Insiders-Outsiders, Transparency, and the Value of the Ticker *

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Abstract

We consider a multi-period rational expectations model in which informed traders (speculators) differ in their access to information on past transaction prices (the ticker). Insiders observe the entire price history before trading whereas outsiders observe past prices with a delay. We show that the fee that an outsider is willing to pay to become an insider decreases with the number of insiders and increases with latency in information dissemination. Moreover, speculators' welfare decreases with the number of insiders. Thus, both speculators and data vendors can benefit from restrictions on access to information on past trades. Last, we find that an increase in latency impairs market liquidity but has no effect on the informativeness of the ticker.

Keywords: Market Data Sales, Latency, Transparency, Price Discovery, Liquidity.

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1 Introduction

Information about trades and quotes is not equally distributed among traders. For instance, in an open-outcry market, floor traders enjoy a faster access to information on ongoing trades than traders outside the floor. More generally, in many markets real time information on transactions and quotes is not free.[1] De facto, there is a segmentation between investors ("insiders") who, at a cost, have access to real time information, and investors ("outsiders") who obtain this information with a delay. This situation is controversial. For instance, NYSE's recent proposal to charge a fee for the dissemination of real time information on quotes and trades in Archipelago (a trading platform acquired by the NYSE in 2006) triggered a strong opposition from some market participants.[2] Similarly, data fees charged by Nasdaq for the dissemination of prices in the U.S. corporate bond market have been the subject of heated debates.[3]

These observations raise intriguing questions. Why do exchanges create unleveled access to real time market data? How do latencies in the dissemination of trade information affect trading strategies, price discovery and liquidity?

We study these questions in a multi-period rational expectations model (in the spirit of Hellwig (1980) and Vives (1995). The model considers the market for a risky security with risk averse speculators who possess private, but imperfect, information on its payoff, and liquidity traders. Speculators submit price contingent orders (limit orders), and effectively play the role of market-makers. They can be "insiders" or "outsiders." Insiders observe the entire price history (the "ticker") when they arrive in the market, while outsiders observe prices with a delay.

In this setting, transaction prices are informative about the liquidation value of the security and each transaction price brings "fresh" information to the market. Thus, observing transaction prices in real time is more informative than just observing lagged transaction prices. Accordingly, insiders have a more precise estimate of the liquidation value. Thus, other things equal, they bear less risk and enjoy a higher

[1]Free information on past trades can be obtained only after some delay (e.g., twenty minutes on the NYSE, fifteen minutes on Nasdaq and Euronext). For an exhaustive list of the delay with which information on transaction prices can be obtained for free from yahoo.com for major stock exchanges and U.S derivatives markets, see http://finance.yahoo.com/exchanges.

[2]See "Latest Market Data Dispute Over NYSE's Plan to Charge for Depth-of-Book Data Pits NSX Against Other U.S. Exchanges," Wall Street Technology, May 21, 2007. See also the letter to the SEC of the Securities Industry and Financial Markets Association (SIFMA) available at http://www.sifma.org/regulatory/comment_letters/41907041.pdf

[3]See, for instance, "Trace Market Data Fees go to SEC," Securities Industry News, 6/3/2002.

expected utility.

The value of the real time ticker is the maximum fee that an outsider is willing to pay to become an insider. We show that this value declines with the proportion of insiders. This finding follows from two observations. First, the clearing price in a given period aggregates the information inferred by insiders from prices yet unobserved by outsiders. This information is more efficiently aggregated when the number of insiders increases, which reduces insiders' informational advantage. Second, surprisingly, the informativeness of the real time ticker does not increase with the proportion of insiders. Thus, the net utility gain of being an insider is unambiguously smaller when the proportion of insiders increases.

We also find that the value of the ticker increases with the latency in information dissemination (i.e., the delay with which outsiders observe past transaction prices). Indeed, the larger is this delay, the noisier is the information provided by the current clearing price about the information contained in transaction prices yet unobserved by outsiders. Thus, a larger latency increases the net benefit of being an insider.

We also identify an externality associated with the acquisition of ticker information by speculators. Namely, an increase in the proportion of insiders lower all speculators' welfare. Actually, insiders take a larger long or short position than outsiders for a given expected return on this position. Hence, when the proportion of insiders increases, speculators are on average more responsive to differences between their forecasts of the value of the security and the clearing price. As a consequence, in each trading round, equilibrium prices are closer to the liquidation value of the security when the proportion of insiders enlarges. But precisely for this reason, both insiders and outsiders obtain, on average, smaller returns on their positions. In other words, widening the dissemination of real time information exacerbates competition among speculators, which make them worse off.

Last, we study how market liquidity depends on latencies in information dissemination. We find that an increase in latency decreases liquidity, except if all speculators observe the ticker in real time. Indeed, an increase in latency implies that outsiders draw less precise inferences from the prices that they observe. Accordingly, they bear more risk and take smaller positions when latency in access to price information increases. Thus, prices are more responsive to order imbalances, i.e., market liquidity is smaller. In line with this implication, Easley, Hendershott, and Ramadorai (2007) find an increase in liquidity for stocks listed on the NYSE following a reduction in latency.

Exchanges derive a significant fraction of their revenues from the sale of trade information. For instance, the sale of market information represents 33% of its annual revenues for the London Stock Exchange, 21% for the Nasdaq and 17% for the NYSE (source: Annual Reports). Hence, they care about the pricing of the ticker.[4] Our findings imply that rationing access to real time ticker information and delaying the dissemination of free ticker information are ways to increase the value of the real time ticker. Thus, exchanges' self interest may not naturally lead to more transparent and more liquid markets. In this way, our analysis contributes on the literature on market transparency.[5]

Our model also contributes to the literature on markets for information (e.g., Admati and Pfleiderer (1986, 1987, 1990), Garcia and Vanden (2005), Veldkamp (2007), Cespa (2007)). In particular, Admati and Pfleiderer (1986) study the sale of financial information, i.e., a signal on the payoff of a risky asset. They show that there is a dilution in the value of this signal due to its leakage through prices and that the information seller can control this dilution by restricting the number of buyers.

As prices aggregate information, they constitute one type of signals on a security's payoff. However, the sale of price information does not fit the canonical framework for the sale of financial information for several reasons. If prices become more informative as more traders buy price information then they are more attractive. Thus, the value of market data may well increase with the number of buyers. Moreover, the precision of prices cannot be directly controlled by the information seller as prices are determined by market forces. Thus, the sale of price information deserves a specific analysis.

There are very few papers on this question. Mulherin et al.(1992) and Pirrong (2002) focus on the allocation of property rights on stock prices. Our analysis is closer to Boulatov and Dierker (2007). However, they do not allow traders to condition their demand on the current clearing price as we do.[6] Thus, in their model, observing

[4] For 2003, the sale of market data generated a revenue of \$386 million for U.S. equity markets for a cost of dissemination estimated at \$38 million (see Exchange Act Rel N°$49,325$ -February, 26,2004 available at http://www.sec.gov/rules/proposed/34-49325.htm).

[5] There is a large literature on the effect of transparency in financial markets (see Biais (1993), Madhavan (1995,1996), Pagano and Roëll (1996)). Most related to our paper are studies of the effect of delays in trade reporting (see Madhavan (1995) and Naik, Neuberger and Viswananathan (1999)). Typically, delays in trade reporting imply that parties in a transaction are more informed than the rest of the market on the details of this transaction and can then exploit this information. However, to our knowledge, this literature has not analyzed the value of getting access to real time information, as we do here.

[6] For this reason, our paper also differs from Hellwig (1982) who considers a multi-period rational

past prices is valuable because it reduces the uncertainty on traders' execution price ("execution risk"). In our model, execution risk is not a concern since traders submit price contingent orders. Rather, the value of price information derives from the fact that prices contain information on future payoffs.[7]

The paper is organized as follows. We describe the model in the next section. We analyze the effect of a change in the proportion of insiders on speculators' welfare and the value of ticker information in Section 3. In Sections 4, we study the effect of an increase in the latency in information dissemination. Section 5 consider additional implications of the model and Section 6 concludes.

2 Model

We consider the market for a risky asset with liquidation value $v \sim N(\bar{v}, \tau_v^{-1})$. The liquidation value is realized at date $N+1$. Trades in this market take place at dates $1, 2, ...N$ between two types of traders: (i) a continuum of speculators (indexed by i) who submit demand functions ("limit orders") and (ii) liquidity traders with inelastic demands ("market orders"). We denote by u_n the aggregate demand of liquidity traders at date n. Liquidity demands are independently and normally distributed with mean zero and precision τ_u^{-1}. Speculators do not observe order imbalances.

Speculator i arriving at date n receives a private signal s_{in} about the value of the security with

$$s_{in} = v + \epsilon_{in}, \tag{1}$$

where $\epsilon_{in} \sim N(0, \tau_{\epsilon_n}^{-1})$. We assume that v and ϵ_{in} are independent for all i, n and error terms are also independent across time and across agents. Furthermore, given v, the average signal $\int_0^1 s_{in} di$ equals v almost surely in every period n (i.e., errors cancel out in the aggregate: $\int_0^1 \epsilon_{in} di = 0$). The model does not require speculators to be informed at each date (i.e., $\tau_{\epsilon_n} = 0$ is possible). However, to fix things, we assume that $\tau_{\epsilon_1} > 0$.

We denote by p_n the clearing price at date n and by p^n the record of all transaction prices up to date n: $p^n = \{p_t\}_{t=0}^{t=n}$ with $p_0 = \overline{v}$. Speculators differ in their access to

expectations model in which some traders condition their demand on past prices, only.

[7] There are other important differences between our approach and Boulatov and Dierker (2007). First they use a reduced form approach to model the idea that observing past prices. Second, in our model traders are homogeneous (same preferences, same precision of private information). Thus, other things equal, they have the same valuation for ticker information. Yet, even in this setting, we show that restricting access to real time information can enhance revenues from the sale of real time information.

ticker information. Specifically, speculators with type I (the *insiders*) observe the ticker in real-time while speculators with type O (the *outsiders*) observe the ticker with a lag equal to $l \geq 2$ periods. That is, insiders arriving at date n observe p^{n-1} before submitting their orders and outsiders arriving at date n observe p^{n-l}. Formally

$$p^{n-l} = \begin{cases} \{p_1, p_2, ..., p_{n-l}\} \text{ if } n > l \\ \overline{v} \text{ if } n \leq l \end{cases}$$

We refer to p^n as the "*real time ticker*" and to p^{n-l} as the "*lagged ticker*". Finally, we refer to $\{p^{n-l+1}, p^{n-l+2}, ..,\}$ as the "*delayed ticker*". The delayed ticker is the set of prices unobserved by outsiders. We refer to l as the *latency in information dissemination*. Insiders represents a fraction μ of all speculators.

Each speculator has a CARA utility function with risk tolerance γ. Thus, if speculator i holds x_{in} shares of the risky security at date n, her expected utility is

$$E[U(\pi_{in})|s_{in}, \Omega_n^k] = E[-\exp\{-\gamma^{-1}\pi_{in}\}|s_{in}, \Omega_n^k],$$

where $\pi_{in} = (v - p_n)x_{in}$ and Ω_n^k is the price information available at date n to a speculator with type $k \in \{I, O\}$.

In period n, insiders and outsiders can submit orders contingent on the price at date n and their information. Insiders however observe the ticker up to date $n-1$ while outsiders observe the ticker up to date $n-l$ only. Thus, in period $n \geq 2$, we denote the demand function of an insider by $x_n^I(s_{in}, p^n)$ and that of an outsider by $x_n^O(s_{in}, p^{n-l}, p_n)$. In the first period, we denote the demand function of speculator i by $x_1(s_{i1}, p_1)$. In each period, the clearing price is such that the net demand for the security is nil, i.e.

$$\int_0^{\mu} x_{in}^I di + \int_{\mu}^1 x_{in}^O di + u_n = 0. \tag{2}$$

Note that speculators can be viewed as market-makers since, eventually, they absorb liquidity traders' net order imbalance.

Parameters μ and l control the level of market transparency. When the proportion of insiders increases, market transparency is larger since more speculators observe the ticker in real time. When latency decreases (l becomes smaller), market transparency increases since outsiders observe past transaction prices more quickly. To isolate the effect of the proportion of insiders on the value of the ticker, we first focus on the case in which $l = 2$ (insiders observe all transaction prices but the last). Then, in Section 4.1, we analyze the effect of increasing latency in information dissemination on the value of the ticker.

Our framework is similar to Vives (1995) but there are some important differences. First, in line with sequential trade models (e.g., Glosten and Milgrom (1985)), speculators can trade only in the period in which they arrive (re-trading is not allowed). More importantly for our purposes, we consider the possibility of unequal access to ticker information among speculators (Vives (1995) focuses on the case $\mu = 1$). Moreover, there are no risk neutral market-makers in our set-up. In this way, in contrast to Vives (1995), the clearing price in each period is not a sufficient statistics for past prices.[8] For this reason, even though traders submit limit orders, observing past prices in real time is valuable (see below).

3 The value of the ticker and the proportion of insiders

3.1 Equilibrium

As speculators are informed, the clearing price in each period reflects speculators' information. Speculators take into account this information in formulating their demands (as in Grossman (1976) or Hellwig (1980)). As usual in the literature, we focus on rational expectations equilibria in which speculators' order placement strategies are linear in their signals and prices.

The next proposition provides a characterization of the unique linear rational expectations equilibrium of the model. We refer to $\tau_n \overset{def}{=} (\mathrm{Var}[v|p^n])^{-1}$ as the informativeness of the real time ticker at date n and we denote the precision of outsiders' forecast at date n by $\hat{\tau}_n \overset{def}{=} (\mathrm{Var}[v|p^{n-2}, p_n])^{-1}$.

Proposition 1 *When $l = 2$, in each period, there is a unique rational expectations equilibrium. In this equilibrium, the price in each period is*

$$p_n = A_n v + B_n u_n + C_n u_{n-1} + D_n E(v \mid p^{n-2}), \text{ for } n \geq 2 \tag{3}$$

$$p_1 = A_1 v + B_1 u_1 + D_1 \overline{v} \tag{4}$$

where $\{A_n, B_n, C_n, D_n\}$ are constants characterized in the proof of the proposition with $D_n = 1 - A_n$. Moreover, $A_n > 0$ iff (a) $\tau_{\epsilon_n} > 0$ or (b) $\tau_{\epsilon_{n-1}} > 0$ and $\mu > 0$. In

[8] This property is not specific to our model. See for instance Brown and Jennings (1989).

this equilibrium speculators' trading strategies in period n are

$$\begin{aligned} x_1(s_{i1}, p_1) &= \gamma(\tau_1 + \tau_{\epsilon_1})(E[v|s_{i1}, p_1] - p_1), \\ x_n^I(s_{in}, p^n) &= \gamma(\tau_n + \tau_{\epsilon_n})(E[v|s_{in}, p^n] - p_n), \\ x_n^O(s_{in}, p^{n-2}, p_n) &= \gamma(\widehat{\tau}_n + \tau_{\epsilon_n})(E[v|s_{in}, p^{n-2}, p_n] - p_n), \end{aligned} \tag{5}$$

where $\tau_n = \tau_v + \tau_u \sum_{t=1}^{n} a_t^2$ *with* $a_n = \gamma\tau_{\epsilon_n}$.

In order to gain intuition on the equilibrium, it is useful to consider some special cases.

Case 1. No "fresh" information is available at date $n-1$ and n (for $n \geq 3$). That is $\tau_{\epsilon_n} = 0$ and $\tau_{\epsilon_{n}-1} = 0$. In this case, $A_n = 0$, $C_n = 0$ and $B_n = (\gamma\tau_{n-2})^{-1}$. Thus, the equilibrium price at date n can be written

$$p_n = E(v \mid p^{n-2}) + (\gamma\tau_{n-2})^{-1}u_n$$

In this case, the price at date n is equal to the expected value of the security conditional on the lagged ticker adjusted for the compensation required by speculators to accommodate liquidity traders' order imbalance. For a given order imbalance, the size of this compensation is smaller when (i) speculators are more risk tolerant (γ large) or (ii) the uncertainty on the asset value is smaller (τ_{n-2} large).■

Case 2. "Fresh" information is available at date $n-1$ but not at date n. That is $\tau_{\epsilon_n} = 0$ but $\tau_{\epsilon_{n}-1} > 0$. If there are no insiders in the market ($\mu = 0$) then the expression for the equilibrium price is unchanged. Indeed, the last transaction price contains information but no speculator observes this price.

The situation is different if $\mu > 0$. Intuitively, insiders at date n can extract a noisy signal on the liquidation value from the price realized at date $n-1$. As they trade on this signal, part of the information contained in the $(n-1)^{th}$ transaction price transpires in the price at date n. In fact, in this case, $A_n > 0$ and $C_n > 0$ and the equilibrium price at date n can be written

$$p_n = E(v \mid p^{n-2}) + A_n(z_{n-1} - E(z_{n-1} \mid p^{n-2})) + B_n u_n$$

where $z_{n-1} = v + \frac{C_n}{A_n}u_{n-1}$ and $\frac{C_n}{A_n} = (\gamma\tau_{\epsilon_{n-1}})^{-1}$. Thus, the price at date n is equal to the expected liquidation value of the security conditional on speculators' common information (the lagged ticker) plus an innovation that corresponds to the signal inferred from the $(n-1)^{th}$ transaction price by insiders, i.e., z_{n-1}.

Outsiders do not directly observe this signal. However, they can extract from the clearing price at date n, a signal $\widehat{z}_n$ such that

$$\widehat{z}_n = v + A_n^{-1}(B_n u_n + C_n u_{n-1}) = z_{n-1} + (A_n^{-1}B_n)u_n.$$

Thus, outsiders obtain new information from the clearing price at date n, beyond and above that in the lagged ticker. However, the signal they extract from the clearing price is noisier than that possessed by outsiders because the current clearing price depends both on (i) the innovation in insiders' expectations and (ii) liquidity traders' order imbalance at date n.$\blacksquare$

In the general case, fresh information is available at date n and $n-1$. Thus, the price at date n contains information on the liquidation value ($A_n > 0$) because (i) speculators trade on new signals received at date n and (ii) insiders trade on the information contained in the price at date $n-1$. Outsiders obtain from the clearing price the signal $\widehat{z}_n$, which is noisier than that extracted from the clearing price by insiders.

We denote the informativeness of this signal by $\tau_n^m = (\text{Var}[v|\widehat{z}_n])^{-1}$. Intuitively, τ_n^m is the contribution of the n^{th} clearing price to the precision of outsiders' forecast of the liquidation value, i.e., $\hat{\tau}_n$, as shown in the next corollary. With a slight abuse of language, we refer to τ_n^m as the informativeness of the n^{th} clearing price.[9]

Corollary 1 *At date n, the precision of outsiders' estimate of the liquidation value conditional on their price information, $\hat{\tau}_n$, is*

$$\hat{\tau}_n = (\text{Var}[v|p^{n-2}, p_n])^{-1} = \tau_{n-2} + \tau_n^m.$$

1. *It increases with the proportion of insiders iff $\tau_{\epsilon_{n-1}} > 0$;*

2. *It is smaller than the precision of the estimate of the liquidation value conditional on the real time ticker, i.e., $\hat{\tau}_n \leq \tau_n$. This inequality is strict iff $\tau_{\epsilon_{n-1}} > 0$.*

As explained previously (case 2), insiders have an information advantage over outsiders iff the delayed ticker contains fresh information, i.e., iff $\tau_{\epsilon_{n-1}} > 0$. This observation explains why outsiders' forecast of the liquidation value is strictly less precise than insiders' forecast when $\hat{\tau}_n < \tau_n$. As insiders trade on the information in the delayed ticker, they incorporate this information in the clearing price at date n.

[9] This is the informativeness of the n^{th} clearing price from the point of view of insiders after accounting for the information contained in the lagged ticker.

Naturally, the price at date n aggregates better this information as more speculators trade on it. For this reason, the contribution of the n^{th} clearing price to the precision of outsiders' forecast, τ_n^m, enlarges in the proportion of insiders. This yields the first part of the corollary.

Intuitively, an increase in τ_n^m reduces the informational advantage of the insiders, i.e., the difference $(\tau_n - \hat{\tau}_n)$. In fact

$$\begin{aligned}\tau_n - \hat{\tau}_n &= (\tau_n - \tau_{n-2}) - (\hat{\tau}_n - \tau_{n-2}) \\ &= ((\gamma\tau_{\epsilon_{n-1}})^2 + (\gamma\tau_{\epsilon_n})^2)\tau_u - \tau_n^m.\end{aligned}$$

Thus, an increase in the proportion of insiders reduces insiders' informational advantage because (i) it increases the informational contribution of the n^{th} clearing price to the precision of outsiders' forecast and (ii) it leaves unchanged the difference between the informativeness of the ticker at date n and its informativeness at date $n-2$. More generally, the informativeness of the ticker at each date τ_n does not depend on the proportion of insiders, as claimed in the next corollary.

Corollary 2 *The informativeness of the real time ticker at any date, τ_n, does not depend on the proportion of insiders. Thus, the informational advantage of insiders, $\tau_n - \hat{\tau}_n$, decreases with the proportion of insiders.*

Thus, a broader dissemination of the real time ticker (μ increases) has no impact on its informativeness. At first glance, this finding is very surprising. Actually, a change in the proportion of insiders affects the informativeness of a "truncated" record of prices. Indeed we have observed that the informativeness of the n^{th} clearing price for an outsider increases in the proportion of insiders. In this condition, one could expect the informativeness of the whole record of prices to increase in the proportion of insiders. But this is not the case.

The explanation of this paradoxical findings is as follows. The contribution of the fresh information available in a given period (i.e., $\int s_{in} di$) to the price in this period increases when speculators' demand is more sensitive to their private signal, s_{in}. This sensitivity is $a_n = \gamma\tau_{\epsilon_n}$, which is independent of μ (See the proof of Proposition 1). Thus, an increase in the proportion of insiders does not change the weight of fresh information incorporated in the price at each date. In contrast, the weight on the information contained in the delayed ticker is increasing in the proportion of insiders. However, this information is useless for insiders since they directly observe the delayed ticker. Thus, the informativeness of the signals that they extract from prices only depend on the weight of fresh information in each price. As this weight is independent from the proportion of insiders, the informativeness of the ticker does not depend on the scope of dissemination of ticker information.

3.2 The ticker externality

Insiders expect larger gains from participating to the market than outsiders because they have a more precise estimate of the liquidation value. To see this, let C^I and C^O be insiders' and outsiders' participation cost, respectively. At the end of the proof of Proposition 1, we show that the ex-ante expected utilities for speculators entering the market at date n are[10]

$$E\left[U\left(\pi^I_{in} - C^I\right)\right] = -\left(\frac{\mathrm{Var}[v|s_{in}, p^n])}{\mathrm{Var}[v - p_n]}\right)^{1/2} \exp\left\{C^I/\gamma\right\}, \tag{6}$$

$$E\left[U\left(\pi^O_{in} - C^O\right)\right] = -\left(\frac{\mathrm{Var}[v|s_{in}, p^{n-2}, p_n])}{\mathrm{Var}[v - p_n]}\right)^{1/2} \exp\left\{C^O/\gamma\right\}. \tag{7}$$

In equilibrium, we have

$$(\mathrm{Var}[v|s_{in}, p^{n-2}, p_n])^{-1} = \tau_{\epsilon_n} + \hat{\tau}_n \leq \mathrm{Var}[v|s_{in}, p^n])^{-1} = \tau_{\epsilon_n} + \tau_n,$$

and the inequality is strict iff $\tau_{\epsilon_{n-1}} > 0$. Hence, we obtain the following result.

Proposition 2 *Suppose $C^I = C^O$. At each date $n \geq 2$, insiders' ex-ante expected utility is strictly larger than outsiders' expected utility iff $\tau_{\epsilon_{n-1}} > 0$,.*

Thus, timely access to ticker information is valuable. This observation, however, does not imply that *collectively* speculators benefit from a wider dissemination of the real time ticker, i.e., that speculators' welfare increases when μ increases. In fact, the opposite is true. We now examine this point.

As insiders have a more precise estimate of the liquidation value of the security, they bear less liquidation risk. Consequently, their demand is more responsive than outsiders' demand to deviations between their estimate of the fundamental value conditional on prices and the current clearing price (the "risk premium"). Indeed,

$$\frac{\partial x^I_n}{\partial(E[v|s_{in}, p^n] - p_n)} = \gamma(\tau_n + \tau_{\epsilon_n}) > \frac{\partial x^O_n}{\partial(E[v|s_{in}, p^{n-2}, p_n] - p_n)} = \gamma(\hat{\tau}_n + \tau_{\epsilon_n}),$$

[10] These expressions derive from Admati and Pfleiderer (1987), Proposition 3.1.

when $\tau_{\epsilon_{n-1}} > 0$. Thus, an increase in the proportion of insiders makes speculators' aggregate demand function more responsive to a change in the risk premium. There are three reasons for this. First, such an increase shifts speculators from the population with a relatively low responsiveness to the population with a relatively high responsiveness to a change in the risk premium. Second, an increase in the proportion of insiders increases the precision of outsiders' estimate at date n, $\hat{\tau}_n$. Hence, outsiders take a larger position for a given risk premium. Last, insiders' willingness to bear risk is unchanged since the informativeness of the real time ticker (τ_n) does not depend on the proportion of insiders.

In sum, an increase in the proportion of insiders intensifies competition among speculators, which intuitively should narrow the difference between the clearing price and the liquidation value of the security. To formalize this intuition, we measure the distance between the clearing price in the second period and the liquidation value of the security by $\text{Var}[v - p_n]$.

Corollary 3 *In equilibrium,* $\text{Var}[v - p_n]$ *decreases with the proportion of insiders at each date* $n \geq 2$.

Armed with this result, we can analyze the effect of a change in the proportion of insiders on speculators' expected utilities at date $n \geq 2$. First, as the proportion of insiders increases, competition among speculators intensifies and the n^{th} clearing price is closer to the payoff of the security ($\text{Var}[v - p_n]$ decreases). As shown by equations (6) and (7), this effect has a negative impact on all speculators' ex-ante expected utility. Second, an increase in the proportion of insiders increases the informativeness of the n^{th} clearing price. This effect however does not affect the precision of insiders' estimate of the final value ($(\text{Var}[v|s_{in}, p^n])^{-1} = \tau_{\epsilon_n} + \tau_n$ is independent from μ, see corollary 2). Thus, insiders' welfare declines with the proportion of insiders. In contrast, the improvement in the informativeness of the n^{th} clearing price enables outsiders to draw more precise inferences (since $(\text{Var}[v|s_{in}, p^{n-2}, p_n]) = \tau_{\epsilon_n} + \hat{\tau}_n$ increases in μ). Thus, they can appropriate a larger share of the gains associated with market-making. This effect counterbalances the negative impact of the increase in competition on their expected utility. Yet, it does not outweigh it as shown by the following proposition.

Proposition 3 *In equilibrium, speculators' ex-ante expected utilities at date* $n \geq 2$ *decline with the proportion of insiders.*

Thus, acquisition of ticker information by one speculator exerts a *negative externality* on other speculators because it intensifies competition among speculators. Thus, dealers quote prices closer to the true value in the second period, implying that both insiders and outsiders earn a smaller return on their positions. Thus, speculators' welfare is *maximal* when the market is *fully opaque*.

This finding has an intriguing implication: collectively speculators would like to commit *not* to use ticker information. However, if ticker information is available, this commitment is *not* self enforcing because any speculator obtains a higher expected utility when she uses this information (*ceteris paribus*, $E[U(\pi_{i2}^I - C^I)] > E[U(\pi_{i2}^O - C^O)])$. There are two ways to enforce this commitment. On the one hand, trading can be organized in such a way that it is costly for speculators to observe transactions in real time, either because they need to use an agent to monitor transactions (as in the case of trading firms with agents on the floor) or they need to search for information (as in OTC markets). On the other hand, the exchange can charge a fee for ticker information (so that $C^I > C^O$). Obviously, if the fee is large enough, no speculator buys information and speculators' preferred outcome would obtain.

But why would a profit-maximizing exchange choose to restrict the dissemination of ticker information in the first place? We now turn to this question by analyzing the effect of the proportion of insiders on the value of the ticker.

3.3 The value of the ticker

Let $\phi_n(\mu)$ be the maximum fee that a speculator entering the market at date n is willing to pay to observe the real time ticker. We assume that otherwise participation costs are identical for insiders and outsiders, so that

$$C^I = C^O + \phi_n(\mu).$$

We call $\phi_n(\mu)$ the value of the ticker at date n. By definition, it solves[11]

$$E\left[U\left(\pi_{in}^I - (C^O + \phi_n(\mu))\right)\right] = E\left[U\left(\pi_{in}^O - C^O\right)\right].$$

Using equations (6) and (7) and solving the last equation for $\phi_n(\mu)$, we obtain

$$\phi_n(\mu) = \frac{\gamma}{2}\ln\left(\frac{\tau_{\epsilon_n} + \tau_n}{\tau_{\epsilon_n} + \hat{\tau}_n}\right). \tag{8}$$

[11] We assume that C^O is small enough so that outsiders are better off paying the participation cost.

Thus, as expected, the value of the real time ticker at date n is strictly positive iff $\tau_n > \hat{\tau}_n$. This condition is satisfied iff $\tau_{\epsilon_{n-1}} > 0$ (Corollary 1). In line with intuition, speculators are willing to pay for the real time ticker iff the delayed ticker provides "fresh" information relative to the lagged ticker.

Equation (8) can be written

$$\phi_n(\mu) = \frac{\gamma}{2} \ln \left(1 + \frac{\tau_n - \hat{\tau}_n}{\tau_{\epsilon_n} + \hat{\tau}_n}\right). \tag{9}$$

Hence, the value of the ticker increases with insiders' informational advantage relative to outsiders, i.e., $\frac{\tau_n - \hat{\tau}_n}{\tau_{\epsilon_n} + \hat{\tau}_n}$. As explained previously, this relative advantage declines when the proportion of insiders increases. Thus, we obtain the following result.

Proposition 4 *The value of the ticker at date $n \geq 2$ decreases with the proportion of insiders.*

Key to this finding is the fact that an in the proportion of insiders does not affect the informativeness of the delayed ticker while it increases the precision of the signal conveyed by the n^{th} clearing price. Thus, an increase in the proportion of insiders reduces the net benefit of getting real time ticker information at any point in time.

The last result implies that rationing access to ticker information is a way for an exchange to increase the price of the ticker and its revenues. To see this point, assume that the cost of disseminating information does not depend on the proportion of insiders (we set it equal to zero) and that $N = 3$. The profit that the exchange derives from the sale of ticker information is[12]

$$\Pi(\mu) = \mu \phi_2(\mu) \tag{10}$$

An interesting question is whether the exchange has an incentive to create unequal access to price information. That is to choose μ so that $\mu < 1$ (not all traders are insiders) and $\mu > 0$ (not all traders are outsiders). The answer to this question is positive at least for some parameter values, as shown by Figure 1.

[Figure 1 about here.]

[12] We assume that ex-ante speculators know whether they will arrive at date 1 or 2. The conclusions are robust when the date at which a given cohort of speculators arrives is chosen randomly since $\phi_n(\mu)$ decreases with μ.

Figure 1, panel (a), shows the exchange profit as a function of μ for specific parameter values (namely $\tau_v = 1$, $\tau_{\epsilon_1} = 0.1$, $\tau_{\epsilon_2} = 0.2$, $\tau_u = 5$ and $\gamma = 1$) when the exchange does not charge an entry fee. In this case the exchange profit peaks for a value of μ that is strictly less than one ($\mu^* = 0.89$).

Of course in reality exchanges get revenues from other sources. In particular, they obtain revenues from trading fees and listing fees. Revenues from listing fees are not likely to depend on the proportion of insiders. In contrast revenues from trading fees could depend on the proportion of insiders influence the trading volume. Introducing trading fees in the analysis is difficult, however. The exchange can also charge an entry fee to recover part of the rents earned by speculators. Let $C^O(\mu)$ be the fee paid by all speculators. This entry fee is paid by all speculators and is chosen to make outsiders indifferent between trading or not, i.e., it solves

$$-\left(\frac{\mathrm{Var}[v|s_{i2}, p_2])}{\mathrm{Var}[v - p_2]}\right)^{1/2} \exp\left\{C^O/\gamma\right\} = -1$$

The entry fee $C^O(\mu)$ decreases with μ since speculators' welfare decreases with μ. The exchange's profit is then

$$\Pi(\mu) = \mu\phi_2(\mu) + C^O(\mu). \tag{11}$$

The exchange charges a fee $\phi_2(\mu)$ for ticker information and an entry fee $C^O(\mu)$ to recover part of the rents earned by speculators.

Figure 1, panel (b), considers the same parameter values as in panel (a) when the exchange charge an entry fee equal to $C^O(\mu)$. As it can be seen on the figure the expected profit peaks at $\mu^{**} = 0.04$. The exchange optimally rations even more access to ticker information because the entry fee declines with the proportion of insiders. Yet, the optimal value of μ remains strictly positive. Thus, even when an exchange can charge an entry fee, it may want to create unequal access to price information. Intuitively, this "divide and conquer" strategy enables the exchange to exploit the fact that ticker information has value and can thereby generate extra revenues.[13]

For the discussion, we assumed that the exchange can directly choose the proportion of insiders. Observe however that since ϕ and C^O are strictly decreasing in μ, the exchange can implement a specific value of μ by setting the corresponding values for ϕ and C^O. Thus, the choice of a fee for ticker information is equivalent to the choice of a level for transparency.

[13] Extensive simulations reveal that we do not obtain a corner solution ($\mu = 0$ or $\mu = 1$) for large values of traders' signal precision, low dispersion of noise traders' demand, and high levels of risk tolerance.

4 Latency

4.1 Latency and the value of the ticker

Until this point, we have assumed that outsiders observe past transaction prices with a delay of one period (in transaction time). We now consider the effect of increasing this delay on the value of the ticker and market liquidity. Namely, we assume that at date n, outsiders observe transaction prices with a lag of $l \geq 2$ period. Hence at date, $n > l$, outsiders observe $\{p_1, p_2, ..., p_{n-l}\}$ and at date $n \leq l$ outsiders have no information on past transaction prices. We denote the precision of outsiders' forecast at date n by $\hat{\tau}_n(l) \stackrel{def}{=} (\text{Var}[v|p^{n-l}, p_n])^{-1}$. In this case, we can generalize Proposition 1 as follows.

Proposition 5 *When $l \geq 2$, in each period, there is a unique rational expectations equilibrium. In this equilibrium, the price in each period is*

$$p_n = A_n(l)v + \sum_{j=0}^{j=l-1} B_j^n(l)u_{n-j} + D_n(l)E(v \mid p^{n-l}) \text{ for } n \geq l \tag{12}$$

$$p_n = A_n(l)v + \sum_{j=0}^{j=n-1} B_j^n(l)u_{n-j} + D_n(l)\overline{v} \text{ for } 1 \leq n < l \tag{13}$$

where $\{A_n(l), B_n(l), C_n(l), D_n(l)\}$ are constants characterized in the proof of the proposition and $D_n(l) = 1 - A_n(l)$. Moreover, $A_n > 0$ iff (a) $\tau_{\epsilon_n} > 0$ or (b) $\tau_{\epsilon_n - 1} > 0$ and $\mu > 0$. In this equilibrium speculators' trading strategies in period n are

$$\begin{aligned} x_1(s_{i1}, p_1) &= \frac{\gamma(E[v|s_{i1}, p_1] - p_1)}{\tau_1 + \tau_{\epsilon_1}}, \\ x_n^I(s_{in}, p^n) &= \frac{\gamma(E[v|s_{in}, p^n] - p_n)}{\tau_n + \tau_{\epsilon_n}}, \\ x_n^O(s_{i2}, p^{n-2}, p_n) &= \frac{\gamma(E[v|s_{in}, p^{n-l}, p_n] - p_n)}{\widehat{\tau}_n + \tau_{\epsilon_n}}, \end{aligned} \tag{14}$$

where $\tau_n = \tau_v + \tau_u \sum_{t=1}^n a_t^2$ with $a_n = \gamma\tau_{\epsilon_n}$.

Using this result, we can study the effect of an increase in latency on the value of the real time ticker. We denote this value at date n by $\phi_n(\mu, l)$. Proceeding as in the case in which $l = 2$, we obtain that

$$\phi_n(\mu, l) = \frac{\gamma}{2} \ln \left(\frac{\tau_{\epsilon_n} + \tau_n}{\tau_{\epsilon_n} + \hat{\tau}_n(l)} \right). \tag{15}$$

The informativeness of the real time ticker, τ_n, does not depend on latency. However, intuitively, an increase in latency reduces the precision of outsiders' forecast. For instance, suppose that latency is increased from $l = 3$ to $l = 4$. Until date 3, outsiders' information set is unchanged and hence the equilibrium is unchanged. Thus, for $n \leq 3$, $\hat{\tau}_n(4) = \hat{\tau}_n(3)$, which implies $\phi_n(\mu, 3) = \phi_n(\mu, 4)$. At date $n = 4$, however, outsiders observe the first transaction price when $l = 3$ but not when $l = 4$. Thus, $\hat{\tau}_4(4) < \hat{\tau}_4(3)$, which implies that $\phi_4(\mu, 3) > \phi_4(\mu, 4)$. The next proposition establishes formally that the value of the real time ticker decreases with the delay with which outsiders observe past transaction prices.

Proposition 6 *When τ_{ϵ_n} is constant and strictly positive, the value of the real time ticker weakly decreases with the length of the delayed ticker, l. More precisely*

$$\begin{aligned} \phi_n(\mu, l) &< \phi_n(\mu, l+1) \text{ for } l \geq n \\ \phi_n(\mu, l) &= \phi_n(\mu, l+1) \text{ for } l < n \end{aligned}$$

Thus, an exchange has two instruments to control the value of the ticker: (i) the scope of its dissemination, μ and (ii) latency, l. Intuitively, increasing latency is a way to decrease the information provided by the current clearing price on the information contained in the delayed ticker. Thus, it makes acquisition of information on the real time ticker more attractive.

The last result has an interesting implication. In our model, we measure latency in transaction time. In reality, investors have access to the lagged ticker for free after a fixed amount of time (e.g., 20 minutes). The more active is a security, the larger the number of transactions in a fixed time interval. Hence, latency increases with trading activity when the delay after which the ticker is free is fixed in calendar time. Proposition 6 suggests that the value of the ticker is larger, other things equal, for more actively traded stocks. Thus, when the fee for market data is fixed across stocks, the model implies that the number of subscribers to a real-time data feed should be larger for more actively traded stocks.

4.2 Latency and Liquidity

We now analyze the effect of latency on market liquidity. As usual, we measure market liquidity by the sensitivity of the clearing price to the net order imbalance, i.e., by B_n in our model. Specifically, the smaller is B_n, the greater is market liquidity in period n and the lower are liquidity traders' expected trading losses since[14]

$$E[(p_n - v)u_n] = B_n/\tau_u.$$

The next corollary shows that an increase in latency reduces the liquidity of the market at each date $n > l$ as shown by the next corollary.

Proposition 7 *When τ_{ϵ_n} is constant and strictly positive and $\mu < 1$, the liquidity of the market at date n weakly decreases with the latency in information dissemination, l. More precisely*

$$\begin{aligned} B_n(l) &< B_n(l+1) \text{ for } l \geq n \\ B_n(l) &= B_n(l+1) \text{ for } l < n \end{aligned}$$

Intuitively, an increase in latency increases outsiders' uncertainty on the liquidation value. Thus, they require a larger compensation to absorb order imbalances when latency enlarges. This finding yield two testable implications. First, stocks experiencing a reduction in latency should become more liquid. Second, for a fixed reduction in latency measured in calendar time, more actively traded stocks should experience a larger improvement in liquidity. Actually, consider two stocks A and B. Transactions occur at a rate of two transaction per minute for stocks A and one transaction per minute for stock B. Now consider a reduction in latency for both stocks from twenty minutes to ten minutes. In transaction time, the reduction in latency for stock A is twice that of stock B. Thus, other things equal, the model implies that the improvement in liquidity for stock A should be larger.[15]

5 Conclusions

In this paper, we study the value of real time information on transactions in a multi-period rational expectations model. Our model features liquidity traders and speculators who, respectively, play the role of liquidity demanders and liquidity providers. All speculators have private information of equal quality. Speculators are either (i) insiders (they observe transactions in real time) or (ii) outsiders (who observe information on transactions with a delay). Our main findings are as follows.

[14]This expression is obtained using the expression for the clearing price in period n (equation (3))

[15]Of course, one difficulty for testing this prediction is that in reality transaction rates are endogenous and could depend on latency.

1. The value of real time information decreases with the proportion of speculators acquiring real time information and increases with the delay with which information on past trades is disseminated for free.

2. Speculators' welfare declines with the proportion of insiders. Speculators are better off in an environment in which access to information on past trades is delayed because it relaxes competition among speculators.[16]

3. The informativeness of the ticker does not depend on the proportion of insiders or the delay with which information on trades is disseminated. However, market liquidity declines with this delay.

The first result implies that even when traders are homogeneous (i.e., have the same valuation for price information), exchanges can raise their revenues from data sales by restricting the dissemination of price information. Thus, creating unleveled access to market data can be in an exchange's best interest. The second result implies that broadening the scope of information dissemination can trigger exit of some liquidity providers (as the benefit of market participation is too large compared to the cost). Combined with our last result, this observation implies that market liquidity and asset prices are affected by the speed with which traders access to real time data.

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Appendix A

Proof of proposition 1

We prove this proposition in three steps (Part 1). In the second part, we derive the expressions for speculators' welfare that appear in the text (equations (6) and (7)).

Step 1. In a symmetric linear equilibrium, speculators' order placement strategies in period $n \geq 2$ can be written

$$\begin{aligned} x_n^I(s_{in}, p^n) &= a_n^I s_{in} - \varphi_2^I(p^n), && (16) \\ x_n^O(s_{in}, p^{n-2}, p_n) &= a_n^O s_{in} - \varphi_2^O(p^{n-2}, p_n), && (17) \end{aligned}$$

The clearing condition in period n is

$$\int_0^{\mu} x_{in}^I di + \int_{\mu}^1 x_{in}^O di + u_n = 0.$$

Thus, using equations (16) and (17), we deduce that at date n

$$a_n v + u_n - \varphi_2^I(p^n) - \varphi_2^O(p^{n-2}, p_n) = 0, \ \forall n \geq 2, \tag{18}$$

with $a_n \overset{def}{=} \mu a_n^I + (1-\mu)a_n^O$. A similar argument shows that

$$a_1 v + u_1 - \varphi(p_1) = 0 \tag{19}$$

Thus, combining equations (18) and (19), we deduce that p^n is observationally equivalent to $z^n = \{z_1, z_2, ..., z_n\}$ with $z_n = a_n v + u_n$.

Step 2. Equilibrium in period n $\geq$ 2.

Insiders. An insider's demand function in period n, $x_2^I(s_{in}, p^n)$, maximizes

$$E[-\exp\{-(v - p_2)x_{i2}^I/\gamma\}|s_{in}, p^n].$$

We deduce that

$$x_2^I(s_{in}, p^n) = \gamma \frac{E[v - p_n|s_{in}, p^n]}{\text{Var}[v - p_n|s_{in}, p^n]} = \gamma \frac{E[v|s_{in}, p^n] - p_n}{\text{Var}[v|s_{in}, p^n]}.$$

As p^n is observationally equivalent to z^n, we deduce (using well-known properties of normal random variables)

$$\begin{aligned} E[v|s_{in}, p^n] &= E[v|s_{in}, z^n] = (\tau_n + \tau_{\epsilon_n})^{-1}(\tau_n E[v|z^n] + \tau_{\epsilon_2} s_{in}), \\ \text{Var}[v|s_{in}, p^n] &= \text{Var}[v|s_{in}, z^n] = (\tau_n + \tau_{\epsilon_n})^{-1}. \end{aligned}$$

where $\tau_n \stackrel{def}{=} (\text{Var}[v|p^n])^{-1} = (\text{Var}[v|z^n])^{-1} = \tau_v + \tau_u \sum_{t=1}^{2} a_t^2$. Thus,

$$\begin{aligned} x_n^I(s_{in}, p^n) &= \gamma(\tau_n + \tau_{\epsilon_n})(E[v|s_{in}, p^n] - p_n) \\ &= a_n^I(s_{in} - p_n) + \gamma\tau_n(E[v|p^n] - p_n) \end{aligned} \tag{20}$$

where $a_n^I = \gamma\tau_{\epsilon_n}$.

Outsiders. An outsider's demand function in period n, $x_n^O(s_{in}, p^{n-2}, p_n)$, maximizes:

$$E\left[-\exp\left\{-(v - p_2)x_{in}^O/\gamma\right\}|s_{in}, p^{n-2}, p_n\right].$$

We deduce that

$$x_n^O(s_{in}, p^{n-2}, p_n) = \gamma\frac{E[v - p_n|s_{in}, p^{n-2}, p_n]}{\text{Var}[v - p_n|s_{in}, p^{n-2}, p_n]} = \gamma\frac{E[v|s_{in}, p^{n-2}, p_n] - p_n}{\text{Var}[v - p_n|s_{in}, p^{n-2}, p_n]}.$$

In equilibrium, outsiders correctly anticipate that the second and the first period prices are related to the value of the security as follows

$$p_n = A_n v + B_n u_n + C_n u_{n-1} + D_n E(v \mid p^{n-2}) \text{ for } n \geq 2, \tag{21}$$

$$p_1 = A_1 v + B_1 u_1 + D_1. \tag{22}$$

Let

$$\widehat{z}_n = \frac{p_n - D_n E(v \mid p^{n-2})}{A_n}$$

Using equation (21), we deduce that $\{s_{in}, p^{n-2}, p_n\}$ is observationally equivalent to $\{s_{in}, p^{n-2}, \widehat{z}_n\}$ and that

$$\widehat{z}_n|v \sim N(v, A_n^{-2}(B_n^2 + C_n^2)\tau_u^{-1}).$$

Hence, using well known properties of normal random variables, we obtain

$$\begin{aligned} E[v|s_{in}, p^{n-2}, p_n] &= (\hat{\tau}_n + \tau_{\epsilon_n})^{-1}(\hat{\tau}_n E[v|p^{n-2}, p_n] + \tau_{\epsilon_n} s_{in}), \\ \text{Var}[v|s_{in}, p^{n-2}, p_n] &= (\hat{\tau}_n + \tau_{\epsilon_n})^{-1}, \end{aligned}$$

where

$$\hat{\tau}_n \stackrel{def}{=} (\text{Var}[v|p^{n-2}, p_n])^{-1} = (\text{Var}[v|z^{n-2}, \widehat{z}_n])^{-1} = \tau_{n-2} + A_n^2(B_n^2 + C_n^2)^{-1}\tau_u. \tag{23}$$

Thus,

$$\begin{aligned} x_n^O(s_{in}, p^{n-2}, p_n) &= \gamma\left(\hat{\tau}_n + \tau_{\epsilon_n}\right)\left(E[v|s_{in}, p^{n-2}, p_n] - p_n\right), \\ &= a_n^O(s_{in} - p_n) + \gamma\hat{\tau}_n(E[v|p^{n-2}, p_n] - p_n). \end{aligned} \tag{24}$$

with $a_n^O = a_n^I = \gamma\tau_{\epsilon_n}$. Thus, $a_n = \mu a_n^I + (1-\mu)a_n^O = \gamma\tau_{\epsilon_n}$.

Clearing price in period n≥2. The clearing condition in period $n \geq 2$ imposes

$$\int_0^\mu x_{in}^I di + \int_\mu^1 x_{in}^O di + u_n = 0.$$

Using equations (20) and (24), we solve for the equilibrium price and we obtain

$$p_n = \frac{1}{K_n}\left(z_n + \mu\gamma\tau_n E[v|p^n] + (1-\mu)\gamma\hat{\tau}_n E[v|p^{n-2}, p_n]\right), \tag{25}$$

where $K_n = a_n + \gamma(\mu\tau_n + (1-\mu)\hat{\tau}_n)$. Observe that

$$\begin{aligned} E[v|p^{n-2}, p_n] &= E[v|p^{n-2}, \widehat{z}_n] = \hat{\tau}_n^{-1}\left(\tau_{n-2}E[v|p^{n-2}] + A_n^2(B_n^2 + C_n^2)^{-1}\tau_u \widehat{z}_n\right) \\ E[v|p^n] &= E[v|p^{n-2}, z_{n-1}, z_n] = \tau_n^{-1}\left(\tau_{n-2}E[v|p^{n-2}] + \tau_u \sum_{t=n-1}^{n} a_t z_t\right). \end{aligned}$$

Substituting $E[v|p^{n-2}, p_n]$ and $E[v|p^n]$ by these expressions in equation (25), we can express p_n as a function of v, u_n, u_{n-1}, and $E[v|p^{n-2}]$. In equilibrium, the coefficients on these variables must be identical to those in equation (21). This condition imposes

$$A_n = \frac{(1+\mu\gamma\tau_u a_n)a_n + \mu\gamma a_{n-1}^2\tau_u + (1-\mu)\gamma A_n^2(B_n^2 + C_n^2)^{-1}\tau_u}{K_n}, \tag{26}$$

$$B_n = \frac{1 + \mu\gamma a_n\tau_u + (1-\mu)\gamma A_n B_n(B_n^2 + C_n^2)^{-1}\tau_u}{K_n}, \tag{27}$$

$$C_n = \frac{\mu\gamma a_{n-1}\tau_u + (1-\mu)\gamma A_n C_n(B_n^2 + C_n^2)^{-1}\tau_u}{K_n}, \tag{28}$$

$$D_n = \frac{\gamma\tau_{n-2}}{K_n}. \tag{29}$$

The three first equations define a system with three unknowns A_n, B_n and C_n. Solving this system of equations, we obtain (after tedious calculations)

$$A_n = \frac{a_n + \mu\gamma(a_{n-1}^2 + a_n^2)\tau_u}{K_n}\left(1 + \frac{(1-\mu)\gamma\tau_u(a_n + \mu\gamma(a_{n-1}^2 + a_n^2)\tau_u)}{(1+\mu\gamma a_n\tau_u)^2 + (\mu\gamma a_{n-1}\tau_u)^2}\right), \tag{30}$$

$$B_n = \frac{1 + \mu\gamma a_n\tau_u}{K_n}\left(1 + \frac{(1-\mu)\gamma\tau_u(a_n + \mu\gamma(a_{n-1}^2 + a_n^2)\tau_u)}{(1+\mu\gamma a_n\tau_u)^2 + (\mu\gamma a_{n-1}\tau_u)^2}\right), \tag{31}$$

$$C_n = \frac{\mu\gamma a_{n-1}\tau_u}{K_n}\left(1 + \frac{(1-\mu)\gamma\tau_u(a_n + \mu\gamma(a_{n-1}^2 + a_n^2)\tau_u)}{(1+\mu\gamma a_n\tau_u)^2 + (\mu\gamma a_{n-1}\tau_u)^2}\right), \tag{32}$$

$$D_n = \frac{\gamma\tau_{n-2}}{K_n}, \tag{33}$$

where $K_n = a_n + \gamma(\mu\tau_n + (1-\mu)\hat{\tau}_n)$, and

$$\hat{\tau}_n = \tau_{n-2} + A_n^2 \left(B_n^2 + C_n^2\right)^{-1} \tau_u \tag{34}$$

$$= \tau_{n-2} + \frac{(a_n(1+\mu\gamma a_n\tau_u) + a_{n-1}(\mu\gamma a_{n-1}\tau_u))^2}{(1+\mu\gamma a_n\tau_u)^2 + (\mu\gamma a_{n-1}\tau_u)^2}\tau_u. \tag{35}$$

Note that coefficient D_n depends on K_n, which is determined by $\hat{\tau}_n$ and therefore ultimately by A_n, B_n and C_n.

Step 3. Equilibrium in period 1. Following the same steps as in period 2, we obtain that in the first period there is a unique linear rational expectations equilibrium with

$$x_1(s_{i1}, p_1) = a_1(s_{i1} - p_1) + \gamma\tau_1(E[v|p_1] - p_1),$$

and

$$p_1 = A_1^* v + B_1^* u_1 + D_1^* \overline{v},$$

with $a_1 = \gamma\tau_{\epsilon_1}$, $\tau_1 \overset{def}{=} (\mathrm{Var}[v|p_1])^{-1} = \tau_v + \tau_u a_1^2$ and

$$\begin{aligned} A_1^* &= \frac{(1+\gamma\tau_u a_1)a_1}{(a_1+\gamma\tau_1)}, \\ B_1^* &= \frac{(1+\gamma\tau_u a_1)}{(a_1+\gamma\tau_1)}, \\ D_1^* &= (1 - A_1^*). \end{aligned}$$

Part 2.

The expressions for speculators' welfare in equations (6) and (7) derive from Proposition 3.1 in Admati and Pfleiderer (1987). They derive the expected utility of a trader who submit limit orders and who receives a vector of signals. It is easy to check that all the distributional hypotheses necessary for applying their proposition are satisfied in our model. Moreover in our model, the unconditional expected difference between the payoff of the security and the equilibrium price at any date (which is denoted μ in Admati and Pfleiderer (1987)) is nil, that is

$$E(\widetilde{v} - p_n) = 0$$

To see this point observe that

$$E(\widetilde{v} - p_n) = \overline{v} - (A_n + D_n)\overline{v}.$$

Now, using Equation (26) in the proof of Proposition **1**, we have

$$A_n = \frac{a_n + \mu\gamma(\tau_n - \tau_{n-2}) + (1-\mu)\gamma(\widehat{\tau}_n - \tau_{n-2})}{K_n}.$$

Thus, $(A_n + D_n) = 1$, which implies $E(\widetilde{v} - p_n) = 0$. Using this observation and Proposition 3.1 of Admati and Pfleiderer (1987), the result is then immediate.QED

Proof of corollary 1

From equation (23) in the proof of Proposition 1, we deduce that

$$\begin{aligned} \hat{\tau}_n &= \tau_{n-2} + \tau_n^m \\ & \tau_{n-2} + \frac{(a_n(1+\mu\gamma a_n\tau_u) + a_{n-1}(\mu\gamma a_{n-1}\tau_u))^2}{(1+\mu\gamma a_n\tau_u)^2 + (\mu\gamma a_{n-1}\tau_u)^2}\tau_u, \end{aligned}$$

where $a_t = \gamma\tau_{\epsilon_t}$. As

$$\frac{\partial\hat{\tau}_n}{\partial\mu} = \frac{2\gamma^2 a_{n-1}^2\tau_u^2(\tau_{\epsilon_2} + \mu(a_{n-1}^2 + a_n^2))}{((1+\mu\gamma\tau_u a_n)^2 + (\mu\gamma\tau_u a_{n-1})^2)^2} > 0,$$

we deduce that $\hat{\tau}_n$ increases with μ.

Last, $\hat{\tau}_n$ can be written as follows

$$\hat{\tau}_n = \tau_{n-2} + \tau_u\left(\frac{(\rho_{n-1}a_{n-1} + \rho_n a_n)^2}{\rho_{n-1}^2 + \rho_n^2}\right),$$

with $\rho_n = (1+\mu\gamma a_n\tau_u)$ and $\rho_{n-1} = (\mu\gamma a_{n-1}\tau_u)$. It is then direct to show that $\hat{\tau}_n \leq \tau_n$. Moreover the inequality is strict iff $a_{n-1} > 0$, i.e., iff $\tau_{\epsilon_1} > 0$. QED

Proof of Corollary 2

We have obtained in the proof of proposition 1 that

$$\tau_n = \tau_v + \tau_u\sum_{t=1}^{n} a_t^2.$$

As a_t does not depend on μ, it is immediate that τ_n does not depend on μ. The second part of the corollary is then immediate.

Proof of corollary 3

Observe that v, p_n, p_{n-2} are normally distributed. Thus,

$$Var(v - p_n) = Var(v - p_n \mid p^{n-2}) + Var(E(v - p_n \mid p^{n-2}))$$

We also observe that

$$E(v - p_n \mid p_{n-2}) = E(v \mid p^{n-2}) - (A_n + D_n)E(v \mid p^{n-2})$$

As $A_n + D_n = 1$ (see the proof of Lemma 2),

$$E(v - p_n \mid p^{n-2}) = 0.$$

We deduce that

$$\begin{aligned} Var(v - p_n) &= Var(v - p_n \mid p^{n-2}) \\ &= (1 - A_n)^2 \tau_{n-2}^{-1} + (B_n^2 + C_n^2)\tau_u^{-1} \end{aligned}$$

Differentiating the above function with respect to μ, we obtain that

$$\frac{\partial Var(v - p_n)}{\partial \mu} < 0, \forall n \geq 2.$$

QED

Proof of proposition 3

to be completed QED

Proof of proposition 4

Immediate from the arguments in the text.

Proof of Proposition ??

From the proof of Proposition 1 we know that for $n \geq 2$

$$B_n^*(\mu) = \frac{1 + \mu\gamma a_n \tau_u}{K_n}\left(1 + \frac{(1-\mu)\gamma\tau_u(a_n + \mu\gamma(a_{n-1}^2 + a_n^2)\tau_u)}{(1 + \mu\gamma a_n \tau_u)^2 + (\mu\gamma a_{n-1}\tau_u)^2}\right),$$

with $K_n = \gamma(\mu(\tau_n + \tau_{\epsilon_n}) + (1-\mu)(\hat{\tau}_n + \tau_{\epsilon_n}))$. For $\mu = 1$, we obtain

$$B_n^*(1) = \frac{1 + \gamma a_n \tau_u}{\gamma(\tau_n + \tau_{\epsilon_n})},$$

while for $\mu = 0$, we obtain

$$B_n^*(0) = \frac{1 + \gamma \tau_u a_n}{\gamma(\hat{\tau}_n + \tau_{\epsilon_n})}.$$

Thus, $B_n^*(1) < B_n^*(0)$ since $\hat{\tau}_n < \tau_n$, $\forall \mu$.

Proof of Proposition 5

The proposition can be proved by following exactly the same steps as those followed in the proof of Proposition 1. Thus, we omit the proof for brevity. Details can be obtained upon request. We just provide the expressions for the coefficients in equations (12) and (13). Let $l^*(n) = Min\{l, n\}$ and recall that $a_n = \gamma\tau_{\epsilon_n}$.

$$\begin{aligned}
A_n(l) &= \frac{a_n + \mu\gamma(\tau_n - \tau_{n-l^*})\tau_u}{K_n}\left(1 + \frac{(1-\mu)\gamma\tau_u(a_n + \mu\gamma(\tau_n - \tau_{n-l^*})\tau_u)}{(1+\mu\gamma a_n\tau_u)^2 + \sum_{j=1}^{l^*-1}(\mu\gamma a_{n-j}\tau_u)^2}\right) \\
B_0^n(l) &= \frac{A_n(l)(1+\mu\gamma a_{n-j}\tau_u)}{a_n + \mu\gamma(\tau_n - \tau_{n-l^*})\tau_u} \\
B_j^n(l) &= \frac{A_n(l)(\mu\gamma a_{n-j}\tau_u)}{a_n + \mu\gamma(\tau_n - \tau_{n-l^*})\tau_u} \qquad \forall j,\ 1 \le j \le l-1 \\
D_n &= \frac{\gamma\tau_{n-l^*}}{K_n}
\end{aligned}$$

with $\tau_n = \tau_v + \sum_{t=1}^{t=n} a_t^2$ and $\tau_0 = \tau_v$. Moreover $K_n = a_n + \gamma(\mu\tau_n + (1-\mu)\hat{\tau}_n(l))$ where

$$\hat{\tau}_n(l) \stackrel{def}{=} (\mathrm{Var}[v|p^{n-2}, p_n])^{-1} = \tau_{n-l} + A_n^2(l)\left(\sum_{j=0}^{j=l^*-1}(B_j^n(l))\right)^{-2}\tau_u. \qquad (36)$$

QED

Proof of Proposition 5

Using equation (36) and the expressions for $A_n(l)$ and $B_j^n(l)$, observe that

$$\hat{\tau}_n(l) = \begin{cases} \tau_v + \frac{(a_n+\mu\gamma(\tau_n-\tau_v))^2}{(1+\mu\gamma a_n\tau_u)^2+\sum_{j=1}^{n-1}(\mu\gamma a_{n-j}\tau_u)^2}\tau_u & \text{for } n \le l \\ \tau_{n-l} + \frac{(a_n+\mu\gamma(\tau_n-\tau_{n-l}))^2}{(1+\mu\gamma a_n\tau_u)^2+\sum_{j=1}^{l-1}(\mu\gamma a_{n-j}\tau_u)^2}\tau_u & \text{for } n > l \end{cases}$$

Thus, for $n \le l$, we have $\hat{\tau}_n(l) = \hat{\tau}_n(l+1)$. This implies $\phi_n^c(\mu, l) = \phi_n^c(\mu, l+1)$ for $l < n$. When τ_{ϵ_n} is constant, we have $a_n = a$, $\forall n$. Then for $n > l$

$$\hat{\tau}_n(l) = \tau_v + (n-l)a^2\tau_u + \frac{(a + l\mu\gamma a^2\tau_u)^2}{(1+\mu\gamma a\tau_u)^2 + (l-1)(\mu\gamma a\tau_u)^2}\tau_u.$$

Long but simple calculations show that $\hat{\tau}_n(l) > \hat{\tau}_n(l+1)$ for $n > l$. We deduce that $\phi_n^c(\mu, l) > \phi_n^c(\mu, l+1)$ for $n > l$ when τ_{ϵ_n} is constant.QED

Proof of Proposition 7

Using the expressions for $A_n(l)$ and $B_0^n(l)$ in the proof of Proposition 5, we obtain that

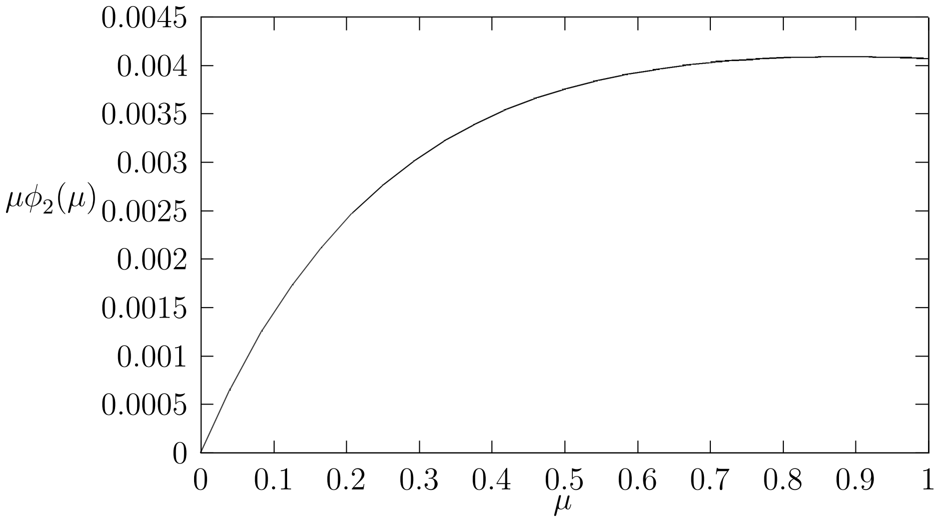
$$B_0^n(l) = \begin{cases} \frac{(1+\mu\gamma a_n\tau_u)}{a_n+\gamma(\mu\tau_n+(1-\mu)\hat{\tau}_n(l)}\left(1 + \frac{(1-\mu)\gamma\tau_u(a_n+\mu\gamma(\tau_n-\tau_v)\tau_u)}{(1+\mu\gamma a_n\tau_u)^2+\sum_{j=1}^{n-1}(\mu\gamma a_{n-j}\tau_u)^2}\right) & \text{for } n \le l \\ \frac{(1+\mu\gamma a_n\tau_u)}{a_n+\gamma(\mu\tau_n+(1-\mu)\hat{\tau}_n(l))}\left(1 + \frac{(1-\mu)\gamma\tau_u(a_n+\mu\gamma(\tau_n-\tau_{n-l})\tau_u)}{(1+\mu\gamma a_n\tau_u)^2+\sum_{j=1}^{l-1}(\mu\gamma a_{n-j}\tau_u)^2}\right) & \text{for } n > l \end{cases}$$

For $n \leq l$, $\hat{\tau}_n(l)$ does not depend on l. Thus, $B_0^n(l) = B_0^n(l+1)$ for $n < l$. When τ_{ϵ_n} is constant, we have $a_n = a$, $\forall n$. Then for $n > l$

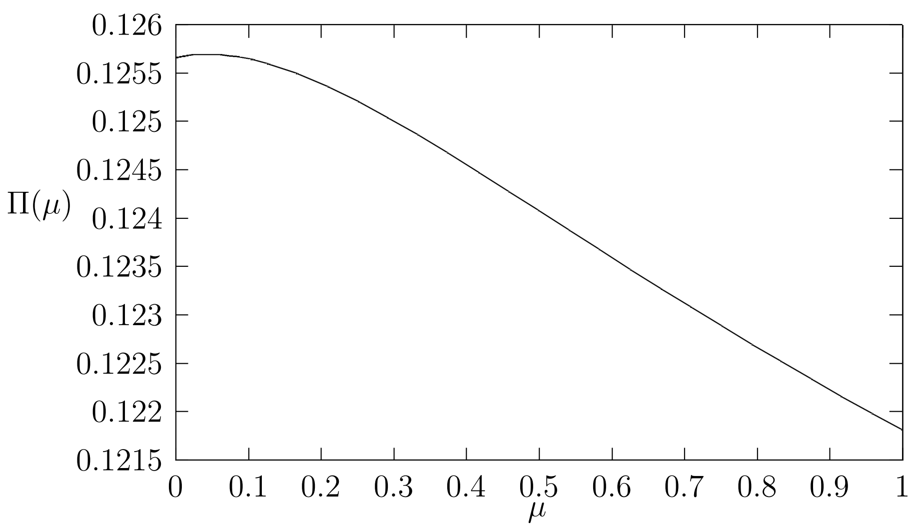
$$B_0^n(l) = \frac{(1 + \mu\gamma a\tau_u)}{a + \gamma(\mu\tau_n + (1-\mu)\hat{\tau}_n(l))}\left(1 + \frac{(1-\mu)\gamma\tau_u a(1+\mu\gamma l a\tau_u)}{(1+\mu\gamma a\tau_u)^2 + (l-1)(\mu\gamma a\tau_u)^2}\right)$$

Recall that for $n > l$, we have $\hat{\tau}_n(l+1) < \hat{\tau}_n(l)$. Thus, using the last equation, we deduce that for $n > l$, we have $B_0^n(l) < B_0^n(l+1)$.

Figure 1: Rationing access to ticker information. Parameters' values $\tau_v = \gamma = 1$, $\tau_u = 5$, $\tau_{\epsilon_1} = .1$, and $\tau_{\epsilon_2} = .2$



(a) Profit from information sales



(b) Profit from information sales and entry fee