# Reputation and Collusion in Procurement*

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**Abstract**

When gains from trade exist both along contractible and non-contractible dimensions and procurement is repeated, non-contractible gains from trade can be realized through reputation/relational contracting. A buyer may restrict participation to his recurrent auctions to a subset of potential sellers and threaten replacement of sellers that perform poorly along non-contractible dimensions. In such a dynamic procurement framework, keeping the optimal number of eligible sellers endogenous, we find that: a) *there is a general trade-off between reputation for quality and collusion*: shorter contracts - more frequent re-auctioning - and restricted participation facilitate non-contractible quality provision, but also collusive agreements among suppliers; and b) when non-contractible quality and variability in suppliers' efficiency are both important, *short contract duration and a collusive agreement between a few eligible sellers may maximize welfare and leave the buyer better off.*

*JEL* Classification Numbers: ...

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# 1 Introduction

Non-contractible dimensions are present in different measure in every economic exchange.[1] For example in procurement of complex IT services it is often impossible to specify all the requirements that are of value for a buyer. Similarly, in health procurement it is often the case that the more critical is quality of procured goods and services, the more difficult is to correctly specify required properties of services to be procured and whether procurement comply with them. In the provision of innovative goods or services, R&D activity generally impacts on the matching between the buyer's needs and the product's characteristics and it is often difficult, if not impossible, to check whether the contractor has performed at its best to this end.

It is well known that when these dimensions are important in terms of gains from trade, letting suppliers compete - say, in an auction - may lead to a very inefficient outcome, for a buyer and in general.[2] Exchanges, however, are often regularly repeated. Reputational forces may then help governing transactions on non-contractible dimensions. An opportunistic supplier that overstates the non-contractible quality of an experience good or that purposely reduces non-verifiable but ex-post observable qualitative aspects to cut costs and bust profits can be punished by its buyer(s). Clearly, this cannot take place under any infringement of contractual terms, but the buyer can exercise some of is discretion to deliberately hinder an "unfaithful" contractor. Within this context the most effective and, probably, the most natural punishment consists in excluding from (some) future trade(s). This form of punishment is certainly available in private contracting where the buyer his generally free to exclude any buyer from its selecting process. In the case of public procurement this decision is still viable to some extent, but it may be partially limited by ruling laws which often restrict civil servants discretionally so as to avoid corruption. However, some national legislations do provide room for discretion in exclusion of dubious providers and give some accountability in the hands of public buyers.[3]

This paper analyzes repeated procurement processes (recurrent auctions or other forms of search) where non-contractible dimensions are an important source of gains from trade and suppliers' relative efficiency changes over time. Some important dimensions of the trade relationship are contractible, such as the length of supplying contract and the price paid for it. On the contrary, quality is non-contractible in our analysis and will be interpreted in broad sense capturing all decisions that a supplier is free to take during the contract execution and which may enhance the value of procurement but which the buyer cannot enforce, like effort, investment and R&D activities.

Other dimensions influencing the trade relationship will remain in the hand of the buyer who

[1] Reasons why some dimensions of exchanges are not explicitly contractible include complexity and prohibitive legal cost of verification; see Hart (1995) for an in depth discussion and Tirole (1999) for an evaluation of the debate on contracts incompleteness.

[2] See Manelli and Vincent (1995), among others.

[3] This is the case in the US and UK. See for example the 2004 US Public Procurement Guidelines. Recently, a new two-stages procedure for EU procurement has been introduced. It contemplates a pre-qualification stage where the public buyer has some discretion to exclude suppliers, followed by an auction. See EC, Directive 2004/18/EC, "On the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts".

can thus implicitly condition the procurement economic environment. In particular, in our analysis the buyer can decide to vary the amount of search done before proposing a contract, namely it has the possibility to fix the number of suppliers in the pool admitted to participate at the recurrent auctions. As previously discussed, we will also allow the buyer to punish with exclusion from future procurement contests firms that have decided to offer low levels of quality along the contractual relationship.

We show that, in a dynamic procurement process the buyer may want to restrict the number of potential trading partners at the cost of reduced screening and savings on contractible dimensions. This is done to boost non-contractible quality. Indeed, by restricting competition, the buyer leaves firms sufficient future rents so that they can find profitable to build reputational commitments for future interactions and prefer to provide acceptable levels of non-contractible quality in the current relationship (and refrain from moral hazard).[4]

Duration of supply contracts is also a crucial aspect in dynamic procurement. Abstracting from (important) technological aspects such as the rate of obsolescence, a shorter duration of supply contracts implies more frequent re-selection/search. We then show that with higher frequency of interaction it is easier for a buyer to obtain high non-contractible quality levels from sellers by threatening exclusion from future trade. Indeed, with more frequent contracting the threat of exclusion is closer in time and gains from "cheating" are smaller so that larger implementable quality can be expected. With this respect we also consider that if shortening contract length reduces the buyer's risk to be locked-in with undesirably low quality, this has to be contrasted with the buyer's costs of running and organizing more frequent auctions and firms' inability to spread possible fixed (and relation-specific) costs of procurement over longer periods of contracting.

Hence, these considerations show that in repeated procurement a buyer may profit from suppliers' stake on future profits and induce larger non-contractible quality by restricting the pool of firms admitted to compete for the procurement contract and reducing the contract length or, which is equivalent, increasing the frequency of recurrent auctions.

On the other hand, for analogous reasons, it is also well known that more frequent procurement auctions with a limited number of competing firms represents an environment most favorable for inducing and sustaining collusive behavior between the selected suppliers! Hence, we find a rather general trade-off between reputation for non-contractible quality and collusion in dynamic procurement. Longer duration of supply contracts - less frequent auctions - together with larger pool of competing suppliers both deter collusion among eligible suppliers but also reduce non-contractible quality levels obtainable from them. Symmetrically, shorter contracts - more frequent auctions - and a smaller pool of suppliers both facilitate suppliers' collusion but also the enforcement of non-contractible quality standards.

This trade-off may show up being rather disappointing because it seems to mark limits on the remedies that can be put at work for non-contractible quality provision. However, our analysis clarifies that collusion itself can directly interact with firms' incentives to provide non-contractible

[4] Fehr et al. (2004) show experimentally how in a dynamic exchange environment, when non contractible aspects become important, agents do not search for the best offer each period but rather stick to the same partner with whom they try to cooperate.

quality. In fact, by increasing the selling price, collusion increases the expected gains from participating in procurement auctions which, as usual, can also be seen as the cost of being excluded if the buyer reacts to low levels of non-contractible quality. Hence, we show that the seeming trade-off between non-contractible quality and collusion, that naturally seems to emerge in repeated procurement, may in fact reveal to be only apparent because larger future rents associated with collusion make firms ready to offer larger non-contractible quality. This somehow provocative result suggests that the buyer may not be necessarily concerned by the spontaneous formation of (legal) consortia among firms that may alternatively compete. Indeed, reinterpreting our result on collusion and non-contractible quality, firms consortia are made stable by frequent interaction among a limited number of potential competitors and, in addition, consortia are also ready to provide larger levels of quality. We show that in this situation, if firms in a cartel or consortium are able to share information on each other's costs, then lessening competition by reducing the pool of potential suppliers and shortening contract duration that both induce *a collusive or legal agreement between eligible suppliers may leave the buyer better off.*

Further exploring this point, we also note that it is often the case in repeated procurement that participating firm have better knowledge on their relative abilities and efficiency than the buyer. Hence, in addition to the previously discussed effect of agreements among firms, consortia and collusion may also have a desirable sorting effect whereby more efficient firms may be selected for supplying. This may not necessarily translate into lower prices for the buyer, and certainly does not in the case of collusion. Rather, larger efficiency in production in these cases tend to increase firms rent and then further boosts implementable non-contractible quality.

In a world in which firms' cost efficiency changes over time, a buyer may use frequent auctions to select the most efficient supplier available each period from a large pool of competitors, instead of being locked-in with inefficient contractors for long periods of times; but the strong competition induced by auctions drives down future rents making reputation ineffective and non-contractible quality hard to obtain. When both gains from trade on non-contractible quality and the variability in firm cost efficiency across time are sufficiently large, one would like at the same time to restrict competition to generate sufficient future rents so that reputational mechanisms work and to select the most efficient firm from a sufficiently large pool of firms. We conclude our analysis discussing how a buyer who is concerned by non-contractible quality, the price paid for procurement and, possibly, also directly by efficiency in supplying should design the procurement environment (i.e. in terms of number of competing firms and exclusion for low quality provision), contractible dimensions of the contractual relationship (i.e. the contract duration and auction frequency) to effectively compose its clashing objectives.

## 1.1 Relation with the literature

Our work is related to several strands of literature.

The adverse effects of competition in procurement of non-contractible quality have been discussed by Manelli and Vincent's (1995) with an analysis of optimal trading mechanisms in presence of non-contractible qualitative aspects of the traded good. They cast their model in terms of adverse selection (sellers are of different quality which is reflected in the produced good), focus on single transaction, and show that when gains from trade are concentrated on the non-contractible qualitative dimension, auctions deliver the worst possible outcome, as they select firms producing the good at the lowest cost but with the lowest level of non-contractible quality. Sequential take-it-or-leave-it offers to randomly selected sellers is then a better mechanism. We clearly differentiate from this paper by considering an environment where quality is an effort decision of procuring firms and not an innate characteristic of suppliers. Reaching similar conclusions on the desirability to limit competition in a repeated procurement contexts but for a different reason than the "bad selection" effect in Manelli and Vincent (1995), our paper complements their analysis.[5] Bajari and Tadelis (2001) also show that with relevant non-contractible dimensions auctioning with fixed-price contracts may be dominated by bilateral negotiation over cost-plus contracts.

The literature on optimal procurement of innovative goods, where pre-auction non-contractible R&D investments are crucial, including for example Taylor (1995), Fullerton and McAfee (1999), and Che and Gale (2003), is also related to our work, at least in spirit. In these analyses even though procurement is not repeated, limiting participation is optimal, as in our model, but for a different reason than relational contracting. Reducing the number of participant increases each participant's probability of winning the award, and thereby encourages pre-auction non-contractible R&D effort-investment.

Our emphasis on repeated purchase with non-contractible quality relates our analysis to the industrial organization literature on "reputation and competition" which studies whether firms' reputational commitments to high non-contractible quality can be compatible with a competitive environment and that has been initiated by the seminal works of Macaulay (1963), Klein and Leffler (1981), Shapiro (1983), and Allen (1984). This early analysis were concerned with the compatibility of "quality-assuring" reputational equilibria - requiring rents that make the effort of maintaining reputation worthwhile also with free entry in the market - but did not analyze in detail firms' competitive interaction (firms' incentives to steal business from each other).[6] Stiglitz (1989) raised the question how could reputation be compatible with perfect competition, which driving firms to undercut each other should eliminate any future supracompetitive gains that could motivate costly investment in reputation. Kranton (2003) offers a model that captures this dilemma. In presence of moral hazard on quality and competition, high quality equilibria are unfeasible and he suggests restricting competition in industries where non-contractible quality is important. In a different model Bar-Isaac (2005) confirms Kranton and Stiglitz's view at the limit, but shows that at intermediate levels of competition a further increase in competition (number of firms and substitutability) may well increase equilibrium product quality. But it is Hoerner (2002) that first offers an elegant answer to Stiglitz's question: in his model with heterogeneous consumers, adverse selection and moral hazard high prices signal high quality and make competition compatible with

[5] On the desirability of auction when all relevant dimension of the trade relationship are contractible see Bulow and Klemperer (1996). Laffont and Tirole (1993) provide a general overview of the broad themes on incentives in procurement.

[6] In Klein and Leffler and Shapiro firms face a perfectly elastic demand at the quality assuring price; in Allen consumers are randomly allocated among the firms charging the lowest price weakly above the "quality-assuring" one.

(in fact necessary for) reputation to work.[7]

These ideas have been also employed in procurement by Kim (1998) and Doni (2004), who study a repeated auctions model with moral hazard on non-contractible quality, and show that it may be good to restrict participation and threaten exclusion if the level of non-contractible quality is too low. Contrary to these papers, we consider the possibility to adjust the frequency of procurement and its interaction with the decision to restrict the pool of potential suppliers. Even if Shapiro (1983) noted that the frequency of interaction may facilitate the operation of reputational mechanisms in experience good markets, the effects of contract duration and auction frequency have been largely neglected in the procurement literature.[8] We also investigate the effects induced by the possibility of legal and illegal agreements emerging as a consequence of these decisions and the trade-offs on quality, price for procurement and efficiency.[9]

Our paper also contributes to the literature on incomplete explicit contracts (the "Hart and Moore paradigm"), and in particular on how dynamic interaction allows to complement these by implicit/relational contracts, from the early contributions of Bull (1987) and McLeod and Malcomson (1989, 1998), to the recent ones like Levin (2003) and Fuchs (2005). As in our paper, the focus of this literature is how relational contracting allows parties to enforce/govern agreements on observable but not verifiable dimensions (effort, investment, quality). Within this rich and growing literature, our work is closest in spirit to the contributions more directly focussing on the interaction between explicit and implicit contracts, i.e. on how explicit contracts should be structured/modified to optimize the joint outcome of explicitly contracted dimensions and implicit effects, like Baker, Gibbons and Murphy (1994, 2002), Pearce and Stacchetti (1998), Halonen (2002), Che and Yoo (2001), Blonski and Spagnolo (2003), and Rayo (2004), among others. To our knowledge, none of these studies considers how the design of procurement process may be influenced by relational contracting, nor deals with how repeated screening through auctions, the number of eligible suppliers, the length of explicit contracts, and collusion interact with relational quality commitments.

We conclude this literature review with noticing that there are at least two approaches to reputation in markets, as convincingly emphasized by Bar-Isaac (2003). A first approach views "reputation as beliefs" where uninformed players infer intrinsic qualities of contracting parties by their behavior so that a firm's reputation consists in buyer's beliefs about its quality-type. See for example Kreps and Wilson (1982), Milgrom and Roberts (1982). A second non-exclusive view, which is the one employed in our paper, considers reputation as a self-sustaining commitment to provide desirable but non-contractible quality by a credible threat on the side of the other partner in case reputational commitment is violated, along the lines suggested in the literature on experience goods we have discussed above.

The rest of the paper is organized as follows. Section 2 presents the model setup. Section 3 analyzes procurement and quality with competing firms. Section 4 discusses the effect of collusion

[7] Note that in our model, and with auctions in general, (price) signalling is impossible because the lower price is chosen by the mechanism, and the trade off quality-competition reappears.

[8] The importance of contracting timing has been recently emphasized by Guriev (2005).

[9] Some authors have explicitly dealt with collusion in repeated auctions among asymmetrically informed firms but with no concerns for non-contractibility. See for example Aoyagi (2003) and Blume and Heidhues (2004).

on implementable quality. Section 5 illustrates optimal procurement. Section 6 extends the base model and discusses its main assumptions. Finally, Section 7 concludes.

## 2 Model Setup

A buyer $B$ needs a unit of a good (or a service) at any period (we will refer to buyer, auctioneer and procurer as synonymous) and $N$ firms can procure the good. Time horizon is infinite and all the players have a constant common discount factor equal to $\delta$.

The buyer cares for the quality of the good. His valuation of the good at any period $V(q)$ depends on quality $q$, with $V(0) = 0$ and $V' \geq 0$.

Any firm $i$ can procure a unit of the good with quality $q_i$ by incurring in a per-period cost $\theta_i + \psi(q_i)$ where $\theta_i$ is an (in-)efficiency parameter and $\psi(\cdot)$ is a positive real valued function with $\psi' \geq 0$, $\psi'' \geq 0$ and $\psi(0) = 0$.

The buyer is not fully informed on each firm's cost $\theta_i$ so that she runs auctions to procure the goods. The auction awards a procurement contract that requires the winning supplier to procure the good for $x \geq 1$ periods with the contract length $x$ determined by the buyer. Contracts that last more than one period cannot be unilaterally renegotiated (i.e. reneging is ruled out by law), but bilateral renegotiation is admissible. Any firm participating an auction makes an offer $b_i$. The contract winner is the bidder that offers the lowest acceptable price and the auction rules map the vector of all bids $\mathbf{b}$ into the per-period payment $p$ that the winner receives for any of the $x$ periods contemplated by the contract.[10] The buyer may set a reservation price $r$ so that acceptable bids must satisfy $r \geq b_i$ and, if $r < b_i$ for any $i$, the buyer does not award the contract. Furthermore, the buyer can decide to limit to $n \leq N$ the number of bidders that are eligible and admitted to the auction process. Organizing an auction costs a fixed amount $K \geq 0$ to the buyer which can be seen also the cost of setting up contractual relation-specific investments.[11]

The firm that is awarded the contract sets the level of quality it will provide once and for all the duration $x$ of the contract.[12] Quality provided by the contract awarded firm is observable but not verifiable so that it is not contractible. However, if the quality provided by the auction winning firm does not satisfy the buyer, in a sense that we now clarify, then the latter can discretionally exclude the contractor for some future auction rounds.[13] We model this exclusion rule with a minimum quality requirement $\underline{q}$ so that if the firm procures a good of quality $q < \underline{q}$, the buyer can discretionally decide to exclude this seller for the next $T$ auctions.

[10] To simplify exposition, we let the price summarize contractible qualitative features. A more general interpretation of our model is that the buyer evaluates all contractible elements in firms' offers with a scoring rule that is here represented by the price, which can be thought of as a score.

[11] In the sequel we will provide another alternative interpretation to this term $K$.

[12] Our result would be qualitatively unaffected if firms could be free to choose the quality level for any period. In this case, if a firm decided not to provide the minimum required quality, then it would certainly do at the first period and for all the duration of the contract, exactly as in the case we study.

[13] As already discussed in the Introduction, this is certainly the case for private procurement. In public procurement this possibility may be partially limited by national laws for public procurement.

Hence, the relationship between the buyer and the contractor is composed by a part that is verifiable and court-enforced, namely contract duration $x$, the reservation price $r$ and payment to the contract awarded firm $b_w$, together with a part that is non-contractible and discretionary, i.e. the provided quality $q$ and the exclusion rule $\sigma = (\underline{q}, T)$.[14]

The **timing of the game** is as follows.
At $t = -2$: Once and for all, the buyer sets the contract length $x$, the number of bidders $n \leq N$ that are admitted to the auction stage and a reservation price $r$ and announces an exclusion rule $\sigma = (\underline{q}, T)$.
At $t = -1$: The buyer randomly selects the $n$ firms amongst the $N$ potential sellers.
At $t = 0$: An infinite repetition of the following stage game (or auction game) takes place:

**Stage Game**

*At time $t_1$, an auction is run that awards the procurement contract, all firms observe who wins and at what price; the winning contractor sets quality $q$ for the good it will provide;*

*At any period $t \in [t_1 + 1, t_1 + x]$ the winner procures the good of quality $q$;*

*At time $t_1 + x$, the buyer observes the quality provided during the contract and excludes the winning firm $i$ for $T$ periods in case $q_i < \underline{q}$.*

*In case of exclusion a new firm enters, otherwise the pool $n$ remains unchanged, and a new stage game starts.*

For the sake of simplicity we make the following working assumptions.

**Assumption 1** *(i) The buyer knows $f$ and $\Theta$ but she does not know the realization of $\theta_i$;*

*(ii) In each period, the cost parameter $\theta_i$ is drawn anew from a distribution $f(\theta_i)$ on $\Theta = [\underline{\theta}, \overline{\theta}]$;*

*(iii) Firms are fully informed.*

**Assumption 2** *The number of potential suppliers $N$ is infinite.*

**Assumption 3** *The buyer needs to be served at any period, otherwise she obtains a per-period payoff equal to $-k$ with $k$ positive and large.*

Assumption 1 (i) is standard. Assumption 1 (ii) could be equivalently substituted by considering a $\theta_i$ which is drawn anew at any auction stage but that remains the same for the contract duration $x$. On the contrary, a model where each firm is characterized by a given level of efficiency $\theta_i$ forever would be much less tractable because the buyer would then learn firms' efficiency auction after auction and firms would anticipate that their actions signal information. Also assumption 1 (iii) is certainly not without loss of generality. It greatly simplifies our analysis, but as we will further discuss in Section 6, our results and the underlying trade-offs seem to be robust to asymmetrically informed firms. Assumption 2 will be relaxed and discussed in Section 6. With Assumption

[14] In Section 6 we explore the possibility for the buyer to use scoring rules on non-contractible quality.

3 interrupting the flow of goods / services is extremely costly which simplifies the treatment of firms participation. It will be also further discussed in Section 6.

A strategy for the buyer includes all the contractible elements of the procurement relationship $(r, n, x, b_w)$ and also the non-contractible exclusion rule $\sigma = (\underline{q}, T)$.[15] In Section 5 we will consider the possibility that the buyer asks a participation fee to the firms that are admitted in the pool of $n \leq N$ potential suppliers.

A strategy for a firm $i$ is composed by a participation decision, a bid $b_i$ and a decision on quality $q_i$ procured within the contract, in case the firm wins an auction.

In Sections 3-5 we will consider the toughest exclusion rule available to the buyer that punishes quality cheating firms with exclusion forever from future auctions. This exclusion rule is credible because the (infinite) firms are ex-ante identical, so that replacing a firm has no cost for the buyer, and constitutes an optimal penal code in the sense of Abreu (1988). Hence, from now on (except for Section 6 where we will relax Assumption 2) the exclusion rule $\sigma$ will be denoted uniquely by the quality requirement $\underline{q}$ with the understanding that $T$ is optimally set at $T^* = \infty$.

# 3 Implementable Quality with Competing Suppliers

In our stationary environment, the buyer's information is the same at any stage game so that her optimal strategy is time invariant and she sets the contractible elements of the procurement process and the quality requirement once and for all stage games. It follows that we can indicate the buyer' surplus for a given awarded contract where the supplier offers a quality $q$ as

$$s_t = \frac{1-\delta^x}{1-\delta} V(q) - b_w - K$$

and her overall surplus as $S = \sum_{k=0}^{\infty} \delta^{kx} s_k$. Similarly, welfare generated by procurement is of a given contract is

$$w(q, x, n) \equiv \frac{1-\delta^x}{1-\delta} \left[ V(q) - \theta'(n) - \psi(q) \right] - K$$

where $\theta'(n) \equiv \min\{\theta_1, ..., \theta_n\}$ is the cost of the most efficient firm in the pool of $n$ potential suppliers admitted at the auction stage and the minimum cost statistics $\theta'(n)$ is a decreasing function of $n$ for any given vector of firms' costs $\boldsymbol{\theta} = (\theta_1, ..., \theta_n)$. Overall welfare then is

$$W(q, x, n) = w(q, x, n) \sum_{k=0}^{\infty} \delta^{kx} = \frac{1}{1-\delta} \left[ V(q) - \theta'(n) - \psi(q) \right] - \frac{1}{1-\delta^x} K.$$

Being quality not-contractible, any firm $i$ that wins an auction may choose to satisfy the quality requirement $\underline{q}$, i.e. to provide a quality $q \geq \underline{q}$ or not. Clearly, none of the firms has incentive to

[15] For simplicity in the exposition we will treat $n$ and $x$ as continuous variables.

provide any quality larger than the minimum requirement so that provided quality will be either $q = \underline{q}$, if the firm's decision is to satisfy the quality requirement, or $q = 0$ if the suppliers decides to cheat on quality.

We start by considering first a (stationary) equilibrium where firms prefer to satisfy the quality requirement. At any auction, the winning firm can decide to comply with the buyer's quality requirement thus obtaining an expected profit equal to

$$b_w - \theta'(n) - A + E\left[\Pi | q \geq \underline{q}\right] \frac{\delta^x}{1-\delta^x}$$

where $A \equiv \delta \frac{1-\delta^{x-1}}{1-\delta} E[\theta] + \frac{1-\delta^x}{1-\delta} \psi\left(\underline{q}\right)$ is the sum of expected cost of production for the $x-1$ periods of supply after the first one and the cost of procuring quality $q$, whilst the term $E\left[\Pi | q \geq \underline{q}\right]$ represents the expected profits from any future auctions of a firm always complying with the quality requirement $q \geq \underline{q}$.[16]

Alternatively, then the winning firm can decide to shirk on quality, thus providing $q = 0$. By so doing the firm saves on production costs $\frac{1-\delta^x}{1-\delta} \psi\left(\underline{q}\right)$ but it induces exclusion from future auctions, with an overall profit equal to

$$b_w - \left[\theta'(n) + \delta \frac{1-\delta^{x-1}}{1-\delta} E[\theta]\right]$$

Furthermore, by standard arguments on price competition, if any firm is awarded the contract at a given auction, the most efficient firm (i.e. the one with cost $\theta'(n)$) is the winner with a bid $b$ which is equal to the minimum between the reservation price $r$ and the second most efficient firm's cost,

$$b_w \equiv \min\{r, \theta''(n) + A\}$$

where $\theta''(n) \equiv \min\left\{\boldsymbol{\theta}/\theta'(n)\right\}$ is the second lowest efficiency parameter in vector $\boldsymbol{\theta}$.

Comparing these profits for the most efficient firm, the following Lemma illustrates the maximal implementable quality for given $n$ and $x$..

**Lemma 1 (Maximal implementable quality)** *A necessary condition for a quality requirement $\underline{q}$ to be implementable is $\underline{q} \leq \overline{q}(x,n)$ where $\overline{q}(x,n)$ is the **maximal implementable quality** that solves*

$$\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)} = \psi\left(\overline{q}\right)\frac{1-\delta^x}{1-\delta}, \tag{1}$$

*with*

$$E\left[\Delta\theta(n)\right] \equiv \begin{cases} E[\theta''(n) - \theta'(n)] & \text{if } n > 1 \\ \overline{\theta} - E[\theta] & \text{if } n = 1 \end{cases}$$

*and $\overline{q}(x,n) = 0$ if either $n = \infty$ or $x = \infty$.*

*In all cases, $\frac{\partial \overline{q}}{\partial x} \leq 0$, $\frac{\partial \overline{q}}{\partial n} \leq 0$ and the negative effect of $x$ $(n)$ on $\overline{q}$ reduces with $n$ $(x)$.*

[16] As we will discuss if a firm prefers to comply with quality it will do so forever.

**Proof.** First we need to discuss the optimal reservation price for the buyer. Being no procurement extremely costly for the buyer, she sets a reservation price $r$ in order to guarantee firms' participation for any realization of costs. To this end $r$ must satisfy the following participation condition[17]

$$r-\overline{\theta}-A\geq 0$$

where $A\equiv \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta]+\frac{1-\delta^{x}}{1-\delta}\psi\left(\underline{q}\right).$

Hence, the optimal reservation price is $r^{s}=\overline{\theta}+A$ and at any auction stage the winning bid of the most efficient firm is

$$b_{w}=\min\{\overline{\theta}+A,\theta''\left(n\right)+A\}=\theta''\left(n\right)+A.$$

For what stated in the text, the winning firm prefers to provide the required quality if

$$b_{w}-\theta'\left(n\right)-\delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta]-\psi\left(\underline{q}\right)\frac{1-\delta^{x}}{1-\delta}+E\left[\Pi|q\geq\underline{q}\right]\frac{\delta^{x}}{1-\delta^{x}}\geq b_{w}-\theta'\left(n\right)-\delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta]$$

or

$$E\left[\Pi|q\geq\underline{q}\right]\frac{\delta^{x}}{1-\delta^{x}}\geq\psi\left(\underline{q}\right)\frac{1-\delta^{x}}{1-\delta}.$$

From independence, for a given $\theta_i$, we have $\Pr(\theta_i\leq\theta_j,\forall j\neq i|\theta_i)=[1-F(\theta_i)]^{n-1}$. Hence, ex-ante the probability of being the lowest cost firm at any stage game $p_w\left(n\right)$ is $\Pr(\theta_i\leq\theta_j,\forall j\neq i)=\int\left[1-F(\theta_i)\right]^{n-1}f\left(\theta_i\right)d\theta_i=1/n$. Moreover, applying the optimal reservation price, in any future auction the winning firm which satisfies the required quality obtains a profit equal $\Delta\theta\left(n\right).$ We can then rewrite the expected profits for any future auction as

$$E\left[\Pi|q\geq\underline{q}\right]=E\left[\Delta\theta\left(n\right)\right]/n$$

so that the firm prefers to satisfy the quality requirement if

$$\frac{E\left[\Delta\theta\left(n\right)\right]\delta^{x}}{n(1-\delta^{x})}\geq\psi\left(\underline{q}\right)\frac{1-\delta^{x}}{1-\delta}$$

Hence, any quality requirement $\underline{q}>\overline{q}\left(x,n\right)$ as defined in (1) will be never satisfied by the most efficient firm.

Note that if $n=1$, the unique firm with cost $\theta$ will ask a price equal to the reservation price $r=\overline{\theta}+A$ and prefers to provide required quality if

$$\left(\overline{\theta}-E[\theta]\right)\frac{\delta^{x}}{1-\delta^{x}}\geq\frac{1-\delta^{x}}{1-\delta}\psi\left(\underline{q}\right)$$

where $\overline{\theta}-E[\theta]=E\left[\Pi|q\geq\underline{q}\right]$ in case $n=1$.

[17] Note that we assume an interim participation decision on the part of the firms, i.e. a firm accepts a contract if, knowing its type $\theta$ for the first period it accepts a contract even if it does not know its costs for subsequent periods in the contract.

We now show that by offering a price $b_w = \theta''(n) + A$, the most efficient firm indeed wins the auction, i.e. none of less efficient firm will have any incentive to offer a lower price and shirk on quality. For this it suffices to consider the second most efficient firm. This will not undercut the most efficient one by planning to provide a quality lower than the requirement if and only if the following is verified:

$$\begin{aligned} \theta''(n) + A - \theta''(n) - \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta] - \frac{1-\delta^x}{1-\delta}\psi(0) \quad &\leq \quad E\left[\Pi | q \geq \underline{q}\right]\frac{\delta^x}{1-\delta^x} \Longleftrightarrow \\ \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta} \quad &\leq \quad E\left[\Pi | q \geq \underline{q}\right]\frac{\delta^x}{1-\delta^x}. \end{aligned}$$

Hence, if (1) is verified then also the previous inequality is verified and can then be disregarded.

The value of $\overline{q}(x,n)$ for $x=\infty$ is immediate from (1). As for the comparative statics on $\overline{q}$ w.r.t. $x$ and $n$, differentiate (1) to obtain

$$\frac{\partial \overline{q}}{\partial n} = \frac{\delta^x(1-\delta)}{(1-\delta^x)^2}\left[\frac{\partial E[\Delta\theta(n)]}{\partial n}\frac{1}{n} - \frac{E[\Delta\theta(n)]}{n^2}\right]\frac{1}{\psi_q(\overline{q})}$$

with $\frac{\partial E[\Delta\theta(n)]}{\partial n} \leq 0$ so that $\frac{\partial \overline{q}}{\partial n} \leq 0$, from which it also follows that $\overline{q}(x,n) = 0$ for $n = \infty$. Furthermore, we also have

$$\frac{\partial \overline{q}}{\partial x} = \frac{(1-\delta)(1+\delta^x)\delta^x Log[\delta]}{(1-\delta^x)^3}\frac{E[\Delta\theta(n)]}{\psi_q(\overline{q})} \leq 0.$$

Finally, we also notice that

$$\frac{\partial^2\overline{q}(x,n)}{\partial x\partial n} = \left[\frac{\partial E[\Delta\theta(n)]}{\partial n}\frac{1}{n} - \frac{E[\Delta\theta(n)]}{n^2}\right]\left[\frac{(1-\delta)(1+\delta^x)\delta^x Log[\delta]}{(1-\delta^x)^3\psi_q(\overline{q})} - \frac{\delta^x(1-\delta)}{(1-\delta^x)^2\psi_{qq}(\overline{q})}\frac{\partial\overline{q}}{\partial x}\right] \geq 0$$

so that a larger $n$ makes $\frac{\partial \overline{q}}{\partial x}$ less negative and a larger $x$ makes $\frac{\partial \overline{q}}{\partial n}$ less negative: with a larger $n$ ($x$) the negative effect of $x$ ($n$) on quality is reduced. ■

The term $E[\Delta\theta(n)]/n$ is the informational rent that firms can expect when winning future auctions. The Lemma clearly shows that with uncontractible quality, firms are ready to provide quality in return of future profits so that if expected profits are small then the implementable quality is also small.[18] Furthermore, if all firms are admitted, i.e. $n=\infty$, any firm faces a negligible probability of being again the most efficient firm in tomorrow auctions and the expected cost difference between the most and the second most efficient firm $E[\Delta\theta(n)]$ is small. In this case future rents are null and there is no incentive to provide quality. This has been already pointed by Stiglitz (1989) who first showed that perfect competition clashes with firms' incentives to provide quality.

Furthermore, the condition on the maximal implementable quality (1) shows that if firms' heterogeneity is small, i.e. the expected cost difference between the most and the second most

[18] With a single firm admitted (i.e. $n=1$) buyer is obliged to buy at the reservation price so that auction profits are equal to $\max\theta - E[\theta] \geq \Delta\theta(n)$ for any $n>1$.

efficient firm $E[\Delta\theta(n)]$ is small, or the number of admitted firms is large (i.e. there is strong competition in the bidding process), then the buyer can only implement a low level of quality. In addition, the longer is the contract length $x$ the smaller is the maximal implementable quality for exclusion of a quality shirking firm is retarded when $x$ is large. In the limit, when the buyer sets a once-and-for-all contract with $x = \infty$, the unique implementable quality is the null quality $\underline{q} = 0$.

Lemma 1 illustrates the negative effect of $x$ and $n$ on the maximal implementable quality. However, $x$ and $n$ have also other effects on the buyer's payoff that we now illustrate.

First, a longer contract (i.e. larger $x$) implies smaller auction costs $K$, but it also implies that along the contract the buyer is stuck with a firm that may no longer be the most efficient firm in the pool of $n$ admitted suppliers.

Second, a larger $n$ implies that the price asked by the most efficient firm $b_w = \theta''(n) + A$ decreases because $\theta''(n)$ is a decreasing function of $n$.

These trade-offs are captured by the buyer's optimization program $(\mathcal{P}_c)$ where the winning firms are induced to comply with the quality requirement,

$$(\mathcal{P}_c) \quad \begin{cases} \max\limits_{\{\underline{q},x,n\}} \quad S\left(\underline{q}, x, n\right) \\ s.t. \\ S\left(\underline{q}, x, n\right) \equiv \frac{1}{1-\delta}\left[V(\underline{q}) - \psi\left(\underline{q}\right)\right] - \frac{E[\theta''(n)]}{1-\delta^x} - \frac{\delta\left(1-\delta^{x-1}\right)E[\theta]}{(1-\delta^x)(1-\delta)} - \frac{1}{1-\delta^x}K \\ \overline{q}(x,n) \geq \underline{q} \end{cases}$$

which is obtained by simply substituting the winning price $b_w$ into $S\left(\underline{q}, x, n\right)$ and that arises whenever the strategy of all firms contemplates the provision of required quality. Instead of providing an explicit solution to the program $(\mathcal{P}_c)$, we now illustrate an important property of the optimal solution that follows from Lemma 1.

The buyer may accept a good of null non-contractible quality instead of no procurement at all (by assumption 3). When she sets a null quality requirement $\underline{q} = 0$, she also maximizes her surplus $S(0, x, n)$ by optimizing the number of admitted firms $n$ and the contract length $x$. In this case then we immediately have

**Lemma 2** *If the buyer (optimally) sets a nil quality standard $\underline{q} = 0$, then the optimal number of admitted firms is $n_{\underline{q}=0} = \infty$ and the optimal contract length is $x_{\underline{q}=0} = \infty$ if $K \geq E[\theta] - \underline{\theta}$ and $x_{\underline{q}=0} = 1$ otherwise.*

**Proof.** The function $S\left(\underline{q}, x, n\right)$ with $\underline{q} = 0$ is non increasing in $n$ and it is non increasing in $x$ if $K \geq E[\theta] - \underline{\theta}$ and increasing in $x$ otherwise. Furthermore, we know from Lemma 1 that $\overline{q}(x,n)$ is nil if at least one amongst $x$ and $n$ is infinite. Hence, the result follows. ■

Let $S(0, x, n)$ denote the maximal surplus associated with procurement of nil quality. We then have

$$S(0,x,n) = \begin{cases} S(0,\infty,\infty) = -\underline{\theta} - \frac{E[\theta]\delta}{1-\delta} - K & \text{if } K \geq E[\theta] - \underline{\theta} \\ S(0,1,\infty) = -\frac{\underline{\theta}}{1-\delta^x} - \frac{\delta\left(1-\delta^{x-1}\right)E[\theta]}{(1-\delta^x)(1-\delta)} - \frac{1}{1-\delta^x}K & \text{otherwise} \end{cases}$$

Indeed, $S(0,x,n)$ is increasing in $n$ so that the optimal $n$ is always infinite when the procured quality is nil (i.e. $n=\infty$) and the optimal contract length depends on the sign of the derivative of $S(0,x,\infty)$ w.r.t. $x$ which is proportional to $K-[E[\theta]-\underline{\theta}]$. Even if the procured quality is nil and competition is maximal with all potential suppliers competing at any auction, still the buyer trades off the cost of running auctions more frequently (i.e. $K$) with the cost of being stuck with an inefficient firm for a long period (i.e. $E[\theta]-\underline{\theta}$). To see the latter, consider a contract that lasts two periods, i.e. $x=2$, and is assigned, say, to firm $i$. In the second period the buyer knows that with $n=\infty$ in the pool of $n$ firms there will be one with cost $\underline{\theta}$ which is more efficient than the procuring firm $i$ and with an efficiency gap equal to $E[\theta]-\underline{\theta}$. Note that the expected maximal surplus associated with $\underline{q}=0$ can be also rewritten with the following intuitive expression,

$$S(0,x,n)=S(0,\infty,\infty)+\min\{0,K-[E[\theta]-\underline{\theta}]\}\frac{\delta}{1-\delta}.$$

If quality is not particularly important for the buyer, then she admits all the firms at the auction stage, sets quality requirement as indicated in Lemma 2 and the implemented quality is nil. However, when quality is valuable then at the optimum the buyer may be induced to reduce both the number of firms thus reducing competition and the contract length so that she can react promptly against a quality shirking firm.

To investigate the effects of these changes of $x$ and $n$ on her payoff, we can decompose the difference buyer's difference in surplus in several terms, as follows

$$\begin{array}{l} S(q,x,n)-S(0,x,n)=\frac{1}{1-\delta}\left[V(q)-\psi(q)\right]+ \\ -\left[E\left[\theta''(n)\right]-\underline{\theta}\right]+\left[E\left[\theta''(n)\right]-E[\theta]\right]\frac{\delta^x}{1-\delta^x} \\ -K\frac{\delta^x}{1-\delta^x}-\min\{0,K-[E[\theta]-\underline{\theta}]\}\frac{\delta}{1-\delta} \end{array} \tag{2}$$

The first term is clearly the positive and direct effect of quality induced by relaxing the implementable quality constraint with a reduction of $x$ and or $n$. The second line shows the cost of reducing $n$ in terms of being served by a firm which is not the most efficient one (in the first period of the first auction when $x=\infty$ and in the first period of any future auction when $x=1$). The third line illustrates that more frequent auctions imply larger organization costs (the first term when $x=\infty$ and also the last one when $x=1$ because from tomorrow and for any period, the buyer organizes an auction at any period with a net cost $K-[E[\theta]-\underline{\theta}]$).

It is worth here to provide an alternative interpretation for term $K$ in the expression (2). Cost $K$ may well also be interpreted as a fixed set-up cost that each contractor pays to provide the good of whatever quality to the buyer and that has to be paid anew for any contractual relationship. Given that the act of provision of the good is verifiable, it is unavoidable for any winning supplier, as well as the decision for this set-up procurement cost $K$. However, it is also clear that each firm can entirely pass-through to the buyer the cost $K$ via the winning bid $b_w$ and our analysis remains unchanged both for the buyer and for any potential supplier. Under this view of $K$, our expression (2) can then be read as a balance between benefits of quality with economies of scale in contracting due to the fixed cost $K$. A shorter $x$ lets sustaining a larger implementable quality, but it comes at the cost of inefficient scale of procurement which is described by the third line in (2).

Expression (2) summarizes the costs and benefits of restricting competition through $n$ and reducing contract duration $x$ taking into account that these decisions help relaxing the constraint on implementable quality $\overline{q}\,(x,n)$. A strictly larger than zero minimum quality requirement $\underline{q}$ is then clearly desirable if the (net) per period value of quality $V(q)-\psi\,(q)$ is sufficiently large for certain values of $q$. In particular, we consider the following measure of how much non-contractible quality is important in terms of producing welfare. We state that *quality is "important"* if increasing quality from a zero level significantly augments the welfare, i.e. $V_q(0)-\psi_q\,(0)$ is large. We can now state the following result.

**Proposition 1 (Optimal Procurement with Competing Firms)** *If non-contractible quality is sufficiently important (i.e.* $V_q(0)-\psi_q\,(0)$ *is positive and large), then*

*(i) the buyer optimally sets quality* $\underline{q}$ *at its maximal implementable level* $\overline{q}\,(x,n)\neq 0$ *and restricts the pool and contract duration, i.e.* $n^s<\infty$ *and* $x^s\leq x_{\underline{q}=0}$*;*

*(iii) optimal* $n^s$ *and* $x^s$ *are complement.*

**Proof.** Consider $S\left(\underline{q},x,n\right)$ evaluated at $\underline{q}=0$ $x_0^s$, $n_0^s$ so that the maximal implementable quality constraint $\overline{q}\,(x,n)\geq\underline{q}$ is trivially binding. We now check whether the buyer can increase her payoff $S\,(0,x,n)$ by reducing $x$ and / or $n$.

For what stated in Lemma 1, reducing $x$ by a small amount $\epsilon$ (in case $x_0^s=1$, simply leave $x$ unchanged) and/or $n$ by a small amount $\varepsilon$, the buyer can relax the implementable quality constraint by an amount $\Delta>0$. The net effect of these changes in $\overline{q}\,(x,n)\,,x,n$ in the buyer's payoff is approximated by the following expression

$$
\begin{aligned}
&\frac{1}{1-\delta}\left[V_q(0)-\psi_q\,(0)\right]\Delta+\\
&-\left[E\left[\theta''\left(n_0^s-\varepsilon\right)\right]-\underline{\theta}\right]+\\
&-\left\{K-\left[E[\theta]-E\left[\theta''\left(n_0^s-\varepsilon\right)\right]\right]\right\}\frac{\delta^{x_0^s-\epsilon}}{1-\delta^{x_0^s-\epsilon}}+\\
&-\min\left\{0,K-\left[E[\theta]-\underline{\theta}\right]\right\}\frac{\delta}{1-\delta}.
\end{aligned}
$$

The first term is positive, the second is negative, the third has ambiguous sign and the forth is positive. Hence, if $V_q(0)-\psi_q\,(0)$ is sufficiently large then the buyer prefers to restrict the number of $n$ and $x$.

In order to check that a reduction of $n$ and $x$ is indeed optimal we need to check whether an equilibrium can emerge where, even if the buyer sets a strictly positive minimum quality requirement $\underline{q}>0$, still all the firms offer nil quality. It is immediate to check that this set of strategies cannot be an equilibrium because the winning bid would be $b_w=\theta''\,(n)+A$ for a nil quality so that the buyer would deviate optimally setting $\underline{q}=0$ obtaining the same level of quality but at a smaller price.

We are now interested in analyzing the relationship between optimal $x$ and $n$ when the implementable quality constraint binds. Differentiating with respect to $n$ the first order condition for

$x$, $\frac{\partial S(\overline{q}(x,n),x,n)}{\partial x} = 0$ we obtain $\frac{\partial x}{\partial n} = -\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n} / \frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial x}$ where $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial x} \leq 0$ for the second order condition. The expression $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n}$ can be decomposed as follows

$$\begin{aligned} \frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n} &= \frac{1}{1-\delta}\left[V_q(\overline{q}\,(x,n)) - \psi_q\,(\overline{q}\,(x,n))\right]\frac{\partial^2 \overline{q}(x,n)}{\partial x \partial n} + \\ &+ \frac{1}{1-\delta}\left[V_{qq}(\overline{q}\,(x,n)) - \psi_{qq}\,(\overline{q}\,(x,n))\right]\frac{\partial \overline{q}(x,n)}{\partial n}\frac{\partial \overline{q}(x,n)}{\partial x} \\ &- \frac{\delta^x Log(\delta)}{(1-\delta^x)^2}\frac{\partial E[\theta''(n)]}{\partial n} \end{aligned}$$

where $\frac{\partial^2 \overline{q}(x,n)}{\partial x \partial n} \geq 0$ as shown in the proof of Lemma 1. Then, if $V_q - \psi_q$ is large enough, $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n} \geq 0$ and $\frac{\partial x}{\partial n} \geq 0$, i.e. $n$ and $x$ are complements, otherwise, if the second and third term in $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n}$ prevail, we have $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n} \leq 0$ so that $\frac{\partial x}{\partial n} \leq 0$ and the two are substitutes. ■

This result shows that when non-contractible quality matters the buyer prefers excluding some firms thus reducing competition and also limiting the length of the procurement contract.[19] In fact, it is only by so doing that she can obtain a strictly positive maximal implementable quality $\overline{q}\,(x,n)$ and then also set a strictly positive minimum quality requirement $\underline{q}$. Reducing firms' profits, competition also reduces incentives to maintain their commitment for "reputation" by complying with the buyer's quality requirement. In addition, the buyer has another instrument to improve quality, namely contract length, so that when quality is a real concern, she can increase quality by reducing the length of the contract. As we have discussed above, this is not without costs for running auctions more frequently is costly both in terms of organizing auctions and in terms of lost economies of scale, but it also has the advantage of avoiding to end up stuck with inefficient firms for long periods.

We have shown that, in the analysis of $x$ and $n$ when the buyer implements a nil quality, the optimal values of these two strategic variables are independent. In fact, the buyer always admits all the firms and sets a contract length uniquely on the basis of the sign of the difference $K - [E[\theta] - \underline{\theta}]$ which is independent of $n$. This independence property of the optimal $x$ and $n$ breaks down when quality is a concern and the buyer prefers to obtain a strictly positive quality. In this case the two instruments can be both complements or substitutes. However, if quality is sufficiently important for the buyer they turn out to be complements so that any event which causes an optimal reduction of firms admitted at any auction also fosters a reduction of the contract length and *viceversa.*

# 4 Quality and Collusion

In this section we explicitly account for the possibility that firms may collude. At $t = -1$ firms decide whether to collude in the auction supergame. If a collusive agreement is reached, the most efficient firm (that with cost $\theta'$) is awarded the contract, procures the good for $x$ periods and receives

[19] As mentioned in the introduction, the negative effect of competition on quality and reputation is also emphasized by Stiglitz (1989). By comparing an auction with all $N$ participating firms against bilateral negotiation Manelli and Vincent (1995) show that it is better to have auction (i.e. a "larger" $N$) if the procured good is a standardized one and bargaining (i.e. smaller $N$) when the value of $q$ is large (they have a trade off between screening and quality implementation).

the payment by the buyer which the firm sets at the highest admissible price, i.e. the reservation price $r$. All the other firms abstain from bidding or submit not acceptable / winning bids so that collusion takes place with bid rotation. Deviation from a collusive agreement is punished in the harshest way. If a defection is observed, firms compete forever in all the following auctions (i.e. we employ grim trigger strategy). All the other features of the game are unchanged and described in Section 2.[20]

Consider the optimal procurement strategies described in the previous section, including the maximum implementable quality $\overline{q}(x,n)$ and exclusion rule $\sigma$ discussed therein. We now study firms' incentives to collude under these strategies.

For the collusive agreement to be sustainable at any auction stage, the second most efficient firm with cost $\theta''$ is the one with the highest incentive to deviate and should prefer not to undercut the most efficient one. If it does not deviate from the collusive agreement, this firm as well as any firm other than the most efficient one, can expect a collusive payoff $\Pi^C$.

Firms' costs are drawn anew at any auction stage from the interval $[\underline{\theta},\overline{\theta}]$ so that, contrary to standard models of collusion, incentives to deviate are not fixed once and for all but are stochastic. Hence, we need to consider a more sophisticated collusive agreement that also contemplates temporary phases of competition when firms' costs make too strong incentives to deviation. More precisely, colluding firms observing a cost vector realization $\boldsymbol{\theta}$ know that with a collusive pricing $r$ the second most efficient firm is too prone to deviation and to avoid a break down of the cartel prefer to temporarily revert to competition, until in a subsequent auction stage costs allow to sustain high collusive pricing. Clearly, all this does not affect firms' ability to detect deviations when costs status would concede collusive pricing and to punish such deviation. Hence, the expected future collusive profit $\Pi^C$ is

$$\begin{aligned}\Pi^C = &\tfrac{\delta^x}{n(1-\delta^x)}E_\theta\left[\overline{\theta}+A-\theta'(n)-\delta\tfrac{1-\delta^{x-1}}{1-\delta}\theta-\psi\left(\underline{q}\right)\tfrac{1-\delta^x}{1-\delta}\right]\Pr(b_w=r)+\\ &+\tfrac{\delta^x}{n(1-\delta^x)}E_\theta\left[\theta''(n)+A-\theta'(n)-\delta\tfrac{1-\delta^{x-1}}{1-\delta}\theta-\psi\left(\underline{q}\right)\tfrac{1-\delta^x}{1-\delta}\right][1-\Pr(b_w=r)]=\\ &\tfrac{\delta^x}{n(1-\delta^x)}\left\{\left[\overline{\theta}-E[\theta'(n)]\right]\Pr(b_w=r)+E\left[\Delta\theta(n)\right][1-\Pr(b_w=r)]\right\}\end{aligned}$$

where $\Pr(b_w=r)$ represents the probability that costs allow collusive pricing and $[1-\Pr(b_w=r)]$ is the probability associated to competitive pricing. Note that we are assuming that colluding firms agree to comply with the quality requirement and in the following we will study whether this is indeed the case or not.[21]

[20] Alternatively to collusion with bid rotation, firms may be able to use undetectable side transfers so that the most efficient firm at any stage game always wins and share the collusive surplus with all the other firms. In case the winning firm shirks on quality it will be excluded from future auctions but can be still compensated also in the future by the firms that are allowed by the buyer to participate. The number of firms that belongs to the collusive ring increases with time but collusion with quality shirking can be still an equilibrium if firms use transfers that also decline with time.

[21] It is well known (Rotemberg and Saloner 1986) that fixed price collusive agreements need not be optimal in a stochastic environment. As it is customary in this literature, we will not consider more sophisticated collusive schemes that smoothly vary with the realizations of costs. They are extremely difficult to derive analytically and are expected to imply similar results.

On the contrary, by deviating the second most efficient firm can ask a price slightly smaller than $r$ thus winning the auction. This firm may decide to deviate from the collusive agreement but to comply with quality so that it obtains the following payoff,

$$\Pi^D_{q>0} = \overline{\theta} - \theta''(n) + \frac{E[\Delta\theta(n)]\,\delta^x}{n(1-\delta^x)}$$

where again $\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$ is the expected payoff for future auctions upon deviation. Alternatively, the deviating firm can also decide to shirk on quality so that its profit turns out to be

$$\Pi^D_{q=0} = \overline{\theta} - \theta''(n) + \psi(\underline{q})\,\frac{1-\delta^x}{1-\delta}$$

Hence, the second most efficient firm at any auction does not deviate if the following incentive compatibility constraint is satisfied[22]

$$\Pi^C \geq \max\left\{\Pi^D_{q>0}, \Pi^D_{q=0}\right\} = \overline{\theta} - \theta''(n) + \max\left\{\frac{E[\Delta\theta(n)]\,\delta^x}{n(1-\delta^x)}, \psi(\underline{q})\,\frac{1-\delta^x}{1-\delta}\right\} \tag{3}$$

where the right hand side shows that the smaller is $\theta''$ the more profitable is the deviation for this firm and less probable is that collusion can take place. For a given procurement rules $(r, x, n)$ and quality requirement $\underline{q}$, (3) may or may not be satisfied depending on the value of $\theta''$. This shows that an optimal collusive agreement has to specify what firms should do when the value of $\theta''$ is small and does not satisfy (3). To this respect, consider the boundary $\hat{\theta}$ which is implicitly defined by the following indifference condition for collusion

$$\begin{aligned}&\tfrac{\delta^x}{n(1-\delta^x)}\left\{\left[\overline{\theta} - E[\theta'(n)]\right]\Pr(\theta'' \geq \hat{\theta}) + E[\Delta\theta(n)]\Pr(\theta'' < \hat{\theta})\right\} = \\ &\overline{\theta} - \hat{\theta} + \max\left\{\tfrac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}, \psi(\underline{q})\,\tfrac{1-\delta^x}{1-\delta}\right\}\end{aligned} \tag{4}$$

where the left hand side is a rewriting of $\Pi^C$ with $\Pr(b_w = r) = \Pr(\theta'' \geq \hat{\theta})$ and $1 - \Pr(b_w = r) = \Pr(\theta'' < \hat{\theta})$. When firms collude, they agree to do refrain from competition at the bidding phase only when $\theta'' \geq \hat{\theta}$ and, on the contrary, when firms observe that $\theta'' < \hat{\theta}$ (recall that firms are fully informed), they know that the collusive agreement would induce the second most efficient firm to deviate and then prefer to temporarily revert to competition (till the next auction stage when $\theta'' \geq \hat{\theta}$). In these cases, the agreement requires that firms compete so that the most efficient firm wins at a price $b_w = \theta''(n) + A$, as in the previous section. However, the buyer may make this event less or more probable by varying her strategy and then the boundary $\hat{\theta}$.

Consistently with the literature on collusion in stochastic environments (e.g. Rotemberg and Saloner 1986), in the following we will indicate that the buyer is able to deter collusion only if her strategy is such that there are no realizations of costs which satisfy incentive compatibility, i.e. uniquely when $\hat{\theta} \geq \overline{\theta}$. With this respect, note that from (4) we have that, if the contract lasts for infinitely many periods (e.g. if $x \to \infty$), or similarly, if all firms are admitted at the auction (i.e. if $n \to \infty$), then $\hat{\theta} > \overline{\theta}$ so that $\theta'' \geq \hat{\theta}$ is never met and collusion is certainly deterred.

We can now state our first result concerning collusion.

[22] It can be shown that if a deviating firm prefers also to shirk on quality, then it does so immediately.

**Proposition 2 (Collusion inducing $n$ and $x$.)** *Assume non-contractible quality is sufficiently important. While increasing implementable quality, small $x$ and $n$ facilitate collusion among suppliers.*

**Proof.** Collusion holds if the incentive compatibility constraint (3) is verified. Furthermore, $\Pi^C$ is larger the smaller is the threshold $\hat{\theta}$ because $\left[\overline{\theta} - E[\theta'(n)]\right] \geq E\left[\Delta\theta(n)\right]$. Hence, collusion is facilitated by a smaller $\hat{\theta}$ as defined in (4). We then need to show that smaller $x$ and $n$ imply a larger implementable quality $\overline{q}(x,n)$ but also a smaller $\hat{\theta}$.

Clearly, if the buyer wants to reduce $n$ and $x$, she does so in order to increase the implementable quality by relaxing the quality constraint $\underline{q} \leq \overline{q}(x,n)$. Hence, we will consider $\hat{\theta}$ defined by (4) when $\underline{q} = \overline{q}(x,n)$.

$$\begin{gathered}\tfrac{\delta^x}{n(1-\delta^x)}\left\{\left[\overline{\theta} - E[\theta'(n)]\right]\Pr(\theta'' \geq \hat{\theta}) + E\left[\Delta\theta(n)\right]\Pr(\theta'' < \hat{\theta})\right\} = \\ \overline{\theta} - \hat{\theta} + \max\left\{\tfrac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}, \psi\left(\overline{q}(x,n)\right)\tfrac{1-\delta^x}{1-\delta}\right\}\end{gathered}$$

If $\max\left\{\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}, \psi\left(\overline{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}\right\} = \frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$ then (4) can be written as

$$\frac{\delta^x}{n(1-\delta^x)}\left\{\overline{\theta} - E[\theta''(n)]\right\}\Pr(\theta'' \geq \hat{\theta}) = \overline{\theta} - \hat{\theta} \tag{5}$$

A small $x$ increases the L.H.S. so that to preserve the equality, $\hat{\theta}$ must reduce. The effect of $n$ is similar but more complex. On one side, a small $n$ increases the probability that a firm wins the auction, thus increasing the L.H.S. in (5). On the other side, a small $n$ means that the winning firm, on average is less efficient than it could have expected to be when $n$ is large. This is captured by the fact that the term $E[\theta''(n)]$ in (5) is increasing in $n$ thus implying that a small $n$ implies a small L.H.S. However, the net effect is necessarily positive. In fact, for a given path of cost realizations for any firm $i$, reducing the number of competitors makes this firm the most efficient one in the same cases as with the large $n$ plus some additional cases (in which it would not have been the most efficient firm with $n$ large).

Note also that with a smaller $\hat{\theta}$, firms can secure the larger collusive rents with higher probability thus increasing expected profits.

If $\max\left\{\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}, \psi\left(\overline{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}\right\} = \psi\left(\overline{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}$ the effects described above are enhanced because we know that $\overline{q}(x,n)$ increases with smaller $x$ and $n$ so that, to restore the equality in (4) $\hat{\theta}$ has to further reduce. ∎

Proposition 1 illustrates that prominence of quality may induce the buyer to both restrict the pool $n$ of bidders at any auction and reduce the length of contracts $x$, thus having more frequent auctions. However, we know that a small number of firms that frequently compete may be induced to collude. Indeed, Proposition 2 shows that the smaller are $n$ and $x$, the larger is the scope for collusion for the firms admitted in the pool and the easier is to sustain collusive agreements.

This Proposition illustrates a seeming trade-off between quality implementable through reputation and the risk of inducing collusion among suppliers. However, it is first instructive to check when the collusive ring prescribes to offer the required quality or not. For what we have stated above, we need to check colluding firms' incentive to provide quality both when the cost structure $\boldsymbol{\theta}$ allows for collusive pricing and when it does not so, i.e. respectively when $\theta'' \geq \hat{\theta}$ and $\theta'' < \hat{\theta}$.

When $\theta'' \geq \hat{\theta}$ so that collusive pricing takes place, one needs to compare the profit for the most efficient firm with the profit this firm can obtain when it shirks on quality. The former is clearly $\overline{\theta} - \theta'(n) + \Pi^C$, on the contrary, the latter is $\overline{\theta} + \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta} - \theta'(n)$. With $\theta'' < \hat{\theta}$ competitive pricing takes place and, by providing quality, the most efficient firm obtains a profit $\theta''(n) - \theta'(n) + \Pi^C$ and $\theta''(n) - \theta'(n) + \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}$ if it shirks on quality. Hence, in any event, the firm prefers to provide the required quality if

$$\Pi^C \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}. \tag{6}$$

We are now in a position to compare the maximal implementable quality when firms collude with that analyzed in Section 3 and associated with competition.

**Proposition 3 (Implementable Quality with Collusion)** *For given contract length $x$, number $n$ of firms admitted to the auction pool and reservation price $r$, the maximal implementable quality when firms collude is larger than with competing firms.*

**Proof.** We need to compare inequality $\Pi^C \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}$ with the equivalent one in the case of (always) competing firm, i.e.

$$\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)} \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}$$

Note that $\Pi^C$ is composed by two terms weighted with different probabilities. The first component is simply $\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$ and the second is $\left[\overline{\theta} - E[\theta'(n)]\right]\frac{\delta^x}{n(1-\delta^x)} \geq \frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$. Hence, for any $\Pr(\theta'' \geq \hat{\theta})$ we immediately have $\Pi^C \geq \frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}(\geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta})$. ■

The result is an immediate consequence of the fact that when firms collude they can expect a larger profit as compared with competition so that they are more reluctant to give up those larger (future) profits for an immediate but once and for all gain by shirking on quality. Note also that in principle the implementable quality is also constrained by incentive compatibility for collusion. In fact, a larger quality requirement $\underline{q}$ increases $\hat{\theta}$ so that with a larger probability the second most efficient firm is induced to deviate from collusion. However, for any probability associated to the event $\theta'' \geq \hat{\theta}$, the expected profit from collusion $\Pi^C$ is always (weakly) larger than the expected profit with competing firms $\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$ and this gives the result.

Proposition 3 shows that the seeming trade-off between implementable quality and collusion may well be misleading, as we now discuss in the next Section.

# 5 Optimal Procurement

Proposition 1 shows that, if the buyer is worried about quality, she may want to restrict participation $n$ and contract length $x$. However, by Proposition 2, both these two decisions tend to induce collusion. In addition, as shown in Proposition 3, the buyer may not be necessarily impaired by collusion because this may allow implementing higher quality. At first grasp this second possibility may sound dubious because, in general, collusion makes the buyer paying a larger price for the good. On the other hand, it should be noticed that when the buyer pays a higher price, she leaves larger rents to the contractor. Hence, there are several effect at play with collusion when quality is not contractible. Indeed, the buyer may optimally want to further restrict $n$ and/or $x$, exactly because this induces collusion and increases the implementable quality. We now explore how the emerging trade-off between collusion and competition is solved by the buyer.

If firms have the ability to collude, the buyer has to keep this possibility into account and adjust her strategies accordingly. Namely, she may chose a procurement contract $(r^s, x^s, n^s)$, a quality requirement $\underline{q}^s$, so as to systematically prevent collusion. In this case, in addition to what we have studied in the previous Section, the buyer must be sure that her strategies do not induce collusion, i.e. that with this procurement contract and $\underline{q}^s$, then constraint (3) is not verified.

Alternatively, the buyer may want to set procurement rules which do induce collusion. As stated in the Introduction, this possibility can be literally taken in terms of cartel formation, or alternatively, one can interpret the collusive pact as a metaphor for lawful agreements such as consortia among potential suppliers. With this interpretation incentive compatibility can then be seen as internal incentives for the stability of the consortium which is independent of any legal obligation among the partners in the agreement.

When the buyer anticipates firms' collusion, she sets a contract $(r^c, x^c, n^c)$ and quality requirement $\underline{q}^c$ so that $\overline{\theta} < \hat{\theta}^c$ where the boundary $\hat{\theta}^c$ is defined by

$$\begin{aligned} &\tfrac{\delta^{x^c}}{n^c(1-\delta^{x^c})} \left\{ \left[\overline{\theta} - E[\theta'(n^c)]\right] \Pr(\theta'' \geq \hat{\theta}^c) + E\left[\Delta\theta(n^c)\right] \Pr(\theta'' < \hat{\theta}) \right\} = \\ &\overline{\theta} - \hat{\theta}^c + \max\left\{ \tfrac{E[\Delta\theta(n^c)]\delta^{x^c}}{n^c(1-\delta^{x^c})}, \psi\left(\underline{q}^c\right) \tfrac{1-\delta^{x^c}}{1-\delta} \right\} \end{aligned} \tag{7}$$

The left hand side of (7) shows that when a firm deviates, this overtakes both the other firms and the buyer. However, from the auction just after the deviation, the firms recognize that a deviation occurred and adapt their strategies accordingly reverting to competition.[23]

Even if collusion may allow the buyer to increase the implementable quality, the analysis of the optimal procurement contract in this case is even subtler than this. In fact, if the buyer's unique concern were the highest non-contractible quality, then Lemma 1 tells us that she should set $n = 1$, possibly also with a short contract (i.e. $x$ small). Clearly, with a single firm, collusion would not be an issue and the implementable quality would be larger than with collusion associated with any number of firms $n^c > 1$. However, this strategy of restricting $n$ to a single firm implies a large cost in terms of *inefficiency in production*. Indeed, as we have discussed in the analysis of the surplus with condition (2), by restricting the number of admitted firms $n$, the expected efficiency parameter

[23] The R.H.S. in (7) shows that the buyer sticks to her collusion-inducing strategy even if she observes a defection. In Section 6 we discuss this behavior and show that, whenever the buyer has the commitment power to leave the contractual terms and the quality requirement unchanged upon cartel defection, then she has interests to do so and restrict to a stationary strategy. Alternatively, implementable quality with collusion would be lower but still larger than with competing firms.

of the winning firm reduces. With this respect, the advantage of collusion then is in allowing a large implementable quality associated with a (moderately) large $n$. In other terms, for any level of $n > 1$, colluding firms are ready to provide larger quality than when they compete and, at the same time, they produce more efficiently than a single firm would do. I our model the cartel is particularly powerful to sort out the most efficient firm also because firms are fully informed. With this respect it is worth recalling that firms repeatedly participating procurement contests have a lot of information on the other potential competitors and this largely eases their ability to obtain maximal rents from collusion also by increasing efficiency.

This consideration brings in a new ingredient in our analysis. There are several reasons that may induce a buyer to be concerned also with efficiency in production. For example and to be consistent with the objective we have stated for our buyer, here we assume that the buyer can ask for a participation fee to the sellers who are admitted in the restricted pool of $n$ firms (as in the case of "selective tendering"). We denote with $\tau^s$ the price the buyer asks for participation when the procurement contract induces competition and with $\tau^c$ when it induces collusion. Firms have a zero outside option if they are not allowed to participate so that we define the set of rational prices for participation with $\tau^i = \gamma\Pi^i$, $i = s, c$, where $\Pi^i$ represents the profit in the two regimes and the parameter $\gamma$ captures the fraction of firms' surplus that can be appropriated by the buyer.[24]

**Proposition 4 (Optimal procurement)** *(i) Assume the buyer does not care for efficiency in production (i.e. $\gamma = 0$). Then, the buyer prefers negotiating with a single firm when "quality is important". Otherwise, she prefers running auctions with many competing firms.*

*(ii) If efficiency is important for the buyer (i.e. $\gamma$ is large), then the buyer prefers negotiating with a single firm when "quality is important"; she prefers running collusion-inducing auctions for intermediate relevance of quality and auctions with many competing firms when quality is not important.*

**Proof.** The first statement of the proposition is immediate. Consider the optimal collusion-inducing contract with $n^c$, $x^c$ and associated implementable quality $\overline{q}^c$. Then, by setting $n = 1$ and keeping the contract length at $x^c$, the maximal implementable quality becomes larger than $\overline{q}^c$ because the single firm can expect larger rents than the $n^c$ colluding firms. Not caring for efficiency, the buyer is better off by restricting $n$ to 1 for she obtains a larger quality at the same price which, in both cases, is equal to the reservation price $r = \overline{\theta} + A\left(\underline{q}\right)$ with $A\left(\underline{q}\right) = \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta] + \frac{1-\delta^x}{1-\delta}\psi\left(\underline{q}\right)$. Recall that this is indeed the unique reservation price that guarantees provision of the good with any cost realization.

Assume now that $\gamma > 0$. We compare the buyer's payoff with and without collusion and when the buyer admits a single firm (i.e. $n = 1$). To simplify the analysis assume that

$$V(q) - \psi(q) = \lambda \geq 0$$

[24] This is certainly not the unique possibility for having the buyer also caring for efficiency. Alternatively, relaxing Assumption 3 the buyer is no more constrained to set the reservation price $r$ at the highest value in order to always guarantee procurement. In this case, setting a smaller reservation price may serve the buyer to appropriate part of the sellers' surplus which increases with their efficiency.

for any $q$, so that in any case the buyer wants to implement the maximal quality. The proof also holds for a strictly concave $V(q) - \psi(q)$, as long as this function reaches the maximum for a sufficiently large $q$.

The buyer avoids collusion if $\hat{\theta} < \overline{\theta}$ with $\hat{\theta}$ defined by (4) (which here does not depend on $\overline{q}$) and the maximal implementable quality $\overline{q}$ is defined by

$$\frac{E\left[\Delta\theta\left(n\right)\right]\delta^{x}}{n(1-\delta^{x})} = \psi\left(\overline{q}\right)\frac{1-\delta^{x}}{1-\delta}$$

In this case, welfare is

$$\begin{aligned} S\left(\overline{q},x,n\right) &= \frac{1}{1-\delta}\left[\lambda\overline{q} - \frac{\delta\left(1-\delta^{x-1}\right)}{1-\delta^{x}}E[\theta] - \frac{1-\delta}{1-\delta^{x}}K\right] + \\ &\quad -\frac{1}{1-\delta^{x}}\left\{\left(1-\gamma\right)E[\theta''\left(n\right)] + \gamma E[\theta'\left(n\right)]\right\} \end{aligned}$$

With a single firm $n=1$ the maximal implementable quality $\overline{q}(n=1)$ is defined by

$$\frac{\delta^{x}}{1-\delta^{x}}\left\{\overline{\theta} - E[\theta]\right\} = \psi\left(\overline{q}\right)\frac{1-\delta^{x}}{1-\delta}$$

(recall that $E[\theta'\left(n=1\right)] = E[\theta]$) and welfare is

$$\begin{aligned} S\left(\overline{q}(n=1),x,1\right) &= \frac{1}{1-\delta}\left[\lambda\overline{q}(n=1) - \frac{\delta\left(1-\delta^{x-1}\right)}{1-\delta^{x}}E[\theta] - \frac{1-\delta}{1-\delta^{x}}K\right] + \\ &\quad -\frac{1}{1-\delta^{x}}\left\{\left(1-\gamma\right)\overline{\theta} + \gamma E[\theta]\right\} \end{aligned}$$

Finally, collusion holds if $\hat{\theta}^{c} \geq \overline{\theta}$ (recall that for what stated above, here $\hat{\theta}^{c}$ does not depend on the implementable quality without collusion) and the associated maximal implementable quality $\overline{q}^{c}$ is defined by setting the R.H.S. equal to the L.H.S. in (6), i.e.

$$\frac{\delta^{x}}{n(1-\delta^{x})}\left\{\overline{\theta}\Pr(\theta'' \geq \hat{\theta}^{c}) + E[\theta''\left(n\right)]\Pr(\theta'' < \hat{\theta}^{c}) - E[\theta'\left(n\right)]\right\} = \psi\left(\overline{q}^{c}\right)\frac{1-\delta^{x}}{1-\delta}$$

In this case welfare can be written as

$$\begin{aligned} S\left(\overline{q}^{c},x,n\right) &= \frac{1}{1-\delta}\left[\lambda\overline{q}^{c} - \frac{\delta\left(1-\delta^{x-1}\right)}{1-\delta^{x}}E[\theta] - \frac{1-\delta}{1-\delta^{x}}K\right] + \\ &\quad -\frac{1}{1-\delta^{x}}\left\{\left(1-\gamma\right)\left[\Pr(\theta'' \geq \hat{\theta}^{c})\overline{\theta} + \Pr(\theta'' < \hat{\theta}^{c})E[\theta''\left(n\right)]\right] + \gamma E[\theta'\left(n\right)]\right\} \end{aligned}$$

We can now compare the buyer's expected surplus with and without the collusive agreement. To induce collusion we know that the buyer must set a number of bidders and a contract length such that $n^c \leq n^s$, $x^c \leq x^s$. The difference in the expected surplus can then be written as

$$\begin{aligned}
&S\left(\overline{q}^c, x^c, n^c\right) - S\left(\overline{q}^s, x^s, n^s\right) = \tfrac{1}{1-\delta}\lambda\left(\overline{q}^c - \overline{q}^s\right) + \\
&-\tfrac{\delta E[\theta]}{1-\delta}\left[\tfrac{1-\delta^{x^c-1}}{1-\delta^{x^c}} - \tfrac{1-\delta^{x^s-1}}{1-\delta^{x^s}}\right] + \\
&-K\left[\tfrac{1}{1-\delta^{x^c}} - \tfrac{1}{1-\delta^{x^s}}\right] + \\
&-\tfrac{1}{1-\delta^{x^c}}\left\{(1-\gamma)\left[\Pr(\theta'' \geq \hat{\theta}^c)\overline{\theta} + \Pr(\theta'' < \hat{\theta}^c)E[\theta''(n^c)]\right] + \gamma E[\theta'(n^c)]\right\} + \\
&+\tfrac{1}{1-\delta^{x^s}}\left\{(1-\gamma)E[\theta''(n^s)] + \gamma E[\theta'(n^s)]\right\}
\end{aligned}$$

The positive effects of collusion are indicated by the first and the second term. The first clearly relates to the positive effect that collusion has on quality because $\overline{q}^c \geq \overline{q}^s$. As for the second term, recall that for any period after the first in any contract, firms do not earn any rent because they do not have private information with respect to the buyer for those periods who has to reimburse the expected cost $E[\theta]$ for any period. Hence, the longer is the contract, the larger is this reimbursement, as we have in the comparison between collusion and competition, $x^c \leq x^s$. On the contrary, the negative effects of collusion are represented by all the other terms. In particular, collusion requires more frequent auctions and this is costly (the third term). In addition, collusion is costly because it implies a larger price (this is effect is captured by setting $\gamma = 0$ in the last two terms) and / or it implies that producing firms are less efficient (this is described by setting $\gamma = 1$). In fact, for any $n^c \leq n$ we have $E[\theta''(n)] \leq E[\theta''(n^c)] \leq \overline{\theta}$ and $E[\theta'(n)] \leq E[\theta'(n^c)]$. Note also that the these costs of collusion are strengthened by the fact that they arise (more) frequently because $x^c \leq x^s$ implies $\frac{1}{1-\delta^{x^c}} \geq \frac{1}{1-\delta^{x^s}}$. In any case, if $\lambda$ is sufficiently large, i.e. quality is important, then the first term dominates.

Finally, we compare collusion against negotiation with a single firm. In this latter case, the buyer sets $n = 1$ and let the implementable quality be $\overline{q}^1$ and the contract length $x^1$, Then, considering for simplicity $x^c = x^1$ we then have

$$\begin{aligned}
&S\left(\overline{q}^c, x^c, n^c\right) - S\left(\overline{q}^1, x^c, 1\right) = \tfrac{1}{1-\delta}\lambda\left(\overline{q}^c - \overline{q}^1\right) + \\
&-\tfrac{1}{1-\delta^{x^c}}\left\{(1-\gamma)\left[\Pr(\theta'' \geq \hat{\theta}^c)\overline{\theta} + \Pr(\theta'' < \hat{\theta}^c)E[\theta''(n^c)]\right] + \gamma E[\theta'(n^c)] - (1-\gamma)\overline{\theta} - \gamma E[\theta]\right\}
\end{aligned}$$

The first term is negative because $\overline{q}(n = 1) \geq \overline{q}^c$. On the contrary, the second is positive. In fact, for any $n^c$ we have $E[\theta''(n^c)] \leq \overline{\theta}$ and $E[\theta'(n^c)] \leq E[\theta]$. Indeed, reducing to $n = 1$ the number of firms implies both that the buyer will have to pay a larger price (on average) and that the producing firm will be less efficient, as compared with collusion and $n^c > 1$. It follows that if $\lambda$ is sufficiently large, then it is better to set $n = 1$ than having more and colluding firms and vice versa. ■

As we have discussed above, collusion leaves large rents to the firms and rents are necessary to induce them to provide high level of quality. However, if the buyer does not care for efficiency in production, then all what she can obtain in terms of quality with collusion can be replicated by admitting a single firm to the auction. Indeed, this guarantees the maximal rent and then the highest implementable quality. However, a single firm comes at the cost of inefficient production because the expected cost of the firm is higher than the expected cost of the most efficient firm when $n$ firms are admitted at the auction stage, i.e. $E[\theta] \geq E[\theta'(n)]$. If the buyer does care for efficiency, then a trade off arise. In this case, restricting $n$ and $x$ so that collusion emerges may become optimal because it allows for balance between higher implementable quality as compared with competition and higher efficiency in production as compared with $n = 1$. Also note that when comparing optimal procurement inducing collusion with optimal procurement with competing firms, these two schemes are equivalent with respect to efficiency in production which, in both cases, is allocated to the most efficient firm. Hence, it follows that comparison between these two alternative procurement process ultimately rests uniquely on their properties in terms of implementable quality and price for procurement.

Finally we also note that if firms are homogenous as for costs so that $E[\Delta\theta(n)]$ is small, then the implementable quality with competition is also small (see condition (1)) because informational rents (profits) are necessary to provide firms' incentives in quality procurement. Hence, we have the following:

**Corollary 1** *If firms' cost heterogeneity is small, then auctioning with competing firms is dominated either by bilateral negotiation or collusion-inducing procurement.*

If firms are very homogeneous in terms of costs so that $E\left[\theta''(n)\right] \simeq E\left[\theta'(n)\right]$, competing firms earn very low rents and the implementable quality is then very low. This is also the case where restricting competition to $n = 1$ may turn to be optimal because the loss in terms of efficiency in production is small. On the other hand, if firms are very heterogeneous, then the buyer can implement a high quality also with many competing firms. Hence, collusion inducing procurement shows its maximal strength for intermediate values of heterogeneity in costs, exactly because it mediates quality with efficiency.

We think the analysis in this section contributes to the literature on the mode of transaction for procurement. On the two extremes, one could seek several suppliers thus relying on the benefits of competition, or, otherwise, one could bargain with a single seller to avoid the drawbacks of competition when quality is not contractible. If auction mechanisms are the natural implementation of competitive tendering, there are several protocols the buyer may rely on to bargain with a single seller. Manelli and Vincent (1995) have analyzed sequential bargaining with take-it-or-leave-it offers designed by the buyer. In a different context, Bulow and Klemperer (1996) reach the conclusion that a seller should prefer using auctions to the best possible negotiation with one less buyer. Bajari and Tadelis (2001) bundle the choice between auctions and negotiation with the choice of the contractual form (with the two extremes of fixed-price and cost-plus contracts). They argue that fixed-price contracts typically lend themselves to competitive bidding in auctions whilst cost-plus contracts are normally negotiated.

It is important to notice that the few papers that have dealt with the choice of the mode of transaction have limited the analysis mainly to a framework with no repetition. However, as we have previously emphasized, a main ingredient of procurement is the need to repeat the procuring process over time and, with non-contractible quality, the level of competition (i.e. the number $n$ of firms admitted at the auction) is only one relevant dimension in the procurement process. Indeed, the duration of the relationship is important. A long term relationship creates an implicit incentive so that procuring firms have incentives to establish reputation and the buyers may prefer long lasting contracts when quality is not contractible. This creates the bridge between our analysis and the important strand of literature dealing with trust and reputation formation in long-term relationships (Fehr, Brown and Falk 2004). Hence, our analysis introduces the novelty of combining these two elements in the choice of a trading procedure, the degree of competition and the length of the awarded contract.

## 6 Extensions

In this section we consider a number of extensions of our base model and check the robustness of the previous results.

**Finite number $N$ of firms.** The analysis in the previous section has relied on Assumption 2 stating that the number of potentially active firms $N$ is infinite. Clearly, when the buyer prefers to negotiate with a single firm, then it is immaterial whether $N$ is finite or not.

When the buyer induces competition between $n > 1$ firms, assume first that the firms admitted at the auction are $n < N$ and consider a candidate strategy for firms such that at any auction stage the most efficient firm provides the minimum required quality and the buyer's exclusion rule $\widetilde{\sigma} = (\tilde{T}, \underline{\tilde{q}})$ with $\tilde{T} > 1$ contemplates the replacement of cheating firms with one among the $N - n$ firms that were previously not allowed to participate. Otherwise, if the firm procures at least the minimum quality, then all firms are kept into the pool of qualified $n$ firms, as in the base model. We now check whether this can be an equilibrium of the game finite $N$. Verifying if a single firm may have any incentive to deviate and cheat on quality we note that, if the firm does so, according to $\widetilde{\sigma}$ the buyer will replace it with one among those $N - n$ firms excluded from the pool of potential suppliers and, given that all other firms will provide the required quality, the pool of active firms $n$ will remain the same for all the subsequent auction stages. Hence, the deviating firm that cheats on quality will be excluded forever, because in the candidate equilibrium at hand all other firm except the deviating one will provide minimum quality. Note that for this to hold true it simply suffices that the length of the exclusion $\tilde{T}$ is at least one auction stage, i.e. $\tilde{T} > 1$. This implies that as long as $n < N$ and with an exclusion rule $\widetilde{\sigma}$ such that $\tilde{T} > 1$, the maximal implementable quality for the buyer is again $\overline{q}\,(x, n)$ as defined in Lemma 1.

Importantly, note also that $n < N$ makes the threat of exclusion of cheating firms by the buyer a credible one so that the buyer has no incentive to deviate from the exclusion rule $\widetilde{\sigma}$ discussed above. Indeed, at any subsequent auction all firms are identical from the buyer's view point so that replacing one firm with another in the pool of $N - n$ firms is costless. Hence, the main difference with the case $N = \infty$ is that, here, a necessary condition for $\overline{q}\,(x, n) > 0$ is $n < N$. It is also clear that if $n = N$ with $N$ finite, there is no exclusion rule $\sigma$ that can guarantee a strictly positive maximal implementable quality.[25,26]

It may be also possible that the coalition of all firms shirks on the quality requirement (both with and without collusion) providing nil quality. In this case, the minimal exclusion of $N - n$ auction rounds becomes also the maximal exclusion length.[27] Interestingly, this shows that a finite $N$ introduces another motive for reducing the pool of admitted firms. In fact, a smaller $n$ now makes the punishment for quality shirking tougher so that the buyer may be induced to further restrict $n$.

As previously discussed, a possible interpretation of the buyer inducing collusion can be that she allows the formation of consortia. It is the interesting to note that with finite $N$, the buyer cannot rely on consortia involving all the potential firms because, otherwise, there would be no room for punishment.

**Non-stationary strategies.** Our environment is stationary as for exogenous variables and this justifies our choice to analyze stationary strategies for the firms and the buyer. Concerning the case of procurement that induces firms agreements, we should note that in what illustrated above, the buyer does not change her strategy when she realizes that collusion has broken down due to a cartel deviation by one of its members (see the right hand side of equation (7)). This is clearly not the unique behavior one could envisage for the buyer on this occurrence. Indeed, assume now that, on the contrary, the buyer may react when she observes a deviation. In particular, by observing a low winning bid, the buyer learns a deviation occurred and then reverts to the optimal contract for competition. This is the case when $b_w$ is such that $b_w \geq \hat{\theta}^c + \psi\left(\underline{q}^c\right)\frac{1-\delta^{x^c}}{1-\delta}$ where the right hand side is the maximum price that would emerge under the collusion agreement when firms temporarily abstain from colluding. Note that if this condition is not satisfied, the buyer is not even able to detect a deviation from the cartel.

We now check whether the buyer has any incentive to react differently with respect to what previously illustrated. Recall that the buyer may want to induce collusion only if this allows to increase the implementable quality. It then follows that the required quality with collusion $\underline{q}^c$ cannot be afforded by firms when they are induced to competition by a defection. Hence, when collusion breaks down, if the buyer sticks to her collusion-inducing strategies, firms will be induced

[25] Assume $n = N$ and consider the exclusion rule $\sigma'$ such that a firm is excluded forever in case he cheats on quality and retained otherwise (similarly if exclusion takes place for a shorter and finite number of periods). By excluding a cheating firm the buyer limits the number of firms admitted to the next auction stage which becomes $N-1$. However, this exclusion at the following auctions is not credible because the buyer gains by increasing competition with a larger number of bidders and would like to admit the cheating firm at next auctions. Any firm anticipates that there will be no punishment for quality cheating and the quality provided in equilibrium will be nil.

[26] With $N$ finite, an equilibrium always exists where all firms provide nil quality, the buyer admits all the firms (i.e. $n = N$) and the exclusion rule $\sigma$ keeps all firms independently of the procured quality or, equivalently, the minimum quality requirement is kept at $\underline{q} = 0$. Indeed, admitting all firms at the auction is optimal for the buyer given this behavior by the firms, becuase expected cost and price are obviously decreasing in the number of potential suppliers. This type of equilibrium is clearly of limited interest to our analysis.

[27] A coalition-proof exclusion rule is easily obtained by complementing the initial rule with one that says that if more firms deviate, firms excluded before are reintegrated in the auction process only after all never-excluded firms have been chosen, and in order of exclusion. The punishment for multiple deviations would then be exclusion for at least $N - n$ periods.

to shirk on quality so that they will be excluded from future auctions. This, in turn, implies that firms' payoff following a deviation are very low and collusion is strengthened. In other words, when the buyer can commit and collusion is desirable, it is in the buyer's interest not to revert to competition in future auctions if a deviation from collusion occurs. This provides support to our choice of considering stationary strategies in the previous Sections.

If, on the other hand, the buyer cannot commit to her strategies, then her sequentially optimal strategy upon (detectable) cartel defection is the optimal strategy with competition. In this case, the payoff of a firm deviating from the cartel would be larger and collusion more difficult to sustain. It is however clear that collusion can still allow for implementable quality larger than competition, albeit smaller than when the buyer can commit to her strategies. Hence, our main results qualitatively hold.

**Discontinuing procurement.** Assumption 3 requires that the buyer procures the good at any point in time. This clearly puts her in a very weak position with colluding firms. In fact, to guarantee procurement, she has to set a reservation price $r$ which is sufficient to pay back the cost of the most inefficient firm for any level of required quality. Clearly, colluding firms can extract all the surplus by asking a price equal to $r$. It is then immediate that abandoning this assumption, $r$ can be optimally set at a lower value even if this may discontinue procurement for certain realization of costs. Consider now a (small) reduction of $r$ from its value in the previous analysis $r < \overline{\theta} + A$. Given that the winning bid with competition is $b_w = \theta'' + A$, this reduction of $r$ has no effect on firms' rents with competition and then also in implementable quality. Consider now collusion where firms are able to with a price equal to $r$. It is then clear that in this case reducing $r$ has a direct effect in limiting colluding firms' rents. We then have that the buyer pays less for procurement but implementable quality also reduces. Note however that as long as the price paid with collusion is larger than with competing firms (a necessary condition for collusion itself), our qualitative results on optimal procurement still hold.

**Scoring rules.** In principle, firms' offers could be formed by a price bid and a quality level that the firm is ready to supply. The buyer could then rank offers according to a scoring rule function of price and of contractible together with non-contractible quality dimensions offered by each firm. The buyer may then exclude the winning firm in case the latter does not provide the promised non-contractible quality. This form of competition with bid-quality offers and scoring rule may be (although it is not necessarily the case) preferable to the one we study only if there is heterogeneity in firms' cost for quality, i.e. in case the per-period cost of firm $i$ is $\psi(q,\theta)$ with $\frac{\partial^2\psi}{\partial q\partial\theta} \neq 0$. This is an interesting line for further extending our analysis to scoring rules.

**Asymmetrically informed firms.** In our analysis we have assumed that firms are fully informed (Assumption 1). This is certainly a simplifying assumption as the literature on repeated games with asymmetric information shows (see for example Compte 1998 and Kandori and Matsushima 1998) together with the few paper dealing with collusion in auction within such setting (see Aoyagi 2003 and Blume and Heidhues 2004). This is also a possible extension for our model. However, notwithstanding the intricacies of collusion in auction with asymmetrically informed bidders we expect that our simple and direct trade-offs still hold in such a sophisticated environment.

# 7 Concluding Remarks

In this paper we have analyzed the relationships between reputation, non-contractible quality and collusion in a repeated procurement context. Repetition in the procurement relationship allow the emergence of reputation as an incentive device inducing firms to supply acceptable levels of quality. Restricting participation and contractual length, the buyer increases firms' incentives to provide quality and hence, maximal implementable quality. On the other hand, running more frequent auctions among few bidders facilitates collusive agreements among suppliers. We have analyzed this trade-off showing that when non-contractible quality and variability in suppliers' efficiency are both important, short contract duration and a collusive agreement between a few eligible sellers may maximize welfare and leave the buyer better off. If quality is a major concern, the buyer can do even better by negotiating with a single firm, even if this may clash with efficiency in production. Hence, we show that the optimal procurement strategy involves a subtle balance between firms' rents, incentives for quality, collusion and efficiency.

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