Asymmetric Equilibria and Non-cooperative Access Pricing in Telecommunications

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Abstract

This paper looks at competition in the Telecommunication industry with non-linear tariffs and network based price discrimination where one of the networks has a relative advantage. We investigate profit-maximizing network pricing behaviour, in particular competitively chosen unregulated non-cooperative access prices at potentially asymmetric market equilibira.

1 Introduction

The recent literature on the network interconnection and pricing strategies in the Telecommunications Industry originating in the work of Armstrong (1998) and Laffont, Rey, and Tirole (LRT 1998a,b) has generically assumed that competition takes place between *symmetric* networks. Within this symmetric framework the analysis of Gans and King (2001) has shown that with non-linear tariffs and network based price discrimination the optimal (i.e. profit-maximizing) choice of negotiated reciprocal access charges will imply a negative markup so that call termination is in fact subsidized.

This theoretical finding is not warranted by the empirical findings however and does not explain the rising concern among competition authorities about the welfare effects of such charges being 'too high'. The German Monopolkommission (2003, p.91) based on a study by wik-Consult, a consulting firm, has published the following table for average access charges (in cent per minute) charged by the four mobile phone networks that are hitherto unregulated:

	1998	1999	2000	2001	2002
T-Mobile	22,86	27,86	17,09	14,39	14,30
Vodafone	28,44	28,44	28,51	15,42	14,30
E-Plus	42,60	42,60	42,68	19,03	16,94
O_2	29,24	29,24	29,32	18,77	17,88

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Based on a study by the Competition Commission (2003) for the UK, the Monopolkommission suggests that the cost for such termination services are only about half of the charges, i.e. between 7 and 7,8 cent. A recent paper by Behringer (2004a) has shown that allowing for non-cooperatively chosen access charges does not preclude the existence of equilibrium in the symmetric model and that such profit-maximizing access charges will imply a positive markup on termination cost. The latter has been noted to be the consequence of the "double marginalization" of complementary product pricing by two firms in LRT, 1998a and Gans & King, 2001 who refrain from calculating this markup explicitly. The scepticism of competition authorities with regard to negotiated access charges is warranted by the results in Behringer (2004b) who finds that the total welfare consequence of such negotiated charges are detrimental as compared to non-cooperatively chosen charges or a 'bill-and-keep' regime.

The assumption of symmetry of the two networks in the previous models is a most welcome simplifying device to keep the analysis of the optimal pricing vectors that form the Nash equilibrium of the game tractable. However, in many cases of regulatory concern it is a new entrant who competes against an incumbent with an established market share and hence the assumption seems to be unfortunate. A tractable theoretical analysis of non-reciprocal access charges in asymmetric settings is thus considered "one of the most valuable areas for future research", (see Armstrong, 2003, p.373). Previous research in asymmetric environments is scarce however. Carter & Wright, 2003 is the most complete and the closest to our model but as Gans and King, 2001 they are only able to show that firms would want to deviate from cost based access prices and cannot give a prediction of how the *asymmetry* of the setting is reflected in the equilibrium non-cooperative access charges and thus how to judge the observed data in the table above. Peitz, 2005, also investigates the issue but focuses on the impact of asymmetric regulation.

The present paper sets out to determine the profit-maximizing choice of unregulated non-reciprocal access prices using the technology of Armstrong (1998). In addition, one of the networks has an inherent advantage over the other that is not based on its technology but on the perception of its service by consumers.

In practice it is often argued that the possibility of high termination charges was important for the business strategy of late entrants such as E-Plus and O_2 that were competing with the established networks T-Mobile and Vodafone (formerly Mannesmann). High termination revenue is seen as allowing the disadvantaged, small networks to offer for example pre-paid phones with no monthly charge and where the cost of the technical unit is often subsidized. As a result an ex-ante (downward) regulation of termination charges would hurt disadvantaged firm disproportionately and may even lead to market exit. Our analysis enables us to judge the validity of such claims within the framework of the formal model provided.

2 The Model

The setup builds upon LRT, 1998b in using a product differentiation model with mass one consumers distributed uniformly on the unit interval with two networks located at the extremes.

The additively separable quadratic utility function of a consumer located at some $x \in [0,1]$ purchasing from network j located at unity is

$$u(q,x) = q(1 - \frac{1}{2}q) + \eta x t \tag{1}$$

with a horizontal preference parameter $t > 0$ the benefit of which is independent of the amount of costly calls initiated q, and an asymmetry parameter $\eta > 0$, where $\eta > 1$ implies an exogenous advantage for the (incumbent) network j over network i located at the origin. This advantage has a multiplicative feature and thus affects those closer to their preferred brand more than those further afar. Whence an advantage for one network implies an increase in the *perceived product differentiation* for that product, i.e. consumers that are located close to unity and use network $j's$ services are even more happy to do so if η increases.

This modelling is slightly different from Carter & Wright, 2003 where the advantage is additive to the location and thus affects all consumers of that network equally (but can be scaled with the product differentiation parameter) which they interpret as "brand loyalty". Note that both specifications share the feature that within a simple Hotelling product differentiation model without (or with equal) prices, a large asymmetry implies that the disadvantaged firm is driven out of the market.

We order the unit mass of consumers such that the difference of their 'address' x to a network is proportional to their individual fixed benefit from being connected, with the consumer furthest away from the network at the origin receiving exactly zero fixed benefit. Hence t represents the *maximum pure benefit of being connected* to a network (without initiating any costly calls but including calls received) and is assumed to be exogenous.

Utility maximization implies that individual demand for the service is

$$q(p) = 1 - p \tag{2}$$

i.e. linear as in Armstrong (1998) and the indirect utility function is

$$v(p) \equiv \int_p^\infty q(\zeta)d\zeta = q(p)\left(1 - \frac{1}{2}q(p)\right) - pq(p) \tag{3}$$

which (with $p \leq 1$ for non-negative quantities) is decreasing and strictly convex in price.

The networks use a two-part tariff consisting of a unit price (e.g. per minute of the service) p and a fixed charge (e.g. a monthly rental charge) G and thus the total per capita consumer j valuation given location $x \in [0,1]$ is

$$U_x = v(p) - G + \eta x t \tag{4}$$

A consumer is indifferent between the two networks given his location $x \in [0,1]$ with networks using two-part tariffs and *network-based price discrimination* if and only if the utility of this marginal consumer satisfies

$$\begin{aligned} U_x &= (1-x)v(p^j_{on}) + xv(p^j_{off}) - G^j + \eta x t = \\ &\quad xv(p^i_{on}) + (1-x)v(p^j_{off}) - G^i + (1-x)t \end{aligned} \tag{5}$$

which we call the '*Hotelling indifference condition*'. The introduction of network-based price discrimination implies that despite interconnection there are 'tariff-mediated network externalities' present given that prices for on- and off-net calls differ. Consumers of network i are better off if more consumers joint the network if on-net prices are below off-net prices and vice versa. The location of the indifferent consumer who expects a given market share gives the network's equilibrium market shares if all expectations are correctly fulfilled at the equilibrium price vector.

We specify marginal costs as $c \equiv 2c_0 + c_1 < 1$ for a call within one network resulting from origination and termination (c_0) and the intermediate line service cost c_1 which we assume to occur at the originating end of the call. Network $i's$ marginal cost for a call from its network to the other network are $c + a^j - c_0$ as it has to pay the access charge a^j to network j, whereas the actual cost of the call is c due to the networks' identical technologies. Firms also face a fixed cost $F > 0$ and a per-capita cost $H > 0$.

The two networks are assumed to be playing a *non-cooperative two-stage game* in which they first choose their optimal access price parameter a^k simultaneously and in the second stage their price vector

$$\Xi^k \equiv \left\{p^k_{on}(a^k, a^{-k}), p^k_{off}(a^k, a^{-k}), G^k(a^k, a^{-k})\right\} \;\; k = i, j \tag{6}$$

simultaneously in order to maximize profits $\Pi^k(\Xi^k,\Xi^{-k})$ (where with two players $-k = j$ if $k = i$ or vice versa), taking as given the parameter vector of the other network. A vector Ξ^k is a *best response* for player k to his rivals' vector Ξ^{-k} if

$$\Pi^k(\Xi^k; \Xi^{-k}) \geq \Pi^k(\Xi'^k; \Xi^{-k}) \tag{7}$$

for all $\Xi'^k \neq \Xi^k$ in the multidimensional support of the price space.

The solution concept for the full game is *pure strategy subgame perfect Nash-equilibrium (SPNE)* and the game is solved by backward induction. The price vector Ξ^* and the access charge a^* constitute a subgame perfect Nash equilibrium *strategy* of the game iff

$$\Pi^k(\Xi^{k*}, a^{k*}; \Xi^{-k*}, a^{-k*}) \geq \Pi^k(\Xi^k, a^k; \Xi^{-k*}, a^{-k*}) \ \forall \ \Xi^k, a^k \text{ and } \forall k. \tag{8}$$

2.1 Solving the second stage

Network j will choose to solve the program

$$\max_{\Xi^j} \left\{ \Pi^j(\Xi^j; \Xi^i) \right\} = \left\{ \begin{array}{c} (1-x) \times \left[\begin{array}{c} G^j - H + \left(p^j_{on} - c\right)(1-x)q^j_{on} + \\ \left(p^j_{off} - (c + a^i - c_0)\right) x q^j_{off} \end{array} \right] + \\ x(1-x)(a^j - c_0)q^i_{off} - F \end{array} \right\} \tag{9}$$

subject to the 'Hotelling indifference condition' (5)

$$x = \frac{v(p^j_{on}) - v(p^i_{off}) - G^j + G^i - t}{v(p^i_{on}) + v(p^j_{on}) - v(p^j_{off}) - v(p^i_{off}) - t(1+\eta)}$$

and given the vector Ξ^i of network i. Here $(1-x)q^j_{on}$ gives the individual demand for on-net calls for a customer of the second network under a *balanced-traffic assumption* and $(1-x)$ is network $j's$ scale or market share. The pure termination profit for j is denoted as $x(1-x)\pi^j(a^j) \equiv x(1-x)(a^j - c_0)q^i_{off}$.

Note that under a balanced-traffic assumption the net number of calls from the incumbent's network to the second network (or vice versa) is zero if $p^i_{off} = p^j_{off}$ even if networks differ in their respective scales. A larger advantage for network j, i.e. a large choice of η will then push the marginal consumer closer to the origin of the unit interval leading to a larger market share for the advantaged network. The choice of network i with scale x is symmetric.

We first determine the network's optimal on-net and off-net prices.

Lemma 1 *Any best response of network i to network j satisfies*

$$\Pi^i(p^{i*}_{on} = c, p^i_{off}, G^i; \Xi^j) \geq \Pi^i(p'^i_{on}, p^i_{off}, G^i; \Xi^j)$$

for all $p'^i_{on} \neq p^{i*}_{on}$ in the support of the price vector space. Similarly, for given access charges $\bar{a}^i, \bar{a}^j$ any best response of network i to network j satisfies

$$\Pi^i(p^i_{on}, p^{i*}_{off} = c_0 + c_1 + \bar{a}^j, G^i; \Xi^j) \geq \Pi^i(p^i_{on}, p'^i_{off}(\bar{a}^j), G^i; \Xi^j)$$

for all $p'^i_{off} \neq p^{i*}_{off}$ in the support of the price vector space. The symmetric result holds for network j.

Proof:
Standard.■

We thus find that it is a *dominant strategy* for any network to set its own on-net price and off-net price markup at the cost levels. In other words, setting the on-net price equal to cost, i.e. $p_{on}^{k*} = p_{on}^{-k*} \equiv p_{on}^{*} = c$ and the off-net price equal of network k to perceived marginal cost $p_{off}^{k*} = c_0 + c_1 + \bar{a}^{-k}$ will be optimal for each network k *independently* of the price vector of the other network $-k$.

We now proceed to calculate the equilibrium fixed charge for each network.

Proposition 2 *Any best response of network i to network j concerning its fixed charge must satisfy*

$$\begin{aligned} G^j &= H + (1-4x)v(p_{on}^*) + 2xv(p_{off}^{j*}) + \\ &\quad (2x-1)v(p_{off}^{i*}) + (2x-1)\pi_T^i(a^i) + (2x(\eta+1)-1)\,t \end{aligned}$$

and any best response of network j to network i concerning its fixed charge must satisfy

$$\begin{aligned} G^i &= H + (4x-3)v(p_{on}^*) + 2(1-x)v(p_{off}^{i*}) + \\ &\quad (1-2x)v(p_{off}^{j*}) + (1-2x)\pi_T^j(a) + (2+\eta-2x(\eta+1))\,t \end{aligned}$$

where from the 'Hotelling indifference condition' (5)

$$x = \frac{v(p_{on}^*) - v(p_{off}^{i*}) - G^j + G^i - t}{2v(p_{on}^*) - v(p_{off}^{j*}) - v(p_{off}^{i*}) - t(1+\eta)}$$

Proof:
Network $j's$ total profit is given as

$$\Pi^j = \left\{ \begin{array}{c} (1-x)\times \\ \left[\begin{array}{c} \left[\begin{array}{c} (1-x)\left(v(p_{on}^j) - v(p_{off}^i)\right) + \\ x\left(v(p_{off}^j) - v(p_{on}^i)\right) \end{array} \right] + \\ t(x(1+\eta)-1) + G^i - H \end{array} \right] + \\ x(1-x)\pi_T^j(a) - F \end{array} \right\} \tag{10}$$

where $x(1-x)\pi_T^j(a)$ denotes the termination profit of network j. We now take the derivative with respect to the optimal scale of network j

$$\begin{aligned} \frac{\partial \Pi^j}{\partial x} = - &\left[\begin{array}{c} \left[\begin{array}{c} (1-x)\left(v(p_{on}^j) - v(p_{off}^i)\right) + \\ x\left(v(p_{off}^j) - v(p_{on}^j)\right) \end{array} \right] + \\ t(x(1+\eta)-1) + G^i - H \end{array} \right] + \\ (1-x) &\left[\left[\begin{array}{c} \left(v(p_{off}^j) - v(p_{on}^i)\right) - \\ \left(v(p_{on}^j) - v(p_{off}^i)\right) \end{array} \right] + (1+\eta)t \right] + (1-2x)\pi_T^j(a) \overset{!}{=} 0 \end{aligned} \tag{11}$$

which by using optimal pricing parameters and realizing that $v(p_{on}^{i*}) = v(p_{on}^{j*}) = v(p_{on}^{*})$, $j \neq i$ yields the best response for G^j which is implicit in x. The Proposition follows from symmetry.■

The system of equations in the above proposition is linear and has a unique solution. Taking the derivative with respect to the optimal scale given the Hotelling indifference and solving for scale is isomorphic to the solution of the first order necessary condition for the optimal choice of the fixed charge (holding the other networks fixed charge constant) using their connection via the Hotelling indifference condition but more convenient when we look at second order conditions below. Clearly the assumption of full market coverage allows us to interchange the scales of the two networks as choice variables of the program.

Also

Lemma 3 *For given access charges $\bar{a}^i, \bar{a}^j$ and sufficiently large t, the equilibrium scale x^* is strictly decreasing in η and has a strictly positive lower bound.*

Proof:
See Appendix.■

This lemma has an important implication for the form that competition between two networks takes. For any magnitude of the relative initial advantage of one of the two networks (possibly the incumbent), there is always a strictly positive market share $x \geq 1/3$ for the second network which is unlike in the standard Hotelling model where a large advantage of one firm drives the other out of the market.

Note that this finding does not depend on the simple linear demand specification we have used but also holds in the setting of LRT with constant elasticity demand. A constant elasticity setting does not allow us to check for second order sufficient conditions when we consider potentially asymmetric equilibria however. This can be done using our linear demand specification and is undertaken below.

2.2 Solving the first stage

We now look at a network's choice of the unregulated *non-reciprocal access charge markup* Δ in this model of network competition knowing from above that the off-net retail prices are functions of this markup.

Lemma 4 *Given the advantage of network j $(\eta > 1)$ is large, non-cooperative access charges can be approximated by*

$$\Delta^{j*} \equiv a^{j*} - c_0 \approx \frac{1}{2}(1-c) > 0$$

and

$$\Delta^{i*} \equiv a^{i*} - c_0 \approx \frac{2}{7}(1-c) > 0.$$

Proof:
See Appendix.■

We are able to look at the components of each firm's price vector for extreme asymmetry. This gives an indication about a strategy that strongly disadvantaged late entrants such as E-Plus and O_2 followed when entering the German Telecommunications markets. We find the following result:

Lemma 5 *Given the advantage of network j $(\eta > 1)$ is large we find that the components of the price vectors satisfy*

$$\Delta^j > \Delta^i$$

and

$$\pi_T^j(a^i) > \pi_T^i(a^j)$$

and

$$G^j > G^i$$

and

$$\Pi^j > \Pi^i.$$

Proof:
See Appendix.■

The analysis thus shows that for large asymmetries a disadvantaged network which targets a smaller market scale will chose a relatively *lower* access charge than an advantaged network unlike the charges observed in the table in the introduction. On the other hand a disadvantaged firm will also set a *lower* fixed charge. Hence the initial strategy of late entrants gaining market share in the German Telecommunications industry with a relatively low fixed charge as observed in practice finds theoretical support.

3 Equilibrium

We first present another comparative statics result:

Lemma 6 *The equilibrium scale x^* is strictly increasing in a^i and decreasing in a^j for sufficiently large t.*

Proof:
See Appendix.■

The intuition for this result is simple: A higher access charge raises the other network's perceived cost that will be passed on to consumers via the off-net call price thus moving the marginal consumer closer to the other network and increasing own market share.

Performing a local analysis around the symmetric equilibrium using a Taylor approximation we find that

Proposition 7 *At the symmetric equilibrium $\eta = 1$ both firms will charge a strictly positive non-cooperative access charge markup. In a neighbourhood of the symmetric equilibrium both networks will optimally increase their access charge markups for $\eta > 1$ and the advantaged network has the higher increase.*

Proof:
See Appendix.■

We therefore find that, as in the investigations of the symmetric setting, the non-cooperative access charge markup with asymmetric networks is positive, both in the *limit analysis* (where contrary to the previous findings of the "double marginalization" effect a limiting access charge can be approximated) and in the *local analysis* around the symmetric equilibrium case $\eta = 1$. Hence our theoretical results are in line with observed behaviour as exhibited in the introduction.

More interestingly in both analyses, given a relative advantage $\eta \neq 1$ for any network, an advantaged network has the larger market share and will optimally choose a relatively *larger* access charge than a disadvantaged network. This result is to be contrasted with that of Carter & Wright, 2003, Proposition 1 for *reciprocal* access charges where both firms, the smaller and the larger firm prefer a zero markup.

3.1 Sufficiency Conditions

We now show that the vector of potentially asymmetric pricing parameters Ξ^* is indeed maximizing the profit of each network for any access charges given that the pure benefit of being connected t is sufficiently large and/or the degree of asymmetry η is sufficiently large.

The sufficient condition of the maximization problem at the second stage is

Lemma 8 *The own second partial derivatives with regard to scale is negative for any access charges (a^i, a^j) if*

$$t(1+\eta) > 2v(p_{on}^*)$$

and the condition is sufficient for post-entry profits for networks i,j, to be strictly positive.

Proof:
See Appendix.■

The sufficient condition for the maximization problem at the first stage and hence for the whole game is

Lemma 9 *The own second partial derivatives with regard to the access charges (a^i, a^j) given the price vector choices Ξ^i, Ξ^j at stage one are negative if the value of the pure benefit of being connected t and/or the degree of asymmetry η is sufficiently large.*

Proof:
See Appendix.■

4 Conclusion

In the preceding analysis we find that, as in the previous symmetric "double marginalization" results of LRT, 1998a and Gans & King, 2001 competitively chosen access charges imply a positive markup on cost also in an *asymmetric* setting where one network competes at a perceived disadvantage and we are able to approximate this markup explicitly. We are thus able to bring the theoretical analysis in line with observed behaviour as exemplified in the introduction. Extending the analysis to asymmetric settings, which are highly prevalent in practice, we are able to shed some light on questions of optimal pricing behaviour that were previously left open.

Unlike in the simple Hotelling model of horizontal product differentiation the advantaged network will not cover the full market, no matter how large the relative advantage becomes. This implies that due to the high dimensionality of the optimal pricing strategy involved, the Telecommunications market may be more 'contestable' than previous studies have suggested. Additionally we find that for a sufficiently pronounced asymmetry, optimal fixed charges and profits for *both* firms are increasing in the asymmetry parameter, i.e. that due to the strategic interaction of the firms the advantage and the implied asymmetry becomes an advantage for the "disadvantaged" firm too.

We eventually find that around the symmetric equilibrium the disadvantaged network will optimally set access charges *below* that of an advantaged network covering a larger market in the asymmetric equilibrium and also that a strongly disadvantaged network charges a strictly lower fixed charge. The former result conflicts with, the latter is in accordance with data from the introduction and hence it seems that there remains room for disadvantaged new entrants in the Telecommunications industry to optimize on their pricing strategies.

5 References

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6 Appendix

Proof of Lemma 3:

As any equilibrium must satisfy the three equations in the previous Proposition simultaneously we can derive the equilibrium scale of network i (and thus of network j from full participation) in implicit form. From above we have

$$x^*(\bar{a}^i,\bar{a}^j)=\frac{\begin{array}{c}-3v(p^*_{on})+2v(p^{i*}_{off}(\bar{a}^j))+v(p^{j*}_{off}(\bar{a}^i))+\\ \pi^i_T(\bar{a}^i)+\pi^j_T(\bar{a}^j)+t\eta+2t\end{array}}{\begin{array}{c}-6v(p^*_{on})+3v(p^{j*}_{off}(\bar{a}^i))+3v(p^{i*}_{off}(\bar{a}^j))+\\ 2\pi^j_T(\bar{a}^j)+2\pi^i_T(\bar{a}^i))+3t\eta+3t\end{array}} \tag{12}$$

taking the derivative we find that $x^*(\bar{a}^i,\bar{a}^j)$is strictly decreasing in η if

$$t>v(p^*_{on})-v(p^{i*}_{off}(\bar{a}^j))-\frac{1}{3}(\pi^i_T(\bar{a}^i)+\pi^j_T(\bar{a}^j)) \tag{13}$$

and as $\lim_{\eta\to\infty}x^*=1/3$ it has a strictly positive lower bound in η.$\blacksquare$

Proof of Lemma 4:

Using the best responses G^i and G^j from Proposition 2 we find an optimal scale x^* that solves the system of equations simultaneously. Firm $i's$ post-entry profit level at this first stage can be written as

$$\Pi^i+F=\left\{(x^*(a^i,a^j))^2\times\left[\begin{array}{c}t(1+\eta)-2v(p^*_{on})+\\ v(p^{j*}_{off}(a^i))+v(p^{i*}_{off}(a^j))+\pi^i_T(a^i)\end{array}\right]\right\} \tag{14}$$

Using the product rule we find that optimal non-reciprocal access charges of firm i necessarily satisfies

$$\begin{array}{l}2x^*(a^i,a^j)\dfrac{\partial x^*(a^i,a^j)}{\partial a^i}\left[\begin{array}{c}t(1+\eta)-2v(p^*_{on})+\\ v(p^{j*}_{off}(a^i))+v(p^{i*}_{off}(a^j))+\pi^i_T(a^i)\end{array}\right]=\\ -(x^*(a^i,a^j))^2\dfrac{\partial(v(p^{j*}_{off}(a^i))+\pi^i_T(a^i))}{\partial a^i}\end{array} \tag{15}$$

Note that

$$\frac{\partial(v(p^{j*}_{off}(a^i))+\pi^i_T(a^i))}{\partial a^i}=c_0-a^i \tag{16}$$

so that as η goes out of bounds we have that

$$2(\frac{1}{3})\frac{\partial x^*(a^i,a^j)}{\partial a^i}\left[\begin{array}{c}t(1+\eta)-2v(p^*_{on})+\\ v(p^{j*}_{off}(a^i))+v(p^{i*}_{off}(a^j))+\pi^i_T(a^i)\end{array}\right]\approx(\frac{1}{3})^2(a^i-c_0) \tag{17}$$

has to hold. As now

$$\frac{\partial x^*(a^i,a^j)}{\partial a^i}\left[\begin{array}{c} t(1+\eta)-2v(p^*_{on})+ \\ v(p^{j*}_{off}(a^i))+v(p^{i*}_{off}(a^j))+\pi^i_T(a^i) \end{array}\right]\approx\frac{1}{9}(1-c)-\frac{7}{18}(a^i-c_0) \tag{18}$$

we find

$$a^i\approx c_0+\frac{2}{7}(1-c) \tag{19}$$

approximates the equilibrium access charge in our setting.

Similarly for the strongly advantaged firm j post-entry profits are

$$\Pi^j+F=\left\{(1-x^*(a^i,a^j))^2\times\left[\begin{array}{c} t(1+\eta)-2v(p^*_{on})+ \\ v(p^{i*}_{off}(a^j))+v(p^{j*}_{off}(a^i))+\pi^j_T(a^{j*}) \end{array}\right]\right\} \tag{20}$$

so that the optimal access charge $a^{\,j}$ necessarily satisfies

$$\begin{aligned} &2(1-x^*(a^i,a^j))\frac{\partial(-x^*(a^i,a^j))}{\partial a^j}\left[\begin{array}{c} t(1+\eta)-2v(p^*_{on})+ \\ v(p^{j*}_{off}(a^i))+v(p^{i*}_{off}(a^j))+\pi^j_T(a^j) \end{array}\right]= \\ &(1-x^*(a^i,a^j))^2\frac{\partial(v(p^{i*}_{off}(a^j))+\pi^j_T(a^j))}{\partial a^j} \end{aligned} \tag{21}$$

and hence as η goes out of bounds we have that

$$2(1-\frac{1}{3})\frac{\partial(-x^*(a^i,a^j))}{\partial a^j}\left[\begin{array}{c} t(1+\eta)-2v(p^*_{on})+ \\ v(p^{j*}_{off}(a^i))+v(p^{i*}_{off}(a^j))+\pi^j_T(a^j) \end{array}\right]\approx(\frac{2}{3})^2(a^j-c_0) \tag{22}$$

has to hold. As now

$$\frac{\partial(-x^*(a^i,a^j))}{\partial a^j}\left[\begin{array}{c} t(1+\eta)-2v(p^*_{on})+ \\ v(p^{j*}_{off}(a^i))+v(p^{i*}_{off}(a^j))+\pi^j_T(a^j) \end{array}\right]\approx\frac{2}{9}(1-c)-\frac{1}{9}(a^j-c_0) \tag{23}$$

we find the approximation

$$a^j\approx c_0+\frac{1}{2}(1-c) \tag{24}$$

as was to be shown.■

Proof of Lemma 5:

Let the network with the exogenous advantage $\eta > 1$ be network j. Then given the advantage is sufficiently large we find from the previous result that the marginal consumers is located at 1/3 on the unit interval. The optimal access charge markups approximately satisfy

$$\Delta^{j*}(x) \approx \frac{1}{2}(1-c) > \Delta^{i*}(x) \approx \frac{2}{7}(1-c) \tag{25}$$

and by replacing the arguments we find

$$v(p^*_{on}) \approx \frac{1}{2}(1-c)^2 \tag{26}$$

and

$$v(p^{i*}_{off}(a^{j*})) \approx \frac{1}{8}\left(1-c\right)^2 \tag{27}$$

and

$$v(p^{j*}_{off}(a^{i*})) \approx \frac{25}{98}\left(1-c\right)^2 \tag{28}$$

and

$$\pi^j_T(a^{i*}) \approx \frac{10}{49}\left(-1+c\right)^2 \tag{29}$$

and

$$\pi^i_T(a^{j*}) \approx \frac{1}{4}\left(1-c\right)^2 \tag{30}$$

If η is sufficiently large, optimal fixed charges satisfy

$$G^{j*} > G^{i*} \tag{31}$$

as the last terms in $G^{j*}$and G^{i*} dominate and

$$\left(2(\frac{1}{3})(\eta+1)-1\right)t > \left(2+\eta-2(\frac{1}{3})(\eta+1)\right)t \tag{32}$$

or

$$\frac{2}{3}\eta - \frac{1}{3} > \frac{1}{3}\eta + \frac{4}{3} \tag{33}$$

always holds. Trivially total profits are larger for the advantaged firm.■

Proof of Lemma 6:
From (12) we have the equilibrium scale as

$$x^*(a^i, a^j, \eta) = \frac{\begin{array}{c} -3v(p^*_{on}) + 2v(p^{i*}_{off}(a^j)) + v(p^{j*}_{off}(a^i)) + \\ \pi^i_T(a^i) + \pi^j_T(a^j) + t\eta + 3t \end{array}}{\begin{array}{c} -6v(p^*_{on}) + 3v(p^{j*}_{off}(a^i)) + 3v(p^{i*}_{off}(a^j)) + \\ 2\pi^j_T(a^j) + 2\pi^i_T(a^i)) + 3t\eta + 3t \end{array}} \tag{34}$$

Taking derivatives we have

$$\frac{\partial x^*(a^i, a^j, \eta)}{\partial a^i} = -2\frac{-a^i(a^j)^2 + (c-1)(a^i)^2 + \left(2(c-1)^2\right)a^j + 6t\left(a^i - 1 + c\right)}{\left(-(a^i)^2 - (a^j)^2 - 2(1-c)(a^i + a^j) + 6t(1+\eta)\right)^2} > 0 \tag{35}$$

and

$$\frac{\partial x^*(a^i, a^j, \eta)}{\partial a^j} = 2\frac{-(a^i)^2 a^j + (c-1)(a^j)^2 + \left(2(c-1)^2\right)a^i + 6t\left(a^j - 1 + c\right)}{\left(-(a^i)^2 - (a^j)^2 - 2(1-c)(a^i + a^j) + 6t(1+\eta)\right)^2} < 0 \tag{36}$$

for sufficiently large t as $a^i, a^j < 1 - c$ by assumption.■

Proof of Proposition 7:
Again firm $i's$ post-entry profit level at the first stage can be written as

$$\Pi^i + F = \left\{ (x^*(a^i, a^j, \eta))^2 \times \left[\begin{array}{c} t(1+\eta) - 2v(p^*_{on}) + \\ v(p^{j*}_{off}(a^i) + v(p^{i*}_{off}(a^j)) + \pi^i_T(a^i) \end{array} \right] \right\} \quad (37)$$

The optimal non-reciprocal access charge of firm i for *any degree of asymmetry* $\eta > 0$ necessarily satisfy

$$\frac{\partial \Pi^i(\eta)}{\partial a^i} = 2x^*(a^i, a^j, \eta) \frac{\partial x^*(a^i, a^j, \eta)}{\partial a^i} \left[\begin{array}{c} t(1+\eta) - 2v(p^*_{on}) + \\ v(p^{j*}_{off}(a^i)) + v(p^{i*}_{off}(a^j)) + \pi^i_T(a^i) \end{array} \right] +$$
$$(x^*(a^i, a^j, \eta))^2 \frac{\partial (v(p^{j*}_{off}(a^i)) + \pi^i_T(a^i))}{\partial a^i} \stackrel{!}{=} 0 \quad (38)$$

Setting $c_0 = 0$ from here to simplify notation somewhat we find that a solution necessarily has to satisfy

$$x^*\left(a^i, a^j, \eta\right) \left(-a^i x^*\left(a^i, a^j, \eta\right) + \frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^i} \left(a^j \left(a^j - 2(1-c)\right) - (a^i)^2 + 2t(1+\eta)\right) \right) \stackrel{!}{=} 0 \quad (39)$$

and for an interior solution

$$\Phi^i(\eta) \equiv -a^i x^*\left(a^i, a^j, \eta\right) + \frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^i} \left(a^j \left(a^j - 2(1-c)\right) - (a^i)^2 + 2t(1+\eta)\right) \stackrel{!}{=} 0 \quad (40)$$

Post-entry profits for the other firm are

$$\Pi^j + F = \left\{ (1 - x^*(a^i, a^j, \eta))^2 \times \left[\begin{array}{c} t(1+\eta) - 2v(p^*_{on}) + \\ v(p^{i*}_{off}(a^j)) + v(p^{j*}_{off}(a^i)) + \pi^j_T(a^{j*}) \end{array} \right] \right\} \quad (41)$$

Note that the profit terms are *fully symmetric* except for the own scale scalar. We find that a solution to the optimal non-reciprocal access charge of firm j necessarily satisfies the first order condition

$$(x^*\left(a^i, a^j, \eta\right) - 1) \left(a^j((1 - x^*\left(a^i, a^j, \eta\right)) + \frac{\partial (x^*\left(a^i, a^j, \eta\right))}{\partial a^j} (a^i(a^i - 2(1-c)) - (a^j)^2 + 2t(1+\eta)) \right) \stackrel{!}{=} 0 \quad (42)$$

or for an interior solution

$$\Phi^j(\eta) \equiv a^j((1 - x^*\left(a^i, a^j, \eta\right)) + \frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^j} (a^i(a^i - 2(1-c)) - (a^j)^2 + 2t(1+\eta)) \stackrel{!}{=} 0 \quad (43)$$

Using *first-order Taylor expansions* of the form

$$\Phi^k(\eta) \approx \Phi^k(\eta)\big|_{\eta=1} + \left.\frac{\partial \Phi^k(\eta)}{\partial \eta}\right|_{\eta=1} (\eta - 1) \forall\ k = i, j \text{ and } i \neq j \tag{44}$$

for η close to 1, i.e. around the *symmetric* equilibrium, we find the linearized simultaneous equation system for an interior solution can be approximated by

$$0 = -x^*\left(a^i, a^j, \eta\right) a^i + \left.\frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^i}\right|_{\eta=1} \left(4t - (a^i)^2 - a^j(2(1-c) - a^j)\right) + \tag{45}$$

$$\begin{pmatrix} \partial \frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^i} / \partial \eta \Big|_{\eta=1} \left(4t - (a^i)^2 - a^j(2(1-c) - a^j)\right) \\ - \left.\frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial \eta}\right|_{\eta=1} a^i + \left.\frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^i}\right|_{\eta=1} 2t \end{pmatrix} (\eta - 1)$$

and symmetrically

$$0 = (1 - x^*\left(a^i, a^j, \eta\right))a^j + \left.\frac{\partial x^*\left(a^i, a^j, \eta)\right)}{\partial a^j}\right|_{\eta=1} \left(4t - (a^j)^2 - a^i(2(1-c) - a^i)\right) + \tag{46}$$

$$\begin{pmatrix} \partial \frac{\partial x^*\left(a^i, a^j, \eta)\right)}{\partial a^j} / \partial \eta \Big|_{\eta=1} \left(4t - (a^j)^2 - a^i(2(1-c) - a^i)\right) \\ - \left.\frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial \eta}\right|_{\eta=1} a^j + \left.\frac{\partial x^*(a^i, a^j, \eta)}{\partial a^j}\right|_{\eta=1} 2t \end{pmatrix} (\eta - 1)$$

From the above Lemma for the symmetric equilibrium $\eta = 1$ we find that for large t we have

$$0 = -x^*\left(a^i, a^j, \eta\right) a^i + \underbrace{\left.\frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^i}\right|_{\eta=1}}_{+} \underbrace{\left(4t - (a^i)^2 - a^j(2(1-c) - a^j)\right)}_{+} \tag{47}$$

and the RHS is strictly decreasing in a^i for given a^j and

$$0 = (1 - x^*\left(a^i, a^j, \eta\right))a^j + \underbrace{\left.\frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^j}\right|_{\eta=1}}_{-} \underbrace{\left(4t - (a^j)^2 - a^i(2(1-c) - a^i)\right)}_{+} \tag{48}$$

has the RHS strictly decreasing in a^j for given a^i. Thus for large t, $c_0 \geq 0$ and $\eta = 1$ we find

$$a^i - c_0, a^j - c_0 > 0 \tag{49}$$

has to hold for the system to be satisfied so that markups are strictly positive.

For any $\eta > 0$ and large t we have the approximate system of (45) and (46) given by the best response functions

$$\begin{aligned} 0 \quad = \quad & -\frac{1}{2}a^i + \underbrace{\left.\frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^i}\right|_{\eta=1}}_{+} \underbrace{\left(4t - (a^i)^2 - a^j(2(1-c) - a^j)\right)}_{+} + \\ & \frac{1}{18}\left(-1 + c + \frac{1}{2}a^i\right)(\eta - 1) \end{aligned} \tag{50}$$

and symmetrically

$$\begin{aligned} 0 \quad = \quad & \frac{1}{2}a^j + \underbrace{\left.\frac{\partial x^*\left(a^i, a^j, \eta\right)}{\partial a^j}\right|_{\eta=1}}_{-} \underbrace{\left(4t - (a^j)^2 - a^i(2(1-c) - a^i)\right)}_{+} \\ & + \frac{1}{18}\left(-1 + c + \frac{1}{2}a^j\right)(\eta - 1) \end{aligned} \tag{51}$$

as by assumption, $a^i, a^j < 1 - c$ we find that the additional term for $\eta > 1$ is strictly negative. Hence for η slightly above one, a^j has to increase by more than a^i in order to keep the simultaneous equation system satisfied. If the advantage is reversed, i.e. $\eta < 1$ then the additional term is positive and a^i has to increase by more than a^j.■

Proof of Lemma 8:

The own second partial derivatives with respect to the optimal scale are

$$\frac{\partial^2 \Pi^i}{\partial x^2} = 4v(p^*_{on}) - 2t(1+\eta) - 2v(p^{i*}_{off}(a^j)) - 2v(p^{j*}_{off}(a^i)) - 2\pi^i_T(a^i) \tag{52}$$

and

$$\frac{\partial^2 \Pi^j}{\partial x^2} = 4v(p^*_{on}) - 2t(1+\eta) - 2v(p^{j*}_{off}(a^i)) - 2v(p^{i*}_{off}(a^j)) - 2\pi^j_T(a^j) \tag{53}$$

so that the condition is necessary and sufficient for the second derivative to be negative for *any* a^i and a^j. By observation, given the optimal post-entry profit functions (14) and (20) we find that the sufficiency result follows.■

Proof of Lemma 9:

Taking derivatives we find

$$\begin{aligned}
\lim_{\eta\to\infty}\left(\frac{\partial^2 \Pi^i(p^{i*}_{on}, p^{i*}_{off}, G^{i*}; \Xi^{j*}, a^j)}{\partial (a^i)^2}\right) &= \qquad (54)\\
\lim_{\eta\to\infty}\left(\begin{array}{c} -(x^*\left(a^i,a^j,\eta\right))^2 - 4\frac{\partial x^*\left(a^i,a^j,\eta\right)}{\partial a^i} a^i x^*\left(a^i,a^j,\eta\right) + \\ \left(x^*\left(a^i,a^j,\eta\right)\partial\frac{\partial x^*\left(a^i,a^j,\eta\right)}{\partial a^i}/\partial a^i + \left(\frac{\partial x^*\left(a^i,a^j,\eta\right)}{\partial a^i}\right)^2\right)\times \\ \left(2t + 2t\eta - 2a^j + 2ca^j + (a^j)^2 - (a^i)^2\right)\end{array}\right) &= \\
\lim_{\eta\to\infty}\left(\frac{\partial^2 \Pi^j(p^{j*}_{on}, p^{j*}_{off}, G^{j*}; \Xi^{i*}, a^i)}{\partial (a^j)^2}\right) &= \\
-\frac{16}{27} &< \quad 0\ \forall\ t
\end{aligned}$$

Also

$$\lim_{t\to\infty}\left(\frac{\partial^2 \Pi^j(p^{j*}_{on}, p^{j*}_{off}, G^{j*}, a^j; \Xi^{i*})}{\partial (a^j)^2}\right) = -\frac{1}{27}\frac{16\eta^2 + 22\eta + 7}{(1+\eta)^2} < 0\ \forall\ \eta \tag{55}$$

and

$$\lim_{t\to\infty}\left(\frac{\partial^2 \Pi^i(p^{i*}_{on}, p^{i*}_{off}, G^{i*}, a^j; \Xi^{j*})}{\partial (a^i)^2}\right) = -\frac{1}{27}\frac{7\eta^2 + 22\eta + 16}{(1+\eta)^2} < 0\ \forall\ \eta \tag{56}$$

and thus sufficiency follows.■