Environmental Risk Regulation and Liability under Adverse Selection and Moral Hazard \(^1\)

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Abstract: This paper analyzes the impact of risk regulation and extended liability on private contracting when production creates an environmental risk on third-parties. We start by deriving the optimal regulation of a buyer-seller (principal-agent) relationship under adverse selection on the seller’s production costs, moral hazard on his safety care and limited liability. This optimal regulation must trade off allocative efficiency against rent extraction in a framework where the firm is rewarded only with moral hazard rent. As a result, the optimal regulation induces some form of complementarity between care and output, even in the absence of any technological interaction. Output distortions are stronger when there is no limit on liability. This optimal regulation can sometimes be implemented with a simple ex post liability rule, provided that the buyer has enough wealth. When liability is extended towards a shallow-pocket buyer, the optimal contract between the buyer and the seller must also avoid any accident to secure gains from trade. Optimal contracts in the extended liability regime are quite comparable with those in the optimal regulation and still exhibit a complementarity between outputs and levels of care even though output distortions are now severe.

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1 Introduction

The importance of an efficient design of liability and risk regulations for environmentally risky ventures is now well recognized and has been highlighted by the lively debate which took place in the U.S. around the 1980 Comprehensive Environmental Response Compensation and Liability Act (CERCLA). Among other things, this act discusses what should be the allocation of liability between a firm which has caused an environmental damage and its various contractual partners in settings where the venture’s assets and profits fall short of covering the full harms caused on third-parties. Conventional wisdom suggests that the contracts signed by such risky ventures with various stakeholders take into account that allocation of liability. To assess the full impact of risk regulation and liability rules on social welfare, public policies aimed at correcting environmental externalities should thus be designed with an eye on the contracting possibilities available to firms involved in environmentally risky activities. It is thus important to delineate circumstances under which contracts are modified by liability rules and risk regulations and to understand the directions of those distortions if any.

Of course, in a world without transaction costs in private contracting, the corrective policies aimed at reducing the likelihood of an environmental damage, be they regulations or liability rules, would not have any impact on contracting. Stakeholders and the risky venture with whom they contract would always reach an efficient agreement. Conditionally on the level of care induced by public policies towards risk, contracts would achieve an efficient allocation of resources within the private sector. The only interesting issue is thus to assess the impact of ex ante regulation and ex post liability rules when private transactions are plagued by informational problems. This paper analyzes the full impact of risk regulation and extended liability on care and output distortions in a framework where private transactions are perturbed by the private information that the risky venture retains at the time of contracting with stakeholders. Private information takes the form of both adverse selection on production cost and hidden choice of the level of care. It is shown that contract distortions significantly depend on whether risk regulation or liability rules are used.

To exemplify some of the issues involved, let us consider a buyer (the principal) contracting with an independent seller (the agent). The seller can be viewed as a production unit or subsidiary providing an essential input for the buyer. Production creates an environmental hazard on third-parties because, for instance, it involves a long-lasting contamination of the production site. However, the seller can take, at a cost, non-observable

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1 CERCLA has also inspired the Canadian legal framework for contaminated sites, and the European Community is also considering to develop environmental liability with the 2002 Community Directive. See Boyer and Laffont (1996) and Strasser and Rodosevitch (1993).
actions which reduce the likelihood of such a damage. To correct this environmental externality, either an ex ante regulation or an ex post liability rule may be used depending on the institutional context.

When the polluting agent has unlimited liability, fines can be made large enough to align the agent’s private incentives to exert care with the socially optimal ones. In that case, the design of the transaction between the buyer and the seller can be disentangled from the problem of inducing safety care. Outputs for an environmentally risky venture are the same as for a non-risky firm. Depending on the informational context surrounding private contracting, those contracts may not reach an efficient outcome but, at least, they are constrained efficient taking into account the informational constraints faced by the buyer. In any case, distortions are the same as in the absence of any environmental risk. In particular, when the seller is privately informed on his production cost, it is well-known that contracts trade off the extraction of the seller’s information rent against productive efficiency to reach an interim efficient outcome.\(^2\) Efficient sellers get an information rent from retaining private information, and reducing that rent requires some output distortions. Even though informational constraints in private contracting impose some distortion away from first-best production, contracting remains unaffected by the provision of incentives for care.

With limited liability, the picture is quite different. Fines cannot exceed the agent’s total profits from his relationship with his principal. Of course, those profits depend on the contract signed with the buyer. Because of adverse selection, part of those profits are available to the seller under the form of adverse selection rent which can never be seized. This possibility to hide rent away from the eyes of the public authority reduces the incentives for care of the most efficient sellers. To compensate this effect, an efficient seller must, at the optimal regulation, receive an extra reward when no damage occurs. With limited liability, an agent is rewarded for truth-telling by means of moral hazard rent. Adverse selection incentive constraints are thus relaxed by increasing effort for the efficient seller. This creates an endogenous positive correlation between care and output even though care provision does not conflict with the minimization of short-run cost.

To sum up, at the optimal regulation with limited liability, outputs distortions are weaker than without limited liability. Intuitively, relaxing the adverse selection incentive constraint requires to distort effort upwards for the most efficient firms. This has a positive social value since it reduces the likelihood of an accident. The shadow cost of the adverse selection incentive constraint has a lower value than with unlimited liability and output distortions are less attractive.

In many practical circumstances, an ex ante regulation is not feasible or not even

\(^2\)See Laffont and Martimort (2002, Chapter 2).
conceivable. Such ex ante regulation must then be replaced by ex post liability rules enforced by Courts. What are the consequences of those rough rules on private contracting. Of course, the complementarity between safety care and output found under ex ante regulation remains an attractive qualitative property which should be looked for under alternative legal regimes.

In this respect, we first delineate circumstances where the optimal regulation can still be implemented under adverse selection, moral hazard and limited liability, by simply imposing a liability payment equal to the full damage to the uninformed stakeholders of the risky venture. Under this extended liability regime, both the buyer and the seller are found liable for the damage generated by the seller. When the buyer is also protected by limited liability, fines cannot exceed the whole gains from trade achieved by the private transaction. This threat of losing the benefits from transacting with the seller makes the buyer somewhat internalize the environmental externality, and somehow aligns his own incentives to promote care with those of a regulator, even though imperfectly. The seller’s contract has indeed to fill a new objective: avoiding any accident to secure gains from trade. This creates a new channel by which output distortions are affected by the liability regime. These gains depend of course on the seller’s cost, i.e., on the adverse selection variable. When uncertainty on costs is sufficiently small, the gains from trade with nearby types of the seller are also close enough. Different types choose almost the same levels of care, making impossible to achieve truth telling without further distortions on effort levels. The complementarity between care and output is maintained. Extending liability towards the buyer preserves then the most important property of the optimal regulation. However, rent extraction, output and care distortions are now excessive compared with the socially optimal ones. This points at an obvious weakness of this ex post liability regime in comparison with the optimal ex ante regulation.

By endogenizing the gains from trade in vertical relationships subject to risk regulation, this paper fills a gap in the literature. First, some authors, following Pitchford (1995), have analyzed how incentive problems between a principal and an agent are affected by the liability environment. These authors focus mostly on the case of financial relationships between a polluting borrower and his lender. They analyze the impact of bargaining power at the contracting stage on the financial transaction (Balkenborg (2001)), the impact of the initial resources of the lender (Lewis and Sappington (2001)), and the degree of control that the lender exerts on the borrower (Boyer and Laffont (1997)). This literature has focused on pure moral hazard environments where the level of safety care is non-verifiable. In such contexts, there exists a conflict of interests between the lender and

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3By implementation, we mean that the allocative consequences of regulation and extended liability are the same although they may differ with respect to the distribution of surplus they involve.

the regulator in the level of safety care they would like to induce. The choice of a liability rule might reduce, at least partially, this conflict. A caveat of this approach is that it takes as given the value of the private transaction and assumes away the possible effects of this moral hazard problem on the size of the financial returns. In other words, this approach implicitly assumes that there is no adverse selection between the principal and his agent, so that liability rules and risk regulation have no impact on these financial returns. Our paper focuses instead on the endogeneity of these returns. We show that the lessons of this earlier literature should be taken with caution when there is adverse selection between the risky venture and its stakeholders. This endogeneity should be recognized at the time of assessing the performances of various regulatory and legal frameworks. As soon as there is adverse selection, moral hazard and limited liability, output distortions are affected by public policies towards care provision sometimes in a rather complex manner.

The impact of liability rules on the whole set of transactions a risky venture is part of has been first investigated by Boyd and Ingberman (1997). In a complete information environment where a first-best regulation could be feasible if policy instruments were unrestricted, they analyze how different liability regimes affect cost minimization and buyer-seller transactions. Laffont (1995) analyzes instead the regulation of a public utility which exerts safety care to avoid an environmental accident in a model involving, like ours, adverse selection and moral hazard. An important technological assumption he makes is that a positive effort level increases production costs and reduces thus output, even in a first-best world. The analysis becomes extremely complex under asymmetric information because of the substitutability between safety care and cost minimizing effort. Dionne and Spaeter (2003) propose also a pure moral hazard model in which there is such a multitask externality. The agent (a borrower) can allocate his investment between directly improving the distribution of the returns of his project and reducing also, in a stochastic sense, the distribution of damages. In fact, our analysis shows that limited liability creates instead some complementarity between output and safety care, even when cost minimization is not directly affected by safety care. By simplifying the technological side, we are able to go further towards characterizing optimal contracting under adverse selection, moral hazard and limited liability. Finally, in a companion paper (Hiriart and Martimort (2003)), we investigate also the impact on liability rules on contracting but with a different timing where safety care has to be incurred before information on cost parameter is learned. Output distortions due to the liability regime arise then only when the buyer (principal) has no bargaining power and must recover the extra liability cost through price distortions.

Section 2 presents the model. Section 3 characterizes the optimal regulation of a buyer-seller relationship when the polluting seller has unlimited liability. In this benchmark, we show that outputs are set at the same level as if the seller’s activity created no
risk for the environment. There is a dichotomy between regulating output and inducing safety care. Section 4 focuses on the optimal regulation when the seller is protected by limited liability. Now, output and care distortions are endogenously linked and some kind of complementarity appears. We show how the optimal regulation can sometimes be implemented with extended liability if the buyer has enough wealth. Section 5 analyzes the benefits of extending liability when, instead, the buyer has limited wealth. We compare the qualitative results obtained there with those of the optimal regulation and show that the complementarity between care and output still prevails. Section 6 concludes. Proofs are relegated to an Appendix.

2 The Model

Following the analysis of Boyd and Ingberman (1997 and 2001), we consider a buyer-seller relationship. However, the lessons of our work are more general and apply to other vertical relationships between an agent exerting an environmentally risky activity and some stakeholder with whom he is linked through contract. One may think for instance of shareholders-workers relationships, regulators-public utilities hierarchies or lender-borrower transactions. The buyer derives a monetary benefit $S(q)$ from consuming $q$ units of the good, with $S' > 0$, $S'' < 0$, and $S(0) = 0$. To always ensure positive and interior outputs, we also assume that the Inada conditions $S'(0) = +\infty$ and $S'(+\infty) = 0$ both hold. The buyer's utility function is:

$$V = S(q) - t,$$

where $t$ is the payment made to the seller. The buyer is risk-neutral and has a reservation payoff exogenously normalized at zero.

The risk-neutral seller has a constant marginal production cost $\theta$ that he privately knows. This random variable is distributed on $\Theta = \{\bar{\theta}, \tilde{\theta}\}$ with respective probabilities $\nu$ and $1 - \nu$. We denote by $\Delta \theta = \bar{\theta} - \tilde{\theta} > 0$ the size of cost uncertainty.

The production process generates an environmental hazard. The seller can nevertheless exert a level of safety care which reduces the probability of an accident. We assume that the damage size $h$ is greater than the first-best surplus in both states of nature. We focus thus on accidents which have a substantial size. If a comprehensive ex ante regulation is not available, this may justify using extended liability towards deep-pocket stakeholders if needed in order to compensate (even if it is partially) harmed third-parties.

Production is exchanged even if an accident occurs; this is not output per se which is risky but the actual production process. For instance, there may be pollution leakages during or after the production process which affect a nearby river or contaminate the
production site without undermining the ability of the seller to produce.

The probability of an environmental damage is \( 1 - e \) where \( e \) is the agent’s effort level which costs him a non-monetary disutility \( \psi(e) \). We assume that \( \psi' > 0, \psi'' > 0, \psi''' > 0 \), with the Inada conditions \( \psi'(0) = 0 \) and \( \psi'(1) = +\infty \) to ensure that effort is always interior and avoid uninteresting corner solutions. The seller’s expected utility can thus be written as:

\[
U = t - \theta q - ez_a - (1 - e)z_a - \psi(e),
\]

where \( z_a \) (resp. \( z_n \)) is the (regulatory or the liability) payment made in case of an (resp. no) environmental damage. We should stress at this point that the level of safety care has no direct technological impact. Contrary to Laffont (1995) and Dionne and Spaeter (2003), exerting an effort to prevent an accident neither increases production cost nor decreases the damage size. To motivate this assumption, note that in many circumstances, technological choices which put a risk on the environment are related to sunk costs (choice of a production site, of a technological process, etc..) and not to short-run variable costs.

When an ex ante incentive regulation is used to correct the externality, a risk-neutral regulator maximizes a social welfare function that takes into account both the net cost of the environmental damage and the buyer’s and the seller’s utilities, namely:

\[
W = -(1 - e)h + e z_n + (1 - e)z_a + \alpha(U + V),
\]

where \( \alpha < 1 \) represents the weight given to the private sector by the regulator. We follow Baron and Myerson (1982) in specifying such a social welfare function with redistributive concerns towards the private sector. Among other things, those concerns can be justified when the regulator is somewhat captured by the industry. In that respect, a case of particular interest is when \( \alpha = 0 \); the regulator can then be interpreted as an uncorruptible judge.\(^5\)

3 Regulation with Unlimited Liability

Let us first consider the normative case in which a regulatory authority offers a comprehensive grand-contract to both the buyer and the seller before any environmental damage occurs. Of course, this complete contractual setting is highly hypothetical. It supposes that the regulator has a strong commitment power to design ex ante the rewards and fines offered to the private sector. It also assumes that private transactions can be regulated and thus, implicitly, that economic and environmental regulations are jointly designed.

\(^5\)See Boyer and Porrini (2001) for a model which distinguishes between the judge and the regulator along similar lines.
Nevertheless, this normative setting gives us an important benchmark before analyzing extended liability in a similar environment (see Section 5).

3.1 Full Information

To start with, we suppose that the level of safety care $e$ and the seller’s marginal cost $\theta$ are both observable by the regulator who can recommend how much output should be traded. The regulator’s problem can then be written as:

$$(P^*): \max_{\{e,q,t,z_n,z_a\}} -(1-e)h + ez_n + (1-e)z_a + \alpha(U(\theta) + V(\theta))$$

subject to

$$V(\theta) \geq 0,$$
$$U(\theta) \geq 0.$$  

where (1) and (2) are the respective participation constraints of both the buyer and the seller. Replacing transfers $t$, $z_n$ and $z_a$ by their values as a function of $U$ and $V$, the regulator’s problem can be rewritten as:

$$\max_{\{e,q,U,V\}} S(q) - \theta q - (1-e)h - \psi(e) - (1-\alpha)(U(\theta) + V(\theta))$$

subject to (1) and (2).

Since the rents left to the private sector are viewed as socially costly, both participation constraints above must be binding at the optimum. The first-best outputs $q^*(\theta)$ and levels of safety care $e^*$ (independent of the seller’s cost) are thus respectively given by:

$$S'(q^*(\theta)) = \theta, $$
$$\psi'(e^*) = h.$$  

Under full information, the marginal surplus of the buyer is equal to the marginal cost of production, and the marginal disutility of safety care covers exactly the damage. To implement this outcome, the regulator can simply recommend the first-best allocation $(q^*(\theta), e^*)$ and punish harshly the agent if the recommended output, safety care or transfers are not observed.

When (1) is binding, the payment $t$ from the buyer to the seller is equal to the gross surplus from trade, namely $S(q) = t$. Everything happens thus as if the optimal regulation
shifted all bargaining power in favor of the informed party in private contracting. Given that value of the price paid by the buyer for the good, (2), when it is binding, only defines the expected regulatory payment $ez_n + (1 - e)z_a$ paid by the seller to the regulator. Many such payments are thus feasible as long as the seller breaks even in expectation.

3.2 Asymmetric Information with Unlimited Liability

Let us now assume that neither the level of care $e$ nor the marginal cost $\theta$ are observable by the regulator and the buyer. If instead the buyer could observe those variables, the regulator could use a “revelation scheme” à la Maskin (1999) to have both the buyer and the seller revealing these pieces of shared information at no cost. Then, clearly, the first-best optimal outcome would be implemented, and contracting between the buyer and the seller would be efficient. Of course, such a complete contracting environment is highly hypothetical. However, as long as exogenous constraints on contracting are not imposed, complete contracts cannot be ruled out a priori. That extreme efficiency result shows that, within the realm of complete contracting, the most interesting case to study is when the buyer is uninformed on the seller’s cost, so that private contracting is plagued by an adverse selection problem.

Under complete contracting and asymmetric information, a regulatory contract specifies ex ante the transfers made to the seller in any event. The optimal regulation thus stipulates a system of fines and rewards depending on whether an accident occurs or not. By the Revelation Principle, there is no loss of generality in assuming that an incentive regulation is a mechanism $\\{t(\hat{\theta}), z_a(\hat{\theta}), z_a(\hat{\theta}), q(\hat{\theta})\}_{\theta \in \Theta}$ stipulating a price paid by the buyer to the seller, regulatory transfers in case an accident occurs or not and an output as function of the seller’s report $\hat{\theta}$ on his cost.

The regulatory contract must first satisfy the uninformed buyer’s participation constraint

$$E_{\theta}(V(\theta)) \geq 0,$$  \hspace{1cm} (5)

where $E(\cdot)$ is the expectation operator with respect to $\theta$, and

$$V(\theta) = S(q(\theta)) - t(\theta)$$

is the buyer’s net profit in state $\theta$.

Second, the seller’s ex post participation constraints must hold since the seller is privately informed on his cost at the time of accepting the regulatory contract. To write

\footnote{See Hiriart and Martimort (2003) for a similar result.}
down these conditions, let us first notice that his expected profit in state $\theta$ is:

$$U(\theta) = t(\theta) - \theta q(\theta) - \min_e \{ e z_n(\theta) + (1 - e) z_a(\theta) + \psi(e) \}.$$  

Clearly, the optimal effort level induced by an incentive compatible mechanism solves:

$$\psi'(e(\theta)) = z_a(\theta) - z_n(\theta). \quad (6)$$

This effort level trades off, from the seller’s point of view, the cost of marginally increasing effort with the benefit of reducing the expected payment he makes to the regulator.

To get a more compact expression of $U(\cdot)$, it is useful to define the seller’s moral hazard rent as:

$$R(e) = e \psi'(e) - \psi(e).$$

Note that $R(\cdot)$ is increasing and convex with the assumptions made on $\psi(\cdot)$.

Then, the seller’s total rent in state $\theta$ can be written as:

$$U(\theta) = t(\theta) - \theta q(\theta) - z_a(\theta) + R(e(\theta)) \geq 0, \text{ for all } \theta \in \Theta. \quad (7)$$

This is the sum of his adverse selection rent coming from private information on the technology and his moral hazard rent coming from his non-observable effort level.

The more stringent participation constraint is, of course, that of the least efficient seller

$$U(\bar{\theta}) \geq 0. \quad (8)$$

Finally, the regulatory contract must be incentive compatible to induce the seller to truthfully reveal his marginal cost. This yields:

$$U(\theta) \geq t(\hat{\theta}) - \theta q(\hat{\theta}) - z_a(\hat{\theta}) + R \left( \varphi(z_n(\hat{\theta}) - z_a(\hat{\theta})) \right), \text{ for all } (\theta, \hat{\theta}) \text{ in } \Theta^2,$$

where $\varphi = \psi'^{-1}$. Putting it differently, we get

$$U(\theta) \geq U(\hat{\theta}) + (\hat{\theta} - \theta) q(\hat{\theta}) \text{ for all } (\theta, \hat{\theta}) \text{ in } \Theta^2.$$

As usual in two-type adverse selection problem,\(^7\) the relevant incentive compatibility constraint corresponds to an upward deviation where an efficient seller wants to mimic an inefficient one, namely:

$$U(\hat{\theta}) \geq U(\bar{\theta}) + \Delta\theta q(\bar{\theta}). \quad (9)$$

\(^7\)See Laffont and Martimort (2002, Chapter 2).
Indeed, by pretending to be a less efficient seller, the efficient one can produce at a lower cost the same amount and save some extra rent.

Therefore, under asymmetric information, the optimal incentive regulation must solve:

\[
(R) : \max_{\{U(\theta), V(\theta), q(\theta), z(\theta), e(\theta)\}} \mathbb{E}\left[-(1 - e(\theta))h + S(q(\theta)) - \theta q(\theta) - (1 - \alpha)(U(\theta) + V(\theta))\right],
\]

subject to (5), (8) and (9).

We can summarize this optimization in the next proposition.

**Proposition 1**: With unlimited liability, the optimal regulation with adverse selection and moral hazard entails:

- An efficient level of care for both types, \(e^{SB}(\theta) = e^*,\) for all \(\theta\) in \(\Theta\).
- The first-best output for an efficient seller and a downward distortion below the first-best for an inefficient one:
  \[
  q^{SB}(\theta) = q^*(\theta)
  \]
  and
  \[
  S'(q^{SB}(\bar{\theta})) = \bar{\theta} + \frac{\nu}{1 - \nu}(1 - \alpha)\Delta \theta.
  \]
- The buyer’s expected profit is zero, \(E(V(\hat{\theta})) = 0\). Only the efficient seller gets a positive rent, \(U^{SB}(\theta) = \Delta \theta q^{SB}(\bar{\theta})\) and \(U^{SB}(\bar{\theta}) = 0\).

Since the weight of the private sector in the social welfare is less than one, transferring wealth from the rest of society towards the private sector is socially costly. The regulated prices of the transaction in both states of nature can be fixed so that the buyer’s participation constraint is binding. Instead, the optimal regulatory policy under asymmetric information must leave some rent to the efficient seller to induce him to reveal his cost parameter. This rent is increasing in the inefficient seller’s level of output. Hence, to reduce the socially costly adverse selection rent, the regulator distorts downwards the production of the inefficient seller. As shown in (10), the marginal benefit of consumption for the buyer is now equal to the virtual cost of the inefficient seller. As it is standard in the literature, this virtual cost captures the existing extra cost of informational rents. Intuitively, starting from the first-best output \(q^*(\bar{\theta})\) and reducing the inefficient agent’s production
by a small amount $dq$ is beneficial since it reduces the efficient agent’s information rent to the first-order and it has only a second-order impact on efficiency in state $\bar{\theta}$. Hence, the virtual costs depends on the ratio between the probabilities of having an efficient or an inefficient seller, $\frac{\nu}{1-\nu}$. Note that the virtual cost decreases with $\alpha$, the weight of the seller’s utility in the social welfare function. Indeed, as $\alpha$ increases, the private sector receives more weight and giving up information rent to the seller is viewed as being less costly by the regulator and output distortions are less needed.

Given these output distortions, the regulator can structure the regulatory payments $z_n(\theta)$ and $z_a(\theta)$ to induce the first-best level of safety care. Typically, a differential $z_a(\theta) - z_n(\theta)$ just equal to the harm level $h$ exerted on third-parties is enough to achieve an efficient level of care. Moreover, structuring rewards and punishments so that this condition holds is costless for the risk-neutral regulator since only the expected regulatory payment he receives matters from his own point of view.

As a matter of fact, all the randomness in the seller’s payments $s$ needed to induce effort can be included into the regulatory payments $z_n(\theta)$ and $z_a(\theta)$. The price $t(\theta)$ paid by the buyer for the good can be made independent on whether an accident occurs or not.\textsuperscript{8} As a result, when the seller has unlimited wealth, there is a complete dichotomy between output distortions and incentives for safety care. These distortions are the same as those that would arise in the optimal economic regulation of a firm generating no environmental risk.

It is worth stressing that the prices $t^{SB}(\bar{\theta})$ and $t^{SB}(\theta)$ are not uniquely pinned down at the optimal regulation above. Indeed, as long as the buyer breaks even in expectation, many such pairs are possible. One possibility is that the buyer gets zero profit in each state of nature, i.e., $V^{SB}(\theta) = 0$ for all $\theta$ in $\Theta$. The prices paid by the buyer to the seller are thus defined by $t^{SB}(\theta) = S(q^{SB}(\theta))$. Two simple justifications can be given for this choice. First, the buyer may be competing à la Bertrand with other similar buyers so that the buyer’s profit in each state is driven to zero. Second, the buyer may have a tiny degree or risk-aversion and full insurance requires that his returns in front of different types of sellers are the same and thus identically equal to zero. This simple choice shows that, at the optimal regulation, the liability constraints of the principal are never relevant even when he has no asset on his own. This feature will of course remain even at the optimal regulation with limited liability on the seller’s side.

For further references, we will sometimes mention the optimal regulation in the absence of liability constraint as an interim efficient outcome, since it maximizes a weighted sum of all the utilities subject to the regulator’s informational constraints.

\textsuperscript{8}The fact that prices are non-conditional on the shock $\theta$ is particularly attractive when trade between the buyer and the seller takes place long before any pollution leakage takes place.
4 Regulation with Limited Liability

It is well known that, in pure moral hazard environments, inducing the first-best level of care may not always be feasible when the seller is protected by limited liability. To see that point in our context, notice that the participation constraint of the inefficient seller is binding at the optimum of Proposition 1. This yields

\[ t^{SB}(\bar{\theta}) - \bar{\theta}q^{SB}(\bar{\theta}) - z^{SB}_{a}(\bar{\theta}) = -R(e^*) < 0, \]

and thus the inefficient seller, if case an accident occurs, must pay a fine \( z^{SB}_{a}(\bar{\theta}) \) so large that he gets bankrupt (assuming he has no asset to start with).

Instead, for an efficient seller, the existence of an adverse selection rent \( \Delta \theta q^{SB}(\bar{\theta}) \) creates a buffer of liabilities which reduces the risk of bankruptcy. More precisely, we have

\[ t^{SB}(\bar{\theta}) - \theta q^{SB}(\theta) - z^{SB}_{a}(\theta) = \Delta \theta q^{SB}(\bar{\theta}) - R(e^*) < 0 \]

only if the damage \( h \) and thus the first-best level of care \( e^* \) are large enough or, alternatively, if the uncertainty on cost \( \Delta \theta \) is small enough.

In the sequel, we will assume that \( h \) is large enough so that bankruptcy in case of an accident is a concern whatever the type of the seller. This assumption simplifies the analysis by getting rid of mixed cases.\(^9\)

To avoid bankruptcy, the following seller’s limited liability constraints must thus be satisfied:

\[ u_{a}(\theta) = t(\theta) - \theta q(\theta) - z_{a}(\theta) \geq 0, \text{ for all } \theta \text{ in } \Theta. \quad (11) \]

4.1 Pure Moral Hazard

Let us start by considering the case where the marginal cost \( \theta \) is common knowledge. Since \( h \) is large enough; (11) will be binding in both states of nature. We can thus rewrite

\[ U(\theta) = \max_{e} \{ e(t(\theta) - \theta q(\theta) - z_{n}(\theta)) - \psi(e) \} \]

or

\[ U(\theta) = R(e(\theta)) \quad (12) \]

where \( \psi'(e(\theta)) = u_{n}(\theta) = t(\theta) - \theta q(\theta) - z_{n}(\theta) \) is positive to induce a positive level of care.\(^9\)

\(^9\)Laffont (1995) makes a similar assumption.
With limited liability and complete information on $\theta$, we can rewrite the regulator’s problem as

$$(R) : \max_{\{e(\theta), U(\theta), V(\theta)\}} -(1 - e(\theta))h - \psi(e(\theta)) + S(q) - \theta q - (1 - \alpha)(U(\theta) + V(\theta)),$$

subject to (1) and (12).

**Proposition 2**: With limited liability and moral hazard only, the optimal regulation entails:

- **The first-best production levels** $q_{MH}(\theta) = q^*(\theta)$ for all $\theta$ in $\Theta$.

- **A downward distortion in the level of care** $e_{MH}(\theta) = e_{MH} < e^*$ which is the same for both seller types:

  $$h = \psi'(e_{MH}) + (1 - \alpha)e_{MH}\psi''(e_{MH}). \quad (13)$$

- **Both types of the seller receive the same limited liability rent**

  $$U_{MH}(\theta) = U_{MH} = R(e_{MH}). \quad (14)$$

Under pure moral hazard, the second-best effort trades off the social benefit of diminishing the probability of an accident against the cost of doing so. This cost has two components: first, as under complete information, the disutility of effort incurred by the seller; second, the cost of leaving a moral hazard rent to the seller to induce his effort when it is non-observable. Indeed, because of moral hazard and limited liability, the regulator can no longer threaten the seller with large fines in case of an accident to provide him costless incentives towards safety care. Only rewards are available and a moral hazard rent $U = R(e)$ must be left to the seller to induce him to exert effort. This rent is again socially costly (with a negative weight $-(1 - \alpha)$). To reduce the social cost of this rent, the second-best effort $e_{MH}$ must be downward distorted below the first-best level. This distortion is greater when the seller’s utility has little weight in the social welfare function ($e_{MH}$ increases with $\alpha$).

### 4.2 Moral Hazard and Adverse Selection

Let us now suppose that the regulator does not observe the firm’s marginal cost $\theta$. As before, adverse selection has, as before, an impact on the quantity that should be traded. However, this imperfect knowledge of the seller’s profit affects also the amount that can be seized by the regulator when an accident occurs. Indeed, when he considers overstating
his marginal cost, the efficient seller takes into account the fact that, if a damage occurs, a lower profit can be seized. In fact, upon such an event, the efficient seller can still save the adverse selection rent $\Delta \theta q(\bar{\theta})$ that he may grasp from mimicking an inefficient seller. This leaves only the inefficient seller’s profit as possible liability payments. The possibility of saving this adverse selection rent when an accident occurs undermines much of the efficient seller’s incentives to exert care.

With both adverse selection and moral hazard, incentive compatibility for an efficient seller can be written as:

$$U(\theta) = \max_e \{ e(t(\theta) - \theta q(\bar{\theta}) - z_n(\bar{\theta})) - \psi(e) \}$$

or putting it differently,

$$U(\theta) = R(e(\theta)) \geq \Delta \theta q(\bar{\theta}) + R(e(\bar{\theta})).$$

This incentive compatibility constraint is important and drives much intuition behind the forthcoming results. Compared with the case with unlimited liability, the price received by the seller for the good is a less effective tool to induce revelation since, with some probability, the sales revenue will be seized by the regulator. The seller has to be rewarded for truth-telling by means of moral hazard rents. These rents are less efficient means of transferring wealth to the private sector to relax incentive constraints since they have also an allocative impact on the levels of safety care. The incentive constraint (15) shows that the adverse selection and moral hazard parts of the incentive problem cannot be disentangled under limited liability.

The relevant participation constraint for the inefficient seller is

$$U(\bar{\theta}) \geq R(e(\bar{\theta})).$$

This participation constraint is also hardened with respect to the case with unlimited liability. There must be a positive rent left even to the least efficient seller if one wants any effort to be exerted.

The regulator’s problem can now be rewritten as:

$$(R) : \max_{\{e(\theta), q(\theta), U(\theta)\}} E \left( (1 - e(\theta))h - \psi(e(\theta)) + S(q(\theta)) - \theta q(\theta) - (1 - \alpha)(U(\theta) + V(\theta)) \right),$$

subject to (5)-(15) and (16).

\[10\] It can be checked ex post that this is the only relevant constraint.
Proposition 3: Assume that $h$ is large enough so that the seller’s limited liability constraints are binding whatever his type. Then, the optimal regulation entails:

- All constraints (5)-(15) and (16) are binding.
- There exists $\lambda > 0$, the multiplier of the incentive constraint (15), such that the optimal effort levels $e^R(\theta)$ and $e^R(\tilde{\theta})$ verify $e^R(\theta) > e^R(\tilde{\theta})$ and satisfy
  \begin{align*}
  h &= \psi'(e^R(\theta)) + \left(1 - \alpha - \frac{\lambda}{\nu}\right) e^R(\theta)\psi''(e^R(\theta)), \\
  h &= \psi'(e^R(\tilde{\theta})) + \left(1 - \alpha + \frac{\lambda}{1 - \nu}\right) e^R(\tilde{\theta})\psi''(e^R(\tilde{\theta})),
  \end{align*}

and
  \begin{align*}
  R(e^R(\theta)) &= \Delta \theta q^R(\tilde{\theta}) + R(e^R(\tilde{\theta})).
  \end{align*}

- The efficient seller produces the first-best output $q^R(\theta) = q^*(\theta)$ whereas the inefficient one’s output is downwards distorted, $q^R(\tilde{\theta}) < q^R(\theta)$ with
  \begin{align*}
  S'(q^R(\tilde{\theta})) &= \tilde{\theta} + \frac{\lambda}{1 - \nu} \Delta \theta.
  \end{align*}

- The buyer obtains zero rent $E_{\theta}(V^R(\theta)) = 0$. The seller gets a positive rent whatever his type
  \begin{align*}
  U^R(\theta) &= \Delta \theta q^R(\tilde{\theta}) + R(e^R(\tilde{\theta})) > 0, \\
  U^R(\tilde{\theta}) &= R(e^R(\tilde{\theta})) > 0.
  \end{align*}

Note first that in a pure moral hazard environment, the same moral hazard rents are left to both seller types. When costs are instead non-observable, this is no longer possible. Doing so would indeed always make attractive for an efficient seller to underestimate his profit in order to systematically “save” the adverse selection rent $\Delta \theta q(\tilde{\theta})$. forces the regulator to give an extra reward to the efficient agent on top of the amount given under pure moral hazard. This extra reward corresponds to the non-verifiable informational rent $\Delta \theta q(\tilde{\theta})$ that can never be seized by the regulator.

With limited liability, these extra rewards increase in fact the level of care exerted by an efficient seller. At the same time, the efficient seller is less tempted to mimic an inefficient one if the latter’s moral hazard rent is downwards distorted. The level of care exerted by this agent is thus reduced to facilitate truth-telling.

Finally, as with unlimited liability, reducing the production of the inefficient seller also helps relaxing the incentive constraint (15). However, limited liability impacts on
this output distortion as it can be seen by comparing the r.h.s. of (10) and (15). At a rough level, it is still true that the regulator trades off the efficiency gain from raising \(q(\bar{\theta})\) against its incentive cost. The marginal cost of raising \(q(\bar{\theta})\) is however given now by the shadow cost \(\lambda\) of the incentive constraint (15). With unlimited liability, this shadow cost is simply the social cost of the efficient firm’s information rent, namely \(\nu(1 - \alpha)\). Instead, with limited liability, efforts and outputs are linked altogether. Raising the output \(q(\bar{\theta})\) has also an extra social value which is to increase the effort \(e(\bar{\theta})\) performed by the most efficient firm and to reduce the likelihood of an accident by this type.

Through the limited liability constraints, everything happens thus as if the buyer’s value of trade was made explicitly dependent on the probability of accident. To get further intuition on the nature of the output distortion, it is useful to rewrite (20) taking into account (19). We get:

\[
(1 - \nu)(S'(q^R(\bar{\theta})) - \bar{\theta}) + \nu \left( \frac{h - \psi'(e^R(\bar{\theta}))}{e^R(\bar{\theta})} \right) \Delta \theta = \nu(1 - \alpha) \Delta \theta. \tag{23}
\]

The first term on the l.h.s. of (23) is the marginal surplus an inefficient seller times the probability that marginal cost is high. If output \(q^R(\bar{\theta})\) is increased by \(dq\), the expected surplus increases thus by this term multiplied by \(dq\). At the same time, such an increase raises the socially costly rent of an efficient seller by an amount \(\nu(1 - \alpha) \Delta \theta dq\) which explains the r.h.s. of (23). However, with limited liability, the efficient seller can only be rewarded for truthtelling in terms of moral hazard rent and thus the effort of an efficient seller is also increased, reducing thereby the likelihood of an accident for that type. This second effect appears as the second term on the l.h.s. of (23).

Increasing production of the inefficient seller has not only an impact on productive surplus but it has also an environmental value. Everything happens as if output had an environmental impact directly incorporated into the consumer’s utility function. It is striking that even though production and care do not interact directly in the production function, incentive compatibility creates such an endogenous link through the liability constraints. One cannot design an environmental policy without keeping an eye on its impact on production. With risk regulation, this impact is positive and output distortions are reduced.

At the optimal regulation, the shadow cost of the incentive constraint with limited liability is lower than without limited liability, at least for small cost uncertainty. Far from exacerbating output distortions, limited liability reduces them. The marginal price paid for production by the inefficient firm is thus closer to its value under complete information. However, this seemingly efficiency gain is somewhat of a mirage. Under asymmetric information, the right notion of efficiency is interim efficiency which should account for the existing informational constraints. Compared to the interim efficient
outcome obtained in Proposition 1, the binding liability constraints move us away from the optimal outcome.

**Comparative Statics:** The difficulty in computing explicitly the value of the shadow cost $\lambda$ of the incentive constraint (15) makes it hard to get general comparative statics. However, we have:

**Proposition 4:** With adverse selection, moral hazard and limited liability, more efficient sellers produce more and exert more care than less efficient ones. Moreover

- $e^R(\theta) > e^{MH} > e^R(\bar{\theta})$
- $q^*(\bar{\theta}) > q^R(\bar{\theta}) > q^{SB}(\bar{\theta})$ when $\Delta \theta$ is small enough.

Our model predicts therefore that efficient sellers are less likely to create an environmental damage than inefficient ones under risk regulation. Because of the complementarity between care and output, production and safety care are positively correlated under limited liability. Note that there does not exist such a correlation without limited liability. Indeed, both types of the seller exert then the same first-best level of safety care even though they produce different outputs.

To understand the lesser magnitude of the output distortion under limited liability, it is useful to come back to (24) to explain better the social value of raising output. As long as $e^R(\bar{\theta})$ remains below the first-best level of effort $e^*$ (and this is the case for instance when $\Delta \theta$ is small enough since then $e^R(\bar{\theta})$ is close to $e^{MH}$), the environmental benefit of raising $q^*(\bar{\theta})$ and thus, by the same token $e(\bar{\theta})$, is positive. This extra value of production justifies less output distortions than without liability constraint.

**Implementation:** The optimal regulation found above is quite demanding. Indeed, it requires communication between the regulation and the seller, observability of the private transaction between the buyer and the seller (and most noticeably control of output), and also commitment to a regulatory scheme. Nevertheless, this regulation can sometimes be implemented by using only ex post liability even when such comprehensive grand-contract is no longer feasible (maybe because economic and environmental regulations are split or because output is hardly verifiable by the regulator).

Suppose that, ex post, a judge imposes a fines equal to the damage, $z_a = h$ on the seller. Assume also that the buyer has all bargaining power in designing the private transaction with the seller and that he has unlimited wealth so that he may end up paying whatever damage is realized. Then, it is easy to see that everything happens as if the fine is paid by the buyer himself.\(^{11}\) The design of the private transaction solves a problem very similar to

\(^{11}\)See Segerstrom and Tietenberg and (1992) and Bontemps, Dubois and Vukina (2003) for this “equivalence principle”.

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(R) except that the buyer does not take into account the social value of the seller’s rent in his own objective function. Only if $\alpha = 0$, it is the case that the optimal regulation can be implemented with an *ex post* liability rule asking the seller (or the buyer) to pay for the full damage. When the regulator has no redistributive concerns, the buyer shares with the latter the same concerns for extracting the seller’s rent and will thus implement the same output. Of course, contrary to the case of regulation, the buyer’s expected payoff is non-zero in the liability regime.

The case $\alpha = 0$ corresponds actually to a regulator who does not give any weight to the private sector in his objective function. He is thus only interested in collecting liabilities payments and counting the expected damage caused to third-parties. This leads exactly to the same objective as if he was acting as a judge forced to balance the cost of this damage with the payments requested from the private sector.

For $\alpha > 0$, the buyer definitively extracts too much rent from a social welfare viewpoint. Equation (23) is still useful to understand how the liability rule might be modified in this environment. Indeed, to find out the optimal output and effort chosen by the buyer for the efficient seller when the former must pay a liability payment $D$ in case of a damage, it is enough to replace respectively $h$ by $D$ and $\alpha$ by zero in that formula. Diminishing $D$ below the full damage, reduces efforts on both types and, because of the convexity of $R(\cdot)$, hardens the efficient seller’s truthtelling incentive constraint, making output distortions even more valuable. This would suggest that reducing the liability of stakeholders would be of little help to reduce output distortions.

5 Extended Liability with Shallow Pocket

Even when $\alpha = 0$, the simple *ex post* liability rule proposed above may not be feasible when the buyer-seller coalition has not enough assets to pay for the damage $h$. This will typically be the case when the buyer has himself few assets available or can easily hide them and the level of harm is much larger than the first-best surplus, $S(q^*(\theta)) - \theta q^*(\theta)$. Then, the *ex post* intervention of the judge can at most seize from the private sector the total value of the gains from trade.

In such an environment, the expected payoff of the buyer (still assuming he has all bargaining power in designing private transaction) becomes

$$E_{\theta} (e(\theta)(S(q(\theta)) - \theta q(\theta) - u_n(\theta)))$$

where $u_n(\theta) = t_n(\theta) - \theta q(\theta) = \psi'(e(\theta))$ is the seller’s payoff when no accident occurs.

The key impact of imposing liability on the buyer up to the whole value of the gains
from trade is that now the buyer may want to distort production to protect the benefits of his transaction with the seller from the threat of being seized.

To understand this new distortion it is useful to start with the case of pure moral hazard.

5.1 Pure Moral Hazard

Suppose that the marginal cost $\theta$ is common knowledge within the buyer-seller coalition. The benefit of a transaction effectively accrues to the buyer only if an accident does not occur so that the optimal contract solves:

$$(P) : \max_{\{e_n(\theta), q(\theta)\}} e(\theta)(S(q(\theta)) - \theta q(\theta) - \psi'(e(\theta))).$$

**Proposition 5**: Assume that the buyer-seller coalition is subject to ex post environmental liability but that both the buyer and the seller are protected by limited liability. Then the optimal private transaction entails:

- A level of care $e^L(\theta)$ such that $e^L(\bar{\theta}) < e^L(\theta)$ with
  $$S(q^*(\theta)) - \theta q^*(\theta) = \psi'(e^L(\theta)) + e^L(\theta)\psi''(e^L(\theta)).$$

- The first-best outputs $q^L(\theta) = q^*(\theta)$.

Conditionally on the fact that no accident takes place, the buyer finds no reason to distort output under complete information. Trade remains always efficient. Imposing liability on both the buyer and the seller has no impact on the traded volume under complete information. The often heard criticism that extending liability towards principals modifies contracting and output should be qualified. This is not the case when the stakeholder has complete information on the agent’s adverse selection parameter. Complete information between the buyer and the seller gives thus some foundations to the assumption made in the earlier literature that modifying the level of care exerted by the seller has no impact on the value of the transaction.

However, under the extended liability regime, the levels of care are far too low with respect to their levels at the optimal regulation (even in the most extreme case where $\alpha = 0$). The private value of the gains from trade is, by assumption, less than the social value of the damage. Protecting those gains does not give enough stake to incentivize the seller to exert effort. The levels of care are even far below the second-best levels found in Section 4.
Note also that different seller types choose different effort levels because the first-best surpluses associated with those types are different. With extended liability and pure moral hazard, we recover the positive correlation between care and output even though its origins are quite different from what we found in Section 4.2. This is now the fact that a more efficient seller creates more surplus that increases his incentives for care within a buyer-seller coalition protected by limited liability. Private contracts have an impact on care but note the reverse under complete information on costs.

5.2 Moral Hazard and Adverse Selection

Let us now turn to the case where $\theta$ is not known by the buyer. The buyer’s problem becomes:

$$(P): \max_{(e(\theta),q(\theta))} E \left( e(\theta)(S(q(\theta)) - \theta q(\theta)) - \psi'(e(\theta)) \right) ,$$

subject to (15).

Since the first-best surpluses for $\theta$ and $\bar{\theta}$ may be far away from each other, it is not immediately clear whether the incentive constraint is binding or not at the optimum of $(P)$. When the uncertainty on cost is not significant, however, the incentive constraint (15) is in fact always binding at the optimum of $(P)$. Indeed as uncertainty decreases, $e^L(\theta)$ and $e^L(\bar{\theta})$ come close to each other, and even though the adverse selection information rent $\Delta\theta q^*(\bar{\theta})$ becomes small, one can show that the first effect dominates so that (15) is violated by the solution proposed when neglecting this constraint.

**Proposition 6**: Assume that $h$ is large enough. Then, for $\Delta\theta$ small enough, the optimal contract between the buyer and the seller is such that there exists $\mu > 0$, the multiplier of the adverse selection incentive constraint (15), such that:

- Only the efficient seller produces the first-best output, $q^A(\theta) = q^*(\bar{\theta})$. For the inefficient seller, production is downward distorted with

  $$S'(q^A(\bar{\theta})) = \bar{\theta} + \frac{\mu}{(1 - \nu)e^A(\theta)} \Delta\theta. \quad (25)$$

- The levels of care $e^A(\theta)$ and $e^A(\bar{\theta})$ are respectively above and below their values in the pure moral hazard case; $e^A(\theta) > e^L(\theta) > e^L(\bar{\theta}) > e^A(\bar{\theta})$.

$$S(q^*(\theta)) - \theta q^*(\theta) = \psi'(e^A(\theta)) + (1 - \mu)e^A(\theta)\psi''(e^A(\theta)), \quad (26)$$

$$S(q^A(\bar{\theta})) - \theta q^A(\bar{\theta}) = \psi'(e^A(\bar{\theta})) + (1 + \mu)e^A(\bar{\theta})\psi''(e^A(\bar{\theta})). \quad (27)$$

12Which was an implicit assumption made when we looked at the conditions under which both types may get bankrupt at the optimal regulation.
Finally, (15) is binding so that
\[
R(e^A(\bar{\theta})) = R(e^A(\bar{\theta})) + \Delta q^A(\bar{\theta}).
\]

The qualitative features of the solution are quite similar to those of the optimal regulation. There still exists a positive correlation between effort and output which is reinforced by the fact that the adverse selection incentive compatibility constraint is now binding. The most efficient seller also exerts more care.

Again, as in the optimal regulation, the buyer solves the adverse selection problem by rewarding an efficient seller through an extra moral hazard rent, whereas an inefficient seller sees that rent being reduced to facilitate truthtelling.

Simultaneously, the buyer reduces the inefficient seller’s output to relax (15). However, since the benefits from trade only go to the buyer when there is no environmental damage, the efficiency cost of distorting the inefficient seller’s output downwards is not viewed as so important by the buyer. Indeed, with some probability trade with this inefficient seller will not be beneficial to the buyer. This forces him to reduce output more than what a regulator would do (even in the extreme case where \( \alpha = 0 \)). The marginal price paid for the output of an inefficient seller is quite low because the buyer has to account for a premium paid for the risk of losing all the benefits of the transaction. Adverse selection introduces a feed-back effect of care on output distortions which are now exacerbated.

With liability being extended to the buyer, and under the conditions of small cost uncertainty, strong allocative distortions appear and contracting forms that look quite inefficient from an interim efficiency viewpoint emerge.

6 Conclusion

In this paper, we have first explored the optimal risk regulation of a buyer-seller hierarchy in a framework with limited liability, moral hazard and adverse selection on some technological parameter which is a priori unrelated to care. At the optimal regulation, one cannot solve separately the moral hazard and the adverse selection sides of the incentive problem. Even in the absence of any technological interaction, the second-best optimal policy endogenously creates such a positive relationship between care and output. Efficient sellers are exert also more care.

Starting from this characterization of the optimal regulation, we then asked under which conditions can it be implemented through a simple liability rule. Such an implementation requires that the regulator has no redistributive concerns at all towards the private sector of the economy. In that case, the optimal regulation can be implemented
with a liability rule imposing to either trading partner (the principal or the agent) a fine equal to the damage incurred by third-party. Whenever the principal has enough wealth, a fine just equal to the full damage induces the second-best optimal level of care even when the agent is protected by limited liability.

However, when the size of the damage is large with respect to the gains from trade and the principal has a limited amount of assets that can be seized, such a liability rule cannot be used to the same extent. We investigated the impact of having both the buyer and the seller being subject to limited liability on the design of a private transaction subject to ex post legal intervention. In such a context, the principal and the agent may lose all their gains from trade if an accident occurs. The private transaction is designed with an eye on that threat. Extended liability still distorts contracting. Even though, they are qualitatively similar and exhibit again a complementarity between care and output, distortions are more severe than at the optimal regulation.

The directions in which output is distorted by risk regulation and liability are not as intuitive as it could seem at first glance. Risk regulation tends to reduce output distortions compared with the interim efficient outcome obtained in the absence of liability constraints. Instead, extended liability tends to increase those distortions quite significantly. This points at the different impacts that risk regulation and liability rules have on production.

References


APPENDIX
• Proof of Proposition 1: As standard in two-type adverse selection model (see Laffont and Martimort (2002), Chapter 2, for instance), (8) and (9) are both binding at the optimum. Moreover, (5) is also obviously binding. From those binding constraints, we derive $U(\theta) = \Delta\theta q(\theta), \ U(\bar{\theta}) = 0$ and $E(V(\theta)) = 0$. Inserting into the principal’s objective function and optimizing with respect to efforts and outputs yields Proposition 1.

Note that $S(q(\theta)) - t(\theta) = 0$ and $U(\theta) = \Delta\theta q^{SB}(\theta)$ define only the expected payment of the efficient seller:

$$e^*z_n(\theta) + (1 - e^*)z_a(\theta) = S(q^*(\theta)) - \theta q^*(\theta) - \psi(e^*) - \Delta\theta q^{SB}(\bar{\theta}).$$

Given this expected value, we can find the values of $z_n(\theta)$ and $z_a(\theta)$ also satisfying $z_n(\theta) - z_a(\theta) = h = \psi'(e^*(\theta))$, as it is needed to implement the first-best effort.

Similarly, $V(\bar{\theta}) = 0 = S(q(\bar{\theta})) - t(\bar{\theta})$ and $U(\bar{\theta}) = 0$ define only the expected payment to the inefficient seller

$$e^*z_n(\bar{\theta}) + (1 - e^*)z_a(\bar{\theta}) = S(q^{SB}(\bar{\theta})) - \bar{\theta} q^{SB}(\bar{\theta}) - \psi(e^*).$$

Again, we can easily find the values of $z_n(\bar{\theta})$ and $z_a(\bar{\theta})$ satisfying also $z_n(\bar{\theta}) - z_a(\bar{\theta}) = \psi'(e^*) = h$.

• Proof of Proposition 2: (1) is obviously binding. Moreover inserting (12) into the objective function and optimizing yields first-best outputs and the distorted effort given by (13).

• Proof of Proposition 3: First observe that (5) must necessarily be binding. Also if (15) was slack, optimization would lead to $e(\bar{\theta}) = e(\theta) = e^{MH}$ and we would get a contradiction when $q(\bar{\theta}) = 0$. Hence, (15) is also necessarily binding.

Denote by $\lambda$ the corresponding positive multiplier. The Lagrangean writes as:

$$E(\theta)(-h(1 - e(\theta)) - \psi(e(\theta)) + S(q(\theta)) - \theta q(\theta) - (1 - \alpha)U(\theta)) + \lambda \left( R(e(\theta)) - R(e(\bar{\theta})) - \Delta\theta q(\bar{\theta}) \right).$$

Optimizing and using the slackness condition yields (17) to (20).

• Proof of Proposition 4: Because $\lambda > 0$, we have

$$h - \psi'(e^R(\theta)) - (1 - \alpha)e^R(\theta)\psi''(e^R(\bar{\theta})) < 0 < h - \psi'(e^R(\bar{\theta})) - (1 - \alpha)e^R(\bar{\theta})\psi''(e^R(\bar{\theta})).$$
Using the fact that \((\psi'(e) + (1 - \alpha)e\psi''(e))' > 0\), we immediately get \(e^{R(\bar{\theta})} > e^{MH} > e^{R(\bar{\theta})}\).

Let us show also that \(\lambda < \nu(1 - \alpha)\) when \(\Delta \theta\) is small enough.

First, let us make explicit for \(e^{R(\bar{\theta}, \lambda)}, e^{R(\theta, \lambda)}\) and \(q^{R(\bar{\theta}, \lambda)}\) the dependence on \(\lambda\) obtained through equations (17), (18) and (20).

The value of \(\lambda\) is then obtained from solving

\[
H(\lambda) = R(e^{R(\bar{\theta}, \lambda)}) - R(e^{R(\bar{\theta}, \lambda)}) - \Delta \theta q^{R(\bar{\theta}, \lambda)} = 0.
\]  

(A.1)

Note of course that \(H'(\lambda) > 0\) and thus that the solution to (A.1) is unique.

By definition, we have \(H(0) = -\Delta \theta q^*(\bar{\theta}) < 0\). Moreover, for \(\lambda = \nu(1 - \alpha)\), we have \(e^{R(\bar{\theta}, \lambda)} = e^*, q^{R(\bar{\theta}, \lambda)} = q^{SB(\bar{\theta})}\) and

\[
h = \psi'(e^{R(\bar{\theta}, \lambda)}) + \frac{1 - \alpha}{1 - \nu}e^{R(\bar{\theta}, \lambda)}\psi''(e^{R(\bar{\theta}, \lambda)}),
\]

thus \(e^{R(\bar{\theta}, \lambda)} > e^{R(\bar{\theta}, \lambda)}\). Finally, \(H(\nu(1 - \alpha)) > 0\) when \(\Delta \theta\) is small enough.

\* Proof of Proposition 5: It is immediate and follows from direct optimization.

\* Proof of Proposition 6: Suppose that the solution is given as in Proposition 6. Denote \(F(e) = \psi'(e) + e\psi''(e)\) and \(G = F^{-1}\), we want to prove that, for \(\Delta \theta\) small enough,

\[
R(G(W^*(\bar{\theta}))) - R(G(W^*(\bar{\theta}))) < \Delta \theta q^*(\bar{\theta}),
\]  

(A.2)

where \(W^*(\theta) = S(q^*(\theta)) - \theta q^*(\theta)\) so that we will have a contradiction with the fact that (15) cannot be slack.

By the Theorem of Intermediate Values, there exists \(\tilde{W} \in (W^*(\bar{\theta}), W^*(\bar{\theta}))\) such that

\[
R(G(W^*(\bar{\theta}))) - R(G(W^*(\bar{\theta}))) = (R \circ G)'(\tilde{W})(W^*(\bar{\theta}) - W^*(\bar{\theta})).
\]

Moreover, we have \(R'(e) = e\psi''(e)\)

\[
G'(\tilde{W}) = \frac{1}{2\psi''(\tilde{e}) + \hat{e}\psi'''(\tilde{e})}
\]

for some \(\tilde{e}\) in \((e^*(\bar{\theta}), e^*(\bar{\theta}))\). Hence, we get

\[
(R \circ G)'(\tilde{W}) = \frac{\hat{e}\psi'''(\tilde{e})}{2\psi''(\tilde{e}) + \hat{e}\psi'''(\tilde{e})} \leq \frac{1}{2}
\]

because \(e \in [0, 1]\).
Finally, (A.2) holds when
\[ \frac{1}{2}(W^*(\theta) - W^*(\tilde{\theta})) < \Delta \theta q^*(\tilde{\theta}). \] 

(A.3)

but for $\Delta \theta$ small, we have
\[ W^*(\theta) - W^*(\tilde{\theta}) \approx \frac{|SD(q^*(\theta))|}{2} (q^*(\theta) - q^*(\tilde{\theta}))^2 + \Delta \theta q^*(\tilde{\theta}) \]

and (A.3) is clearly satisfied because $(q^*(\theta) - q^*(\tilde{\theta}))^2$ is $O(\Delta \theta^2)$. 

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