Optimal pricing and commitment in two-sided markets[*]

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Abstract

This paper proposes a model of Bertrand competition between platforms and analyzes the sustainability of dominant platform equilibria in two-sided markets with the following characteristics: i) platforms are essential bottlenecks for buyers (users) to access the products offered by sellers (developers); ii) sellers enter the market before buyers; iii) only sellers can multihome; iv) platforms can charge fixed fees on both sides and variable fees (royalties) to sellers. The central issue arising in such a context is the ability of platforms to credibly commit to the price they will charge buyers when they set their prices for sellers. The possibility of commitment changes the pricing game substantially by enlarging the set of pricing strategies available to platforms and we investigate its effect on the sustainability of dominant platform equilibria and resulting profits, both when sellers are bound to exclusivity and when they are allowed to multihome.

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"Olaf [Olafsson] was about two-thirds of the way through his speech when he said, "I would like to call up Steve Race to tell you a little bit more about the Sony Playstation." So I walked up. I had a whole bunch of sheets of paper in my hands, and I walked up, put them down on the podium, and I just said, "$299," and walked off stage to this thunderous applause."[1]

Steve Race, president of Sony Computer Entertainment of America, at the first E3 (Electronic Entertainment Expo), May 11-13, 1995, Los Angeles

1 Introduction

Most of the recent literature on two-sided markets has modelled the two "sides" or categories of agents as arriving at the same time and therefore playing a simultaneous coordination game. While no one would argue that all agents in such markets arrive at the same time in a literal sense, there is a widely held view, according to which the equilibria of the simultaneous-move game represent the equilibria which would arise in a more realistic, sequential-move game. This is probably true as long as agents of the two categories or "sides" arrive in a sufficiently alternated fashion or as long as demands by the two sides are sufficiently elastic so that the order of arrival is irrelevant. Some two-sided markets fit this descrition: credit cards (merchants and consumers); yellow page directories, TV, newspapers (advertisers and viewers/readers); real-estate agents and other intermediaries (buyers and sellers), etc.

However, there are several prominent categories of two-sided markets, for which this stylized representation does not seem particularly well-suited, because there is a natural and well-defined order of arrival of the two sides, in the sense that *most* members of one side of the market arrive before *most* members of the other side. For example, in the software and videogame markets, most application and game developers join platforms (operating systems and game consoles) before most users do. This is for technological reasons: application and game development are long and costly processes, therefore platform vendors in these markets have to start courting developers almost at the same time as they begin developing the platform, in order

[1] Quoted by Kent (2003), p.516.
Note: Olaf Olafsson, president of Sony Electronic Publishing.

to ensure that enough application support will be available for it at launch. An operating system or game console could not be launched simultaneously to both users and developers, because no users would buy it without applications or games and by the time any become available, users would be gone to another platform.

In this paper we depart from previous literature by focusing on the strategic issues arising when the two sides of a market arrive in a clear and well-defined order. This stylized fact raises a number of important questions. First, is it possible and/or desirable for platforms to credibly commit to the price they will charge the side arriving later when trying to attract the side arriving earlier? Second, what is the pricing structure in such markets? In particular, does sequential arrival imply that the pricing structure will be biased in favor of the side arriving earlier?

We propose a model for studying Bertrand competition among platforms and the sustainability of dominant platform equilibria in two-sided markets with the following characteristics: i) platforms are essential bottleneck inputs for buyers (users) to access the products offered by sellers (developers); ii) sellers arrive before buyers; iii) only sellers can multihome[2]; iv) platforms can charge fixed fees on both sides and variable fees (royalties) on the seller side. The central issue arising in this context is the ability of platforms to commit to the price they will charge buyers when they set their prices for sellers. The possibility of commitment changes the pricing game substantially by enlarging the set of pricing strategies available to platforms and we investigate its effect on profits in dominant platform equilibria, both when sellers are bound to exclusivity and when they are allowed to multihome.

Perhaps the main characteristic of two-sided markets is the presence of bilateral indirect network effects giving rise to a "chicken-and-egg" problem. However, in our case, since developers arrive before users, this implies that indirect network effects are asymmetric: once sellers have decided which platform to support, the coordination problem on the buyer side vanishes. Buyers will simply adopt the platform offering the largest surplus, taking into

[2]Indeed, mainly for cost reasons, not many people buy two computers or two videogame consoles. However independent application and game developers generally support more than one platform.

account the price it charges and the number of supporting sellers. Thus, the only coordination game is played by sellers. It would then seem natural that platforms should concentrate all their efforts on attracting developers, or, in other words, on capturing the "chicken side" of the market. One would consequently expect platform profit structures to be generally biased in favor of developers, i.e. that the share of net revenues from the developer side of the market in total platform profits will be *lower* than the share of net revenues from the user side. It turns out that this is not necessarily true. On the one hand, most operating system vendors have adopted a business model, which involves subsidizing the developer side of the market and making most profits on users; on the other hand however, virtually all videogame console manufacturers[3] have chosen the opposite business model: they sell consoles to users at or below cost and make the bulk of their revenues on the developer side of the market. More specifically, operating system vendors generally subsidize developers through tools, support, conferences, etc.[4] and they seldom charge variable fees. By contrast, console makers derive their revenues from royalties charged to game developers. As an example, for their most recent console releases, the royalties charged by Sony for PS2, Nintendo for GameCube and Microsoft for XBox range from $8 to $10 per game sold[5].

Furthermore and perhaps not coincidentally, commitment to user prices seems to play a significant strategic role in videogame console makers' announcements prior to console launch, most notably at the E3 conference[6]. By contrast, it does not seem to be such an important factor in the software market. Indeed, major operating system vendors such as Microsoft, Sun, IBM, etc., are not particularly well-known for announcing user prices of their upcoming versions prior to launch or at developer conferences, where they generally try to lure developers by focusing mainly on application programming interfaces and other software development issues.

Consistent with these observations, commitment and the determination

[3] At least since 1989, the year when Nintendo first launched the NES in the United States.

[4] See Evans (2003).

[5] See Kent (2001).

[6] See Kent (2001).

of variable fees are strongly interdependent in our model. Royalties link the pricing game for sellers and the subsequent pricing game for buyers. Specifically, positive royalties announced in the first stage act as negative "marginal costs" for platforms when they compete for buyers in the second stage. There are two conflicting effects of royalties, which drive most of the mechanisms in our model: on the one hand, *absent commitment* to buyer (user) prices, high royalties provide a better competitive position vis-a-vis buyers in the second stage by allowing platforms to price more aggressively, but on the other hand, from an *ex ante* (i.e. prior to stage one) perspective, a platform expecting to extract most of the surplus created by sellers' products has an incentive to set low royalties in order to maximize this surplus. Furthermore, the pricing structure may or may not favor developers depending on the rules of the pricing game being played, in particular the feasibility of platform commitment, the possibility for sellers to multihome and sellers' fixed cost structure.

In this paper we restrict attention to dominant platform equilibria, i.e. equilibria in which one platform benefits from favorable seller expectations and corners both sides of the market. There are two reasons for this choice. First, it can be shown that more balanced, market-sharing equilibria are either unstable or non-existent in our model because platforms are perfect substitutes for each other; moreover, the study of market sharing equilibria does not offer any additional insights relative to the study of dominant platform equilibria. Second and perhaps most important, in the markets we are interested in, dominance by a single platform seems to be a pervasive feature: Campbell Kelly (2003) for example states that 80% market share for a single platform is very common in both the software and the videogame markets.

The overarching idea arising from our model is that platforms' (partial) inability to extract surplus from both sides of the market leads to welfare-reducing pricing distorsions. In the case of a monopoly platform, we show that if it faces favorable developer expectations then it is able to extract the entire social surplus and therefore its pricing will be socially optimal. If on the other hand it is confronted with unfavorable developer expectations, its inability to extract the full social surplus leads to inefficient pricing and lower social welfare.

In the case of two competing platforms, we show that when credible commitment to user prices is not feasible, a dominant platform equilibrium is always sustainable, both under a developer exclusivity regime as well as when multihoming is permitted. In this case, the challenger's only feasible pricing strategy is to do whatever it takes in order to attract developers (by subsidizing their fixed costs) and then hope to recoup on users. The dominant platform may find it more profitable to be in a multihoming regime; however, the resulting level of social welfare is always higher under exclusivity. Moreover, socially optimal pricing never arises in this case.

When commitment is feasible, the game becomes significantly more complex because the challenger can use divide-and-conquer pricing strategies on *both* sides of the market. In particular, rather than using direct subsidies to fight off developers' unfavorable expectations, it can commit to very low prices for users, which signals to developers that it will win the user battle. This enables the challenger to charge developers higher prices than it could without credible commitment. In this case, a dominant platform equilibrium is no longer necessarily sustainable under an exclusivity regime, however it is always sustainable when developers are allowed to multihome. From a social welfare perspective, commitment by the dominant platform is always better than no-commitment. However, platform competition introduces a misalignment between the dominant platform's objectives and social welfare maximization, which is why the dominant platform may sometimes find it more profitable not to commit.

The paper is organized as follows: the next section presents a brief discussion of the relevant literature, section three sets up the modelling framework and the fourth section is devoted to the analysis of the monopoly platform case. The fifth section provides a full characterization of the second stage pricing game for users. Section six analyzes the full pricing game between two competing platforms with a dominant platform, with and respectively without platform commitment to user prices. Section seven concludes.

2 Relevant literature

Our paper belongs to the very recent literature on two-sided markets, pioneered by Armstrong (2002) (hereafter A), Rochet and Tirole (2003) (hereafter RT) and Caillaud and Jullien (2002) (hereafter CJ). RT and A emphasize the role of relative price elasticities of demand on the two sides of the market in determining platform pricing structures. However, by assuming elastic demands on both sides, they abstract from issues of coordination, multiple equilibria and market dominance by a single platform. The paper closest in spirit to ours is CJ: they study competition among intermediaries with homogeneous populations on both sides of the market and the sustainability of dominant platform and market sharing equilibria, which arise endogenously as a consequence of indirect network effects. By contrast with RT and A, who assume platforms can only use fixed fees, CJ allow for more sophisticated pricing instruments, most notably variable fees conditional on succesful matching. However, in all these papers the volume of transactions between the two sides of the market is not directly affected by platforms' prices: RT and A essentially assume that each member of one side interacts with an exogenously given proportion of members on the other side, whereas in CJ, each member of one side transacts with only one member of the other side (in case matching is succesful). In our model the variable fees charged by the platform (royalties) play a central role, because they affect the prices and volumes of trade between developers and users and therefore social welfare.

Our solution and equilibrium concept are directly adapted from CJ; we also borrow their terminology for pricing strategies, in particular *divide-and-conquer*. However, in their model the two sides are essentially symmetric from all points of view and are assumed to arrive and coordinate simultaneously, therefore timing or commitment issues do not arise. By contrast, we assume sequential arrival (developers arrive before users), implying that the coordination game is played by developers, and that only developers can multihome and be charged variable fees, for the reasons exposed above. Finally, another innovation of our framework is to study the influence of "economies of scale" resulting from multihoming: the fixed cost of porting an existing application to another platform can be lower than the initial fixed cost of developing from scratch. The developer fixed cost structure (aside from payments to platforms) plays an important role when multihoming is possible, which up to now has not been studied by the two-sided market literature.

Lastly, our paper is related in an interesting way to Stahl (1988). His paper studies two-stage price competition between merchants, who must compete both for input suppliers and for consumers. He analyzes two pricing games: in the first game, there is winner-take-all competition for supplies in the first stage and competition for users in the second stage, given the capacities acquired in the first stage. In the second game, the timing is reversed. Stahl shows that all these games have a unique subgame perfect Nash equilibrium, which most of the time yields Walrasian prices. This is in stark contrast with the games we study here, which exhibit multiple equilibria. The difference is due to the fact that in Stahl's model there are no indirect network effects between the two sides of the market, i.e input suppliers (akin to developers in our model) and consumers (users). Indeed, since merchants compete in bids for suppliers, the latter only care about the price they are being offered and not about the decisions of other suppliers. In our model, by contrast, developers' profits depend crucially on the final user market share obtained by the platforms(s) they support, which gives rise to indirect network effects. Thus, Stahl's framework is very similar to a modified version of our model, in which platforms would compete in buyout bids for developers rather than in access charges. Moreover, in Stahl's model, the only link between the two sequential pricing stages is the capacity acquired by merchants in the first stage, whereas in our model, the link is provided by the royalty rates charged by platforms. Lastly, the reversal of the pricing stages in Stahl (1988) (merchants sell forward contracts to consumers first) corresponds in a way to our introduction of the possibility of commitment to user prices, except that our commitment game is more general, since platforms can choose whether or not to commit, whereas in Stahl, the timing and rules of the game are exogenously imposed.

3 Model

There are three types of agents in our model: consumers, developers and platforms. Platforms are indispensable to consumers in order to use the applications produced by developers; however, purchasing a platform only provides access to the applications supported by that platform (there is incompatibility between platforms). Moreover, developers cannot make applications compatible with a platform without the latter's consent.

We assume there is a continuum of users, normalized at $[0, 1]$, identical in their tastes for platforms and with independent and identically distributed preferences for applications. The platform offers no standalone utility to users: this assumption simplifies the exposition but nothing essential would change if we allowed users' valuations for each platform to be greater than 0[7]. Users' valuations for any given application are randomly drawn from an interval $[0, \overline{v}]$, according to a cumulative distribution G, where $G' > 0$, $G(0) = 0$, $G(\overline{v}) \leq 1$[8].

Developers are also modeled as a continuum $[0, N]$, where N is to be interpreted as the number of developers per user. They are assumed to act independently of each other. Developing an application has a fixed initial development cost of $\dot{f}$ and, consistent with our focus on software-like markets, 0 marginal cost. However, we assume that an application developed for a platform can be *ported* to another platform at an additional fixed cost of γf, with $0 < \gamma \leq 1$. The case $\gamma < 1$ is equivalent to a sort of increasing returns to scale when developers support more than one platform. A smaller γ means that porting is less costly relative to development "from scratch". We assume away any differences in developing and porting costs *across* platforms, that is development and porting costs are the same, regardless of the platform for which the application has been initially developed.

[7] This would capture the existence of in-house (i.e. produced by the platform) applications or games for example.

[8] $G(\overline{v}) < 1$ corresponds to the case, in which a fraction $(1 - G(\overline{v}))$ of users is not interested in the application, so that they do not purchase it even at a price of 0.

We define the *per user* demand for any application priced at p:

$$d(p) = \begin{cases} G(\overline{v}) & \text{if } p \leq 0 \\ G(\overline{v}) - G(p) & \text{if } 0 \leq p \leq \overline{v} \\ 0 & \text{if } \overline{v} \leq p \end{cases}$$

We assume that users do not know their valuation for the applications developed for a platform prior to making their platform adoption decisions, which implies that user choices are made based on expected surplus. The consequnce of this assumption is that platforms are able to serve user demand optimally. Any user's *expected* surplus from an application priced at p and available on the platform he has purchased is:

$$S(p) = \int_p^{\overline{v}} (G(\overline{v}) - G(v))\,dv = \int_p^{\overline{v}} d(\rho)\,d\rho$$

Denote by D^U the overall user demand for a platform. Since users' preferences for any given application are uncorrelated, total user demand for an application developed for the platform is:

$$D(p) = d(p)\,D^U$$

Platforms have three pricing instruments at their disposition: fixed fees for users P^U, fixed fees P^D and variable fees or royalties r for developers. Thus, although marginal cost is 0, developers face a variable fee r charged by the platform. In all the pricing games we consider platforms *publicly* announce their prices, i.e. we assume away the possibility of secret offers and discrimination on either side. Moreover we restrict attention to "simple" price vectors $\left(P^U, r, P^D\right)$, thus ruling out more sophisticated pricing such as exclusivity discounts or multihoming penalties or any sort of other contingent clauses.

Net profits of a developer for a single platform are then:

$$(p - r)\,d(p)\,D^U - P^D - f$$

Therefore each developer for that platform will set a price $p(r)$ given by:

$$p(r) = \arg\max_p\,(p - r)\,d(p) \tag{1}$$

Define:

$$\pi^D(r) = (p(r) - r)\, d(p(r))$$

Then profits from developing for a single platform are:

$$\pi^D(r)\, D^U - P^D - f$$

In our model users always adopt only one platform. However developers may sometimes *multihome*, i.e. port their application to more than one platform. Suppose a developer chooses to support two competing platforms 1 and 2, where platform $i = 1, 2$ charges prices $\left(r_i, P_i^D\right)$ and attracts D_i^U users, with $D_1^U + D_2^U = 1$. Then his total profits from multihoming are:

$$\pi^D(r_1)\, D_1^U + \pi^D(r_2)\, D_2^U - P_1^D - P_2^D - (1+\gamma) f$$

Turning now to plaforms, the expression of total profits for a platform charging prices $\left(P^U, r, P^D\right)$ and serving N developers and D^U users is:

$$\Pi^P = P^U D^U + Nrd(p(r))\, D^U + NP^D$$

We will oftentimes discuss both price structure and profit or revenue structure. By pricing structure we mean the price configuration $\left(P^U, r, P^D\right)$. Define:

$$\begin{aligned}
\Pi^{PD} &= NP^D + Nrd(p(r))\, D^U \\
\Pi^{PU} &= P^U D^U
\end{aligned}$$

Π^{PD} is the part of net platform profits obtained on the developer side of the market, while Π^{PU} is the part obtained on the user side. The profit structure is then the configuration $\left(\Pi^{PD}, \Pi^{PU}\right)$.

3.1 Definitions and technical assumptions

We make the usual assumptions on $d(p)$, which ensure the concavity of the maximization problem (1) and imply that $\pi^D(r)$ is decreasing and $p(r)$ increasing in r.

Furthermore, define:

$$r^* = \sup\{r, p(r) = 0\}$$

r^* is the maximum royalty rate for which it is optimal for developers to price their applications at 0. Clearly $r^* < 0$ and $p(r) = 0$ for all $r \leq r^*$.

We denote by $W(p(r))$ the total social surplus *per user* created by an application priced at $p(r)$, the sum of user surplus $S(p(r))$ and developer revenues per user $p(r)\,d(p(r))$:

$$W(p) = S(p) + pd(p)$$

Since $p(r)$ is decreasing in r, both $S(p(r))$ and $W(p(r))$ are decreasing in r and are maximized for $r = r^*$, i.e. when the application is sold at marginal cost, which is equal to 0.

Also, in order to simplify the exposition, from now on we write everything as a function of r alone, i.e.:

$$\begin{aligned} d(r) &\equiv d(p(r)) \\ \pi^D(r) &\equiv (p(r) - r)\,d(r) \\ S(r) &\equiv S(p(r)) \\ W(r) &\equiv W(p(r)) \end{aligned}$$

Let us now define a function $\Phi(r)$, which plays a central role in our model:

$$\Phi(r) = \begin{cases} S(r^*) + rd(r^*) & \text{if } r \leq r^* \\ S(r) + rd(r) & \text{if } r^* \leq r \leq \overline{v} \\ 0 & \text{if } \overline{v} \leq r \end{cases}$$

$\Phi(r)$ represents the sum of consumer surplus and the platform's royalty revenue per application and per user, when the royalty rate is r. We will see below that $n\Phi(r)$ can be interpreted as the strength of a platform's competitive position vis-a-vis users, when it has attracted n developers and has not yet committed to user prices.

We make the following assumptions:

Assumption 1 *Φ admits a unique maximum on the interval $]r^*, \overline{v}[$*[9].

[9] It is straightforward to show that:

$$\Phi'(r) = (1 - G(p(r)))\left[1 - p'(r)\left(1 + r\frac{g(p(r))}{1 - G(p(r))}\right)\right]$$

This assumption of a unique maximum is quite appealing intuitively. When the royalty rate is too high, the application price is high and thus the demand for the aplication is low, so that both royalty revenues and consumer surplus are low. In turn, when the royalty rate is strongly negative, consumer demand and surplus are high but the negative royalty revenue outweighs this positive effect.

Let then:

$$r_\Phi = \arg\max_r \Phi(r)$$
$$\overline{\Phi} = \Phi(r_\Phi)$$

Moreover, since $\Phi(r) \longrightarrow -\infty$ when $r \rightarrow -\infty$ and $\Phi(0) = S(0) > 0$, there exists a unique $r_0 \in]-\infty, 0]$ such that:

$$\Phi(r_0) = 0$$

The typical shape of Φ is depicted in figure 1.

Assumption 2 $\pi^D(r_\Phi) \geq f$

This assumption simply requires fixed development costs to be low enough so that development is still profitable, even for "high" royalty rates.

Assumption 3 $\pi^D(0) \geq \overline{\Phi} \geq \pi^D(r_\Phi)$

Assumption 3 is purely technical and simplifies calculations.

It is straightforward to verify that assumptions 1 and 3 are satisfied by the following family of demand functions[10]:

$$d(p) = (1-p)^\theta \text{ for } \theta \in [0, +\infty[$$

which implies that this assumption is satisfied as long as the hazard rate $\frac{g(p(r))}{1-G(p(r))}$ is increasing and $p'(r)$ is increasing, constant (but different than 0) or only "slowly" decreasing in r relative to the function between paranthesis, which is increasing in r. Then $\Phi'(r) < 0$ when $r \rightarrow \overline{v}$ and $\Phi'(r) = 0$ for r low enough (negative).

[10] We obtain:

$$r^* = -\frac{1}{\theta}; r_0 = \frac{-\theta}{\theta^2+\theta+1}; r_\Phi = \frac{1}{\theta^2+\theta+1}$$

$$\Phi(r) = \frac{\theta^\theta}{(1+\theta)^{\theta+2}}(1-r)^\theta\left(\theta + \left(\theta^2+\theta+1\right)r\right)$$

$$\pi^D(0) = \frac{\theta^\theta}{(1+\theta)^{\theta+1}} > \overline{\Phi} = \frac{\theta^{2\theta}}{(\theta+1)\left(\theta^2+\theta+1\right)^\theta} > \pi^D(r_\Phi) = \frac{\theta^{2\theta+1}}{\left(\theta^2+\theta+1\right)^{\theta+1}}$$

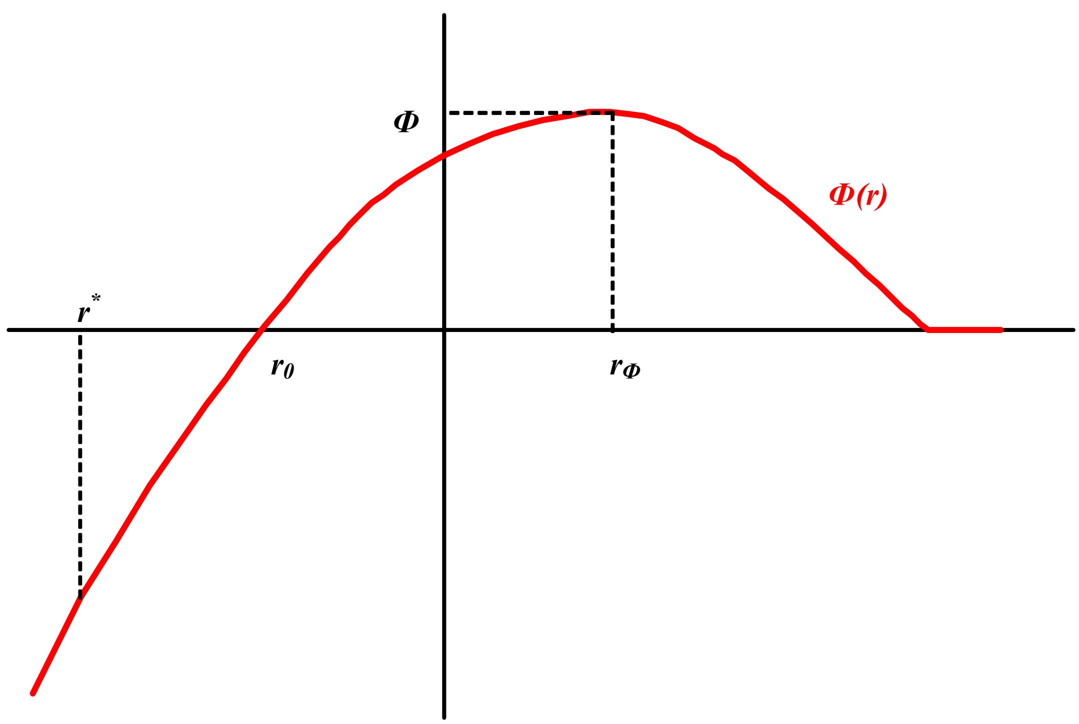

Figure 1:

Also, note that assumptions 2 and 3 imply:

$$\pi^D(0) \geq \overline{\Phi} \geq \pi^D(r_\Phi) \geq f$$

3.2 Timing and commitment

We assume the timing of arrival is well defined: *all* application developers arrive in the first stage and *all* users arrive in the second stage.

Given non-simultaneous arrival of the two sides, there are two scenarios to consider, according to whether platforms have or not the possibility to *credibly* commit to their prices P_i^U for users in the first stage, i.e. when they set their prices for developers. If commitment is feasible then each platform i can either announce its full set of three prices $\left(P_i^U, r_i, P_i^D\right) \in \mathbb{R}^3$ in the first stage or announce only $\left(r_i, P_i^D\right) \in \mathbb{R}^2$ and wait until the second stage to set P_i^U. If not, both platforms are limited to the second option.

There are thus two types of pricing games:

The no-commitment pricing game

Stage 1) Platforms $i = 1, 2$ simultaneously announce their prices $\left(r_i, P_i^D\right)$ for developers.

Developers simultaneously and non-cooperatively decide which -if any- platform to develop for. Let N_i denote the number of developers supporting platform i.

Stage 2) Platforms $i = 1, 2$ simultaneously announce their prices P_i^U for users, taking P_i^D, r_i and N_i for $i = 1, 2$ as given.

Stage 3) Developers announce their prices and users decide which -if any- platform to adopt and which applications to purchase, among those supported by the platform they have chosen.

The commitment pricing game

Stage 1) Platforms $i = 1, 2$ decide whether to commit to user prices or not and simultaneously announce their prices: platforms j having chosen to commit announce $\left(P_j^U, r_j, P_j^D\right)$; platforms k having chosen not to commit announce $\left(r_k, P_k^D\right)$.

Developers simultaneously and non-cooperatively decide which -if any- platform to develop for. Let N_i denote the number of developers supporting platform i.

Stage 2) Platforms k simultaneously announce their prices P_k^U for users, taking P_j^U and $\left(P_i^D, r_i, N_i\right)$ for $i = 1, 2$ as given.

Stage 3) Developers announce their prices and users decide which -if any- platform to adopt and which applications to purchase, among those supported by the platform they have chosen.

4 Monopoly platform

It is useful to start by analyzing optimal pricing and commitment by a single monopoly platform in order to gain some insight into the mechanisms at work in our model.

First note that commitment to user prices is always a weakly dominant strategy for a monopoly platform. Indeed, any outcome of the pricing game with no commitment can be replicated in the pricing game with commitment simply by committing to the user price the platform would charge anyway in the second stage. However, credible commitment to user prices may not be feasible, therefore we will treat both cases.

In the second stage, total user demand as a function of platform prices P^U and r and of the number n of developers supporting the platform is:

$$D^U\left(P^U, r, n\right) = \begin{cases} 1 \text{ if } nS\left(r\right) - P^U \geq 0 \\ \\ 0 \text{ if } nS\left(r\right) - P^U < 0 \end{cases}$$

P^U is either determined from stage 1 in the commitment pricing game, in which case the platform has no strategic variables to choose in the second stage, or it is the only strategic variable that the platform can set in stage 2 in case commitment is not feasible.

Platform profits in stage 2 are then:

$$\Pi_2^P = \left(P^U + nrd\left(r\right)\right) D^U\left(P^U, r, n\right)$$

If the platform has not committed to its user price in stage 1 then it maximizes its second stage profits by setting:

$$P^U = \begin{cases} nS(r) \text{ if } \Phi(r) \geq 0 \\ \\ \infty \quad \text{if } \Phi(r) < 0 \end{cases}$$

which yields:

$$\Pi_2^P = n \max(\Phi(r), 0)$$

Thus, if the monopoly platform cannot commit to user prices, the royalty rate r it sets in the first stage has to satisy $\Phi(r) \geq 0$ or, equivalently, $r \geq r_0$. Indeed, no developer will ever sign up to a royalty rate such that $r < r_0 < 0$ because they correctly anticipate in this case that the platform will set a prohibitively high user price in the second stage in order to make 0 sales and avoid having to pay the costly *negative* royalties it has announced. When credible commitment is feasible however, the platform no longer faces this constraint and its second stage profits may well turn out to be negative[11].

In order to capture both of these possibilities, denote by $\mathbf{P}$ the vector of prices announced by the platform in the first stage. If the platform commits to user prices, then $\mathbf{P} = (P^U, r, P^D) \in \mathbb{R}^3$. If not, $\mathbf{P} = (r, P^D) \in \mathbb{R}^2$. Then we can write user demand from the perspective of stage 1 as a function of $\mathbf{P}$ and n:

$$D^U(\mathbf{P}, n) = \begin{cases} 1_{\{nS(r) - P^U \geq 0\}} \text{ if } \mathbf{P} = (P^U, r, P^D) \\ \\ 1_{\{n\Phi(r) > 0\}} \quad \text{if } \mathbf{P} = (r, P^D) \end{cases} \tag{2}$$

where

$$1_{\{condition\}} = \begin{cases} 1 \quad \text{if } condition \\ \\ 0 \text{ if not } condition \end{cases}$$

Working our way backwards to the beginning of stage 1, the platform maximizes total profits under the developer participation constraint. However, the strategic complementarities between developers' participation decisions give rise to indirect network effects and therefore multiple equilibria, in the sense that for any given price vector $\mathbf{P}$ announced by the platform in the first stage, there may be several equilibrium configurations of developers'

[11] They are generally outweighed by positive revenues from fixed fees in the first stage.

participation decisions. Formally, given $\mathbf{P}$, there exists an equilibrium with n developers supporting the platform, $0 \leq n \leq N$, if and only if:

$$\begin{cases} n > 0 \Longrightarrow \Pi^{D}\left(\mathbf{P}, n\right) \geq 0 \\ \\ n < N \Longrightarrow \Pi^{D}\left(\mathbf{P}, n\right) \leq 0 \end{cases}$$

where $\Pi^{D}\left(\mathbf{P}, n\right)$ are developer profits:

$$\Pi^{D}\left(\mathbf{P}, n\right) = \pi^{D}\left(r\right) D^{U}\left(\mathbf{P}, n\right) - P^{D} - f$$

We can then define the *developer demand function* as a mapping $n\left(.\right)$, which associates to each price vector $\mathbf{P}$ an equilibrium number of developers $n\left(\mathbf{P}\right)$ supporting the platform. Note that these definitions are general enough to include both the commitment and the no-commitment pricing games.

It is apparent that in general there exist multiple developer demand functions. Each of them describes developers' (interdependent) adoption decisions for every price vector $\mathbf{P}$ announced by the platform in the first stage.

An equilibrium is then a pair $\left(\mathbf{P}, n\left(.\right)\right)$ where $n\left(.\right)$ is a developer demand function and $\mathbf{P}$ maximizes the profits of a platform facing developer demand function $n\left(.\right)$.

Given the simultaneous nature of the developer coordination game, this equilibrium concept can be interpreted as a *rational expectations equilibrium*, in which, given $\mathbf{P}$, each infinitesimal developer has expectations about all developers' adoption decisions, namely the fraction choosing to support the platform, and in equilibrium expectations are common and fulfilled.

Using this interpretation, we focus here on two polar developer demand functions, stemming from two types of developer expectations.

The first one is *favorable developer expectations*: each infinitesimal developer expects all developers to support the platform, as long as they obtain non-negative profits by doing so at the prices announced by the platform. This type of expectations may arise for example if the platform in question is a long-standing incumbent or benefits from outstanding reviews in specialized magazines; it could be Microsoft's Windows, Sony's Playstation, Palm, etc. Formally:

$$n^F(\mathbf{P}) = N1_{\{\Pi^D(\mathbf{P},N)\geq 0\}}$$

It follows that the platform sets $\mathbf{P}$ in order to maximize its profits subject to the developer participation constraint, which in this case is:

$$P^D \leq \pi^D(r)\, D^U(\mathbf{P}, N) - f \tag{3}$$

The second demand function we analyze stems from *unfavorable developer expectations*: each infinitesimal developer expects no developer will support the platform as long as this is consistent with the price vector announced. This type of expectations might prevail if the platform is a new entrant or if does not benefit from good reviews. Examples could be Palm's 1993 debut with Zoomer, IBM's failure with OS/2, etc. Formally:

$$n^{NF}(\mathbf{P}) = N1_{\{\Pi^D(\mathbf{P},0)>0\}}$$

In this case, the platform sets $\mathbf{P}$ to maximize profits subject to:

$$P^D < \pi^D(r)\, D^U(\mathbf{P}, 0) - f \tag{4}$$

We treat each of these two cases in turn and show that they lead to very different pricing structures.

a) Favorable developer expectations

If the monopoly platform benefits from favorable developer expectations, then the relevant constraint is (3).

Assume first that commitment to user prices is feasible. It is then easily seen that the optimal solution is to set $P^U = NS(r)$ and $P^D = \pi^D(r) - f$. The royalty rate r is thus chosen to maximize $\Pi^P = NW(r) - Nf$ and the solution is $r = r^*$ yielding:

$$\Pi^P = NW(r^*) - Nf$$

In particular, $p(r) = 0$ so that optimal pricing by the platform induces socially optimal pricing by developers (marginal costs are equal to 0). This result is due to the fact that the platform extracts all the surplus from both sides of the market, therefore its profits are equal to total social welfare[12]. More precisely, given that developers have market power, the platform will subsidize them through negative royalties in order to maximize the total surplus created by each application, which it can subsequently extract through fixed fees on both sides of the market. Net revenue from the developer side is negative:

$$\begin{aligned}\Pi^{PD} &= N\left(P^D + r^* d\left(r^*\right)\right) = N\left(p\left(r^*\right) d\left(r^*\right) - f\right) \\ &= -Nf < 0\end{aligned}$$

Meanwhile, net revenues on the user side are positive:

$$\Pi^{PU} = NS\left(r^*\right) = NW\left(r^*\right) > 0$$

Now assume the platform is unable to credibly commit to its price for users. Developers correctly anticipate that the platform will attract all users by charging $P^U = NS\left(r\right)$ in the second stage, but only *as long as* $\Phi\left(r\right) \geq 0$. Therefore the platform charges them $P^D = \pi^D\left(r\right) - f$ and maximizes $NW\left(r\right) - Nf$ subject to $r \geq r_0$. Since $W\left(r\right)$ is decreasing in r, the solution is to set $r = r_0$, yielding:

$$\Pi^P = NW\left(r_0\right) - Nf$$

which is strictly lower than under commitment, as expected. The profit structure is more balanced:

$$\begin{aligned}\Pi^{PD} &= Np\left(r_0\right) d\left(r_0\right) - Nf \gtrless 0 \\ \Pi^{PU} &= NS\left(r_0\right) > 0\end{aligned}$$

Thus, in this case the ability to commit raises social welfare.

We summarize all the above results in the following proposition.

[12] This is where the assumption that users do not know their valuations for applications *ex-ante*, i.e. before purchasing the platforms, is important. However, the strong result above matters only as a reference point against which we can compare all of the subsequent results, therefore their substance does not hinge on this simplifying assumption.

Proposition 1 *A monopoly platform facing favorable developer expectations extracts the entire social surplus. If commitment to user prices is feasible, its pricing is socially optimal: it requires charging a royalty rate equal to $r^* < 0$, thus inducing developers to sell their applications at the socially optimal price of 0. The resulting pricing structure involves a net subsidy to the developer side of the market and recoupment on the user side. If credible commitment to the price for users is not feasible then the platform charges a royalty rate equal to $r_0 > r^*$ and obtains strictly lower profits, so that social welfare is lower.*

b) Unfavorable developer expectations

In this case, the platform is constrained by (4).

Assume first that the platform cannot credibly commit to user prices. Then, using (2), (4) simplifies to:

$$P^D < -f$$

This price attracts *all* developers, because although they expect the platform to completely fail in the marketplace ($D^U(\mathbf{P}, 0) = 0$), they will still support it in order to collect the net reward $-P^D - f \geq 0$. But then the platform will recoup on users by charging:

$$P^U = NS(r)$$

Profits are then:

$$\Pi^P = N\Phi(r) - Nf$$

and are maximized for $r = r_\Phi > r_0$.

Assume now that credible commitment to user prices is possible. (4) becomes:

$$P^D < \pi^D(r)\, 1_{\{P^U<0\}} - f$$

This expression implies that, in order to solve the chicken-and-egg problem, the platform vendor has exactly two pricing strategies at its disposition: either $P^U < 0$ or $P^D < -f$. Following Caillaud and Jullien (2001), we call these strategies *divide-and-conquer* (DC). This terminology is self-explanatory and will become clear below.

i) DC strategy targetted at developers

By charging $P^D < -f$ the platform attracts all developers, which then enables it to charge $P^U = NS(r)$. Therefore this pricing strategy is exactly identical to the unique strategy available to the platform when commitment is not feasible. Profits are again:

$$\Pi^P = N\Phi(r) - Nf$$

and are maximized for $r = r_\Phi$

ii) DC strategy targetted at users

Now the platform subsidizes users by *committing to* $P^U \leq 0$. Such a price *credibly signals* to developers that the platform will attract all users, *irrespective* of their decisions. Developers understand this, therefore the platform can charge them:

$$P^D = \pi^D(r) - f$$

Profits are then:

$$\Pi^P = Np(r)\,d(r) - Nf$$

and are maximized for $r = 0$.

Note that, since developers arrive before users, although we say that this strategy is "targetted at users", it is in fact still *indirectly* targetted at developers: a negative P^U *signals* to developers that all users will be attracted.

The following proposition synthesizes the preceding analysis.

Proposition 2 *A monopoly platform facing unfavorable developer expectations is unable to extract the entire social surplus.*

If credible commitment to user prices is feasible then optimal platform pricing requires $r = 0$*,* $P^D = \pi^D(0) - f$ *and* $P^U = 0$*, yielding profits* $\Pi^P = N\pi^D(0) - Nf$ *and a profit structure biased in favor of users:* $\Pi^{PD} = N\pi^D(0) - Nf > 0$ *and* $\Pi^{PU} = 0$*. Social welfare is* $SW = NW(0) - Nf$*.*

If credible commitment to user prices is not feasible then the optimal pricing solution is to set $r = r_\Phi$*,* $P^D = -f$ *and* $P^U = NS(r_\Phi)$*, yielding profits* $\Pi^P = N\overline{\Phi} - Nf$ *and the following profit structure:* $\Pi^{PD} = Nr_\Phi d(r_\Phi) - Nf \gtrless 0$ *and* $\Pi^{PU} = NS(r_\Phi) > 0$*. Social welfare is* $SW = NW(0) - Nf$*.*

■

It is interesting to note that, when commitment is feasible, the profit structure shifts *away* from subsidizing developers when it becomes "harder" to attract them, i.e. when developer expectations change from favorable to unfavorable: indeed, the two corresponding revenue structures are opposite. This counterintuitive result suggests that the timing of arrival of chicken and eggs is less important than market structure and the pricing instruments available.

Under unfavorable developer expectations, both platform profits and social welfare are higher when commitment is possible[13]. However, this is "pure luck" here in the sense that, because of unfavorable developer expectations, the platform is no longer able to extract the entire social surplus created by applications, implying that its incentives are no longer aligned with those of a hypothetical social planner. This can be seen by comparing the differences in platform profits and social welfare under commitment and no-commitment:

$$\Delta\Pi^P = N\left(\pi^D(0) - \overline{\Phi}\right) \gtrless N\left(W(0) - W(r_\Phi)\right) = \Delta SW$$

It is insightful to illustrate the preceding analysis graphically. One can view the difference between platform profits and total social welfare as a premium that the platform has to pay in order to overcome unfavorable developer expectations. Indeed, given that it cannot charge the first best optimal prices corresponding to favorable expectations and thereby extract the entire social welfare, the platform has to give up a part of this surplus in order to make any profits at all. The question is which part of social surplus should it sacrifice?

In figure 2, total social surplus per application $W(r)$ is broken down into developer profits $\pi^D(r)$ and second period surplus $\Phi(r)$, i.e. the sum of user surplus and platform royalty revenue. By subsidizing developers the platform chooses to give up developer profits and therefore it chooses $r = r_\Phi$ in order to maximize the red curve corresponding to $\Phi(r)$, as opposed to maximizing total social surplus $W(r)$, i.e. the black curve.

[13] This is easily seen by recalling assumption 3 and that $W(r)$ is decreasing in r.

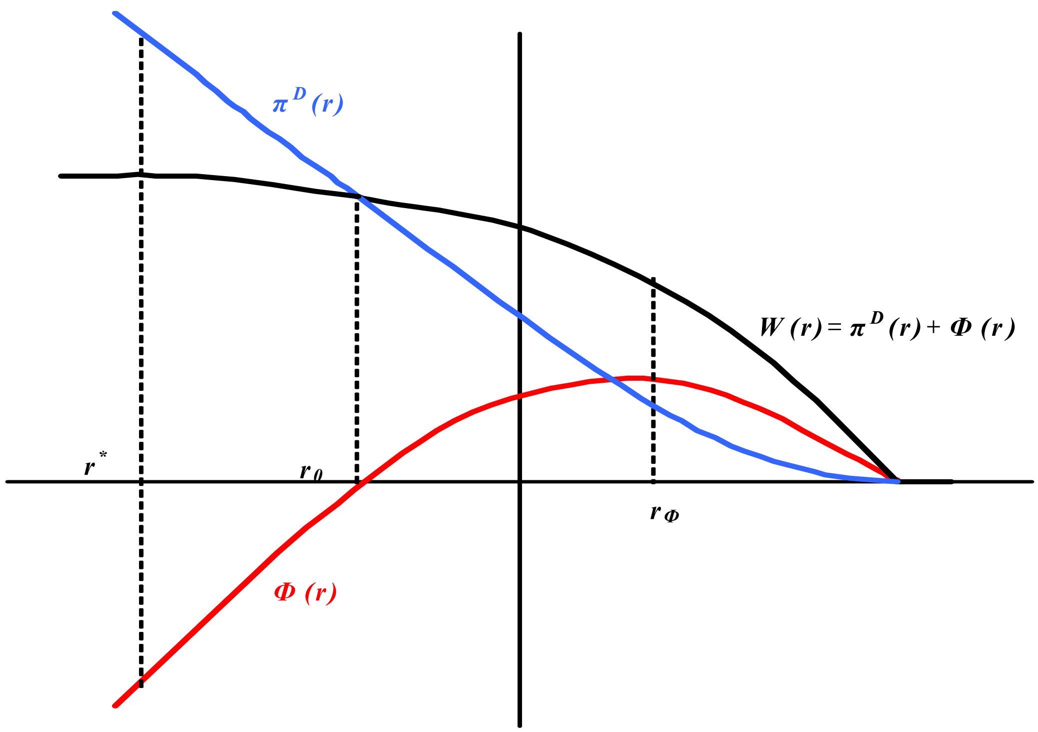

Figure 2:

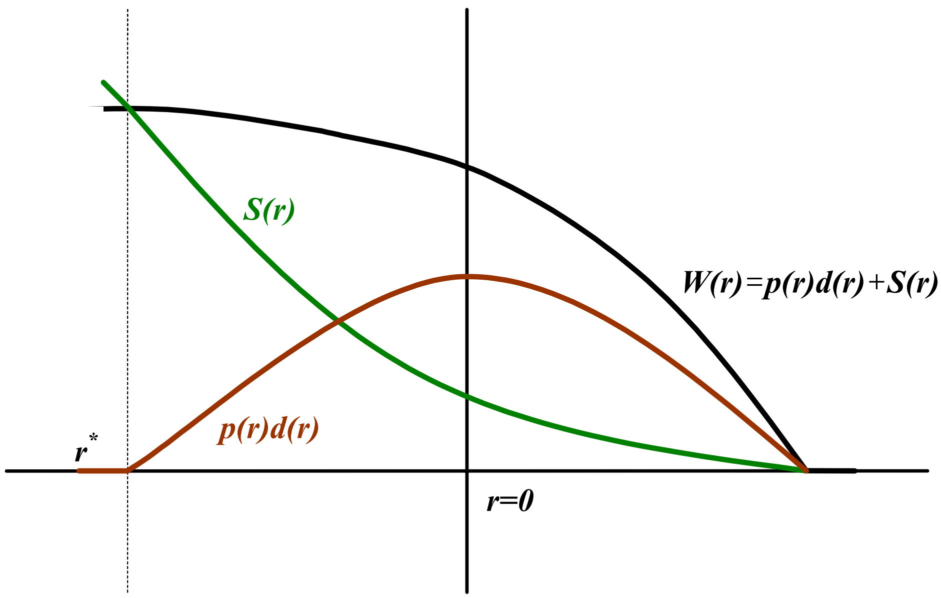

Figure 3:

In figure 3, total social surplus per application is now split into developer profits gross of fixed cost f plus platform royalty revenues $(= p(r)\,d(r))$ and user surplus $S(r)$. Thus, when the platform chooses to subsidize users, it chooses to give up user surplus and finds itself on the brown curve, which once again leads to an inefficient royalty rate $(r = 0)$.

Both these diagrams illustrate the misalignments between the platform's objectives and the social planner's, caused by unfavorable developer expectations and leading to inefficiencies: indeed, social welfare is always strictly lower than under favorable developer expectations. We will see in the next sections how competition between platforms can lead to similar inefficiencies, as it forces platforms to give up parts of total social surplus and therefore entail potentially inefficient pricing[14].

[14]Note that total welfare depends solely on the royalty rate in our simple model because fixed fees are just transfers with no effect on the quantities traded.

5 Platform competition for users

We now turn to the analysis of the pricing game played by two competing platforms. The Bertrand pricing game played in stage 2 (competition for users) is entirely determined by the outcome of stage 1, most notably developers' adoption decisions. We will therefore first fully characterize the outcome of the battle for users under all possible scenarios; we will use this characterization in all subsequent subsections in order to solve the entire pricing game.

Let $\mathbf{N} = (N_{e_1}, N_{e_2}, N_m)$ be the distribution of developers among the two platforms at the beginning of stage 2. N_{e_i} is the number of developers supporting platform i *exclusively* and N_m is the number of developers supporting both platforms (i.e. *multihoming*)[15]. It is then convenient to define:

$$N_i = N_{e_i} + N_m$$

the total number of developers supporting platform i.

Also, let $\mathbf{P} = \{\mathbf{P}_1, \mathbf{P}_2\}$ be the vector of prices announced by the platforms in the first stage. If platform i credibly commits to a user price P_i^U in the first stage, then $\mathbf{P}_i = \left(P_i^U, r_i, P_i^D\right) \in \mathbb{R}^3$ and P_i^U can no longer be modified in the second stage. If platform i makes positive sales then $P_i^U \leq N_i S\left(r_i\right)$. Its second stage profits are $\left(P_i^U + N_i r_i d\left(r_i\right)\right) D_i^U$, where D_i^U is the number of users joining platform i. Note in particular that P_i^U may be lower than $-r_i N_i d\left(r_i\right)$, implying that platform i makes negative profits in the second stage.

If on the other hand platform i has not committed to its user price in the first stage then $\mathbf{P}_i = \left(r_i, P_i^D\right) \in \mathbb{R}^2$ and P_i^U is determined during the second stage Bertrand pricing game. However, in this case platform i will never price below the "marginal cost" $-N_i r_i d\left(r_i\right)$, meaning that, absent commitment to user prices, second stage platform profits cannot be negative. Consequently, the highest utility platform i can offer users conditional on *not* having committed to user prices is $N_i S\left(r_i\right) + N_i r_i d\left(r_i\right) = N_i \Phi\left(r_i\right)$, which is also equal to the highest profits platform i can earn in the second

[15] Note that under exclusivity $N_m \equiv 0$ and $N_{e_1} + N_{e_2} = N$, whereas under multihoming $0 \leq N_{e_1} + N_{e_2}, N_m \leq N$.

stage, obtained by setting $P_i^U = N_i S(r_i)$. It follows that in this case the royalty rate announced to developers has to satisfy:

$$\Phi(r_i) \geq 0 \Longleftrightarrow r_i \geq r_0 \tag{5}$$

Indeed, if this condition did not hold then platform i would be certain to make negative profits in the second stage, which it can simply avoid by charging an astronomical user price P_i^U. This ensures it will make 0 sales and will not have to pay the costly negative royalty rate it has announced to developers[16]. Knowing this, such an r_i will attract no developers.

Before proceeding, we need to specify how users distribute themselves when they are indifferent between the 2 platforms:

Assumption 3 *If users are indifferent between platforms 1 and 2 then they split in proportions* (λ_1, λ_2) *with* $\lambda_1 + \lambda_2 = 1$ and $\lambda_1 \lambda_2 > 0$.

Note that we assume the split is constant, i.e. independent of the distribution of developers among the 2 platforms, as long as the latter offer the exact same utility to users.

There are three possible scenarios to consider: both platforms have committed to user prices in the first stage, only one platform has committed to its user price and lastly, none of the two platforms has committed to user prices.

5.0.1 Both platforms have committed to user prices

In this case platforms do not make any strategic choices in the second stage and the first stage price vector is $\mathbf{P} = \left\{ \left(P_i^U, r_i, P_i^D\right)_{i=1,2} \right\}$.

User demand for platform i is then:

$$D_i^U(\mathbf{P}, \mathbf{N}) = \begin{cases} 1 \text{ if } \begin{array}{l} N_i S(r_i) - P_i^U > N_j S(r_j) - P_j^U \\ \text{and } N_i S(r_i) - P_i^U \geq 0 \end{array} \\ \\ \lambda_i \text{ if } N_i S(r_i) - P_i^U = N_j S(r_j) - P_j^U \geq 0 \\ \\ 0 \text{ if } N_i S(r_i) - P_i^U < N_j S(r_j) - P_j^U \end{cases} \tag{6}$$

[16] Recall that $\Phi(r_i)$ is negative if and only if:

$$r_i < r_0 < 0$$

and total profits are:

$$\Pi_i^P = N_i P_i^D + \left(P_i^U + N_i r_i d\left(r_i\right)\right) D_i^U$$

5.0.2 Platform i has committed, platform j has not committed

The vector of prices announced by platforms in the first stage is now $\mathbf{P} = \left\{\left(P_i^U, r_i, P_i^D\right), \left(r_j, P_j^D\right)\right\}$.

Platform i wins the user battle if and only if the utility it offers users is non-negative and higher than the maximum utility that platform j *could potentially* offer, which translates into[17]:

$$\left(D_i^U\left(\mathbf{P},\mathbf{N}\right), D_j^U\left(\mathbf{P},\mathbf{N}\right)\right) = \begin{cases} (1,0) & \text{if } N_i S\left(r_i\right) - P_i^U > N_j \Phi\left(r_j\right) \\ \left(\lambda_i, \lambda_j\right) & \text{if } N_i S\left(r_i\right) - P_i^U = N_j \Phi\left(r_j\right) \geq 0 \\ (0,1) & \text{if } N_i S\left(r_i\right) - P_i^U < N_j \Phi\left(r_j\right) \end{cases} \tag{7}$$

In stage 2 platform j sets:

$$P_j^U = \max\left(-N_j r_j d\left(r_j\right), N_j S\left(r_j\right) - \max\left(N_i S\left(r_i\right) - P_i^U, 0\right)\right)$$

Total profits are:

$$\begin{aligned} \Pi_i^P &= N_i P_i^D + \left(P_i^U + N_i r_i d\left(r_i\right)\right) D_i^U \\ \Pi_j^P &= N_j P_j^D + \left(N_j \Phi\left(r_j\right) - \max\left(0, N_i S\left(r_i\right) - P_i^U\right)\right) D_j^U \end{aligned}$$

5.0.3 Neither platform has committed

In this case the first stage price vector becomes $\mathbf{P} = \left\{\left(P_i^U, r_i, P_i^D\right)_{i=1,2}\right\}$.

Platform i wins if and only if the maximum utility it could potentially offer users is higher than what j could offer. Thus[18]:

$$D_i^U\left(\mathbf{P},\mathbf{N}\right) = \begin{cases} 1 & \text{if } N_j \Phi\left(r_j\right) > N_j \Phi\left(r_j\right) \\ \lambda_i & \text{if } N_i \Phi\left(r_i\right) = N_j \Phi\left(r_j\right) \\ 0 & \text{if } N_i \Phi\left(r_i\right) < N_j \Phi\left(r_j\right) \end{cases} \tag{8}$$

[17] For simplicity, we already assume that $\Phi\left(r_j\right) \geq 0$.

[18] We already assume $\Phi\left(r_i\right)$, $\Phi\left(r_j\right) \geq 0$.

In stage 2 platform i will set:

$$P_i^U = \max\left(-r_i N_i d\left(r_i\right), N_i S\left(r_i\right) - N_j \Phi\left(r_j\right)\right)$$

Thus, total profits from the perspective of stage 1 are:

$$\Pi_i^P = N_i P_i^D + \left(N_i \Phi\left(r_i\right) - N_j \Phi\left(r_j\right)\right) D_i^U$$

With these characterizations in hand, we can now turn to the analysis of the full pricing game between two-competing platforms.

6 Platform competition with a dominant platform

Just like in the case with a monopoly platform, the coordination game played by developers in the first stage admits multiple equilibria for any given price vector $\mathbf{P}$. In order to formalize the definition of our equilibrium concept, we adapt the definitions from Caillaud and Jullien (2002).

Denote by $\mathbf{S}^D$ the set of strategies available to developers:

$$\mathbf{S}^D = \begin{cases} \{0, e_1, e_2\} & \text{if multihoming is not permitted} \\ \\ \{0, e_1, e_2, m\} & \text{if multihoming is feasible} \end{cases}$$

where 0 stands for no development, e_i for exclusive development for platform i and m means that the developer supports both platforms.

The expression of developer profits for each strategy available as a function of the vector of prices $\mathbf{P} = (\mathbf{P}_1, \mathbf{P}_2)$ and the distribution of developers $\mathbf{N} = (N_{e_1}, N_{e_2}, N_m)$ is then:

$$\Pi^D\left(s, \mathbf{P}, \mathbf{N}\right) = \begin{cases} \pi^D\left(r_i\right) D_i^U\left(\mathbf{P}, \mathbf{N}\right) - P_i^D - f & \text{if } s = e_i \\ \\ \begin{array}{r} \pi^D\left(r_1\right) D_1^U\left(\mathbf{P}, \mathbf{N}\right) + \pi^D\left(r_2\right) D_2^U\left(\mathbf{P}, \mathbf{N}\right) \\ -P_1^D - P_1^D - \left(1+\gamma\right) f \end{array} & \text{if } s = m \\ \\ 0 & \text{if } s = 0 \end{cases}$$

where $D_i^U\left(\mathbf{P}, \mathbf{N}\right)$, $i = 1, 2$, are defined in the previous section.

Definition 3 *A distribution of developers $\mathbf{N} = (N_{e_1}, N_{e_2}, N_m)$ is an equilibrium distribution given $\mathbf{P} = (\mathbf{P}_1, \mathbf{P}_2)$ if and only if:*

$$N_s > 0 \Longrightarrow \Pi^D\left(s, \mathbf{P}, \mathbf{N}\right) \geq \max_{\substack{s' \in \mathbf{S}^D \\ s' \neq s}} \Pi^D\left(s', \mathbf{P}, \mathbf{N}\right)$$

A system of developer demand functions is a mapping $\mathbf{N}(.)$*, which associates to every price vector* $\mathbf{P}$ *an equilibrium distribution of developers* $\mathbf{N}(\mathbf{P})$*.*

Definition 4 *An equilibrium is a pair* $(\mathbf{N}(.), \mathbf{P})$*, where i)* $\mathbf{N}(.)$ *is a system of developer demand functions and ii)* $\mathbf{P}$ *is a Nash equilibrium of the reduced form pricing game induced by* $\mathbf{N}(.)$*.*

Note again that these definitions are general enough to cover both the commitment and the no-commitment pricing games. Also, as noted before, this equilibrium concept can be interpreted as a *rational expectations equilibrium*, in which developer expectations are common and fulfilled.

Clearly, there exist multiple equilibria stemming from different systems of developer demand functions. Caillaud and Jullien (2002) study the sustainability of two types of equilibria: dominant platform equilibria, in which one platform covers both sides of the market, and market-sharing equilibria, in which both platforms are active.

In all that follows we focus our attention exclusively on dominant platform equilibria. There are two reasons for doing so.

First, from a theoretical standpoint, the matching model proposed by Caillaud and Jullien (2001) exhibits complementarities between matchmakers due to the imperfection of the matching technique. Indeed, if one platforms does not perform a match, the other platform may do so, therefore it is only natural that market-sharing equilibria arise, some of them being asymmetric, with one platform acting as a first source and charging 0 transaction fees and the other acting as a second source with positive transaction fees. By contrast, in our model platforms are perfect substitutes and it can be shown (at the price of lengthy calculations, which do not offer any additional insights) that market-sharing equilibria are either unstable or non-existent. Moreover, since only developers play a coordination game, we cannot easily adopt Caillaud and Jullien's monotonicity refinement for systems of developer demand functions. Finally, our focus is on commitment issues and it turns out that analyzing only dominant platform equilibria is sufficient in order to gain the main insights.

Secondly and perhaps most importantly, there is a strong empirical reason for this restriction: in the markets we are interested in, dominance by a single platform at any given point in time seems to be the norm. Indeed, in most operating systems markets there exists one platform with an 80% plus market share (Microsoft's Windows for PCs, Palm for PDAs, Symbian for smartphones) and the same is true in the videogame market (Nintendo's NES circa 1986-1991, Sega's Genesis circa 1992-1994, Sony's Playstation circa 1995-2001). This phenomenon is in contrast with other two-sided markets, in which one typically observes several platforms sharing the market on more equal terms: Visa, Mastercard and American Express in the credit card market; e-Bay, Yahoo and Amazon in the on-line auctions market, etc.

Consequently, in all that follows we assume *platform 1 is dominant and platform 2 is dominated*, in the sense that platforms 1 benefits from *favorable developer expectations*: each infinitesimal developer expects all developers to support platform 1 *exclusively* whenever this is an equilibrium given the vector of prices announced by the two platforms in the first stage. Formally:

$$\mathbf{N}(\mathbf{P}) = (N,0,0) \Longleftrightarrow \Pi^D(e_1,\mathbf{P},(N,0,0)) \geq \max_{\substack{s\in\mathbf{S}^D \\ s\neq e_1}} \Pi^D(s,\mathbf{P},(N,0,0)) \tag{9}$$

In words, a price vector is a dominant platform equilibrium if and only if each individual developer's preferred strategy at these prices and *given that he expects all other developers to develop exclusively for platform 1* is to also develop exclusively for platform 1.

Lastly, we define sustainability of a dominant platform equilibrium as follows:

Definition 5 *A dominant platform equilibrium is sustainable if and only if there exists* $\mathbf{P}_1$ *such that platform 2 cannot make non-negative profits whenever platform 1 announces* $\mathbf{P}_1$ *and the system of developer demand functions* $\mathbf{N}(.)$ *satisfies (9).*

6.1 Dominant platform equilibrium with no commitment to user prices

We start with the (simpler) case in which platforms cannot credibly commit to user prices in the first stage, so that the price vector is always $P = \left\{ \left(r_i, P_i^D\right)_{i=1,2} \right\}$ and (5) must hold for $i = 1, 2$. We also need to distiguish between two possible regimes: developers are bound to exclusivity with one platform and developers are allowed to multihome.

6.1.1 Exclusivity

If developers can only develop exclusively for one platform, a necessary condition for a set of prices $\left(P_i^D, r_i\right)_{i=1,2}$ to be a dominant equilibrium for platform 1 is:

$$\pi^D (r_1) - P_1^D - f \geq \max \left(0, -P_2^D - f\right)$$

This condition says that each individual developer has to earn non-negative profits from developing for platform 1 and have no incentive to switch to platform 2 *if he expects* all other developers to register with 1. Indeed, in this case, he can be sure that 1 will win the second period battle for users, since $N\Phi(r_1) \geq 0$.

Let us determine platform 2's best response given 1's prices P_1^D and r_1.

Given unfavorable expectations against it, the only way platform 2 can attract any developers and subsequently gain positive user market share is by charging:

$$P_2^D < P_1^D - \pi^D (r_1) \tag{10}$$

By doing so, it is certain to attract *all* developers and obtain profits:

$$\Pi_2^P = NP_2^D + N\Phi (r_2)$$

which it maximizes subject to (10) and[19]:

$$\Phi (r_2) > 0$$

We obtain:

[19] Note that since $\pi^D (r_1) - P_1^D - f \geq 0$ on the equilibrium path, (10) also ensures that $P_2^D \leq -f$, so that developers are sure to make non-negative profits by registering with 2.

Lemma 6 *Platform 2's best response to P_1^D and r_1 when developer expectations favor platform 1 is to set $r_2 = r_\Phi$ and $P_2^D = P_1^D - \pi^D(r_1)$ (slightly less), obtaining $\Pi_2^P = NP_1^D - N\pi^D(r_1) + N\overline{\Phi}$.*

■

In order for the dominant platform equilibrium to be sustainable, platform 1's prices have to be such that platform 2's best response is unprofitable, i.e.:

$$NP_1^D < N\pi^D(r_1) - N\overline{\Phi} \tag{11}$$

Platform 1 maximizes its profits $\Pi_1^P = NP_1^D + N\Phi(r_1)$ subject to (11), $\Phi(r_1) \geq 0$ and $\pi^D(r_1) - P_1^D - f \geq 0$. Recalling assumption 3, these three constraints reduce to:

$$\Phi(r_1) \geq 0$$

$$P_1^D < \pi^D(r_1) - \overline{\Phi}$$

yielding the following optimization problem for platform 1:

$$\max_{r_1} \left\{ NW(r_1) - N\overline{\Phi} \right\}$$

subject to $\Phi(r_1) \geq 0$.

Recalling that $W(r)$ is decreasing in r and using assumptions 2 and 3[20], we obtain the following characterization of the dominant firm equilibrium.

Proposition 7 *A dominant platform equilibrium always exists when developers cannot multihome and platforms cannot commit to user prices. The dominant platform 1 sets $r_1 = r_0$ and $P_1^D = \pi^D(r_0) - \overline{\Phi}$ and obtains profits $\Pi_1^P = NW(r_0) - N\overline{\Phi}$. The profit structure is: $\Pi_1^{PU} = P_1^U = NS(r_0) > 0$ and $\Pi_1^{PD} = N\left(p(r_0)d(r_0) - \overline{\Phi}\right) \gtrless 0$.*

Social welfare is: $SW = NW(r_0) - Nf$.

■

[20] We use these assumptions to prove dominant platform profits are positive:

$$W(r_0) - \overline{\Phi} = \pi^D(r_0) - \overline{\Phi} > \pi^D(0) - \overline{\Phi} \geq 0$$

6.1.2 Multihoming

With multihoming, each developer has now three options (aside from no development) instead of just two: exclusivity with platform i, $i = 1$ or 2 and multihoming. Multihoming is preferred to exclusivity with platform j if and only if:

$$\pi^D(r_i)\, D_i^U - P_i^D - \gamma f \geq 0$$

This condition shows that there is a sense in which platform i benefits from the fact that a developer *already* supports platform j. Indeed, if he did not, the participation condition for the developer to platform i would be:

$$\pi^D(r_i)\, D_i^U - P_i^D - f \geq 0$$

which is more restrictive if $\gamma < 1$.

In order for the dominant platform equilibrium to be sustainable, we have to impose that developers prefer exclusivity with platform 1 to *both* exclusivity with platform 2 *and* multihoming.

Therefore, an equilibrium set of prices $\left(P_i^D, r_i\right)_{i=1,2}$ has to satisfy (5) and:

$$\pi^D(r_1) - P_1^D - f \geq \max\left(0, -P_2^D - f\right)$$

$$-P_2^D - \gamma f \leq 0$$

Using the same procedure as in the exclusivity case, we start by determining platform 2's best response when facing unfavorable developer expectations and given platform 1's prices. In order to have any chance of obtaining any market share, 2 has to charge:

$$P_2^D \leq -\gamma f$$

This ensures that, developers prefer multihoming to exclusivity with platform 1, even if platform 2 makes no sales to users. Otherwise, each individual developer believes 2 will not attract any developer support and therefore will not make any sales to users, so that it is rational not to develop for 2.

Furthermore, if $\Phi(r_2) < \Phi(r_1)$ then platform 2 makes 0 sales to users and therefore earns strictly negative profits. Thus, in order for the above subsidy to make any sense, 2 has to charge r_2 such that:

$$\Phi(r_2) > \Phi(r_1) \tag{12}$$

If $r_1 = r_\Phi$, platform 2 makes positive profits if and only if developers register *exclusively* with 2. Indeed, if they multihome, then regardless of how users split between platforms, neck-to-neck competition for users results in 0 profits for both platforms in the second stage, which leaves platform 2 with total profits of $-N\gamma f < 0$. And developers prefer exclusivity with platform 2 to multihoming in this case if and only if:

$$\lambda_1 \pi^D(r_\Phi) - P_1^D - \gamma f < 0 \Longleftrightarrow P_1^D > \lambda_1 \pi^D(r_\Phi) - \gamma f$$

However, 1 may not have to choose $r_1 = r_\Phi$ in order to keep platform 2 out: indeed, we know that absent competition, a platform has every interest in setting the royalty rate it charges developers as low as possible. Then, if $\Phi(r_1) < \overline{\Phi}$, platform 2 can charge $\Phi(r_2) > \Phi(r_1)$ and win the battle for users even if all developers keep multihoming.

The following lemma fully characterizes 2's best response.

Lemma 8 *Platform 2's best response to 1's prices is to set $r_2 = r_\Phi$ and $P_2^D = -\gamma f$. The resulting best response profits are:*

$$\Pi_2^P = \begin{cases} N\overline{\Phi} - N\gamma f - N\Phi(r_1) & \text{if} \quad \begin{array}{l} (\Phi(r_1) < \overline{\Phi} \text{ and } P_1^D \leq -\gamma f) \text{ or} \\ (\Phi(r_1) = \overline{\Phi} \text{ and } P_1^D \leq \lambda_1 \pi^D(r_1) - \gamma f) \end{array} \\ \\ N\overline{\Phi} - N\gamma f & \text{if} \quad \begin{array}{l} (\Phi(r_1) < \overline{\Phi} \text{ and } P_1^D > -\gamma f) \text{ or} \\ (\Phi(r_1) = \overline{\Phi} \text{ and } P_1^D > \lambda_1 \pi^D(r_1) - \gamma f) \end{array} \end{cases}$$

Proof. See appendix. ■

Now that we have determined platform 2's best response, it remains to impose that platform 1's prices do not allow 2 to obtain positive profits with its best response. The following proposition fully characterizes the dominant firm equilibrium.

Proposition 9 *A dominant platform equilibrium always exists when developers can multihome and platforms cannot commit to user prices. The dominant platform charges* $r_1 = r_\Phi$ *and* $P_1^D = \min\left(\lambda_1 \pi^D(r_\Phi) - \gamma f, \pi^D(r_\Phi) - f\right)$ *obtaining profits* $\Pi_1^P = N\overline{\Phi} - N\gamma f + N\min\left(\lambda_1\pi^D(r_\Phi), \pi^D(r_\Phi) - (1-\gamma) f\right) \geq 0$ *and the following revenue structure:* $\Pi_1^{PU} = P_1^U = NS(r_\Phi) > 0$ *and* $\Pi_1^{PD} = Nr_\Phi d(r_\Phi) - N\gamma f + N\min\left(\lambda_1\pi^D(r_\Phi), \pi^D(r_\Phi) - (1-\gamma) f\right) \gtreqless 0.$

Social welfare is: $SW = NW(r_\Phi) - Nf$

Proof. See appendix. ■

The interesting feature of this equilibrium is the influence of γ. Intuitively, one would expect the dominant platform's equilibrium profits to be increasing in γ: indeed, the higher γ, the more costly porting is and therefore the easier it should be to fend off the challenging platform. The reason this intuition turns out to be incorrect[21] is that the dominant platform is forced to resort to a "defensive strategy" (i.e. set $P_1^D = \min\left(\lambda_1\pi^D(r_\Phi) - \gamma f, \pi^D(r_\Phi) - f\right)$), whose cost may well be strictly *increasing* in γ. We call this strategy "defensive" because its purpose is to render multihoming preferrable to exclusivity with the challenger rather than to render exclusivity with the dominant platform preferrable to multihoming.

Comparing propositions (7) and (9), it appears that resulting social welfare is lower under multihoming. This is because when multihoming is possible, it is relatively easier for the challenger to attract developers, which prevents the dominant platform from extracting developer profits and consequently the latter sets its royalty rate so as to maximize the sum of user surplus and royalty revenue, i.e. $\Phi(r)$ (recall figure 2). However, the dominant platform may sometimes make higher profits under multihoming than under exclusivity[22]. This illustrates an important insight of our model, namely that competitive pressure from a challenging platform in this type of two-sided market introduces a misalignment between the dominant platform's objectives and social welfare maximization, thereby inducing pricing inefficiencies and lowering social welfare.

[21] Note indeed that for $\lambda_1\pi^D(r_\Phi) \leq \pi^D(r_\Phi) - (1-\gamma) f$, Π_1^P is *decreasing* in γ.

[22] This is true for example when λ_1 is close to 1 and $W(r_0) < W(r_\Phi) + \overline{\Phi} - f$.

Finally, note that the socially optimal pricing ($r = r^*$) never arises in equilibrium in this case.

6.2 Dominant platform equilibrium when commitment to user prices is feasible

Let us now turn to the case in which platforms have the option of credibly committing to user prices in the first stage. This option makes the game significantly more complex, as we have to analyze both commitment and no-commitment strategies for both the dominant platform and the challenger. Moreover, credible commitment to user prices makes it possible for the challenger to use divide-and-conquer pricing strategies targetted to users, which were not feasible before.

As we will see below however, the desirability of commitment depends on whether developers are under a regime of exclusivity or multihoming. In particular, whereas under exclusivity commitment is *always* a weakly dominant strategy for the challenger, this is not necessarily the case under multihoming. For the dominant platform, the choice between commitment and no-commitment turns out to be complex and depends on the parameters of the model. However, the intuition for the relevant tradeoff is quite simple and appealing: if the dominant platform does not commit, then it runs the risk of seeing the challenger taking over the market by announcing low user prices. If on the other hand it commits, then it can make sure the entrant stays out by committing to low user prices. But in this case, come stage 2, the dominant platform no longer faces any competition for users and it is unable to exploit this adavantageous position, therefore the user prices it has committed to *ex-ante* turn out to be inefficiently low *ex-post*.

In order to analyze the dominant platform equilibrium we use the same procedure as above, except that here platform 2's best response involves *both* a commitment *and* a pricing strategy, which we have to determine given platform 1's commitment *and* pricing strategies.

6.2.1 Exclusivity

In each of the pricing games with commitment (under exclusivity and multihoming) we have to treat two subgames, according to whether or not platform 1 commits to its user price.

Platform 1 does not commit to its user price This requires $\Phi(r_1) \geq 0$. Now consider platform 2's options.

If it chooses not to commit to its user price, then the pricing game is identical to the one we analyzed above, when commitment was not feasible. We know from Lemma 6 that in this case platform 2's best pricing strategy is to set $r_2 = r_\Phi$ and $P_2^D = P_1^D - \pi^D(r_1)$, obtaining $\Pi_2^P = NP_1^D - N\pi^D(r_1) + N\overline{\Phi}$.

Suppose now that platform 2 commits to user price P_2^U in the first stage. There are now two ways for platform 2 to gain market share: a DC strategy targeted at users or a DC strategy targeted at developers.

i) DC strategy targetted at developers

The idea behind this strategy is exactly the same as under no-commitment: platform 2 seeks to attract developers despite unfavorable beliefs. This can only be done by charging:

$$P_2^D < P_1^D - \pi^D(r_1)$$

By charging such a price, 2 is certain to obtain the support of all developers, which which in turn allows it to commit to the same user price it ended up charging without committing to user prices, namely:

$$P_2^U = NS(r_2)$$

It is then clear that in this case platform 2's best response is exactly the same as without commitment, i.e. charge $r_2 = r_\Phi$ for profits:

$$\Pi_2^P = NP_1^D - N\pi^D(r_1) + N\overline{\Phi} \tag{13}$$

We can therefore conclude that commitment to user prices is a weakly dominant strategy for platform 2 in this case, since a DC strategy on developers exactly replicates its only available strategy without commitment.

ii) DC strategy targetted at users

The point of this strategy for platform 2 is to convince developers that it will win the user battle *even without any developer support.* 2 can achieve this by commiting to P_2^U low enough to overcome unfavorable developer expectations, which, using (7), yields the following condition:

$$P_2^U < -N\Phi(r_1)$$

Indeed, given that platform 1 has not committed to user prices, developers anticipate that the latter will be able to compete for users by offering a surplus up to $N\Phi(r_1)$. But once it has set such a price for users, developers *know* that platform 2 will win the user battle, so that 2 can charge them:

$$P_2^D \leq \pi^D(r_2) + \min\left(P_1^D, -f\right)$$

This condition ensures that developers prefer supporting 2 rather than 1 and that they obtain non-negative profits by doing so. We therefore obtain that 2's best response with a DC strategy aimed at users is to set $r_2 = 0$, obtaining profits of:

$$\Pi_2^P = N\pi^D(0) + N\min\left(P_1^D, -f\right) - N\Phi(r_1) \tag{14}$$

Platform 1 maximizes its profits subject to the developer rationality constraint and to the condition that platform 2 is unable to make non-negative profits with its best response strategy, i.e. both (13) and (14) must be negative. Formally, 1 solves:

$$\max\left\{NP_1^D + N\Phi(r_1)\right\}$$

subject to:

$$0 \leq \Phi(r_1) \tag{15}$$

$$P_1^D \leq \pi^D(r_1) - \overline{\Phi} \tag{16}$$

$$\min\left(P_1^D, -f\right) \leq \Phi(r_1) - \pi^D(0) \tag{17}$$

Note that without (17) we would obtain exactly the same result as in Proposition 7. The new constraint (17) corresponds to the possibility for platform 2 of using a DC strategy on users, which platform 1 has to render unprofitable. In fact, if it chooses not to commit, platform 1's profits are strictly lower than when commitment was not feasible for either platform: indeed, plugging r_0 into the constraints above, it is easily seen that the solution from Proposition 7 is not admissible here.

Proposition 10 *Given that platform 1 does not commit to its user price, its optimal pricing is:*

If $\overline{\Phi} + f < \pi^D(0)$ *and* $2\overline{\Phi} \leq \pi^D(0) + \pi^D(r_\Phi)$ *then set* $r_1 = r_\Phi$ *and* $P_1^D = \overline{\Phi} - \pi^D(0)$ *for profits* $\Pi_1^P = 2N\overline{\Phi} - N\pi^D(0)$*. Social welfare is:* $SW = NW(r_\Phi) - Nf$*.*

If $\overline{\Phi} + f < \pi^D(0)$ *and* $2\overline{\Phi} > \pi^D(0) + \pi^D(r_\Phi)$ *then set* $r_1 = \widehat{r}$ *and* $P_1^D = \overline{\Phi} - \pi^D(0)$ *for profits* $\Pi_1^P = 2N\Phi(\widehat{r}) - N\pi^D(0)$*, where* $\widehat{r}$ *is uniquely defined by* $\Phi(\widehat{r}) - \pi^D(0) = \pi^D(\widehat{r}) - \overline{\Phi}$*. Social welfare is:* $SW = NW(\widehat{r}) - Nf$*.*

If $\overline{\Phi} + f \geq \pi^D(0)$ *then set* $r_1 = \widetilde{r}$ *and* $P_1^D = \pi^D(\widetilde{r}) - \overline{\Phi}$ *for profits* $\Pi_1^P = NW(\widetilde{r}) - N\overline{\Phi}$*, where* $\widetilde{r}$ *is uniquely defined by* $\Phi(\widetilde{r}) = \pi^D(0) - f$*. Social welfare is:* $SW = NW(\widetilde{r}) - Nf$*.*

Proof. See appendix. ■

Platform 1 commits to its user price Just like in the previous case, it is easily seen that platform 2 does weakly better by committing to its user price, because its DC strategy on developers is identical to its only available strategy when it does not commit to user prices. Therefore we will directly assume platform 2 commits to it user price P_2^U.

i) DC strategy aimed at developers

$$P_2^D < P_1^D - \pi^D(r_1)$$[23]

Since this price secures all developers for platform 2, it can commit to a user price (almost) equal to (recall (6)):

$$P_2^U = NS(r_2) + \min\left(0, P_1^U\right)$$

These prices enable 2 to capture both sides of the market entirely and obtain profits of $N\Phi(r_2) + \min\left(0, P_1^U\right) + NP_1^D - N\pi^D(r_1)$, which are maximized by setting $r_2 = r_\Phi$:

$$\Pi_2^P = N\overline{\Phi} + \min\left(0, P_1^U\right) + N\left[P_1^D - \pi^D(r_1)\right]$$

[23]Note that this directly implies that platform 2's prices satisfy the developer rationality constraint if it is satisifed by platform 1's prices:

$$P_2^D < P_1^D - \pi^D(r_1) \leq -f \leq \pi^D(r_2) - f$$

The expression above is interpreted as follows: the first term is the maximum revenue that 2 can make on users given its choice of a DC strategy targetted at developers; the second term is the cost of attracting users once full developer support has been secured (this cost is positive only if P_1^U is negative); the third term is the cost of the DC strategy on developers, i.e. the total amount of the subsidy needed to divert developers away from platform 1.

ii) DC strategy aimed at users

Here, in order to convince developers that it will win the user battle even without their support, platform 2 needs to commit to (slightly less than):

$$P_2^U = \min\left(0, P_1^U - NS\left(r_1\right)\right) = P_1^U - NS\left(r_1\right)$$

where the second equality comes from the fact that on the equilibrium path users must derive non-negative utility from joining platform 1.

Given this user price, developers are certain that platform 2 will win all users, therefore the latter can charge:

$$P_2^D = \pi^D\left(r_2\right) + \min\left(P_1^D, -f\right)$$

obtaining total profits:

$$\Pi_2^P = Np\left(r_2\right) d\left(r_2\right) + P_1^U - NS\left(r_1\right) + \min\left(P_1^D, -f\right)$$

Thus, platform 2's best response using a DC strategy aimed at users is to set $r_2 = 0$, obtaining:

$$\Pi_2^P = N\pi^D\left(0\right) + P_1^U - NS\left(r_1\right) + \min\left(P_1^D, -f\right)$$

A quick interpretation of the expression above runs as follows: the first term is the maximum revenues that platform 2 can make on developers given that it uses a DC strategy aimed at users (i.e. subsidizes users); the second term is negative and represents the cost of the DC strategy (i.e. the subsidy needed to divert them away from platform 1); the third term is the cost of the strategy on developers, *once* users are *already* secured, which explains why this cost is different from $-f$ only when P_1^D is very low.

Turning now to platform 1, its prices need to be such that none of these two DC strategies is profitable for platform 2. Relegating the algebra in the appendix, we obtain the following proposition.

Proposition 11 *Given that platform 1 commits to its user price, its optimal pricing strategy is the best of the two following options:*

1) $P_1^U = Nf - N\overline{\Phi}$, $r_1 = \min(0, \overline{r})$ *and* $P_1^D = \pi^D(r_1) - f$, *yielding* $\Pi_1^P = Np(r_1)\,d(r_1) - N\overline{\Phi}$, *where* $\overline{r}$ *is uniquely defined by* $S(\overline{r}) = \pi^D(0) - \overline{\Phi}$. *Social welfare is:* $SW = NW(\min(0, \overline{r})) - Nf$.

2) $P_1^U = NS(r^*) + Nf - N\pi^D(0)$, $r_1 = r^*$ *and* $P_1^D = N\pi^D(r^*) - N\overline{\Phi}$, *yielding* $\Pi_1^P = NW(r^*) - N\overline{\Phi} + Nf - N\pi^D(0)$. *Social welfare is:* $SW = NW(r^*) - Nf$.

Propositions 10 and 11 show that the dominant platform equilibrium is not necessarily sustainable when commitment to user prices is feasible and developers are bound to eclusivity. This is in contrast to proposition 7, which showed that the dominant platform equilibrium was always sustainable with exclusivity when user prices commitment was not feasible. The reason is that commitment opens up the possibility for new and more agressive pricing strategies by the challenger (platform 2), which lower the dominant platform's profits.

Social welfare is always higher when platform 1 commits, but the latter may choose no commitment for strategic reasons -namely, in order to avoid having to commit to low user prices, which *ex-post* turn out to be inefficiently low from the platform's perspective-, resulting in inefficiently high royalty rates. Once again, absent the competitive pressure of the challenger, the dominant platform would always prefer commitment, resulting in higher profits *and* social welfare.

6.2.2 Multihoming

One of the most important changes with respect to the previous case, when developers were bound to exclusivity, is that now commitment to user prices is no longer a weakly dominant strategy for platform 2.

Platform 1 does not commit to its user price If platform 2 does not commit to user prices either, then we are in a scenario we have already analyzed above: platform 2's best response without commitment is given by lemma 8. Specifically, platform 2 sets $r_2 = r_\Phi$ and $P_2^D = -\gamma f$. The resulting best response profits are:

$$\Pi_2^P = \begin{cases} N\overline{\Phi} - N\gamma f - N\Phi(r_1) & \text{if} \quad \begin{array}{l} \left(P_1^D \leq -\gamma f \text{ and } \Phi(r_1) < \overline{\Phi}\right) \text{ or} \\ \left(\Phi(r_1) = \overline{\Phi} \text{ and } P_1^D \leq \lambda_1 \pi^D(r_1) - \gamma f\right) \end{array} \\ \\ N\overline{\Phi} - N\gamma f & \text{if} \quad \begin{array}{l} \left(\Phi(r_1) < \overline{\Phi} \text{ and } P_1^D > -\gamma f\right) \text{ or} \\ \left(\Phi(r_1) = \overline{\Phi} \text{ and } P_1^D > \lambda_1 \pi^D(r_1) - \gamma f\right) \end{array} \end{cases} \tag{18}$$

Suppose now that platform 2 commits to its user price. Then it has the choice between using a DC pricing strategy targetted at developers and one targetted at users.

i) DC strategy targetted at developers

The first step of this strategy for platform 2 is the same as under no commitment. It sets $P_2^D \leq -\gamma f$ in order to ensure that developers prefer multihoming to exclusivity with platform 1. Given such a price, in order to win the battle for users *and* credibly signal this to developers, platform 2 has to charge (see 7):

$$P_2^U < NS(r_2) - N\Phi(r_1) \tag{19}$$

Indeed, if P_2^U did not satisfy this condition then, consistent with platform 1 dominance, developers would coordinate on multihoming, in which case platform 2 would lose the user battle.

There are now two cases to consider, depending on whether $P_1^D \gtrless -\gamma f$.

If $P_1^D > -\gamma f$ then, since it is certain that platform 1 does not gain any users, developers will prefer exclusivity with platform 2 to multihoming. Therefore P_2^D has to satisfy:

$$P_2^D \leq \min\left(-\gamma f, \pi^D(r_2) - f\right) \tag{20}$$

Combining (20) and (19), it follows that platform 2's best response in this case is to set $r_2 = r_\Phi$ obtaining:

$$\Pi_2^P = N\overline{\Phi} - N\gamma f - N\Phi(r_1) \tag{21}$$

If $P_1^D \leq -\gamma f$ then developers prefer multihoming to exclusivity with platform 2, so that the relevant constraint on P_2^D is now:

$$P_2^D < \min\left(-\gamma f, \pi^D(r_2) - P_1^D - (1+\gamma) f\right) = -\gamma f$$

where the equality follows from the observation that on the equilibrium path platform 1's prices must satisfy developer rationality, i.e.:

$$\pi^D(r_1) - P_1^D - f \geq 0$$

The solution is then identical to the one we have found above: $r_2 = r_\Phi$ and $\Pi_2^P = N\overline{\Phi} - N\gamma f - N\Phi(r_1)$.

Comparing this result with (18), it appears that there are cases -depending on the prices charged by 1- in which no commitment is strictly better than commitent for platform 2. In particular this is true only if $P_1^D \geq -\gamma f$ and $\Phi(r_2) > \Phi(r_1)$. The reason is that in some cases, by not committing, platform 2 avoids binding itself ex-ante to an inefficiently low user price from an ex-post perspective. Indeed, suppose platform 2 can manage to "convince" developers that it will win the battle for users even under the most unfavorable circumstances for it (i.e. when developers multihome) and that platform 1's fixed fees ar too high $(P_1^D > -\gamma f)$. In such a scenario commitment would require platform 2 to set a low user price, consistent with expectations of multihoming, whereas if 2 waits until the second stage before announcing its price for users, developers correctly anticipate that platform 2 will attract all users and will consequently give up the expensive (and useless) registration with platform 1, leaving platform 2 in an uncontested position to collect the entire user surplus.

ii) DC strategy targetted at users

Using (7), platform 2 is certain to attract all users despite unfavorable developer expectations if and only if:

$$P_2^U < -N\Phi(r_1)$$

Once again, the maximum fixed fee that 2 can charge developers depends on 1's fixed fee.

If $P_1^D \geq -\gamma f$, then developers prefer exclusivity with 2 to multihoming. Therefore, platform 2's price have to be such that exclusivity with 2 is rational and preferred to exclusivity with 1:

$$P_2^D < \min\left(\pi^D(r_2) - f, \pi^D(r_2) + P_1^D\right) = \pi^D(r_2) - f \qquad (22)$$

In turn, if $P_1^D < -\gamma f$, then developers will prefer multihoming to exclusivity with 2. Platform 2's prices have to be such that multihoming is rational and preferred to exclusivity with 1:

$$P_2^D < \pi^D(r_2) - \gamma f - \max\left(0, P_1^D + f\right) \tag{23}$$

Combining (22) and (23), we obtain the following expression for platform 2's profit:

$$\Pi_2^P = -N\Phi(r_1) + Np(r_2)d(r_2) - N\begin{cases} f & \text{if } P_1^D \geq -\gamma f \\ P_1^D + (1+\gamma)f & \text{if } -f \leq P_1^D \leq -\gamma f \\ \gamma f & \text{if } P_1^D \leq -f \end{cases}$$

Maximizing yields $r_2 = 0$ and:

$$\Pi_2^P = N\pi^D(0) - N\Phi(r_1) - N\begin{cases} f & \text{if } P_1^D \geq -\gamma f \\ P_1^D + (1+\gamma)f & \text{if } -f \leq P_1^D \leq -\gamma f \\ \gamma f & \text{if } P_1^D \leq -f \end{cases} \tag{24}$$

Turning now to platform 1, its prices need to be such that none of these 3 strategies yields non-negative profits for platform 2. Therefore 1 maximizes $NP_1^D + N\Phi(r_1)$ subject to the constraint that (18), (21) and (24) are all negative and developers make non-negative profits.

Proposition 12 *If the dominant platform 1 does not commit to user prices then it can keep platform 2 out of the market if and only if:*

$$\overline{\Phi} > \pi^D(0) - f$$

When this condition holds, letting $\widehat{r} > r_0$ be the unique solution to $\Phi(\widehat{r}) = \pi^D(0) - f$, platform 1's optimal pricing is to set $r_1 = r_\Phi$ and $P_1^D = \min\left(\lambda_1\pi^D(r_\Phi) - \gamma f, \pi^D(r_\Phi) - f\right)$, for profits:

$$\Pi_1^P = N\overline{\Phi} - N\gamma f + N\min\left(\lambda_1\pi^D(r_\Phi), \pi^D(r_\Phi) - (1-\gamma)f\right) \geq 0$$

Proof. See appendix. ■

Platform 1 commits to its user price If platform 2 does not commit, it starts as above by charging $P_2^D \leq -\gamma f$. Using (7), platform 2 wins the user battle if and only if[24]:

$$N\Phi(r_2) > NS(r_1) - P_1^U \tag{25}$$

If $NS(r_1) - P_1^U > N\overline{\Phi}$ then platform 2 will not make any sales to users (too strong competitive position of platform 1 on users) and consequently it can not make non negative profits without committing.

If $NS(r_1) - P_1^U < N\overline{\Phi}$ then platform 2 sets r_2 in order to satsify (25). Again, the expression of 2's profits depends on whether $P_1^D \gtrless -\gamma f$. If $P_1^D > -\gamma f$ then developers prefer exclusivity with platform 2 to multihoming since they know 2 will attract all users. Therefore platform 2 maximizes:

$$\Pi_2^P = NP_2^D + N\Phi(r_2) + \min\left(0, P_1^U\right)$$

subject to:

$$\begin{aligned} P_2^D &< \min\left(-\gamma f, \pi^D(r_2) - f\right) \\ N\Phi(r_1) &\geq NS(r_1) - P_1^U \end{aligned}$$

The solution is to set $r_2 = r_\Phi$ and $P_2^D = -\gamma f$ entailing:

$$\Pi_2^P = N\overline{\Phi} - N\gamma f + \min\left(0, P_1^U\right) \tag{26}$$

If one the other hand $P_1^D \leq -\gamma f$ then developers coordinate on multihoming and platform 2 maximizes:

$$\Pi_2^P = NP_2^D + N\Phi(r_2) - NS(r_1) + P_1^U$$

subject to:

$$\begin{aligned} P_2^D &\leq \min\left(-\gamma f, \pi^D(r_2) - P_1^D - (1+\gamma) f\right) \\ N\Phi(r_2) &\geq NS(r_1) - P_1^U \end{aligned}$$

The solution is once again $r_2 = r_\Phi$ and $P_2^D = -\gamma f$ but profits are lower:

$$\Pi_2^P = N\overline{\Phi} - N\gamma f - NS(r_1) + P_1^U \tag{27}$$

[24]We do not have to worry about the knife edge situation $N\Phi(r_2) = NS(r_1) - P_1^U$ here, since platform 1 will never let it occur. Indeed, it can slightly lower P_1^U for a discrete positive jump in profits.

Suppose now that platform 2 commits to its user price. The derivation of 2's optimal pricing strategies in this case is very similar to some of the proofs above, therefore we relegate the algebra in the appendix and simply state the results.

Lemma 13 *Platform 2's best possible pricing strategies with commitment in response to the dominant platform committing to its user price are:*

i) DC targetted at developers, with $P_2^D = -\gamma f$, $r_2 = r_\Phi$ *and* $P_2^U = NS(r_\Phi) + P_1^U - NS(r_1)$, *yielding* $\Pi_2^P = N\overline{\Phi} - N\gamma f - NS(r_1) + P_1^U$

ii) DC targetted at users, with $P_2^D = \pi^D(0) - \begin{cases} f & \text{if } P_1^D \geq -\gamma f \\ P_1^D + (1+\gamma) f & \text{if } -f \leq P_1^D \leq -\gamma f \\ \gamma f & \text{if } P_1^D \leq -f \end{cases}$,

$r_2 = 0$ *and* $P_2^U = P_1^U - NS(r_1)$, *yielding:*

$$\Pi_2^P = N\pi^D(0) - NS(r_1) + P_1^U - N \begin{cases} f & \text{if } P_1^D \geq -\gamma f \\ P_1^D + (1+\gamma) f & \text{if } -f \leq P_1^D \leq -\gamma f \\ \gamma f & \text{if } P_1^D \leq -f \end{cases}$$

Proof. See appendix. ■

Once again, comparing the characterization in Lemma 13 with platform 2's best no-commitment strategy (26 and 27), it appears that platform 2 does strictly better in some cases by not committing to user prices. The intuitive argument presented above for the case when platform 1 did not commit to its user price applies here as well.

Let us turn now to platform 1 and determine its optimal pricing under the constraint that it keeps platform 2 out of the market by rendering its best response strategies unprofitable.

Proposition 14 *The dominant platform 1 can always keep the rival platform 2 out of the market and make non-negative profits by committing to user prices. Its optimal price strategy which achieves this is to choose the best option among the follwing two:*

1) $P_1^U = NS(r^*) - N\max\left(\overline{\Phi}, \pi^D(0) - f\right)$*,* $r_1 = r^*$ *and* $P_1^D = \pi^D(r^*) - f$*, which yields* $\Pi_1^P = NW(r^*) - N\max\left(\overline{\Phi} + f, \pi^D(0)\right) > 0$

2) $P_1^U = N\gamma f - N\overline{\Phi}$*,* $r_1 = 0$ *and* $P_1^D = \pi^D(0) - f$*, which yields* $\Pi_1^P = N\pi^D(0) - N(1-\gamma)f - N\overline{\Phi} > 0$

Proof. See appendix. ■

Thus, we have proven that when developers can multihome a dominant platform equilibrium is always sustainable: the dominant platform can guarantee non-negative profits by committing to its user price. From proposition 12 we know that it can also achieve non-negative profits without commitment if and only if $\overline{\Phi} > \pi^D(0) - f$. In this case, the choice between commitment and no-commitment depends on the parameters of the model.

7 Conclusion

We have investigated the sustainability of dominant platform equilibria sustained by favorable developer expectations in two-sided markets with developers and users, in which developers arrive before users. We have shown that the dominant platform equilibrium is always sustainable when platforms cannot commit to user prices, i.e. both under developer exclusivity and multihoming. The dominant platform may find it more profitable to be in a multihoming regime; however, the resulting level of social welfare is always higher under exclusivity. Moreover, socially optimal pricing never arises in this case.

These results change when platform commitment to user prices is feasible. Under exclusivity, the dominant platform equilibrium is not always sustainable. Commitment is a weakly dominant strategy for the challenger, whereas the dominant platform may prefer not to commit to its user prices in order to avoid having to set them too low. On the other hand, under a multihoming regime, the dominant platform equilibrium is always sustainable. The dominant platform can keep the challenger out and make non-negative profits by committing to its user prices; however, there are cases when no-commitment is viable and yields higher profits than commitment. Socially optimal pricing may arise in a dominant platform equilibrium, under exclusivity as well as under multihoming.

Regarding the influence of the developer fixed cost structure, we have shown that dominant platform profits may turn out to be *decreasing* in porting costs, so that the common intuition, according to which the latter act as a barrier to entry protecting the incumbent's position, is misguided.

Both from a social welfare and a consumer welfare perspective[25], commitment by the dominant platform is always better than no-commitment. However, given the misalignment introduced by platform competition between the dominant platform's objectives and social welfare maximization, the dominant platform may sometimes find it more profitable not to commit.

One of the main conclusions emerging from our analysis is that, given the complex balancing act that platforms operating in two-sided markets must perform, their (partial) inability to extract surplus from both sides of the market leads to welfare-reducing pricing distorsions. In particular, this happens when platforms need to overcome unfavorable developer expectations and/or when there is competitive pressure from a challenging platform. In this sense, our results can be interpreted to suggest that platform competition is not necessarily desirable for social welfare.

Also, we have shown that price commitment plays a central role in two-sided markets with sequential arrival of the two sides, most notably in determining platforms' pricing structures. Given the ubiquity of user price announcement battles at developer conferences (E3) in the videogame industry[26] and their relative scarcity in the software industry, our model can be considered to provide a partial solution to the "software-videogames puzzle", i.e. the radically different platform pricing structures and business models observed in these markets, which is surprising given their similarities. However, much more empirical work is needed in this direction, in particular systematic studies of the inner workings of the two industries, platform-developer relations, etc...

Finally, we believe that pricing structure and commitment issues in dynamic models (with properly sequential, one-by-one entry of the two sides) are the proper subjects for future research on two-sided markets.

[25] Indeed, absent commitment, user net surplus is always 0, since the platform who manages to attract developers in the first stage has market power over users in the second stage *and* the ability to exploit it.

[26] See our introductory quote by Steve Race.

8 Appendix

Proof. of Lemma 8

Assume first that $\Phi(r_1) < \overline{\Phi}$. There are two cases to consider, according to whether $P_1^D \gtrless -\gamma f$.

If $P_1^D \leq -\gamma f$, then developers prefer multihoming to exclusivity with platform 2, even when they know that 2 wins the user battle. Then r_2 and P_2^D need to be such that multihoming is both rational and preferred to exclusivity with 1:

$$P_2^D \leq \min\left(\pi^D(r_2) - P_1^D - (1+\gamma) f, \pi^D(r_2) - \gamma f\right)$$

Combined with $P_2^D \leq -\gamma f$, we obtain the following constraint:

$$P_2^D < \min\left(-\gamma f, \pi^D(r_2) - \gamma f - \max\left(0, P_1^D + f\right)\right) \tag{28}$$

Platform 2 maximizes its profit $\Pi_2^P = NP_2^D + N[\Phi(r_2) - \Phi(r_1)]$ subject to (28) and (12). Assume that:

$$\pi^D(r_2) - \gamma f - \max\left(0, P_1^D + f\right) \leq -\gamma f \tag{29}$$

Then 2's best response profits are:

$$\Pi_2^P = NW(r_2) - N\left(\gamma f + \max\left(0, P_1^D + f\right) + \Phi(r_1)\right)$$

This expression is decreasing in r_2, therefore platform 2 will choose the lowest level of r_2 feasible, subject to (29) and (12). Since $\pi^D(r_2)$ is also decreasing in r_2 and $\pi^D(r_\Phi) \geq f \geq$max$\left(0, P_1^D + f\right)$, (29) is violated by all $r_2 \leq r_\Phi$. Therefore, since Φ is decreasing to the right of r_Φ, if there is a solution to the problem above then (29) holds with equality.

We have thus shown that the best response by platform 2 necessarily satisfies:

$$\pi^D(r_2) - \gamma f - \max\left(0, P_1^D + f\right) \geq -\gamma f \tag{30}$$

Constraint (28) becomes:

$$P_2^D \leq -\gamma f$$

It follows immediately that the optimal royalty rate for platform 2 is $r_2 = r_\Phi$ (it satisfies both (12) and (30)) and its best response profits are:

$$\Pi_2^P = N\overline{\Phi} - N\gamma f - N\Phi(r_1)$$

Now assume that $P_1^D > -\gamma f$. In this case, developers prefer exclusive development for platform 2 to multihoming when (12) holds. Therefore r_2 and P_2^D need to be such that exclusivity with 2 is both rational and better than exclusivity with 1:

$$P_2^D < \min\left(-\gamma f, \pi^D\left(r_2\right) + \min\left(P_1^D, -f\right)\right) = \min\left(-\gamma f, \pi^D\left(r_2\right) - f\right)$$

A very similar argument to the one above shows that the best response by 2 necessarily involves:

$$\pi^D\left(r_2\right) - f \geq -\gamma f$$

Thus, just like in the first case, the relevant constraint is $P_2^D \leq -\gamma f$ and the optimal royalty rate is $r_2 = r_\Phi$. Only now, 2 obtains higher profits, since 1 is a non-factor in the user battle (it has no developer support):

$$\Pi_2^P = N\overline{\Phi} - N\gamma f$$

Assume now that $\Phi\left(r_1\right) = \overline{\Phi}$. Platform 2 necessarily has to set $r_2 = r_\Phi$ and attract developers *exclusively* in order to make non-negative profits. However, this is possible if and only if:

$$\lambda_1 \pi^D\left(r_\Phi\right) - P_1^D - \gamma f < 0 \Longleftrightarrow P_1^D > \lambda_1 \pi^D\left(r_\Phi\right) - \gamma f \tag{31}$$

If this condition holds, then developers prefer exclusive development for platform 2 to multihoming. The same argument used above applies here and we obtain the same result:

$$\Pi_2^P = N\overline{\Phi} - N\gamma f$$

Otherwise, if (31) does not hold, then platform 1 dominance requires that developers coordinate on multihoming, in which case platform 2 makes total profits of $-N\gamma f$. Indeed, the rest of the surplus is dissipated through competition for users in the second stage, because both platforms have the same number of developers. ∎

Proof. of Proposition 9

From the characterization of platform 2's best response, it follows that platform 1's prices necessarily satisify: $P_1^D \leq -\gamma f$ or $r_1 = r_\Phi$ and $P_1^D \leq \lambda_1 \pi^D\left(r_\Phi\right) - \gamma f$.

With $P_1^D \leq -\gamma f$, platform 1 maximizes $NP_1^D + N\Phi(r_1)$ subject to $P_1^D \leq \min\left(-\gamma f, \pi^D(r_1) - f\right)$ and $\Phi(r_1) \geq 0$. The solution is easily seen to be $P_1^D = -\gamma f$ and $r_1 = r_\Phi$ leading to $\Pi_1^P = N\overline{\Phi} - N\gamma f \geq 0$.

But when $r_1 = r_\Phi$ platform 1 can charge $P_1^D \leq \min\left(\lambda_1 \pi^D(r_\Phi) - \gamma f, \pi^D(r_\Phi) - f\right)$, obtaining:

$$\Pi_1^P = N\overline{\Phi} - N\gamma f + \min\left(\lambda_1 \pi^D(r_\Phi), \pi^D(r_\Phi) - (1-\gamma) f\right) \geq N\overline{\Phi} - N\gamma f$$

■

Proof. of Proposition 10

First note that either (16) or (17) is binding at the optimum (otherwise platform 1 could increase P_1^D). Suppose (16) is binding; then $\Pi_1^P = NW(r_1) - N\overline{\Phi}$ and platform 1 will seek to set r_1 as low as possible while still satisfying $r_1 \geq r_0$ and (17). But for $r_1 = r_0$, $P_1^D = \pi^D(r_0) - \overline{\Phi} > 0$ and $-\pi^D(0) < -f$, therefore (17) is violated. Thus, (17) necessarily binds at the optimum.

If $\overline{\Phi} + f < \pi^D(0)$ then the only way (17) can bind is if $P_1^D = \Phi(r_1) - \pi^D(0)$ implying $\Pi_1^P = 2N\Phi(r_1) - N\pi^D(0)$. In this case platform 1 will want to set r_1 as high as possible or equal to r_Φ on condition that (16) holds. If $2\overline{\Phi} \leq \pi^D(0) + \pi^D(r_\Phi)$ then the solution is $r_1 = r_\Phi$. If $2\overline{\Phi} > \pi^D(0) + \pi^D(r_\Phi)$ then, since $\Phi(0) - \pi^D(0) < 0 \leq \pi^D(0) - \overline{\Phi}$, there exists a unique $\widehat{r} \in [0, r_\Phi]$ such that $\Phi(\widehat{r}) - \pi^D(0) = \pi^D(\widehat{r}) - \overline{\Phi}$ and the solution is $r_1 = \widehat{r}$.

If $\overline{\Phi} + f \geq \pi^D(0)$ then suppose that at the optimum $\Phi(r_1) - \pi^D(0) \geq -f$. Then (16) necessarily binds so that $\Pi_1^P = NW(r_1) - N\overline{\Phi}$ and platform 1 wants to set r_1 as low as possible. Since $\Phi(r_0) - \pi^D(0) = -\pi^D(0) < -f$, optimality requires $\Phi(r_1) - \pi^D(0) \leq -f$. If $\Phi(r_1) - \pi^D(0) < -f$ then $P_1^D = \Phi(r_1) - \pi^D(0)$ and $\Pi_1^P = 2N\Phi(r_1) - N\pi^D(0)$, so that r_1 will be set as high as possible. And, since $\overline{\Phi} - \pi^D(0) \geq -f$, we conclude that the optimal solution in this case is $r_1 = \widetilde{r}$ where $\Phi(\widetilde{r}) = \pi^D(0) - f$ (note that $\widetilde{r} \in [r_0, r_\Phi]$), which implies $P_1^D = \pi^D(\widetilde{r}) - \overline{\Phi}$. ■

Proof. of Proposition 11

Platform 1 solves:

$$\max_{P_1^U, r_1, P_1^D} \left\{P_1^U + Nr_1 d(r_1) + NP_1^D\right\}$$

subject to:

$$P_1^D \leq \pi^D(r_1) - f \tag{32}$$

$$\min\left(0, P_1^U\right) + NP_1^D < N\pi^D(r_1) - N\overline{\Phi} \tag{33}$$

$$P_1^U + N\min\left(-f, P_1^D\right) < NS(r_1) - N\pi^D(0) \tag{34}$$

There are four possibilities to consider:

i) $P_1^U \leq 0$ and $P_1^D \leq -f$ which imply $P_1^U + NP_1^D \leq -Nf$. (33) and (34) become:

$$\begin{aligned} P_1^U + NP_1^D &< N\pi^D(r_1) - N\overline{\Phi} \\ P_1^U + NP_1^D &< NS(r_1) - N\pi^D(0) \end{aligned}$$

Combining the three constraints we obtain:

$$\begin{aligned} \Pi_1^P &< N\min\left\{r_1 d(r_1) - f, p(r_1)d(r_1) - \overline{\Phi}, \Phi(r_1) - \pi^D(0)\right\} \\ &\leq N\min\left\{r_1 d(p(r_1)) - f, p(0)d(0) - \overline{\Phi}, \overline{\Phi} - \pi^D(0)\right\} \\ &\leq 0 \end{aligned}$$

This possibility is thus to be ruled out.

ii) $P_1^U \leq 0$ and $P_1^D \geq -f$

Then (32), (33) and (34) become:

$$P_1^D \leq \pi^D(r_1) - f \tag{35}$$

$$P_1^U + NP_1^D < N\pi^D(r_1) - N\overline{\Phi} \tag{36}$$

$$P_1^U < NS(r_1) + Nf - N\pi^D(0) \tag{37}$$

First, note that either (35) or (36) or both must be binding at the optimum for platform 1, otherwise it could obtain higher profits by slightly increasing P_1^D without violating the other constraints.

Next, we argue that we can without loss of generality assume (35) is binding. Indeed, assume platform 1 can attain maximum profits with $\left(P_1^U, r_1, P_1^D\right)$ and (35) is not binding; then (36) must be binding. Keeping r_1 fixed, let $P_1^{D'} = P_1^D + \frac{\varepsilon}{N}$ and $P_1^{U'} = P_1^U - \varepsilon$, with ε chosen such that $P_1^{D'} = \pi^D(r_1) - f \geq -f$. Then profits are unchanged and $\left(P_1^{U'}, r_1, P_1^{D'}\right)$ clearly satisfy all the constraints if they are satisfied by $\left(P_1^U, r_1, P_1^D\right)$. We have therefore constructed an optimal solution for which (35) is binding.

Then, plugging $P_1^D = \pi^D(r_1) - f$ in (36) and (37), the remaining constraints are equivalent to:

$$P_1^U < N\min\left(0, f-\overline{\Phi}, S(r_1)+f-\pi^D(0)\right) = N\min\left(f-\overline{\Phi}, S(r_1)+f-\pi^D(0)\right)$$

Assume first that at the optimum $S(r_1)+f-\pi^D(0) \leq f-\overline{\Phi}$. Then platform 1 can obtain profits (almost) equal to $NW(r_1) - N\pi^D(0)$, which implies that it will want to set r_1 as low as possible. Since $S(r^*) = W(r^*) > \pi^D(0) - f$, at the optimum we necessarily have:

$$S(r_1)+f-\pi^D(0) \geq f-\overline{\Phi} \tag{38}$$

Thus $P_1^U = Nf - N\overline{\Phi}$ and platform 1's profits are (almost) equal to:

$$Np(r_1)\,d(r_1) - N\overline{\Phi}$$

which it maximizes subject to (38).

Let then $\overline{r}$ be the unique solution to:

$$S(\overline{r})+f-\pi^D(0) = f-\overline{\Phi} \Leftrightarrow S(\overline{r}) = \pi^D(0) - \overline{\Phi}$$

$\overline{r}$ exists since $S(r^*)+f-\pi^D(0) > 0 \geq f-\overline{\Phi} \geq f-\pi^D(0)$.

The solution is then:

$$r_1 = \min(0, \overline{r}) \in [r^*, 0]$$

If $S(0)+\max\left(\overline{\Phi}, f\right) > \pi^D(0)$ then $r_1 = 0$ and:

$$\Pi_1^P = N\pi^D(0) - N\overline{\Phi} \geq 0$$

iii) $P_1^U \geq 0$ and $P_1^D \leq -f$

$$NP_1^D < N\pi^D(r_1) - N\overline{\Phi}$$
$$P_1^U + NP_1^D < NS(r_1) - N\pi^D(0)$$

The optimal solution requires the second constraint to be binding: it suffices to set P_1^D low enough. We obtain: $r_1 = r_\Phi$ and $\Pi_1^P = N\overline{\Phi} - N\pi^D(0) \leq 0$. This option is to be ruled out as well.

iv) $P_1^U \geq 0$ and $P_1^D \geq -f$

$$NP_1^D < N\pi^D(r_1) - N\overline{\Phi}$$
$$P_1^U < NS(r_1) + Nf - N\pi^D(0)$$

Ignore the first two constraints. Then, setting P_1^U and P_1^D such that the last two constraints are binding:

$$\Pi_1^P = NW(r_1) - N\overline{\Phi} + Nf - N\pi^D(0)$$

which is maximized for $r_1 = r^*$. Note that this is feasible since platforms can commit to user prices, thus we do not have to impose $\Phi(r) \geq 0$ like in the previous section. We now need to check the first two constraints are satisfied:

$$\begin{aligned} P_1^U &= NS(r^*) + Nf - N\pi^D(0) = NW(r^*) + Nf - N\pi^D(0) \\ &\geq NW(r^*) - N\pi^D(0) \\ &\geq 0 \\ P_1^D &= N\pi^D(r^*) - N\overline{\Phi} > N\pi^D(0) - N\overline{\Phi} \\ &\geq 0 \end{aligned}$$

Therefore the solution is to set $r_1 = r^*$, yielding:

$$\Pi_1^P = NW(r^*) - N\overline{\Phi} + Nf - N\pi^D(0)$$

Finally, platform 1 chooses between options ii) and iv). ■

Proof. of Proposition 12

Platform 1's prices have to satisfy $P_1^D \leq \min\left(-\gamma f, \pi^D(r_1) - f\right)$ or $\left(r_1 = r_\Phi\right.$ and $\left.P_1^D \leq \min\left(\lambda_1 \pi^D(r_1) - \gamma f, \pi^D(r_1) - f\right)\right)$. Otherwise platform 2 can make non-negative profits with its no-commitment pricing strategy.

Profit maximization and (**24**) imply that platform 1 will set P_1^D as high as possible, i.e. $P_1^D = -\gamma f$ or $P_1^D = \min\left(\lambda_1 \pi^D(r_1) - \gamma f, \pi^D(r_1) - f\right)$. Once again, if $\overline{\Phi} \leq \pi^D(0) - f$ then platform 1 cannot keep 2 out. If in turn $\overline{\Phi} > \pi^D(0) - f$ then platform 1 can either set $P_1^D = -\gamma f$ in which case profit maximization implies $r_1 = r_\Phi$, or $P_1^D = \min\left(\lambda_1 \pi^D(r_1) - \gamma f, \pi^D(r_1) - f\right) >$

$-\gamma f$, in which case it has to set $r_2 = r_\Phi$ to preclude platform 2 from entering. In any event $N\overline{\Phi} - N\gamma f - N\Phi(r_1) < 0$ is satisfied and therefore the solution for this case is to set $P_1^D = \min\left(\lambda_1 \pi^D(r_1) - \gamma f, \pi^D(r_1) - f\right)$ and $r_1 = r_\Phi$, obtaining:

$$\Pi_1^P = N\overline{\Phi} - N\gamma f + N\min\left(\lambda_1 \pi^D(r_\Phi), \pi^D(r_1) - (1-\gamma) f\right) \geq 0$$

■

Proof. of Lemma 13

i) DC strategy targeted at developers

Given the possibility of multihoming, 2 needs to charge $P_2^D < -\gamma f$. This is low enough to ensure multihoming is preferred to exclusivity with platform 1 by all developers. However, in order to gain positive user market share, 2 also needs:

$$\begin{aligned} P_2^U &< \min\left(NS(r_2), P_1^U - NS(r_1) + NS(r_2)\right) \\ &= P_1^U - NS(r_1) + NS(r_2) \end{aligned}$$

Indeed, if P_2^U violates this condition, then there exists an equilibrium, in which all developers multihome, 2 loses the battle for users and consequently makes negative profits.

Now, if $P_1^D \geq -\gamma f$, then developers prefer exclusivity with 2 to multihoming to exclusivity with 1, therefore platform 2 maximizes:

$$\Pi_2^P = P_1^U - NS(r_1) + N\Phi(r_2) + NP_2^D$$

subject to:

$$P_2^D < \min\left(-\gamma f, \pi^D(r_2) - f\right)$$

By the same argument used in the previous proofs, we know that the optimal solution requires:

$$\pi^D(r_2) - f \geq -\gamma f$$

Therefore, platform 2 can obtain maximum profits of:

$$\Pi_2^P = P_1^U - NS(r_1) + N\overline{\Phi} - N\gamma f$$

by setting $r_2 = r_\Phi$ and $P_2^D = -\gamma f$.

If on the other hand $P_1^D < -\gamma f$, then developers prefer multihoming to both exclusivity with 2 and exclusivity with 1. Platform 2 maximizes:

$$\Pi_2^P = P_1^U - NS\left(r_1\right) + N\Phi\left(r_2\right) + NP_2^D$$

subject to:

$$P_2^D < \min\left(-\gamma f, \pi^D\left(r_2\right) - P_1^D - \left(1+\gamma\right) f\right)$$

If $\pi^D\left(r_2\right) - P_1^D - \left(1+\gamma\right) f \leq -\gamma f$, then 2 will charge r_2 as low as this constraint allows, therefore, since any $r_2 \leq r_\Phi$ violates the constraint, it follows that the optimal solution necessarily involves:

$$\pi^D\left(r_2\right) - P_1^D - \left(1+\gamma\right) f \geq -\gamma f$$

Platform 2 will then choose $r_2 = r_\Phi$ and obtain:

$$\Pi_2^P = P_1^U - NS\left(r_1\right) + N\overline{\Phi} - N\gamma f \tag{39}$$

Finally, we need to check whether 2 could do any better by charging:

$$P_2^D < P_1^D - \pi^D\left(r_1\right) \leq -f < \gamma f$$

Such a price makes both multihoming and exclusivity with 2 more attractive than exclusivity with 1. However, we know that the developers' choice between multihoming and exclusivity with 2 depends solely on 1's prices, therefore the analysis will be exactly the same as above, except that now 2 is charging a lower price P_2^D. Clearly, this cannot yield higher profits.

ii) DC strategy targeted at users

Now platform 2 starts by attracting users at all costs:

$$P_2^U < P_1^U - NS\left(r_1\right)$$

Then it can set prices for developers given that the latter *know* all users will adopt platform 2 irrespective of their decision.

If $P_1^D \geq -\gamma f$, then developers prefer exclusivity with 2 to multihoming. Therefore, platform 2's price have to be such that exclusivity with 2 is rational and preferred to exclusivity with 1:

$$P_2^D < \min\left(\pi^D\left(r_2\right) - f, \pi^D\left(r_2\right) + P_1^D\right) = \pi^D\left(r_2\right) - f \tag{40}$$

In turn, if $P_1^D < -\gamma f$, then developers will prefer multihoming to exclusivity with 2. Platform 2's prices have to be such that multihoming is rational and preferred to exclusivity with 1:

$$P_2^D < \pi^D(r_2) - \gamma f - \max\left(0, P_1^D + f\right) \tag{41}$$

Combining (40) and (41), we obtain the following expression for platform 2's profit:

$$\Pi_2^P = P_1^U - NS(r_1) + Np(r_2)\,d(r_2) - N\begin{cases} f & \text{if} \quad P_1^D \geq -\gamma f \\ P_1^D + (1+\gamma)f & \text{if} -f \leq P_1^D \leq -\gamma f \\ \gamma f & \text{if} \quad P_1^D \leq -f \end{cases}$$

Maximizing with respect to r_2 yields the expression given in the text. ■

Proof. of Proposition 14

Assume first that $\overline{\Phi} < \gamma f$. In this case platform 2 can not make non-negative profits without committing or with commitment to user price but using a DC strategy targetted at developers, as long as $NS(r_1) - P_1^U \geq 0$, which holds on the equilibrium path. Then platform 1 needs only worry about the DC strategy on users and the developer rationality constraint:

$$\begin{aligned} P_1^U &< NS(r_1) + Nf - N\pi^D(0) \\ P_1^D &\leq \pi^D(r_1) - f \end{aligned}$$

Platform 1's optimal pricing is then to charge $r_1 = r^*$ obtaining:

$$\Pi_1^P = NW(r^*) - N\pi^D(0) > 0$$

Assume now that $\overline{\Phi} \geq \gamma f$. Then platform 1's prices have to satisfy $P_1^U < NS(r_1) - N\overline{\Phi}$ or $P_1^U < N\gamma f - N\overline{\Phi}$ or $P_1^D \leq -\gamma f$, in order to prevent platform 2 from taking over the market by using its optimal no-commitment strategy.

Suppose platform 1 charges $P_1^D \leq -\gamma f$. Then it maximizes $\Pi_1^P = P_1^U + Nr_1 d(r_1) + NP_1^D$ subject to:

$$\begin{aligned} P_1^U &< NS(r_1) + N\gamma f - N\overline{\Phi} \\ P_1^U &< NS(r_1) - N\pi^D(0) + \begin{cases} P_1^D + (1+\gamma)f & \text{if} -f \leq P_1^D \leq -\gamma f \\ \gamma f & \text{if} \quad P_1^D \leq -f \end{cases} \end{aligned}$$

Clearly it is then optimal to set $P_1^D = -\gamma f$ and $P_1^U < NS(r_1) + N\min\left(\gamma f - \overline{\Phi}, f - \pi^D(0)\right)$ implying:

$$\begin{aligned}\Pi_1^P &< N\Phi(r_1) - N\gamma f + N\min\left(\gamma f - \overline{\Phi}, f - \pi^D(0)\right)\\ &\leq N\overline{\Phi} - N\gamma f + N\min\left(\gamma f - \overline{\Phi}, f - \pi^D(0)\right)\\ &\leq 0\end{aligned}$$

Therefore any acceptable solution for platform 1 must have $P_1^D > -\gamma f$, which requires $P_1^U < NS(r_1) - N\overline{\Phi}$ or $P_1^U < N\gamma f - N\overline{\Phi}$.

The first alternative is to choose the first of these two constraints, therefore maximizing profits subject to:

$$\begin{aligned}P_1^U &< NS(r_1) - N\max\left(\overline{\Phi}, \pi^D(0) - f\right)\\ P_1^D &\leq \pi^D(r_1) - f\end{aligned}$$

We obtain $P_1^U = NW(r^*) - N\max\left(\overline{\Phi}, \pi^D(0) - f\right)$, $r_1 = r^*$ and $P_1^D = \pi^D(r^*) - f$, yielding:

$$\Pi_1^P = NW(r^*) - N\max\left(\overline{\Phi}, \pi^D(0) - f\right) > 0$$

The second alternative is to maximize profits subject to:

$$\begin{aligned}P_1^U &< N\min\left(\gamma f - \overline{\Phi}, S(r_1) - \pi^D(0) + f\right)\\ P_1^D &\leq \pi^D(r_1) - f\end{aligned}$$

Assume that at the optimum $S(r_1) - \pi^D(0) + f < \gamma f - \overline{\Phi}\,(\leq 0)$. Then $\Pi_1^P = NW(r_1) - N\pi^D(0)$, therefore 1 wants to set r_1 as low as possible. But $S(r^*) - \pi^D(0) + f \geq 0$, therefore it must be that $S(r_1) - \pi^D(0) + f \geq \gamma f - \overline{\Phi}$. We then obtain $P_1^U = N\gamma f - N\overline{\Phi}$, $r_1 = 0$ and $P_1^D = N\gamma f - N\overline{\Phi}$, yielding:

$$\Pi_1^P = N\pi^D(0) - N(1-\gamma) f - N\overline{\Phi}$$

■

9 References

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