Beliefs Based Exchange Rate Overshooting

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Abstract

The paper introduces habit persistence in consumption decisions in an infinitely–lived agents open economy monetary model with a cash–in–advance constraint. We first show that high enough — but still reasonable — values for habit persistence yields indeterminate equilibria. We however show that real indeterminacy is not *per se* sufficient to generate volatile and persistent fluctuations in exchange rate dynamics. The form of the beliefs matters. When agents do not trust in money, the nominal exchange rate essentially mimics the dynamics of money supply growth and never overshoots. Conversely, when beliefs are positively correlated with money supply shock. the model is capable of generating overshooting and therefore volatility and persistence in exchange rate dynamics.

Keywords: Habit persistence, cash–in–advance, beliefs, real indeterminacy, overshooting, exchange rate dynamics.

JEL Class.: E21, E32, F41

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Introduction

In a recent paper, Obstfeld and Rogoff [2000] identify the six major puzzles in international macroeconomics, among which we think one of the most important is to explain *why are exchange rates so volatile and so apparently disconnected from fundamentals?* (Obstfeld and Rogoff [2000], p. 2). Indeed, it turns out that the exchange rate is the key relative price in international transactions which exerts potential feedback in the whole real side of an open economy. Most of existing models in international macroeconomics have difficulties to account for the high volatility of the exchange rate. One explanation for this result lies in the fact that in these models the dynamics of the nominal exchange rate essentially depends on the domestic and foreign real consumption streams and therefore inherits the excess smoothness of consumption. However, some models recently developed have proven to be helpful in accounting for the dynamics of the nominal exchange rate and explaining its high volatility (see Betts and Devereux [1996], Engel [1996], Chari, Kehoe and McGrattan [1998] or Hau [2000] among others). In these models, the nominal exchange rate dynamics is fully related to that of fundamentals and most of the successes in accounting for the nominal exchange rate volatility stem from assumptions put on the structure of international trade (Pricing to market, price stickiness ...). Recently Hairault, Patureau and Sopraseuth [2000] have propose a small open economy version of Fuerst [1992] or Christiano [1991]. Their model generates a persistent liquidity effect, and therefore — through the uncovered interest rate parity — a persistent overshooting of the nominal exchange rate following a money supply injection for high enough adjustment costs on money holdings. This mechanism is sufficient to account for the nominal exchange rate volatility. Common to these classes of successful models is that they both assume the existence of frictions that either affect the price–setting behavior or the revelation of information to obtain a satisfying monetary transmission mechanism to account for high enough exchange rate volatility.

In this paper, we propose to follow another route and go back to the initial monetary models approach, keeping the full *ex–ante* prices flexibility and complete information assumptions. We introduce intertemporal complementarities in consumption decisions in an open economy monetary model where money is held in the economy because households face a cash–in–advance constraint. More important is the fact that households' preferences are characterized by habit persistence, introducing time non–separability in the model. Habit persistence has proven to be a relevant assumption for representing preferences, and helpful for understanding several puzzles (see e.g. Deaton [1992], Beaudry and Guay [1996], Lettau and Uhlig [1995] or Boldrin, Christiano and Fisher [1999]), in particular asset pricing puzzles (see e.g. Constantidines [1990], Campbell and Cochrane [1995]).

We first show that high enough habit persistence generates real indeterminacy in our monetary economy, resulting from the interplay between habit persistence and the cash–in–advance constraint. Indeed, when individuals face the same positive belief on future inflation, higher expected inflation leads to them to substitute current for future consumption, thus increasing their habits. This translates into higher money demand for tomorrow when habit persistence is strong enough, putting upward pressure on prices. Then, inflation expectations become self–fulfilling. One interesting feature of this result lies in its ability to account for the disconnection between the nominal (and the real) exchange rate and the underlying fundamentals such as interest rates, output and money supply. Indeed, when the equilibrium paths are not determined beliefs matter. In others, there exists an infinite number of beliefs functions which are consistent with the rationale expectation equilibrium. Nevertheless, we show that real indeterminacy is not sufficient *per se* to account for the dynamics of the exchange rate. When beliefs are not correlated with money injection — when individuals do not trust in money — the model generates perfect price flexibility and money is neutral. Then the nominal exchange rate behaves exactly as in the price flexible monetary model, and does not display enough volatility. Conversely, when beliefs are perfectly correlated with money injections — when individuals trust in money — the model displays endogenous price stickiness and can enhance the volatility of the nominal exchange rate. Moreover, when agents trust in money, the model generates a strong enough propagation mechanism that both yields persistence of nominal exchange rate dynamics, and can create overshooting. The model therefore highlights the importance of beliefs in the determination of the nominal exchange rate.

The paper is organized as follows. A first section presents our benchmark model economy, insisting on the individuals behavior. Section 2 characterizes the local dynamic properties of the model and discusses the conditions under which real indeterminacy occurs. After explaining the failure of the basic flexible prices cash–in–advance model, section 3 discusses the role of beliefs in exchange rate dynamics. A last section offers some concluding remarks.

1 The model economy

This section describes the main ingredients that characterize our open economy.

1.1 The domestic economy

The domestic economy is comprised of a unit mass continuum of identical infinitely lived agents, so that we will assume that there exists a representative household in the economy. This household consumes a consumption bundle (c_t) composed of both domestic ($c_{h,t}$) and foreign ($c_{f,t}$) produced goods:

$$c_t = \left[\omega^{\frac{1}{\rho}} c_{h,t}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} c_{f,t}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{1}$$

where $\rho \in (0, \infty)$ is the elasticity of substitution between foreign and domestic goods. $(1-\omega) \in [0;1]$ is the import share. Implicit in this specification is that each country specializes in the production of a local good, which is imperfectly substitutable with the good produced abroad. The optimal composition of the consumption basket is determined by minimizing the overall expenditures, taking (1) into account. This yields the following demand functions

$$c_{h,t} = \left(\frac{P_{h,t}}{P_t}\right)^{-\rho} \omega\, c_t \text{ and } c_{f,t} = \left(\frac{e_t P^\star_{f,t}}{P_t}\right)^{-\rho} (1-\omega)\, c_t \tag{2}$$

where $P_{h,t}$ and $P^\star_{f,t}$ respectively denote the domestic and foreign production prices expressed in the currency of the producer. e_t is the nominal exchange rate and P_t is the consumption price which expresses as

$$P_t = \left[\omega P_{h,t}^{1-\rho} + (1-\omega)(e_t P^\star_{f,t})^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{3}$$

Given this intratemporal allocation of resources, the household has now to allocate intertemporally her revenues. She enters period t with real balances m_t/P_t brought into period t from the previous period and net foreign assets (b_t), as a mean to transfer wealth from one period to another. The household supplies her hours on the labor market at the real wage W_t. During the period, she also receives a lump–sum transfer from the monetary authorities in the form of cash equal to N_t/P_t and interest rate payments from bond holdings ($(R_{t-1}-1)b_t$). All these revenues are then used to purchase a consumption bundle c_t, money balances and net foreign assets for the next period. Therefore, the budget constraint simply writes as

$$b_{t+1}+\frac{m_{t+1}}{P_t}+c_t \leqslant R_{t-1}b_t+W_th_t+\frac{m_t}{P_t}+\frac{N_t}{P_t} \tag{4}$$

Money is held because the household must carry cash — money acquired in the previous period and the money lump sum transfer — in order to purchase goods. She therefore faces a standard cash–in–advance constraint of the form:

$$c_t \leqslant \frac{m_t+N_t}{P_t}+R_{t-1}b_t-b_{t+1} \tag{5}$$

Note that below, we will focus on equilibria in which the constraint binds. Implicit in this is that we will consider equilibria where the gross nominal interest rate exceeds unity or equivalently where the inflation rate is strictly positive, which will turn out to be the case for the specification of the shock we will consider.

Each household has preferences over consumption and leisure represented by the following intertemporal utility function:

$$E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left(\log\left(s_\tau\right)-\kappa h_\tau\right) \tag{6}$$

where $\beta\in(0,1)$ is the discount factor, h_t denotes the number of hours supplied by the household. s_t is a consumption index on which we will come back and from which the household derives utility. E_t denotes the expectation operator conditional on the information set available in period t. The linearity of disutility in labor is assumed for simplicity[1] and corresponds to Hansen's [1985] labor indivisibility assumption. An

[1] Auray, Collard and Fève [2000] show that the main dynamic result are left unaffected by considering a more general utility function.

attractive feature of this specification is that the model can be directly compared against the standard cash–in–advance economy considered by Cooley and Hansen [1989] and Cooley and Hansen [1995].

Besides, this specification of the utility function allows for habit persistence in the consumption behavior, and therefore introduce time non–separability in the utility function. Indeed, following Constantidines and Ferson [1991], Braun, Constantidines and Ferson [1993], we consider that s_t takes the form

$$s_t = c_t - \theta c_{t-1} \text{ with } \theta \in (0,1) \tag{7}$$

such that the household preferences are characterized by internal habit persistence specified in difference with one lag in consumption.[2] Note that setting θ to zero, we retrieve a standard cash–in–advance model for an open economy.

Habit persistence is introduced as in Constantidines and Ferson [1991]. We first show that high enough habit persistence generates real indeterminacy in our monetary economy. This results from the interplay between habit persistence and the cash–in–advance constraint, given a specific environment on the labor and asset markets. Indeed, when individuals face the same positive belief on future inflation, higher expected inflation leads to them to substitute current for future consumption, thus increasing their habits. This translates into higher money demand for tomorrow when habit persistence is strong enough, putting upward pressure on prices. Then, inflation expectations become self–fulfilling. We then show that real indeterminacy is robust against the introduction of additional goods and/or asset that can be used to avoid the inflation tax, and against more general specification of the technology than the one considered in our benchmark model. Nevertheless, we show that real indeterminacy is not sufficient *per se* to generate the monetary transmission mechanism we are mainly interested in, the form of the beliefs matters a lot. Indeed, when beliefs are not correlated with money injection — when individuals do not trust in money — the model generates perfect price flexibility and money is

[2] In Auray et al. [2000] we investigate a bit further the implications of this assumption on the overall dynamic behavior of a closed monetary economy.

neutral. Conversely, when beliefs are perfectly correlated with money injections — when individuals trust in money — the model displays endogenous price stickiness associated with a positive and persistent response of output.

The household then determines her optimal consumption/money/assets holdings and labor supply plans maximizing (6) subject to the budget (4) and cash–in–advance (5) constraints.

The technology in the domestic economy is described by the constant return to scale production function $y_t = h_t$, such that in equilibrium the real wage is $W_t = P_{h,t}$. Finally, money is exogenously supplied by the central bank according to the following money growth rule:

$$M_{t+1} = g_t M_t \tag{8}$$

where $g_t \geqslant 1$ is the exogenous gross rate of growth of money, such that $N_t = M_{t+1} - M_t = (g_t - 1)M_t$. Hereafter, we will assume, following Cooley and Hansen's [1989], that g_t evolves as an exogenous AR(1) process

$$\widehat{g}_{t+1} = \rho_g \widehat{g}_t + \varepsilon^g_{t+1}$$

where $|\rho_g| < 1$ and ε^g is a centered gaussian white noise with variance σ^2_ε.

1.2 The rest of the world

As we consider an open economy model, we need to specify the behavior of the rest of the world. Like in Cole and Ohanian [1900], we simplify the analysis and abstract from production and consumption/money/assets holdings decisions. At a first glance, this might be considered as considering a small open economy model, in which the rest of the world is taken as given. However, our model economy departs from a small open economy as we allow for some feedback from the domestic economy to the rest of the world. Indeed, the rest of the world chooses how optimally allocate their income between goods produced in both economies. Hence, the representative household in the rest of the world consumes a consumption bundle ($c^\star_t$) composed of both domestic ($c^\star_{h,t}$) and foreign

($c^\star_{f,t}$) produced goods:

$$c^\star_t = \left[(1-\omega)^{\frac{1}{\rho}} c^{\star\frac{\rho-1}{\rho}}_{h,t} + \omega^{\frac{1}{\rho}} c^{\star\frac{\rho-1}{\rho}}_{f,t}\right]^{\frac{\rho}{\rho-1}} \tag{9}$$

where the elasticity of substitution between goods ($\rho \in (0,\infty)$) and the import share ($(1-\omega) \in [0;1]$) are assumed to be the same as in the domestic economy. $\{c^\star_t\}_{t=0}^{\infty}$ is an exogenous sequence of real aggregate consumption purchases in the rest of the world. Implicit in this assumption is that the aggregate demand from the rest of the world is a strong exogenous variable, which is therefore left unaffected by changes occurring in the domestic economy. The optimal composition of the consumption basket is determined by minimizing the overall expenditures, taking (9) into account. This yields the following demand functions

$$c^\star_{h,t} = \left(\frac{P_{h,t}}{e_t P^\star_t}\right)^{-\rho}(1-\omega)\, c^\star_t \text{ and } c^\star_{f,t} = \left(\frac{P^\star_{f,t}}{P^\star_t}\right)^{-\rho}\omega\, c^\star_t \tag{10}$$

where $P^\star_t$ is the consumption price which expresses as

$$P^\star_t = \left[(1-\omega)\left(\frac{P_{h,t}}{e_t}\right)^{1-\rho} + \omega P^{\star 1-\rho}_{f,t}\right]^{\frac{1}{1-\rho}} \tag{11}$$

Output in the rest of the domestic economy is given by an exogenous sequence $\{y^\star_t\}_{t=0}^{\infty}$. This hypothesis amounts to impose that production capacities in the rest of the world are left unaffected by a shock occurring in the domestic economy.

The exogeneity assumption imposed on the world demand and world production capacities does not imply that the rest of the world is totally non–responsive to shocks in the domestic economy. Indeed, any domestic shock will translate into a reallocation of consumption purchases between goods produced in the domestic economy and in the rest of the world, since such a shock affects the relative price between foreign and domestic goods. Further, this specification also implies that the CPI will respond in the rest of the world. Therefore, this modelling cannot be associated to the small open economy model, since price effects still exert an impact in the rest of the world. Among other, this implies that such a modelling does not yield the unpleasant unit root property that small open economy models usually exhibit.

1.3 The Equilibrium

An equilibrium of this economy is a sequence of prices $\{P_t, P_t^\star, P_{h,t}, P_{f,t}^\star, e_t, W_t\}_{t=0}^{\infty}$ and a sequence of quantities $\{c_t, c_{h,t}, c_{f,t}, c_{h,t}^\star, c_{f,t}^\star, y_t, h_t, m_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ which solve the maximization problem and satisfy market clearing conditions. Goods market clearing condition implies:

$$y_t = c_{h,t} + c_{h,t}^\star \tag{12}$$

$$y_t^\star = c_{f,t} + c_{f,t}^\star \tag{13}$$

The labor market clearing in the domestic economy imposes $Y_t = h_t$ and the money market clearing requires

$$m_{t+1} = M_{t+1} = M_t + N_t \tag{14}$$

The domestic current account is given by

$$B_{t+1} = R_{t-1}B_t + \frac{P_{h,t}}{P_t}c_{h,t}^\star - \frac{e_t P_{f,t}^\star}{P_t}c_{f,t} \tag{15}$$

and the current account in the foreign economy then satisfies

$$B_{t+1}^\star = R_{t-1}B_t^\star + \frac{P_{f,t}^\star}{P_t^\star}c_{f,t} - \frac{P_{h,t}}{e_t P_t^\star}c_{h,t}^\star \tag{16}$$

Asset market clearing finally imposes that $B_{t+1} + B_{t+1}^\star = 0$ in each and every period, which translates in the world aggregate resources constraint

$$P_{h,t}y_t + e_t P_{f,t}^\star y_t^\star = P_t c_t + e_t P_t^\star c_t^\star$$

In order to study the dynamic properties of the model, we deflate each nominal variable for nominal growth. We therefore define the set of variables $p_t = P_t/M_t$, $p_t^\star = e_{t-1}P_t^\star/M_t$, $p_{h,t} = P_{h,t}/M_t$, $\Delta e_t = e_t/e_{t-1}$ and $p_{f,t}^\star = e_{t-1}P_{f,t}^\star/M_t$. Note that one of the prices cannot be determined in this setting because we did not specify neither producer price setting behavior nor any cash–in–advance constraint in the rest of the world, such that one equation is missing. Therefore, $p_{f,t}^\star$ will be assumed to be exogenous in the sequel.[3]

Hereafter, and to keep things simple, we will assume that all exogenous sequences will be set to constants such that $c_t^\star = c^\star$, $y_t^\star = y^\star$ and $p_{f,t}^\star = p_f^\star = 1$.

[3] Note that by the homogeneity of degree one of the price indices, the choice of the price we decide to make exogenous does not matter, since what really determines the CPI is the terms of trade $P_{h,t}/(e_t P_{f,t}^\star)$.

2 Dynamic properties

This section establishes the dynamic properties of our model economy, and more precisely characterizes conditions on the level of habit persistence for real indeterminacy, and discuss these results.

2.1 Habit persistence and indeterminacy

The dynamic properties of output are strongly related to the perfect foresight version of the model economy. First of all, note that when the two economies are perfectly symmetric, we have

$$y = y^{\star} = h = h^{\star} = c = c^{\star}$$

further, we have

$$p = p^{\star} = p_h = p_f^{\star} = \Delta e = 1$$

Therefore, in the steady state, output is given by:

$$y^{\star} = \frac{\beta}{g} \frac{1-\beta\theta}{\kappa(1-\theta)}$$

The local dynamic properties of our model economy may then be investigated taking a first order log-linear approximation about the deterministic steady state, which yields the following linear second order finite difference equation:

$$\widehat{y}_{t+2} - \left[\frac{1+\beta}{\beta} + \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{2\omega\rho\beta\theta}\right]\widehat{y}_{t+1} + \left[\frac{1}{\beta} - \frac{(1-\theta)(1-\theta\beta)}{\beta\theta}\right]\widehat{y}_t = 0 \quad (17)$$

where $\widehat{y}_t = (y_t - y^{\star})/y^{\star}$. Equation (17) can be expressed in the more compact form

$$(1-\lambda L)(1-\mu L)\widehat{y}_{t+2} = 0$$

where L denotes the lag operator. The position of λ and μ around the unit circle determines the local dynamic properties of the log–linear economy. The model satisfies a saddle path property iff both λ and μ are of modulus greater than one. Conversely, if at least one of the eigenvalues lies inside the unit circle the equilibrium is locally indeterminate, *i.e.* there exists a continuum of equilibria in the neighborhood of the steady state. First of all, we can establish the following property.

Proposition 1 *There exists $\theta^\star \in (0,1)$ such that for all $\theta \geqslant \theta^\star$ one and only one eigenvalue lies inside the unit circle.*

Proposition 1 establishes that — for a given value of the discount factor (β) — there exists a value of the weight of habit persistence above which prophecies become self–fulfilling.

Figure 1: Roots of the characteristic polynomial

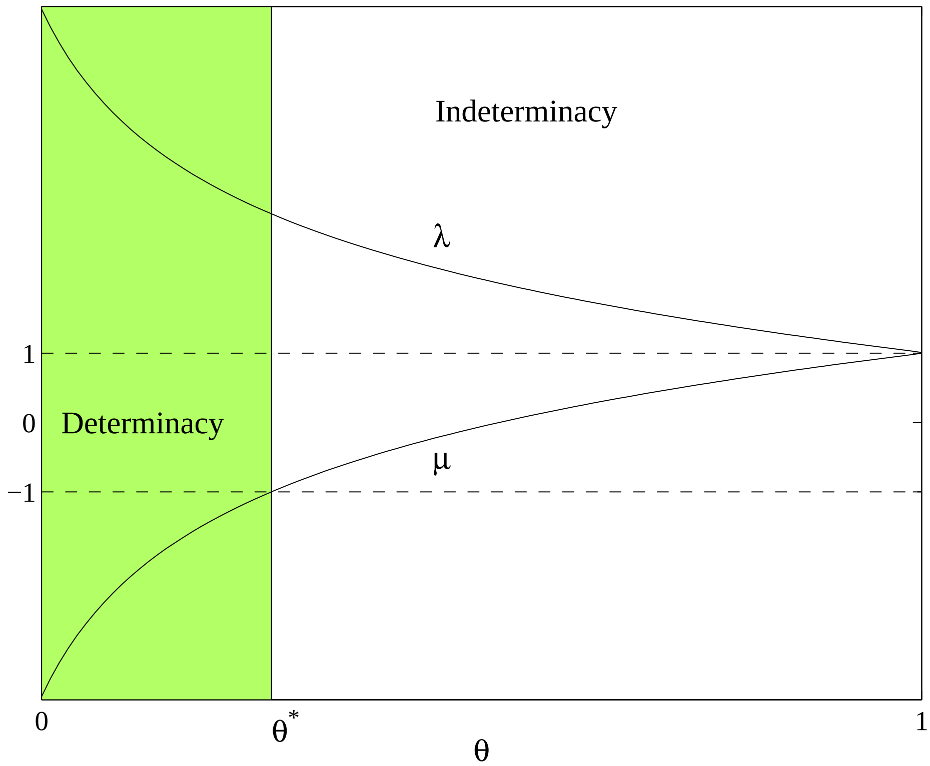


Figure 1 illustrates proposition 1 and the above discussion. The two curves represents the evolution of the two roots of the characteristic polynomial and shows that one of the two roots always remain greater than unity. The shaded area corresponds to values of θ for which the equilibrium is totally determinate. Above $\theta^\star$ the equilibrium becomes indeterminate. It is worth noting that as θ tends to 1, the stable root tends to one. Further, as can be seen from figure 1, the stable root is positive for high level of habit persistence. This leads to the following proposition.

Proposition 2 *There exists a threshold $\widetilde{\theta} \in (\theta^\star, 1)$ such that the stable root is strictly positive.*

An implication of this last result is that there exists values for the habit persistence parameter such that output is positively serially correlated, which is consistent with the observed persistence in aggregate data. This contrasts with the standard cash–in–advance model that generates no persistence, when the money growth process — if assumed to be exogenous — is serially uncorrelated. The latter proposition therefore establishes that the cash–in–advance model, when coupled with habit persistence, possesses internal propagation mechanisms strong enough to generate persistence. Beyond, this proposition states that, in this model, persistence goes together with real indeterminacy, as $\widetilde{\theta} > \theta^{\star}$. Another interesting result is that oscillatory sunspot equilibria may also occur as for values of $\theta \in (\theta^{\star}, \widetilde{\theta})$, the stable eigenvalue is strictly negative. It should however be noted, that, in this case, output is negatively serially correlated which is not empirically appealing.

2.2 Discussion

This section sheds light on the underlying mechanisms that are at work in the model. We first investigate the real indeterminacy result. Then, we characterize the behavior of all other aggregates of interest.

In an equilibrium, the dynamics of the economy may be rewritten as

$$\kappa = \beta E_t \vartheta_{t+1} \left[\frac{1}{c_{t+1} - \theta c_t} - \frac{\beta\theta}{c_{t+2} - \theta c_{t+1}} \right] \tag{18}$$

where ϑ_{t+1} denotes the inflation tax in the economy — taken as given by the individuals when determining their optimal plans. The inflation tax can be viewed as an increasing marginal tax function of future aggregate consumption — external to individuals. Let us assume that individuals behavior is characterized by a weak habit persistence ($\theta \simeq 0$) and that they all have the same positive belief on future inflation. This leads every individual to increase current consumption. But, as intertemporal substitution is high, individual consumption drops in the next period. Since all individuals are identical and face the same belief, aggregate consumption drops in the next period. Therefore, the inflation tax shall decrease, which cannot support inflation beliefs. Any changes in beliefs can only be due to monetary policy, and is therefore related to fundamental shocks.

Let us now consider the case where habit persistence is large enough to weaken intertemporal substitution motives ($\theta \gg 0$) and that all individuals again face the same positive belief on future inflation. Like in the previous case, individuals consume more today. But, contrary to the preceding case, the irreversibility in consumption decisions associated with habit persistence leads the agents to increase their future individual consumption too. Since, they are all identical and face the same belief, aggregate future consumption eventually increases. It follows that the aggregate inflation tax increases, therefore supporting the initial individual beliefs. Note that these beliefs may now depart from fundamentals — even though they can be arbitrarily correlated to fundamentals.

We now turn to the analysis of the other aggregates of our model economy, given the consumption dynamics in response to a monetary shock. Let us first consider the domestic PPI. The log–linearized PPI is simply given by

$$\widehat{p}_{h,t} = \widehat{g}_t - \widehat{c}_t \tag{19}$$

Consider first a situation where money is totally neutral in the economy, any money injection translates into a one to one increase in PPI. Now consider that the equilibrium is indeterminate and agents have positive beliefs in face of a money injection, then consumption may rise sufficiently and weakens the inflationary effects of monetary policy. There may also be a situation where beliefs are strong enough to totally offset inflationary pressure, such that the PPI is rigid in the short–run. Since the CPI is given by

$$\widehat{p}_t = \widehat{g}_t - \left(1 - \frac{1-\omega}{2\omega\rho}\right)\widehat{c}_t \tag{20}$$

it will respond to a money injection even if the PPI remains unchanged, when $\omega \in (0,1)$ and $\rho < \infty$. Indeed, the latter equation may be rewritten as

$$\widehat{p}_t = \widehat{p}_{h,t} + \frac{1-\omega}{2\omega\rho}\widehat{c}_t$$

In others, let us assume that we are in a situation where the PPI is rigid in the short run ($\widehat{p}_{h,t} \simeq 0$), the response of the CPI is essentially given by $(1-\omega)\widehat{c}_t/(2\omega\rho)$, which accounts for imported inflation. Indeed, the only way to break this effect is either to set $\omega = 1$ — autarky — or to let $\rho \to \infty$ — which corresponds to a perfect substitutability between goods — in which case any increase in foreign prices translates into a drop in the consumption of goods produced in the rest of the world. This imported effect can be found — with the opposite sign — in the CPI of the rest of the world which expresses as

$$\widehat{p}_t^\star = -\frac{1-\omega}{2\omega\rho}\widehat{c}_t \tag{21}$$

Note that the absence of any PPI effect stems from our assumption that $p_{f,t}^\star = p_f^\star$. The observed decline in the CPI in the rest of world results from the depreciation of the nominal exchange rate following a money injection. Indeed, changes in the nominal exchange rate write as

$$\widehat{\Delta e}_t = \widehat{g}_t + \frac{1-2\omega\rho}{2\omega\rho}\widehat{c}_t \tag{22}$$

which may be better understood as[4]

$$\widehat{\Delta e}_t = (\widehat{g}_t - \widehat{g}_t^\star) + \frac{1-2\omega\rho}{2\omega\rho}(\widehat{y}_t - \widehat{y}_t^\star)$$

where, from our assumptions $\widehat{g}_t^\star = 0$ — no/or constant monetary policy in the rest of the world — and $\widehat{y}_t^\star = 0$ — constant world production capacities. We therefore find the standard monetary model of nominal exchange rate determination. Consider first a situation where money is neutral, a positive money injection yields a one for one depreciation of the exchange rate, which instantaneously shifts to its long–run level. If we now consider a situation where beliefs matter and are positively correlated with money injections, then the response is ambiguous. It crucially depends on the elasticity of substitution between goods. When this elasticity is low enough ($\rho < 1/(2\omega)$), the nominal exchange rate depreciation is magnified. Indeed, if money leads individual to increase their consumption purchases and if goods are complement, the increase in consumption yields an increase in the demand for both goods, which creates an upward pressure on both $\widehat{p}_{h,t}$ and $\widehat{\Delta e}_t + \widehat{p}_f^\star$. Conversely, when goods are substitutable, substitution effects imply that nominal exchange rate depreciation is weakened and may even be totally offset in the short–run. If we now consider the real exchange rate $Q_t = e_t P_t^\star / P_t$, it writes as

$$\widehat{q}_t = \frac{2\omega - 1}{2\omega\rho}\widehat{c}_t \tag{23}$$

in equilibrium. Like many other variables, it does not react when money neutral. When a money injection matters and yields an increase in consumption, the real exchange rate

[4] We made use of the fact that in the log–linearized equilibrium, $\widehat{c}_t = \widehat{y}_t$. See appendix B.

depreciates provided $\omega > 1/2$ — the degree of openness is not too large. The limit case where $\omega = 1/2$ corresponds to a situation where

$$\widehat{\Delta e}_t = \widehat{p}_t - \widehat{p}^\star_t$$

or otherwise stated where purchasing power parity (PPP) holds as $\widehat{q}_t = 0$ — or $Q_t = 1$. Also note that the PPP also holds when goods are perfect substitutes ($\rho \to 0$). Finally, terms of trade, $\tau_t = e_t P^\star_{f,t}/P_{h,t}$, are given by

$$\widehat{\tau}_t = \frac{1}{2\omega\rho}\widehat{c}_t \tag{24}$$

Terms of trade vary in a similar way to the real exchange rate in response to a money injection, but the magnitude of their fluctuations differs.

3 Exchange rate dynamics

This section attempts to shed light on the nominal exchange rate dynamics. More precisely, we characterize the long–run and the short–run properties of nominal exchange rate, focusing on the so–called overshooting property. We first show how the standard cash–in–advance model fails in accounting for the main features of the nominal exchange rate dynamics, and then turn to our model economy.

3.1 The Failure of the Cash–in–advance Economy

To provide with a better understanding with the major failures of the standard cash–in–advance open economy model, we now propose a benchmark experiment where we set $\theta = 0$. Then the equilibrium consumption choice writes as

$$\kappa = \frac{\beta}{g_t} E_t \frac{p_{h,t}}{p_{t+1}c_{t+1}}$$

which admits, in general equilibrium, the log–linear representation

$$\widehat{c}_t = -\rho_g \widehat{g}_t + \mu E_t \widehat{c}_{t+1} \text{ with } \mu = -\frac{1-\omega}{2\omega\rho}$$

The equilibrium path is locally unique when $-1 < \mu < 1$. First note that since $\omega \in [0, 1]$ and $\rho > 0$, the equilibrium is determinate provided $\rho > (1-\omega)/(2\omega)$ — the economy has to display high enough substitutability between domestic and foreign goods.[5] The solution to the log–linear representation of the economy is then given by

$$\widehat{c}_t = -\frac{2\omega\rho\rho_g}{2\omega\rho+\rho_g(1-\omega)}\widehat{g}_t \equiv \gamma_0\widehat{g}_t$$

When money displays positive serial correlation[6], then consumption converges back to its steady state monotonically and so do changes in the nominal exchange rate:

$$\widehat{\Delta e}_t = \widehat{g}_t + \frac{1-2\omega\rho}{2\omega\rho}\widehat{c}_t = \left(1+\frac{1-2\omega\rho}{2\omega\rho}\gamma\right)\widehat{g}_t \equiv \psi_0\widehat{g}_t$$

However, note that since ρ is positive, $\omega \in (0,1)$ and $|\rho_g| < 1$, γ_0 will always be negative, such that consumption, and therefore output, drops following a monetary injection. The standard cash–in–advance economy cannot, as well–known, generate the monetary transmission mechanism.

Proposition 3 *The domestic currency depreciates following a monetary injection provided the elasticity of susbtitution between foreign and domestic goods satisfies*

$$\rho > \frac{\rho_g}{2(1+\rho_g)}$$

When money shocks are *iid* — ρ_g=0 —, the constraint of proposition 3 always holds. When ρ_g is positive, then foreign and domestic goods have to display enough substitutability. The result may be understood recalling that consumption drops following a money injection. Therefore, an individual will demand less of the two goods. But, in the domestic domestic prices increase more than one to one following the money injection (see equation (19)). Since goods are substitutable enough, there is a shift away from domestic goods toward goods produced in the rest of the world. Therefore, domestic households are willing to import more, which require the sale of bonds denominated in local currency. The demand for domestic money drops and the nominal exchange rate depreciates. Note that when ρ_g is large, the inflation tax displays more persistence, such that consumption drops to a larger extent such that the demand for domestic bonds increases as bonds are

[5]Note that the converse situation is eventually of low interest since it is associated to a negative eigenvalue such that consumption displays negative serial correlation. This is highly counterfactual and we therefore do not investigate this situation any further.

[6]We do not investigate the case of negative serial correlation, which is counterfactual.

a way to escape the inflation tax. Therefore, higher substitutability is needed to counter this effect and the constraint on ρ is more stringent.

Proposition 4 *The nominal exchange rate cannot overshoot its long term level — when it depreciates — when $\rho_g \geqslant 0$.*

In order to provide with some intuition, let us consider the case where the monetary shock is *iid*. In this case, money is neutral and consumption does not react to a monetary injection. The shock is totally accommodated by changes in nominal variables:

$$\widehat{\Delta e}_t = \widehat{p}_{h,t} = \widehat{p}_t = \widehat{g}_t$$

Furthermore, while the model implies that the domestic currency depreciates in face a positive monetary injection, it does not generate overshooting as the nominal exchange rate instantaneously shifts to its new level.

Figure 2 reports impulse response functions (IRF) to a positive money injection when $\rho_g = 0.5$. This illustrates our previous statements that output drops in face the money supply shock, and the nominal exchange rate does not overshoot its long–run level when the inflation tax is persistent. This analysis illustrates the well–known drawbacks of the standard cash–in–advance model for the analysis of exchange rate dynamics. In the next section we will show the potential of our augmented cash–in–advance model.

3.2 The merit of Beliefs

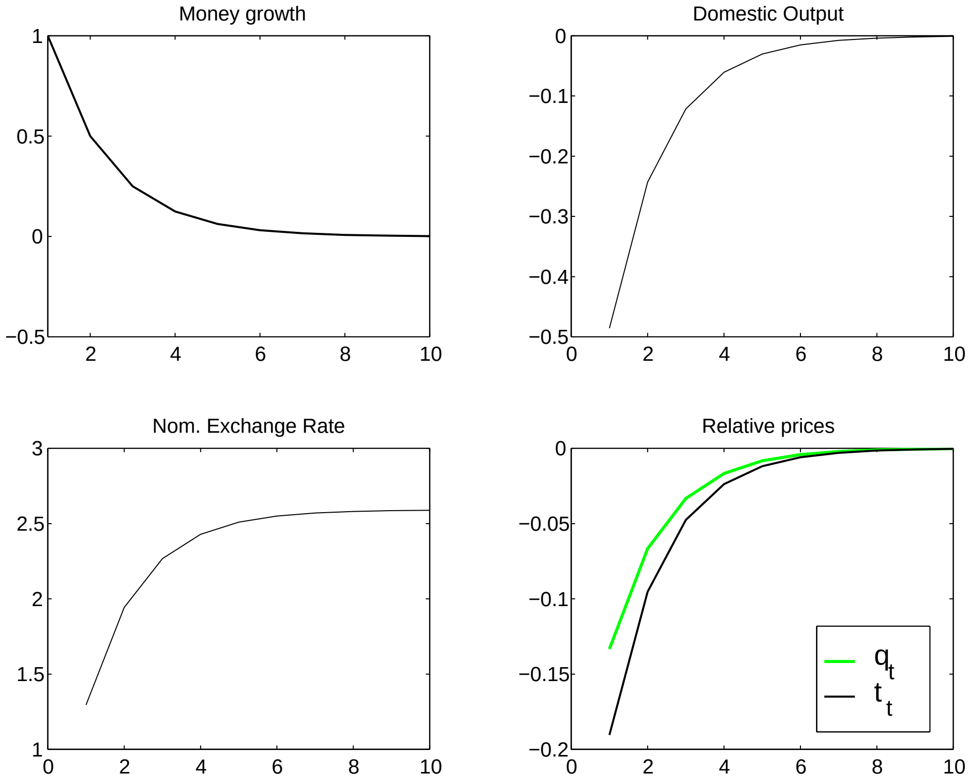
In this section, we go back to our specification and consider cases where $\theta > 0$. In this case, output dynamics is described by the second–order finite difference equation

$$\begin{aligned} E_t\widehat{y}_{t+2} - \left[\frac{1+\beta}{\beta} + \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{2\omega\rho\beta\theta}\right] E_t\widehat{y}_{t+1} & \\ + \left[\frac{1}{\beta} - \frac{(1-\theta)(1-\theta\beta)}{\beta\theta}\right]\widehat{y}_t - \frac{(1-\theta)(1-\theta\beta)}{\beta\theta}E_t\widehat{g}_{t+1} &= 0 \end{aligned}$$

In this case, the threshold values for θ, $\theta^\star$ and $\widetilde{\theta}$, take the simple forms

$$\begin{aligned} \theta^\star &= \frac{3(1+\beta) - \sqrt{9(1+\beta)^2 - 4\beta}}{2\beta} \\ \tilde{\theta} &= \frac{2+\beta - \sqrt{\beta^2+4}}{2\beta} \end{aligned}$$

Figure 2: Impulse responses (ρ_g=0.5)



Reasonable values for β (β close to unity) imply that $\theta^\star \approx 0.17$ and $\widetilde{\theta} \approx 0.38$. Therefore, real indeterminacy occurs rather easily in this economy, but more remarkable is that the model generates positive serial correlation in output dynamics with a value of θ which is not too high with respect to existing point estimates. Indeed, empirical studies suggest parameter estimates for θ that exceed significantly the minimal value that yields indeterminacy. For instance, Constantidines and Ferson [1991] and Braun et al. [1993] obtain an estimated value of θ that lies within $[0.5; 0.9]$ on macro data. Habit persistence appears to be lower but still significant on micro data. Naik and Moore [1996] report estimates for habit persistence on food consumption data of 0.486 which is far above the threshold value of θ yielding indeterminacy and exceeds that needed for positive persistence. We now focus on solutions associated with indeterminate equilibrium. In such cases, we have

$$\widehat{y}_t = \mu \widehat{y}_{t-1} + \gamma \widehat{g}_{t-1} + \varepsilon_t^y \text{ where } \gamma = -\frac{\rho_g}{\lambda - \rho_g} \frac{(1-\theta)(1-\theta\beta)}{\beta\theta} \tag{25}$$

where ε_t^y is a martingale difference sequence that can be related to fundamental shocks (money shocks), depending on individuals beliefs about monetary policy, such that it writes

$$\begin{aligned} \varepsilon^y_t &= b(g_t - E_{t-1}g_t) + \nu_t \\ &= b\varepsilon^g_t + \nu_t \end{aligned} \quad (26)$$

with $E_{t-1}\nu_t = 0$ and $|b| < \infty$. ν_t denotes pure extrinsic beliefs that are unrelated to fundamentals. The parameter b rules the dependency of agents beliefs to fundamentals. It is worth noting that this parameter is an extrinsic characteristic of agents beliefs, which, as shown in the previous section is critical for the properties of the equilibrium. Then, the nominal exchange rate dynamics takes the simple ARIMA(2,1,1) form:

$$(1-\mu L)(1-\rho_g L)(1-L)\widehat{e}_t = (1+\psi b + (\psi(\gamma - \rho_g b) - \mu)L)\,\varepsilon_t \text{ where } \psi = \frac{1-2\omega\rho}{2\omega\rho} \quad (27)$$

To keep the exposition simple, let us consider the case where $\rho_g = 0$ and $\nu_t = 0$, $\forall t$. Then, (25) and (27) reduce to

$$\begin{aligned} \widehat{y}_t &= \mu\widehat{y}_{t-1} + b\varepsilon^g_t \\ \Delta\widehat{e}_t &= \mu\Delta\widehat{e}_{t-1} + (1+\psi b)\varepsilon_t - \mu\varepsilon_{t-1} \end{aligned}$$

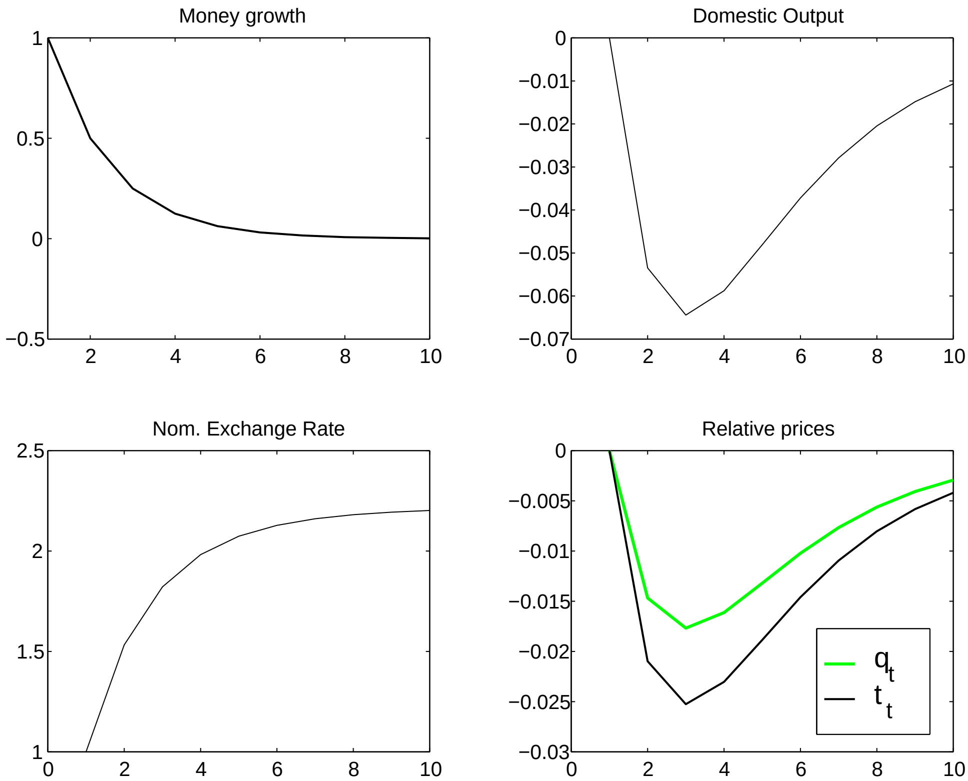
Note that the two last equations just show that the model can generate persistence, provided $\mu > 0$. But they also show that real indeterminacy is not *per se* sufficient to generate the monetary transmission mechanism (output increases in face of a positive money injection) or the overshooting of nominal exchange rate, additional assumption are to be placed on individuals' beliefs — in particular how they comove with money supply shocks— as we now illustrate.

Let us first consider the case where b=0, such that the above system reduces to

$$\begin{aligned} \widehat{y}_t &= \mu\widehat{y}_{t-1} \\ \Delta\widehat{e}_t &= \varepsilon_t \end{aligned}$$

which implies that money is neutral. Indeed, following a money injection, output remains at its steady state level ($\widehat{y}_t = 0$) and $\widehat{y}_t$ is a degenerated stochastic variable. Conversely, changes in the nominal exchange rate respond one for one to a money injection, hence fully absorbs the shock and so does all other nominal variables. This case corresponds to a full price flexibility situation where the nominal exchange instantaneously shifts to its new long run level and cannot overshoot. We therefore retrieve the quantity theory of money, which can then be associated with a particular form of beliefs where agents do not trust in money. This also imply that the volatility of changes in the nominal exchange rate is the same as the one of the money supply shock. The model behaves as badly as the standard cash–in–advance model. Figure 3 reports IRF in the more general case $\rho_g = 0.5$. As stated above, the model resembles the standard cash–in–advance model, in

Figure 3: Impulse responses



Note: These IRF are obtained with $\rho_g = 0.5$, $\beta = 0.99$, $\omega = 0.85$, $\rho = 1.5$ and $\theta = 0.75$.

that it fails to account for the monetary transmission mechanism and the exchange rate dynamics. The only notable difference stems from the higher persistence of adjustment dynamics.

We now investigate a situation where individuals' beliefs are perfectly positively correlated with the money supply shock — agents trust in money — such that b=1. The output/changes in the nominal exchange rate dynamics then rewrites

$$\begin{aligned}
\widehat{y}_t &= \mu\widehat{y}_{t-1}+\varepsilon_t^g \\
\Delta\widehat{e}_t &= \mu\Delta\widehat{e}_{t-1}+(1+\psi)\varepsilon_t-\mu\varepsilon_{t-1}
\end{aligned}$$

Let us first analyze the impact effect of a positive money supply shock in the domestic economy ε_t on the nominal exchange rate. A positive money supply shock always yields a depreciation of the domestic currency as the impact effect is given by $1+\psi=1/(2\omega\rho)>0$, which eventually corresponds to the change in the terms of trade. But the magnitude of the depreciation depends on the substitutability between goods. When goods are perfect substitute, $\rho\rightarrow\infty$, changes in the nominal exchange rate essentially correct any change in the terms of trade arising from a money supply shock since the household is able to reallocate her consumption purchases between the two goods. But, on impact, production prices are left unaffected by a monetary injection since consumption responds one for one to a money injection and the domestic PPI is given by (19). Hence, the model is found to generate endogenous production price rigidity. Suppose now that the nominal exchange rate depreciates (appreciates), given perfect substitutability between goods, there will be a perfect switch away toward domestic (rest of the world) goods. Therefore, in order to acquire domestic (rest of the world) goods, the rest of the world must instantaneously sell (purchase) bonds denominated in foreign currency which yields an appreciation (depreciation) that offset the initial depreciation (appreciation). Therefore the nominal exchange rate does not respond. Terms of trade and real exchange rate are left unaffected. Hence, besides PPI rigidity, this version may account for complete short–run nominal rigidity in an open economy. When goods are not perfect price substitute, the initial increase in consumption is associated with an increase in the willingness to consume the two types of goods. This, in particular, leads the domestic household to sell bonds denominated in domestic currency in order to purchase goods produce in the rest of the world, yielding a depreciation of the nominal exchange rate. The lower the substitutability, the greater the willingness to consume both types of goods, and therefore the greater is this effect. The magnitude of the depreciation is therefore magnified.

A widely discussed question in the literature dealing with nominal exchange rate is the long–run effect of a money injection on its dynamics — the so–called $\mathcal{A}(1)$. Beyond it provides a way to characterize the dynamic properties of the nominal exchange rate as it is related to the size of the unit root. In the case we investigate $\mathcal{A}(1)$ is simply given by

$$\mathcal{A}(1) = 1 + \frac{\psi}{(1-\mu)}$$

and we can establish the following proposition.

Proposition 5 *When agents are fully confident in money, a positive money injection can yields either $\mathcal{A}(1) \geqslant 1$ when goods are complement enough or $\mathcal{A}(1) < 1$ when they display substitutability.*

A first implication of the last proposition is that when individuals put confidence in money (b=1), the model may either generate a long–run effect of money greater than that obtained in the simple random walk model, or a long–run effect lower than 1 reflecting a anti–persistent property. It can eventually become negative when goods are substitute enough. In order to understand this result, let us consider the case of perfect substitutability. In face of a positive money supply shock, the terms of trade are left unaffected so that

$$\widehat{\Delta e}_t = \widehat{p}_{h,t} = \widehat{g}_t - \widehat{c}_t$$

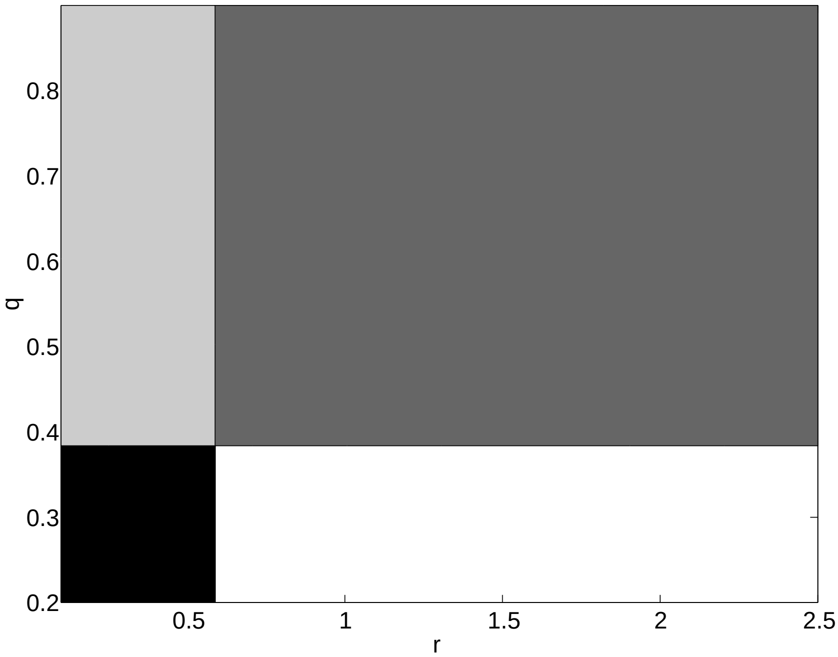
Since the shock is iid, the only effect that plays the following period is the effect transiting through consumption. Since consumption responds positively on impact and displays positive persistence (for $\theta > \widetilde{\theta}$), changes in the nominal exchange rate are negative from the second period on, such that the domestic currency appreciates in the long–run. Conversely, when goods are complement, the increase in overall domestic consumption translates in higher demand for both domestic and foreign goods. Since, foreign production prices (deflated for domestic money) are left unaffected the extra demand for foreign goods leads the domestic currency to depreciate, because of bonds trading, this effect being stronger as complementarity increases. Since consumption displays positive persistence (for $\theta > \widetilde{\theta}$), this effect persists too such that the $\mathcal{A}(1)$ is greater than 1.

Endowed with these preliminary results on short–run and long–run effects of money shocks, we now tackle the key question of overshooting.

Proposition 6 *When agents trust in money, the nominal exchange rate overshoots its long–run value if (i) $\rho > 1/(2\omega)$ when $\theta < \widetilde{\theta}$, or (ii) $\rho < 1/(2\omega)$ when $\theta > \widetilde{\theta}$.*

Proposition 6 makes it explicit that overshooting is fundamentally related to goods substitution on the one hand, therefore appealing to intratemporal substitution effects, and habit persistence on the other hand, therefore reflecting the fundamental role of intertemporal motives in the determination of exchange rate properties. It may indeed either be obtained for high θ (positive persistence) and high substitution between foreign and domestic goods or low θ (negative persistence) and low elasticity of substitution. Intratemporal motives essentially have to do with the level of the $\mathcal{A}(1)$, and therefore with the long–run effect. Indeed, proposition 5 has established that when ρ is high enough — when goods are enough substitutable — the money shock can yield a long–run appreciation of the domestic currency when individuals trust in money, while we get a short–run depreciation (or small appreciation) on impact. Therefore, the nominal exchange rate overshoots. On the other hand, situations where goods are high substitutable are associated with negative persistence such that changes in the nominal exchange rate are necessarily negative in the second period. This however leaves opened the question of the coincidence between a positive (and higher than 1) $\mathcal{A}(1)$ and overshooting. This is reported in figure 4 which characterizes pairs (θ,ρ) yielding both overshooting and a high value for the long–run effect of money on the nominal exchange rate. It is worth noting that overshooting occurs for a wide range of values for habit persistence and substitutability parameters. For example, in the Backus, Kehoe and Kydland's [1992, 1995] calibration of $\rho = 1.5$, overshooting occurs for θ greater than 0.38, which is in accordance with empirical findings on habit persistence. We also report in figure 5, the overshooting area in the case of a persistent money injection ($\rho_g = 0.5$). At a first glance, it appears that the region of overshooting is narrowed compared to the *iid* case. For instance, when goods are substitutable, the required values of habit persistence to get overshooting are much higher than in the *iid* case. The main reason for this result may be found in the

Figure 4: Zone of overshooting (ρ_g=0)

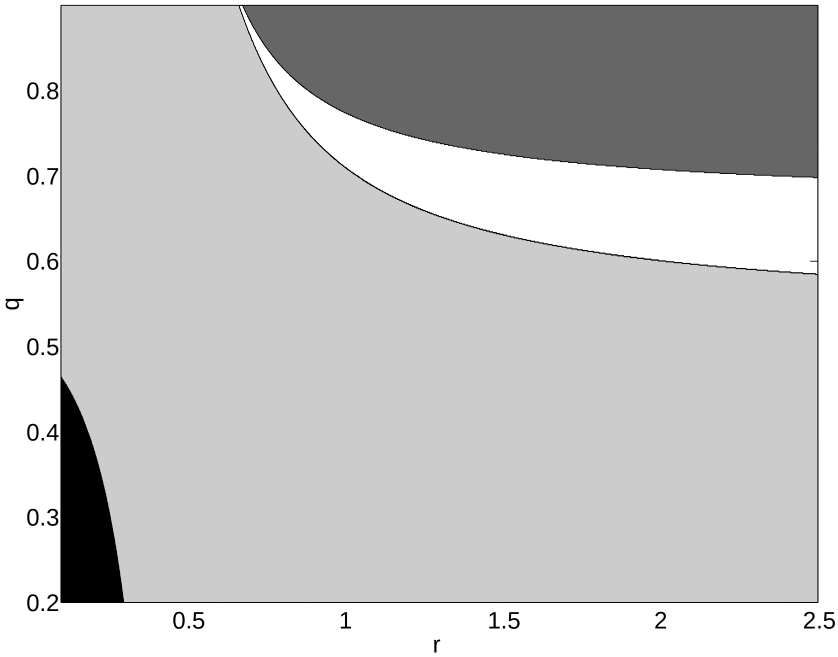


Note: white: Neither overshooting nor $\mathcal{A}(1) > 1$, light gray: No Overshooting, but $\mathcal{A}(1) > 1$, dark gray: overshooting but $\mathcal{A}(1) < 1$, black: both overshooting and $\mathcal{A}(1) > 1$.

greater persistence of the inflation tax. Indeed, persistent inflation leads the household to reduce her consumption along the transition path if habit persistence is not sufficient — the inflation tax dominates intertemporal complementarity in consumption decisions. The demand for goods produced in the rest of the world shifts downward so that the relative demand for foreign assets decreases. Therefore, the depreciation is weakened. Conversely, when habit persistence is large enough with regard to the persistence of the inflation tax, households will be able to maintain their consumption plans and will therefore, among other, maintain their demand for goods produced in the rest of the world. Depreciation of the nominal exchange rate is therefore magnified.

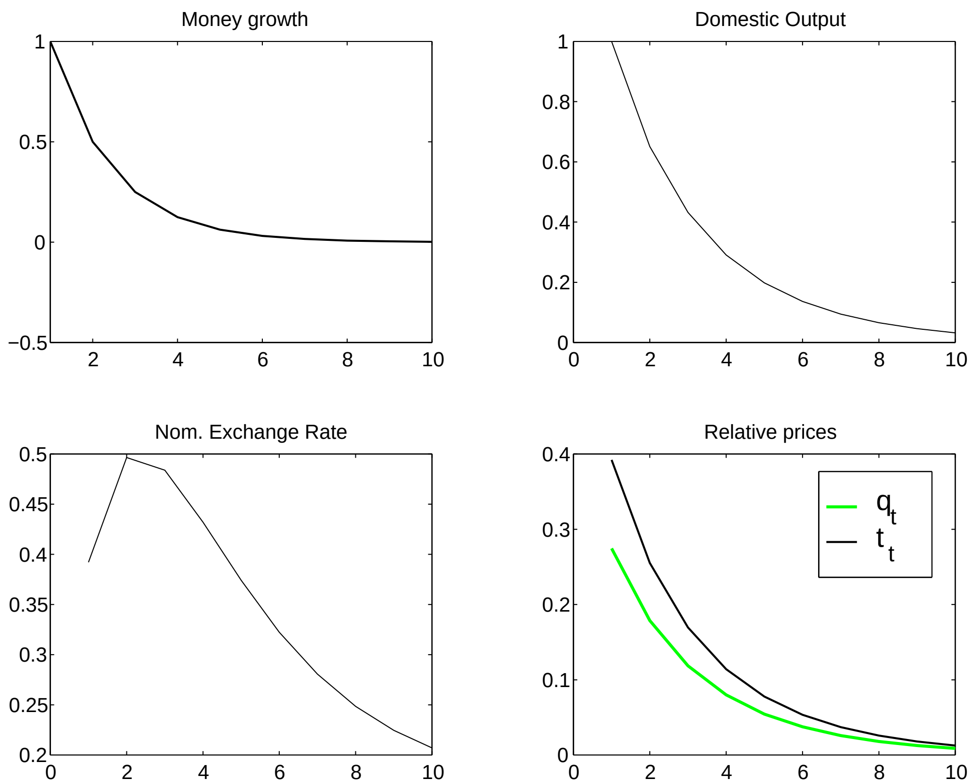
Figure 6 reports IRF when money supply shocks are persistent ($\rho_g = 0.5$). Contrary to the standard cash–in–advance model and the model with no confidence in money, the model is capable of generating both the monetary transmission mechanism and overshooting. Note that, in this case, even if the inflation is persistent, the effect of large habit is sufficient to offset its effect on consumption decision, as output is always above its steady state value. Therefore, households are willing to consume more of each type of good,

Figure 5: Zone of overshooting (ρ_g=0.5)



Note: white: Neither overshooting nor $\mathcal{A}(1) > 1$, light gray: No Overshooting, but $\mathcal{A}(1) > 1$, dark gray: overshooting but $\mathcal{A}(1) < 1$, black: both overshooting and $\mathcal{A}(1) > 1$.

Figure 6: Impulse responses



Note: These IRF are obtained with $\rho_g = 0.5$, $\beta = 0.99$, $\omega = 0.85$, $\rho = 1.5$ and $\theta = 0.75$.

so that the demand for foreign currency rises such that the depreciation is magnified. Overshooting follows.

We finally investigate the volatility implications of the model for the nominal exchange rate. The MA(∞) representation of $\widehat{\Delta e}_t$ is given by

$$\widehat{\Delta e}_t = g_t + \psi \sum_{i=0}^{\infty} \mu^i g_{t-i}$$

such that its volatility writes as

$$\sigma_{\Delta e} = \left(1 + 2\psi + \frac{\psi^2}{1-\mu^2}\right)^{\frac{1}{2}} \sigma_g$$

The model can endogenously amplify the volatility of the money supply shocks when the household trusts in money. This is established by the following proposition.

Proposition 7 *When agents trust in money, $\zeta \equiv \sigma_{\Delta e}/\sigma_g > 1$ if*

(i) domestic and foreign goods are complement enough, $\rho < 1/(2\omega\rho)$, or

(ii) domestic and foreign goods are substitute enough, $\rho < 1/(2\omega\rho)$, and the following inequality holds

$$2 + \psi > 2\mu^2$$

When goods are complement enough *(i)*, this precisely corresponds to the zone where the model generates a long–run effect of money supply shocks $\mathcal{A}(1)$ greater than 1, in the iid case. This suggests that most of the volatility in the nominal exchange rate dynamics may be found in low frequency phenomena rather than high frequency. In other words, the ability of the model to account for low frequency properties of the nominal exchange rate seems to be more important than its ability to generate an overshooting property. When goods are substitute enough, this only occurs for high enough persistence(μ), which corresponds to situations where the model, despite it generates a $\mathcal{A}(1)$ lower than 1, generates overshooting. This then corresponds to a situation where most of the nominal exchange rate volatility is accounted for by high frequency phenomena. Note that in this experiment, we omitted the extrinsic uncertainty steaming from the pure extrinsic belief

ν_t. As soon as ν_t is brought back into the model the volatility of the exchange rate is given by

$$\sigma_{\Delta e} = \left(\left(1 + 2\psi + \frac{\psi^2}{1-\mu^2} \right) \sigma_g^2 + \frac{\psi^2}{1-\mu^2} \sigma_\nu^2 \right)^{\frac{1}{2}}$$

where $\sigma_\nu^2 = E(\nu_t^2)$. Therefore, the model can generate any level of nominal exchange rate volatility when agents have extrinsic beliefs. Note that the same level of volatility may be achieved with a particular indexation level of beliefs to money shocks (b), since

$$\sigma_{\Delta e} = \left(1 + 2b\psi + \frac{b^2\psi^2}{1-\mu^2} \right)^{\frac{1}{2}} \sigma_g$$

Hence for $b \gg 0$, the model can generated high volatility in the nominal exchange rate. Indeed, confidence in money is so high that it induces very strong amplification mechanisms of money shocks in the economy.

4 Concluding remarks

The paper has shown that, introducing time non–separability in consumption decisions, an infinitely–lived agents monetary model with a cash–in–advance constraint may be helpful to understand nominal exchange rate dynamics. We assume that one period lagged consumption produces service flows, that are perfectly internalized by the representative household. We first show — in a simple open economy model where fiat money is used to allocate wealth intertemporally — that high enough habit persistence yields self–fulfilling prophecies. Depending on the form of the beliefs, the model can account for greater volatility and persistence in the exchange rate dynamics. Implicit in this result is that real indeterminacy is not *per se* sufficient to explain exchange rate dynamics. Two conditions have to be full–filled: *(i)* beliefs should matter and *(ii)* beliefs should be positively related to money injection. *(i)* essentially states that real indeterminacy is a necessary condition. *(ii)* states that there must be some confidence in money.

Several issues may be then worth considering. First of all, one may check the robustness of our results against other specifications for the money demand, in particular with money in the utility function. Our intuition is that when money and consumption are gross complement, our results should still hold as a cash–in–advance constraint reveals a strong complementarity between money and consumption. Another route that may be worth pursuing is to provide with a systematic quantitative evaluation of the time series implications of the mechanism we discussed. In particular, it may be interesting to assess the ability of the monetary economy we consider to account quantitatively for observed volatility and persistence in the nominal and real exchange rate dynamics.

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A Equilibrium conditions

Then the deflated equilibrium is characterized the following system of dynamic equations:

$$v'(h_t) = \frac{\beta}{g_t}\frac{p_{h,t}}{p_{t+1}}E_t\left[u'(c_{t+1}-\theta c_t)-\beta\theta u'(c_{t+2}-\theta c_{t+1})\right] \tag{A.1}$$

$$v'(h_t)i_t = \frac{p_{h,t}}{p_t}E_t\left[u'(c_t-\theta c_{t-1})-\beta\theta u'(c_{t+1}-\theta c_t)\right] \tag{A.2}$$

$$h_t = \left(\frac{p_{h,t}}{p_t}\right)^{-\rho}\omega\, c_t + \left(\frac{p_{h,t}}{\Delta e_t p_t^\star}\right)^{-\rho}(1-\omega)\, c^\star \tag{A.3}$$

$$y^\star = \left(\frac{\Delta e_t p_f^\star}{p_t}\right)^{-\rho}(1-\omega)\, c_t + \left(\frac{p_f^\star}{p_t^\star}\right)^{-\rho}\omega\, c^\star \tag{A.4}$$

$$p_t c_t = g_t + (\Delta e_t p_f^\star)^{1-\rho}p_t^\rho(1-\omega)c_t - p_{h,t}^{1-\rho}(\Delta e_t p_t^\star)^\rho(1-\omega)c^\star \tag{A.5}$$

$$p_t = \left[\omega p_{h,t}^{1-\rho} + (1-\omega)(\Delta e_t p_f^\star)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{A.6}$$

$$p_t^\star = \left[(1-\omega)\left(\frac{p_{h,t}}{\Delta e_t}\right)^{1-\rho} + \omega p_f^{\star 1-\rho}\right]^{\frac{1}{1-\rho}} \tag{A.7}$$

B Log–linear representation

the log–linear representation of the above system is

$$0 = \widehat{p}_{h,t} - \widehat{g}_t - \widehat{p}_{t+1} + \frac{1}{(1-\theta)(1-\beta\theta)}E_t\left[\beta\theta\widehat{c}_{t+2} - (1+\beta\theta^2)\widehat{c}_{t+1} + \theta\widehat{c}_t\right] \tag{B.8}$$

$$\widehat{i}_t = \widehat{p}_{h,t} - \widehat{p}_t + \frac{1}{(1-\theta)(1-\beta\theta)}E_t\left[\beta\theta\widehat{c}_{t+1} - (1+\beta\theta^2)\widehat{c}_t + \theta\widehat{c}_{t-1}\right] \tag{B.9}$$

$$\widehat{h}_t = \omega\rho\widehat{p}_t - \rho\widehat{p}_{h,t} + \omega\widehat{c}_t + \rho(1-\omega)(\widehat{p}_t^\star + \widehat{\Delta e}_t) \tag{B.10}$$

$$(1-\omega)\widehat{c}_t - \rho(1-\omega)(\widehat{\Delta e}_t - \widehat{p}_t) + \omega\rho\widehat{p}_t^\star = 0 \tag{B.11}$$

$$\widehat{p}_t + \omega\widehat{c}_t = \widehat{g}_t + (1-\rho)(1-\omega)(\widehat{\Delta e}_t - \widehat{p}_{h,t}) + \rho(1-\omega)(\widehat{p}_t - \widehat{p}_t^\star - \widehat{\Delta e}_t) \tag{B.12}$$

$$\widehat{p}_t = \omega\widehat{p}_{h,t} + (1-\omega)\widehat{\Delta e}_t \tag{B.13}$$

$$\widehat{p}_t^\star = (1-\omega)(\widehat{p}_{h,t} - \widehat{\Delta e}_t) \tag{B.14}$$

Plugging (B.14)and (B.13) into (B.11), we get:

$$\widehat{\Delta e}_t = \widehat{p}_t + \frac{1}{2\rho}\widehat{c}_t \tag{B.15}$$

Then, using the latter result in (B.13), we get

$$\widehat{p}_{h,t} = \widehat{p}_t - \frac{1-\omega}{2\omega\rho}\widehat{c}_t \tag{B.16}$$

Now, note that using (B.13), (B.14) rewrites as

$$\widehat{p}_t^{\star} = \widehat{p}_{h,t} - \widehat{p}_t \tag{B.17}$$

from which we then get

$$\widehat{p}_t^{\star} = -\frac{1-\omega}{2\omega\rho}\widehat{c}_t \tag{B.18}$$

Plugging (B.15), (B.16) and (B.18) into (B.10), we get

$$\widehat{h}_t = \widehat{c}_t \tag{B.19}$$

Likewise, using the same equation in (B.12) we get

$$\widehat{p}_t = \widehat{g}_t - \left(1 - \frac{1-\omega}{2\omega\rho}\right)\widehat{c}_t \tag{B.20}$$

therefore

$$\begin{aligned}
\widehat{\Delta e}_t &= \widehat{g}_t + \frac{1-2\omega\rho}{2\omega\rho}\widehat{c}_t && \text{(B.21)}\\
\widehat{p}_{h,t} &= \widehat{g}_t - \widehat{c}_t && \text{(B.22)}\\
\widehat{p}_t^{\star} &= -\frac{1-\omega}{2\omega\rho}\widehat{c}_t && \text{(B.23)}
\end{aligned}$$

Then, using (B.19), (B.20) and (B.22) into (B.8) we get

$$\begin{aligned}
&E_t\widehat{c}_{t+2} + \frac{\varphi(1-\theta)(1-\beta\theta)-(1+\beta\theta^2)}{\beta\theta}E_t\widehat{c}_{t+1}\\
&+\left(\frac{1}{\beta} - \frac{(1-\theta)(1-\beta\theta)}{\beta\theta}\right)\widehat{c}_t = \frac{(1-\theta)(1-\beta\theta)}{\beta\theta}E_t\widehat{g}_{t+1}
\end{aligned} \tag{B.24}$$

where $\varphi = 1-(1-\omega)/2\omega\rho$. Finally, using (B.20) and (B.22) the nominal gross interest rate is given by

$$\begin{aligned}
\widehat{i}_t &= \frac{(\varphi-1)(1-\theta)(1-\beta\theta)-(1+\beta\theta^2)}{(1-\theta)(1-\beta\theta)}\widehat{c}_t\\
&\quad+\frac{\beta\theta}{(1-\theta)(1-\beta\theta)}E_t\widehat{c}_{t+1} + \frac{\theta}{(1-\theta)(1-\beta\theta)}\widehat{c}_{t-1}
\end{aligned} \tag{B.25}$$

Proof (Proposition 1): See Auray et al. [2000].

$\square$

Proof (Proposition 2): See Auray et al. [2000].

$\square$

Proof (Proposition 3): Recall that in equilibrium, we have

$$\Delta\widehat{e}_t = \widehat{g}_t + \frac{1-2\omega\rho}{2\omega\rho}\widehat{c}_t = \psi_0\widehat{g}_t$$

The nominal exchange rate depreciates following a monetary injection iff $\psi_0 > 0$

$$1 - \frac{\rho_g(1-2\omega\rho)}{2\omega\rho+\rho_g(1-\omega)} > 0$$

which is equivalent to

$$\frac{2\omega\rho+2\omega\rho\rho_g-\rho_g\omega}{2\omega\rho+\rho_g(1-\omega)} > 0$$

which reduces to

$$2\rho\left(1+\rho_g\right) > \rho_g$$

$\square$

Proof (Proposition 4): Recall that overshooting occurs following a monetary injection if the exchange rate is above its steady state level on impact. Since the impact effect of a unitary money supply shock is equal to ψ_0, and the long–run effect of this shock is given by

$$\mathcal{A}(1) = \frac{\psi_0}{1-\rho_g}$$

Overshooting occurs if

$$\psi > \frac{\psi_0}{1-\rho_g}$$

Since we are restricting ourselves to situations where the domestic currency depreciates we necessarily have $\psi_0 > 0$, such that the last inequality holds if and only if $\rho_g < 0$.

$\square$

Proof (Proposition 5): Since $|\mu| < 1$, $1-\mu > 0$ such that $\mathcal{A}(1) > 1$ as soon as $\psi > 0$, or otherwise stated when foreign and domestic goods do not display too much substitutability ($\rho < 1/2\omega$).

$\square$

Proof (Proposition 6): Note that when $\rho_g = 0$, $\gamma = 0$ such that $\widehat{e}_0 - \mathcal{A}(1)$ reduces to

$$\widehat{e}_0 - \mathcal{A}(1) = -\frac{\psi\mu}{1-\mu} \tag{B.26}$$

Two cases are then to be considered

- $\theta < \widetilde{\theta}$, in which case $-1 < \mu < 0$. Therefore, (B.26) reduces to

$$\psi > 0 \Longleftrightarrow \rho < \frac{1}{2\omega}$$

- $\theta > \widetilde{\theta}$, $\mu \in (0,1)$, therefore the nominal exchange rate overshoots if

$$\psi < 0 \iff \rho > \frac{1}{2\omega}$$

$\square$

Proof (Proposition 7): $\zeta > 1$ is equivalent to

$$2\psi + \frac{\psi^2}{1-\mu^2} > 0 \iff \psi\left(2(1-\mu^2)+\psi\right) > 0$$

Two cases are then to be considered:

1. $\psi > 0 \iff \rho < 1/(2\omega)$, in which case the second term of the above inequality is also positive since $|\mu| < 1$.
2. $\psi < 0 \iff \rho > 1/(2\omega)$, in which case the second term of the above inequality has to be negative for the inequality to be satisfied — *i.e.* $2 + \psi > 2\mu^2$.

$\square$