The Employment–Productivity Tradeoff around the 1980s:

A Case for Medium Run Theory.

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Abstract

Over the last few decades, the pattern of productivity growth across industrialized countries has been quite uneven. In the late seventies and throughout the eighties, countries like Germany and Japan experienced substantial growth in labor productivity, while the US did rather poorly. In contrast, by the mid–nineties, productivity patterns had reversed and it was now the US that was experiencing high growth. The main claim of this paper is that such cross–country differences are hard to understand without a theory of the medium run. To understand this phenomena, we present a simple model of growth which, in the long run, works like a Solow growth model, but in the medium run involves cycles of technological adoption where the timing is influenced by country specific factors. We then use the model to show how medium run adoption cycles can quantitatively explain the differential growth experiences of industrialized countries since the 1970s and in particular explain the strong tradeoff between productivity growth and employment growth (or labor force growth) that arose from the late seventies to the mid–nineties.

Key Words: Cross–Country Growth, Technological Adoption, Labor Force growth, Medium Run.

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Introduction

The growth performances of industrial countries during the last 30 years of the XXth century have been quite uneven and often staggered. For example, in the late 70s and throughout the eighties, countries like Germany and Japan experienced substantial improvements in labor productivity, while the US performance was rather poor. In contrast, by the mid–nineties the situation had reversed itself with the US experiencing high growth, while Japan and Germany were lagging. These differential growth performances have led some economic commentators to suggest that the institutions favorable for growth have changed over this period. According to this view, the eighties were a period where the institutional structure of Japan and Germany — which is highly corporatist — was more favorable to growth than that of the US; while in the nineties the growth process had changed and the institutions of the US — which are more decentralized — were most favorable for growth. Such a view, whereby the determinants of growth change quite radically over short periods, contrasts rather sharply with the more stable view of the growth process developed over the years by the growth literature. The question that thereby arises is: *can we reconcile the uneven growth pattern among industrial countries observed in the 70s, 80s and 90s with a long term view of stable determinants of growth?*; or instead *should we conclude from the experiences of the last 30 years that the growth process is rather unstable with the main determinants of growth changing substantially from decade to decade?*

In this paper, we will argue that the long term determinants of growth may well be stable, as suggested by traditional of growth theory, but that medium run dynamics may render inferences regarding growth using a short panel of cross-country data difficult or hazardous. Presently, when making cross-country growth comparisons, it is common practice to take account of catch-up forces — considered as *long–term* phenomena — and to control for different phases of the business cycle — taken as *short–run* phenomena. In this paper we argue that one should also recognize the potential role of medium run dynamics explicitly when comparing economic performance across countries, that is, recognize the possible presence of temporary growth cycles that may be staggered across countries over periods of 10 to 20 years.[1] In our view, failing to account for medium run phenomena may cause the determinants of long term growth to appear to change for spurious reasons. In effect, we will show that some correlates of growth do change over time and we will present a simple model of endogenous technological adoption that illustrates why country specific factors may influence the timing of growth, and thereby create the illusion that the determinants of long term growth have

[1] This view echoes that presented in Blanchard [1997].

changed, when they have, in fact, remained constant.[2]

In order to motivate our interest in medium run dynamics, we will begin by documenting an intriguing correlation pattern that arose among industrialized countries over the last quarter of a century, and which is hard to understand without a theory of medium run growth. In particular, we will show that during the period of the late seventies to the mid–nineties, an extremely strong negative correlation between the growth in labor productivity and the rate of employment and/or labor force growth emerged. For example, a difference in labor force growth of 1% per year over this period was associated with poorer performance in labor productivity growth of almost 1% per year. Moreover, we show that this correlation is very robust and we present instrumental variable estimates to support the view that this link is likely causal. However, we will also show that this pattern is not ubiquitous, as it was not present in the sixties and started to disappear in the latter half of the nineties. Based on this observation, we seek to reconcile a very strong negative tradeoff between labor force growth and productivity growth that lasted for about 15 to 20 years (around the eighties), with the view that labor force growth is likely to have negligible effects on labor productivity growth in the long run. To this end, we present a model of endogenous technological adoption which behaves like a neo–classical growth model in the long run but exhibits adoption dynamics with a timing that is influenced by country specific factors such as labor force growth. The main goal of the model is to illustrate how something as trivial as labor force growth can cause substantial differences in medium run growth patterns while simultaneous having no effect on long run growth. Hence, our model will highlight the potential importance of medium run growth phenomena and its sensitivity to factors that may be considered unimportant in either the long run or over the business cycle.

The theoretical model we present is kept as simple as possible. As in Solow [1956], growth in technological opportunities is exogenous and the rate of savings is kept constant. However, at every point in time, there is a set of technologies that can be adopted.[3] The technologies differ in terms of their potential for improvement. When a given technology stops improving,

[2]One of the messages of this paper is that the use of short observation on growth (less that 20 years) as a means of detecting the determinants of future long term growth may be unwise and even misleading since it may be significantly affected by medium run factors. This type of warning is particularly relevant given the common practice in economic journalism of using indiscriminately recent cross-country observations on economic growth as a means of identifying new growth paradigms.

[3]The view of technological change upon which we build is in the spirit of the General Purpose Technology literature (see e.g. Bresnahan and Trajtenberg [1995]) as we model the growth process as composed of a sequence of episodes associated with the adoption of different technologies. In effect, one of the objectives of our modelling strategy is to present a simple model of technological change, which maintains the properties of a Solow growth model in the long run, but captures tradeoffs associated with technology adoption in the medium run.

agents must decide whether to abandon it and start adopting a new technology which still has potential for improvement.[4] However, the adoption decision in the model is not trivial as there is a tradeoff created by the amount of investment needed to productively use the next technology. As time elapses and the new technology improves, the tradeoff changes in favor of the improving technology and adoption becomes clearly dominant. The interaction of adoption decisions with such technological opportunities leads to three growth phases: *(i)* a stagnant phase when a country produces with a technology that is no longer improving, *(ii)* a high growth phase when the country is adopting the new technology and *(iii)* a mature phase when the country is producing the more advanced technology while it is still improving.

The effective production function in the model — that is, the production function which incorporates the optimal technological adoption decision — moves in a neutral fashion in the long run. Therefore all countries eventually benefit equally from a new technology. However, the medium–run will be characterized by endogenous adoption decisions that will cause countries to "take off" in the new technology at different times depending on their individual characteristics. Hence, the model creates what Matsuyama [2002] calls a *flying geese* pattern, that is, a period were some countries grow rapidly then slowly, while other countries do the reverse. Given this feature, we focus on how different rates of labor force growth can affect the timing of the take–off and subsequent slowdown in labor productivity growth. It will be seen that countries with low rates of population growth begin adopting a new technology sooner as they find the required capital outlays less constraining. In contrast, countries with high rates of population growth wait until efficiency in the new technology improves before adopting the new technology. Since the mechanism emphasized in the model is extremely simple, it is easy to examine its quantitative properties, as most of the parameters in the model have rather unambiguous empirical counterparts. As we will show, the effects of labor force growth in this model can explain the timing of recent growth experiences among industrial countries, and in particular is capable of explaining the negative co–movement between labor force growth and productivity growth observed around the 1980s.[5]

The remaining sections of the paper are structured as follows. In section 1, we present evidence from the late seventies to the early nineties that highlights a strong negative correlation

[4] There is a large literature on technological adoption and aggregate dynamics. (see e.g. Gort and Klepper [1982], Jovanovic and MacDonald [1994], Andolfatto and MacDonald [1998], Helpman and Trajtenberg [1998] among others)

[5] It is interesting to note that the onset of a technological transition in the model is caused by the termination of technological improvement in the old technology. Hence, the model has the property that growth is slow at the beginning of a technological transition, since production is concentrated in the non–improving old technology. However, contrary to many existing models, it is not the new technology that causes slow growth; but instead the slow growth in the old technology that makes adopting the new technology attractive.

between labor force growth and labor productivity growth which is hard to explain without a theory of the medium run. Moreover, we show that this link appears temporary as it was not present earlier and that it started to diminish in importance by the mid–nineties. In section 2 we present our model of technological transition which emphasizes how technological adoption decisions can interact with country specific factors as to generate medium run growth cycles that are staggered across countries. In section 3, we perform a quantitative exploration of the extent to which the model is capable of reproducing the co–movements between labor force growth and productivity growth observed from the early seventies to the late nineties. Finally, a last section offers concluding remarks.

1 Some Intriguing Observations

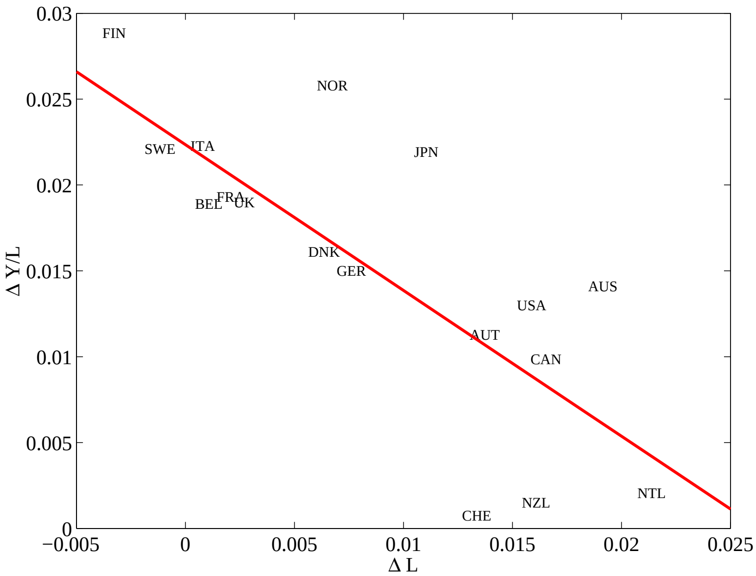
In this section we highlight what we see as an intriguing medium–run observation between the growth of labor productivity and growth of employment that arose among industrial countries from the late seventies to the mid–nineties. We focus on the experience of the major industrialized countries, that is, countries with more than a million people and with GDP–per–capita in 1980 greater than 50% of the US level.[6] We choose to focus only on the richest countries since it is a set for which assuming common access to frontier technological opportunities appears most plausible. The 17 countries forming our sample are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, United Kingdom and the United States. The data are taken from the 6.0 version of the Heston and Summers data set and the OECD statistical compendium. In particular, the output data is taken from the Heston Summers data set to assure international comparability, while employment and labor force data are taken from the OECD statistical compendium. Details of data construction are presented in appendix A. Note that in the Heston-Summers data, employment is proxied by the number of people between the ages of 15 and 64. Since this is a poor proxy for employment, we do not use this proxy in constructing output-per-worker but instead use the employment data available in the OECD statistical compendium.

[6] We found it natural to cut the sample at this point since it is where there was a rather large break in the data. For example, the next richest industrialized countries had per–capita–incomes below one third of the US level in 1980.

1.1 The rise of demographic forces

This section aims at illustrating the potential importance of employment growth in affecting medium–run growth in output-per-worker. As a first illustration, Figure 1 reports the relationship between annualized labor productivity growth and annualized employment growth for the period 1978 to 1995. As can be clearly seen from this figure, there is a very strong negative correlation between these variables over this period. This is confirmed by the examination of the regression coefficient on employment growth which is -0.85 with a standard error of 0.17. Despite the existence of such a strong negative relationship, a first impression

Figure 1: Labor Productivity Growth and Employment Growth (1978–1995)

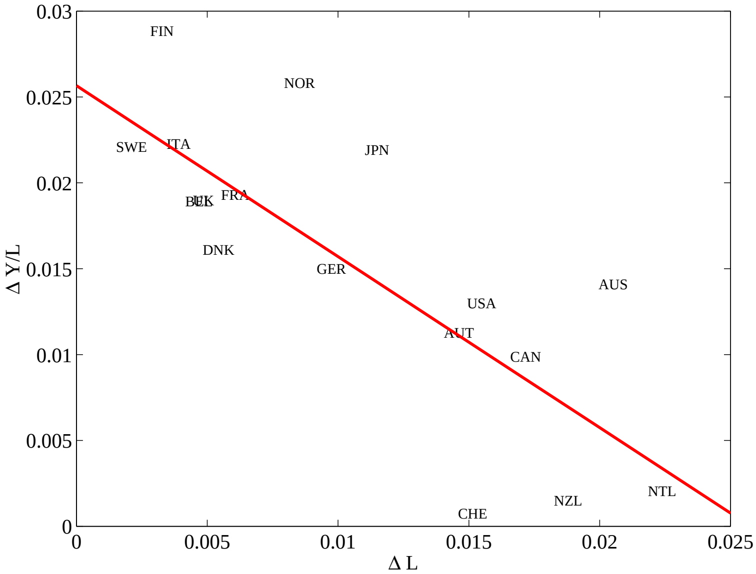


Note: The plain line corresponds to the regression model:

$$\Delta \log(Y/L) = \underset{(0.002)}{0.022} - \underset{(0.169)}{0.849}\, \Delta \text{Emp},\ \overline{R}^2 = 0.69$$

one may have when seeing this observation is that it may mainly reflect differences in unemployment trends. Therefore, as a first pass to show that this observation is unlikely driven by differences in unemployment, in Figure 2 we plot the relationship between the growth in labor productivity and the growth in labor force. As can be seen from the figure, the relationship between labor productivity growth and labor force growth is almost identical to that observed between labor productivity and employment growth. One should actually not be surprised by the similarity between these two figures given the magnitudes involved. For example, when comparing countries with the highest and lowest rates of employment growth

Figure 2: Labor Productivity Growth and Labor Force Growth (1978–1995)



Note: The plain line corresponds to the regression model:

$$\Delta \log(Y/L) = \underset{(0.002)}{0.026} - \underset{(0.194)}{0.996}\, \Delta \mathrm{LF},\ \overline{R}^2 = 0.71$$

over the entire period (not annualized), we are discussing a magnitude of about 35%. In contrast, when we compare differences in changes in rates of unemployment we are discussing magnitudes of about 5%. Hence changes in rates of unemployment are just not of the right magnitude to be driving the observed negative correlation between labor productivity growth and employment growth. In what follows, we will focus mainly on the negative relation presented in Figure 2 — that is between labor productivity growth and labor force growth — since we want to examine whether the differences in labor force growth across countries may have caused different growth patterns.

It is important to recognize the magnitudes involved in Figure 2 (or in Figure 1). In particular the slope of the regression line is almost exactly -1 (-0.996 with a standard deviation of 0.194). An implication of this observation is that over a period of 17 years, GDP (not per worker terms) grew at approximately the same rate across these countries independently of the growth in the labor force; even though the labor force grew by more than 35% in some countries versus others. Clearly, if this relationship is robust, it is worth understanding. Hence, we now turn to examining the robustness of this observation as well as examining the potential of interpreting it in a causal fashion.

Table 1: Benchmark regression

	OLS (1)	WLS1 (2)	WLS2 (3)	OLS (4)	WLS (5)	OLS (6)	WLS (7)	OLS (8)	WLS (9)
Const.	0.145	0.120	0.134	0.090	0.064	0.077	0.084	0.013	0.011
	(0.086)	(0.076)	(0.082)	(0.085)	(0.079)	(0.079)	(0.075)	(0.085)	(0.080)
$(Y/L)_0$	-0.012	-0.009	-0.010	-0.004	-0.002	-0.002	-0.003	0.003	0.003
	(0.008)	(0.007)	(0.008)	(0.009)	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)
$\Delta L(\%)$	-0.858	-0.818	-0.835	-0.939	-0.905	-0.942	-0.922	-0.993	-0.979
	(0.208)	(0.202)	(0.206)	(0.196)	(0.195)	(0.142)	(0.144)	(0.162)	(0.167)
I/Y	—	—	—	0.013	0.011	0.017	0.014	0.019	0.018
				(0.007)	(0.007)	(0.005)	(0.005)	(0.006)	(0.006)
H_1	—	—	—	—	—	0.027	0.028	—	—
						(0.017)	(0.016)		
H_2	—	—	—	—	—	—	—	0.004	0.005
								(0.004)	(0.004)
R^2	0.66	0.69	0.67	0.72	0.73	0.82	0.88	0.84	0.85

Note: WLS1: The weights correspond to the log of country's total population. WLS2: The weights correspond to the log of country's GDP.

In Table 1, we report a set of results associated with running cross–country regressions of labor productivity growth using standard controls suggested by the growth literature. In the first Column, in addition to labor force growth we include as a regressor the country's initial level of labor productivity. In columns (2) and (3), we run this regression using weighted least squares (WLS) were the first set of weights are the log of the country's total population (WLS1) and the second set of weights is the log of the countries GDP in 1978 (WLS2). In columns (4) and (5) we further include the countries' average investment rates over the period. We now only report the OLS results and one WLS result to minimize repetition, as the weighting does not alter the results. In particular, we only report WLS results using the log of total population as weights. In columns (6), (7), (8) and (9), we control for differences in human capital accumulation by including the countries' student enrolment rates, denoted by H_1 (Columns (6) and (7)), or alternatively by including the average years of education of individuals in each country, denoted by H_2.[7] The main observation we want to highlight across this set of regressions is the robustness of the partial correlation between labor force growth and labor productivity growth. In particular, taking convergence forces into account — through the introduction of the initial level of labor productivity in the regression — does not bring the population growth effect down significantly. The OLS regression indicates that

[7] Details on human capital measures are given in appendix A.2.

partial correlation between labor productivity and labor force growth is still -0.86 with a standard deviation of 0.2. Weighting the data by either GDP or the size of the population does not affect significantly the size of the population growth effect which remains at -0.82. Controlling for differences in investment rates in investment capital slightly magnifies the effect, which rises to -0.94 for OLS (with standard deviation 0.2) and remains close to -0.9 under WLS. Controlling for education does not change the results either as the magnitude of the effects of population growth remains in a neighborhood of -0.9 when controlling for our two measures of investment in education.[8] It is also wroth noting that over this period, there was no evidence of significant convergence forces across these countries, regardless of the set of controls. We now examine the robustness of these observations.

1.2 Robustness of the results

In Table 2, we reproduce the sequence of regressions of Table 1 where we now include the percentage change in country's unemployment rate as an additional regressor. This is actually

Table 2: Controlling for unemployment changes

	OLS (1)	OLS (2)	OLS (3)	OLS (4)
Const.	0.158	0.091	0.089	0.045
	(0.076)	(0.067)	(0.087)	(0.086)
$(Y/L)_0$	-0.012	-0.004	-0.003	-0.000
	(0.007)	(0.007)	(0.009)	(0.008)
$\Delta L(\%)$	-0.866	-0.968	-0.939	-0.983
	(0.182)	(0.154)	(0.142)	(0.162)
$\Delta U(\%)$	-0.063	-0.074	-0.012	-0.034
	(0.028)	(0.024)	(0.038)	(0.037)
I/Y	—	0.016	0.017	0.017
		(0.006)	(0.005)	(0.006)
H_1	—	—	0.025	—
			(0.018)	
H_2	—	—	—	0.005
				(0.004)
R^2	0.74	0.83	0.85	0.86

aimed at addressing whether the pattern we are documenting could simply be capturing the indirect effects of certain labor market policies which may have favored labor productivity at

[8]Note that neither measure of education seem to be significant both under OLS and WLS.

the cost of increasing unemployment.[9] If this were the whole story, it would have nothing to do with the type of interaction between technological adoption and demographically driven employment growth which we believe has been important.[10] As can be seen, the introduction of the change in unemployment does not affect the observed negative correlation between labor force growth and labor productivity growth, whether additional regressors such as investment rates or educational effects are included or not. In fact, our point estimates for the effects of labor force growth are almost unaffected by the inclusion of the change in the rate of unemployment, both in terms of significance and magnitude. Hence, the pattern we are emphasizing does not appear to be driven simply by the behavior of unemployment: something more akin to an increased tradeoff between employment growth and productivity growth appears to be at work. In order to see whether this partial correlation may be driven

Table 3: Controlling for Institutional differences

	OLS	OLS	OLS	OLS Europe
	(1)	(2)	(3)	(4)
Const.	0.146	0.132	0.131	0.160
	(0.086)	(0.086)	(0.086)	(0.089)
$(Y/L)_0$	-0.012	-0.010	-0.010	-0.013
	(0.008)	(0.008)	(0.008)	(0.009)
$\Delta L(\%)$	-0.894	-0.783	-0.824	-1.024
	(0.227)	(0.225)	(0.236)	(0.235)
AS	0.001	—	0.002	—
	(0.003)		(0.003)	
SC	—	0.003	0.003	—
		(0.003)	(0.003)	
R^2	0.67	0.68	0.68	0.77

mainly by some culture–based institutional differences, in Table 3, we report a set of results where we include a dummy variable for predominantly anglo–saxon countries (Australia, Canada, New Zealand, United Kingdom and the USA), denoted AS, or one for scandinavian countries (Denmark, Norway and Sweden), denoted SC. Finally, in the last column of the table we report results where we limit the sample exclusively to European countries. Once again the negative correlation between labor productivity growth and labor force growth is

[9] See Blanchard [1997] for a discussion along these lines.

[10] Many economists argue that high labor productivity growth in Europe may have been caused mainly by institutions that have restricted employment. Our view is that this interpretation of the European experience is much too restrictive in that it does not recognize that demographic factors may have been an even more important factor for high labor productivity growth than what can be accounted for by the increase in unemployment.

robust, as the negative effects of labor force growth remains high and statistically significant.

It is often suggested that the age structure of the population, and more specifically changes in the age structure, may matter for growth.[11] It is therefore worth exploring whether our results are robust to controlling for such an effect. To this end, in Table 4 we include as additional regressors two measures which capture changes in the age structure of the population. The first measure relates to the percentage change in ratio of the child population to the total population, denoted $\Delta(C/T)$. The second one corresponds to the percentage change in the ratio of the elderly to the total population, denoted $\Delta(E/T)$. As can be seen in Table 4, the inclusion of the regressors does not affect our observation regarding the importance of labor force growth on labor productivity growth. In fact, the point estimates become slightly larger in size reaching -1.

Table 4: Controlling for age structure effects

	OLS (1)	OLS (2)	OLS (3)	OLS (4)
Const.	0.100	0.069	-0.030	0.008
	(0.083)	(0.078)	(0.093)	(0.075)
$(Y/L)_0$	-0.007	-0.002	0.008	0.005
	(0.008)	(0.008)	(0.009)	(0.007)
$\Delta L(\%)$	-1.000	-1.017	-1.000	-0.967
	(0.204)	(0.187)	(0.142)	(0.153)
$\Delta(C/T)(\%)$	0.148	0.277	0.432	0.378
	(0.226)	(0.219)	(0.232)	(0.195)
$\Delta(E/T)(\%)$	0.344	0.307	0.301	0.246
	(0.161)	(0.148)	(0.178)	(0.125)
I/Y	–	0.012	0.021	0.020
		(0.007)	(0.005)	(0.005)
H_1	–	–	-0.014	–
			(0.026)	
H_2	–	–	–	-0.001
				(0.004)
R^2	0.74	0.78	0.88	0.88

Given the potential endogeneity of the regressors used in Table 1, in Table 5 we present a set of results based on instrumental variable estimation. In particular, we want to explore the robustness of results related to taking into account the potential endogeneity of both labor force growth and of the investment rates. To begin, in column (1) of Table 5 we instrument

[11] See for example Feyrer [2002].

the growth in the labor force over the period by the growth of the population aged 15 to 64 (active population) over the same period. This strategy is meant to control for biases induced by the potential endogeneity of the participation decision. In column (2), we use an alternative instrument for the growth of the labor force which is the growth in the population of all ages over the period 1960 to 1970. The attractive feature of this IV strategy is that it uses population growth in a previous period to predict current labor force growth and therefore uses mainly differences in births in the sixties — which are likely to be uncorrelated with residual determinants of labor productivity in the 1978-95 period — to identify the relationship. The interesting aspect worth noticing from these two sets of IV results is that the coefficients on labor force growth are actually slightly larger after instrumentation as compared with the OLS result in column (1) of Table 1 and furthermore suggests that the relationship may be causal.

Table 5: Benchmark regression (IV)

	IVB1	IVB2	IVI1	IVI2	IVH	IVH
	(1)	(2)	(3)	(4)	(5)	(6)
Const.	0.135	0.136	0.066	0.094	0.090	0.023
	(0.089)	(0.093)	(0.092)	(0.088)	(0.086)	(0.095)
$(Y/N)_0$	-0.011	-0.011	-0.001	-0.005	-0.003	0.002
	(0.009)	(0.009)	(0.009)	(0.009)	(0.009)	(0.009)
$\Delta L(\%)$	-0.909	-0.907	-0.936	-0.925	-0.875	-0.930
	(0.237)	(0.279)	(0.227)	(0.218)	(0.171)	(0.195)
I/Y	—	—	0.020	0.012	0.018	0.020
			(0.009)	(0.009)	(0.006)	(0.006)
H_1	—	—	—	—	0.028	—
					(0.025)	
H_2	—	—	—	—	—	0.005
						(0.005)

Note: IVB1: Labor force growth is instrumented by active population growth, IVB2: Labor force growth is instrumented by total population growth between 1960 and 1970. IVI1: Labor force growth is instrumented by active population growth and the investment ratio by the beginning of period investment ratio, IVI2: Labor force growth is instrumented by active population growth and the investment ratio by the investment ratio of the earlier period. IVH: Labor force growth is instrumented by active population growth, the investment ratio by the beginning of period investment ratio and the education variable by the years of schooling in the period 1960–1978.

In columns (3) and (4) of Table 5, we include the countries' average investment rates in physical capital as an additional regressor and explore two IV strategies. Similarly, in columns (5) and (6), we add our two human capital variables (H_1 and H_2) as regressors and pursue two more IV strategies. In all these four cases, we instrument the rate of labor force growth by the growth rate in active population. In the case of columns (3) and (4), we also instrument the investment rate in physical capital by respectively the initial investment rate (rate in 1978) and the average investment rate over the period 1960–78. In the case of columns (5) and (6), the investment rate in physical capital is instrumented by the rate in 1978 and the human capital variable is instrumented by the country's average number of years of schooling. Given the strong negative correlation between labor force growth and labor productivity growth observed even after instrumenting, the results of Table 5 suggest that, over the period 1978–1995, differences in labor force growth may have actually caused countries to experience different growth patterns. In particular, we believe that these results suggest that the 1978–95 period was an era where countries faced a large negative tradeoff between labor force growth and labor productivity growth. However, before exploring a potential explanation of this observation, we want to highlight the timing of this phenomena and thereby document its medium–run nature.

1.3 The Timing

As shown in Tables 1 through 5, there was a very strong and robust negative partial correlation between labor force growth and productivity growth across industrial countries over the period 1978 to 1995. Moreover the IV estimates suggest that this correlation may plausibly be interpreted as reflecting causation. We now want to document why this pattern should be viewed as a medium–run phenomenon which arose somewhere around the mid seventies and started to reverse about 20 years later. To this end, we begin in Tables 6 and 7 by showing that the determinants of labor productivity growth over period 1978–95 were statistically different from those observed over the period 1960–78.

In particular, in Table 6, we want to highlight that the importance of labor force growth in affecting labor productivity changed drastically over the period 1960–1995; going from being statistically insignificant to causing a large negative effect. For instance, let us consider the first two columns of Table 6 where we report OLS estimates of the relationship between labor productivity growth and our standard co–variates for the two sub-periods: 1960–78 and 1978–95. The bottom of the table reports test statistics for coefficient stability, with associated p–values in brackets. In calculating the stability tests, we allowed for the country specific error term to be correlated across periods. The first statistic, $\mathcal{Q}$(total), tests for the overall stability of all coefficients (excluding the constant) over the two sub–periods. The second test, $\mathcal{Q}(\Delta L)$, corresponds to the test of whether the coefficient on the growth in

Table 6: Benchmark regression

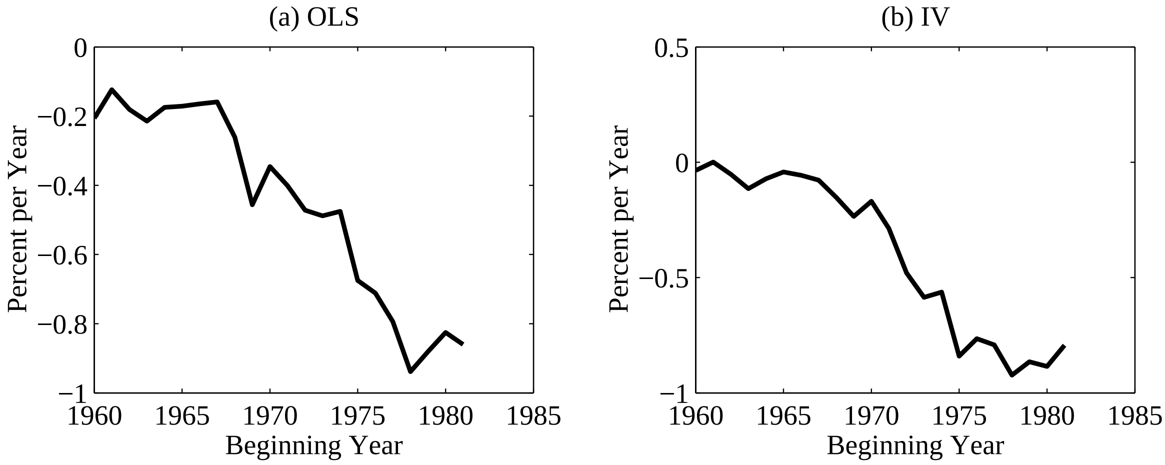
	OLS		WLS		IV	
	(1)	(2)	(3)	(4)	(5)	(6)
	60–78	78–95	60–78	78–95	60–78	78–95
Const.	0.377	0.145	0.391	0.120	0.380	0.135
	(0.060)	(0.086)	(0.055)	(0.076)	(0.061)	(0.089)
$(Y/L)_0$	-0.035	-0.012	-0.036	-0.009	-0.035	-0.011
	(0.006)	(0.008)	(0.006)	(0.007)	(0.006)	(0.009)
$\Delta L(\%)$	-0.127	-0.858	-0.030	-0.818	-0.112	-0.909
	(0.221)	(0.208)	(0.219)	(0.202)	(0.229)	(0.237)
R^2	0.77	0.66	0.84	0.69	–	–
$\mathcal{Q}$(Total)	8.866	[0.031]	12.631	[0.006]	8.647	[0.034]
$\mathcal{Q}(\Delta L)$	6.644	[0.010]	7.981	[0.005]	6.697	[0.010]
$\mathcal{Q}(Y/L)$	6.095	[0.014]	10.051	[0.002]	6.145	[0.013]

Note: WLS: The weights correspond to the log of country's total population. IV: Lafor force growth is intrumented by active population.

labor force changed over the period. The third test corresponds to whether the speed of convergence (the coefficient on $(Y/L)_0$) changed over the two sub–samples. As we can see from these test statistics, the determinants of labor productivity growth changed considerably over the two sub–periods. In the first period, 1960 to 1978, the main determinant of growth in labor productivity was initial conditions. In other words, the period 1960–1978 is an era where convergence forces seemed to play an important role. In contrast, the effect of labor force growth was statistically irrelevant. Over the second period, the convergence forces had become very modest and instead there emerged a strong negative correlation between labor force growth and labor productivity growth. The same results obtain when we use WLS (columns (3) and (4)). In columns (5) and (6), we re–estimate the relationships by instrumental variables and find a similar pattern. Furthermore, in Table 7 we show that this instability is robust to controlling for differences in the investment rate and for human capital differences measured by either the student enrollment rate or the average years education of the population.

The goal of these two tables is not to suggest that there was a structural break in 1978, but rather that there was an underlying process that caused the importance of labor force growth in affecting labor productivity to change over this period. In order to get a better sense of the process underlying the preceding results, in Figure 3 we report the estimated effect of labor force growth on labor productivity derived from a set of rolling regressions. Our basic regression consists of regressing the growth in output–per–worker on the growth in labor force, the investment rate and the initial level of output–per–worker, with each variable measured in relation to the given window. We chose a size of window of 17 years such that it includes 1978–95 as one point. Nevertheless, in order to check the robustness of our results, we also explored window sizes between 15 to 25 years, all of which led to similar conclusions. Panel (a) reports the evolution of the labor force growth coefficient when the estimation is carried in OLS with the same co-variates as in the first column of Table 7. In panel (b) we report results associated with IV estimation as in columns (3) and (4) of Table 7. Hence, the first point on this graph corresponds to the estimated effect of labor force growth over the period 1960–77, while the last point corresponds to the period 81–98. Note that we end this exercice in 1998 since the Heston–Summers data we are using to make valid international comparisons ends in 1978. As we can see from the figures, the effect of labor force growth was small and rather stable of the first windows of the sample. Then, over a period starting somewhere around the early to mid–seventies, the effect of labor force grew in importance reaching a peak over windows that contained all the eighties. Finally over the last few windows, that is when the data of the later nineties come in to play, we see the importance of labor force growth recede.

Figure 3: Effect of Labor Force growth (Rolling regressions)



The pattern in Figure 3 suggests an intriguing medium–run phenomenon that we believe has gone almost entirely unnoticed in the literature. In particular, this figure indicates the emergence of a type of cycle associated with the importance of labor force growth on productivity growth. Given such an observation, we now turn to examining theoretically why such medium run dynamics may arise.

Table 7: Controlling for Investment Effects (both in physical and human capital)

	OLS		IV		OLS		IV		OLS		IV		OLS		IV	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	60–78	78–95	60–78	78–95	60–78	78–95	60–78	78–95	60–78	78–95	60–78	78–95	60–78	78–95	60–78	78–95
Const.	0.377	0.145	0.380	0.135	0.387	0.090	0.407	0.066	0.433	0.077	0.468	0.090	0.456	0.013	0.481	0.023
	(0.060)	(0.086)	(0.061)	(0.089)	(0.066)	(0.085)	(0.068)	(0.092)	(0.071)	(0.079)	(0.125)	(0.086)	(0.060)	(0.085)	(0.063)	(0.095)
$(Y/N)_0$	-0.035	-0.012	-0.035	-0.011	-0.036	-0.004	-0.039	-0.001	-0.041	-0.002	-0.053	-0.003	-0.041	0.003	-0.043	0.002
	(0.006)	(0.008)	(0.006)	(0.009)	(0.007)	(0.009)	(0.008)	(0.009)	(0.008)	(0.008)	(0.016)	(0.009)	(0.007)	(0.008)	(0.007)	(0.009)
$\Delta L(\%)$	-0.127	-0.858	-0.112	-0.909	-0.124	-0.939	-0.082	-0.936	-0.120	-0.942	0.451	-0.875	0.183	-0.993	0.339	-0.930
	(0.221)	(0.208)	(0.229)	(0.237)	(0.220)	(0.196)	(0.228)	(0.227)	(0.242)	(0.142)	(0.573)	(0.171)	(0.223)	(0.162)	(0.260)	(0.195)
I/Y	—	—	—	—	-0.003	0.013	-0.008	0.020	-0.004	0.017	-0.027	0.018	-0.008	0.019	-0.010	0.020
					(0.009)	(0.007)	(0.010)	(0.009)	(0.009)	(0.005)	(0.022)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)
H_1	—	—	—	—	—	—	—	—	0.003	0.027	-0.106	0.028	—	—	—	—
									(0.021)	(0.017)	(0.079)	(0.025)				
H_2	—	—	—	—	—	—	—	—	—	—	—	—	-0.016	0.004	-0.018	0.005
													(0.006)	(0.004)	(0.007)	(0.005)
R^2	0.77	0.66	–	–	0.78	0.72	—	—	0.47	0.85	–	–	0.87	0.84	–	–
$\mathcal{Q}$(Total)	8.866	[0.031]	8.647	[0.034]	12.220	[0.016]	12.362	[0.015]	19.787	[0.001]	12.337	[0.030]	25.346	[0.000]	22.007	[0.001]
$\mathcal{Q}(\Delta L)$	6.644	[0.010]	6.697	[0.010]	8.047	[0.005]	6.951	[0.008]	8.658	[0.003]	7.677	[0.006]	17.165	[0.000]	15.470	[0.000]
$\mathcal{Q}(Y/N)$	6.095	[0.014]	6.145	[0.013]	8.333	[0.004]	9.505	[0.002]	12.009	[0.001]	4.677	[0.031]	18.309	[0.000]	15.320	[0.000]

Note: IV: Labor force growth is instrumented by active population growth, the investment ratio by the beginning of period investment ratio and the education variable by the years of schooling.

2 A Model of Technological Transitions

The goal of this section is to illustrate why certain country specific factors — like labor force growth — may affect medium run growth even if their effect in the long run is likely negligible. To this end, we develop a simple model of technological adoption which behaves like a Solow growth model in the long–run, but which generates medium–run dynamics associated with technological choice.[12] In the model, countries choose to adopt a new technology at different times depending on their capital–labor ratios. Since our claim is that medium run cycles are a potentially important phenomenon, much of the emphasis of the section will be to illustrate quantitatively why medium run growth performance — that is performance over periods of 15 to 20 years — may be greatly effected by factors that are traditionally thought to be as inconsequential as labor force growth. In particular, we will explore the extent to which such a simple model of technology adoption is capable of explaining the negative correlation between labor force growth and labor productivity observed across industrialized countries from 1978 to 1995.

2.1 The technology

In the model economy, growth arises because of two phenomena. First of all, as in the standard neo–classical growth model, any given technology may improve due to disembodied technological progress. Second, agents have the ability to switch from technologies that are no longer improving towards ones that are still improving. The model imbeds a non–trivial technological choice problem by assuming that leading edge technologies require minimum outlays per worker to be productive. We model this constraint as a minimum capital–labor ratio requirement, but what is actually needed is very little substitution between labor and capital at low levels of the capital–labor ratio. Hence, when a new technology becomes available it may not be immediately adopted since the cost of equipping a worker — even with the minimal amount of capital required for operation — may not be profitable. However, over time, technological improvements within the leading edge technology will make it both more productive and more flexible in the sense that its total factor productivity improves and its minimal capital–labor requirements become less stringent. Therefore, over time a leading edge technology will become increasingly attractive to adopt and every country will

[12] One may argue that a simpler model with embodied technology may account for the observations we made in the previous section. We however show in a technical appendix (available from the authors upon request) that a model of embodied technological progress would, in a similar framework, generate inverse results with regards to the empirical evidence.

eventually adopt it.

Now consider a set of economies which all produce the same homogenous good that can be either consumed or invested. At each point in time, there exists a sequence of technologies indexed by i, which depends on capital, $K_{i,t}$, and unskilled labor, $L_{i,t}$, according to the following production function

$$Y_{i,t} = (\theta_{i,t}K_{i,t})^{\alpha} L_{it}^{1-\alpha} \quad \text{if} \quad \frac{\theta_{i,t}K_{i,t}}{L_{i,t}} \geqslant \kappa_i$$

where $\alpha \in (0,1)$ and $\theta_{i,t}$ is a technology parameter. In the above, κ_i corresponds to the minimum effective capital–labor ratio required to use technology i efficiently. In the case where $\frac{\theta_{i,t}K_{i,t}}{L_{i,t}} < \kappa_i$, production is impossible. However, in such a case, firms can actually use this technology by leaving some workers idle in order to reach the minimum required capital–labor ratio — therefore employing $\widetilde{L}_{i,t} = \theta_{i,t}K_{i,t}/\kappa_i$ workers. Then, the level of production attained in this situation is equal to

$$Y_{i,t} = (\theta_{i,t}K_{i,t})^{\alpha} \widetilde{L}_{it}^{1-\alpha} = \theta_{i,t}K_{i,t}\kappa_i^{\alpha-1}$$

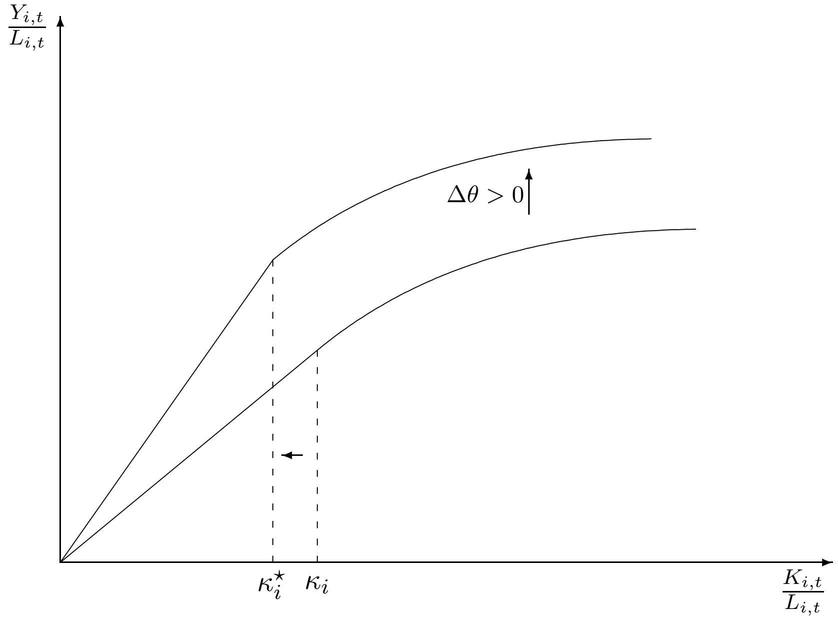
Hence, the relevant production function actually takes the form

$$Y_{i,t} = \begin{cases} (\theta_{i,t}K_{i,t})^{\alpha} L_{it}^{1-\alpha} & \text{if} \quad \frac{\theta_{i,t}K_{i,t}}{L_{i,t}} \geqslant \kappa_i \\ \\ \theta_{i,t}K_{i,t}\kappa_i^{\alpha-1} & \text{if} \quad \frac{\theta_{i,t}K_{i,t}}{L_{i,t}} < \kappa_i \end{cases} \tag{1}$$

The production function for technology i is illustrated in Figure 4. As long as the capital–labor ratio remains below the minimum requirement κ_i, the economy will leave idle some workers and therefore production is on a AK segment of the technology. Once the minimum requirement is attained, the firm can plainly use the available technology. The figure also illustrates that an increase in productivity, θ, causes *(i)* the production function to shift up while simultaneously causing *(ii)* the minimum capital–labor requirement to decrease, therefore making the technology more flexible. It is worth immediately noting that in the equilibrium we will emphasize, firms will not be operating in the segment where workers are idle and therefore the unappealing properties of this segment will not be an issue.

Technologies are differentiated along two margins: *(i)* the potential for their improvement and *(ii)* their effective capital–labor ratio requirement. On the first margin, the technologies are assumed to differ in the sense that a less sophisticated technology stops improving before a more sophisticated technology. In particular, if we let γi represent the time at which

Figure 4: Technology i



technology i stops improving, the dynamics of θ_i are given by:

$$\theta_{i,t} = \begin{cases} \exp(\rho\, t) & \text{if } t \leqslant \gamma i \\ \\ \exp(\rho\gamma\, i) & \text{if } t > \gamma i \end{cases} \tag{2}$$

where $\rho > 0$ denotes the improvement rate of technology. Once technology i attains its maturity stage ($t > \gamma i$) it stops improving. Note that the greater the i, the longer is the time span over which $\theta_{i,t}$ increases. In other words, the more advanced technology has a longer life of improvement. Also note that this specification implies that all technologies that keep on improving actually grow at the same rate and are therefore equivalent in terms of productivity.

The second margin along which technologies differ is their minimum capital requirements κ_i, as we assume that the minimum requirement increases with i as follows:

$$\kappa_i = \kappa_0 \exp(\eta i) \tag{3}$$

where $\eta > 0$ is the rate at which the effective-capital–labor constraint becomes more stringent the more advanced the stage of technology.

Given these specifications, a technology with a higher i can unambiguously be considered more advanced in the sense that it improves over a longer period of time, while at the same time being a technology that requires a higher ratio of effective capital–labor ratio to be productive.

Firms in this economy are price takers and choose inputs and technologies as to maximize profits. Beside the standard production decisions, firms also have to take a decision with respect to which technology to use and therefore to decide whether to adopt a new technology. Despite the fact that firms face a whole menu of technologies, the technological choice problem at any given point in time can be reduced to one of choosing between two technologies provided that $\eta \leqslant \frac{\rho\gamma}{1-\alpha}$ — *i.e.* the minimum effective capital–labor ratio does not grow at a too fast pace. In order to see this, let $\widetilde{\imath}(t)$ denote the greatest i such that $t > \gamma i$ — that is, $\widetilde{\imath}(t)$ is the most recent technology that stopped improving. Under the assumption that $\eta \leqslant \frac{\rho\gamma}{1-\alpha}$, it is easy to verify that technology $\widetilde{\imath}(t)$ dominates all technologies i satisfying $i < \widetilde{\imath}(t)$ — *i.e.* technologies that have stopped improving before it. Furthermore technology $\widetilde{\imath}(t)+1$, which we will refer to as the *leading edge technology* at time t, weakly dominates all technologies with $i > \widetilde{\imath}(t)+1$ since it is as productive as technologies with greater i (since it keeps on growing), while having a lower capital requirement. Hence, if $\eta \leqslant \frac{\rho\gamma}{1-\alpha}$, at any time t a firm's technology decision consists of choosing only between technologies $\widetilde{\imath}(t)$ and $\widetilde{\imath}(t)+1$. In what follows, we will assume that $\eta = \frac{\rho\gamma}{1-\alpha}$ as to keep the technological choice problem binary. Under this assumption, the parameter γ can be interpreted as the length of time for which a technology is a leading edge technology.

2.2 Households

The population in a country j, $N_j(t)$, which is also its labor force, is assumed to grow at rate $\nu_j > 0$ such that[13]

$$N_j(t) = N_j(0)\exp(\nu_j t) \tag{4}$$

Households in this economy are assumed to behave like Solow households.[14] We do not introduce optimizing behavior on the part of the household in the model simply to make the analysis more transparent. Instead we assume households consume a fraction $(1-s)$ of production each period, with all savings being directed toward capital accumulation. Finally, the capital stock depreciates at the rate δ, implying that the capital accumulation dynamics

[13]Note that in order to keep notations simple, we omitted the j index, it should nevertheless be clear to the reader that by $N(t)$ we mean $N_j(t)$. This will also apply to the capital stock

[14]The assumption here is that financial markets are not integrated across countries. In an earlier version of this paper, we allowed for international financial flows subject to an adjustment costs. Results with imperfect international capital flows gave similar results to that presented here.

in country j are

$$\dot{K}(t) = sY(t) - \delta K(t) \tag{5}$$

2.3 Main properties of the equilibrium

There are two main equilibrium properties of this model which we want to highlight. First, we want to characterize the nature of the growth process. In particular, the growth process of this economy will be shown to generally involve three phases. A second issue we want to address is the timing of these phases, and more specifically how this timing will be affected by differences in rates of labor force growth.

As a preliminary step towards understanding the nature of the growth process, it is helpful to describe the technology adoption decision. Indeed, the growth process, and more specifically the different phases it goes through, depends crucially on which technologies are selected at any given point in time by a country. As shown in Proposition 1[15], the technological choice decision is directly related to a country's capital–labor ratio.

Proposition 1 *At each point in time, there exists a capital–labor ratio* $\widehat{k}_t \equiv \widehat{\frac{K_t}{L_t}}$*, such that*

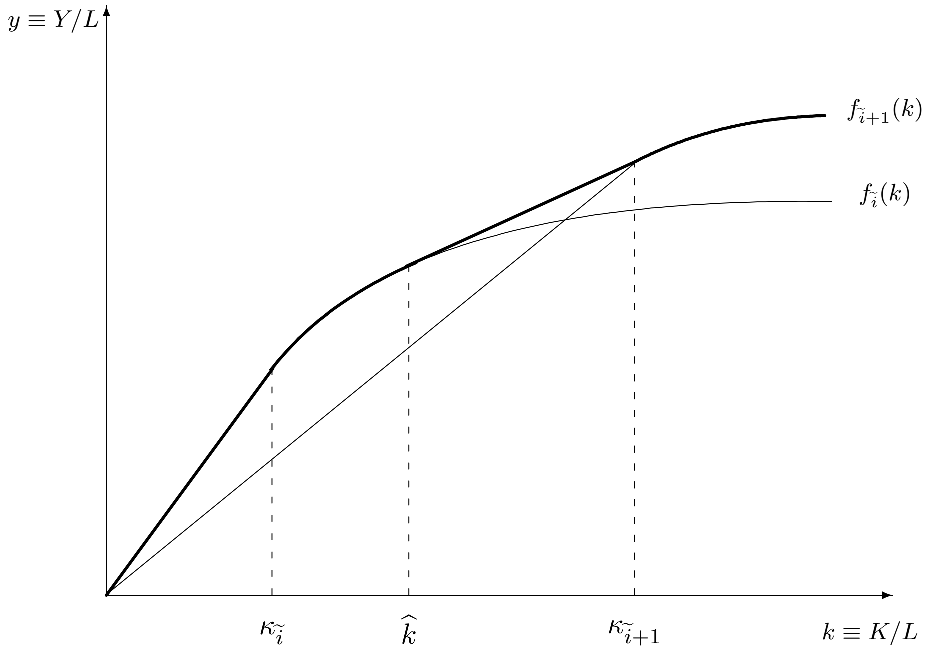
1. *if* $\frac{K_t}{L_t} \leqslant \widehat{k}_t$*, then only technology* $\widetilde{\imath}(t)$ *is in use.*
2. *if* $\widehat{k}_t < \frac{K_t}{L_t} < \frac{\kappa_{i+1}}{\theta_{i+1,t}}$*, then technologies* $\widetilde{\imath}(t)$ *and* $\widetilde{\imath}(t)+1$ *are in use.*
3. *if* $\frac{\kappa_{i+1}}{\theta_{i+1,t}} \leqslant \frac{K_t}{L_t}$ *then only technology* $\widetilde{\imath}(t)+1$ *is in use.*

Moreover, when $\widehat{k}_t < \frac{K_t}{L_t} < \frac{\kappa_{i+i}}{\theta_{i+1,t}}$ *then output–per–worker is linear in* $\frac{K_t}{L_t}$*.*

Figure 5 illustrates Proposition 1 in the case where $\widehat{k} > \frac{\kappa_{\widetilde{i}}}{\theta_{\widetilde{i}}}$. As can be seen, if a country has a very high capital labor ratio, $k > \kappa_{\widetilde{\imath}+1}$, then it is clearly optimal to use the leading edge technology, $f_{\widetilde{\imath}+1}(k)$. In contrast, if a country has a low capital labor ratio, $k \leqslant \widehat{k}$, then the leading edge technology cannot be used in its most efficient manner. It is therefore optimal to use the stagnant technology, $f_{\widetilde{\imath}}(k)$. Finally, there is a middle range of capital–labor ratios, $\widehat{k} < k < \kappa_{\widetilde{\imath}+1}$, where it is optimal for the representative firm to allocate capital and labor among the two technologies. By so doing it uses both technologies simultaneously, taking advantage of the improvements in the leading edge technology while simultaneously employing all factors

[15] The interested reader is referred to the appendix of this paper for a proof of each proposition.

Figure 5: Technology Adoption



and avoiding the effects of decreasing returns in the old technology. Over time, these regions will change as the leading edge technology evolves and its capital requirement becomes less stringent. Such adoption decisions have direct implications for the growth process. In effect, a country which is using the stagnant technology at a point in time will eventually start using the leading edge technology, since the capital labor requirement becomes less and less stringent, while the productivity of the leading edge technology makes it more and more attractive. Similarly, a country which is simultaneously using both technologies (the stagnant and the leading edge) will continually increase its use of the leading edge technology up to the point where it eventually stops using the stagnant technology entirely.

The technology adoption process described in Proposition 1 actually defines a reduced form production function which incorporates the optimal choice of technologies. This process of technology adoption is crucial in determining the properties of the overall growth process. Indeed, by combining the output determination given in Proposition 1 with the capital accumulation process in (5), we obtain a simple dynamic system that jointly determines the growth of capital and output. The analysis of this system enables us to characterize the growth process. In particular, we state in Propositions 2 and 3 two important properties of this growth process.

Proposition 2 *If the initial capital–labor ratio requirement is sufficiently small:*

$$\kappa_0 < \left(\frac{s}{\delta+\nu}\right)^{\frac{1}{1-\alpha}},$$

then a country will grow indefinitely by sequentially adopting each new leading edge technology.

This property is central to the model as it actually gives a condition under which the economy will grow indefinitely due to *(i)* improvements within technologies and *(ii)* movements across technologies. Basically, an economy will grow without bounds provided it saves sufficiently to cover the minimal capital requirements for each technology, that is κ_0 is small enough $\left(\kappa_0 < \frac{s}{\delta+\nu}^{\frac{1}{1-\alpha}}\right)$. In this case, a country will benefit from advances in productivity by continuously adopting new leading edge technologies. The next proposition characterizes the stages of the growth process.

Proposition 3 *If a country's propensity to accumulate capital is not too strong :*

$$\frac{s}{\delta+\nu} < \kappa_0^{1-\alpha}\exp(\rho\gamma) \qquad (i)$$

and if κ_0 is sufficiently small:

$$\kappa_0 < \left(\frac{s}{\delta+\nu+\frac{\alpha\gamma}{1-\alpha}}\right)^{\frac{1}{1-\alpha}} \qquad (ii)$$

then a country will not immediately adopt the leading edge technology when the previous one becomes stagnant. Instead it will experience at least the following two phases:

1. *A phase where only a stagnant technology is used,*
2. *A phase where both the currently leading edge technology and previous leading edge technology (which is now stagnant) are used.*

Proposition 3 characterizes two phases associated with technological choices. In particular, as long as the country does not favor capital deepening two much — as given by condition (i) — it will not adopt the leading edge technology immediately when the previous leading edge technology stops improving. Instead, it continues with the old now–stagnant technology, waiting for the capital requirement of the leading edge technology to decrease sufficiently. Then, once the capital requirements are not too demanding, it starts adopting the leading edge technology. This adoption is gradual as both technologies remain in use for a while. Finally, when the leading edge technology is sufficiently productive, the country may completely abandon the previous technology and will experience growth due entirely to improvements in the leading edge technology. If, in contrast, the country favors very high capital labor ratios and the capital-labor requirements are not very demanding, then it will find it optimal to always adopt the leading edge technology each time the previous technology becomes stagnant. In such a case the growth process is precisely the same as the standard Solow growth model. Obviously, this case is not of great interest to us.

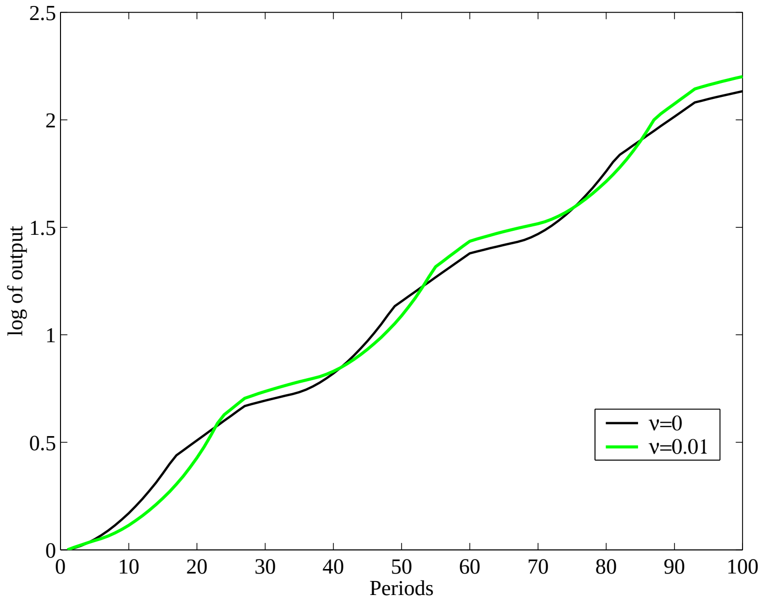
As shown in the last two propositions, a country will exhibit different phases of technological adoption and growth. It is therefore natural to ask how these growth phases are affected by country specific factors. More specifically, we are interested in understanding how these phases will differ across countries that have different rates of labor force growth. In this respect, the first aspect to notice is that, given identical initial conditions and the same savings rate, a country with a lower rate of labor force growth will always have a higher capital–labor ratio than a country with a higher rate of labor force growth. Hence, from Proposition 1 and Figure 5, we can immediately infer that differences in rates of labor force growth will interact with timing decisions for technology adoption. These differences are expressed in the next proposition.

Proposition 4 *Given two countries that differ only in their rates of labor force growth, and assuming that the conditions of Proposition 3 hold, the country with the lower rate of labor force growth will:*

1. *use a stagnant technology for less time, start adopting the leading edge technology sooner and will entirely abandon the stagnant technology sooner,*
2. *in the long run both economies will share the same average growth rates.*

Proposition 4 expresses the main insight of the model. It indicates that differences in rates of labor force growth can cause countries to behave differently in the medium run but similarly in the long run. Accordingly, since countries will be in different growth phases at any point in time, this type of model predicts that the relationship between labor productivity growth and labor force growth will be changing over time. For example, when a leading edge technology is first being adopted, it will be adopted by countries with low rates of labor force growth. Meanwhile, countries with a high rate of labor force growth will continue using the stagnant technology for some time. Hence, during such a period, greater labor force growth will be negatively correlated with output growth. However, over time, this pattern must disappear as countries with high rates of labor force growth eventually start adopting the new technology and therefore start to catch–up. This type of staggered growth pattern is illustrated in Figure 6 where growth in output–per–worker is depicted for two countries that differ only by rates of labor force growth. The time paths in the figure — which for now should be considered

Figure 6: Staggered Growth Patterns



only as illustrative — correspond to the outcomes of a simple parameterized version of the model.[16] As can be seen from the figure, the two countries share the same growth trend but exhibit growth cycles which are out of phase. Indeed, as can be inferred by the growth pattern, the economy with a low rate of labor force growth starts adopting the leading edge technology sooner than the high labor force growth economy. It therefore grows at a faster pace for a while, benefiting from technological advances in the leading edge technology as well as the productivity gains associated with adopting the new technology. But then, when the leading edge technology has improved sufficiently and becomes more attractive, the economy with the lower rate of labor force growth starts adopting it and therefore starts a high growth phase. Eventually, both countries entirely shift to the leading edge technology and grow at similar rates. This cycle starts again once the leading edge technology stops improving and becomes a stagnant technology. From a qualitative point of view, such a pattern has the potential to explain why during certain periods — as for example from the late seventies

[16]The parametrization used to produce Figure 6 sets the saving rate to 0.2 and the capital depreciation rate to 0.03. On the technological side of the model, the rate of growth of technology, ρ, is set such that long term growth rate is 2%, the capital share, α, is 0.35 and γ the length of life of a leading edge technology is 33 years. Also, as noted we set $\eta = \gamma\rho/(1-\alpha)$.

through to the mid-nineties — countries with low rates of labor force growth may have done particularly well; such countries may have been adopting the leading edge technology faster and sooner than their high labor force growth counterparts. However, it remains to be shown whether such effects are quantitatively relevant and, in particular, of sufficient magnitude to explain the strong negative correlation between labor force growth and labor productivity growth observed around the 1980s. This is the focus of the next section.

3 The medium run phenomenon: A quantitative assessment

In this section we examine whether, within the confines of the model developed in the previous section, small differences in labor force growth can actually lead to quantitatively important medium run differences in growth patterns of the type identified in Section 1. To this end, we only need to focus on one single technological cycle, as the sample we considered only covers a time period of 40 years. As will become clear, focusing on a single cycle greatly simplifies the calibration exercise.

3.1 Calibration of the model

Let us consider a case where countries are initially at a steady state in an $\widetilde{\imath}(t)$ technology — that is, they have reached a steady state in a technology which has recently become stagnant. However, even though they have not yet begun to adopt it, the technology $\widetilde{\imath}(t)+1$ is available and improving. The goal of this exercise is to compare the growth performances of two countries which start in this position and differ only in terms of having a one percent difference in rates of population growth. Since we want to make a quantitative assessment of the model, we need to select reasonable values for the model's parameters for the model. There are 6 parameters in the model (α, s, δ, ρ, γ, and κ_0), many of which are rather easy to calibrate. In particular, following Bernanke and Gürkaynak [2001], we set the capital share, α, to 0.35. The savings rate is set to 0.2 and the depreciation rate of capital to 0.03 per year. All these values are commonly used in the growth literature. Parameter ρ is set at 0.04 so that the long term growth generated by the model is 2% per year.

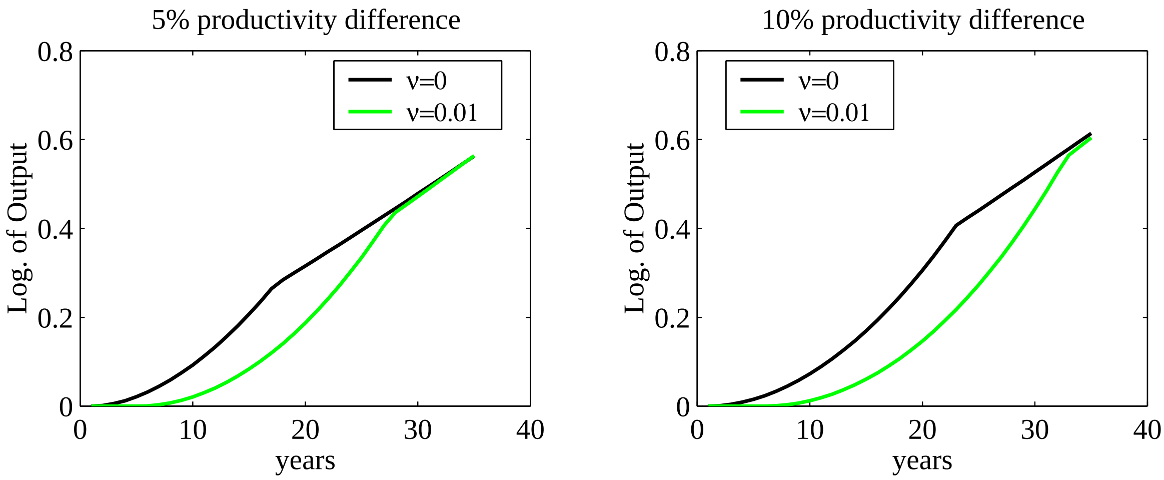
In principle, there are three remaining parameters to calibrate: γ, the time during which a technology i improves, η, the rate of growth of the capital–labor requirement, and κ_0, the initial capital–labor ratio requirement. However, since we are looking only at one growth cycle, we actually only need a function of both these parameters. In effect, instead of focusing on γ, η and κ, we can directly focus on the relative value of productivity in the leading edge technology, $\theta_{\tilde{\imath}(t)+1}$, versus the stagnant technology, $\theta_{\tilde{\imath}(t)}$ at a particular point in time. For example, we can focus on the value of $\theta_{\tilde{\imath}(t)+1}$ versus $\theta_{\tilde{\imath}(t)}$ at the time when technology $\theta_{\tilde{\imath}(t)+1}$ starts being adopted by the low labor force growth country. Since we have no obvious way of calibrating this value, we will explore the model's properties under two scenarios: one where the difference in productivity is 5% and one where the difference is 10%.

3.2 Quantitative analysis

In this section, we perform two exercises. The first one gives a quantitative assessment of the magnitude of the medium run phenomenon by focusing on output differences within one cycle of technological adoption. The second one directly examines the ability of the model to replicate the pattern and magnitude of labor force growth effects we identified in Section 1.

As a first exercise, we plot in Figure 7 the path of output–per–worker (in logs) in two countries — one with a low rate of labor force growth ($\nu = 0$) and one with a higher rate of labor force growth ($\nu = 0.01$) — over a 35 years period, where the first year is the beginning of the adoption phase of the new technology for the country with zero labor force growth. The

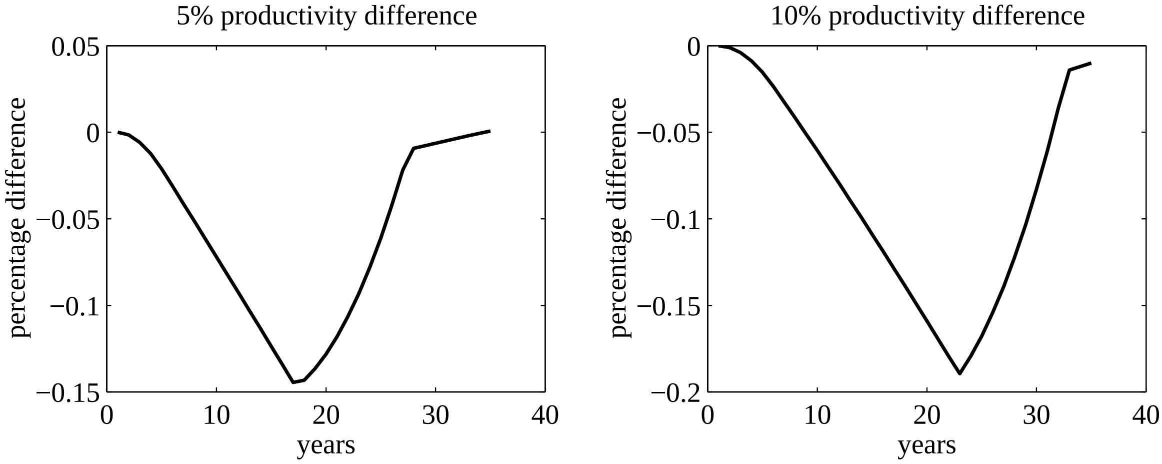
Figure 7: Output paths



left panel displays the time paths for both economies under the assumption that the new technology is 5% more productive initially — *i.e*, $\theta_{\tilde{\imath}(t)+1} = 1.05 \times \theta_{\tilde{\imath}(t)}$ — when adoption begins, while the right panel considers a difference of 10%. Recall that initially capital labor ratios in the countries are not sufficiently high to immediately switch to the new technology, they only start adopting the technology gradually as it becomes more productive. In both cases, for comparative purposes, we normalized the starting output–per–worker log levels in each country to 0.

In Figure 8, we plot the difference in growth performance across the two countries, that is, we plot the difference between the profiles depicted in Figure 7. The main aspect we

Figure 8: Output difference



want to highlight is that countries that differ only in terms of a 1% difference in labor force growth actually exhibit very different medium run growth. In particular, let us focus on a 5% productivity difference between the stagnant and the leading edge technology (left panel). As can be seen from Figure 7, the economy with a low rate of population growth (L–economy) starts adopting the leading edge technology immediately (by construction) whereas the economy with a 1% higher rate of population growth, because its capital–labor ratio is lower, waits before starting to adopt the leading edge technology. It then takes 10 years for the L–economy to fully adopt the leading edge technology, while the H–economy adopts it at a much slower pace, as it takes it 20 years to fully adopt technology $\tilde{\imath}(t)+1$. As reported in Figure 8, this translates into large quantitative differences in terms of output–per–worker, as when the L–economy has fully adopted technology $\tilde{\imath}(t)+1$ while the H–economy is still adopting it, the L–economy has an output per worker about 15% higher than the H–economy. In other words, the model implies that small differences in population growth rates (1%) can have large temporary consequences in terms of output–per–worker (15% in this case). These effects are magnified when the leading edge technology is more productive. If we now consider a 10% productivity difference between the stagnant and the leading edge technology, it now takes 21 years for the L–economy to adopt technology $\tilde{\imath}(t)+1$ entirely, while it takes 31 years for the H–economy. More interestingly, the output–per–worker gap is now maximal at about 20%. Hence, this experiment illustrates that the medium run effect of small labor force growth differences can be of very significant order.

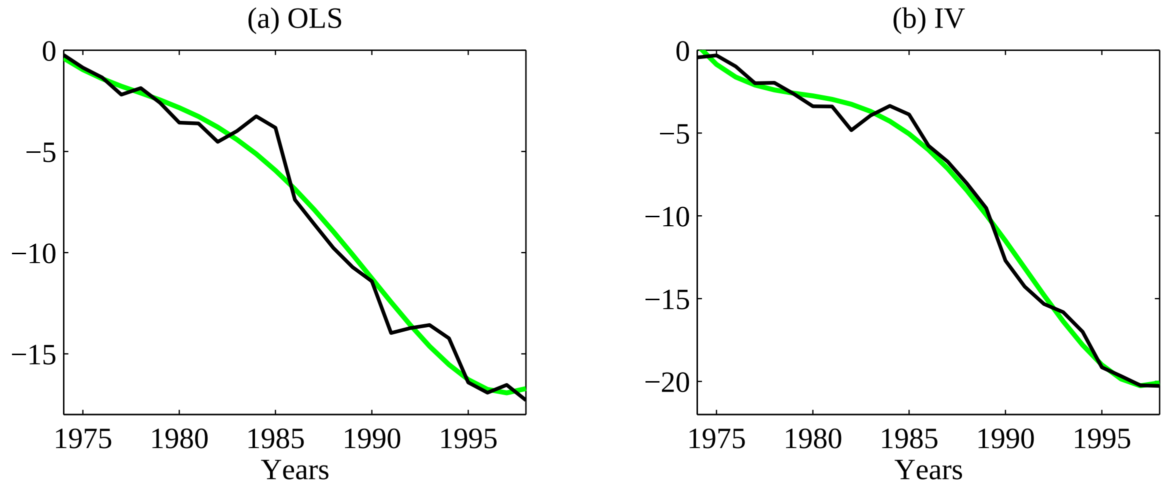
Given the patterns observed in Figure 7, we want to ask whether these are of a magnitude capable of explaining the relationship between labor force growth and labor productivity growth we documented in Section 1. To this end, we ran the following sequence of univariate regressions (on the original data) starting in 1974, and plot the resulting series for β_n.

$$\log\left(\left(\frac{Y}{L}\right)_{74+n}\right) - \log\left(\left(\frac{Y}{L}\right)_{74}\right) = \alpha_n + \beta_n \frac{1}{n}[\log(LF_{74+n}) - \log(LF_{74})] \qquad (6)$$

The resulting series of βs gives the cumulative effect of a 1% difference in labor force growth on labor productivity growth since 1974, therefore providing an empirical counterpart to the information reported in Figure 8. We start in 1974 for three reasons. First, from the rolling regressions (Figure 3) we see that the importance of labor force growth started to become important somewhere close to 1974. Second, many papers in the literature (see Greenwood and Yorukoglu [1997] among others) take 1974 as a plausible breaking date associated with the beginning of an important technological transition. Third, starting around 1974 or after, it is reasonable to look at this relationship without controlling for the initial level of output–per–worker since this factor appears negligible afterwards for these countries.

The series of βs is plotted in Figure 9. Panel (a) and (b) reports respectively the results for estimation by OLS and IV (where the rate of growth of active population is used as an instrument for labor force growth). In this figure, we overlay a fitted quartic trend (gray line) as to highlight the low frequency movements in the evolution of the βs. As we can see from

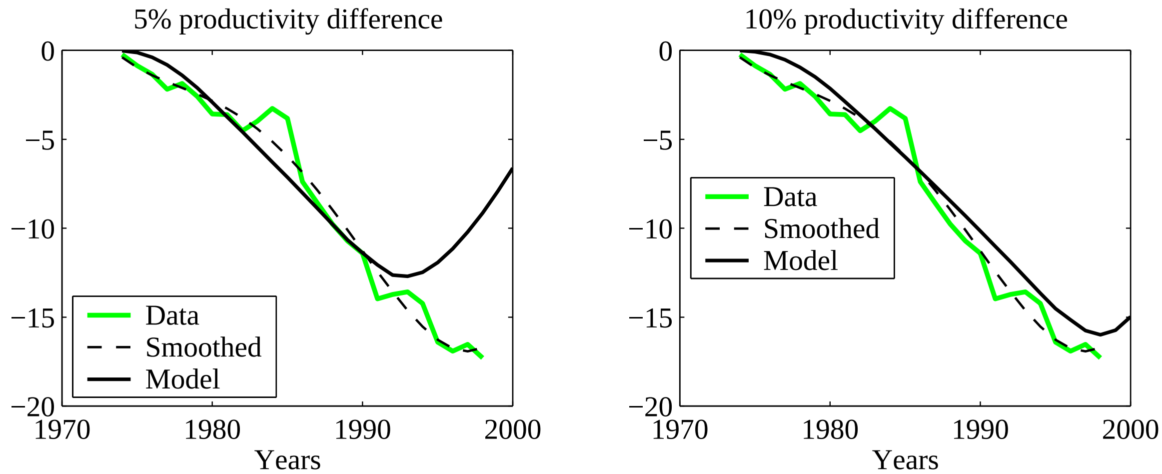
Figure 9: Cumulative effects of Labor Force Growth



both panels, a 1% difference in labor force growth is associated with a substantial difference in labor productivity growth over the period. For example, the OLS estimates (panel (a)) suggest the effect reached a peak around 1995 at about 17% while the IV estimates suggest a peak at about the same date but with an effect that is 30% stronger (about 22%).

Finally, in Figure 10, we overlay the content of panel (a) of Figure 9 with the series of βs obtained from data generated with the model. More precisely, we generate data from a set of 17 countries at steady state levels in the stagnant technology in 1974. All countries differ only in terms of their rate of labor force growth, which are taken from the data. We then obtain a series of theoretical βs from the same regression as equation (6). Figure 10 clearly shows that the model with a 10% productivity difference can replicate both the size and persistence of the observed effects of labor force growth and labor productivity growth over a period of 25 years. Although we must recognize that the model has one degree of freedom — the initial difference in $\theta_{\tilde{\imath}(t)+1}$ versus $\theta_{\tilde{\imath}(t)}$ — it is nevertheless remarkable to find such a close fit between model and data over an extended period of time. In particular, it is surprising to see that countries which differ by only one percent in labor force growth may choose the timing and speed of technological adoption in a manner sufficiently different so as to create growth differences of the order of 15 to 20% over a period of 15 to 20 years.

Figure 10: Theoretical effects of Labor Force Growth



In order to complete the argument, we want to show how our simple model of technological adoption can generate data that look like what we initially documented in Figure 2 of Section 1. For this exercise, we start with a set of 17 countries at steady state levels in the stagnant technology in 1974, where countries differ only in terms of their rate of labor force growth. The experiment is conducted under the hypothesis that the productivity of the leading edge technology is 10% higher than that of the stagnant technology at that point in time. We then run the model for each of these countries for 24 years where we feed into the model the actual labor force growth observed in each of our countries in the sample. We then look at the cross plot between output growth and labor force growth for the set of countries, where

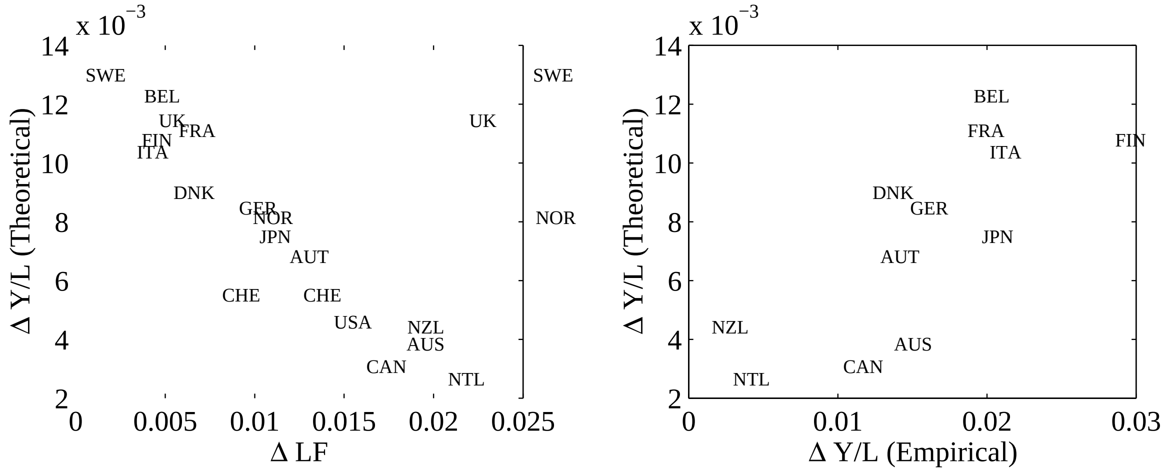


Figure 11, where each point is associated with the country for which the labor force growth

$\overline{R}^2 = 0.61$),

run. Second, we have presented a very simple model of technological adoption that is capable of explaining – both qualitatively and quantitatively – the observed pattern. As a result, the model gives a new interpretation of the differential growth performances observed across industrial countries since the 1970s. Our results suggest that the different rates of labor force growth across countries caused staggered growth patterns, with low labor force growth countries doing particularly well in the 1980s and higher labor force growth countries doing better in the 1990s. We believe these results will provide further interest in studying and identifying medium run phenomena using cross-country growth regressions.

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A Data

A.1 Main data

The output data are taken from the latest version of the Penn World Table 6.0 downloadable from `http://webhost.bridgew.edu/baten/`.

In particular, we build out labor productivity measure using the real GDP chain series in the Penn World Tables, and the employment data from the OECD statistical compendium.

Labor Force, Employment and Unemployment rate data are all taken from the OECD compendium. Likewise, data on population under 15, between 15 and 64, and above 64 are also taken from the OECD compendium. When necessary the sample was completed relying on online data available from the OECD web site (`http://www.oecd.org`)

The corresponding annualized average rate of growth for the variable Z within the sub–sample [t;t+n] is computed as

$$\Delta z = \frac{\log(Z_{t+n}) - \log(Z_t)}{n}$$

The investment ratio at constant prices corresponds to the variable `KI` in the PWT 6.0 and is divided by 100. In the regressions, i/y then refers to the logarithm of this variable.

A.2 Human capital data

Education data are taken from the Barro and Lee [1993] dataset, which is downloadable from

`http://www.nuff.ox.ac.uk/Economics/Growth/datasets.htm`.

The education variables are borrowed from Barro and Lee [1993]. We consider essentially 2 measures of human capital. The first one is related to the overall enrollment rate in education. Assuming that, on average, most people spend 6 years in primary schooling, 6 years in secondary schooling and 4 years in higher schooling, we first define the index

$$H_1 = \log\left(\frac{6 \times \mathtt{P} + 6 \times \mathtt{S} + 4 \times \mathtt{H}}{16}\right)$$

where `P`, `S` and `H` respectively denote the total gross enrollment ratio for, respectively, primary, secondary and higher schooling.

Our second measure of the education variable relates to average schooling years in the total population over 25.. It is built as $H_2 = \log(\texttt{HUMAN})$.

Note that these measures are only reported every 5 years in the database from 1960 to 1985. We actually used an average of years 1960 and 1965 for the first sub–sample and 1975 and 1980 for the second sub–sample. We restrict ourselves to 2 periods for the average for data availability purposes.

B Proof of propositions

B.1 Proposition 1

Since there are no market imperfections, the technological choice decision can be found by solving a social planner's problem. This problem actually amounts to deciding on the optimal allocation of input between technologies $\widetilde{\imath}(t)$ and $\widetilde{\imath}(t)+1$. The problem is then to solve

$$\max_{\sigma_1,\sigma_2,\gamma_1,\gamma_2} (\theta_1\sigma_1 K)^\alpha(\gamma_1 L)^{1-\alpha} + (\theta_2\sigma_2 K)^\alpha(\gamma_2 L)^{1-\alpha}$$

subject to

$$\begin{array}{llll} \text{(1a)} & 0 \leqslant \sigma_1 \leqslant 1 & \text{(2a)} & 0 \leqslant \gamma_1 \leqslant 1 \\ \text{(1b)} & 0 \leqslant \sigma_2 \leqslant 1 & \text{(2b)} & 0 \leqslant \gamma_2 \leqslant 1 \\ \text{(1c)} & \sigma_1+\sigma_2 \leqslant 1 & \text{(2c)} & \gamma_1+\gamma_2 \leqslant 1 \\ \text{(3a)} & \sigma_1 K \geqslant \frac{\kappa_1}{\theta_1}\gamma_1 L & \text{(3b)} & \sigma_2 K \geqslant \frac{\kappa_2}{\theta_2}\gamma_2 L \end{array}$$

where $\kappa_1 \equiv \exp(\eta\widetilde{\imath}(t))$, $\kappa_2 \equiv \exp(\eta(\widetilde{\imath}(t)+1))$, $\theta_1 \equiv \exp(\rho\gamma\widetilde{\imath}(t))$, and $\theta_2 \equiv \exp(\rho t)$ with $t > \gamma\widetilde{\imath}$. The set of first order optimality conditions is given by

$$\alpha(\theta_1 K)^\alpha \sigma_1^{\alpha-1}(\gamma_1 L)^{1-\alpha} + \lambda^0_{\sigma_1} - \lambda^1_{\sigma_1} - \zeta_\sigma + \mu_1 K = 0 \quad (7)$$

$$\alpha(\theta_2 K)^\alpha \sigma_2^{\alpha-1}(\gamma_2 L)^{1-\alpha} + \lambda^0_{\sigma_2} - \lambda^1_{\sigma_2} - \zeta_\sigma + \mu_2 K = 0 \quad (8)$$

$$(1-\alpha)(\theta_1\sigma_1 K)^\alpha L^{1-\alpha}\gamma_1^{-\alpha} + \lambda^0_{\gamma_1} - \lambda^1_{\gamma_1} - \zeta_\gamma - \mu_1\frac{\kappa_1}{\theta_1}L = 0 \quad (9)$$

$$(1-\alpha)(\theta_2\sigma_2 K)^\alpha L^{1-\alpha}\gamma_2^{-\alpha} + \lambda^0_{\gamma_2} - \lambda^1_{\gamma_2} - \zeta_\gamma - \mu_2\frac{\kappa_2}{\theta_2}L = 0 \quad (10)$$

where λ^0_x, λ^1_x denote the lagrange multipliers associated with the positivity and less than unity constraints for variable $x \in \{\sigma_j, \gamma_j; j=1,2\}$, ζ_x is the lagrange multiplier associated with the full utilization constraints (1c and 2c) for variable $x \in \{\sigma,\gamma\}$ and μ_j is the lagrange multiplier associated with the capital labor ratio constraint in technology $j=\{1,2\}$. All these multipliers are non–negative.

Several cases are then to be considered.

1) We first consider the case where the capital labor ratio satisfies

$$\frac{K}{L} > \frac{\kappa_2}{\theta_2}$$

In this case, the firm can satisfy the minimal capital requirements (3a) and (3b) — implying $\mu_1=\mu_2=0$ — and it is optimal to use all available resources such that (1c) and (2c) are binding — implying $\zeta_\sigma>0$ and $\zeta_\gamma>0$ and $\sigma_2=1-\sigma_1$ and $\gamma_2=1-\gamma_1$. We will now show that in this case, since $\theta_2>\theta_1$ it is optimal for the firm to use only technology 2 — *i.e.*

$\widetilde{\imath}(t)+1$. Let us therefore assume that the firm decides to mix the two technologies, such that $\sigma_1 \in (0,1)$ and $\gamma_1 \in (0,1)$. Full utilization of resources implies that the first order conditions can be rewritten as

$$\begin{aligned}
\alpha(\theta_1 K)^\alpha \left(\frac{\sigma_1}{\gamma_1}\right)^{\alpha-1} L^{1-\alpha} &= \alpha(\theta_2 K)^\alpha \left(\frac{1-\sigma_1}{1-\gamma_1}\right)^{\alpha-1} L^{1-\alpha} \\
(1-\alpha)(\theta_1 K)^\alpha \left(\frac{\sigma_1}{\gamma_1}\right)^{\alpha} L^{1-\alpha} &= (1-\alpha)(\theta_2 K)^\alpha \left(\frac{1-\sigma_1}{1-\gamma_1}\right)^{\alpha} L^{1-\alpha}
\end{aligned}$$

from which we get $\sigma_1/\gamma_1 = (1-\sigma_1)/(1-\gamma_1)$. But plugging this latter result in one of the two equations of the system, we end up with

$$\alpha(\theta_1 K)^\alpha \left(\frac{\sigma_1}{\gamma_1}\right)^{\alpha-1} L^{1-\alpha} = \alpha(\theta_2 K)^\alpha \left(\frac{\sigma_1}{\gamma_1}\right)^{\alpha-1} L^{1-\alpha} \Longleftrightarrow \theta_1 = \theta_2$$

which is a contradiction since $\theta_2 > \theta_1$ as $\theta > \gamma\widetilde{\imath}$. Hence, it is always optimal for the firm to use technology 2 — *i.e.* $\widetilde{\imath}(t)+1$ — since its productivity is higher.

2) We now consider the case where the capital labor ratio satisfies

$$\frac{K}{L} < \frac{\kappa_1}{\theta_1}$$

In such a case — which is actually the polar opposite of the preceding — the firm can only satisfy the minimal capital–labor constraint by leaving workers idle and therefore the marginal product of labor will be zero. Hence, the production function will take an AK form. We will now show that the marginal efficiency, A, is maximized by producing with technology 1 only — *i.e.* technology $\widetilde{\imath}$. Let us assume that the firm decides to use both technologies, devoting a share σ of its capital to technology $\widetilde{\imath}(t)$. The problem of the firm then reduces to

$$\max_{\{0\leqslant\sigma\leqslant1\}} \theta_1 \sigma K \kappa_1^{\alpha-1} + \theta_2(1-\sigma)K\kappa_2^{\alpha-1}$$

In this case, the problem is linear in σ and can be rewritten as:

$$\max_{\{0\leqslant\sigma\leqslant1\}} \theta_2 \kappa_2^{\alpha-1} + (\theta_1\kappa_1^{\alpha-1} - \theta_2\kappa_2^{\alpha-1})\sigma K$$

The solution of this problem is then

$$\sigma = \begin{cases} 0 & \text{if } \theta_1\kappa_1^{\alpha-1} - \theta_2\kappa_2^{\alpha-1} < 0 \\ 1 & \text{if } \theta_1\kappa_1^{\alpha-1} - \theta_2\kappa_2^{\alpha-1} \geqslant 0 \end{cases}$$

From the definition of κ_j and θ_j, $j=1,2$, we get

$$\theta_1\kappa_1^{\alpha-1} - \theta_2\kappa_2^{\alpha-1} = 1 - \exp(\rho(t - \gamma(\widetilde{\imath}(t)+1)))$$

Remembering that the technology choice between technologies $\widetilde{\imath}(t)$ and $\widetilde{\imath}(t)+1$ only takes place within the time interval $\gamma\widetilde{\imath}(t) < t < \gamma(\widetilde{\imath}(t)+1)$, it is clear that the solution of the problem is $\sigma = 1$. Therefore, only technology 1 — *i.e* $\widetilde{\imath}(t)$ — will be in use.

Let us now define $k^\star = \max\left\{\frac{\kappa_1}{\theta_1}, \widehat{k}\right\}$, where $\widehat{k}$ is the solution to[17]

$$(1-\alpha)\theta_1^\alpha \widehat{k}^\alpha + \alpha\theta_1^\alpha \frac{\kappa_2}{\theta_2}\widehat{k}^{\alpha-1} - \kappa_2^\alpha = 0$$

3) Let us first consider the case where $k^\star = \frac{\kappa_1}{\theta_1}$. We now show that if $k^\star \leqslant \frac{K}{L} \leqslant \frac{\kappa_2}{\theta_2}$, it is optimal to use both technologies. If the firm follows this strategy, the total output will be

$$Y = \frac{\kappa_2^\alpha - \kappa_1^\alpha}{\frac{\kappa_2}{\theta_2} - \frac{\kappa_1}{\theta_1}}K + \frac{\frac{\kappa_2}{\theta_2}\kappa_1^\alpha - \frac{\kappa_1}{\theta_1}\kappa_2^\alpha}{\frac{\kappa_2}{\theta_2} - \frac{\kappa_1}{\theta_1}}L \text{ or } y = \frac{\kappa_2^\alpha - \kappa_1^\alpha}{\frac{\kappa_2}{\theta_2} - \frac{\kappa_1}{\theta_1}}k + \frac{\frac{\kappa_2}{\theta_2}\kappa_1^\alpha - \frac{\kappa_1}{\theta_1}\kappa_2^\alpha}{\frac{\kappa_2}{\theta_2} - \frac{\kappa_1}{\theta_1}}$$

Since $\frac{\kappa_1}{\theta_1} \geqslant \widehat{k}$, this strategy is better than using only technology 1 (or $\widetilde{\imath}(t)$). Indeed, for $k < \frac{\kappa_1}{\theta_1}$, the firm uses the technology

$$Y_1 = \theta_1 K \kappa_1^{\alpha-1}$$

which from the definition of θ_1, κ_1 and $\eta = \gamma\rho/(1-\alpha)$ can be rewriten as $Y_1 = K$ or $y_1 = k$. It is then straightforward to verify, using the definition of κ_i and θ_i, i=1,2, that

$$y = \frac{\kappa_2^\alpha - \kappa_1^\alpha}{\frac{\kappa_2}{\theta_2} - \frac{\kappa_1}{\theta_1}}k + \frac{\frac{\kappa_2}{\theta_2}\kappa_1^\alpha - \frac{\kappa_1}{\theta_1}\kappa_2^\alpha}{\frac{\kappa_2}{\theta_2} - \frac{\kappa_1}{\theta_1}} > k \quad \forall k^\star < k < \frac{\kappa_1}{\theta_1}$$

In other words, technology 1 is dominated by a linear combination of the technologies 1 and 2.

Moreover, it produces more that using only technology 2 (or $\widetilde{\imath}(t)+1$) when satisfying (3b). Indeed, in this case, technology 2 can be written as

$$Y_2 = \theta_2 K \kappa_2^{\alpha-1}$$

which from the definition of θ_2, κ_2 and $\eta = \gamma\rho/(1-\alpha)$ becomes $Y_2 = \exp(\rho(t-\gamma(\widetilde{\imath}(t)+1)))K$ or $y_2 = \exp(\rho(t-\gamma(\widetilde{\imath}(t)+1)))k$, which for $t < \widetilde{\imath}(t)+1$ is such that $y_2 < k$ and therefore reinforces the argument developed for technology 1.

[17]The equation admits a unique solution. Indeed, the problem amounts to solving $g(\widehat{k}) = 0$. First note that$g(\kappa_2/\theta_2) = (\theta_1^\alpha - \theta_2^\alpha)\kappa_2/\theta_2 < 0$ since $\theta_2 > \theta_1$ and $\lim_{\widehat{k}\downarrow 0} g(\widehat{k}) = +\infty$ since $\alpha \in (0,1)$. Furthermore, we have

$$g'(\widehat{k}) \equiv \alpha(1-\alpha)\theta_1^\alpha \widehat{k}^{\alpha-1} + \alpha(\alpha-1)\frac{\kappa_2}{\theta_2}\widehat{k}^{\alpha-2} = 0 \Longleftrightarrow \widehat{k} = \frac{\kappa_2}{\theta_2}$$

and $g'(\widehat{k}) < 0$ for all $\widehat{k} \in (0, \kappa_2/\theta_2)$. Hence the curve is monotonically decreasing within the range $(0, \kappa_2/\theta_2)$, which together with the two limit conditions at the bounds of the interval implies that $\widehat{k}$ is unique.

Hence, it is optimal to mix the two technologies, and the resulting production function is an affine function of the capital–per–worker.

4) Let us now consider the case where $k^\star = \widehat{k}$. If $\widehat{k} < \frac{K}{L} < \frac{\kappa_2}{\theta_2}$, we want to show that it is optimal to mix the two technologies while satisfying the minimum capital–labor constraint (3b) with equality, in which case $\mu_1 = 0$ and $\mu_2 > 0$. The set of first order conditions then becomes, assuming an interior solution

$$\begin{aligned}
\alpha(\theta_1 K)^\alpha \sigma_1^{\alpha-1}(\gamma_1 L)^{1-\alpha} - \zeta_\sigma &= 0 && (11)\\
\alpha\theta_2 K \kappa_2^{\alpha-1} - \zeta_\sigma + \mu_2 K &= 0 && (12)\\
(1-\alpha)(\theta_1\sigma_1 K)^\alpha L^{1-\alpha}\gamma_1^{-\alpha} - \zeta_\gamma &= 0 && (13)\\
(1-\alpha)\kappa_2^\alpha L - \zeta_\gamma - \mu_2\frac{\kappa_2}{\theta_2}L &= 0 && (14)
\end{aligned}$$

From the first and third conditions, we see that constraint (1c) and (2c) bind, such that the optimal mix has the property $\sigma_2 = 1 - \sigma_1$ and $\gamma_2 = 1 - \gamma_1$. Combining the first order conditions we end up with

$$(1-\alpha)\theta_1^\alpha\left(\frac{\sigma_1 K}{\gamma_1 L}\right)^\alpha + \alpha\theta_1^\alpha\frac{\kappa_2}{\theta_2}\left(\frac{\sigma_1 K}{\gamma_1 L}\right)^{\alpha-1} - \kappa_2 = 0$$

which implies that $\frac{\sigma_1 K}{\gamma_1 L} = \widehat{k}$. This relation together with (3b), $\sigma_2 = 1 - \sigma_1$ and $\gamma_2 = 1 - \gamma_1$ yields

$$\sigma_1 = \frac{\widehat{k}}{K}\frac{\kappa_2 L - \theta_2 K}{\kappa_2 - \theta_2\widehat{k}} \text{ and } \gamma_1 = \frac{1}{L}\frac{\kappa_2 L - \theta_2 K}{\kappa_2 - \theta_2\widehat{k}}$$

where σ_1 and γ_1 satisfy (1a), (1b), (2a) and (2b) if $\frac{K}{L} > \widehat{k}$. Such a solution is optimal since it solves the following set of first order conditions with all the multipliers having the positive sign. In this case, the production function is an affine function of capital–per–worker and takes the form

$$Y = \frac{\theta_1^\alpha\widehat{k}^\alpha\kappa_2 - \kappa_2^\alpha\theta_2\widehat{k}}{\kappa_2 - \theta_2\widehat{k}}L + \frac{\theta_2\kappa_2^\alpha - \theta_1^\alpha\widehat{k}^\alpha\theta_2}{\kappa_2 - \theta_2\widehat{k}}K$$

5) We finally consider the case $k^\star = \widehat{k}$ and $\frac{\kappa_1}{\theta_1} < k < \widehat{k}$. Given σ_1 and γ_1 determined in **4)**,we know that it is not feasible to mix the two technologies while strictly satisfying (3b) as this mixing is only optimal for $K/L > \widehat{k}$. Therefore, either it is optimal to use only technology 1 — *i.e.* $\widetilde{\imath}(t)$ — or to use only technology 2 — *i.e.* $\widetilde{\imath}(t)+1$ — or to mix the two technologies while satisfying both (3a) and (3b) with equality. From **1)** it is clear that it cannot be optimal to use only technology 2 only while satisfying (3b) as technology 1 would dominate it. The only remaining alternative to using only technology 1 would be to use a mix of technology

1 and 2 while satisfying both (3a) and (3b) with equality. But since $K/L > \kappa_1/\theta_1$ it is not optimal for firm 1, from the definition of the production function, to keep on using the linear part of the production function, this case is therefore not relevant. The last remaining possibility is therefore to produce using only technology 1.

QED $\square$

B.2 Proposition 2

We want to show that if the initial capital–labor ratio requirement — κ_0 — is sufficiently small, then a country will grow indefinitely by sequentially adopting each new leading edge technology. Let us denote $k \equiv K/L$ in a given economy, such that we have

$$\frac{\dot{k}(t)}{k(t)} = s\frac{f(k(t), \theta)}{k(t)} - (\delta + \nu)$$

Any increase in θ leads to a shift in the rate of growth of capital implying that the capital–labor ratio keeps on growing. Therefore, the only requirement for the economy to grow and adopt new leading edge technology is that the rate of growth of capital is positive in a neighborhood of $k = 0$. Further, since $\kappa_0 > 0$, any $k \in \mathscr{K}_0 = \{x \in \mathbb{R}_+,\ \varepsilon > 0, \|x\| \leqslant \varepsilon\}$ is associated with the use of technology $\widetilde{\imath}(t)$. Therefore, we need to find a condition on κ_0 such that

$$\gamma_k \equiv \frac{\dot{k}(t)}{k(t)} = s\frac{f_{\widetilde{\imath}(t)}(k(t), \theta_{\widetilde{\imath}(t)})}{k(t)} - (\delta + \nu) > 0 \text{ for } k \in \mathscr{K}_0$$

which becomes, taking into account that for $k \in \mathscr{K}_0$, $k < \kappa_0$,

$$s\theta_{\widetilde{\imath}(t)}\kappa_{\widetilde{\imath}(t)}^{\alpha-1} - (\delta + \nu) > 0$$

Plugging the definition of $\theta_{\widetilde{\imath}(t)}$ and $\kappa_{\widetilde{\imath}(t)}$ into the preceding equation, we get

$$s\exp(\rho\gamma\widetilde{\imath}(t))\kappa_0^{\alpha-1}\exp((\alpha-1)\eta\widetilde{\imath}(t)) - (\delta + \nu) > 0$$

since we restrict our attention to $\eta = \gamma\rho/(1-\alpha)$, this becomes

$$s\kappa_0^{\alpha-1} - (\delta + \nu) > 0$$

implying that

$$\gamma_k > 0 \Longleftrightarrow \kappa_0 < \left(\frac{s}{\delta + \nu}\right)^{\frac{1}{1-\alpha}}$$

QED $\square$

B.3 Proposition 3

1) Given that the system starts with an initial low capital–labor ratio, $K_0/L_0 \leqslant \kappa_0$, an upper bound for the capital–labor ratio the economy can reach at the very moment the technology becomes stagnant ($t = \gamma\widetilde{\imath}(t)$) is the steady state level of K/L that could be attained if the technology $\widetilde{\imath}$ were available for ever, that is

$$\overline{\frac{K}{L}} = \left(\frac{s}{\delta+\nu}\right)^{\frac{1}{1-\alpha}} \exp\left(\frac{\alpha}{1-\alpha}\rho\gamma\widetilde{i}\right)$$

Recall that from Proposition 1 if $\overline{\frac{K}{L}} < \frac{\kappa_{\widetilde{\imath}(t)+1}}{\theta_{\widetilde{\imath}(t)+1}}$, the leading edge technology will not be adopted immediately, since $k^\star = \widehat{k} = \frac{\kappa_{\widetilde{\imath}+1}}{\theta_{\widetilde{\imath}+1}}$. Note in particular that at this moment, $t = \gamma\widetilde{\imath}(t)$, we have $\theta_{\widetilde{\imath}(t)+1} = \exp(\rho t) = \exp(\rho\gamma\widetilde{\imath}(t)) = \theta_{\widetilde{\imath}(t)}$. Hence, the condition $\overline{\frac{K}{L}} < \frac{\kappa_{\widetilde{\imath}(t)+1}}{\theta_{\widetilde{\imath}(t)+1}}$ becomes

$$\left(\frac{s}{\delta+\nu}\right)^{\frac{1}{1-\alpha}} \exp\left(\frac{\alpha}{1-\alpha}\rho\gamma\widetilde{i}\right) < \frac{\kappa_0\exp(\eta(\widetilde{\imath}(t)+1))}{\exp(\rho\gamma\widetilde{\imath}(t))}$$

Remembering that $\eta = \frac{\rho\gamma}{1-\alpha}$, we obtain condition *(i)* of the proposition

$$\frac{s}{\delta+\nu} < \kappa_0^{1-\alpha}\exp(\rho\gamma)$$

Therefore, if this condition is satisfied, a country will necessarily go through phases where it is only using the stagnant technology.

2) If a country does not immediately adopt the leading edge technology, then the only reason it would never use both technologies simultaneously is if K/L is always smaller than $k^\star$ (defined in Proposition 1). This can only happen if K/L is always lower then $\kappa_{\widetilde{\imath}(t)}/\theta_{\widetilde{\imath}(t)}$. But this implies that the economy is always leave workers idle and is therefore producing with an AK production function where $A = \kappa_0^{\alpha-1}$.[18] The economy, and therefore capital per worker, then grows at rate $s\kappa_0^{\alpha-1} - (\delta+\nu)$. Now, if the capital per worker grows at a faster pace than does the capital–labor requirements — that is

$$s\kappa_0^{\alpha-1} - (\delta+\nu) > \frac{\gamma\rho}{1-\alpha} \Longleftrightarrow \kappa_0 < \left(\frac{s}{\delta+\nu+\frac{\gamma\rho}{1-\alpha}}\right)^{\frac{1}{1-\alpha}}$$

which is condition *(ii)* — then the country cannot remain indefinitely in the case $\frac{K(t)}{L(t)} < \frac{\kappa_{\widetilde{\imath}(t)}}{\theta_{\widetilde{\imath}(t)}}$ as it would be growing faster than the leading edge technology. Hence under condition *(ii)*, a country will experience phases where it uses both technologies.

QED □

[18] Remember that in this case, the technology is of the form $\theta_{\widetilde{\imath}}K\kappa_{\widetilde{\imath}}^{\alpha-1}$. Plugging the form of $\theta_{\widetilde{\imath}}$ and $\kappa_{\widetilde{\imath}}$, the result obtains.

B.4 Proposition 4

1) Since the accumulation for $k \equiv K/L$ is of the form

$$\frac{\dot{k}(t)}{k(t)} = s\frac{f(k(t),\theta)}{k(t)} - (\delta + \nu)$$

it is clear, from the standard neoclassical growth model, that a country with a higher rate of labor force growth has a lower value of capital per worker, k, at each point in time. Since the decision to start adopting the new leading edge technology depends on k (see Proposition 1), a country with a lower rate of labor force growth will attain the condition $k \geqslant k^\star$ sooner than one with higher labor force growth. Therefore it will use the stagnant technology for less time.

2) We will prove the second point of the proposition by contradiction. Assume first that the two countries do not share the same long–run rate of growth. A simple and direct implication of this is that the difference between their respective level of output–per–worker will grow without bounds. However, since condition *(ii)* of Proposition 3 holds for both countries — that is

$$\kappa_0 < \left(\frac{s}{\delta + \nu + \frac{\gamma\rho}{1-\alpha}}\right)^{\frac{1}{1-\alpha}}$$

both countries eventually adopt each new leading edge technology, and therefore benefit from the latest technology improvement. Therefore, they both eventually face the same capital–labor requirements and technology improvements, implying that the differences between their respective levels of output–per–worker eventually vanish, which is a contradiction. Therefore, the two countries necessarily grow at the same rate.

QED □