power does not increase the net social gains from piracy, because it is associated with greater complementarities between goods which also makes innovation more valuable.
REFERENCES


Goh, A. and J. Olivier (2002), "Free Trade and Protection of Intellectual Property Rights: Can We Have One Without the Other?", mimeo, HEC.


8 Appendix

Let us first characterize an equilibrium where country B only produces 1-goods. We will normalize the wage of workers in country A to $w_A = 1$. Country B can produce any good competitively. Therefore, its producers will pin down the price of 1-goods:

$$p_1(k) = w_B/b = p_1, \forall k.$$ 

2-goods can be produced in country B at cost $w_B/a$. Therefore,

$$p_2(k) = p_2 = \min \left( \frac{w_B}{a}, \mu w_A \right),$$

where $\mu = \sigma/(\sigma - 1)$ is the monopoly markup one would observe absent competition from country B.

In equilibrium, it must be better to buy goods from competitive producers in country B than from the patent holder in country A. Therefore, one must have

$$w_B/b < \mu w_A = \mu.$$ 

To compute equilibrium quantities, note that national income in country B is

$$Y_B = w_B L_B$$

Country A potentially produces both types of goods. Let $L_{iA}$ be the labor input in country A devoted to $i$-goods. Then:

$$Y_A = p_2 L_{2A} + \frac{w_B}{b} L_{1A}$$

That equation is the GDP at market prices national income identity, saying that GDP is the total value of final goods being sold.

Equilibrium of supply and demand for either type of good leads to the following equality, which must hold in equilibrium:
\[ \alpha p_2 L_{2A} = w_B \left[ \frac{1 - \alpha}{b} L_{1A} + (1 - \beta)L_B \right] \] (13)

Let us first look for an equilibrium with monopoly pricing for the 2-goods, and no production of good 1 in country A. Then \( w_B \) must be such that

\[ \mu < w_B/a, \] (14)

and

\[ w_B/b < w_A = 1. \] (15)

The first condition states that the monopoly price of 2-goods is lower than their production cost in country B. The second condition says that the cost of 1-goods in country A is higher than in country B, so that no holder of a patent for a 1-good in country A wants to actually produce.

We then have \( L_{1A} = 0, L_{2A} = L_A \). Consequently, eq. (13) implies

\[ w_B = \frac{\alpha \mu L_A}{(1 - \beta)L_B}. \]

Substituting into conditions (14) and (15), we find that this regime holds iff

\[ a < \frac{\alpha L_A}{(1 - \beta)L_B} < \frac{b}{\mu} \]

which characterizes regime (P1). Note that this regime may only hold if \( b > a \mu \).

Let us now look for a regime where country A produces some 1-goods, assuming again monopoly pricing in the 2-goods. This will be the case if (14) holds and if

\[ w_B \geq b. \] (16)

Given that \( p_1 = w_B/b \), if \( p_1 > 1 \), then it is profitable for producers of 1-goods in country A to cover the whole market, leaving no output for producers in country B (the standard dominant firm result). That cannot be true in
equilibrium, as it contradicts the full employment condition in country B and
the assumption that country B only produces 2-goods. Therefore in such a
regime it must be that $w_B = b$. Profits are equal to zero for producers of
1-goods in country A. $L_{2A}$ is then determined by (13), yielding

$$L_{2A} = \frac{(1 - \alpha)L_A + b(1 - \beta)L_B}{\alpha \mu + 1 - \alpha}.$$ 

For this regime to hold, it must be that (14), holds, i.e. $\mu < b/a$, and that
$L_{2A} \leq L_A$, that is

$$\frac{b}{\mu} < \frac{\alpha L_A}{(1 - \beta)L_B}.$$ 

This therefore characterizes regime P2.

Now let us consider the possibility of limit pricing in 2-goods, and no pro-
duction of 1-goods in country A. That corresponds to the following conditions:

$$p_2 = \frac{w_B}{a} < \mu$$

and

$$w_B < b.$$ 

One then has $L_{1A} = 0, L_{2A} = L_A$ Plugging these conditions into (13), we
find that this is a knife-edge case which can only hold if $\alpha L_A/a = (1 - \beta)L_B$. We shall therefore ignore it.

Now consider the case of limit pricing in 2-goods, and some production of
1-goods in country A. We again have $p_2 = \frac{w_B}{a} < \mu$ and $w_B = b$. Substituting
into (13) we get

$$L_{2A} = \frac{(1 - \alpha)L_A + b(1 - \beta)L_B}{\alpha b/a + 1 - \alpha}.$$ 

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This regime holds if \( b/a < \mu \) and \( L_{2A} < L_A \), or equivalently

\[
a < \frac{\alpha L_A}{(1 - \beta)L_B}.
\]

Thus we have characterized regime P3.

Now let us look at an equilibrium where country B produces both goods.

It must be that \( p_1 = w_B/b \) and \( p_2 = w_B/a \).

For country B to produce some of the 2-goods, it must be that \( p_2 \leq w_A = 1 \), otherwise monopolists in country A would flood the market.

Assume \( p_2 < 1 \), then a fortiori \( p_1 < 1 \), as \( b > a \). Then, no producer in country A would be profitable, which contradicts the full employment condition there. Thus we must have \( p_2 = 1 \), implying \( w_B = 1 \). Then \( p_1 = a/b < 1 \), implying that country A does not produce 1-goods.

The equilibrium is then easily characterized. Profits in country A are equal to zero, GDP in country A is

\[
Y_A = w_A L_{2A} = L_A;
\]

and GDP in country B is given by

\[
Y_B = w_B L_B = a L_B.
\]

Equilibrium in the goods markets can be written

\[
\frac{\alpha}{p_1} Y_A + \frac{\beta}{p_1} Y_B = b L_{1B},
\]

implying

\[
L_{1B} = \frac{\alpha L_A + \beta a L_B}{a}.
\]

For this regime to hold we must have \( L_{1B} < L_B \), i.e.

\[
\frac{\alpha L_A}{(1 - \beta)L_B} < a.
\]
This therefore characterizes regime P4.

We now turn to the enforcement regime.

In this regime all production units are owned by the monopolist in country 2, regardless of where they are located. Prices are thus always equal to the markup times the wage in the country where the good is being produced.

First consider the case where country A only produces 2-goods and country B only produces 1-goods. We then have

\[ p_1 = \mu w_B / b_i \]  
(17)

\[ p_2 = \mu w_A = \mu. \]  
(18)

GNP for country A’s residents is

\[ Y_A = p_2 L_A + (\mu - 1) w_B L_B, \]  
(19)

where the second term represents profits repatriated from country B.

GNP in country B is given by

\[ Y_B = w_B L_B. \]  
(20)

Equilibrium in goods markets is given by

\[ (1 - \alpha) \frac{Y_A}{p_2} + (1 - \beta) \frac{Y_B}{p_2} = L_A. \]  
(21)

Substituting (17)-(20) we get

\[ w_B = \frac{\alpha \mu L_A}{(1 - \beta)L_B + (1 - \alpha)(\mu - 1)L_B}. \]

For this regime to be an equilibrium, it must be that firms do not want to relocate in the other country. Consider first a monopoly based in country A. Its profit is given by

\[ \pi_2 = (\mu - 1)c_2, \]
where \( c_2 = L_A/N_2 \) is the amount sold of each 2-good.

If it decides to relocate in country B, its unit cost is \( w_B/a \). It will charge \( p_2' = \mu w_B/a \), and sell a quantity equal to \( c_2' = c_2(p_2'/p_2)^{\sigma} \), so that its profits are

\[
\pi_2' = c_2(\mu - 1)(w_B/a)^{1-\sigma}.
\]

We must have \( \pi_2' \leq \pi_2 \), i.e. \( w_B/a > 1 \). Similarly, for producers of 1-goods not to want to relocate in country A, we need \( w_B/b < 1 \). These two inequalities are equivalent to

\[
a < \frac{\alpha \mu L_A}{[(1-\beta) + (1-\alpha)(\mu-1)]L_B} < b,
\]

and this characterizes regime E1. It is then straightforward to compute profits.

Now consider an equilibrium where country B produces 1-goods and 2-goods, and country A produces only 2-goods. Producers of 2-goods must be indifferent between locating in country A and locating in country B, therefore we must have \( w_B/a = w_A = 1 \). Hence

\[
w_B = a.
\]

Consequently, \( p_2 = \mu \) and \( p_1 = \mu a/b \).

Equations (19) and (20) still hold, while (21) must be replaced by

\[
(1 - \alpha) \frac{Y_A}{p_2} + (1 - \beta) \frac{Y_B}{p_2} = L_A + aL_2B.
\]

Substituting, this allows to compute \( L_{2B} \):

\[
L_{2B} = \frac{(1 - \beta)aL_B + (1 - \alpha)(\mu - 1)aL_B - \alpha \mu L_A}{\mu a}.
\]
For this regime to be an equilibrium, one must have $0 < L_{2B} < L_B$, which, as the second inequality $L_{2B} < L_B$ is always satisfied, is equivalent to

$$\frac{\alpha \mu L_A}{[(1 - \beta) + (1 - \alpha)(\mu - 1)] L_B} < a.$$ 

This characterizes regime E4.

Finally, consider a regime where country A produces 1-goods and 2-goods, and country B only produces 1-goods. Equality of unit costs across countries for 1-goods implies

$$w_B = b.$$ 

Hence, $p_1 = p_2 = \mu$; and equilibrium in goods markets can be obtained by substituting $L_{2A}$ for $L_A$ into the RHS of (21), yielding

$$L_{2A} = (1 - \alpha)L_A + \frac{bL_B}{\mu} \left[ (1 - \beta) + (1 - \alpha)(\mu - 1) \right] .$$ 

This regime holds if $L_{2A} < L_A$, or equivalently

$$\frac{\alpha \mu L_A}{[(1 - \beta) + (1 - \alpha)(\mu - 1)] L_B} > b.$$ 

This therefore characterizes regime E2.
Figure 1: specialization and pricing regimes, piracy
Figure 2: specialization and pricing regimes, Enforcement
Figure 3: Country A
Figure 4: Net gains, country B
Figure 5: Critical discount rate, country B
Figure 6: Country A's gains from piracy

-25 -20 -15 -10 -5 0 5

1 1,05 1,1 1,15 1,2 1,25 1,3 1,35 1,4 1,45 1,5 1,55 1,6 1,65 1,7 1,75 1,8

Gains

Markup

Static
Dynamic
Figure 7: Critical discount rate, country A
Figure 8: Net gains, country B