Money Growth and Interest Rate Rules: Is There an Observational Equivalence?

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Abstract

In this paper, we discuss the observational equivalence between two representations of monetary policy: a stationary stochastic process of the growth rate of money supply and a Taylor type rule, *i.e.* a relationship between interest rate and expected inflation. We show that the equivalence between money growth rule and interest rate rule depends on the relative size of the sunspots associated to nominal and/or real variables.

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JEL Classification: E4, E5.

Introduction

In this paper, we discuss the observational equivalence between two representations of monetary policy. In a first case, monetary policy is represented as a stationary stochastic process of the growth rate of money supply. In the second, monetary policy is represented as a Taylor type rule, *i.e.* a relationship between interest rate and expected inflation. We do so using a general equilibrium monetary model – a cash–in–advance economy – that is sufficiently simple to permit a closed form analysis of equilibrium conditions.[1] Given the solution in the two cases, we obtain *(i)* the Taylor rule parameter under the model with

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[1] Minford, Perugini and Srinivasan [2002] and Auray and Fève [2002] showed that the equilibrium conditions of a monetary model with exogenous money growth rule may be rewritten as a Taylor type rule. However they do not study the equivalence between the two monetary policies.

exogenous money supply and *(ii)* the parameter of money growth under the model with a Taylor type rule. We then compare these two cases in order to evaluate some equivalence property.

It is worth noting that introducing an interest rate rule – a Taylor type rule – leads to multiple equilibria in a cash–in–advance economy (see Carlstrom and Fuerst [2000]). We then distinguish our results through two stochastic environments. When we only consider the sunspot variable that modifies the inflation behavior, there is no equivalence between exogenous money growth and interest rate rules. Conversely, when we only consider the sunspot variable that modifies the real variables, there exists perfect equivalence between exogenous money growth and interest rate rules. Therefore the equivalence between money growth rule and interest rate rule depends on the relative size of the sunspot variables associated to nominal and real variables.

The paper is organized as follows. A first section presents a monetary model with flexible prices. In the second section we discuss the equivalence between the two monetary policies taking into account the case of multiple equilibria. A last section offers some concluding remarks.

1 The monetary economy

This section is devoted to the exposition of the model. We set up a cash–in–advance model with perfect prices flexibility. The model is deliberately stylized in order to deliver basic results on aggregate co-movements.

Households

The economy is comprised of a unit mass continuum of identical infinitely lived agents. A representative household enters period t with real balances m_t/P_t brought from the previous period and nominal bonds b_t. The household supplies labor at the real wage W_t/P_t. During the period, the household also receives a lump–sum transfer from the monetary authorities in the form of cash equal to N_t and real interest rate payments from bond holdings ($(R_{t-1} - 1)b_t/P_t$). All these revenues are then used to purchase a consumption bundle, money balances and nominal bonds for the next period. Therefore, the budget constraint simply writes as

$$b_{t+1} + m_{t+1} + P_t c_t = W_t h_t + R_{t-1} b_t + m_t + N_t$$

Money is held because the household must carry cash — money acquired in the previous period and the money lump sum transfer — in order to purchase goods. She therefore faces a cash–in–advance constraint of the form :

$$P_t c_t \leqslant m_t + N_t + R_{t-1} b_t - b_{t+1}$$

Each household has preferences over consumption and leisure represented by the following intertemporal utility function :

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\log(c_{t+i} - h_{t+i}\right]$$

where $\beta \in (0,1)$ is the discount factor, h_t denotes the number of hours supplied by the household. E_t denotes the expectation operator conditional on the information set available in period t. The household determines her optimal consumption/saving, labor supply and money and bond holdings plans maximizing utility subject to the budget and cash–in–advance constraint. Consumption behavior together with labor supply yields

$$\frac{P_t}{W_t} = \beta E_t \frac{P_t}{P_{t+1}} \frac{1}{c_{t+1}}$$

whereas nominal return of bond holdings is given by :

$$R_t = \frac{W_t}{P_t c_t}$$

This last equation together with the CIA constraint determines the money demand where the real balances are a decreasing function of the nominal interest rate for a given real wage.

Firms and Government Budget Constraint

The technology is described by the following constant returns to scale production function :

$$Y_t = A h_t$$

such that in equilibrium the real wage is $W_t/P_t = A$ where A is a strictly positive scale parameter. The government issues nominal bonds B_t to finance open market operations. The government budget constraint is $M_{t+1} + B_{t+1} = M_t + R_{t-1} B_t + N_t$ with M_0 and B_0 given.

Monetary Rules

We consider two representations of monetary policy. In the first case, monetary policy is described by an exogenous money growth rule. This way of describing monetary policy is standard in monetary economics. This is analogous to the “helicopter drop”. In the second case, monetary policy is represented by a Taylor type rule that describes how a central bank sets the nominal interest rate in response to economic variables.

Case 1: Money is exogenously supplied according to the following money growth rule

$$M_{t+1} = \gamma_t M_t \tag{1}$$

where γ_t follows an AR(1) process :

$$\log(\gamma_t) = \rho_\gamma \log(\gamma_{t-1}) + (1 - \rho_\gamma)\log(\bar{\gamma}) + \sigma_{\varepsilon^\gamma}\varepsilon_t^\gamma$$

ε_t^γ is a white noise with unit variance, $\sigma_{\varepsilon^\gamma} > 0$ and $|\rho_\gamma| < 1$. In this case, the Central Bank could implement what is essentially the classic textbook policy of dropping freshly printed money from a helicopter. A money-financed tax cut is then essentially equivalent to Milton Friedman's famous "helicopter drop of money".

Case 2: We specify the following Taylor type rule

$$\widehat{R}_t = \eta E_t \widehat{\pi}_{t+1} \tag{2}$$

where the hat denotes the percentage of deviation from the long run value. This specified Taylor type rule (equation 2) is similar to the one introduced by Batini and Haldane [1999] and Clarida, Gali and Gertler [2000]. It incorporates only the expected inflation rate and aims at describing the joint behavior of the nominal interest rate and the expected inflation. We choose this Taylor type rule for several reasons. First, the empirical finding with this rule are actually well documented. The works of Batini and Haldane [1999], and Clarida, Gali and Gertler [1998] and [2000] provide strong evidence of an increase in the real interest rate facing higher expected inflation. Second, we restrict our analysis to a single parameter in order to deliver a clear result. The idea here is to deliver a simple one–to–one relation between the two monetary policy rule parameters. Third, previous empirical results suggest that the estimated parameter of (expected) output gap is marginally significant for the Volcker–Greenspan era. Conversely, the estimates of the expected inflation parameter are significant, positive and exceeds unity in most cases (see Taylor [1999] and Clarida et al. [2000]).

Equilibrium

An equilibrium is a sequence of prices and allocations, such that given prices, allocation maximizes profits (when taking technological choice into account) and maximizes utility (subject to the savings behavior), and all markets clear.

2 Observational equivalence

The equilibrium conditions are approximated using a log-linearization about the deterministic steady state. The log-linear solution depends only on the parameters of the forcing variable (in the case of exogenous money supply) or of the parameter that represent the central bank reaction function (in the case of a Taylor rule). Given the solution, we consider *(i)* the Taylor rule parameter (η_ρ) under the flexible price model with exogenous money supply and *(ii)* the autoregressive process parameter (ρ_η) of the AR(1) money growth process under the model with a Taylor rule. We then compare the two parameters in order to evaluate some equivalence property.

<u>Case 1: Exogenous Money Supply</u>

The log-linear approximation of the flexible price economy is given by :

$$\widehat{\pi}_t = \widehat{\gamma}_t + \widehat{y}_{t-1} - \widehat{y}_t \tag{3}$$

$$\widehat{y}_t = -\rho_\gamma \widehat{\gamma}_t \tag{4}$$

$$\widehat{R}_t = -\widehat{y}_t \tag{5}$$

$$\widehat{\gamma}_t = \rho_\gamma \widehat{\gamma}_{t-1} + \varepsilon_t^\gamma \tag{6}$$

Using equations (3)–(6), we define inflation and nominal interest rate in terms of the forcing variable :

$$\widehat{\pi}_t = (1+\rho_\gamma)\widehat{\gamma}_t - \rho_\gamma \widehat{\gamma}_{t-1} \tag{7}$$

$$\widehat{R}_t = \rho_\gamma \widehat{\gamma}_t \tag{8}$$

Rewriting equation (7) in period $t+1$ and taking expectation yields $E_t\widehat{\pi}_{t+1} = \rho_\gamma \widehat{R}_t$. Substituting in (8) leads to:

$$\widehat{R}_t = \frac{1}{\rho_\gamma} E_t \widehat{\pi}_{t+1}$$

The Taylor rule parameter ($\widehat{\eta}_\rho$) under the model with exogenous money supply is given by:

$$\widehat{\eta}_\rho = \frac{1}{\rho^\gamma}$$

In this case, the parameter of the Taylor type rule is a non–linear decreasing function of ρ_γ that accounts for the persistence of money injections. When money injection are very persistent ($\rho_\gamma \to 1$), the nominal interest rate weakly reacts to expected inflation and the real interest rate remains almost constant. Conversely, when the money injection is almost a white noise ($\rho_\gamma \to 0$), the estimated central bank reaction function implies that the nominal interest rate strongly responds to the expected inflation. It follows that an estimated "active" Taylor rule is associated to weak persistence of money injection, whereas an estimated "passive" Taylor rule corresponds to persistent money injection.

Finally, this result possesses empirical contents. Given some previous estimates of ρ_γ, it follows that the value of η is greater than one and is close to the ones of estimated

Taylor rule (see Clarida et al. [2000], tables II and III, p 157 and 160). For example, $\rho_\gamma \in (1/2, 2/3)$ – which corresponds to the range of estimates – implies a parameter value of the Taylor rule between 1.5 and 2.

Case 2: Taylor type Rule

We now consider the stochastic process of money growth under the flexible prices model with a Taylor rule. We will seek to verify if there exist or not observational equivalence between the two monetary rules (see Christiano, Eichenbaum and Evans [1998] for a discussion). In this case, the log-linear approximation of the economy is given by :

$$\widehat{\pi}_t = \widehat{\gamma}_t + \widehat{y}_{t-1} - \widehat{y}_t \tag{9}$$

$$\widehat{y}_t = -E_t \widehat{\gamma}_{t+1} \tag{10}$$

$$\widehat{R}_t = -\widehat{y}_t \tag{11}$$

$$\widehat{R}_t = \eta E_t \widehat{\pi}_{t+1} \tag{12}$$

From these expressions (9)–(12), one obtains the output dynamics:

$$E_t \widehat{y}_{t+1} = \frac{1}{\eta} \widehat{y}_t \tag{13}$$

The dynamic properties of the equilibrium critically depends on the value of η with respect to the unit circle. When $|\eta| < 1$, the equilibrium is locally determinate. Conversely, the equilibrium is locally indeterminate when $|\eta| > 1$. This means that agressive policies ($\eta > 1$) leads to real indeterminacy. As stated by Carlstrom and Fuerst [2000], this aggressive monetary policy is the basis of indeterminacy as it implies that nominal and real interest rate moves in the same line. Suppose a sunspot-driven increase in current consumption. The intertemporal allocation of saving lowers the real interest rate and thus the nominal interest rate when $\eta > 1$. From the money demand (11), consumption increases. This completes the circle and the initial beliefs are therefore rational. Equation (13) rewrites:

$$\widehat{y}_t = \frac{1}{\eta} \widehat{y}_{t-1} + \varepsilon_t^y$$

where ε_t^y is a martingale difference sequence that satisfy $E_{t-1}\varepsilon_t^y = 0$. This term is a sunspot variable that is consistent with rational expectations. When indeterminate, this model with a Taylor rule implies only one type of sunspot variables that affect real variables.[2] Moreover, nominal inderteminacy[3] occurs for any value of η:

$$\widehat{\gamma}_t = \varepsilon_t^\pi - \widehat{y}_{t-1}$$

[2] It is worth noting that the stochastic dimensions implied by the sunspot variables increases when we consider the sticky prices version of the model (see Auray and Fève [2003]).

[3] By nominal indeterminacy, we mean that inflation rate is free, *i.e.* there is nothing to pin down the initial growth rate of money.

where $E_{t-1}\varepsilon_t^\pi = 0$. A supplementary sunspot variable enters in the determination of the growth rate of money. The stochastic process of money growth is given by:

$$\widehat{\gamma}_t = \frac{1}{\eta}\widehat{\gamma}_{t-1} + \varepsilon_t^\pi - \frac{1}{\eta}\varepsilon_{t-1}^\pi - \varepsilon_{t-1}^y$$

Consider now the autoregressive parameter of order one of money growth:

$$\widehat{\rho}_\eta = \frac{Cov(\widehat{\gamma}_t, \widehat{\gamma}_{t-1})}{V(\widehat{\gamma}_{t-1})}$$

The autoregressive parameter $\widehat{\rho}_\eta$ of money growth under the model with a Taylor type rule is given by :

$$\widehat{\rho}_\eta = \frac{1}{\eta}\left[\frac{\eta^2\sigma_{\varepsilon^y}^2}{(\eta^2-1)\sigma_{\varepsilon^\pi}^2 + \eta^2\sigma_{\varepsilon^y}^2}\right]$$

where $\sigma_{\varepsilon^y}^2$ and $\sigma_{\varepsilon^\pi}^2$ denote the variance of ε_t^y and ε_t^π, respectively. The parameter $\widehat{\rho}_\eta$ depends on the relative size of the sunspots associated to nominal and/or real variables. When we only consider the sunspot variable that modifies the inflation behavior ($\sigma_{\varepsilon^\pi}^2 > 0$ and $\sigma_{\varepsilon^y}^2 = 0$), there is no equivalence between exogenous money growth and interest rate rules as $\widehat{\rho}_\eta = 0$. However, as pointed out by Carlstrom and Fuerst [2000], nominal indeterminacy is of no importance since it has no effect on real variables. Moreover, the quantitative implications of ε_t^π are counterfactual for nominal variables and especially for money growth. Conversely, when we only consider the sunspot variable that affects the real variables ($\sigma_{\varepsilon^y}^2 > 0$ and $\sigma_{\varepsilon^\pi}^2 = 0$), there exists a perfect equivalence between exogenous money growth and interest rate rules as $\widehat{\rho}_\eta = \frac{1}{\eta}$. The interpretation of the parameter value of the Taylor rule or the parameter value of the process of money growth leads to the same conclusion, *i.e.* an agressive Taylor rule can be viewed as weekly persistent money growth and *vice versa*.

3 Concluding remarks

In this paper, we show that the equivalence between money growth rule and interest rate rule in a flexible prices economy depends on the relative size of the sunspots associated to nominal and/or real variables. When we only consider the sunspot variable that modifies the real variables, we get the result of a perfect observational equivalence between the two monetary policies. Conversely, when we only consider the sunspot variable that affects the inflation behavior, any change in the Taylor rule parameter has no effect on the autoregressive parameter of money growth.

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