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Exclusive vs Overlapping Viewers: Two-Sided Multi-Homing in Media Markets

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Abstract

This paper investigates competition for advertisers in media markets when viewers can subscribe to multiple channels. A central feature of the model is that channels are monopolists in selling advertising opportunities towards their exclusive viewers, but can only obtain a competitive price for advertising opportunities to multi-homing viewers. Strategic incentives of firms in this setting are different than in previous models of media markets, like Anderson and Coate (2005). If viewers can only watch one channel then firms compete for marginal consumers by reducing the amount of advertising on their channels. In our model channels might increase levels of advertising to reduce the overlap in viewership. For example, in case of two-sided multi-homing stations always overadvertise. We take an account of the differences between the predictions of the two types of models and find that our model is more consistent with recent developments in the European broadcasting market. We also show that if channels can charge subscription fees on viewers, then ex ante symmetric firms can end up in an asymmetric equilibrium in which one collects all or most of its revenues from advertisers, while the other channel collects most of its revenues via viewer fees.

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1 Introduction

Advertising expenditures for television commercials have grown sharply in recent years in almost all industrialized countries. In Germany expenditures for television advertising amounted to 7.74 Mrd. Euro in 2004 which is in real terms three times higher than the expenditures in 1990.[1] Similar numbers can be found in almost all western countries. At the same time there has always been a discussion if regulation concerning the amount and the content of commercials is necessary.[2] In the European Union there exists nowadays an advertising ceiling of 12 minutes per hour but the media commission of the European Union is discussing whether to abolish this law. By contrast, in USA an advertising cap was abolished already in 1981 by the Federal Communication Commission.[3] So the question arises if there is too much advertising in an unregulated broadcasting market which would render a regulation necessary or if such a regulation would be harmful.

In a recent paper Anderson and Coate [2005] provide a careful analysis of these issues. They model markets for media advertising as a two-sided market with both positive and negative externalities: advertisers care positively about the number of viewers on the same platform, but viewers dislike more advertising. Advertising is assumed to be informative. The paper provides useful insights in understanding strategic interaction in media markets and the welfare properties of equilibrium under different market configurations. It also inspired a fastly growing literature (see a brief summary of this below) that builds on the model of Anderson and Coate.

In this paper we revisit the analysis of markets for media advertising and point out that some of the results from the previous literature only apply to settings in which viewers are not allowed to multi-home, i.e. to connect to multiple platforms.[4] What makes this observation particularly relevant is that in most media markets restricting viewers to single-home is not a realistic assumption: viewers can subscribe to multiple TV channels, listeners can buy equipment to listen to many different radio stations, and readers can subscribe to multiple magazines with similar content.[5] We show that adding this extra element of realism changes the nature of competition in the market substantially, and in

[1] It is also 44% of all advertising expenditures (including radio, newspapers, and magazines) in 2004 while it accounted only for 25% in 1990 (http://www.agf.de/daten/werbemarkt/werbespendings/).

[2] See Motta & Polo (1997) for a history and the problems concerning advertising regulation on TV.

[3] There is still a ceiling on the amount of advertising during children's programmes: 12 minutes per hour on weekdays and 10 minutes per hour on weekends.

[4] Most of the time we will use the terminology "viewers" for ease of exposition, but the model we present applies to various forms of broadcasting, where the precise terminology would be "listeners" or "readers".

[5] Of course, at one moment of time a viewer can only watch one programme (and a listener only listen to one radio station) but the relevant time period in these markets is normally the course of a day or an evening in witch a viewer (or listener) multi-homes a lot.

some cases reverses the results obtained in previous models. The implication of this is that it is important to make a distinction between exclusive viewerships and overlapping viewership when investigating firms' decisions in media markets.

A central feature of the model is that overlapping viewers are less valuable for platforms than exclusive ones, for two reasons. The first is a direct effect: viewers subscribing to a second channel might substitute time away from watching the first channel, which makes the first channel less attractive in a setting of informative advertising (there is less chance of "hitting the target"). The second effect comes from the fact the platforms provide alternative ways of reaching a multi-homing viewer, and therefore compete with each other in selling advertising opportunities to these viewers. Therefore they can only obtain a competitive price for selling advertising opportunities to these viewers, which is equal to the incremental value of trying to reach a viewer through a second channel. As opposed to this, platforms are monopolists with respect to selling advertising opportunities towards their exclusive viewers, and they can extract all the surplus for this transaction from advertisers.[6]

If viewers can only single-home, then platforms compete with each other for viewers (and ultimately for advertisers) by reducing the amount of advertising on the platform and thereby stealing potential consumers away from the other platform. This consideration is not valid when viewers can multi-home. In general, the reduced value of multi-homing viewers induces platforms to distort their advertising decisions, in order to reduce or eliminate the overlap of either on the viewer side or on the advertiser side. Depending on which side it is less costly to reduce overlap, advertising levels can be distorted both upwards or downwards. We conduct the analysis for both when platforms are owned by competing firms, and when they are owned by a monopolist provider. In the latter case we distinguish between the case when the monopolist can charge a different price for multi-homing advertisers (discriminating monopolist) and when it cannot. We show that overlapping viewers are more valuable for a discriminating monopolist than either to a nondiscriminating monopolist or to competing firms, and therefore the former distort their advertising decisions less.

We address whether the entrance of a competing station increases or decreases advertising on a platform, whether in the presence of multiple platforms competition or monopolist ownership implies higher levels of advertising, and whether market equilibrium implies too much or too little advertising. We show

[6]That multi-homing viewers are worth less to advertisers is also an empirically well documented fact. In 1980 already Fisher et al. found that "a spot on a large audience provides a greater number of different viewers than two spots on a smaller audience, because the smaller audience may have some viewers in common," and so advertisers are willing to pay more for it than for the two spots together. For a more recent estimation with a similar conclusion see Chwe (1998).

that for each of these questions we get different answers in a model in which viewers can multi-home than in one in which viewers are restricted to single-home. There is a particularly stark difference in how the level of advertising responds to entrance of a competitor. When platforms are weakly differentiated in a single-homing model entrance decreases the level of advertising on a platform. This is because the incumbent firm is forced to make its channel more appealing to viewers to avoid losing its marginal viewers. However, if viewers can multi-home and the value of overlapping viewers is low platforms increase advertising after the entry, to reduce the overlap in viewership. Moreover, we get a clear welfare result in the case of multi-homing of both sides which is probably the most relevant case in practice. in this case there is overadvertising in both duopoly and monopoly and the number of advertising is smaller in duopoly than in monopoly.

We also point out that in a model with viewer multi-homing platforms' profits are not always monotonically increasing in the attractiveness of viewers. The intuition behind this is that if there is overlapping viewership, then an increase in the attractiveness also increases the overlap, which for a region of parameter values dominates the positive effect of channels becoming more desirable for viewers.

Finally, we show that if platforms are allowed to charge viewer fees, then in the model with viewer multi-homing a certain type of asymmetric equilibrium can arise in a perfectly symmetric setting. In these equilibria one platform collects most or all of its revenues from advertising, while the other platform collects most of its revenues via viewer fees and provides little advertising. These phenomenon can be observed for example in the cable television market ("standard" cable channels and HBO).

Our model is consistent with recent developments in the European broadcasting market that are difficult to reconcile with models of media markets in which viewers can only single-home. One is the ITV premium puzzle in the UK broadcasting market. ITV is the leading commercial channel in the UK but in recent years it lost some audience share due to competition of many new channels. Despite this fact it is prevalent that ITV is enjoying a premium in its advertising price per 10,000 eye balls over the newer channels and that this premium has even increased lately. A common explanation that is given in a Commission Report (2003) and that is addressed in our model is that reaching 10 mill. eye balls on ITV is worth more than reaching 10 mill. eye balls on the smaller channels. The reason is that it is likely that on the smaller channels an advertiser reaches less that 10 mill. people because some of them watched the commercial twice.
A recent development in the German broadcasting market is that in the last five years the amount of time people spent on watching TV increased steadily, while the price of commercials significantly decreased. This is consistent with our result that an increase in the desirability of channels might decrease the

advertising revenues of broadcasters, because of increasing the overlap in viewership. Another stylized fact we point out is the strong resistance of private channels against lifting advertising regulations on public broadcasters. In a model in which viewers can only single-home it is always beneficial for a platform if there is more advertising on another platform, because it becomes easier to steal viewers from it. However, in a model with multi-homing on the viewer side a platform can generate more advertising revenue if the other platform is not allowed to broadcast commercials.

There is an enormous and diverse literature on advertising. For a comprehensive summary, see Bagwell (2005). There is also a large and growing literature on two sided markets. For a survey of this literature, see Rochet and Tirole (2005). Below we only mention some recent paper which are in the intersection of the above lines of research, since they are the closest to our work. The recent literature on media markets started with the already mentioned paper by Anderson and Coate (2005). We use their model framework, with some modifications. Particularly related to our investigation is section 7 of their paper, in which they briefly investigate a two-period model in which viewers can switch from one channel to the other after period 1. It is shown that in this case the possibility of underadvertising is mitigated. Choi (2004) uses the Anderson and Coate framework to investigate how regulating the number of stations or the amount of advertising affects welfare. Kind et al. (2003) and Barros et. al. (2003) consider vertical mergers between stations and advertisers. Crampes et. al. (2005) compare price competition and quantity competition in media markets. Dukes and Gal-Or (2003) and Peitz and Valletti (2004) extend the analysis by examining location choices of stations. Finally, Armstrong (2005), using a two-sided market model with no negative externalities, investigates whether stations prefer charging advertisers on a per-consumer basis or on a lump-sum basis.

2 The Model

The model is a version of the one in Anderson and Coate (2005), with two-sided multi-homing and homogenous advertisers. It is a two-stage game with three different types of players: platforms owner(s), viewers, and advertisers.

Platforms

There are two platforms (channels), indexed by $i \in \{0, 1\}$. We will investigate both the case in which the same owner operates both platforms, and the case in which the channels are competing. The owners of platforms set the advertising levels a_0 and a_1, with the aim of maximizing profits.[7] In the

[7] Alternatively, we could consider a model in which platforms set prices. This leads to similar conclusions as the quantity competition we investigate, but the analysis (in particular characterizing the range of parameter values resulting in different types of equilibria) is technically more difficult.

monopoly case we also investigate the possibility that the monopolist can sell joint advertising slots for the two platforms. In this case the monopolist sets three quantities: $\overline{a}_0$, $\overline{a}_1$ and $\overline{a}_{01}$, where $\overline{a}_0$ and $\overline{a}_1$ are the quantities of single advertising slots on the two platforms, while $\overline{a}_{01}$ is the quantity of joint advertising slots. The resulting total amount of advertising on the channels is then $a_0 = \overline{a}_0 + \overline{a}_{01}$ and $a_1 = \overline{a}_1 + \overline{a}_{01}$. We refer to the latter case as a price discriminating monopolist (since the equilibrium price of a joint advertising slot will typically be different than the sum of the prices of single advertising slots), while we refer to the case when a monopolist can only set two quantities as a nondiscriminating monopolist.

Viewers

There is a continuum of viewers with mass M, uniformly distributed on $[0, 1]$. Viewers can decide to watch both channels. A viewer who is located at position x_j obtains a net viewing benefit of $\beta - \gamma a_0 - \tau x_j$ if only watching channel 0, and $\beta - \gamma a_1 - \tau(1 - x_j)$ if only watching channel 1. Viewers have heterogeneous tastes, with those located at point 0 liking channel 0 most and those located at point 1 liking channel 1 most. The marginal travel cost is τ. β represents the base level of utility from listening to one's ideal channel. Finally, viewers dislike advertising and γ represents the nuisance cost parameter concerning commercials. As in Anderson and Coate (2005), viewers do not get any positive value from advertising, because advertisers are monopolist producers of differentiated products and therefore they can extract all the consumer surplus (see below in more detail).

A viewer who watches both channels obtains utility $u(\beta - \gamma a_0 - \tau x_j, \beta - \gamma a_1 - \tau(1 - x_j))$. We assume that u is increasing in both variables, and that $u(\beta - \gamma a_0 - \tau x_j, 0) = \beta - \gamma a_0 - \tau x_j$ and $u(0, \beta - \gamma a_1 - \tau(1 - x_j)) = \beta - \gamma a_1 - \tau(1 - x_j)$. The above specification implies that a viewer watches channel 0 iff $\beta - \gamma a_0 - \tau x_j > 0$, and watches channel 1 iff $\beta - \gamma a_1 - \tau(1 - x_j) > 0$ (in particular if both of the above terms are positive then the viewer watches both channels).

Advertisers

There is a continuum of advertisers with mass N. To simplify the analysis, we restrict attention to the case when advertisers are homogeneous. In Section 6 we shortly discuss how are results extend to the case of heterogeneous advertisers.

Advertising is informative. Each advertiser is a monopolist producer of a differentiated product, with constant marginal cost zero. Consumers have to get informed of the product through advertising in order to be able to buy it. For each product, a fraction q of the viewers (randomly selected) has a reservation value normalized to be A, while the rest of the viewers have reservation value

0. Because the advertiser is a monopolist producer of its product, it charges a price of exactly A.

The gross value of an advertising slot on a platform for an advertiser is equal to the expected increase in sales revenues that the ad generates, which depends on the number of viewers who watch this channel, the time they spend on watching it, and whether the same viewers can get informed about the product through watching the other channel. If the amount of time a viewer spends on watching a channel is $t \in [0,1]$, there is $\varepsilon \in [0,1]$ probability that a viewer does not pay attention to a commercial when it comes on, and commercials appear at random times on the channel, then the expected value of advertising to this consumer is $t(1-\varepsilon)qA$. If the same viewer watches two channels, and spends t amount of time for watching each channel, then advertising to this viewer through both channels yields an expected value $[t(1-\varepsilon)+t(1-\varepsilon)(t\varepsilon)]qA$. In what follows we simplify the model by assuming that each viewer who only watches one channel spends the same amount of time, t' on watching the channel. Similarly, each viewer who watches both channel spends the same amount of time, t'' on watching each channel.[8] Furthermore, we assume that $t'' \in [t'/2, t']$. In the limit case $t'' = t'$ a viewer does not substitute any time away from the first channel when subscribing to a second channel.

This formulation leads to a reduced form in which the expected value of advertising to a consumer who watches exactly one channel is ω, the expected value of advertising through exactly one channel to a viewer who watches two channels is $\omega' \in [\omega/2, \omega]$, while the expected value of advertising through both channels to a viewer who watches two channels is $\omega' + \omega''$ where $\omega'' \in (0, \omega')$.[9] The incremental value of reaching the viewer with a commercial through a second channel (ω'') is smaller than the value of reaching her through the first channel (ω') because there is a chance that the viewer already got informed about the product through the first channel.[10]

Let n_0 denote viewers who watch channel 0 (independently of also watching channel 1), n_1 denote viewers who only watch channel 1 (independently of also watching channel 0), and n_{01} denote viewers who watch both channels. Let p_i denote the unit price of an advertising slot on platform i. Then the profit of an advertiser is:

[8] More general formulations, in which the amount of time a viewer spends on watching a channel depends on the utility the viewer obtains from connecting to the channel, would imply the same qualitative conclusions.

[9] In particular, $\omega = t'(1-\varepsilon)qA$, $\omega' = t''(1-\varepsilon)qA$ and $\omega'' = [t''(1-\varepsilon)(t''\varepsilon)]qA$.

[10] The same applies for when the same advertiser buys a second advertising slot on the same platform. The latter phenomenon does not arise in equilibrium in our model if there are enough potential advertisers, that is why we do not formulate the value of broadcasting multiple advertising spots for the same product on a given channel.

$$\pi = \begin{cases} 0 \text{if he does not advertise} \\ n_i\omega + n_{01}\omega' - p_i \text{ if he advertises only on platform } i \\ (n_0 + n_1 - 2n_{01})\omega + n_{01}(\omega' + \omega'') - p_0 - p_1 \text{ if he advertises on both.} \end{cases}$$

Timing of the game

First platform owners simultaneously set advertising levels (number of advertisement slots) on their platforms, that is $\overline{a}_0$, $\overline{a}_1$ and $\overline{a}_{01}$ in the case of a discriminating monopolist, and a_0 and a_1 in all other cases. The chosen advertising levels determine the prices that clear the market. In the next section we show that these market clearing prices are unique. Finally, viewers observe the levels of advertising on the platforms and decide whether to watch both channels, or only one, or neither of them. This specification assumes that platforms collect all their revenues from advertisers. In Section 5 we consider the case when platforms can also charge subscription fees on viewers.

3 Equilibrium Levels of Advertising

In this section we solve for subgame perfect Nash equilibria of the game introduced in the previous section, both for when the platforms are owned by a monopolist provider and for the case of competing platforms. For the characterization of the equilibrium of the corresponding model when viewers are not allowed to multi-home (which is an equivalent model to the one in Anderson and Coate (2005) with homogenous advertisers), see Appendix A.

3.1 General considerations

First, note that the amount of viewers is determined uniquely by the advertising quantity decisions of channels. In the nondiscriminating monopoly case, and in the duopoly case it is given by: $n_0 = \max(0, \min(\frac{\beta-\gamma a_0}{\tau}, 1)M)$ for platform 0, and $n_1 = \max(0, \min(\frac{\beta-\gamma a_1}{\tau}, 1)M)$ for platform 1. The marginal viewers in an inner solution are $x_{m0} = \frac{\beta-\gamma a_0}{\tau}$ and $x_{m1} = 1 - \frac{\beta-\gamma a_1}{\tau}$. The overlapping viewership is $n_{01} = \max(0, x_{m0} - x_{m1})M$, which is equal to $\max(0, \frac{2\beta-\gamma(a_0+a_1)}{\tau} - 1)M$ for an inner solution. In the discriminating monopoly case the viewerships are $n_0 = \max(0, \min(\frac{\beta-\gamma(a_0+a_{01})}{\tau}, 1)M)$ and $n_1 = \max(0, \min(\frac{\beta-\gamma(a_0+a_{01})}{\tau}, 1)M)$ for platforms 0 and 1 respectively. The marginal viewers and the viewerships are displayed in Figure 1.

Since the number of viewers on each platform is uniquely determined after any quantity announcement, equilibrium price is uniquely determined for any $a_0, a_1 \in (0, N)$ too, in the nondiscriminating monopoly case and in the duopoly case. For such advertising levels the unique market clearing prices are:

$$p_i = \begin{cases} \omega n_i \text{ if } n_0 + n_1 \leq M \\ \omega(n_i - n_{01})\omega + n_{01}\omega' \text{ if } n_0 + n_1 > M \text{ and } a_0 + a_1 \leq N \\ \omega(n_i - n_{01})\omega + n_{01}\omega'' \text{ if } n_0 + n_1 > M \text{ and } a_0 + a_1 > N \end{cases}$$

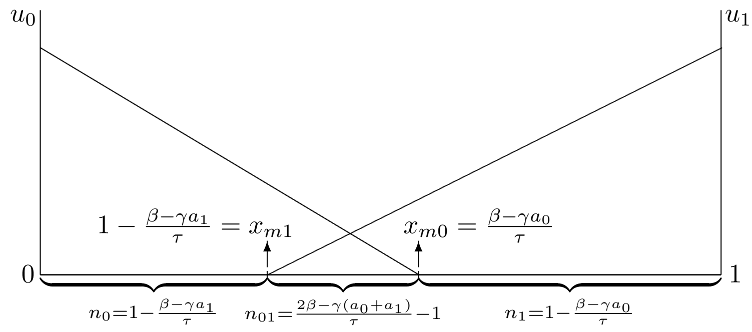

Figure 1: Viewerships (if viewers overlap)

In words, the equilibrium price depends on the number of overlapping and nonoverlapping viewers, and on whether advertisers overlap or not. The value of a nonoverlapping viewer is ω, while the value of an overlapping viewer is ω' if advertisers do not overlap, and ω'' if advertisers overlap. If $a_i = 0$ for some $i \in \{1, 2\}$ then we assume that p_i is the minimal price that clears the market (there is a multiplicity in this case because any price larger than this would also clear the market). If $a_i = N$ then we assume that p_i is the maximal price that clears the market (there is a multiplicity in this case because any price less than this would also clear the market).

Analogously, the equilibrium price is uniquely determined in the nondiscriminating monopoly case for any $a_0, a_1, a_{01} > 0$ such that $a_0 + a_{01} < N$ and $a_1 + a_{01} < N$. If $n_0 + n_1 \leq 1$ then the price for an advertising spot on platform i is $p_i = \omega n_i$, and the price of a joint advertising spot on both platforms is $p_{01} = \omega(n_0 + n_1)$. If $n_0 + n_1 \leq M$ then the price for an advertising spot on platform i is $p_i = \omega(n_i - n_{01}) + \omega' n_{01}$, and the price of a joint advertising spot on both platforms is $p_{01} = \omega(n_0 + n_1 - 2n_{01}) + (\omega' + \omega'')n_{01}$. Just like before, if assume that the minimal market clearing price prevails for cases $a_0 = 0$, $a_1 = 0$ and $a_{01} = 0$, while the maximum market clearing price prevails for cases $a_0 + a_{01} = N$ and $a_1 + a_{01} = N$.

For ease of exposition, we impose the following two parameter restriction for the subsequent analysis:

$$\max(\frac{\beta}{2\gamma}, \frac{2\beta - \tau}{2\gamma}) \leq N \tag{1}$$

$$\beta - \gamma \frac{N}{2} \leq \tau \tag{2}$$

The first one rules out boundary cases in which all potential advertisers advertise on a platform. The second one implies that the nuisance parameter is high enough such that if the amount of advertising is at least $\frac{N}{2}$ (half of the potential advertisers are present) then not every viewer would watch both channels. This assumption rules out boundary cases in which even for relatively high levels of advertising all viewers watch both channels in equilibrium. The above restrictions are only made for analytical convenience, in order to avoid checking whether the corresponding boundary conditions $a_0, a_1 \leq N$ and $x_0 \leq 1$, $x_1 \geq 0$ bind. Dropping these restrictions does not change the qualitative features of the analysis.

3.2 Non-discriminating Monopolist

We need to consider three cases: when there is no overlap of viewers, when viewers overlap but advertisers do not, and finally when both viewers and advertisers overlap.

First assume that $n_{01} = 0$, which means no viewer overlap. The profit function of station i is then given by

$$\Pi_i = p_i a_i = \omega M(\frac{\beta - \gamma a_i}{\tau})a_i.$$

Solving this for a_i yields

$$a_i = \frac{\beta}{2\gamma}$$

giving a total profit of $\frac{\omega M \beta^2}{2\tau\gamma}$.

The condition for which the above values are compatible with the starting assumption $n_{01} = 0$ is $\beta \leq \tau$. If $\beta > \tau$ then the best advertising level choices that lead to nonoverlapping viewership are $a_0 = a_1 = \frac{2\beta - \tau}{2\gamma}$ (resulting in $n_0 = n_1 = M/2$). The profit in the latter case is:

$$\Pi_M^* = \frac{\omega M(2\beta - \tau)}{2\gamma}.$$

From now on assume $\beta > \tau$. To analyze the monopolist's decision in this parameter region, we distinguish between the case when $\frac{2\beta - \tau}{\gamma} \leq N$ and the case when $\frac{2\beta - \tau}{\gamma} > N$.

Consider first $\frac{2\beta - \tau}{\gamma} \leq N$. In this case it cannot be that in equilibrium both advertisers and viewers overlap, since overlapping viewership implies $a_0 + a_1 \leq N$. suppose first that $n_{01} > 0$. Then the profit function in this region is given by:

$$\Pi_M = Ma_0 \left(\omega \max(1 - A_1, 0) + \omega' \min(A_0 + A_1 - 1, 1)\right) +$$

$$+Ma_1\left(\omega\max(1-A_0,0)+\omega'\min(A_0+A_1-1,1)\right),$$

with $A_i = \frac{\beta-\gamma a_i}{\tau}$.

Solving this for a_0 and a_1 yields:

$$a_0 = a_1 = \max\left(\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}, \frac{\beta-\tau}{\gamma}\right). \tag{3}$$

The monopolist's profit is then $\Pi_M = \min\left(\frac{M\omega'(\omega'(2\beta-\tau)-\omega(\beta-\tau))^2}{\gamma\tau(2\omega'-\omega)^2}, \frac{(\beta-\tau)2M\omega'}{\gamma}\right)$

The above levels of advertising are consistent with the starting assumption $n_{01}>0$ iff $\omega' > \frac{\omega\beta}{2\beta-\tau}$. If $\omega' \leq \frac{\omega\beta}{2\beta-\tau}$ then establishing an overlapping viewership is not optimal for the monopolist because ω' is too small and so $n_{01}>0$. The optimal choice is $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$.

Consider next $\frac{2\beta-\tau}{\gamma} > N$. Assume first that $n_{01} > 0$, but $a_0 + a_1 < N$. Then equilibrium advertising levels are given by (3). This is only consistent with the starting assumption $a_0 + a_1 < N$ if $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)} < N$. In the latter case (3) is a global optimum for the monopolist (note that $n_{01} > 0$ holds because $\frac{2\beta-\tau}{\gamma} > N$). In this case only viewers multi-home while there are some potential advertisers who do not advertise on either of the channels. If $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)} \geq N$ then the optimal choice for the monopolist under which $n_{01} > 0$ and $a_0 + a_1 \leq N$ is setting $a_0 = a_1 = \frac{N}{2}$. This gives a profit of:

$$\Pi^{**}_M = \frac{MN(2\tau\omega - 2\beta\omega + 4\beta\omega' - 2\tau\omega' + N\gamma\omega - 2N\gamma\omega')}{2\tau}.$$

Here again viewers multi-home but the overall number of potential advertisers is low enough such that they all advertise but none of them is multi-homing.

If $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)} \geq \frac{2\beta-\tau}{2\gamma}$, which holds when $\omega' \leq \frac{\beta\omega}{2\beta-\tau}$, the optimal decision for the monopolist is $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ (establishing as before viewerships that just do not overlap).

If $\omega' > \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$ then we have to consider three scenarios: nonoverlapping viewership, overlapping viewership and nonoverlapping advertisers, and overlap on both sides. Consider the latter scenario first: $n_{01} > 0$ and $a_0 + a_1 > N$. The profit function in this case is given by:

$$\Pi_M = Ma_0\left(\omega(1-\frac{\beta-\gamma a_1}{\tau}) + \omega''(\frac{2\beta-\gamma(a_0+a_1)}{\tau}-1)\right)+$$

$$+Ma_1\left(\omega(1-\frac{\beta-\gamma a_0}{\tau}) + \omega''(\frac{2\beta-\gamma(a_0+a_1)}{\tau}-1)\right).$$

Given the constraints $x_0 \leq 1$ and $x_1 \geq 0$ the optimal solution is:

$$a_0 = a_1 = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{2\gamma(2\omega'' - \omega)}.$$

This gives a profit of

$$\Pi_M^{***} = \frac{M[\omega''(2\beta - \tau) - \omega(\beta - \tau)]^2}{2\gamma\tau(2\omega'' - \omega)}.$$

This solution is only consistent with the assumptions $n_{01} > 0$ and $a_0 + a_1 > N$ if $\omega'' > \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega''-\omega)}$. It can be shown that in the relevant parameter range $\frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega''-\omega)} > \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$, therefore $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$ implies $N \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega''-\omega)}$. So if $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega''-\omega)}$ is the optimla choice we have two-sided multi-homing. but still it remains to check that this more profitable than reducing advertisers to $a_0 = a_1 = \frac{N}{2}$ and with that receiving a higher surplus from each one of them. This depends on the relative magnitudes of Π_M^{**} and Π_M^{***}. If on the other hand $\omega'' \leq \frac{\beta\omega}{2\beta-\tau}$ then the optimal choice for the monopolist is either $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ or $a_0 = a_1 = \frac{N}{2}$, depending on the relative magnitudes of Π_M^{*} and Π_M^{**}. This reveals that $\Pi_M^{*} \geq \Pi_M^{**}$ if $N \leq \frac{\omega\tau}{\gamma(2\omega'-\omega)}$, which means that if N is small viewers do not overlap while if N is large than advertisers do not overlap. Overlapping of both sides is not optimal because ω'' is too low.

We can summarize the solution for different parameter values as follows:

Setting $a_1 = a_2 = \frac{\beta}{2\gamma}$ is optimal for the monopolist if $\beta < \tau$. If the travel cost parameter is high enough, then there is no overlap in the equilibrium viewerships of channels.

Setting $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ is optimal for the monopolist if $\beta \geq \tau$ and either of the following conditions hold:
(i) $\omega' \leq \frac{\beta\omega}{2\beta-\tau}$,
(iii) $\omega' > \frac{\beta\omega}{2\beta-\tau}$, $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$, $\omega'' \leq \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega\tau}{\gamma(2\omega'-\omega)}$.
For low values of ω' the monopolist sets advertising such that exactly half of the viewers connect to each channel while some advertisers multi-home.

Setting $a_0 = a_1 = \max(\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}, \frac{\beta-\tau}{\gamma})$ is optimal for the monopolist if $\beta \geq \tau$, $\omega' \geq \frac{\beta\omega}{2\beta-\tau}$ and $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)} \leq N$. For high enough ω' and N the monopolist establishes overlapping viewership and nonoverlapping advertisers.

Setting $a_0 = a_1 = N/2$ is optimal for the monopolist if $\beta \geq \tau$ and either of the following conditions hold:

(i) $\omega'' > \frac{\beta\omega}{2\beta-\tau}$, $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$ and $\Pi_M^{**} \geq \Pi_M^{***}$,[11]
(ii) $\omega' > \frac{\beta\omega}{2\beta-\tau}$, $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$, $\omega'' \leq \frac{\beta\omega}{2\beta-\tau}$ and $N > \frac{\omega\tau}{\gamma(2\omega'-\omega)}$
For high values of ω' and low values of N and ω'' the monopolist sets advertising levels such that advertisers just do not overlap while viewerships do.

Setting $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega''-\omega)}$ is optimal for the monopolist if $\beta \geq \tau$, $\omega'' > \frac{\beta\omega}{2\beta-\tau}$, $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$ and $\Pi_M^{***} \geq \Pi_M^{**}$. For high values of ω' and ω'' and low values of N the monopolist chooses overlap on both sides.

It is straightforward to verify that the resulting profit function of the monopolist is continuous in all parameters.

3.3 Discriminating Monopolist

If the monopolist can offer a price discount to advertisers who advertise on both platforms, then it can extract the whole surplus $\omega' + \omega''$ (as opposed to $2\omega''$, the amount that a nondiscriminating monopolist can extract) from advertising a product to a multi-homing viewer through both channels. This only makes a difference for cases when both viewers and advertisers overlap, since the value of advertising two different products on the two different channels to a multi-homing viewer is $2\omega'$, which exceeds $\omega' + \omega''$. Therefore the optimal advertising levels a_0, a_1 chosen by a discriminating monopolist are exactly the same as the ones for a nondiscriminating monopolist (and $\overline{a}_{01} = 0$) for parameter values outside the region where $\beta > \tau$, $\omega' > \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$. The derivation of the optimal advertising region is given in Appendix B.

The solution for different parameter values is as follows:

Setting $a_1 = a_2 = \frac{\beta}{2\gamma}$ is optimal for the monopolist if $\beta < \tau$.

Setting $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ is optimal for the monopolist if $\beta \geq \tau$ and either of the following conditions hold:
(i) $\omega' \leq \frac{\beta\omega}{2\beta-\tau}$,
(ii) $\omega' > \frac{\beta\omega}{2\beta-\tau}$, $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$, $\omega'' \leq \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega\tau}{\gamma(2\omega'-\omega)}$.

Setting $a_0 = a_1 = \max(\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}, \frac{\beta-\tau}{\gamma})$ is optimal for the monopolist if $\beta \geq \tau$, $\omega' > \frac{\beta\omega}{2\beta-\tau}$ and $N \geq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$.

Setting $a_0 = a_1 = \frac{N}{2}$ is optimal for the monopolist if $\beta \geq \tau$ and either of the following two conditions hold:

[11] Note that $\omega'' > \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$ also imply $\omega' > \frac{\beta\omega}{2\beta-\tau}$ and $N \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega''-\omega)}$.

(i) $\omega' > \frac{\beta\omega}{2\beta-\tau}$, $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$, $\omega'' \leq \frac{\beta\omega}{2\beta-\tau}$ and $N > \frac{\omega\tau}{\gamma(2\omega'-\omega)}$.
(ii) $\omega'' > \frac{\beta\omega}{2\beta-\tau}$, $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$ and $\Pi_M^{**} \geq \Pi_M^{+}$.

Setting $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{2(2\omega''-\omega)\gamma}$ is optimal for the monopolist if $\beta \geq \tau$, $\omega'' > \frac{\beta\omega}{2\beta-\tau}$, $N < \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$ and $\Pi_M^{**} \leq \Pi_M^{+}$.

Again it can be verified that the resulting equilibrium profit function is continuous in the parameters. Furthermore, as opposed to the nondiscriminating monopoly case, the equilibrium levels of advertising are continuous in ω'', because increasing advertising above $\frac{N}{2}$ is not associated with a discontinuity in advertising revenues if the monopolist can sell joint advertising slots with a discount. However, for low levels of ω'' the equilibrium levels of advertising are discontinuous in ω' (if $\frac{2\beta-\tau}{2\gamma} > N$ and ω'' is low enough, then there is a critical value of ω' at which the monopolist switches from nonoverlapping viewership to nonoverlapping advertisers).

3.4 Duopoly

Here we analyze the case when the stations are controlled by different firms.

If $\tau \geq \beta$ is the same a in the monopoly case because viewerships never overlap ($n_{01} = 0$). The interesting cases arise if $\tau < \beta$. in this case the derivation of the equilibria is analytically similar to the monopoly analysis and is therefore relegated to Appendix C. The big difference is that dependent on parameter values we get multiple equilibria in the duopoly case. The reason is that advertising levels are strategic complements. For example, if a_1 is high then the optimal response of station 2 is to set a_2 high as well to avoid multi-homing of viewers. If instead a_1 is low station 2 can avoid multi-homing of viewers only via a very high a_2 which is not optimal because the viewership of station 2 would then be very low. The optimal answer is instead to set a_2 low as well. Both of these equilibria can exist at the same time. Obviously this is not possible in the monopoly case because the monopolist always has a unique optimal policy.

The equilibria for different parameter values are as follows:

Advertising levels $a_0 = a_1 = \frac{\beta}{2\gamma}$ constitute an equilibrium if $\beta < \tau$. This is the same as in the monopoly case.

Advertising levels $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ constitute an equilibrium if $\beta \geq \tau$ and either of the following conditions hold:
(i) $\omega' \leq \frac{\tau\omega}{2\beta-\tau}$
(ii) $\omega' > \frac{\tau\omega}{2\beta-\tau}$, $\omega'' \leq \frac{\tau\omega}{2\beta-\tau}$, $N \leq \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$ and $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$.

Advertising levels $a_0 = a_1 = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$ constitute an equilibrium $\beta \geq \tau$, $\omega' \geq \frac{\tau\omega}{2\beta-\tau}$ and $N \geq \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$.

Advertising levels $a_0 = a_1 = \frac{N}{2}$ constitute an equilibrium if $\beta \geq \tau$, $\omega' \geq \frac{\tau\omega}{2\beta-\tau}$, $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$ and either of the following conditions hold:
(i) $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ and $\omega' \geq \frac{\gamma N-2(\beta-\tau)}{\gamma N}$
(ii) $N \geq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ and $\Pi_i^{dd} \leq \frac{MN(2\tau(\omega-\omega')+2\beta(2\omega'-\omega)-\gamma N(2\omega'-\omega))}{4\tau}$.

Advertising levels $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ constitute an equilibrium if $\beta \geq \tau$, $\omega' \geq \frac{\tau\omega}{2\beta-\tau}$, $N \leq \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$, $N < \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ and $\Pi_i^{dd} \geq \frac{MN(2\tau(\omega-\omega')+2\beta(2\omega'-\omega)-\gamma N(2\omega'-\omega))}{4\tau}$.

So if $\omega' \geq \frac{\gamma N-2(\beta-\tau)}{\gamma N}$, $\omega'' \leq \frac{\tau\omega}{2\beta-\tau}$, $N \leq \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$ and $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ both $a_0 = a_1 = \frac{N}{2}$ and $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ are equilibria. If ω'' stations avoid multi-homing of both sides together. They achieve this either by avoiding overlap of viewers or by avoiding overlap of advertisers while the other side overlaps, respectively.
If $\omega' \geq \frac{\gamma N-2(\beta-\tau)}{\gamma N}$, $\omega'' \geq \frac{\tau\omega}{2\beta-\tau}$, and $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$, both $a_0 = a_1 = \frac{N}{2}$ and $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ are equilibria.

4 Analysing equilibrium advertising levels

In this section we analyze equilibrium levels of advertising under different specifications, and compare the results to the predictions of a model in which viewers can only single-home.

4.1 Entrance of a competitor and advertising

In this subsection we investigate how a channel adjusts the level of advertising after the entrance of a competing channel, in the two-sided multi-homing model introduced in Section 2, and in a corresponding model in which viewers can only single-home.

If only firm 0 is present in the market, it is straightforward to establish that in equilibrium it chooses $a_0 = \max(\frac{\beta}{2\gamma}, \frac{\beta-\tau}{\gamma})$.

Assume now that firm 1 enters the market. If viewers can only single-home, the equilibrium levels of advertising are given by:[12]

[12] See Appendix A for characterization of equilibria in the single-homing model.

$$a_0 = a_1 = \begin{cases} \frac{\beta}{2\gamma} & \text{if } \beta \leq \tau \\ \frac{2\beta-\tau}{2\gamma} & \text{if } \beta > \tau \geq 2/3\beta \\ \frac{\tau}{\gamma} & \text{if } \tau < 2/3\beta \end{cases}$$

This implies that in the single-homing case whether the entrance of a competitor increases the level of advertising depends on the travelling cost parameter. If $\tau > \frac{\beta}{2}$ then the amount of advertising on channel 0 increases (weakly). If differentiation between channels is high, then stealing marginal consumers from the competitor is costly. Then in equilibrium channels advertise a lot, to a smaller audience. However, if $\tau < \frac{\beta}{2}$ then entrance decreases the amount of advertising on channel 0. If channels are less differentiated, then competition for viewers forces the channels to advertise less, to retain marginal consumers switching to the competitor.

If consumers can multi-home, then whether entry increases or decreases advertising depends on whether in the resulting equilibrium there is overlap among viewers and advertisers.

First consider $N \geq \frac{2\beta-\tau}{\gamma}$. Then there cannot be overlapping on both sides in equilibrium. Entry of a competitor implies the following for channel 0:
(i) If $\beta < \tau$ then there is no change in the level of advertising.
(ii) If $\beta \geq \tau$ and $\omega' < \frac{\tau\omega}{2\beta-\tau}$ then advertising increases from $\frac{\beta}{2\gamma}$ to $\frac{2\beta-\tau}{2\gamma}$.
(iii) If $\beta \geq \tau$ and $\omega' \geq \frac{\tau\omega}{2\beta-\tau}$ then advertising increases from $\frac{\beta}{2\gamma}$ to $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)} = \frac{\beta}{2\gamma} + \frac{(\omega-\omega')\tau}{2\gamma(2\omega'-\omega)}$.

If $N < \frac{2\beta-\tau}{\gamma}$, then the previous analysis can be replicated for all cases, implying that competition increases advertising, with the exception of the region where $\beta \geq \tau$, $\omega' \geq \frac{\tau\omega}{2\beta-\tau}$, $\frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)} \leq N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$ and $\Pi_i^{dd} \leq \frac{MN(2\tau(\omega-\omega')+2\beta(2\omega'-\omega)-\gamma N(2\omega'-\omega))}{4\tau}$. In this region advertising levels $a_0 = a_1 = N/2$ constitute an equilibrium. The starting assumption $N < \frac{2\beta-\tau}{\gamma}$ implies that if this equilibrium arises in competition, then competition decreases advertising on channel 0. The channel decreases advertising from $\frac{\beta}{2\gamma}$ to $\frac{N}{2}$, in order to avoid overlapping of advertisers.

Proposition 1: The entrance of a competitor always weakly increases the level of advertising on the platform if $N \geq \frac{2\beta-\tau}{\gamma}$. If $N < \frac{2\beta-\tau}{\gamma}$ then entrance of a competitor can either decrease or increase the level of advertising.

Note that for some parameter regions the difference between predictions of the single-homing and multi-homing models is stark. This is the case when the travel cost parameter is low and either (i) the amount of potential advertisers is large; or (ii) the value of overlapping viewers is low. In these cases in the single-homing model competition for viewers forces a channel to decrease the level of advertising, while in the multi-homing model competition increases the level of advertising on the channel, because of the decreased value of overlapping viewers.

4.2 Ownership structure and advertising

In this subsection we assume that both channels are present in the market, and compare advertising levels when the channels are operated by a monopolist provider to advertising levels arising with competing platforms.

The benchmark is again the model in which viewers can only single-home. In this case a monopolist platform provider always chooses (weakly) higher levels of advertising than the equilibrium levels in duopoly competition. In particular, if $\tau \geq 2/3\beta$ then monopoly and duopoly imply the same levels of advertising, while $\tau < 2/3\beta$ implies that the amount of advertising is strictly smaller in duopoly than in monopoly.[13]

This result no longer holds in the model with viewer multi-homing. Advertising level in a duopoly equilibrium can be both larger or smaller than the advertising levels set by either a nondiscriminating or a discriminating monopolist. This is despite the fact that in the multi-homing model it still holds that competing platform owners, as opposed to a monopolist provider, do not take into account the positive effect of increasing their level of advertising on on revenues obtained on the other platform. This effect can be offset for several different reasons, though. First, in the discriminating monopolist case, the platform owner can extract higher revenues of overlapping viewers in case of two-sided overlapping. Therefore there is a region of parameter values such that the unique symmetric duopoly equilibrium involves high enough advertising levels such that viewerships just don't overlap, while the discriminating monopolist chooses lower levels of advertising that induce two-sided multi-homing. Second, if there are multiple symmetric equilibria in duopoly competition, then one of these equilibria might imply higher levels of advertising than the levels chosen by either the discriminating or a nondiscriminating monopolist.

More formally, in the region $\beta \geq \tau$, $\omega'' > \frac{\beta\omega}{2\beta-\tau}$, $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$, $\Pi_M^{**} \geq \Pi_M^{***}$, $\omega' > \frac{\tau\omega}{2\beta-\tau}$ and $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ a nondiscriminating monopolist chooses advertising levels $\frac{N}{2}$ (advertisers just don't overlap), while there is a symmetric equilibrium in duopoly such that $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma} > \frac{N}{2}$ (viewers just don't overlap). There are several regions of parameter values such that the duopoly game has at least one symmetric equilibrium with higher levels of advertising than advertising levels in an equilibrium with a discriminating monopolist. One is characterized by $\beta \geq \tau$, $\omega'' > \frac{\beta\omega}{2\beta-\tau}$, $\Pi_M^{**} \leq \Pi_M^{+}$, $N \leq \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$, $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ and $N \geq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$. For these parameter values $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{2(2\omega''-\omega)\gamma} < \frac{2\beta-\tau}{2\gamma}$ in equilibrium in the game with a discriminating monopolist, while $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ yields an equilibrium in duopoly competition.

[13]See Appendix A.

Proposition 2: The level of advertising in equilibrium can be higher in duopoly competition than either when the platforms are owned by a nondiscriminating or by a discriminating monopolist.

The above implies that, as opposed to a model in which viewers can only single-home, in the multi-homing model competing firms might find it profitable to establish advertising ceilings (since profits are always higher in the monopoly cases). This might be an explanation why stations in the USA voluntarily established such a ceiling in the 1970s. The National Association of Broadcasters (NAB) had a code of conduct in place which limited the advertising time per hour to 9.5 minutes in prime time and to 16.5 minutes at all other times.[14] There are several other possible explanations though, the most natural one being that the code of conduct was a standard cartel agreement, to keep prices of commercials high.[15] This argument could be valid in a model with heterogeneous advertisers.

There is no clear-cut ordering of the advertising levels chosen by a discriminating and a nondiscriminating monopolist. The only difference between these cases is that overlapping viewers are more valuable for the discriminating monopolist in case of two-sided overlapping, and therefore the discriminating monopolist is more likely to choose that regime. However, for some parameter values for which the nondiscriminating monopolist chooses advertising levels leading to two-sided overlap the discriminating monopolist chooses higher levels of advertising and nonoverlapping viewership, while for other parameter values it chooses lower levels of advertising and nonoverlapping advertisers.

4.3 Social welfare

In Appendix A we show that in a model in which viewers can only single-home and the channels collect their revenues from advertising fees, a monopolist provider always chooses a higher level of advertising than what is socially optimal. The intuition behind this is that the monopolist does not fully internalize the negative externality of advertising on viewers if all its revenues are obtained through advertising fees. On the other hand, whether duopoly competition induces too much or too little advertising relative to the socially optimal level depends on the magnitudes of the nuisance cost parameter and the travelling cost parameter. If both of these parameters are small enough, then competition implies too little advertising.

Below we show that the possibility of two-sided multi-homing the result that a monopolist always chooses too high levels of advertising only holds for the

[14]For further reference see Campbell (1999)

[15]The department of Justice alleged the Code of Conduct and argued that it kept the price of advertising high and therefore violated antitrust laws. As a consequence the NAB voluntarily quit the Code in 1983.

case of the discriminating monopolist. A nondiscriminating monopolist does not take the full social value of advertising into account if both viewers and advertisers overlap, because it cannot extract all the surplus from it. Therefore for small values of the nuisance parameter it might end up advertising less than the socially optimal amount. Similar conclusions apply to advertising levels implied by duopoly competition. Therefore, just like in the model in which viewers can only single-home, the welfare implications of equilibria in duopoly competition are ambiguous, although for different reasons. In the two-sided multi-homing model too little advertising is induced not by a small travel cost, but by a combination of relative scarcity of advertisers and low value of overlapping viewers.

Social welfare in our model is the sum of aggregate viewer surplus and the total surplus from advertising, which can be written as follows:

$$WF = \overline{a}_0 \left(\omega(n_0 - n_{01}) + \omega' n_{01}\right) +$$

$$\overline{a}_1 \left(\omega(n_1 - n_{01}) + \omega' n_{01}\right) +$$

$$\overline{a}_{01}[\omega(n_0 + n_1 - 2n_{01}) + (\omega' + \omega'')n_{01}] +$$

$$M \int_{0}^{(n_0-n_{01})/M} (\beta - \gamma a_0 - \tau x)dx + M \int_{1-(n_1-n_{01})/M}^{1} (\beta - \gamma a_1 - \tau(1-x))dx) +$$

$$\int_{(n_0-n_{01})/M}^{n_0/M} u(\beta - \gamma a_0 - \tau x, \beta - \gamma a_1 - \tau(1-x)dx.$$

where $a_i = \overline{a}_i + \overline{a}_{01}$ for $i = \{0, 1\}$.

Note that the sum of the first three terms is exactly equal to the profit function of a discriminating monopolist, and that the last three terms are decreasing in advertising levels. Below we show that this implies that a discriminating monopolist always chooses a (weakly) higher level of advertising than any social welfare maximizing advertising level.

First note that a welfare maximizing vector of advertising levels $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ cannot imply $n_i = 0$ for some $i \in \{0, 1\}$, otherwise setting $\overline{a}'_0, \overline{a}'_1, \overline{a}'_{01}$ such that $\overline{a}'_{-i} = \overline{a}_{-i} + \overline{a}_{01}$ and $\overline{a}_i = \overline{a}_{01} = 0$ would lead to strict welfare improvement: it would generate positive as opposed to zero social welfare on platform i while leaving social welfare generated on the other platform unchanged. As shown in Subsection 3.1, $n_0 = \max(0, \min(\frac{\beta - \gamma a_0}{\tau}, 1)M)$ and $n_1 = \max(0, \min(\frac{\beta - \gamma a_1}{\tau}, 1)M)$. This can be used to show that if $\overline{a}_0 \neq \overline{a}_1$ and if advertising levels $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ imply $n_i > 0$ for $i \in \{0, 1\}$ then setting $\overline{a}'_0, \overline{a}'_1, \overline{a}'_{01}$

such that $\overline{a}_0' = \overline{a}_1' = \frac{\overline{a}_0+\overline{a}_1}{2}$ and $\overline{a}_{01}' = \overline{a}_{01}$ leads to strict welfare improvement. This complements the result in Subsection 3.3 that if $\overline{b}_0, \overline{b}_1, \overline{b}_{01}$ are equilibrium advertising levels for a discriminating monopolist then $\overline{b}_0 = \overline{b}_1$. Note that if $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ is either a welfare maximizing or a discriminating monopoly equilibrium advertising vector, then the following hold: (i) if $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ implies $n_{01} > 0$ then $\overline{a}_{01} = \max(0, N - a_0 - a_1)$, otherwise social welfare or the monopolist profit could be increased by increasing $\overline{a}_0, \overline{a}_1$ and decreasing $\overline{a}_{01}$; (ii) if $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ implies $n_{01} > 0$ then any $\overline{a}_0', \overline{a}_1', \overline{a}_{01}'$ such that $a_i' = a_i$ for $i \in \{0,1\}$ would imply the same level of social welfare and monopolist profit. The above establish that if $\overline{a}_0, \overline{a}_1, \overline{a}_{01}$ is either a welfare maximizing or a discriminating monopoly equilibrium advertising vector, then it can be described simply by the scalar $a_0 = a_1$.

Let now $a_0 = a_1 = a^*$ in an equilibrium with a discriminating monopolist. This implies that profit function

$$\Pi(a) = 2(a - \max(0, N - 2a))\left(\omega(n_0 - n_{01}) + \omega' n_{01}\right) +$$

$$\max(0, N - 2a)[\omega(n_0 + n_1 - 2n_{01}) + (\omega' + \omega'')n_{01}]$$

is such that $\Pi(a^*) \geq \Pi(a)$ for any $a \geq a^*$. But note that the welfare function

$$WF(a) = \Pi(a) +$$

$$M \int_0^{(n_0-n_{01})/M} (\beta - \gamma a - \tau x)dx + M \int_{1-(n_1-n_{01})/M}^{1} (\beta - \gamma a - \tau(1-x))dx +$$

$$\int_{(n_0-n_{01})/M}^{n_0/M} u(\beta - \gamma a - \tau x, \beta - \gamma a - \tau(1-x)dx,$$

where the last three terms are strictly decreasing in a. Therefore $WF(a^*) > WF(a)$ for any $a \geq a^*$, implying that any welfare maximizing advertising vector has to be such that $a_i \leq a^*$.

Proposition 3: A discriminating monopolist always chooses a weakly higher advertising level than what is socially optimal.

A similar general result cannot be established for the nondiscriminating monopolist and the duopoly cases. To demonstrate this, and to provide intuition when market equilibrium implies too much or too little advertising, we characterize the welfare maximizing advertising levels and compare them to the levels chosen by a nondiscriminating monopolist for a particular specification of the model.[16] The specification is when $\omega' = \omega$, that is when viewers do not substitute time away from viewing a channel if they start watching a second channel, too. In this case there is a natural specification of the utility function of a multi-homing viewer located at x_i, given by:

$$U_i = s_0(\beta - \gamma a_0 - \tau x_i) + s_1(\beta - \gamma a_1 - \tau(1 - x_i)).$$

That is, utility of a viewer is just the sum of utilities obtained from watching the channels.

Characterizing the welfare maximizing solution is a similar exercise to characterizing equilibrium levels for a discriminating monopolist, therefore here we only report the solution:

$$a_i^{WF} = \begin{cases} 0 \text{ if } \omega < \gamma \\ \frac{\beta(\omega-\gamma)}{(2\omega-\gamma)\gamma} \text{ if } \omega \geq \gamma \text{ and } \tau \geq \frac{2\beta\omega}{2\omega-\gamma} \\ \frac{\beta(\omega-\gamma)}{(2\omega-\gamma)\gamma} \text{ if } \omega \geq \gamma,\ \tau < \frac{2\beta\omega}{2\omega-\gamma}, \text{ and } N \geq \frac{2\beta(\omega-\gamma)}{(2\omega-\gamma)\gamma} \\ \frac{N}{2} \text{ if } \omega \geq \gamma,\ \tau < \frac{2\beta\omega}{2\omega-\gamma},\ \omega'' \geq \frac{\omega\beta - \frac{\gamma\tau}{2} - \gamma\omega N}{2\beta-\tau-\gamma N} \text{ and } \frac{\omega''(2\beta-\tau)-\beta\gamma-\omega(\beta-\tau)}{\gamma(\omega''-\frac{\gamma}{2})} \leq N \\ \frac{\beta(\omega-\gamma)-N\gamma(\omega-\omega'')}{\gamma(4\omega''-2\omega-\gamma)} \text{ if } \omega \geq \gamma,\ \tau < \frac{2\beta\omega}{2\omega-\gamma},\ \omega'' \geq \frac{\omega\beta - \frac{\gamma\tau}{2} - \gamma\omega N}{2\beta-\tau-\gamma N} \\ \quad \text{and } N < \frac{\omega''(2\beta-\tau)-\beta\gamma-\omega(\beta-\tau)}{\gamma(\omega''-\frac{\gamma}{2})} \\ \frac{N}{2} \text{ if } \omega \geq \gamma,\ \tau < \frac{2\beta\omega}{2\omega-\gamma},\ \omega'' < \frac{\omega\beta - \frac{\gamma\tau}{2} - \gamma\omega N}{2\beta-\tau-\gamma N} \\ \quad \text{and } N \geq \frac{2\omega\tau-\gamma\tau-2\beta\gamma}{\gamma(4\omega'-2\omega-\gamma)} \\ \frac{2\beta-\tau}{2\gamma} \text{ if } \omega \geq \gamma,\ \tau < \frac{2\beta\omega}{2\omega-\gamma},\ \omega'' < \frac{\omega\beta - \frac{\gamma\tau}{2} - \gamma\omega N}{2\beta-\tau-\gamma N} \\ \quad \text{and } N < \frac{2\omega\tau-\gamma\tau-2\beta\gamma}{\gamma(4\omega'-2\omega-\gamma)} \end{cases}$$

The types of welfare maximizing outcomes and the corresponding parameter restrictions are similar to the types of equilibrium outcomes characterized in Section 4 for $\omega \geq \gamma$. In the region $\omega < \gamma$ the optimal level of advertising is zero, which is never the case in equilibrium.

The optimal advertising levels for the non discriminating monopolist in case of $\omega' = \omega$ are given by:

$$a_i^* = \begin{cases} (i) \text{ If } \tau \geq \beta \text{ and if } \tau < \beta \text{ and } N \geq \frac{\beta}{\gamma} \text{ then } \frac{\beta}{2\gamma} \\ (ii) \text{ If } \tau < \beta, \text{ and } N < \frac{\beta}{\gamma} \text{ then:} \\ \quad \frac{N}{2} \text{ if } \omega'' \geq \frac{\omega\beta}{2\beta-\tau} \text{ and } \Pi_M^{***} \leq \Pi_M^{**} \\ \quad \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega''-\omega)} \text{ if } \omega'' \geq \frac{\omega\beta}{2\beta-\tau} \text{ and } \Pi_M^{***} \geq \Pi_M^{**} \\ \quad \frac{N}{2} \text{ if } \omega'' < \frac{\omega\beta}{2\beta-\tau} \text{ and } N > \frac{\tau}{\gamma} \\ \quad \frac{2\beta-\tau}{2\gamma} \text{ if } \omega'' < \frac{\omega\beta}{2\beta-\tau} \text{ and } N \leq \frac{\tau}{\gamma} \end{cases}$$

In most of the parameter regions, in particular in the region where there is two-sided overlap in equilibrium, the nondiscriminating monopolist chooses a higher level of advertising than what is socially optimal. However, there is a

[16] The intuitions for the welfare implications of duopoly equilibria are similar.

range of parameters characterized by $\omega \geq \gamma$, $N < \frac{\omega''(2\beta-\tau)-\beta\gamma-\omega(\beta-\tau)}{\gamma(\omega''-\frac{\gamma}{2})}$, $\tau < \beta$, $N < \frac{\beta}{\gamma}$, $\omega'' \geq \frac{\omega\beta}{2\beta-\tau}$ and $\Pi_M^{***} \geq \Pi_M^{**}$ such that the welfare maximizing outcome implies two-sided overlapping (ie advertising levels higher than $\frac{N}{2}$) while the monopolist sets advertising levels equal to $\frac{N}{2}$ to avoid overlapping of advertisers. This can happen because the nondiscriminating monopolist does not fully take into account the surplus generated by advertising to overlapping viewers in case of two-sided overlap.

We conclude this section by showing that for the region in which all market configurations imply two-sided overlapping, which is is the case when both ω' and ω'' are high enough and there is a relative scarcity of potential advertisers, then there is overadvertising in all regimes. moreover, advertising levels can be ordered unambiguously.

Note that in the region with two-sided overlap advertising levels are unique in equilibrium for all market configurations. Let a_D denote the equilibrium advertising level chosen by the discriminating monopolist on a platform, a_{ND} denote the equilibrium advertising level chosen by the nondiscriminating monopolist, and a_{duo} denote the equilibrium advertising level chosen by each firm in the duopoly case. The relationship $a_{duo} < a_{ND}$ is immediate from $a_{ND} = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega''-\omega)}$ and $a_{duo} = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$. Comparing a_{duo} with $a_{dis} = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-\gamma N(\omega-\omega'')}{2\gamma(2\omega''-\omega)}$ reveals that $a_{duo} > a_{dis}$ if $\omega''(2\beta-\tau) - \omega(\beta-\tau) < \gamma N(3\omega''-\omega)$. From assumption (1) the right hand side is at least $\frac{2\beta-\tau}{2}(3\omega''-\omega)$, therefore it is always higher than the left hand side if $\omega'' \geq \frac{\omega\tau}{2\beta-\tau}$. But this inequality always holds in case of two-sided multi-homing and so $a_{duo} > a_{dis}$. Finally, from Proposition 3 we know that $a_{WF} < a_{dis}$.

Proposition 4: If there is overlap on both sides in equilibrium, then advertising levels are too high in all three regimes compared to the socially optimal level. The order of welfare levels is $a_{WF} < a_{dis} < a_{duo} < a_{ND}$.

4.4 Comparative statics

In case of a monopolist provider, advertising levels and firm's profits are monotonic in the parameters of the model, with non surprising signs. An increase in β or a decrease in either γ or τ increases advertising levels and profits, because the same viewership sizes are compatible with higher amounts of advertising. Similarly, increases in ω, ω' and ω'' increase advertising levels and profits.

In case of competing channels, these relationships become nonmonotonic in the region $\frac{2\beta-\tau}{\gamma} > N$. This is partly because of the discontinuity of the (symmetric) equilibrium correspondence. For example, it is straightforward to find an example such that $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma} > \frac{N}{2}$ constitutes a duopoly equilibrium, but for any increase in ω' the unique duopoly equilibrium involves advertising levels less than $\frac{N}{2}$.

Perhaps even more surprisingly, the profit function can be nonmonotonic in τ even within a certain type of equilibrium, so even when equilibrium levels of advertising do not change discontinuously. Consider a neighborhood of parameter values such that $\beta > \tau$, $\omega' > \frac{\tau\omega}{2\beta-\tau}$ and $\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)} < \frac{N}{2}$. For these parameter values the unique equilibrium in duopoly competition implies $a_0 = a_1 = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$. The equilibrium profit of a station is given by $\frac{M\omega'[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega'-\omega)^2}$. Differentiating this $\frac{M\omega'[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega'-\omega)^2}$ with respect to τ yields

$$\frac{M\omega'}{\gamma(3\omega'-\omega)^2}[(2(2\beta-\tau)\omega'-2\omega(\beta-\tau))(\omega-\omega')\tau-[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2], \quad (4)$$

After simplifying, the sign of this expression is given by:

$$\text{sign}\left(\frac{\partial\Pi_i}{\partial\tau}\right) = \text{sign}\left((\tau^2-4\beta^2)(\omega')^2+(4\beta^2\omega-2\tau^2\omega)\omega'+\omega^2\tau^2-\omega^2\beta^2\right).$$

This sign is positive if $\max\left(\frac{\omega(4\beta^2-2\tau^2-2\beta\tau)}{8\beta^2-2\tau^2}, \frac{\omega\tau}{2\beta-\tau}\right) \leq \left(\frac{\omega(4\beta^2-2\tau^2+2\beta\tau)}{8\beta^2-2\tau^2}\right)$.

Proposition 5: There is a range of parameter values for which advertising levels are uniquely determined in duopoly competition and they are strictly decreasing in τ.

An increase in τ on one hand has a direct negative effect on profits of the firms, because it reduces the total potential surplus (stations become less valuable for viewers). However, it decreases the amount of overlap in viewerships, which has a positive effect on firms' profits. For the parameter values above, the second effect dominates, implying that a decrease in the appeal of stations might increase their profits.

It is easy to check that the profit functions in duopoly competition are nonmonotonic in τ in a model with viewer single-homing, too. However, in that model whenever a decrease in τ is associated with a decrease in the equilibrium profits, it is also associated with a decrease in equilibrium advertising levels. In the multi-homing model this goes exactly the other way: τ and equilibrium profits can only decrease simultaneously if the equilibrium advertising levels increase.

5 Introducing viewer fees

In this section we analyze the case where stations, in addition to collecting revenues from advertisers, can charge fees on viewers for watching the station. Because of new encryption techniques, viewer pricing becomes more and more important nowadays.[17] We denote the viewer fee on platform i by f_i, and restrict it to be nonnegative for $i = \{0, 1\}$.

To simplify the analysis in this section we assume $\omega' = \omega$ and that a multi-homing viewer's utility is simply the sum of the net benefits obtained from watching each program:

$$u(x) = s_0(\beta - \gamma a_0 - \tau x - f_0) + s_1(\beta - \gamma a_1 - \tau(1-x) - f_1).$$

for a viewer located at x. s_0 and s_1 are binary variables such that $s_i = 1$, if the viewer watches channel i, and $s_i = 0$ otherwise. These utilities imply that $n_0 = M \min(\frac{\beta - \gamma a_0 - f_0}{\tau}, 1)$ and $n_1 = \min(M \frac{\beta - \gamma a_1 - f_1}{\tau}, 1)$.

Below we focus on the case when the platforms are owned by competing firms. The analysis of the case when platforms are operated by a monopolist is analogous.[18]

It is easy to show here that if $\omega \leq \gamma$ then independently of the other parameters it is optimal for each station to set $a_i^* = 0$ and $f_i^* = \frac{\beta}{2}$. This gives a profit of $\Pi_i = \frac{\beta^2 M}{4\tau}$. If the nuisance parameter is too high relative to the value of advertising, then there is no advertising in equilibrium and platforms collect all their revenues through viewer fees.

Assume from now on that $\omega > \gamma$. For the region $N \geq \frac{\beta}{\gamma}$ the platforms can act as local monopolists and analysis remains the same as in Section 3: stations set $f_i^* = 0$ and $a_i^* = \frac{\beta}{2\gamma}$, and earn a profit of $\Pi_i = \frac{\beta^2 \omega M}{4\tau\gamma}$.

Next we analyse the most interesting case, namely if viewers and advertisers potentially multi-home, i.e. $\omega > \gamma$, $\beta \geq \tau$ and $N < \frac{\beta}{\gamma}$. The profit function of a station i when there is overlap on both sides is:

$$\Pi_i = M a_i \left(\omega(1 - \frac{\beta - \gamma a_j - f_j}{\tau}) + \omega''(\frac{2\beta - \gamma(a_i + a_j) - f_i - f_j}{\tau} - 1) \right) +$$
$$+ M f_i (\frac{\beta - \gamma a_i - f_i}{\tau}).$$

The first order conditions would constitute a minimum here because the second principal minor of the Hessian is negative $(-\frac{(\omega - \gamma)^2}{\tau^2})$ independent of a_j and f_j.

[17] For example, in the US many special interest channels, especially sports or movie channels, can only be watched by paying additional fees. But also in many European countries recent movies or popular sport events can only be watched via pay-TV.

[18] This analysis is available from the authors upon request.

Thus one possible solution can be to set $a_i > 0$ and $f_i = 0$. This gives the same solution as in the case without the possibility of a viewer fee, namely

$$a_i^* = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{\gamma(3\omega'' - \omega)}.$$

We know that $N < \frac{\beta}{\gamma}$ and so advertisers only overlap if $2a_i^* > \frac{\beta}{\gamma}$ or $\tau > \frac{\beta}{2}$. Viewers overlap if $\omega'' \geq \frac{\omega\tau}{2\beta-\tau}$.

If one of these conditions does not hold, then there can by three types of equilibrium. Two of these are analogous to equilibria without the possibility of a viewer fee: platforms set their viewer fees to 0 and advertising levels are such that either viewers or advertisers just don't overlap. However, there is a new type of equilibrium, in which at least one of the channels collect revenues from both sides. In these equilibria $a_i^* = \frac{N}{2} + \delta$ and $a_{-i}^* = \frac{N}{2} - \delta$ with $\delta > 0$. Below we focus on these new equilibria.

The above advertising levels (for some fixed δ) imply that station $-i$'s profit is $\Pi_{-i} = M(N/2 - \delta + f_{-i})(\frac{\beta - \gamma(\frac{N}{2} - \delta) - f_{-i}}{\tau})$. Maximizing this with respect to f_{-i} gives

$$f_{-i}^* = \frac{2\beta + (2\delta - N)(\omega + \gamma)}{4}.$$

f_{-i}^* is here always positive even if δ is zero because $N < \frac{\beta}{\gamma}$ and $\omega > \gamma$. This gives station $-i$ a profit of $\Pi_{-i}^* = \frac{M(2\beta+(N-2\delta)(\omega-\gamma))^2}{16\tau}$.

Calculating the optimal fee of station i delivers $f_i^* = \frac{2\beta-(2\delta+N)(\omega+\gamma)}{4}$. But this fee is only positive as long as $\delta < \frac{2\beta-N(\omega+\gamma)}{2(\omega+\gamma)}$ and station i then gets a profit of $\Pi_i^* = \frac{M(2\beta+(N+2\delta)(\omega-\gamma))^2}{16\tau}$. Thus if $\delta \geq \frac{2\beta-N(\omega+\gamma)}{2(\omega+\gamma)}$ then $f_i^* = 0$ and station i only gets revenue from advertising. We still have to check under which conditions the combination $a_i^* = \frac{N}{2} + \delta$ and $a_{-i}^* = \frac{N}{2} - \delta$ is indeed an equilibrium. First, it is easy to see that the profit of station i is bigger than the profit of station $-i$ if δ is strictly positive. Comparing now the profit of station $-i$ in the proposed equilibrium with the deviation profit if station $-i$ only chooses a viewer fee and does no longer advertise yields that a deviation is not profitable if $\frac{M(2\beta+(N-2\delta)(\omega-\gamma))^2}{16\tau} \geq \frac{M\beta^2}{4\tau}$ or $N \geq 2\delta$. But the last inequality always holds since the maximal possible value of δ is $\frac{N}{2}$. Since $\Pi_i^* > \Pi_{-i}^*$ such a deviation is also not profitable for station i. Another possible deviation for a firm is to set the level of advertising in such a way that advertisers overlap. For small enough ω'' however, this is not a profitable deviation for either platform. The last possible deviation is for station $-i$ to set $a_{-i}^{dev} = \frac{4\beta-2\gamma-\gamma N-2\delta}{2\gamma}$ so that to avoid overlap of viewers instead of advertisers. This deviation is not profitable though for example if $\tau \leq \frac{\beta}{2}$ and $\omega'' \geq \frac{\omega\tau}{2\beta-\tau}$. Note that the latter can hold for arbitrarily small positive ω'' if β is large enough. This yields the following conclusion:

Proposition 6: There exist asymmetric equilibria in which station i gets most, or even all, of its revenues from advertising and sets $a_i^* = \frac{N}{2} + \delta$ ($\delta \in (0, \frac{N}{2})$) and $f_j^* = \max(0, \frac{2\beta-(2\delta+N)(\omega+\gamma)}{4})$, while station $-i$ gets most of its revenue via viewer fees and sets $a_{-i}^* = \frac{N}{2} - \delta$ and $f_{-i}^* = \frac{2\beta+(2\delta-N)(\omega+\gamma)}{4}$.

These asymmetric equilibria are interesting because they arise from a perfectly symmetric model. The main intuition for the existence of these equilibria is that $\omega > \gamma$ implies that collecting revenues from advertisers is more profitable than via viewer fees, provided that there is no overlap on both sides of the market. However, if overlapping viewers are not very valuable, then firms want to avoid two-sided overlap. This means that if one platform advertises a lot, then the other platform might prefer only serving the remaining small number of advertisers and switching to charging positive viewer fees.

This type of differentiation can be observed in various settings. For example standard commercial TV channels in the US are cheap to subscribe to and broadcast relatively lot of commercials, while exclusive channels like HBO are more expensive to subscribe to, but broadcast less or no commercials.

We conclude this section by pointing out that the welfare implications of introducing viewer fees are ambiguous. For many parameters viewer fees improve welfare, because they help internalizing the negative externalities on the viewer side. However, there exist also constellations such that viewer fees are welfare reducing. For example, look at the case in which $\omega < \gamma$ and $N \geq \frac{\beta}{\gamma}$. In this case the optimal policy in the duopoly and also in both monopoly regimes is $a_i^* = \frac{\beta}{2\gamma}$ which leads to a welfare of $\frac{\beta^2(2\omega+\gamma)}{4\tau\gamma}$. Instead with the viewer fee, $a_i^* = 0$ and $f_i^* = \frac{\beta}{\gamma}$ giving a welfare of $\frac{3\beta^2}{4\ \gamma}$ which is higher than the one without viewer fee because $\omega < \gamma$. Here the possibility of a viewer fee lead both stations to reduce their advertising levels which improves welfare. On the other hand, in all three regimes there are cases in which the optimal policy without the viewer fee is $a_i^* = \frac{N}{2}$ while with the viewer fee a station leaves its advertising level unchanged but sets $f_i^* > 0$ which is welfare reducing. Here the stations use the viewer fee only to extract more surplus from consumers.

6 Application: recent developments in the British and German Broadcasting Market

In this section we shortly discuss some recent developments in the British and German broadcasting market, and contrast them with different models of media markets.

In the UK, the ITV network (commonly known as 'Channel 3') is the biggest commercial television network which had in the beginning of the 1980s an audience share of around 50 per cent. Because of entry of many new smaller channels

(like 'Channel 4' in 1983 or 'Five' in 1997) the audience share of ITV decreased steadily over the last 15 years.[19] Yet, the percentage of ITV's net advertising revenue out of total TV advertising revenue has decreased by far less. Moreover, from 1992 on the advertising price (per adult impact) of ITV[20] has increased and this increase was even higher from 1997 to 2002, a time in which many new competing channels entered the market. This is known as the ITV premium puzzle.
An explanation for this premium is that adult impact is different for ITV than for the other channels. Although adult impact has decreased over time on ITV (because of decreasing audience share) advertisers are willing to pay more for it on ITV than on competing channels. The reason is that if an advertisement reaches e.g. one million eyeballs on ITV it is likely that it reaches a million different people because ITV has a lot of mass audience programmes and is good at reaching so called 'light' viewers who do not watch television often. Furthermore, there is a small but significant proportion of viewers who only watch ITV and BBC, and so advertisers can only reach them via ITV.[21] On the other hand, the value of a million eyeballs on other channels is much smaller because it is likely that one reaches the same viewer twice or more.
This explanation is fully in line with the predictions of our model in which the reaction of entry of a new station is to increase advertising to avoid viewer overlap. With that the incumbent loses audience but its marginal viewer is of higher value for the advertisers than before.

The German broadcasting market consists of six major channels (ARD, Pro7, RTL, SAT1, ZDF, and the 'third programmes,' which are regional programmes in each state). Each of these channels has a market share between 10% and 14%, leading with ARD with 13.9% and ending with SAT1 with 10.3%.[22] Three of the channels are public channels (ARD, ZDF, and the 'third programmes') and are financed partly by taxes and partly by advertising revenues. The other three channels (Pro7, RTL, SAT1) are private stations and collect revenues only through advertising.

For the private stations there is no law regulating their broadcasting of commercials besides the 12 minutes limit per hour set by the media commission of the European Union. The public stations on the other hand are not allowed to broadcast commercials after 8:00 p.m. The German government is currently discussing to abolish this law and to allow public stations to broadcast advertising after 8:00 p.m. A well-publicized fact concerning this is that the private

[19] The audience share in 1987 was 40 per cent while in 2003 it was only 22.2 per cent. See Competition Commission (2003), p.96.

[20] The advertising price is measured as the expenditure of an advertiser divided by the impact on its targeting group where the latter is measured as percentage of viewers of the targeted group. This variable does not account for double viewing, so if a viewer watches the commercial twice she is counted in the same way as two viewers who have seen the commercial once.

[21] See Competition Commission (2003), p.123 and p.136.

[22] The source of these data is AGF/GfK Fernsehforschung. See also www.agf.de.

stations are strongly against this abolishment, arguing that the public stations have a social mission for their viewers and should therefore be not allowed to fill their viewing time with commercials.[23] This stylized fact would be difficult to reconcile with models in which advertisers can multi-home but viewers can single-home, like the one in Anderson and Coate (2005). In those models more advertising of a rival station makes that station less attractive, therefore increases viewership of the own station and leads to higher profits. Hence, private stations should be supporting an initiation like above. However, their behavior is in line with the results of our model. If viewers multi-home (and indeed there is a large overlapping viewership between public and private channels), then allowing public channels to advertise more freely intensifies competition among channels for advertisers and drives down advertising fees. This decreases the profits of private channels.[24]

Another prevalent development in the German broadcasting market is that viewership has grown steadily in the last 5 years, but this was accompanied by decreasing advertising fees. The percentage of viewers in the population on a regular weekday has increased from 73.7% to 75.4% from 2001 till 2004, and the average minutes per day a viewer spends watching TV have increased from 192 to 210. But during this time commercial prices have decreased by roughly 10%, e.g. the price per 1000 eye-balls of a 30 second commercial has dropped from 10,34 to 9.52 Euros. Furthermore, television advertising expenditures increased from 7.636 bn to 7.744 bn Euros, which together with the decreasing prices indicates that at the same time the level of advertising increased.[25] Our model provides an explanation to this seemingly puzzling observation. During this time period it is notable that the content of public stations has moved closer to the one of private stations, especially in the afternoon and late-afternoon programme. This move was made to attract more viewers, in particular younger ones who are a particularly attractive consumer group for advertisers. In our model this development can be associated with a decrease in τ, which makes the channels more attractive and induces more potential viewers to watch. But this also increases the overlapping viewership among channels, and therefore lowers advertising prices. As shown in Subsection 4.4, this can lead to lower profits despite the channels are more attractive.[26] On the other hand, a similar decrease in τ in a model in which viewers can only single-home cannot lead to increasing advertising levels and decreasing prices, and typically leads to decreasing advertising levels since platforms have to be more attractive in order

[23] For example, in July 2005 the president of the Association of Private Radio and Telecommunication demanded a complete advertising ban for public TV stations. See e.g. www.presseportal.de.

[24] Another, complementary explanation for this behavior is that public stations could obtain higher revenues, which if used to improve the quality of programs, could induce some viewers to substitute time away from watching private channels to watching public channels.

[25] See www.agf.de/daten/werbemarkt/werbespendings/.

[26] A second possible explanation, which is often given in newspapers, is the struggling German economy. This can explain part of the price drop. But it is hardly conceivable that prices have dropped in a growing market by such a large amount because of this reason alone.

to retain their viewers from switching to the other channel.

7 Extensions

Our analysis can be extended in many directions. Here we only briefly discuss a few of them.

If advertisers are heterogenous with respect to the surplus that their product can generate, like in Anderson and Coate (2005), then there is an extra incentive for firms to advertise less. This is the usual downward distortion effect resulting from imperfect competition, both in the monopoly and the duopoly cases (more profitable advertisers are willing to pay a higher advertising fee per viewer). However, most of the qualitative conclusions of our analysis still apply in this framework.[27]

If viewers are heterogeneous with respect to the nuisance parameter for advertising and platforms can charge viewer fees, then various new types of equilibria can arise besides the ones characterized in Section 5. Besides asymmetric equilibria in which one platform chooses a high level of advertising while the other chooses a low level, it becomes possible in equilibrium that the same platform offers different versions of the same content, with different levels of advertising. This phenomenon can be observed among internet based services: there are web sites that offer different subscription fees to their members depending on whether they choose to access the same service with on-screen advertisements or without them.[28]

We do not explicitly model the platforms' decision on the timing of commercials.[29] If channels can synchronize the timing of their commercial blocks, they have an incentive to do so, because it increases the value of overlapping viewers. Sweeting (2005, 2006) provides empirical evidence that competing radio stations coordinate this way. The paper emphasizes the motive of doing this to prevent listeneres from switching channels when commercials come on the air. In our model framework coordinating commercial block times increases the value of multi-homing listeners even if they do not flip channels to avoid commercials. This is because synchronizing makes it less likely that a listener hears a commercial broadcasted on two different channels twice, which brings the value of ω'' closer to ω'.

[27] Proposition 4, that a discriminating monopolist always sets inefficiently high levels of advertising, is no longer true.

[28] See for example Fileplanet at http://www.fileplanet.com/subscribe/subscribe.shtml, a web site that allows members to download pc games. Access with advertisements is $6.95/month, while advertisement-free access is $9.95/month.

[29] This issue is of course only relevant for the case of electronic media.

8 Conclusion

The main point of our paper is that in media markets keeping a distinction between exclusive and overlapping viewership is important, because their values might be very different for the media platforms. The possibility of overlapping viewership changes the nature of competition in these markets. This implies that different qualitative conclusions hold for models that allow viewers to connect to multiple platforms - in most contexts a realistic assumption - than for models in which viewers can connect to at most one platform.

9 Appendix

9.1 Appendix A: Single-Homing Viewers

To create a benchmark for comparison, in this appendix we characterize equilibrium in the same model as the one defined in Section 2 (in particular, assuming that advertisers are homogeneous), with the modification that viewers cannot multi-home. We show that the welfare implications of the resulting model are ambiguous, as in the more general setting of Anderson and Coate (2005). A monopolist platform owner either advertises too much in equilibrium or broadcasts the socially efficient level of advertising. The level of advertising in duopoly competition can be either too high, socially efficient, or too low. Finally, it is ambiguous whether monopoly or duopoly leads to a more efficient outcome. For the intuition behind these results, see Anderson and Coate (2005).

Just like in Section 3, we assume that $N \geq \max(\frac{2\beta-\tau}{2\gamma}, \frac{\beta}{2\gamma})$, to rule out cases when the total number of advertisers is a binding constraint on advertising levels in equilibrium. This is done to simplify the exposition, the propositions below can be easily extended to parameter values not satisfying the above restriction.

If $\tau \geq \beta$ then the equilibrium is the same as before, both for a monopolist platform owner and for competing platforms, since stations are local monopolists and do not compete for the same viewers. If $\beta < \tau$ then the following observations can be made:

(i) if there is a set of viewers who would have positive net utility from watching both channels, then the marginal viewer is $x = \frac{1}{2} + \frac{\gamma(a_1-a_0)}{2\tau}$. Then the profit of station 0 is $\Pi_0 = (\frac{1}{2} + \frac{\gamma(a_1-a_0)}{2\tau})M\omega a_0$ and the profit of station 1 is $\Pi_1 = (\frac{1}{2} - \frac{\gamma(a_1-a_0)}{2\tau})M\omega a_1$.

(ii) if there are no viewers who would have positive net utility from watching both channels, then simple considerations establish that the marginal viewer gets exactly 0 net utility when connecting to either channel, otherwise the channels could increase their profits.

This leads to the following characterization of equilibria:

Monopoly:

The equilibrium level of advertising for a monopolist platform provider if viewers cannot multi-home is given by:

$$a_i^{mon} = \begin{cases} \frac{\beta}{2\gamma} & \text{if } \beta \leq \tau \\ \frac{2\beta-\tau}{2\gamma} & \text{if } \tau < \beta \end{cases}$$

The $\beta \leq \tau$ case is straightforward. If there is a set of viewers who would have positive net utility from watching both channels, then the monopolist could increase its profits by increasing advertising on both channels. Furthermore, the monopolist's profit is maximized if the marginal viewer is $x = \frac{1}{2}$. These imply the result for $\tau < \beta$.

Duopoly:

In this case we only characterize symmetric equilibria. The asymmetric equilibria that arise for a range of parameter values are similar to the ones in the model in which viewers can multi-home.[30] It is straightforward to show that the asymmetric equilibria that can arise imply the same total amount of advertising and smaller total welfare than the (unique) symmetric equilibrium for the corresponding parameter values. This implies that the propositions below hold for asymmetric equilibria, too.

The equilibrium level of advertising in symmetric equilibrium for competing providers if viewers cannot multi-home is given by:

$$a_i^{duo} = \begin{cases} \frac{\beta}{2\gamma} & \text{if } \beta \leq \tau \\ \frac{2\beta-\tau}{2\gamma} & \text{if } \beta > \tau \geq 2/3\beta \\ \frac{\tau}{\gamma} & \text{if } \tau > 2/3\beta \end{cases}$$

The $\beta \leq \tau$ case is straightforward. For $\beta > \tau$, assume first that the marginal viewer gets positive utility from joining a channel. Then $\Pi_0 = (\frac{1}{2} + \frac{\gamma(a_1-a_0)}{2\tau})M\omega a_0$ and $\Pi_1 = (\frac{1}{2} - \frac{\gamma(a_1-a_0)}{2\tau})M\omega a_1$ imply that the best response functions of the platforms are $a_i = \frac{\tau+\gamma a_{-i}}{2\gamma a_{-i}}$ for $i = 0, 1$. This implies that the marginal viewer is $\frac{1}{2}$, and that $a_0 = a_1 = \frac{\tau}{\gamma}$. The condition for viewers located at $\frac{1}{2}$ getting positive utility when $a_0 = a_1 = \frac{\tau}{\gamma}$ is $\tau > 2/3\beta$. For $\beta > \tau \geq 2/3\beta$ the marginal consumer at $\frac{1}{2}$ gets exactly 0 net utility when connecting to a channel, which yields $a_0^{duo} = a_1^{duo} = \frac{2\beta-\tau}{2\gamma}$.

Social optimum:

[30] Just like when viewers can multi-home, it can be shown that asymmetric equilibria can only arise when in equilibrium the marginal viewer gets 0 net utility from connecting to either channel. In this case we focus on the case when the marginal viewer is $x = \frac{1}{2}$, although there is an interval around $\frac{1}{2}$ such that the marginal viewer can be any point of this interval.

The socially optimal level of advertising if viewers cannot multi-home is given by:

$$a_i^{WF} = \begin{cases} 0 & \text{if } \gamma \geq \omega \\ \frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)} & \text{if } \gamma < \omega \text{ and } \frac{2\beta\omega}{2\omega-\gamma} \leq \tau \\ \frac{2\beta-\tau}{2\gamma} & \text{if } \gamma < \omega \text{ and } \frac{2\beta\omega}{2\omega-\gamma} > \tau \end{cases}$$

The case when $\gamma \geq \omega$ and the case when $\gamma < \omega$ and $\frac{2\beta\omega}{2\omega-\gamma} \leq \tau$ are the same as in the model in which viewers can multi-home. If $\frac{2\beta\omega}{(2\omega-\gamma)} > \tau$ then it is easy to establish that in the welfare maximizing outcome all consumers connect to a channel. In this case the welfare function is:

$$WF = (\frac{1}{2} + \frac{\gamma(a_1 - a_0)}{2\tau})M\omega a_0 + (\frac{1}{2} - \frac{\gamma(a_1 - a_0)}{2\tau})M\omega a_1 +$$

$$+M\int_0^{\frac{1}{2}+\frac{\gamma(a_1-a_0)}{2\tau}}(\beta - \gamma a_0 - \tau x)dx + M\int_{\frac{1}{2}+\frac{\gamma(a_1-a_0)}{2\tau}}^{1}(\beta - \gamma a_0 - \tau(1-x))dx.$$

If $\gamma < \omega$ then the above expression is increasing in a_i $(i = 0, 1)$. Furthermore, $WF(a_0, a_1) \geq WF(\frac{a_0-a_1}{2}, \frac{a_0-a_1}{2})$. Then the welfare maximizing advertising levels are the maximum possible symmetric levels that are compatible with all consumers wanting to connect to a channel, which is $\frac{2\beta-\tau}{2\gamma}$.

Using the above results we can compare the socially optimal level of advertising with the equilibrium levels of advertising in monopoly and in duopoly.

First consider a monopolist provider. If $\omega > \gamma$ then the socially efficient amount is given by $\frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)}$ if $\frac{2\beta\omega}{2\omega-\gamma} \leq \tau$ and by $\frac{2\beta-\tau}{2\gamma}$ if $\frac{2\beta\omega}{2\omega-\gamma} > \tau$. Instead a monopolist advertises $\frac{\beta}{2\gamma}$ if $\beta \leq \tau$ and $\frac{2\beta-\tau}{2\gamma}$ if $\tau < \beta$. Comparing this shows that in the region $\frac{2\beta\omega}{2\omega-\gamma} \leq \tau$ a monopolist advertises too much because $\frac{\beta}{2\gamma} > \frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)}$ since $\gamma > 0$. In the region $\beta \leq \tau < \frac{2\beta\omega}{2\omega-\gamma}$, the monopolist also advertises too much because comparing $a_i^{mon} = \frac{\beta}{2\gamma}$ with $a_i^{WF} = \frac{2\beta-\tau}{2\gamma}$ shows that $a_i^{mon} > a_i^{WF}$ because $\beta \leq \tau$ in this region. Lastly, for $\beta > \tau$ both advertising amounts are the same. If $\omega \leq \gamma$, then the socially optimal level of advertising is zero while in monopolist chooses a positive level.

Proposition 7: Assume that viewers cannot multi-home. If $\omega \leq \gamma$, or if $\omega > \gamma$ and $\tau \geq \beta$ then the monopolist advertises more on both platforms than the socially efficient amount. If $\omega > \gamma$ and $\tau < \beta$ then the monopolist in equilibrium chooses the socially efficient level of advertising.

Now consider the duopoly case. Since the advertising amounts here are the same as in monopoly for $\tau \geq 2/3\beta$, the result for this case is analogous to the result for a monopolist platform provider. For $\tau < 2/3\beta$, $a_i^{duo} = \frac{\tau}{\gamma}$ while the socially efficient amount is $\frac{2\beta-\tau}{2\gamma}$. Comparing these two equations shows that

$\frac{\tau}{\gamma} < \frac{2\beta-\tau}{2\gamma}$ if $\tau < 2/3\beta$, which holds in this region. If $\omega \leq \gamma$ then the socially optimal level of advertising is zero while, as for the monopolist provider, it is always positive in duopoly equilibrium.

Proposition 8: Assume that viewers cannot multi-home. If $\omega \leq \gamma$, or if $\omega > \gamma$ and $\tau \geq \beta$ then there is too much advertising on both platforms in symmetric duopoly equilibrium compared to the social optimum. If $\omega > \gamma$ and $\beta > \tau \geq 2/3\beta$ then the level of advertising in symmetric duopoly equilibrium is socially efficient. If $\omega > \gamma$ and $\tau < 2/3\beta$ then the level of advertising is lower on both platforms in symmetric duopoly equilibrium than in social optimum.

The previous results imply that competition among platforms leads to a more efficient outcome than monopoly does if $\tau < 2/3\beta$ and $\gamma \geq \omega$, while monopoly leads to a more efficient outcome than competition if $\tau < 2/3\beta$ and $\gamma < \omega$. In all other cases monopoly and platform competition are equally efficient from the social point of view.

9.2 Appendix B: Discriminating Monopolist

If advertisers overlap, then $\overline{a}_{01} = N - \overline{a}_0 - \overline{a}_1$. From this, the monopolist's profit function can be written as follows:

$$\Pi_m = \overline{a}_0 M \left(\omega(1 - \frac{\beta - \gamma(N - \overline{a}_0)}{\tau}) + \omega'(\frac{2\beta - \gamma(2N - \overline{a}_0 - \overline{a}_1)}{\tau} - 1) \right) +$$

$$\overline{a}_1 M \left(\omega(1 - \frac{\beta - \gamma(N - \overline{a}_1)}{\tau}) + \omega'(\frac{2\beta - \gamma(2N - \overline{a}_0 - \overline{a}_1)}{\tau} - 1) \right) +$$

$$(N - \overline{a}_0 - \overline{a}_1) M[\omega(2 - \frac{2\beta - \gamma(2N - \overline{a}_0 - \overline{a}_1)}{\tau}) + (\omega' + \omega'')(\frac{2\beta - \gamma(2N - \overline{a}_0 - \overline{a}_1)}{\tau} - 1)]$$

The first term is the revenue from advertisers only advertising on platform 0, the second term is the revenue from advertisers who only advertise on platform 1, while the third one is the revenue from multi-homing advertisers. Writing out the first-order conditions and solving for the optimal values of a_0 and a_1 yields:

$$\overline{a}_0 = \overline{a}_1 = \frac{\omega(\beta - \tau) - \omega''(2\beta - \tau) - N\gamma(\omega' + 3\omega'' - 2\omega)}{2(2\omega'' - \omega)\gamma} \tag{5}$$

This implies that total advertising on a platform is:

$$a_i = \overline{a}_i + \overline{a}_{01} = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau) - N\gamma(\omega' - \omega'')}{2(2\omega'' - \omega)\gamma}.$$

Denote the resulting profit of the monopolist by Π^+. The above levels of advertising are only consistent with the assumptions $n_{01} \geq 0$ and $a_0 + a_1 \geq N$ if $\omega'' \geq \frac{\beta\omega}{2\beta - \tau}$ and $N \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{(2\omega''-\omega)\gamma}$. It can be shown that $N \leq$

$\frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{(2\omega''-\omega)\gamma}$ is implied by $N \leq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(2\omega'-\omega)}$. Then $\frac{2\beta-\tau}{2\gamma} \geq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{2(2\omega''-\omega)\gamma}$ implies that the optimal choice for the monopolist is either $\frac{N}{2}$ or the levels specified by (5), depending on the relative magnitudes of Π_M^{**} and Π^{+}. Similarly, $\frac{2\beta-\tau}{2\gamma} \leq \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)-N\gamma(\omega'-\omega'')}{2(2\omega''-\omega)\gamma}$ implies that the optimal advertising level on each platform is either $\frac{2\beta-\tau}{2\gamma}$ or $\frac{N}{2}$, depending on N bigger or smaller than $\frac{\omega\tau}{\gamma(2\omega'-\omega)}$.

9.3 Appendix C: Duopoly

If $\tau \geq \beta$, then there is no difference to the preceding analysis. Since in this case the stations are local monopolists, the equilibrium levels of advertising and per platform profits do not change: $a_i = \frac{\beta}{2\gamma}$ and $\Pi_i = \frac{\omega M\beta^2}{4\tau\gamma}$ for $i = 0, 1$. The interesting case arises when $\tau < \beta$, implying that viewers would overlap at the above advertising levels.

Assume first that viewers overlap, but advertisers do not. The profit function of firm i is then given by:

$$\Pi_i = Ma_i\left(\max(\omega(1-\frac{\beta-\gamma a_j}{\tau}),0)+\min(\omega'(\frac{2\beta-\gamma(a_i+a_j)}{\tau}-1),\omega')\right).$$

Taking the first order condition and solving for a_i yields:[31]
$a_i = \max[\frac{1}{2}\frac{\tau}{\gamma\omega'}\left(\omega\left(-\frac{1}{\tau}(\beta-\gamma a_j)+1\right)+\omega'\left(\frac{1}{\tau}(2\beta-\gamma a_j)-1\right)\right), \frac{\beta-\tau}{\gamma}]$
which is increasing in a_j (levels of advertising are strategic complements). The equilibrium levels of advertising are:

$$a_0 = a_1 = \max(\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}, \frac{\beta-\tau}{\gamma}). \quad (6)$$

This yields equilibrium profits of

$$\Pi_i = \min(\frac{M\omega'[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega'-\omega)^2}, \frac{M\omega'(2\beta-\tau)}{\gamma}).$$

The above equilibrium is only valid as long as at advertising levels given by (6) viewers overlap, but advertisers do not. The condition for no viewer overlap is $\omega' \geq \frac{\tau\omega}{2\beta-\tau}$. If $\omega' < \frac{\tau\omega}{2\beta-\tau}$ and $\tau < \beta$ then the only possibility for equilibrium is when the two platforms set their advertising levels such that viewers just don't overlap. This means that in equilibrium $a_j + a_i = \frac{2\beta-\tau}{\gamma}$ and $n_0 + n_1 = M$.[32] That is, the sum of advertising levels and the sum of viewerships are uniquely pinned down. However, how the share of advertising (and total viewership)

[31] The second order condition for maximum, $-2M\omega'\frac{\gamma}{\tau} < 0$ is satisfied.

[32] Note that the maintained assumption $\frac{\beta}{\gamma} \leq N$ implies $\frac{2\beta-\tau}{2\gamma} \leq N$, therefore there are $a_0, a_1 \in [0, N]$ such that $a_0 + a_1 = \frac{2\beta-\tau}{2\gamma}$.

is divided between channels is not uniquely determined. For any parameter constellation satisfying $\omega' < \frac{\tau\omega}{2\beta-\tau}$ and $\tau < \beta$ there is a $\overline{\delta} \in (0, \frac{1}{2})$ such that $a_0 = \frac{2\beta-\tau}{\gamma} + \delta$, $a_1 = (1-x)\frac{2\beta-\tau}{\gamma} - \delta$ constitute an equilibrium of the duopoly game for any $\delta \in [-\overline{\delta}, \overline{\delta}]$. Among these, we focus on the symmetric equilibrium $a_0 + a_1 = \frac{2\beta-\tau}{2\gamma}$, which is both the most efficient from the social point of view and maximizes the total profit of the platforms. The qualitative conclusions of our model would not change if instead we selected an asymmetric sharing rule of the market for the above parameter values.

The condition for advertisers not to overlap at advertising levels given by (6) is $N \geq \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$. Just like in the case of a monopolist provider, this automatically holds if $\frac{2\beta-\tau}{\gamma} \leq N$.

Assume from now on that $\omega' > \frac{\tau\omega}{2\beta-\tau}$ and $N < \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$. Then there can be three types of equilibria: (i) $a_0 + a_1 > N$ and $n_{01} > 0$ (two-sided overlap); (ii) $a_0 + a_1 > N$ and $n_0 + n_1 = M$ (viewerships just don't overlap); (iii) $a_0 + a_1 = N$ and $n_{01} > 0$ (advertisers just don't overlap).

If $a_0 + a_1 > N$ and $n_{01} > 0$, then profit functions are given by:

$$\Pi_i = Ma_i\left(\omega(1 - \frac{\beta - \gamma a_j}{\tau}) + \omega''(\frac{2\beta - \gamma(a_i + a_j)}{\tau} - 1)\right).$$

Solving for the equilibrium levels yields:

$$a_0 = a_1 = \frac{\omega''(2\beta - \tau) - \omega(\beta - \tau)}{\gamma(3\omega'' - \omega)}. \tag{7}$$

This gives a profit of $\frac{M\omega''[(2\beta-\tau)\omega''-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega''-\omega)^2}$.

As before, for the case $a_0 + a_1 > N$ and $n_0 + n_1 = M$ we only consider the symmetric candidate equilibrium $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$. Similarly, if $a_0 + a_1 = N$ and $n_{01} > 0$ then we only consider the symmetric candidate equilibrium $a_0 = a_1 = \frac{N}{2}$. There can be equilibria in which the firms divide viewers or advertisers asymmetrically, but it can be shown that the existence of such asymmetric equilibria always implies the existence of a symmetric one, and that the latter maximizes both social efficiency and aggregate profits of platforms.[33]

Next we characterize the range of parameter values for which each of the above symmetric profiles constitute equilibrium.

[33] Contact the authors for a complete characterization of asymmetric equilibria.

First we consider the region $N > \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$.[34] Note that $\omega'' < \frac{\tau\omega}{2\beta-\tau}$ implies the above.[35] Then advertising levels in (7) imply $a_0+a_1 < N$. Therefore in this region overlapping on both sides cannot be in equilibrium.

Out of the remaining two possibilities, $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ constitutes an equilibrium if no firm wants to decrease advertising to $N - \frac{2\beta-\tau}{2\gamma}$, the point at which advertisers don't overlap anymore. The profit implied by this deviation is $M(N-\frac{2\beta-\tau}{2\gamma})\left(\omega(1-\frac{\beta-\frac{2\beta-\tau}{2}}{\tau})+\omega'(\frac{2\beta-\gamma N}{\tau}-1)\right)$. Therefore the condition for $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ to constitute an equilibrium is

$$\frac{\omega}{2}\frac{2\beta-\tau}{2\gamma} \geq (N-\frac{2\beta-\tau}{2\gamma})\left(\omega(1-\frac{\beta-\frac{2\beta-\tau}{2}}{\tau})+\omega'(\frac{2\beta-\gamma N}{\tau}-1)\right) \quad (8)$$

This inequality holds whenever $\frac{2\beta-\tau+\tau\frac{\omega}{\omega'}}{2\gamma} \leq N$. By (1) this is always true, therefore for parameter range $\omega' \geq \frac{\tau\omega}{2\beta-\tau}$, $N < \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$ and $\omega'' \leq \frac{\tau\omega}{2\beta-\tau}$ advertising levels $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ always constitute an equilibrium.

Finally, $a_0 = a_1 = \frac{N}{2}$ constitutes an equilibrium if no firm wants to deviate and set $a_i = \frac{4\beta-2\tau-\gamma N}{2\gamma}$ such that viewers just do not overlap (but advertisers do). Advertising levels $a_0 = a_1 = \frac{N}{2}$ imply a profit of

$$\Pi_i = \frac{MN(2\tau(\omega-\omega')+2\beta(2\omega'-\omega)-\gamma N(2\omega'-\omega))}{4\tau}.$$

The above deviation gives a profit of $\Pi_i^{dev} = \frac{\omega M(4\beta-2\tau-\gamma N)(2\tau-2\beta+\gamma N)}{4\tau\gamma}$. Comparing these two profit levels shows that Π_i^{dev} is lower iff $\omega' \geq \frac{\gamma N-2(\beta-\tau)}{\gamma N}$.

Note that there is a range of parameter values for which both $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ and $a_0 = a_1 = \frac{N}{2} < \frac{2\beta-\tau}{2\gamma}$ can be in equilibrium. The intuition behind this multiplicity is that advertising on different platforms are strategic complements.

Consider now $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$. As shown above, this implies $\omega'' \geq \frac{\tau\omega}{2\beta-\tau}$. In this case advertising levels in (7) are consistent with two-sided multi-homing. The condition for this to constitute an equilibrium is that no firm wants to deviate and set $a_i = N - \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$, to avoid overlap in advertisers. This deviation yields:

$$\Pi_j^{dev} = \left(M(N-\frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}\right) \times$$

[34] It can be shown that this cannot occur if $\tau \geq \frac{\beta}{2}$, because in the latter case $\frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)} > N$ implies $\frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)} > N$.

[35] $\omega'' < \frac{\tau\omega}{2\beta-\tau}$ implies $\frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)} < \frac{2\beta-\tau}{2\gamma}$ and that $N > \frac{2\omega'(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$ implies $\frac{2\beta-\tau}{\gamma} < N$.

$$\left(\omega\left(1-\frac{\beta-\frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{(3\omega''-\omega)}}{\tau}\right)+\omega'\left(\frac{2\beta-\gamma N}{\tau}-1\right)\right)$$

The deviation is therefore profitable if $\Pi_j^{dev} > \frac{M\omega''[(2\beta-\tau)\omega''-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega''-\omega)^2}$. Thus, $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ constitute an equilibrium if $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$ and $\Pi_j^{dev} > \frac{M\omega''[(2\beta-\tau)\omega''-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega''-\omega)^2}$.

It is straightforward to show that $a_0 = a_1 = \frac{2\beta-\tau}{2\gamma}$ cannot be in equilibrium if $N \leq \frac{2\omega''(2\beta-\tau)-2\omega(\beta-\tau)}{\gamma(3\omega''-\omega)}$.

Finally, the condition for $a_0 = a_1 = \frac{N}{2}$ to constitute an equilibrium is that no firm has an incentive to deviate to increase advertising (resulting in two-sided overlap). It can be shown that the optimal deviation of this type is $a_i^{dd} = \frac{2\tau(\omega-\omega')+2\beta(2\omega'-\omega)-\gamma N(2\omega'-\omega)}{4\omega'\gamma}$ giving a profit of

$$\Pi_i^{dd} = \frac{(2\tau(\omega-\omega')+2\beta(2\omega'-\omega)-\gamma N(2\omega'-\omega))^2}{16\tau\omega'\gamma}.$$

Therefore the condition for the existence of this type of equilibrium is

$$\Pi_i^{dd} \leq \frac{MN(2\tau(\omega-\omega')+2\beta(2\omega'-\omega)-\gamma N(2\omega'-\omega))}{4\tau}.$$

Again there is a parameter range in which there is multiplicity of equilibria, namely for which both $a_0 = a_1 = \frac{\omega''(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega''-\omega)} > \frac{N}{2}$ and $a_0 = a_1 = \frac{N}{2}$ constitute equilibria.

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