Pensions with endogenous and stochastic fertility[1]

Helmuth Cremer, Firouz Gahvari and Pierre Pestieau

University of Toulouse (IDEI and GREMAQ)
21, allée de Brienne
31000 Toulouse - France

Department of Economics
University of Illinois at Urbana-Champaign
Champaign, IL 61820, USA

CORE, Delta, CEPR and CREPP, University of Liège
7, bd du Rectorat
4000 Liège - Belgique

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Abstract

This paper studies the design of a pay-as-you-go social security system in a society where fertility is in part stochastic and in part determined through capital investment. If parents' investments in children are publicly observable, pension benefits must be linked positively to the the level of investment, and payroll taxes negatively to the number of children. The outcome is characterized by full insurance with all parents, regardless of their number of children, enjoying identical consumption levels. Without observability, benefits must increase, and payroll taxes decrease, with the number of children. The second-best level of investment in children, and the resulting average fertility rate, are less than their corresponding first-best levels.

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1 Introduction

The recent fertility decline in the West is often cited as a major impediment to the fiscal solvency of pay-as-you-go (PAYGO) social security systems. At the same time, the pay-as-you-go feature of the social security systems has partly been blamed for causing the observed fertility decline. The reason for this latter linkage is that in such systems, the size of a person's pension benefits depends on everybody else's fertility decisions leading to a decentralized equilibrium outcome with too few children. This problem is exacerbated by another externality problem associated with the "quality" of children, and their human capital accumulation, through educational decisions of the parents. The rate of return of a pay-as-you-go system depends not just on the fertility rate, but also on productivity growth. The more productive the children, the higher will be their ability to produce and to pay taxes. This reinforces the public good nature of a family's child-rearing activities. It is not surprising then that some economists have recently advocated a policy of linking pension benefits (or contributions) to individuals' fertility choices.[1]

Such a policy raises a number of objections, however. What truly determines fertility, and what accounts for the observed evolution in fertility behavior, are still open questions. What is clear though is that no one can fully control fertility. The actual number of children in a family does not necessarily coincide with the number the parents initially intended to have. Infertility, premature death, misplanning and multiple births are some of the reasons explaining this gap. Similarly, one cannot deterministically determine the future earning abilities of children simply by investing in their education and training. Making benefits independent of the number of children can then be viewed as a mechanism to insure parents against these various random shocks.

This paper studies the design of pension systems in a setting that takes into account the gap between intended and actual quantity, and quality, of children, the implicit free riding, and the non-observability of effort. We use a two-period model in which the actual "number" of children is in part the result of some early investment decisions the

[1] Abio *et al.* (2004), Bental (1989), Cigno *et al.* (2003), Fenge and Meier (2004), Kolmar (1997), van Groezen *et al.* (2000, 2003).

prospective parents make at the beginning of the first period. However, the number of children is observed early and the parents can adjust both their first and the second period consumption levels accordingly. We do not specifically distinguish between fertility and education decisions. Instead, we lump the investments in quantity and quality together as if one decision determines both. In doing this, we use the concept of number of children in "efficiency units" which is widely used in growth theory.[2]

There are two underlying agency problems here: adverse selection and moral hazard. Adverse selection problem arises if individuals differ in child-rearing ability, or in taste for children. To simplify matters, and to distinguish between the implications of the two, here we focus on the moral hazard issue leaving the adverse selection considerations to another paper.[3] This allows us to work with (ex-ante) identical individuals.

If fertility were fully deterministic, the moral hazard (incentive) problem could easily be overcome and all individuals enticed to choose the socially optimal first-best number of children. On the other hand, if fertility were fully random, there would be no moral hazard (incentive) problem and the optimal social security system fully insures parents against the fertility uncertainty. When fertility is determined in part through investment and in part through random elements, there naturally arises a question as to the possible tradeoff between full insurance and incentive considerations. In a first-best environment when the parents' "effort levels" in having and raising (productive) children are publicly observable, however, one may be able to achieve both objectives. We show that this is the case. We also show that the decentralization of the first best requires pension benefits to be linked positively to the parents' level of investment in children (and not

[2] The key distinguishing element between quantity and quality decisions is one of timing. The number of children born is known quite early; the quality of children (i.e. their future earning capacity) is determined much later. To account for both features one needs a model with at least three periods of decision making. This makes the problem more complicated than necessary. Concentrating on a setting with two periods of decision making, as opposed to three, simplifies the modeling substantially. Cigno and Luporini (2003) have a three-period model; however they do not optimize over tax instruments.

[3] See, Cremer *et al.* (2004). The moral hazard problem has also been studied by Sinn (2004), though in a different setting, and Cremer *et al.* (2003). That paper was based on two very restrictive assumptions which we drop here. First, we had ignored all possibilities for private savings, assuming that the only mechanism for transfer of resources to the future is (except for possible "voluntary" arrangements between parents and children whereby children help their retired parents with the expectation that their own children would help them) a PAYGO public pension system. Second, we had assumd that the number of children is observed late in the first period so that the first-period consumption could not vary with the number of children.

to their number), coupled with payroll taxes that vary inversely with the number of children.

When the parents' investment in children is not publicly observable, a tradeoff between insurance and incentive considerations surfaces. We prove that in this case, the optimal level of investment in children, and the resulting average fertility rate, are less than their corresponding first-best values. To attain the second-best, one must institute a pay-as-you-go pension plan under which benefits increase, and payroll taxes decrease, with the number of children. Moreover, families with more children should be more than compensated for the extra cost of children so that they will enjoy a higher level of first-period consumption. Interestingly, with the exception of the last finding, these results carry over to situations where payroll taxes cannot depend on the number of children.

Finally, we examine how the endogeneity of fertility modifies Samuelson's classic requirement for optimality of PAYGO pension plans.[4] To do this, we assume that individuals can transfer resources to the future using a storage technology with a fixed rate of return. We show that the possibility of investing in fertility, and thus making this technology more productive, implies that PAYGO will dominate storage over a higher range of returns. This will be the case in both first- and second-best environments; however, this range will be smaller in the second best.

2 The basics

Consider a two-period overlapping generations model in the steady state. Each generation consists of a continuum of identical individuals. The young have fixed endowments y and the old live on pensions. Preferences of the young depend positively on their consumption in the first period, c, and their consumption in the second period, d. A parent can have either n_1 or n_2 children, with $n_2 > n_1$. The actual realization of n_i depends on an initial "investment in children," k, and on some random shock.[5] Thus when a

[4] This is when the population growth rate (what Samuelson~(1958) called the "biological" rate of interest), which is assumed exogenous, exceeds the interest rate.

[5] The investment may be construed as an investment to enhance "quality" as well as "quantity" of children. With this interpretation, one should think of n_i as being measured in "efficiency units".

parent invests k, he will have n_2 children with probability $\pi(k)$ where $0 \leqslant \pi(k) \leqslant 1$ and $\pi'(k) > 0$ ($\pi''(k) < 0$ and $\pi(0) > 0$). Naturally, the probability of having n_1 children is given by $1 - \pi(k)$. Whenever it makes the notation simple, we substitute $\pi_2(k)$ for $\pi(k)$ and $\pi_1(k)$ for $1 - \pi(k)$. The cost of having children is not limited to the initial investment k. There are other costs that vary proportionately (at the rate of $\theta \geq 0$) to the actual number of children. These costs are also borne in the first period.

To keep the model simple, assume that preferences over (c_i, d_i), $i = 1, 2$, are represented by an additive utility function. Consequently, at the beginning of the first period, the expected utility of the young (i.e. future parents) is written as

$$U = \sum_{i=1}^{2} \pi_i(k) [u(c_i) + v(d_i)], \tag{1}$$

where $u(\cdot)$ and $v(\cdot)$ are strictly concave functions.

There are two potential mechanisms for financing second-period consumptions: storage or a PAYGO pension plan. Under the storage technology, part of the initial endowment is invested yielding a *fixed* rate of return, r.[6] Under a PAYGO scheme, the government collects taxes from the current young and distributes the proceeds to the retired. With the young having, on average,

$$\bar{n}(k) = \pi_1(k) n_1 + \pi_2(k) n_2$$

children, the PAYGO rate of return is $\bar{n}(k) - 1$. This corresponds to what Samuelson called the biological rate of interest.

2.1 Laissez faire

Absent any government intervention, each individual maximizes his expected utility subject to two budget constraints, one of which becoming relevant ex post, depending on the number of children. The Lagrangian expression associated with the individual's problem is

$$\mathcal{L}_L = \sum_i \left\{ \pi_i(k) [u(c_i) + v(d_i)] + \lambda_i \left[y - c_i - \frac{d_i}{1+r} - k - n_i\theta \right] \right\}.$$

[6] The rate of return is net of any "capital depreciation".

It follows from the first-order conditions of this problem with respect to c_i and d_i that

$$\frac{v'(d_i)}{u'(c_i)} = \frac{1}{1+r}, \qquad i = 1, 2. \tag{2}$$

This is the classic condition for optimal intertemporal consumption. We also have

$$\frac{\partial \mathcal{L}_L}{\partial k} = \pi'(k)\left[u(c_2) + v(d_2) - u(c_1) - v(d_1)\right] - \sum_i \pi_i(k)\, u'(c_i). \tag{3}$$

Observe that individuals with n_1 children have higher disposable incomes, net of the cost of children, than individuals with n_2 children. It follows that $u(c_2) + v(d_2) \geq u(c_1) + v(d_1)$: utility is higher with n_1 children than with n_2. From (3) one then obtains that $\partial \mathcal{L}_L / \partial k \leq 0$ and

$$k_L = 0. \tag{4}$$

That under laissez faire $k = 0$, should not be surprising. Children bestow no utility on their parents so that there is no reason to invest in them (given that they are costly to have).[7]

Having characterized the equilibrium under laissez faire, we next characterize the first-best solution for this economy and then turn to the second best.

3 The utilitarian first-best

Assume first that the social planner has perfect information, particularly with respect to the individuals' investment levels in children k, and that he controls all the relevant variables in the economy. The planner determines which technology, storage or PAYGO, is used to finance old-age consumption and sets c_i, d_i and k accordingly. He maximizes the sum of steady-state lifetime utility

$$W = \sum_i \pi_i(k)\left[u(c_i) + v(d_i)\right], \tag{5}$$

[7] The individual's problem has been set up on the assumption that there are no private insurance markets. If individuals can buy fair insurance, they will pool their resources together and thus maximize their expected utility subject to the *single* budget constraint

$$\sum_i \pi_i(k)\left[y - c_i - \frac{d_i}{1+r} - k - \theta n_i\right].$$

In this case, one can easily show that we will continue to have $k = 0$, but that $c_1 = c_2$ and $d_1 = d_2$.

subject to two *alternative* resource constraints:

$$\sum_i \pi_i(k)\left[y - c_i - \frac{d_i}{1+r} - k - n_i\theta\right] = 0, \tag{6}$$

$$\sum_i \pi_i(k)\left[y - c_i - \frac{d_i}{\bar{n}(k)} - k - n_i\theta\right] = 0. \tag{7}$$

We will see below that it will never be optimal to use both technologies simultaneously. The first constraint holds when one saves for his own second-period consumption; the second when old-age consumption is financed through a PAYGO scheme with $\bar{n}(k) \equiv \pi_1(k)n_1 + \pi_2(k)n_2$.[8] The planner's problem is best solved sequentially. First, we find the optimum conditional on the use of storage and PAYGO technologies; then we compare the levels of welfare achieved at these two conditional optima.

3.1 Storage

Under the storage technology, the planner maximizes (5) subject to the resource constraint (6). Deriving the first-order conditions of this problem, one can easily establish that $c_1 = c_2 = c$; $d_1 = d_2 = d$. Thus, not surprisingly, consumption levels are equalized across types. The problem can then be written as

$$\max_{c,d,k} \quad W_S = u(c) + v(d), \tag{8}$$

$$\text{s.t.} \quad y - c - \frac{d}{1+r} - \bar{n}(k)\theta = 0. \tag{9}$$

The first-order conditions imply

$$\frac{v'(d)}{u'(c)} = \frac{1}{1+r}, \tag{10}$$

and

$$k^S = 0. \tag{11}$$

Equation (10) is the optimality condition for intertemporal consumption with the rate of return r on storage. It is also similar to the expression (2) under laissez faire. Condition

[8] As is well-known, one can always increase the steady-state utility level through the imposition of lump-sum taxes on the old and distributing the proceeds to the young (also in a lump-sum manner). However, such increases in steady-state utility are made at the expense of the old generation alive when the policy is instituted. These issues are not pertinent to the main point of this paper and are ignored in our discussions.

(11) obtains because $\partial W_S/\partial k < 0$ (when incorporating the budget constraint). It is identical to condition (4) under laissez faire.[9] As in there, increasing k has only costs and no benefits.[10] The values of c and d that solve this problem depend on (the exogenous) value of r, as does the associated level of welfare, $W_S^*(r)$. Moreover, it follows directly from (9) that W_S^* is an increasing function of r.

3.2 PAYGO

The problem of the social planner under a PAYGO pension plan is to maximize (5) subject to the resource constraint (7). In this case too, one obtains $c_1 = c_2 = c$ and $d_1 = d_2 = d$. The Lagrangian expression of the problem can then be written as

$$\mathcal{L}_P = \left[u(c) + v(d) + \mu\left(y - c - \frac{d}{\bar{n}(k)} - k - \bar{n}(k)\theta\right)\right],$$

with c, d and k as decision variables. One obtains the following optimality conditions[11]

$$\frac{v'(d)}{u'(c)} = \frac{1}{\bar{n}(k)}, \tag{12}$$

and

$$\frac{\partial \mathcal{L}_P}{\partial k} = \mu\left[\frac{\bar{n}'(k)\,d}{\bar{n}^2} - 1 - \bar{n}'(k)\theta\right] = 0. \tag{13}$$

Equation (12) is the counterpart of (10) with $\bar{n}(k) - 1$ as the net rate of return on "savings". The optimal level of investment in children, k^P, is determined according to the tradeoff stated in equation (13). This condition requires that the cost of increasing k, which includes the extra cost of children $\bar{n}'(k)\theta$, equals the benefits associated with the induced increase in the return to PAYGO. The level of welfare achieved at the PAYGO solution is denoted by W_P^*.

3.3 PAYGO versus storage

To determine the first-best solution one must compare the levels of welfare attained at the two conditional optima. First, observe that both conditional solutions imply

[9] The solution is thus identical to that under laissez faire with full private insurance.

[10] This extreme result holds because, as with the laissez faire solution, there are no direct benefits associated with having children in this setting. To capture such benefits, one may include n as a separate argument of the utility function. It would then be possible to have $k^S > 0$.

[11] The first-order condition with respect to k is for an interior solution. If we have a corner solution $k = 0$, the solution is identical to a case with exogenous fertility and one is back to the original Samuelson formulation.

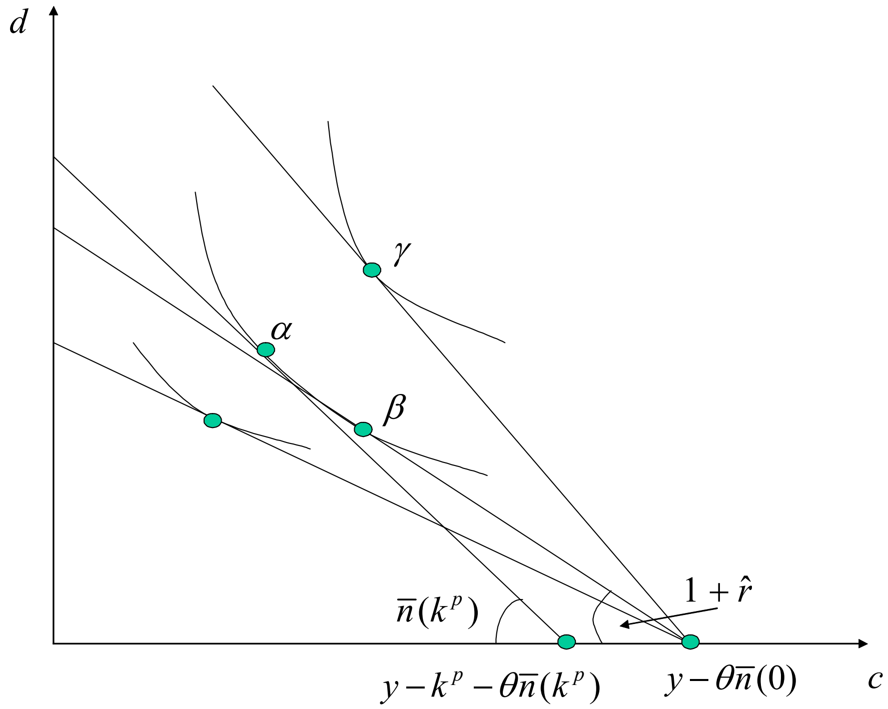

Figure 1: Figure 1: Optimal allocations with storage and PAYGO technologies

equalization of consumption levels across types. The parents are fully insured for the uncertainty they face over the number of children they will have. Given this common property, it is intuitively obvious that the choice between the technologies must depend solely on their respective "rates of return". Specifically, when $1+r \geq \bar{n}(k^p)$, the storage technology dominates. Its return is at least as high as that of PAYGO, but it does not require the initial investment k^P. Put differently, the rate of return for PAYGO must be higher than the rate of return on storage to compensate for the investment k^P. When $1+r < \bar{n}(k^p)$, the choice between the two technologies is more involved. This is best explained using a graphical representation in the (c, d) plane; see Figure 1.

Let α represent the optimal allocation between c and d under PAYGO. It corresponds to a point of tangency between an indifference curve and the resource constraint originating from $y - k^P - \theta\bar{n}(k^p)$ (endowment minus total cost of children) with a slope (in absolute value) of $\bar{n}\left(k^P\right)$. Define $\hat{r}$ as the rate of return on storage at which first-best welfare level under storage equals its PAYGO level: $W_S^*(\hat{r}) = W_P^*$. Graphically, $1+\hat{r}$ is the slope of the budget line (under storage) with horizontal intercept of $y - \theta\bar{n}(0)$ and which is tangent to the indifference curve corresponding to W_P^*.[12] One attains the same utility level under storage at point β, with more first-period and less second-period consumption. Observe also that β is not available unless *everyone* is subjected to PAYGO. That is, it is not optimal to use the two technologies simultaneously. With $r > \hat{r}$, society would opt for the storage technology and the solution γ. With $r < \hat{r}$, the demographic technology dominates storage.

The above discussion reveals how endogenous fertility modifies the Samuelsonian condition for instituting a PAYGO system. Denote the level of welfare under a PAYGO system with an exogenous rate of return equal to $\bar{n}(0) - 1$ by W_0. Clearly, $W_0 < W_P^*$. Now, with $W_S^*(r)$ being an increasing function of r, the value of r that equates the rate of returns on storage and PAYGO technologies in a Samuelsonian world, $\tilde{r} = \bar{n}(0) - 1$, must be less $\hat{r}$. It then follows that for all $1+r \in (\bar{n}(0), 1+\hat{r})$ one would use the storage technology when fertility is exogenous and PAYGO when fertility is endogenous; see Figure 2.[13] Put differently, the endogeneity of n creates more possibilities for PAYGO to outperform storage. Clearly, the more productive is investment in children, the larger will be the region where PAYGO dominates storage (W_P^* will be greater and $1+\hat{r}$ moves to the right). On the other hand, the cost of investment in children works to diminish the advantage of PAYGO. When this cost is high, W_P^* will be smaller and $(\bar{n}(0), 1+\hat{r})$

[12] The graphical representation, and the comparison between $1+r$ and $\bar{n}\left(k^P\right)$, assume no initial fixed investment costs in the storage technology. This is inconsequential; its only import being $\hat{r} < \bar{n}\left(k^P\right) - 1$: the critical technology-switching rate of return on storage is less than the PAYGO rate of return. Allowing for initial investment cost in storage may change this conclusion; but that is all that it may do. To see this, let k^S denote the cost of investment in storage. This changes the position of $y - \theta\bar{n}(0)$ intercept to $y - k^S - \theta\bar{n}(0)$. No further modification is called for as long as $k^S + \theta\bar{n}(0) < k^P + \theta\bar{n}\left(k^P\right)$. On the other hand, if $k^S + \theta\bar{n}(0) > k^P + \theta\bar{n}\left(k^P\right)$, the $y - k^S - \theta\bar{n}(0)$ intercept will be to the left of $y - k^P - \theta\bar{n}\left(k^P\right)$ intercept so that the line that originates from $y - k^S - \theta\bar{n}(0)$, and is tangent to the indifference curve corresponding to W_P^* at β, will have a steeper slope in absolute value than $\bar{n}\left(k^P\right)$ (the absolute value of the line through $y - k^P - \theta\bar{n}\left(k^P\right)$ and tangent to the same indifference curve). In this case, β will be above and to the left of α. Consequently, one chooses PAYGO whenever $1+r \leq \bar{n}\left(k^P\right)$. Otherwise, if $1+r > \bar{n}\left(k^P\right)$, one will opt for one or the other technology depending on how r compares with $\hat{r}$. Put differently, one must have $\hat{r} > \bar{n}\left(k^P\right) - 1$. This will not change any of our conclusions.

[13] This assumes an interior solution for k. With a corner solution at $k = 0$, $\bar{n}(0) = 1 + \hat{r}$ and the original Samuelsonian condition remains intact.

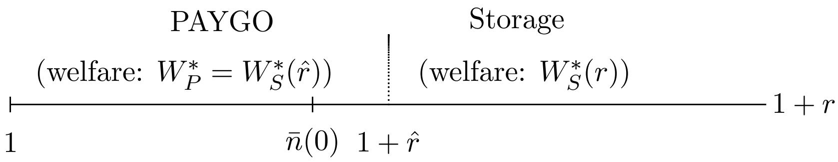

Figure 2: The choice between storage and PAYGO.

shrinks.[14]

3.4 Decentralization of the optimum

We now briefly examine how the first-best optimum can be decentralized. With the first-best consumption levels (in both periods) being independent of the number of children, one may think of the decentralized solution as offering individuals full insurance against the risk (from a personal perspective) of having many children. When the storage technology dominates PAYGO, it suffices to compensate those with n_2 children for their extra cost of $(n_2 - n_1)\,\theta$. This is achieved by levying a lump-sum tax equal to $(n_2 - n_1)\,\theta/2$ on parents with n_1 children, and giving the proceeds to parents with n_2 children. Such a tax and transfer policy fully insures the parents while satisfying the government's budget constraint.

When PAYGO dominates storage, decentralization is somewhat more intricate. First, in order for the payroll taxes (T_i's) and pension benefits (P_i's) to satisfy the government's budget constraint, and with $\bar{n}\,(k)$ young individuals for every old person, we must have $\overline{n}(k)\ \sum \pi_i\,(k)\,T_i = \sum \pi_i\,(k)\,P_i$. Second, as with the storage technology, the individualized payroll taxes must satisfy $T_1 - T_2 = (n_2 - n_1)\,\theta$ in order to equalize first-period consumption levels. Third, to equalize second-period consumption levels, pensions will have to be independent of the number of children: $P_1 = P_2 = P$. Finally, we also need to induce the "correct" level of investment in children and ensure that there are no private savings.

[14] These conclusions are independent of the size of the initial investment in storage technology. This is because such investments are present whether or not fertility is endogenous. On the other hand, the cost of investment in fertility technology does not exist in the Samuelson model.

If there are no benefits associated with investment in children, no one will choose a positive level of k. To ensure some investment, pensions must in part be conditioned on k. Specifically, let k^P and d^P denote the (PAYGO) first-best values of k and d. Set $P = \overline{P} + k\,\overline{n}(k^P)$ where $\overline{P}$ is fixed and satisfies $\overline{P} + k^P\,\overline{n}(k^P) = d^P$. It is easy to show that under this pension scheme, and with the "appropriate" choice of T_1 and T_2, all individuals would choose $k = k^P$, opt for zero private savings (negative savings are not allowed) and choose d^P and c^P as well (c^P is the PAYGO first-best value of c). A subsidy on k at the rate of $\overline{n}(k^P) - 1$ per unit of k (a gross return of $\overline{n}(k^P)$) is necessary because any given individual's choice of k has no impact on the aggregate k and thus on $\overline{n}(k)$, the rate of return of the PAYGO system. Put differently, an individual's investment in children creates a positive externality. Consequently, unless there is a (Pigouvian) subsidy, individuals will set k at zero and there will be no investment in children.

Observe that the Pigouvian subsidy is set at a rate that brings the private cost of investing in k equal to its social cost which is one. To see this, recall that in the above scheme the subsidy on k is paid in the second period as part of one's pension benefits. Discounted to the first period (with a discount rate of $1/\overline{n}(k^P)$), its value will be equal to 1—the marginal cost of k. Observe also that, at the optimal solution, the marginal cost of k equals the marginal (net) social benefits of k. One can see this by rewriting (13) as

$$1 = \frac{\bar{n}'\left(k^P\right) d^P}{[\bar{n}(k^P)]^2} - \bar{n}'\left(k^P\right)\theta,$$

where the right-hand side of this expression is the net social marginal benefit of k. Its first terms measures the induced impact on the return of PAYGO, while the second terms represents the extra cost of raising children.

We summarize the results of this section as

Proposition 1 *(i) The first-best allocation under storage requires that parents do not invest in fertility, have equal consumption levels regardless of their number of children in both periods of their lives, and that their first- and second-period consumption levels satisfy the classic condition for intertemporal consumption at a rate of return determined by the storage technology. The allocation can be decentralized by levying a lump-sum tax* *equal to* $(n_2 - n_1)\,\theta/2$ *on parents with* n_1 *children, and giving the proceeds to parents with* n_2 *children.*

(ii) The first-best allocation under PAYGO requires that parents make an investment in fertility equal to k^p*, the solution to equation (13), have equal consumption levels regardless of their number of children in both periods of their lives, and that their first and second period consumption levels satisfy the condition for intertemporal consumption at the rate of* $\overline{n}(k^P)$*. The allocation can be decentralized by linking pension benefits,* P*, to investment in children according to* $P = \overline{P} + k\,\overline{n}(k^P)$ *where* $\overline{P}$ *is fixed and satisfies* $\overline{P} + k^P\,\overline{n}(k^P) = d^P$ *(the superscript* P *denotes a PAYGO first-best value), coupled with individualized payroll taxes that satisfy* $T_1 - T_2 = (n_2 - n_1)\,\theta > 0$ *and the government's per period budget constraint,* $\sum \pi_i\,(k)\,T_i = \sum \pi_i\,(k)\,P_i/\overline{n}(k)$*.*

(iii) Let $\hat{r}$ *denote the rate of return on storage at which first-best welfare level under storage equals its PAYGO level. Then, for all* $r < \hat{r}$*, PAYGO dominates storage and for all* $r > \hat{r}$*, storage dominates PAYGO.*

(iv) For all $1 + r \in (\bar{n}(0), 1 + \hat{r})$ *one would use the storage technology when fertility is exogenous and PAYGO when fertility is endogenous.*

4 Second-best solution

The first-best characterization rests on the assumption that the government can control k fully, either directly or through a subsidy. This will be the case if k (and thus c) are publicly observable. When the observability assumption is not satisfied, one will be in a second-best environment. Under this circumstance, the full insurance prescriptions of a first-best world may not hold. We shall examine this question below.

Assume that k and c are not publicly observable but n_i's $(i = 1, 2)$ are. Public policy consists of either a tax system while letting the young save for their own retirement;[15] or a pension plan through which the government collects taxes from the current young and distributes the proceeds to the retired. With n_i's being publicly observable, taxes T_i's and transfers P_i's may be conditioned on the number of children. Whether or not

[15] Alternatively, the government may institute a fully-funded pension plan, taxing away all savings, investing (storing) the proceeds and distributing the investments and the returns as pensions.

this should be the case is an interesting policy question which this section attempts to shed light on. Next section studies the settings where contributions and/or pensions are required by law to be uniform.

4.1 Storage

Under the storage technology, the resource constraint is given by

$$\sum_i \pi_i(k)\left[y - c_i - \frac{d_i}{1+r} - k - n_i\theta\right] = 0. \tag{14}$$

This is identical to the first-best constraint. Most significantly, the unobservability of k is of no relevance here. In particular, there is no reason why individuals should be induced to choose a different level of k than they would do otherwise. Indeed, the (conditional) first-best allocation of subsection 3.4 (which requires equalization of consumptions levels for parents with different number of children) is attainable as long as tax payments are not restricted to be uniform. As in the first-best, this is done through levying a lump-sum tax equal to $(n_2 - n_1)\,\theta/2$ on parents with n_1 children, and giving the proceeds to parents with n_2 children. The investment level k is set at zero which is in line with individual incentives. Summing up, with storage and state-dependant contributions, the (conditional) first- and second-best solutions coincide.

4.2 PAYGO

Recall that the first-best outcome entailed two properties: full insurance plus an optimal choice of k such that, given an exogenous rate of return on storage equal to r, $\overline{n}(k^p) > 1 + \hat{r} > 1 + r$. With k being directly "under control," the planner could set it at its optimal level without one having to forgo the full insurance property. In a second best environment, k can no longer be directly controlled and keeping the full insurance property is consistent with $k = 0$ only. Second-best optimality may then require trading off the full insurance property for a positive choice of k (although not at its first-best value). We will see below this is precisely the outcome when one can control k only indirectly, through the incentives that the pension scheme provides.

The young's problem, when facing the policy instruments T_1, T_2, P_1 and P_2, is

$$\max_{c_1,c_2,k} \quad U = \big(1-\pi(k)\big)[u(c_1)+v(d_1)] + \pi(k)[u(c_2)+v(d_2)], \tag{15}$$

$$\text{s.t.} \quad c_i = y - k - T_i - \theta n_i \qquad i = 1,2, \tag{16}$$

$$d_i = P_i \qquad i = 1,2. \tag{17}$$

This yields the following first-order condition for an interior solution for k,

$$\pi'(k)\Big\{u(c_2)+v(d_2) - \big[u(c_1)+v(d_1)\big]\Big\} - \big(1-\pi(k)\big)u'(c_1) - \pi(k)u'(c_2) = 0. \tag{18}$$

Naturally, the second-order condition $\Delta \equiv (d^2U/dk^2)|_{k=\widetilde{k}} < 0$ must also be satisfied; we shall assume throughout the paper that this is the case.[16]

The first term on the left-hand side of (18) measures the benefit (for the individual) of increasing k, while the second term measures the cost. Not surprisingly, an interior solution requires marginal benefits to equal marginal costs. Observe that when the left-hand side of (18) is non-positive at $k = 0$, we have a corner solution and the individual does not invest in k. This occurs for instance when $c_1 = c_2$ (i.e., when $T_1 - T_2 = \theta(n_2 - n_1)$) and $d_1 = d_2$. The solution to the individual's problem, denoted by $\widetilde{k}(T_1, T_2, P_1, P_2)$, describes all possible values of k that the government can induce through its choice of T_1, T_2, P_1 and P_2. The following lemma establishes the comparative static properties of $\widetilde{k}(T_1, T_2, P_1, P_2)$, which will prove useful in studying the government's problem.

[16] We have

$$\Delta = \frac{\pi''(k)}{\pi'(k)} \sum_i \pi_i(k)u'(c_i) + 2\pi'(k)\big[u'(c_1) - u'(c_2)\big] + \sum_i \pi_i(k)u''(c_i).$$

The first and the last expressions in the right-hand side of above are negative due to concavity of $u(.)$ and $\pi(.)$ Consequently, as long as c_2 does not exceed c_1 by "much", Δ will be negative. A sufficient condition is $c_1 \geq c_2 \Rightarrow T_1 - T_2 \leq \theta(n_2 - n_1)$.

Lemma 1 *If* $\widetilde{k}(T_1, T_2, P_1, P_2)$ *is given by an interior solution,*

$$\frac{\partial \widetilde{k}}{\partial T_1} = \frac{\pi'(k)u'(c_1) + [1-\pi(k)]u''(c_1)}{(-\Delta)} \lesseqgtr 0 \tag{19}$$

$$\frac{\partial \widetilde{k}}{\partial T_2} = \frac{-\pi'(k)u'(c_2) + \pi(k)u''(c_2)}{(-\Delta)} < 0 \tag{20}$$

$$\frac{\partial \widetilde{k}}{\partial P_1} = \frac{-\pi'(k)v'(d_1)}{(-\Delta)} < 0 \tag{21}$$

$$\frac{\partial \widetilde{k}}{\partial P_2} = \frac{\pi'(k)v'(d_2)}{(-\Delta)} > 0. \tag{22}$$

At a corner solution, $k = 0$ *and all partial derivatives of* $\widetilde{k}$ *are also equal to zero.*[17]

The inequality signs are as expected. The ambiguity of the first derivative is due to the conflicting income and incentive effects of increasing T_1. For simplicity we shall concentrate on the "normal" case which occurs if $\partial \widetilde{k} / \partial T_1 > 0$.

Consider now the government problem. A first ingredient is the resource constraint. With $\bar{n}(k)$ young individuals for every old person, this is given by

$$\overline{n}(k)\big[\big(1-\pi(k)\big)T_1 + \pi(k)T_2\big] = \big(1-\pi(k)\big)P_1 + \pi(k)P_2. \tag{23}$$

Equation (23) is a rewriting of (7) in terms of the second-best policy instruments. It requires that total contributions equal total pension benefits. A second element is the constraint that $k = \widetilde{k}(T_1, T_2, P_1, P_2)$ which reflects the government's indirect control of the level of investment in children. The conditional second-best problem for the PAYGO case is then summarized by the Lagrangian

$$\begin{aligned}\Gamma^S &= \big(1-\pi(k)\big)[u(c_1)+v(d_1)] + \pi(k)[u(c_2)+v(d_2)] \\ &+ \mu\big\{\big(1-\pi(k)\big)(\overline{n}(k)T_1 - P_1) + \pi(k)(\overline{n}(k)T_2 - P_2)\big\} \\ &+ \eta\big[\widetilde{k}(T_1, T_2, P_1, P_2) - k\big],\end{aligned}$$

[17] At the transition between these two regimes $\widetilde{k}$ may not be differentiable (even though it is continuous as long as the second-order condition holds). We ignore this technical difficulty for the sake of simplicity.

where $d_i = P_i$ and $c_i = y - k - T_i - \theta n_i$ $(i = 1, 2)$. The first-order conditions are[18]

$$\frac{\partial \Gamma^S}{\partial T_1} = [1 - \pi(k)][\mu \overline{n}(k) - u'(c_1)] + \eta \frac{\partial \widetilde{k}}{\partial T_1} = 0, \tag{24}$$

$$\frac{\partial \Gamma^S}{\partial T_2} = \pi(k)[\mu \overline{n}(k) - u'(c_2)] + \eta \frac{\partial \widetilde{k}}{\partial T_2} = 0, \tag{25}$$

$$\frac{\partial \Gamma^S}{\partial P_1} = (1 - \pi(k))[v'(d_1) - \mu] + \eta \frac{\partial \widetilde{k}}{\partial P_1} = 0, \tag{26}$$

$$\frac{\partial \Gamma^S}{\partial P_2} = \pi(k)[v'(d_2) - \mu] + \eta \frac{\partial \widetilde{k}}{\partial P_2} = 0, \tag{27}$$

$$\begin{aligned} \frac{\partial \Gamma^S}{\partial k} &= \mu \big\{ -\pi'(k)(\overline{n}(k)T_1 - d_1) + \big(1 - \pi(k)\big)T_1(n_2 - n_1)\pi'(k) \\ &+ \pi'(k)(\overline{n}(k)T_2 - d_2) + \pi(k)T_2(n_2 - n_1)\pi'(k)\big\} - \eta = 0. \end{aligned} \tag{28}$$

Recall that the definition of $\widetilde{k}(T_1, T_2, P_1,\ P_2)$ encompasses both interior as well as corner solutions (of the individuals' problem). We start by considering the case in which the optimal policy induces an interior solution for $\widetilde{k}$. The case of $k = 0$ will be discussed later.

4.2.1 Interior solution for $\widetilde{k}$

The first-order conditions (24)–(28) indicate that the properties of the second-best solution depend crucially on the sign of η. To the extent that k entails a positive externality so that the individuals tend to choose a level of k that is "too low", one would expect $\eta > 0$. The following lemma shows that this is effectively the case, as long as $\partial \widetilde{k}/\partial T_1 \geq 0$ holds.

Lemma 2 *If $\partial \widetilde{k}/\partial T_1 \geq 0$, then $\eta > 0$.*

Proof. The proof is by contradiction. Assume $\eta \leq 0$. Then the first-order conditions (26)–(27), together with the concavity of $v(\cdot)$, imply $d_2 \leq d_1$. Similarly, (24)–(25), the assumption that $\partial \widetilde{k}/\partial T_1 \geq 0$ and the concavity of $u(\cdot)$ result in $c_2 \leq c_1$. Given these two inequalities, it follows directly from (18) that one cannot have an interior solution for $\widetilde{k}$, and we have a contradiction. ■

[18] In calculating $\partial \Gamma^S / \partial k$, we have utilized the individual's first-order condition (18).

We are now in a position to study the properties of the second-best solution. We are particularly interested in the relationship between payroll taxes and pension benefits on the one hand, and the number of children on the other. Consider the benefits first. With $\eta > 0$, it follows from (21) and (26) that $v'(d_1) - \mu > 0$, and from (22) and (27) that $v'(d_2) - \mu < 0$. The concavity of $v(.)$ then implies $d_2 > d_1$. Regarding payroll taxes, with $\partial\widetilde{k}/\partial T_1 \geq 0$, expression (20) and equations (24)–(25) yield $\mu\overline{n} \leq u'(c_1)$ and $\mu\overline{n} > u'(c_2)$. Concavity of $u(.)$ then implies $c_2 > c_1$, so that $T_1 - T_2 > \theta(n_2 - n_1) > 0$.

The following proposition summarizes the second-best results under storage and PAYGO.

Proposition 2 *(a) Under storage the (conditional) first- and second-best solutions coincide.*

(b) Assume an increase in payroll taxes on parents with small number of children increases (or leaves unchanged) their investment in children ($\partial\widetilde{k}/\partial T_1 \geq 0$). Then in the second-best allocation under a PAYGO pension system: Benefits should increase with the number of children ($P_2 > P_1$); payroll taxes must decrease with the number of children; families with a higher number of children are more than compensated for the extra cost of children ($T_1 - T_2 > \theta(n_2 - n_1) > 0$, and $c_2 > c_1$); the investment in children, and the resulting average fertility rate, are less than their corresponding first-best levels.

To interpret these results, recall that the first-best solution requires full insurance: consumption levels of the young and the retired are independent of the number of children. In a first-best setting, this is provided without preventing k to also be set at its optimal level. As shown earlier, one could induce an optimal level of k, a publicly observable variable, by linking pension benefits to it. When k is not observable, this procedure is no longer feasible. Instead, pension benefits, and contributions, may be linked to the number of children which are observable, and whose realization can be influenced by k.

Specifically, if contributions and benefits entail full insurance, individuals will have no incentive to invest in children and $k = 0$. To induce a positive k, contributions and benefits must be linked to the number of children. In consequence, one loses the full insurance property. The optimal policy then strikes a balance between insurance and incentive considerations. Roughly speaking, if one were to think of k as effort, we have a moral hazard problem which calls for less-than-full insurance. It is thus not surprising that $d_2 > d_1$ and $c_2 > c_1$. The higher consumption levels for parents with a greater number of children, works as an incentive mechanism to induce positive investment in children. Observe that $c_2 > c_1$ does not just require $T_1 > T_2$, it calls for the stronger condition $T_1 - T_2 > \theta(n_2 - n_1)$. In words, a higher number of children implies a reduction in benefits larger than the extra cost of children. This may appear surprising at first, but it is easily understood by realizing that when $T_1 - T_2 \leq \theta(n_2 - n_1)$, one can gain on both the insurance and incentive fronts by widening the gap between T_1 and T_2.

As a final observation, manipulate the first-order conditions (24)–(27) to arrive at

$$\frac{\big(1-\pi(k)\big)v'(d_1) + \pi(k)v'(d_2)}{\big(1-\pi(k)\big)u'(c_1) + \pi(k)u'(c_2)} = \frac{1 - \frac{\eta}{\mu}\big(\frac{\partial \widetilde{k}}{\partial P_1} + \frac{\partial \widetilde{k}}{\partial P_2}\big)}{\overline{n}(k) + \frac{\eta}{\mu}\big(\frac{\partial \widetilde{k}}{\partial T_1} + \frac{\partial \widetilde{k}}{\partial T_2}\big)}. \tag{29}$$

The left-hand side of equation (29) denotes the marginal rate of substitution between d_i's and c_i's. This was set equal to equal to $1/\overline{n}(k)$, their relative "marginal costs", under first best; see equation (12). Consequently, the second best implies that the individuals' life-cycle consumption patterns are distorted. If the difference in the young's consumption levels is "close" to the difference in the old's consumption levels (between people with different number of children),[19] the right-hand side of (29) is greater than $1/\overline{n}(k)$. This means that the marginal rate of substitution between d_i's and c_i's *increases* as one goes from first best to second best. Put differently, *less* resources are transferred to the future for consumption in the second best relative to first best.

[19] Specifically,

$$\frac{1 - \frac{\eta}{\mu}\big(\frac{\partial \widetilde{k}}{\partial P_1} + \frac{\partial \widetilde{k}}{\partial P_2}\big)}{\overline{n}(k) + \frac{\eta}{\mu}\big(\frac{\partial \widetilde{k}}{\partial T_1} + \frac{\partial \widetilde{k}}{\partial T_2}\big)} - \frac{1}{\overline{n}(k)}$$
$$= \frac{-\frac{\eta}{\mu}\Big\{\pi'(k)\big[u'(c_1) - u'(c_2)\big] + \pi'(k)\overline{n}(k)\big[v'(d_2) - v'(d_1)\big] + \sum \pi_i k u''(c_i)\Big\}}{(-\Delta)\overline{n}(k) + \big[\overline{n}(k) + \frac{\eta}{\mu}\big(\frac{\partial \widetilde{k}}{\partial T_1} + \frac{\partial \widetilde{k}}{\partial T_2}\big)\big]}.$$

With $\partial\widetilde{k}/\partial T_1 > 0$, one can show that $\partial\widetilde{k}/\partial T_1 + \partial\widetilde{k}/\partial T_2 > 0$ so that the denominator is positive. The numerator will also be positive as long as $u'(c_1) - u'(c_2) > 0$ is "close," in absolute value, to $\overline{n}(k)\big[v'(d_2) - v'(d_1)\big] < 0$.

4.2.2 Corner solution at $\widetilde{k} = 0$

In describing an individual's behavior facing the policy instruments T_1, T_2, P_1 and P_2 under PAYGO, we pointed out that if the individual is induced to choose $c_1 = c_2$ and $P_1 = P_2$, then he will opt for $k = 0$. Similarly, one can deduce from the first-order conditions (24)–(27) of the second-best problem, that if $k = 0$ is the second-best choice of k, optimality requires $c_1 = c_2$ and $P_1 = P_2$. Consequently, one faces the possibility of having $c_1 = c_2$, $d_1 = d_2$ and $k = 0$ as the second-best solution. To investigate this possibility, evaluate $\partial\Gamma^S/\partial k$ at $c_1 = c_2 = c$, $d_1 = d_2 = d$, $k = 0$ and simplify.[20] We have

$$\frac{\partial\Gamma^S}{\partial k} = -u'(c) + \mu\pi'(0)(n_2 - n_1)\left[\frac{d}{\overline{n}(0)} - \theta\overline{n}(0)\right].$$

One can see from this expression that $\partial\Gamma^S/\partial k$ may take a negative value at $k = 0$. Thus we cannot *a priori* rule out a solution with $d_1 = d_2$, $c_1 = c_2$ and $k = 0$. This would be the case if individuals have a very large degree of risk aversion, if π is not very responsive to k or if θ is "large" . If this happens, the tradeoff between c and d will again be at its first-best value of $1/\overline{n}(k)$, albeit at $k = 0$. [See equation (29) which would then simplify to (12)]. The solution is effectively the same as the first-best outcome under storage, with a rate of return equal to $(1 + r) = \overline{n}(0)$, rather than the PAYGO technology.

4.3 PAYGO versus storage in the second best

Denote the welfare achieved at the second-best solution by W_P^{SB} (for PAYGO) and $W_S^{SB}(r)$ (for storage). As in the first best, the choice between the two technologies hinges on the exogenous level of r. The results obtained in subsections 4.2 and 4.1 imply that $W_P^{SB} \leq W_P^*$,[21] while $W_S^{SB}(r) = W_S^*(r)$ holds for any level of r. Thus, under PAYGO, the unobservability of k results in a welfare loss. Under storage, on the other hand, the first-best outcome is achieved even if k is unobservable. Let r^{SB} denote the critical level of return satisfying $W_S^{SB}(r^{SB}) = W_P^{SB} < W_P^*$. It then immediately follows that $r^{SB} < \widehat{r}$, where $\widehat{r}$ is the critical level in the first best. In other words, the range

[20] The expression for $\partial\Gamma^S/\partial k$ differs from that given by (28) because the latter was derived *assuming* an interior solution for $\tilde{k}$.

[21] The equality sign applies if the first- and second-best outcomes under PAYGO are given by the corner solution $k = 0$, and $c_1 = c_2, d_1 = d_2$.

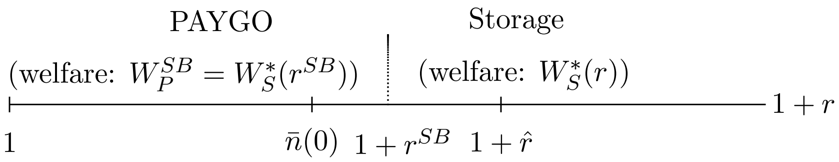

Figure 3: PAYGO versus storage in the second best.

of values of r for which storage dominates PAYGO is larger in the second best than in the first best. Put differently, whenever storage is optimal in the first best, it is also optimal in the second best. On the other hand, when PAYGO is optimal in the first best, it may or may not be optimal in the second best.

Observe also that a simple graphical representation as in Figure 1 can no longer be provided because, in the second best under PAYGO, $c_1 \neq c_2$ and $d_1 \neq d_2$.[22] Nevertheless a graphical representation similar to Figure 2 is possible. Figure 3 depicts the range of values of r for which one or the other technology dominates in the second best. The figure shows that the endogeneity of fertility continues to lead to the superiority of PAYGO over storage for some values of r (at which the reverse would have been true under Samuelson's original model with exogenous fertility). Nevertheless second-best considerations make this range smaller. Recall that in the first best whenever $1 + r \in (\bar{n}(0), 1 + \hat{r})$, PAYGO will outperform storage with endogenous fertility (but not when fertility is exogenous where $1 + \hat{r} = \bar{n}(0)$). In the second best, this interval is shortened to $(\bar{n}(0), 1 + r^{SB})$. Observe also that if we have a corner solution under second best but not under first best, $1 + r^{SB} = \bar{n}(0)$, and the "superiority" of the second best PAYGO disappears completely. If we have a corner solution under first best as well, then $1 + \hat{r} = 1 + r^{SB} = \bar{n}(0)$ and we are back in Samuelson's world.

We end this section with another proposition.

Proposition 3 *Let r^{SB} denote the rate of return on storage at which second-best welfare*

[22]Except, of course, when the PAYGO second best implies a corner solution for $k = 0$. We would then have, for both technologies, budget lines starting from y and it will be sufficient to compare r with $\overline{n}(0)$.

level under storage equals its PAYGO level [$W_S^{SB}(r^{SB}) = W_P^{SB}$]. We have:

(i) $r^{SB} < \hat{r}$, $W_S^{SB}(r^{SB}) = W_S^*(r^{SB})$, and $W_P^{SB} < W_P^*$ [$\hat{r}$ is the welfare-equalizing rate under first best, and $*$ indicates first-best values.]

(ii) For all $1+r \in (\bar{n}(0), 1+r^{SB})$ one would use the storage technology when fertility is exogenous, and PAYGO when fertility is endogenous.

5 Second best PAYGO with state-independent first-period taxes

The discussion thus far has allowed for tax and benefit schemes that are both state dependent. As a policy prescription, however, one may want to restrict pension contributions and/or pension benefits to be independent of the number of children. Indeed, depending on the timing of the decision process, there are circumstances under which differentiation of T_1 from T_2 may not even be possible.[23] With this in mind, we shall now discuss a special case of our model where the tax payments do not vary with the number of children. This policy restriction prevents us from achieving the second best solution of the previous section. However, the setting continues to constitute a departure from the traditional PAYGO pension plans under which it is not just the taxes on the young, but also the pension benefits of the old, that do not vary with the number of children.

Formally, observe first that the equality of first-period tax payments implies, from (16),

$$c_1 - c_2 = \theta(n_2 - n_1) > 0.$$

Thus, individuals who end up with more children would also have to pay in full the corresponding additional costs (this is in addition to k paid by everyone). Now, with an individual treating his tax payments and pension benefits as fixed, he faces the same optimization problem as before. This yields a first-order condition identical to (18).[24]

[23] This will be the case, for example, if taxes are levied before n is realized. It will be a natural timing sequence if n represents the quality (productivity) of children rather than their number. With this interpretation, however, one may also want to set θ at zero.

[24] With $T_1 = T_2$, $c_1 - c_2 = \theta(n_2 - n_1) > 0$ so that $u'(c_1) < u'(c_2)$. It then follows from the expression for Δ in footnote 16 that $\Delta < 0$ and the second-order condition is now necessarily satisfied.

The solution to k in this case will depend on P_1, P_2 and T (with $T_1 = T_2 = T$). It is easy to check that the expressions for $\partial\widetilde{k}/\partial P_1$ and $\partial\widetilde{k}/\partial P_2$ remain unchanged from (21)–(22). One can also easily show that, with $c_1 > c_2$, $\partial\widetilde{k}/\partial T = \partial\widetilde{k}/\partial T_1 + \partial\widetilde{k}/\partial T_2 < 0$.

Regarding the government's optimization, one must now impose an additional constraint ($T_1 = T_2 = T$) on the second-best problem. This is summarized by the Lagrangian

$$\begin{aligned}\Gamma^C &= \big(1-\pi(k)\big)[u(c_1)+v(d_1)] + \pi(k)[u(c_2)+v(d_2)] \\ &+ \mu\left\{\big(1-\pi(k)\big)(\overline{n}(k)T_1 - P_1) + \pi(k)(\overline{n}(k)T_2 - P_2)\right\} \\ &+ \eta\big[\widetilde{k}(T_1,T_2,P_1,P_2) - k\big] + \lambda(T_1 - T_2),\end{aligned}$$

with $d_1 = P_1$ and $d_2 = P_2$. The first-order conditions with respect to P_1, P_2 and k, are as in the unconstrained case. As in that case, we will again have, as long as the solution for $\widetilde{k}$ is interior, $\eta > 0$ and $d_2 > d_1$.[25] The intuition is the same as in the unrestricted case. In particular, the level of differentiation between d_2 and d_1 is determined by trading off incentive with insurance effects. Of course, the levels of d_1, d_2 and k will be different as c_1 now exceeds c_2 rather than the other way around.

To see the implication of $T_1 = T_2$ constraint, assume again that $\partial\widetilde{k}/\partial T_1 \geq 0$. Under this circumstance, $\lambda < 0$ (see the Appendix). Thus reducing T_2 and increasing T_1 from their current equal value are welfare improving. Finally, corresponding to equation (29) under the unconstrained second best, we now have

$$\frac{\big(1-\pi(k)\big)v'(d_1) + \pi(k)v'(d_2)}{\big(1-\pi(k)\big)u'(c_1) + \pi(k)u'(c_2)} = \frac{1 - \frac{\eta}{\mu}\big(\frac{\partial\widetilde{k}}{\partial P_1} + \frac{\partial\widetilde{k}}{\partial P_2}\big)}{\overline{n}(k) + \frac{\eta}{\mu}\big(\frac{\partial\widetilde{k}}{\partial T}\big)} > \frac{1}{\overline{n}(k)}, \tag{30}$$

where the inequality sign follows from the fact that $\partial\widetilde{k}/\partial T = \partial\widetilde{k}/\partial T_1 + \partial\widetilde{k}/\partial T_2 < 0$, and $\partial\widetilde{k}/\partial P_1 + \partial\widetilde{k}/\partial P_2 < 0$.[26] This duplicates the result under the unrestricted second best. However, there, we had to assume that the differences in consumption levels between

[25] Proofs are as in the unrestricted case except that, with $c_1 > c_2$ in this case, the first-order conditions with respect to c_1, c_2 are not utilized. Consequently, the sign of $\partial\widetilde{k}/\partial T_1$ is of no relevance here.

[26] We have

$$\frac{\partial\widetilde{k}}{\partial P_1} + \frac{\partial\widetilde{k}}{\partial P_2} = \frac{\pi'(k)\big[v'(d_2) - v'(d_1)\big]}{(-\Delta)}.$$

With $d_2 > d_1$, the concavity of $v(\cdot)$ implies $v'(d_2) - v'(d_1) < 0$, and the above expression is negative.

people with different number of children were "close" for when they are young and when they are old. We can now state, unambiguously, that as long as taxes are independent of the number of children, *less* resources are transferred to the future relative to the first best.

We summarize these results as

Proposition 4 *Assume $T_1 = T_2 = T$:*

(i) The constraint implies $c_1 > c_2$ thus reversing the corresponding (unconstrained) second-best finding on first-period consumption levels. The other second-best results continue to hold. That is, pension benefits increase with the number of children so that $d_2 > d_1$; and that investment in children and the average fertility rate are less than their corresponding first-best levels.

(ii) Reducing T_2 and increasing T_1 from T are welfare improving (if $\partial\widetilde{k}/\partial T_1 \geq 0$).

(iv) Less resources are transferred to the future relative to the first best.

6 Two polar cases

Finally, it will be instructive to contrast the lessons of our model with those obtained in two polar cases: one where fertility is controlled in a deterministic way, and the other where fertility is random and purely exogenous. Consider first the case where fertility is perfectly controllable. Clearly, in a deterministic environment, there is no need to provide insurance. One only needs to worry about incentives and ensure the "correct" choice of k. A simple formalization of this idea within our model is to assume that k takes only two values $k \in \{k_1, k_2\}$, and that k_1 leads to n_1 and $k_2 > k_1$ to n_2. Then, with children having no intrinsic benefits, parents will choose k_1 even if k_2 happens to be optimal. (We are assuming that the rate of return to storage is low enough that a PAYGO with k_1 is preferable to storage). The optimal policy is then to have $c_i = c(k^*)$, $P_i = d_i = d(k^*)$, and $T_i = P_i/n_i$ where $u'(c_i) = n_i v'(d_i)$ and $k^* \in \{k_1, k_2\}$ yields the higher U. This ensures that individuals will choose the socially optimal (first-best) number of children.

Consider next the other extreme setting in which there is no control over fertility.

There is no moral hazard (incentive) problem here and first-best optimality requires only full insurance. This is precisely the outcome under the structure of our model when $\pi'(k) = 0$. Then, one can easily check that the optimal policy is to set $k = 0$ and $c_1 = c_2$ and $d_1 = d_2$ (we are again assuming that the rate of return on storage is low enough that it is dominated by the PAYGO scheme). The optimal social security system fully insures parents against the fertility uncertainty. The solutions (first-best and second-best) derived in this paper offer a compromise between the two extreme cases.

7 Conclusion

The PAYGO social security has traditionally been studied as if the rate of fertility were given or at least not controllable. More recently, a series of papers have focused on the endogeneity of fertility and the need to make parents responsible when their behavior have social externalities. In the case or PAYGO pension plans, social externalities are positive implying that the *laissez-faire* generates a suboptimal population growth. In other cases, such as the "Tragedy of the Commons," social externalities are negative and population growth is excessive. Making people responsible for their fecundity raises problems when the control is only partial because fecundity involves some randomness. It is then important to insure parents against fertility shocks they are not responsible for.

We have shown that, with positive externalities, one should grant parents who have more children larger pension benefits. At the same time, the parents' contributions must be linked negatively to their *investment* in children if the investments are publicly observable, and to the *number* of children if investments are not observable. With observability, the outcome is characterized by full insurance with all parents enjoying identical consumption levels regardless of their number of children, when they work as well as when they are retired. In the absence of observability, families with more children should be more than compensated for the extra cost of children so that they can enjoy a higher level of first-period consumption. Moreover, the optimal level of investment in children, and the resulting average fertility rate, will be less than their corresponding first-best levels. Except for the extra compensation result, all other second-best results carry over to situations where payroll taxes cannot depend on the number of children.

Finally, we have revisited Samuelson's classic requirement for optimality of PAYGO pension plans when fertility is in part endogenous. We have shown that the possibility of investing in fertility, and thus making this technology more productive, implies that PAYGO will dominate storage over a higher range of rates of return in both first- and second-best environments, with the range being smaller in the second best.

Appendix

Second-best under $T_1 = T_2 = T$ constraint:

(i): The first-order conditions are

$$\frac{\partial \Gamma^T}{\partial T_1} = [1-\pi(k)][\mu\overline{n}(k) - u'(c_1)] + \eta\frac{\partial \widetilde{k}}{\partial T_1} + \lambda = 0, \tag{A1}$$

$$\frac{\partial \Gamma^T}{\partial T_2} = \pi(k)[\mu\overline{n}(k) - u'(c_2)] + \eta\frac{\partial \widetilde{k}}{\partial T_2} - \lambda = 0, \tag{A2}$$

$$\frac{\partial \Gamma^T}{\partial P_1} = (1-\pi(k))[v'(d_1) - \mu] + \eta\frac{\partial \widetilde{k}}{\partial P_1} = 0, \tag{A3}$$

$$\frac{\partial \Gamma^T}{\partial P_2} = \pi(k)[v'(d_2) - \mu] + \eta\frac{\partial \widetilde{k}}{\partial P_2} = 0, \tag{A4}$$

$$\frac{\partial \Gamma^T}{\partial k} = \mu\pi'(k)((n_2 - n_1)T - \eta = 0 \tag{A5}$$

Adding equation (A1) to (A2) leads to,

$$\mu\overline{n}(k) - u'(c_1) = \pi(k)\big[u'(c_2) - u'(c_1)\big] - \eta\Big(\frac{\partial \widetilde{k}}{\partial T_1} + \frac{\partial \widetilde{k}}{\partial T_2}\Big) > 0, \tag{A6}$$

where the sign follows from the fact that $c_1 > c_2$ in this case, the concavity of $u(\cdot)$, and $\partial\widetilde{k}/\partial T_1 + \partial\widetilde{k}/\partial T_2 < 0$. Next, assuming $\partial\widetilde{k}/\partial T_1 \geq 0$, and using (A6) in (A1) we will have,

$$\lambda = -\big[1-\pi(k)\big]\big[\mu\overline{n}(k) - u'(c_1)\big] - \eta\frac{\partial \widetilde{k}}{\partial T_1} < 0. \tag{A7}$$

References

[1] Abio, G., G. Mahieu and C. Patxot, (2004), On the optimality of PAYG pension systems in an endogenous fertility setting, forthcoming, *Journal of Pension Economics and Finance*.

[2] Bental, B., (1989), The old age security hypothesis and optimal population growth, *Journal of Population Economics*, 1, 285-301.

[3] Cigno, A., (1991), *The Economics of the Family*, Clarendon Press, Oxford.

[4] Cigno, A., Luporini, A. and A. Pettini, (2003), Transfers to families with children as a principal-agent problem, *Journal of Public Economics*, 87, 1165–1177.

[5] Cigno, A., and A. Luporini, (2003), Optimal policy with heterogeneous families, asymmetric information and stochastic fertility, paper presented at CESifo Venice Summer Institute.

[6] Cremer, H., F. Gahvari, and P. Pestieau, (2003), Stochastic fertility, moral hazard, and the design of pay-as-you-go pension plans, paper presented at CESifo Venice Summer Institute.

[7] Cremer, H., F. Gahvari, and P. Pestieau, (2004), Pensions with heterogeneous agents and endogenous fertility, mimeo.

[8] Fenge, R. and V. Meier, (2004), Pensions and fertility incentives, forthcoming, *Canadian Journal of Economics*.

[9] Kolmar, M., (1997), Intergenerational redistribution in a small open economy with endogenous fertility, *Journal of Population Economics*, 10, 335-356.

[10] Samuelson, P.A., (1958), An exact consumption-loan model of interest with or without the social contrivance of money, *Journal of Political Economy*, 66, 467–482.

[11] Sinn, H.W., (2004), The pay-as-you-go pension system as fertility insurance and an enforcement device, *Journal of Public Economics*, 88, 1335–1357.

[12] van Groezen, B., T. Leers and L. Meijdam, (2000), Family size, looming demographic changes and the efficiency of social security reform, CentER Working Paper no 2000-27, Tilburg.

[13] van Groezen, B., T. Leers and L. Meijdam, (2003), Social security and endogenous fertility: pensions and child allowances as Siamese twins, *Journal of Public Economics*, 87, 233-251.