When monopoly oversupplies quality*

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Abstract

We consider a monopoly facing a differentiated unit demand where consumers value quality of goods and incur transportation costs. We show the monopoly can oversupply quality contrary to classic models of vertically differentiated unit demand, because here, demand is globally elastic.

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1 Introduction

This paper deals with the optimal choice of quality by a monopoly. We consider a monopoly facing a differentiated demand where consumers value quality of goods and incur transportation costs.

Numerous papers deal with the problem of quality choice by a monopoly. In models where consumers buy at most one unit of a good (unit demand), the results on the quality chosen by the monopoly mainly differ with respect to the assumption on market coverage. When the market is fully covered (i.e. whatever the choice of qualities by the monopoly, the total production remains constant), then the monopoly undersupplies quality as compared with the

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choice of a social planner. Conversely, when the market is not covered, there is no distorsion in the qualities offered.

Thus, when the market is fully covered, in a pure model of vertical differentiation and with the same assumptions as ours regarding to density and costs, Spence (1975) and Lambertini (1998) have shown that a monopoly producing a finite number of qualities undersupplies quality as compared to the social planner[1]. Mussa and Rosen (1978) obtained the same result with an infinite number of qualities. Recently Rochet and Stole (1999) in a context with both horizontal and vertical differentiation show that monopoly undersupplies quality as compared with the social planner choice but in a lower extent than in a pure vertical differentiation model. On the contrary, in a context where market is not fully covered, Lambertini (1998) showed that with a finite number of qualities, the monopoly does not distort quality. Distortion as compared to the social planner choice only affects prices.

Spence (1975) gave the two relevant effects explaining the monopoly quality level compared to the first best one:

- the monopoly has incentive to undersupply quality if the valuation of quality for the average consumer is higher than the one of the marginal consumer;
- the relation between quality and the extent to which the firm restricts quantity due to market power.

In this paper, where the unit demand is differentiated horizontally and vertically, we show that the monopoly can oversupply quality. The double differentiation adds one more degree of freedom in the demand making its price-elasticity higher than in classic model of vertical differentiation. Even if the average consumer valuation of quality is higher than the marginal consumer one, because the demand is globally elastic the monopoly oversupplies quality in order to capture a higher share of the social welfare by restricting quantities sold. The result that the monopoly oversupplies quality does not usually occur in models with a unit demand, but in frameworks where consumers can buy several units of the good (due to this assumption, demand is more elastic).

The next section presents the framework and the demand. Section 3 studies the first best optimum, whereas section 4 analyzes the quality choice of the monopoly when he sells only one product and compares it to the first best one. Section 5 concludes.

[1] See also Gabszewicz & Thisse (1986) and Shaked & Sutton (1982) for a discussion on the finiteness property.

2 The demand

Consumers are located along a segment of size 1 and buy at most one unit of a good. When they buy a product of quality q at price p, their utility is classically modeled by:

$$U(\theta, x) = \theta q - p - tx \tag{1}$$

where θ is the marginal utility for quality and t is defined as the transportation cost from the consumer location (x) to the retailer located at 0 (it can also be interpreted as a valuation of an outside good differing across consumers). We assume that $\theta \in [0, \overline{\theta} = 1]$ and is uniformly distributed on this segment, $x \in [0, L = 1]$. For this reason, consumers do not perceive the good in a identical way as they differ in their taste for quality (parameter θ) and in their location relative to the retailer where the good is sold (parameter x). The good is therefore horizontally and vertically differentiated.

The firm selling the good is a monopoly located at $x = 0$, producing the good of quality q with a quadratic unit cost function defined by: $c(q) = \frac{1}{2}q^2$. The marginal cost is thus constant in quantity[2], but marginal cost on quality is increasing, and convex. This monopoly decides the quality of the good and the price charged to consumers.

The frontier on θ defining consumers located at x who are indifferent between buying one unit of the good of quality q or not buying at all are characterized by $U(\theta_0(x), x) = 0$, that is:

$$\theta_0(x) = \frac{p + tx}{q} \tag{2}$$

The consumer of type $\theta = 1$ and indifferent for consumption is then such that $\theta_0(x) = 1$. Its location, denoted by $\overline{x}$ is defined by:

$$\overline{x} = \frac{q - p}{t} \tag{3}$$

Therefore, two cases may appear according to the behavior of the consumer characterized by $\theta = 1$ and located at $x = 1$ (that is the location of $\overline{x}$ relative to 1). Situation (A) is defined by the fact such consumer has no rent, so there exists consumers characterized with $\theta = 1$ who are not buying the good.

In this case, demand writes as $D^A(p, q) = \frac{(1 - \theta_0(0))\overline{x}}{2}$, that is:

$$D^A(p, q) = \frac{\left(1 - \frac{p}{q}\right)\left(\frac{\overline{\theta} q - p}{t}\right)}{2\overline{\theta} L} \tag{4}$$

[2] In this setting, assuming the monopoly is located at $x = 0$ is not restrictive as there is no interaction with another firm, and due to constant return to scale, location does not matter.

In regime (B), the consumer ($x = 1, \theta = 1$) enjoys a strictly positive utility. The demand is thus $D^B(p,q) = \frac{(2-\theta_0(0)-\theta_0(1))}{2}$ and can be rewritten:

$$D^B(p,q) = \frac{\left(2 - \frac{p}{q} - \frac{p+t}{q}\right)}{2} \tag{5}$$

3 The first-best optimum

The First Best quality and price are defined by the maximization of the consumer surplus: as the rule to achieve a maximal Social Welfare is to set the price to marginal cost, the monopoly profit is thus zero.

In regime (A), consumer surplus is defined by:

$$CS^A(p,q) = \int_0^{\overline{x}(p,q)} \left(\int_{\theta_0(x)}^{\overline{\theta}} (\theta q - p - tx) d\theta \right) dx = \frac{(q-p)^3}{6qt} \tag{6}$$

and its maximization leads to:

$$p^{fb-A} = c(q^{fb-A}) = \frac{8}{25} \text{ and } q^{fb-A} = \frac{4}{5} \tag{7}$$

For regime (B), the consumer surplus is now defined by:

$$CS^B(p,q) = \int_0^1 \left(\int_{\theta_0(x)}^1 (\theta q - p - tx) d\theta \right) dx = \frac{3p^2 + 3pt + t^2 + 3q(q - 2p - t)}{6q} \tag{8}$$

The computations for first best values are tedious, but there are not needed for the comparisons with the quality chosen by the monopoly. Indeed, the index that allows to rank first best qualities and monopoly qualities only relies on the analytical expression of the social welfare evaluated with the quality chosen by the monopoly (see next section).

The Social Planner is in regime (A) as long as $\overline{x}(t^{fb}) < 1$. Given equation (3) computed with p^{fb-A} and q^{fb-A}, it leads to $t > t^{fb} = \frac{12}{25}$.

4 The monopoly optimum and comparisons

Due to the definition of the demand in regime A given in equation (4), monopoly profit is defined by:

$$\pi^A(p,q) = (p - c(q))D^A(p,q) = \left(p - \frac{1}{2}q^2\right) \frac{\left(1 - \frac{p}{q}\right)\left(\frac{q-p}{t}\right)}{2} \tag{9}$$

Maximizing (9) with respect to price[3] and quality gives:

$$p^{m-A}(q) = \frac{q(q+1)}{3}\ ,\ q^{m-A} = \frac{4}{5}, \text{ and a profit } \pi^{m-A} = \frac{32}{3125t} \tag{10}$$

Situation (A) is relevant as long as $\overline{x}(t_m) \leq 1$, that is: $t > t_m = \frac{8}{25}$ (because $p > q - t \Leftrightarrow t > t_m$ as the price is independent of the transportation cost).

In regime (B), the monopolist maximizes $\pi^B(p,q) = (p - c(q))D^B(p,q)$, leading to:

$$p^{m-B}(q) = \frac{q^2 + 2q - t}{4} \text{ and } q^{m-B} = \frac{1 + \sqrt{3t+1}}{3} \tag{11}$$

This equilibrium achieves a monopoly profit equal to:

$$\pi^{m-B} = \frac{(1-9t) + (3t+1)^{\frac{3}{2}}}{27} \tag{12}$$

Contrary to the previous regime, the price and the quality of the good do now depend on the transportation cost. When t increases, it has two effects on the price level. First, it induces an increase in quality so cost arises and as demand elasticity in price decreases, this leads to an increase in price. Second, the increase in t makes elasticity in price increase, so the final price can decrease. Finally, the price increases when the first effect dominates (quality effect) that is as long as $t < 1$. But because regime (B) is only relevant for $t \in [0, \frac{8}{25}]$, the price set by the monopoly is always increasing in the transportation cost for regime (B).

The quality and price of the good are continuous in t (and so is the profit) between the two regimes (A and B)[4].

Proposition: *For any transportation cost, the monopoly chooses a higher or equal quality than the first best one. Quality chosen is strictly higher for* $t \in]0, \frac{12}{25}[$.

Proof:

To compare the first best qualities with the quality level chosen by the monopolist, we do not need to compute first best qualities but an index (see below).

As described in Spence (1975), the incentive to distort quality level depends on how the surplus that the monopoly may appropriate varies with quality. Indeed, two effects must be

[3]It is interesting to note that as quality is constant, the price is also independent of the transportation cost t and equal to $p^{m-A} = p^{m-A}(q^{m-A}) = \frac{12}{25}$. This comes from the fact that in a such model, elasticity in price or in quality does not depend on transportation cost.

[4]The profit is concave in (p,q) in each regime, and differentiable in $t = \frac{8}{25}$.

taken into account simultaneously for explaining the monopoly choice in quality: marginal *versus* average valuation quality, and incentive to restrict quantities sold.

The average consumer valuation of quality is higher than the marginal consumer one. Computing the inverse demand function for both regimes gives (where Q is the total quantity sold):

$$p^{m-A}(Q,q) = q - \sqrt{2Qqt} \text{ and } p^{m-B}(Q,q) = \frac{q(1-2Q)-t}{2}$$

Therefore,

$$\frac{\partial^2 p^{m-A}(Q,q)}{\partial q \partial Q} = -\frac{t}{2\sqrt{2Qqt}} < 0 \text{ and } \frac{\partial^2 p^{m-B}(Q,q)}{\partial q \partial Q} = -1 < 0$$

Given a total quantity Q, the monopoly has incentive to undersupply quality compared to the first-best one (like in classic unit demand models of vertical differentiation). But here, as demand is more elastic because of horizontal differentiation, quantities restrictions must be taken into account for explaining that the monopoly oversupplies quality.

The analytic expressions of the monopoly and the first best are:

$$p^{m-A} = \frac{12}{25}; q^{m-A} = \frac{4}{5}; p^{fb-A} = \frac{8}{25}; q^{fb-A} = \frac{4}{5}; \tag{13}$$

and

$$p^{m-B} = \frac{4-3t+4\sqrt{1+3t}}{18}; q^{m-B} = \frac{1+\sqrt{1+3t}}{3}; \tag{14}$$

When the Social Planner is in regime (A), that is for $t > t^{fb} = \frac{12}{25}$, the monopoly is also in regime (A) as $t > t_m = \frac{8}{25}$. There is no distortion in quality and it is set to $q^{m-A} = q^{fb-A} = \frac{4}{5}$.

When the Social Planner is in regime (B), two cases must be distinguished for the monopoly's regime[5]. The monopoly is still in regime (A) for $\frac{8}{25} < t < \frac{12}{25}$, and switches for regime (B) when $t < \frac{8}{25}$. In order to compare qualities in both situations, we follow the method proposed by Spence (1975) in his Proposition 2 p. 421: the monopoly oversupplies quality when $\beta'(q) > 0$ with $\beta(q) = \frac{\pi^i(p(q),q)}{SW^i(q)}$ and $i = (A)$ or (B) for regimes. $SW^i(q)$ denotes the Social Welfare, that is the Consumer Surplus evaluated with a price equal to the unit cost and at the monopoly's optimum quality.

- When $0 < t < \frac{8}{25}$, the monopoly and the Social Planner are in regime (B). Monopoly profit gives: $\pi^B(p(q),q) = \frac{((q-2)q+t)^2}{16q}$ and $SW^B(q) = \frac{3(q-2)^2q+6(q-2)t+\frac{4t^2}{q}}{24}$. Straightforward

[5]The Social Planner switches for greater values of t than the monopoly from regime (B) to regime (A). In regime (A), there is no distortion in quality but the price of the monopoly is higher than the unit cost. So when t decreases, switching from (A) to (B) occurs for larger values of the transportation cost for the Social Planner. Quality then vary in regime (B).

computations lead to:

$$Sign[\beta'(q(t))] = Sign[6(q(t)-1)t^2((q(t)-2)q(t)+t)] \tag{15}$$

Using (14), the sign of $\beta'(q(t))$ is positive for $t \in [0, \frac{8}{25}]$.The monopoly therefore oversupplies quality compared to the Social Planner (excepted in $t = 0$ where the index $\beta'(q) = 0$).

- For $\frac{8}{25} < t < \frac{12}{25}$, the monopoly is located in regime (A) whereas the Social Planner is still in regime (B). Therefore, $\pi^A(p(q), q) = -\frac{(q-2)^3 q^2}{108t}$ leading to Sign$[\beta'(q(t))]$=Sign$[-4(q(t)-2)^2(q(t)-1)q(t)^2((q(t)-2)q(t)+2t)^2]$=Sign$[192(12-25t)^2]$ by substituting q with $q^{m-B} = \frac{4}{5}$. In this range of transportation costs, the derivative is positive and the monopoly is still oversupplying quality.

□

The monopoly does not distort quality compared to first best as long as $t = 0$ or $t > \frac{12}{25}$. In all other configurations, the quantity restrictions because of demand elasticity overrides the fact that the consumer's average valuation of quality is higher than the marginal one.

5 Conclusion

The result is that under partial market coverage, the monopoly oversupplies quality. This contrasts with the case of only one differentiation dimension and unit demand models where the monopoly sets the same quality than the Social Planner or a lower one. When taking into account the double differentiation demand of consumers, and therefore an always partial market coverage, we find the monopoly can oversupply quality. The quantity restriction effect leads to a higher quality supplied, and thus a higher price.

Considering global elastic demands, without multi-unit buying consumers, adds one more degree of freedom for the demand function inducing a higher global elasticity in price. This improvement reverses the classical conclusion of unit demand models only differentiated vertically as the monopoly restricts more severely the quantities he sells. He is then obliged to set a higher quality level than the first best one in order to capture a higher share of the social surplus.

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