# Emergence and entry of B2B marketplaces

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### Abstract

In a successive vertical oligopoly, a set of "sellers" produce some input to be transformed into a final product by a set of "buyers". On this two-sided market, a firm's profit increases with the number of firms of the other type and decreases with the number of firms of its own type. We examine the emergence or the entry of a new marketplace sponsored by a profit-maximizing intermediary who targets buyers and sellers in sequential way by setting membership fees (or subsidies). **JEL classification codes**: L11, L13, L23 **Keywords**: two-sided markets, vertical oligopoly, B2B

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## 1 Introduction

Business-to-business electronic marketplaces can be defined as *virtual marketplaces where several buyers meet several sellers in order to conduct transactions.* As emphasized by Wichmann (2003), B2B marketplaces (for short) clearly differ from classical e-commerce sites (where a single seller trades with many buyers), from procurement networks (where a single buyer trades with many sellers), and from simple information directories and industry networks (which do not lead to actual transactions).

It is useful to classify B2B marketplaces according to their industry focus and their ownership structure (see Popović, 2002). In terms of industry focus, B2B marketplaces can be either 'vertical' or 'horizontal': vertical marketplaces are established along traditional industry segments, whereas horizontal marketplaces offer services across multiple industries. In terms of ownership structure, one distinguishes between 'third-party' and 'consortia' B2B marketplaces: third-party marketplaces are neutral communities of many sellers and many buyers, with open criteria for entry, whereas consortia marketplaces are built by a small number of industry leaders that dominate their respective industries.<sup>1</sup> At the date of this writing (July 2004), *eMarket Services*<sup>2</sup> list around 1000 international B2B marketplaces. The vast majority of them have a vertical industry focus and are owned by third-parties (Popović, 2002).

According to Lucking-Reiley and Spulber (2001), "[e]xpectations about productivity gains from B2B e-commerce can be usefully divided into four areas: possible efficiencies from automation of transactions, potential economic advantages of new market intermediaries, consolidation of demand and supply through organized exchanges, and changes in the extent of vertical integration of companies." When it comes to the specific case of B2B marketplaces, the third area appears to be dominant. Business analysts

<sup>&</sup>lt;sup>1</sup>Other terminologies are proposed: Kaplan and Sawhney (2000) and Yoo *et al.* (2003) call 'neutral' the marketplaces owned by independent third parties, and 'biased' the marketplaces owned by either suppliers or buyers. Marketplaces in the latter category are also sometimes called 'Industry Sponsored Exchanges (ISE)' (as in Ordanini et al., 2004).

 $<sup>^{2}</sup>eMarket Services$  is an international independent collaboration of trade promotion organisations (www.emarketservices.com).

report indeed that the main motivation for firms to join a B2B marketplace is to enlarge their portfolio of potential trading partners.<sup>3</sup> In other words, it appears that the "liquidity benefits" (induced by bringing together a large number of buyers and sellers) prevail over the "efficiency benefits" (stemming from the automation and streamlining of transactions).

B2B marketplaces (especially vertical and third-party ones) appear thus as typical examples of "two-sided markets". As intuitively defined by Jullien (2004), "the concept of two-sided markets refer to situations where one or several competing 'platforms' provide services that are used by two types of trading partners to interact and operate an exchange." More precisely, Evans (2003) defines two-sided markets by the combination of three main features: first, the presence of two distinct categories of agents; second, the existence of indirect network effects (i.e., the benefits accruing to an agent of one category increase as the pool of members from the other category enlarges); third, the agents' inability to internalize these indirect benefits efficiently and, thereby, the scope for intermediation.

In this paper, we focus on the two-sided aspects of B2B marketplaces. We abstract away the productivity gains and cost savings that B2B e-commerce might entail (the "efficiency benefits") by assuming that transactions are technically equivalent whatever the marketplace they are conducted on. The only source of differentiation between marketplaces stems thus from their respective numbers of sellers and buyers. In such a setting, we examine the incentives for a third-party intermediary to launch a new vertical B2B marketplace within a specific industry. More precisely, we address the following questions: Absent "efficiency benefits", is there scope for profitable intermediation in B2B e-commerce? If so, how does a third-party intermediary maximize its profits? Which side of the market should be attracted first? Which fee structure should be put in place? Should one or the other side of the market be subsidized? Should the intermediary try to attract all firms on both sides of the market?

Questions of this sort are at the center of a recent literature in economics,

<sup>&</sup>lt;sup>3</sup>See the interview of Ph. Nieuwbourg, CEO of AEPDM (Association européenne des places de marché) in Journal du Net (www.journaldunet.com/itws/it\_nieuwbourg.shtml, last consulted 11/08/04).

which examines two-sided markets (or, more generally, multi-sided markets). Seminal contributions are Rochet and Tirole (2003), Evans (2003), Caillaud and Jullien (2003) and Armstrong (2004). This literature draws on the older literature on network effects (e.g., Katz and Shapiro (1985) for direct network effects, or Chou and Shy (1990) for indirect network effects) and on multi-product pricing of complementary goods (e.g., Bulow *et al.*, 1985). Rochet and Tirole (2004) propose a useful introduction and road map to this flourishing literature.

While the two-sided market literature considers B2B marketplaces as one application among many other examples,<sup>4</sup> a number of recent papers (e.g., Milliou and Petrakis, 2004 and Yoo *et al.*, 2003) adopt an opposite stand: they focus on B2B marketplaces in general, but not exclusively on their two-sided nature.<sup>5</sup>

The model developed in this paper has some interesting features which usefully complement the two strands of literature we just described. Mainly, as shown in Section 2, the successive oligopoly model we use allows us to derive, endogenously, the payoffs of both types of agents, and to give a precise structure to the various externalities that exist between firms. In particular, we observe that the "liquidity" of the marketplace has two contrasting effects on both types of firms: (i) a *positive indirect network effect* (a firm's profit increases as the number of firms of the other type increases), and (ii) a *negative competition effect* (a firm's profit decreases as the number of firms of its own type increases). The coexistence of these two effects on each side of the market contrasts with most of the two-sided market literature where

<sup>&</sup>lt;sup>4</sup>Other examples are credit cards, advertising in media markets, matchmakers, systems of hardware and software, etc. (see Amstrong (2004) for a comprehensive list and Evans (2003) for a typology).

<sup>&</sup>lt;sup>5</sup>Milliou and Petrakis (2004) analyse a firm's incentives to join a *public* B2B marketplace (like the ones we consider here) or to create a *private* B2B marketplace (i.e., in their analysis, a procurement network owned by a single buyer and used exclusively for doing business with its established suppliers). Although this paper does not consider thirdparty intermediation, it has a couple of common points with our analysis: firms' payoffs are derived explicitly from a production model and competition exists among buyers and among sellers. On the other hand, Yoo *et al.* (2003) focus on intermediation (they study the impact of the ownership structure-third-party vs. consortia marketplaces) but take an ad hoc formulation for firms' payoffs and allow competition only among sellers.

the competition effect is often assumed away.<sup>6</sup> Moreover, we are able to examine how the results are affected by a change in the relative magnitude of the two effects (which we can induce by varying the degree of differentiation of the final product sold on a marketplace).

We contrast two scenarios. The first scenario corresponds to the *emer*gence of a marketplace: buyers and sellers are assumed to be previously 'unattached' and an intermediary sets up a marketplace to attract some of them. In contrast, the second scenario corresponds to the entry of a marketplace: it supposes, more realistically, that all buyers and sellers already trade on an existing marketplace. Then, a new intermediary enters and tries to divert some firms towards his new marketplace. The main difference between the two scenarios lies in the firms' outside option when they decide not to join the new marketplace: in the emergence scenario, the fall back is exogenous (it is zero for all firms); in the entry scenario, the fall back is endogenous and non-negative. Everything else equal, when more firms of one type switch to the new marketplace, the remaining firms of that type face fewer competitors on the old marketplace, which therefore becomes more attractive. In that scenario, it is less likely that the intermediary tries to attract all firms of one type, because of the strong negative competition effect.

We assume that the intermediary sets membership fees in a sequential way and is not able to commit to the fee it will charge the second type of firms when it sets its fee for the first type of firms. The issue of which group of firms to target first is thus of primary importance. We show that either the intermediary attracts all firms from the group targeted first, or it attracts more firms in the first group than in the second group. The first target chosen by the intermediary is thus used as a commitment vis-a-vis the firms in the second group that they will find a certain number of partners

<sup>&</sup>lt;sup>6</sup>Some papers consider competition between sellers but we have no knowledge of papers also considering competition between buyers. For instance, Rochet and Tirole (2002) and Schmalensee (2002) provide a formal analysis of the credit card payment industry, where merchants compete with one another but cardholders do not. Similarly, in Nocke, Peitz and Stahl (2004) sellers compete on the market for differentiated products, which are sold to independent consumers.

in the new marketplace. As a result the attractiveness of the marketplace increases for the other type of firms.

From the perspective of the firms targeted second, the new marketplace offers also the opportunity to reduce competition. Regarding sellers for instance, only those that join the new marketplace will be able to serve the buyers that have already joined. In order to make this marketplace more attractive, the intermediary attracts a single seller. That seller makes large profits because he faces a large pool of buyers (the network effect) and because he faces no competitor (the competition effect). The intermediary then chooses fees that capture the whole seller's profit (in the emergence scenario), or a large share of it (in the entry scenario). The same result holds if buyers are targeted second and if they produce homogenous varieties: the intermediary attracts only one of them to put it in position of a monopolist and to extract larger rents. The model we use allows to consider an alternative case in which buyers produce goods that are completely differentiated. It is only in that case, and when buyers are targeted second, that the intermediary attracts all firms from the second group because the competition effect among buyers vanishes. In all other cases, competition induces the intermediary to grant a monopoly to the second side of the market.

In the entry scenario, when the fall back positions of the agents are endogenous, competition has an effect on the number of firms attracted first. Whatever the type of firms attracted first, it is in the interest of the new intermediary to induce less than one third of firms to switch to his marketplace if these firms compete with each other. Attracting more firms would be too costly because firms that stick to the old marketplace would have less incentive to leave it as they face fewer competitors.

In the emergence scenario, the intermediary always makes profits and extracts positive fees from both types of firms. This result holds in the entry scenario, unless the intermediary targets buyers first and buyers produce completely differentiated varieties. In that latter case, the new intermediary does not offer the buyers a way to differentiate themselves from the other buyers. Moreover, since he grants a monopoly to one seller in the second stage, his marketplace is not attractive for the buyers. He must therefore subsidize them. If the number of sellers is large, the old marketplace keeps all of them but one and, therefore, stays very attractive for the buyers. In that case, entry of the new marketplace is not profitable.

We also show that for low competition among buyers, the emergence and entry scenarios are similar regarding the decision about which group to target first: the intermediary always prefers to target sellers first. For strong competition among buyers, the intermediary's best option is to target first the side of the market with the largest pool of firms.

Our results contrast thus sharply with the popular view put forward in the business literature that "liquidity" is essential for B2B marketplaces: "To succeed, [neutral] e-hubs must attract both buyers and sellers quickly, creating liquidity at both ends" (Kaplan and Sawhney, 2000); "the first pillar of e-marketplace success is building liquidity" (Brunn *et al.*, 2002). As we show, this view is misleading because it puts too much emphasis on (vertical, two-sided) network effects while neglecting (horizontal) competition effects.

The rest of the paper is organized as follows. In Section 2 we lay out the model. In Sections 3 and 4, we study in turn the emergence and entry scenarios.<sup>7</sup> We conclude and propose some directions for future research in Section 5.

### 2 The model

We consider the market for an intermediate good, which is produced by a set of upstream firms (which we call "sellers") and sold to a set of downstream firms (which we call "buyers"). The latter firms transform the intermediary product, on a one-for-one basis, into a differentiated final product, which is sold to a representative consumer. Transactions take place on a "marketplace", which we define loosely as a common platform on which a number of sellers and a number of buyers trade with one another.

We analyze the following game. In the first step, both types of firms

<sup>&</sup>lt;sup>7</sup>The calculations underlying some of our results are standard but relatively tedious; they are contained in a technical appendix which can be downloaded at www.core.ucl.ac.be/~pbel/mktplace\_app.pdf.

decide to operate on one (and only one) particular marketplace.<sup>8</sup> In the second step, on each marketplace comprising a positive number of firms of each type, production decisions are made, first by sellers and next, by buyers; then, the representative consumer demands the final product and all payoffs accrue.

### 2.1 Production decisions

We solve the game backwards, starting with buyers' decisions. Consider a marketplace with b buyers, each one (i = 1, ..., b) producing one variety of the final product. The representative consumer visiting this marketplace has a quadratic surplus function:

$$U(q_1, q_2, \dots, q_b) = \sum_{i=1}^b q_i - \frac{1}{2} \left( \sum_{i=1}^b q_i^2 + \gamma \sum_{i=1}^b \sum_{j \neq i} q_i q_j \right) - \sum_{i=1}^b p_i q_i$$

where  $0 \leq \gamma \leq 1$  indicates the strength of product substitutability. Maximizing consumer's surplus yields the linear inverse demand schedule  $p_i = 1 - q_i - \gamma Q_{-i}$  (with  $Q_{-i} = \sum_{j \neq i} q_j$ ) in the region of prices where quantities are positive. Suppose that the unit cost has two components: first, the price w paid for the intermediate product and second, the usage fee  $a_b$  charged by the marketplace owner. The first-order condition for profit maximization yields

$$w = 1 - a_b - 2q_i - \gamma Q_{-i}$$

Summing on all firms and taking advantage of symmetry gives

$$w = 1 - a_b - (1/b) (2 - \gamma + \gamma b) Q.$$

Because of the one-for-one transformation technology, the total quantity of the final product (Q) is equal to the total quantity of the intermediary product (X). The previous expression gives thus the inverse demand function for the sellers. So, seller j (with j = 1, ..., s), whose marginal cost of

<sup>&</sup>lt;sup>8</sup>In the jargon of the two-sided markets literature, we say that agents are not allowed to "multi-home", i.e., to conduct transactions simultaneously on different marketplaces (or platforms). The implications of multi-homing are examined in several recent papers (see, e.g., Gabszewicz and Wauthy, 2004).

production is assumed to be restricted to the usage fee  $a_s$  charged by the marketplace owner, has the following first-order condition for profit maximization:

$$1 - a_b - a_s - (2/b) (2 - \gamma + \gamma b) x_j - (1/b) (2 - \gamma + \gamma b) X_{-j} = 0.$$

At the symmetric Cournot equilibrium, each seller produces a quantity

$$x = \frac{b\left(1 - a_b - a_s\right)}{\left(s + 1\right)\left(2 - \gamma + \gamma b\right)},$$

where it is assumed that  $a_b + a_s \leq 1$ . The total quantity exchanged on the marketplace is thus equal to

$$Q(s,b) = X(s,b) = \frac{bs(1-a_b-a_s)}{(s+1)(2-\gamma+\gamma b)}.$$
(1)

Using the latter expression, one computes the firms' profits at the equilibrium:

for sellers, 
$$\pi_s(s,b) = \frac{b(1-a_b-a_s)^2}{(s+1)^2(2-\gamma+\gamma b)} - A_s,$$
 (2)

for buyers, 
$$\pi_b(s,b) = \frac{s^2 (1-a_b-a_s)^2}{(s+1)^2 (2-\gamma+\gamma b)^2} - A_b,$$
 (3)

where  $A_s$  and  $A_b$  stand for the membership fees levied by the marketplace owner on, respectively, sellers and buyers.

### 2.2 Nature of externalities

In their overview of two-sided platforms, Rochet and Tirole (2004) make a key distinction between usage externalities and membership externalities. On the one hand, gains from trade almost always arise from usage of the platform; usage decisions depend on how much the platform charges for usage ( $a_b$  and  $a_s$  in our model). On the other hand, ex ante membership decisions depend on the interaction-independent fixed fees that platforms charge ( $A_b$  and  $A_s$  in our model); in the presence of indirect network effects, membership decisions generate membership externalities.

Regarding usage externalities, Rochet and Tirole (2004) consider that a market is one-sided if the volume of transactions realized on the platform depends only on the aggregate price level  $(a_b + a_s)$  and is two-sided otherwise (i.e., if it is sensitive to reallocations of the total price between the buyer and the seller). In our setting, we observe that X(s, b) only depends on  $(a_b + a_s)$ , meaning that the marketplace is one-sided in terms of usage externalities. In other words, usage fees are neutral in our setting.

On the contrary, the non-neutrality of membership fees is quite obvious. For instance, if  $A_b$  is increased and  $A_s$  decreased so as to keep the marketplace owner's profit constant, fewer buyers will find the marketplace attractive and the volume of trade will change (even if more sellers are likely to join). The marketplace is thus two-sided when we consider membership externalities *across* the two sides of the market. But the distinguishig feature of our setting is that it also implies membership externalities *within* each side of the market. More precisely, one observes that the "thickness" of the marketplace has two contrasting effects:

• a *positive indirect network effect*: a firm's profit increases as the number of firms of the other type increases

$$\pi_{s}(s, b+1) > \pi_{s}(s, b) \text{ and } \pi_{b}(s+1, b) > \pi_{b}(s, b);$$

• a *negative competition effect*: a firm's profit decreases as the number of firms of its own type increases

$$\pi_s (s+1, b) < \pi_s (s, b) \text{ and } \pi_b (s, b+1) \le \pi_b (s, b).$$

The relative magnitude of these effects depends on the degree of substitutability between the varieties of the final product ( $\gamma$ ). As  $\gamma$  decreases, the intensity of competition between buyers decreases as well. On the extreme, when buyers produce totally differentiated varieties (i.e.,  $\gamma = 0$ ), the competition effect disappears for them:  $\pi_b (s, b+1) = \pi_b (s, b)$ .

We will now use the profit functions  $\pi_s(s, b)$  and  $\pi_b(s, b)$  as the primitives to analyze different scenarios of competition between market places.

### 2.3 Marketplace choices

#### 2.3.1 B2B intermediation

We introduce one additional player in the game, namely an "intermediary" whose objective is to start a new B2B marketplace. We assume for simplicity

that setting up this new marketplace does not involve any (fixed nor variable) cost. We also assume that the intermediary's strategy is restricted to setting (fixed) membership fees (or subsidies) for sellers and for buyers, which are denoted as above by  $A_s$  and  $A_b$  respectively. We exclude (variable) usage fees for the following reasons. From an empirical point of view, it is observed that although usage (or transaction) fees are traditionally the most common sources of revenue, an increasing number of B2B marketplaces tend to reduce them while increasing membership (or subscription) fees. Popović (2002, p. 15) invokes the firms' "reluctance to be charged every time they decide to transact", while Rochet and Tirole (2004, p. 19) argue that the platform might be unable to tax the interaction properly: "Buyers and suppliers may find each other and trade once on a B2B exchange, and then bypass the exchange altogether for future trade". There are also more technical reasons. First, we show in Appendix 6.1 that it might be impossible to solve the model when the intermediary is allowed to combine membership and usage fees.<sup>9</sup> Second, as noted above, abstracting usage fees away does not affect the two-sided nature of our setting, for it comes only from membership externalities.

We also assume that the intermediary sets membership fees in a sequential way and is not able to commit to the fee it will charge one type of firms when it sets its fee for the other type of firms. We justify this assumption as follows. The first justification comes from the observation that in several categories of two-sided markets, most agents of one side of the market arrive before most agents of the other side. For example, Hagiu (2004) points that "in the software and videogame markets, most application developers join platforms (operating systems and game consoles) before most users do." Although no such natural order seems to prevail in the cases we consider, there is no more reason to think that the two sides arrive simultaneously. That is why we give the intermediary the possibility to decide which side of the market should join the marketplace before the other. Second, as we

<sup>&</sup>lt;sup>9</sup>In particular, there are instances where the intermediary's optimal profit does not depend on the number of sellers he attracts to the marketplace. Therefore, in the case where the intermediary targets buyers before sellers, buyers are unable to decide whether to join or not, as they cannot anticipate how many sellers will be present on the marketplace.

argue in Appendix 6.2, the presence of indirect network effects makes impractical the simultaneous setting of the two membership fees (resulting in the simultaneous move of the two types of firms). As usual in this type of situations (see, e.g., Katz and Shapiro, 1985 or Gabszewicz and Wauthy, 2004), multiple equilibria might occur for a given pair of fees and there is no obvious way to select among them in order to solve the intermediary's problem.

### 2.3.2 Timing

Under the above assumptions, we examine two scenarios. The first scenario corresponds to the *emergence* of a marketplace: buyers and sellers are assumed to be previously 'unattached'; trading (and thereby achieving positive profits) is conditional on joining the new marketplace. In contrast, the second scenario corresponds to the *entry* of a marketplace: it supposes, more realistically, that all buyers and sellers already trade on an existing marketplace (which is not owned by any intermediary). The main difference between the two scenarios lies in the firms' outside option when they decide not to join the new marketplace: in the emergence scenario, the fall back is exogenous (it is zero for all firms); in the entry scenario, the fall back is endogenous (it is non-negative and it depends on how many firms switch to the new marketplace).

More precisely, the intermediary takes in both scenarios the following sequence of decisions. First, the intermediary decides whether he enters or not the game. The decision to enter is also a commitment to set a marketplace that will comprise at least one buyer and one seller. Second, the intermediary decides which type of firms to target before the other. Say he decides to target buyers first. The next decision consists in setting the membership fee  $A_b$ . After buyers have decided whether or not to join the new marketplace, the intermediary observes the buyers' decisions and sets the membership fee  $A_s$  for sellers. The sellers, observing the whole sequence of decisions, choose then whether or not to join the new marketplace. The production game described above finally follows. Obviously, if the intermediary decides to target sellers first,  $A_s$  will be set before  $A_b$  and sellers move



Figure 1: Timing of decisions

before buyers do. Figure 1 represents the timing of the game.

### 2.3.3 Overview of the analysis

By examining the emergence and entry scenarios in turn, we address several unanswered questions (practitioners have speculated about these questions but little academic research has considered them so far). A first set of issues concern the intermediary's strategy: we want to advise the intermediary about which side of the market to target first (is it more profitable to attract buyers before sellers or the other way round?), and about the optimal mix of buyers and sellers to attract (on each side of the market, is it more profitable to attract all firms, some firms or only one firm?). The second set of issues have a more normative nature: we want to inform the regulator about the effects of the intermediary's activities on global efficiency (does B2B intermediation create value?).

Clearly, the answers to these questions will depend on the economic parameters. We will show how the relative size of the buyers' and sellers' pools (which we denote respectively by B and S) determines the optimal sequence of moves for the intermediary. More importantly, we will stress the crucial role played by the degree of substitutability among final products  $(\gamma)$ . In this respect, we will, for the sake of tractability, contrast the polar cases of  $\gamma = 1$  and  $\gamma = 0$ . When  $\gamma = 1$ , buyers produce a homogeneous

final product. An example of a vertical B2B marketplace corresponding to this setting is *Tomatoland.com*, which is focused on the intermediate market for tomatoes: the sellers are tomato producers (hundreds of farmers and cooperatives) and the buyers transform tomatoes into ketchup, a rather homogeneous product.<sup>10</sup> At the other extreme, when  $\gamma = 0$ , buyers produce completely differentiated varieties. Leatherfashion and more is an example of a B2B marketplace fitting these characteristics: sellers are leather producers while buyers produce transformed goods as differentiated as footwear products, garments, leathergoods, or upholstery.<sup>11</sup> The degree of product substitutability clearly affects the attractiveness of a new marketplace for the buyers: when  $\gamma = 1$ , buyers can alleviate the fierce competitive pressure by moving to a new marketplace counting a smaller number of buyers (and visited by different consumers); this "differentiation incentive" disappears when  $\gamma = 0$ . We can thus anticipate that when  $\gamma = 1$ , the intermediary will find it easier to attract buyers, and thereby to make profit and to create value for the whole industry.

Before examining the two scenarios, we emphasize a result that is common to both. Which group of firms is targeted first is an important decision for the intermediary because the attractiveness of the marketplace for the second group of firms will depend positively on the number of joining firms from the first group. Thus, the first target chosen by the intermediary is a commitment vis-a-vis the firms in the second group that they will find a certain number of partners in the new marketplace. By contrast, the intermediary cannot commit vis-a-vis the firms in the first group that he will attract many firms from the second group in stages 5 and 6 of the game. The commitment vis-a-vis the second group is costly for the intermediary: he must reduce the fee (or increase the subsidy) to the firms in the first group in order to induce them to join the marketplace. However, the commitment allows the intermediary to increase the revenue he can extract from the second group of firms (by attracting more of them and/or by setting higher

 $<sup>^{10}</sup>$ See www.tomatoland.com. See also "600 membres autour de Tomatoland.com, la place de marché du ketchup" (*Journal du Net*, 17/01/02, www.journaldunet. com/0201/020117tomato.shtml, last consulted 12/08/04).

 $<sup>^{11}</sup>$ See www.leatherfashionandmore.com.

membership fees). Therefore, the intermediary sacrifices part of his potential revenues extracted from firms in the first group in order to boost his revenues from firms in the second group. We will indeed observe that in both scenarios, the intermediary always attract more firms from the first group than from the second group.

### **3** Emergence of a marketplace

In this scenario, the only possibility for sellers and buyers to trade (and thus to make positive profits) is to join the marketplace owned by the intermediary (noted I). As a result, the intermediary is in a position to extract the entire profit buyers and sellers can achieve on the marketplace. Starting a new marketplace is thus always a profitable venture in this scenario. The intermediary's problem consists in finding the optimal number of firms to attract on each side of the market and in choosing whether it is more profitable to attract buyers or sellers first.<sup>12</sup> As we now show, the solution depends both on the intensity of competition between buyers (parametrized by the degree of product substitutability,  $\gamma$ ) and on the relative sizes of the buyers' and sellers' pools (i.e., B versus S).

We first contrast the intermediary's optimal choices according to whether buyers or sellers are targeted first.

**Lemma 1** Under the strategy 'sellers first', the intermediary sets membership fees so as to attract as many sellers as possible and, (i) for low competition among buyers ( $\gamma = 0$ ), as many buyers as possible, and (ii) for strong competition among buyers ( $\gamma = 1$ ), only one buyer. Under the strategy 'buyers first', the intermediary sets membership fees so as to attract as many buyers as possible and only one seller.

**Proof.** We solve the game backwards. Consider first the strategy 'sellers first'. In stage 6 (see Figure 1), buyers decide to enter as long as  $\pi_b(s, b) \ge A_b$ . In stage 5, the intermediary is able to extract the whole profit of buyers

<sup>&</sup>lt;sup>12</sup>As suggested by Rochet and Tirole (2004, p. 39), the intermediary can, in the present case, be seen as a competition authority who cares about the benefits associated to competition.

and he chooses the value of  $A_b$  that maximizes  $bA_b = b\pi_b(s, b)$ . For  $\gamma = 0$ , it is readily checked that the entry of an extra buyer is always profitable for the intermediary  $(b^* = B)$  whereas for  $\gamma = 1$ , the intermediary attracts only one buyer  $(b^* = 1)$ .<sup>13</sup> In stage 4, sellers anticipate that  $b^*$  buyers will enter  $(b^* = B \text{ if } \gamma = 0 \text{ and } b^* = 1 \text{ if } \gamma = 1)$ . They decide thus to enter if  $\pi_s(s, b^*) \ge A_s$ . In stage 3, the intermediary chooses the value of  $A_s$  that maximizes  $sA_s + b^*A_b$  under the constraints that  $A_b = \pi_b(s, b^*)$  and  $A_s =$  $\pi_s(s, b^*)$ . The latter constraint indicates that the intermediary extracts the whole profit of sellers, whereas the former constraint indicates that a change in the number of sellers not only affects the profits of buyers but also the rent that the intermediary can extract from them. The maximization amounts to choose the value of s that maximizes  $s\pi_s(s, b^*) + b^*\pi_b(s, b^*)$ . Some computation show that the entry of an extra seller is always profitable for the intermediary who therefore sets a fee that allows the entry of the Ssellers.<sup>14</sup> This proves statements (i) and (ii).

Now we consider the strategy 'buyers first'. We follow the same methodology as above. Since, in stage 6, sellers decide to enter as long as  $\pi_s(s,b) \geq A_s$ , the intermediary chooses in stage 5 the value of  $A_s$  that maximizes  $sA_s = s\pi_s(s,b)$ . A few lines of computation establish that the intermediary sets a fee that attracts only one seller whatever the value of  $\gamma$ .<sup>15</sup> In stage 4, buyers anticipate that only one seller will enter. They thus decide to enter if  $\pi_b(1,b) \geq A_b$ . Hence, in stage 3, the intermediary chooses the value of  $A_b$  that maximizes  $bA_b + A_s$  under the constraints that  $A_s = \pi_s(1,b)$  and  $A_b = \pi_b(1,b)$ . The maximization amounts to choose the value of b that maximizes  $b\pi_b(1,b) + \pi_s(1,b)$ . It is readily checked that the entry of an extra buyer is always profitable for the intermediary who therefore sets a fee that allows the entry of the B buyers, which proves the second part of the

 $<sup>^{13}</sup>b\pi_b(s,b) - (b-1)\pi_b(s,b-1)$  is proportional to  $(2-\gamma)^2 - \gamma^2 b(b-1)$ . For  $\gamma = 0$ , this expression is always positive whereas for  $\gamma = 1$ , it becomes negative for any  $b \ge 1.6$ .

<sup>&</sup>lt;sup>14</sup>For  $\gamma = 0$  and  $b^* = B$ ,  $s\pi_s(s, b^*) + b^*\pi_b(s, b^*) - [(s-1)\pi_s(s-1, b^*) + b^*\pi_b(s-1, b^*)]$  is positive and equal to  $B(2s+1)/4s^2(s+1)^2$  whereas this expression is positive and equal to  $[2s+1+(2s^2-1)(B-1)]/4s^2(s+1)^2$  for  $\gamma = 1$  and  $b^* = 1$ .

 $<sup>^{15}</sup>s\pi_s(s,b) - (s-1)\pi_s(s-1,b)$  is proportional to  $-s^2 + s + 1$ , which is negative for  $s \ge 1.6$ .

lemma.<sup>16</sup>

As discussed above, the choice of the group of firms to target first is a commitment vis-a-vis the firms that are targeted second. The commitment is costly for the intermediary who must reduce the fees from the first group of firms to attract more firms from this group and boost his revenues from firms in the second group. Indeed, the proposition shows that the intermediary attracts all buyers when buyers are targeted first, and he attracts all sellers when sellers are targeted first.

The number of firms that the intermediary tries to attract from the second group depends on the degree of competition between these firms. The intermediary is able to reduce competition if he attracts few firms on his marketplace. A reduction in competition allows him to extract larger rents from firms that come to his marketplace. Competition among sellers is always strong because they produce the same input whereas competition among buyers is strong only if  $\gamma = 1$ . In both cases, the intermediary reduces competition by attracting only one seller ('buyers first') or only one buyer ('sellers first' and  $\gamma = 1$ ). If buyers are targeted second and  $\gamma = 0$ , the intermediary cannot reduce competition among buyers so that he tries to attract all buyers to extract as much rent as possible.<sup>17</sup>

We analyze now the initial decisions of the intermediary. As for stage 1, we know from the above results that the intermediary extracts firms' entire profits, meaning that it always pays off to launch the new marketplace. The remaining issue is to choose between the 'sellers first' and the 'buyers first' strategies. The next proposition states our main result.

**Proposition 2** For low competition among buyers  $(\gamma = 0)$ , or for strong competition and more sellers than buyers  $(\gamma = 1 \text{ and } S > B)$ , the intermediary chooses the 'sellers first' strategy. For strong competition among buyers and fewer sellers than buyers  $(\gamma = 1 \text{ and } S \leq B)$ , the intermediary chooses

 $<sup>\</sup>frac{1}{16} b \pi_b (1,b) + \pi_s (1,b) - [(b-1) \pi_b (1,b-1) + \pi_s (1,b-1)] = [\gamma^2 (1-\gamma) b^2 + \gamma (3\gamma^2 - 9\gamma + 8) b + (3-2\gamma) (2-\gamma)^2] / 4 (2-\gamma+\gamma b)^2 (2-2\gamma+\gamma b)^2$ which is positive.

<sup>&</sup>lt;sup>17</sup>Here is another way to see these results. In stage 5, the intermediary chooses the number of firms that maximises total profit on a particular side of the market. One easily checks that sellers' total profit,  $s\pi_s(s,b)$  is a decreasing function of s, while buyers' total profit,  $b\pi_b(s, b)$  decreases with b for  $\gamma = 1$ , but increases with b for  $\gamma = 0$ .

### the 'buyers first' strategy.

**Proof.** Collecting the results of Lemma 1, we compute the intermediary's profit under the two strategies for  $\gamma = 0$  or 1.

#### 'Sellers first'

$$\begin{split} \gamma &= 0: s^* = S, \ b^* = B, \quad \Pi_I^{sf}\left(S,B\right) = S\pi_s\left(S,B\right) + B\pi_b\left(S,B\right) = \frac{SB}{4}\frac{(S+2)}{(S+1)^2}, \\ \gamma &= 1: s^* = S, \ b^* = 1, \quad \Pi_I^{sf}\left(S,1\right) = S\pi_s\left(S,1\right) + \pi_b\left(S,1\right) = \frac{S}{4}\frac{(S+2)}{(S+1)^2}, \\ \text{`Buyers first'} \\ \gamma &= 0: s^* = 1, \ b^* = B, \quad \Pi_I^{bf}\left(1,B\right) = \pi_s\left(1,B\right) + B\pi_b\left(1,B\right) = \frac{3B}{16}, \\ \gamma &= 1: s^* = 1, \ b^* = B, \quad \Pi_I^{bf}\left(1,B\right) = \pi_s\left(1,B\right) + B\pi_b\left(1,B\right) = \frac{B}{4}\frac{(B+2)}{(B+1)^2}. \end{split}$$

For  $\gamma = 0$ , we observe that  $\Pi_I^{sf}(S, B) - \Pi_I^{bf}(1, B) = B(S+3)(S-1) / 16(S+1)^2 > 0$ , so that the intermediary chooses the strategy 'sellers first' and attracts as many buyers and sellers as possible. For  $\gamma = 1$ , we observe that  $\Pi_I^{sf}(S, 1) - \Pi_I^{bf}(1, B) = (S + B + 2)(S - B) / 4(S + 1)^2(B + 1)^2$ . The intermediary chooses the strategy 'sellers first' and attracts as many buyers and sellers as possible if there are more sellers than buyers (S > B). Otherwise, he chooses the strategy 'buyers first' and attracts as many buyers as possible and only one seller.

For low competition among buyers ( $\gamma = 0$ ), buyers potentially make large profits. As discussed above, the best way for the intermediary to extract these profits is to target buyers second. If competition among buyers is as strong as among sellers ( $\gamma = 1$ ), then the intermediary targets first the group of firms that potentially makes the smallest profits in order to extract larger rents from the other group. In our model, the side of the market that makes the smallest profits is the one with the largest number of competitors. Thus sellers are the first target if S > B whereas buyers are otherwise.

### 4 Entry of a new marketplace

Suppose now that the game starts with one, unsponsored, marketplace being available. All S sellers and all B buyers interact through this marketplace, achieving profits respectively equal to  $\pi_s(S, B)$  and  $\pi_b(S, B)$ . Then the intermediary I launches a new marketplace. As in the previous section, we examine the determinants of the intermediary's optimal strategy: what makes the intermediary prefer to attract buyers or sellers first? Because firms can earn positive profits when they refrain from joining the new marketplace, we also need to examine whether the creation of a new marketplace is a profitable venture. To induce firms to switch, the intermediary may indeed have to subsidize the side of the market it attracts first, and membership fees levied afterwards on the other side of the market may not be sufficient to recoup the initial investment. We thus examine if the new intermediary subsidizes one side of the market and if he is able to enter despite the competition of the existing marketplace.

In case of successful entry, the industry is composed of two marketplaces. On the one hand, each type of firms on each marketplace faces fewer competitors, but on the other hand, sellers can sell to fewer buyers and buyers can buy from fewer sellers. We examine whether the entry of the new marketplace enhances the profits and the production of the whole industry. We show that the answer to these questions depends on the intensity of competition among buyers, and on the numbers of firms in the game.

The game proceeds exactly as in the emergence scenario. However, because a firm's fall back is now given by the profits it makes by staying with the current marketplace (instead of being equal to zero), we need to examine more closely the switching decision of sellers and buyers (stages 4 and 6 in Figure 1). This is done in next section. We then successively study the strategies 'buyers first' and 'sellers first'. Under each strategy, we first determine the number of buyers and sellers attracted by the new intermediary. Second, we examine whether one side of the market is subsidized by the new intermediary. Third, we check whether the new marketplace enhances the profits and total production of the industry. Finally, we examine which strategy is chosen by the intermediary and we compare the entry and the emergence scenario.

### 4.1 The switching decision

Starting with sellers, given the membership fee  $A_s$  set by the intermediary, it is a Nash equilibrium for 1 < s < S sellers to switch if the following two conditions are met: (i) the net profit of a seller on the new marketplace (with s sellers) is larger than (or equal to) the profit that he would get by going back to the existing marketplace (with S - s + 1 sellers); and (ii) the profit of a seller on the existing marketplace (with S - s sellers) is larger than the profit that he would get by moving to the new marketplace (with s + 1 sellers). Algebraically, this gives the following two conditions:<sup>18</sup>

$$\begin{cases} \pi_s (s, b) - A_s \ge \pi_s (S - s + 1, B - b), \\ \pi_s (S - s, B - b) > \pi_s (s + 1, b) - A_s. \end{cases}$$

Defining

$$\hat{A}_{s}(s,b) \equiv \pi_{s}(s,b) - \pi_{s}(S-s+1,B-b),$$

one can rewrite the latter two conditions as the following interval

$$\hat{A}_s(s+1,b) < A_s \le \hat{A}_s(s,b).$$

$$\tag{4}$$

Clearly, the properties of  $\pi_s(s, b)$  imply that  $\hat{A}_s(s, b)$  decreases with s (and increases with b). The two conditions define thus an open interval. Note that all S suppliers switch if  $A_s \leq \hat{A}_s(S, b)$ .

In a similar way, it is a Nash equilibrium for 1 < b < B buyers to switch if the following two conditions are met:

$$\begin{cases} \pi_b (s, b) - A_b \ge \pi_b (S - s, B - b + 1), \\ \pi_b (S - s, B - b) > \pi_b (s, b + 1) - A_b. \end{cases}$$

Defining

$$\hat{A}_{b}(s,b) \equiv \pi_{b}(s,b) - \pi_{b}(S-s,B-b+1),$$

we can rewrite the latter two conditions as the following interval

$$\hat{A}_b\left(s, b+1\right) < A_b \le \hat{A}_b\left(s, b\right). \tag{5}$$

From the expression of  $\hat{A}_s(s, b)$  and  $\hat{A}_b(s, b)$ , it is clear that the intermediary is no longer in a position to extract the entire profits firms would make on the new marketplace if they join. Firms will join only if the fee does not exceed the difference between profits on the new marketplace and on the existing one. Possibly, for given b and s, this fee might be negative.

<sup>&</sup>lt;sup>18</sup>Without loss of generality, we assume that when firms are indifferent between the two marketplaces, they choose to trade on the new one.

### 4.2 Strategy 'buyers first'

In this section, we first show that, for every positive number of buyers he has attracted beforehand, the intermediary always grant a monopoly to a single seller (stage 6). Then, the optimal number of buyers to attract in stage 4 will depend on the balance between two conflicting effects: first, a larger number of buyers increases the seller's incentive to switch and thereby, the rent the intermediary can extract from that seller; second, as more buyers switch, competition increases among them, which implies that a higher subsidy has to be paid to (or a lower rent can be extracted from) buyers. Obviously the latter effect disappears when buyers produce varieties that are completely differentiated ( $\gamma = 0$ ).

Our findings are summarized in the following lemma.

**Lemma 3** Consider the strategy 'buyers first'. (i) The intermediary sets membership fees so as to attract a single seller. (ii) For low competition among buyers ( $\gamma = 0$ ) and for a small pool of sellers ( $S \leq 8$ ) he sets membership fees so as to attract as many buyers as possible and he makes positive profits. (iii) For a larger pool of sellers ( $S \geq 9$ ), he makes losses and he attracts the smallest number of buyers (b = 1). (iv) For strong competition among buyers ( $\gamma = 1$ ), he sets membership fees so as to attract an intermediate number of buyers ( $b^*$  with  $1 \leq b^* < (B+4)/3$ ). This number is non decreasing in the size of the pool of buyers (B) and non increasing in the size of the pool of sellers (S).

**Proof.** We solve the game backwards. Consider stage 6. Suppose that b buyers have switched to the new marketplace (with  $1 \le b \le B$ ) and that the intermediary sets the sellers' membership fee to  $A_s$ . It is a Nash equilibrium for s sellers to switch if the two conditions in (4) are fulfilled (with 1 < s < S).

In stage 5, the intermediary knows that by setting  $A_s = \hat{A}_s(s,b)$ , he induces s sellers to switch given that b buyers have done so before. His problem is thus to choose, for a given b, the value of s that maximizes  $s\hat{A}_s(s,b)$ . The particular form of  $\pi_s(s,b)$  allows us to be more specific about the optimal number of sellers. Indeed, we can check that  $\hat{A}_s(1,b)$  –  $s\hat{A}_{s}(s,b)$  is strictly positive for all s > 1.<sup>19</sup> It follows that for all values of b > 1, the intermediary attracts a single seller,  $s^* = 1$ . This proves statement (i) of the lemma.

In stage 4, buyers base their decision to switch on the membership fee,  $A_b$ , set by the intermediary and on the equilibrium choices made at stages 5 and 6. They thus anticipate correctly that  $s^* = 1$ . It is a Nash equilibrium for b buyers to switch if the two conditions in (5) are fulfilled.

In stage 3, the intermediary maximizes  $\Pi_I(1, b) = A_s(1, b) + bA_b(1, b)$ . To prove statements (ii) and (iii), consider low competition among buyers  $(\gamma = 0)$ . Then, it is readily checked that  $\Pi_I(1, b+1) - \Pi_I(1, b)$  is positive for  $S \leq 8$  and negative otherwise.<sup>20</sup> Thus,  $b^* = B$  if  $S \leq 8$  and  $b^* = 1$  if  $S \geq 9$ . For low competition among buyers and for a small pool of sellers, the intermediary's best strategy is to attract all buyers. For larger pools of sellers, the intermediary prefers to attract a single buyer and a single supplier. However, anticipating somewhat on the solution of the whole game, it is possible to show that the intermediary will never choose the strategy 'buyers first' if  $S \ge 9$  because his total profits would be negative:  $\hat{A}_b(1,1) +$  $\hat{A}_s(1,1) < 0$  for  $S \ge 9$ . This proves statements (ii) and (iii) of the lemma.

To prove statement (iv), consider strong competition among buyers ( $\gamma =$ 1). The full proof of the statement is tedious and left for the technical appendix. The sketch of the proof goes as follows. We first show that when the intermediary selects only one seller, his profit  $\Pi_I(1, b)$  is concave in the number of buyers. Hence, there is only one value of the number of buyers that maximizes the intermediary's profit. This value is approximated by the value of b such that  $d\Pi_I(1,b)/db = 0$  (it is the integer that is the closest to this value). To prove that this number is non decreasing in the size of the pool of buyers (B) and non increasing in the size of the pool of sellers (S), we compute  $d^2\Pi_I(1,b)/dbdB > 0$  and  $d^2\Pi_I(1,b)/dbdS < 0$ . Finally we show that  $\Pi_I(1, (B+4)/3) < \Pi_I(1, (B+1)/3)$ . Combined with the concavity of  $\Pi_I(1,b)$ , this implies that  $b^* < (B+4)/3$ .

Some similarities exist with the emergence scenario. First, the intermedi-

<sup>&</sup>lt;sup>19</sup>The exact value of  $\hat{A}_{s}(1,b) - s\hat{A}_{s}(s,b)$  is  $[b(s-1)^{2} / 4(2+\gamma b-\gamma)(s+1)^{2}] +$  $\{ (B-b) (s-1) \left[ (S+2)^2 - s \right] / (S+1)^2 (S-s+2)^2 \left[ 2 + \gamma (B-b) - \gamma \right] \}.$ <sup>20</sup>  $\Pi_I (1, b+1) - \Pi_I (1, b)$  is proportional to  $-S^4 + 6S^3 + 19S^2 - 4.$ 

ary collects most of his rent from firms targeted second, that is from sellers. Because competition is strong among sellers who sell a homogenous product, the intermediary sets an membership fee for the sellers that is sufficiently large so that only a single seller agrees to join.

Second, when he makes positive profits, the intermediary sets a membership fee that allows the entry of a possibly large number of buyers, so as to be able to extract larger rents from the seller in the last stages of the game. However, it is now more difficult than in the emergence scenario to attract buyers because they enjoy profits on the existing marketplace. Moreover, when there is competition among buyers ( $\gamma = 1$ ), the difficulty to attract buyers increases with the number of buyers who move to the new marketplace because their profit on the existing marketplace negatively depends on the number of buyers who stay on that marketplace. It is therefore not surprising to see that the optimal number of buyers attracted by the intermediary is smaller than in the emergence case when  $\gamma = 1$ .

In the absence of competition among buyers ( $\gamma = 0$ ), buyers' fall back is now positive but remains independent of the number of buyers on each marketplace. In other words, it is more costly for the intermediary to attract them, but this cost remains constant. As a result, the intermediary attracts either all of them ( $S \leq 8$ ) or makes losses ( $S \geq 9$ ). When the pool of sellers is relatively large ( $S \geq 9$ ), the launch of the new marketplace is not a profitable venture because buyers anticipate correctly that a large pool of sellers will remain on the existing marketplace, which makes the option of staying on that marketplace more profitable for them. It is then too expensive for the intermediary to attract buyers.

To attract firms on his marketplace, the new intermediary may have to subsidize one side of the market. The next lemma examines this possibility.

**Lemma 4** Consider the strategy 'buyers first'. (i) For low competition among buyers ( $\gamma = 0$ ), buyers are subsidized by the new intermediary. Sellers pay positive fees to the new intermediary unless  $S \ge 9$  and B > $(S^2 + 2S + 5)/4 \ge 26$  (but then the new marketplace is not profitable). (ii) For strong competition among buyers ( $\gamma = 1$ ), buyers pay positive fees to the new intermediary unless B = 2 and  $S \ge 5$ . Sellers pay positive fees to the new intermediary.

**Proof.** For  $\gamma = 0$  and  $S \leq 8$ , the intermediary attracts all buyers who receive a subsidy equal to  $\hat{A}_b(1, B) = -(3S - 2)(S - 2)/16S^2$  whereas the seller who switches afterwards pays a fee equal to his profits:  $\hat{A}_s(1, B) = \pi_s(1, B) = B/8$ .

For  $\gamma = 0$  and  $S \ge 9$ , the intermediary attracts a single buyer who receives a subsidy  $\hat{A}_b(1,1) = -(3S-2)(S-2)/16S^2$  and he attracts a single seller who pays a fee (or receive a subsidy) equal to  $\hat{A}_s(1,1) = (S^2 + 2S + 5 - 4B)/8(S+1)^2$ . Note that with  $S \ge 9$ , the intermediary makes losses.

For  $\gamma = 1$ , the proof is tedious and left for the technical appendix.

Hence, a profitable marketplace always extracts fees from sellers but may have to pay subsidies to attract a sufficiently large pool of buyers. It is easier to attract buyers when competition is fierce among them. Indeed in case of strong competition, buyers are attracted towards the new intermediary because the new marketplace insulates them from the competition of buyers who stay on the old marketplace. By contrasts, when  $\gamma = 0$ , buyers do not compete with each other and, unless they are subsidized, they prefer to stay where their input is the cheapest, that is, at the proximity of the largest pool of sellers.

From a policy point of view, we want to investigate the extent to which the activity of the intermediary promotes or undermines the production and the profitability of the industry. The answer depends on the degree of competition between buyers and is recorded in the next lemma.

**Lemma 5** Consider the strategy 'buyers first'. For low competition among buyers ( $\gamma = 0$ ) the intermediary reduces the production and the profits of the industry by launching his new marketplace. For strong competition among buyers ( $\gamma = 1$ ), he promotes the production and profits of the industry.

**Proof.** The proof is given in the technical appendix where the production and profits of the industry in each configuration are computed and compared.

When buyers produce homogenous products ( $\gamma = 1$ ), the new marketplace gives them the opportunity to differentiate themselves from their competitors who remain on the existing marketplace. The activity of the intermediary is then beneficial to the industry. By contrast, when buyers produce differentiated products, ( $\gamma = 0$ ), the intermediary does not offer any value added to the industry. Even worse, he attracts all buyers, preventing the existing marketplace to survive, and he attracts only one seller. Allowing more sellers in the new marketplace would increase the buyers' profit but the intermediary would be unable to capture it in stage 3 of the game without inducing some buyers to stay with the existing marketplace. As a result, the intermediary promotes his new marketplace at the expenses of the existing marketplace and of the industry.

We now consider the other strategy where the intermediary determines first the membership fee for sellers and next, the membership fee for buyers.

### 4.3 Strategy 'sellers first'

In this section we show that for low competition among buyers, the intermediary sets membership fees so as to attract as many sellers and buyers as possible whereas for large competition, he tries to attract an intermediate number of sellers and only one buyer. Similarly to the emergence scenario, the first target chosen by the intermediary is a commitment vis-a-vis the firms in the second group that they will find a certain number of partners in the new marketplace. Therefore, the intermediary tries to attract a relatively large number of sellers. In contrast to the emergence scenario, the sellers' fall back is endogenous, positive and increases with the number of sellers who move to the new marketplace. It is thus increasingly difficult to attract sellers to the new marketplace. The number of sellers to attract depends not only on the costs of attracting them, but also on the benefits. As more sellers choose the new marketplace, the intermediary can extract more rents from the buyers in the next stages. The rent extracted from the buyers is lower when they produce homogenous goods ( $\gamma = 1$ ) than when they produce differentiated goods ( $\gamma = 0$ ). It is thus more fruitful to attract all sellers in the latter case, whereas the intermediary attracts fewer sellers in the former case.

The results are summarized in the following lemma.

**Lemma 6** Consider the strategy 'sellers first'. For low competition among buyers ( $\gamma = 0$ ), the intermediary sets membership fees so as to attract as many buyers and sellers as possible. For strong competition among buyers ( $\gamma = 1$ ), the intermediary sets membership fees so as to attract a single buyer and an intermediate number of sellers ( $s^*$  with  $1 \le s^* < (S+3)/3$ ). This number is non decreasing in the size of the pool of sellers (S) and non increasing in the size of the pool of buyers (B).

**Proof.** The proof is relegated to Appendix 8; it follows the same steps as the proof of Lemma 3.

As in the 'buyers first' strategy, we examine to what extend the new intermediary must subsidise one side of the market to attract firms. We also derive the effects of the entry of the new intermediary on the industry' production and profits.

**Lemma 7** Consider the strategy 'sellers first'. Whatever the strength of competition buyers and sellers pay positive fees to the new intermediary.

**Proof.** For  $\gamma = 0$ , the proof is trivial: there is no buyer and no seller on the other marketplace so that the sellers and buyers on the new marketplace do not have any positive fall back; the new intermediary can extract their entire profits. For  $\gamma = 1$ , the proof is more tedious and left for the technical appendix.

**Lemma 8** Consider the strategy 'sellers first'. For low competition among buyers ( $\gamma = 0$ ) the intermediary does not affect the production nor the profits of the industry by launching his new marketplace. For strong competition among buyers ( $\gamma = 1$ ), he promotes the production and profits of the industry.

**Proof.** The proof is given in the technical appendix where the profits of the industry in each configuration are computed and compared.  $\square$ 

As in the 'buyers first' strategy, the activity of the intermediary is beneficial to the industry when buyers produce homogenous products on the existing marketplace ( $\gamma = 1$ ) because the new marketplace offers them an opportunity to differentiate themselves from their competitors who remain on the existing marketplace. When buyers produce differentiated products, ( $\gamma = 0$ ), the intermediary does not offer any value added to the industry, but under this strategy, he does not reduce the profits of the industry because he does not push any firm out of the market.

### 4.4 Buyers or sellers first?

In stage 2 of the game, the intermediary chooses the strategy that gives him the largest profits. As stated in the proposition 9, the optimal strategy depends on the degree of competition among buyers.

**Proposition 9** For low competition among buyers  $(\gamma = 0)$ , or for strong competition and more sellers than buyers  $(\gamma = 1 \text{ and } S > B)$ , the intermediary chooses the 'sellers first' strategy. For strong competition among buyers and fewer sellers than buyers  $(\gamma = 1 \text{ and } S \leq B)$ , the intermediary chooses the 'buyers first' strategy.

**Proof.** (Sketch) When there is no competition among buyers ( $\gamma = 0$ ) and the pool of sellers is large ( $S \ge 9$ ), the 'sellers first' strategy clearly dominates as it allows the intermediary to make positive profits whereas the 'buyers first' strategy entails losses. When the pool of sellers is smaller ( $S \le 8$ ), the optimal strategy is guided by the profits

$$\Pi_{I}^{bf}(1,B) = B\left(-S^{2}+8S-4\right)/(16S^{2}), \qquad (6)$$

$$\Pi_{I}^{sf}(S,B) = BS(S+2) / \left(4(S+1)^{2}\right).$$
(7)

It is readily checked that  $\Pi_{I}^{sf}(S, B) - \Pi_{I}^{bf}(1, B) > 0.^{21}$  Therefore, for  $\gamma = 0$ , the intermediary always prefer the strategy 'sellers first' and he attracts all buyers and sellers.

When competition is fiercer ( $\gamma = 1$ ), the intermediary attracts an intermediate number of firms from the group targeted first, and a single firm from the group targeted second. It is not possible to compute the exact

<sup>&</sup>lt;sup>21</sup>More precisely, this expression is equal to  $B(S-1)[S^2(S+3) + 4(S-1)(S+1)^2] / 16(S+1)^2 S^2$ .

value of the number of firms that the intermediate attracts but it is possible to compare the profits under the two strategies. The comparison is done in the technical appendix. If B < S, the intermediary chooses the 'sellers first' strategy whereas if  $B \ge S$ , the intermediary chooses the 'buyers first' strategy.

The choice of the optimal strategy follows the same rules as in the emergence scenario. The intermediary targets first the side of the market that is the most competitive and then he extracts a larger profit from the other side of the market. Two comments are in order. First, we observe that the intermediary always find a strategy to launch the new marketplace in a profitable way. In other words, the ability to choose which group to target first allows the intermediary to overcome firms' reluctance to leave the existing marketplace where they make positive profits. Second, it can also be observed that the activity of the intermediary is never detrimental to industry production and profits. Better: when competition is fierce between buyers, the entry of the new marketplace allows buyers to split into two markets, which decreases competition and increases total production and profits.

### 4.5 Summary: Emergence vs entry

When comparing the two scenarios, it appears that the degree of competition between buyers plays a crucial role. When buyers produce completely differentiated varieties ( $\gamma = 0$ ), the two scenarios are very much alike when sellers are targeted first: the intermediary attracts all sellers followed by all buyers and extracts the entire profit of the industry. However, the two scenarios differ markedly when buyers are targeted first and the pool of sellers is sufficiently large ( $S \ge 9$ ): intermediation is not profitable in the entry scenario and the intermediary would thus stay out if he was constrained to target buyers before sellers. When the pool of sellers is not too large ( $S \le 8$ ), the intermediary attracts all buyers followed by a single seller and makes positive profits in both scenarios, but he has to pay a subsidy to buyers in the entry scenario. Regarding the decision about which group to target first, the two scenarios are similar: the intermediary always prefers to target sellers first. When competition among buyers is fierce ( $\gamma = 1$ ), the similarities between the two scenarios are the following: (i) the side of the market which is targeted second is organized as a monopoly; (ii) the intermediary's best option is to target first the side of the market with the largest pool of firms (or buyers in case of a tie). The main difference concerns the side of the market which is targeted first: the intermediary induces full participation in the emergence scenario but only partial participation in the entry scenario because of the cost of commitment. The results are summarized in Table 1.

|         | $\gamma = 0$  | $\gamma = 1$                           |
|---------|---|--|
|         | Emergence   | Emergence                              |
|         | $b^* = B, s^* = 1$                                    | $b^* = B, s^* = 1$                     |
| Durrang | Entry   | Entry                                  |
| first   | $b^* = \int B (S \le 8) e^* = 1$                      | $1 \le b^* < \frac{B+4}{3}, s^* = 1$   |
|         | $0 = \begin{cases} 1 & (S > 9) \end{cases}$ , $S = 1$ | all firms pay fees                     |
|         | buyers are subsidized                                 | (unless $B = 2, S \ge 5$ )             |
|         | industry profits $\downarrow$                         | industry profits $\uparrow$            |
|         | Emergence   | Emergence                              |
|         | $s^* = S, b^* = B$                                    | $s^*=S, b^*=1$                         |
| Sellers | Entry   | Entry                                  |
| first   | $s^* = S, b^* = B$                                    | $1 \leq s^* \leq \frac{S}{3}, b^* = 1$ |
|         | all firms pay fees                                    | all firms pay fees                     |
|         | industry profits $\longleftrightarrow$                | industry profits $\uparrow$            |
| Which   | <i>Emergence</i> and <i>entry</i>                     | <i>Emergence</i> and <i>entry</i>      |
| group   |   | Sellers if $S > B$                     |
| first?  | Sellers   | Buyers if $B \ge S$                    |

Table 1: Comparison of the emergence and entry scenarios

## 5 Conclusion

In this paper, we examine the incentives for a third-party intermediary to launch a new vertical B2B marketplace within a specific industry. We focus on the two-sided nature of B2B intermediation: the marketplace benefits accruing to sellers (resp. buyers) increase as the pool of buyers (resp. sellers) enlarges. Alongside these positive indirect network effects, our framework also exhibits negative direct competition effects: other things being equal, sellers and buyers are better off the fewer firms of their own type are present on their marketplace. In this complex web of externalities, we investigate the following issues: the scope for profitable intermediation, the optimal strategy for the intermediary (which side of the market to attract first? which fee structure to put in place?), and the effect of intermediation on firms' profits.

We contrast two scenarios: the new marketplace either 'emerges' (buyers and sellers are previously 'unattached' and make zero profits) or 'enters' (buyers and sellers interact on an existing marketplace where they achieve positive profits). In both scenarios, it appears that the degree of competition between buyers shapes the results. If buyers produce completely differentiated varieties, the two scenarios are very much alike when sellers are targeted first: the intermediary attracts all sellers followed by all buyers and extracts the entire profit of the industry. However, the two scenarios differ markedly when buyers are targeted first and the pool of sellers is sufficiently large: intermediation is not profitable in the entry scenario and the intermediary would thus stay out if he was constrained to target buyers before sellers. When the pool of sellers is not too large, the intermediary attracts all buyers followed by a single seller and makes positive profits in both scenarios, but he has to pay a subsidy to buyers in the entry scenario. Regarding the decision about which group to target first, the two scenarios are similar: the intermediary always prefers to target sellers first. On the other hand, when competition among buyers is fierce, the two scenarios do not differ too much: the intermediary is always advised to target first the side of the market with the largest pool of firms (by attracting all firms in the emergence scenario, but less than a third of them in the entry scenario), and to attract next only a single firm from the other side of the market. We also show that at the equilibrium of the game (and whatever the degree of competition among buyers), the activity of the intermediary (weakly) enhances industry profits.

Our analysis focuses on third-party (or 'neutral') marketplaces. There exists, however, another category of marketplaces, namely consortia (or 'bi-

ased') marketplaces which takes an increasing importance on the B2B scene (see Ordanini *et al.*, 2004). These marketplaces (like *Covisint* in the automotive industry) come directly from the decisions of leaders on one side of the market. In future research, we shall endeavour to analyse the formation of such consortia marketplaces within the framework developed in this paper. We should be able to address unanswered questions such as: What is the equilibrium size of the consortium running the marketplace? Do sellers or buyers have a higher incentive to launch such a marketplace? How do consortium members trade off their own profits with the fees (or subsidies) they collect from (or pay to) firms on the other side of the market? What are the antitrust implications of consortia marketplaces?

Two other areas of further research are likely to require some modifications of the present setting. First, the model proves ill-suited to analyze head-to-head competition between two new intermediaries. Intuitively, as soon as each intermediary finds it optimal to attract more than half of the available sellers and/or buyers, there always exists a profitable way to undercut the rival and no equilibrium in pure strategies exists in this Bertrand competition (based on membership fees). Indeed, when a firm switches marketplaces, it modifies the profit to be made on each marketplace and thereby, creates an endogenous source of horizontal differentiation between marketplaces. Because horizontal differentiation softens price competition, intermediaries manage to avoid the Bertrand Paradox but, as buyers and sellers face switching costs, intermediaries always have some leeway to profitably lure away a firm from the rival marketplace. Some form of heterogeneity among firms on each side of the market (for instance in terms of costs of adopting e-commerce) should solve this problem.

Second, we would also need to extend our model to consider multihoming. It is indeed common for firms to conduct transactions on several marketplaces. However, it is not obvious how to introduce this possibility in our successive oligopoly model.

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# 6 Appendix

### 6.1 Usage fees

In this section, we show that it might be impossible to solve the game by backward induction when the intermediary can combine usage and membership fees. Consider the 'buyers first' game in the emergence scenario. At stage 5, the intermediary chooses the fees  $a_s$  and  $A_s$  so as to maximise profits. The maximization problem writes as follows:

$$\max_{a_s, A_s} \prod_{I} = sA_s + a_s X(s, b) \text{ s.t. } \pi_s(s, b) - A_s \ge 0.$$

The first term in the profit function is the total membership fee collected on sellers, while the second term is the total usage fee (computed as a proportion of the quantity exchanged).

Writing the Lagrangean

$$L = sA_s + a_sX(s,b) + \lambda \left(\pi_s(s,b) - A_s\right)$$

and taking the first-order derivatives, we have

$$\begin{aligned} \frac{\partial L}{\partial A_s} &= 0 \iff s = \lambda, \\ \frac{\partial L}{\partial a_s} &= 0 \iff X(s,b) + a_s \frac{\partial X}{\partial a_s} + \lambda \frac{\partial \pi_s}{\partial a_s} = 0, \\ \frac{\partial L}{\partial \lambda} &= 0 \iff A_s = \pi_s(s,b). \end{aligned}$$

Developping the second equations and using  $s = \lambda$  gives the optimal usage fee:

$$a_s^* = \frac{s-1}{2s} \left(1 - a_b\right) < \left(1 - a_b\right).$$
(8)

Plugging this value into  $\pi_s(s, b)$  and X(s, b) gives the optimal membership fee

$$A_{s}^{*} = \frac{b(1-a_{b})^{2}}{4(2-\gamma+\gamma b) s^{2}}$$
(9)

and the volume exchanged at the optimum

$$X^*(s,b) = \frac{b(1-a_b)}{2(2-\gamma+\gamma b)}.$$

We can now compute the intermediary's profit at the optimum:

$$\Pi_{I}^{*} = s \frac{b\left(1-a_{b}\right)^{2}}{4\left(2-\gamma+\gamma b\right)s^{2}} + \frac{s-1}{2s}\left(1-a_{b}\right) \frac{b\left(1-a_{b}\right)}{2\left(2-\gamma+\gamma b\right)} = \frac{b\left(1-a_{b}\right)^{2}}{4\left(2-\gamma+\gamma b\right)},$$
(10)

which does not depend on s. Hence, the intermediary can secure the level of profit given by (10) by attracting any number of sellers and setting usage and membership fees according to (8) and (9). For example, if the intermediary prefers to attract a single seller, he can do so by setting  $a_s = 0$  and  $A_s = \pi_s(1, b)$ , which is the solution described in Lemma 1. To attract more sellers, the intermediary will decrease the membership fee and compensate by increasing the usage fee. The bottomline is that the optimal number of sellers at stage 5 is indeterminate. Therefore, buyers are unable to make a decision at stage 4 and the backward induction procedure breaks down.

### 6.2 Multiple equilibria if simultaneous moves

We show here (in the case of the emergence scenario) that there is no clear way to solve the game if we assume that the intermediary sets  $A_s$  and  $A_b$ simultaneously and then, that sellers and buyers decide at the same time whether or not to join the new marketplace.

A Nash equilibrium between buyers and sellers, given  $A_s$  and  $A_b$ , is defined as follows: b buyers and s sellers enter the marketplace if and only if

$$\begin{aligned} \pi_s\left(s,b\right) &\geq A_s > \pi_s\left(s+1,b\right), \\ \pi_b\left(s,b\right) &\geq A_b > \pi_s\left(s,b+1\right). \end{aligned}$$

We construct an example with two pairs (s, b) satisfying the latter four conditions. Take  $\gamma = 1$  and show that there are values of  $A_s$  and  $A_b$  for which both (1, 1) and (2, 2) are Nash equilibria. Simple computations establish the following (with all payoffs multiplied by 10,000 to ease the exposition):

- (1, 1) is an equilibrium iff (i)  $1250 \ge A_s > 555.56$ , and (ii)  $625 \ge A_b > 277.78$ .
- (2, 2) is an equilibrium iff (i)  $740.74 \ge A_s > 416.67$ , and (ii)  $493.83 \ge A_b > 277.78$ .

Clearly, for any  $A_s$  comprised between  $\pi_s(2,1) = 555.56$  and  $\pi_s(2,2) =$ 740.74, and any  $A_b$  comprised between  $\pi_b(2,3) = 277.78$  and  $\pi_b(2,2) =$ 493.83, both (1, 1) and (2, 2) are Nash equilibria. To maximize its profit, the intermediary would like to attract 2 buyers and 2 sellers by setting the highest possible fees, i.e.,  $A_s = \pi_s(2,2)$  and  $A_b = \pi_b(2,2)$ . However, if  $A_s = \pi_s(2,2)$  and  $A_b = \pi_b(2,2)$ , the industry is clearly better off if only one seller and one buyer enter. (In both situations, firms staying out make zero profit. If two sellers and two buyers enter, their entire surplus is captured by the intermediary; so, industry profits are equal to zero. On the other hand, if only one seller and one buyer enter, they keep some positive profits. So, from the industry's point of view, the latter situation Pareto-dominates). It follows that the intermediary's and the industry's objectives are in conflict, meaning that there is no a priori argument we could resort to in order to select among the two equilibria.

### 6.3 Proof of Lemma 6

We solve the game backwards. Consider stage 6 of the game. Given the membership fee  $A_b$  set by the intermediary and the number of sellers who have switched to the new marketplace, it is a Nash equilibrium for b buyers to switch if conditions (5) are fulfilled (with 1 < b < B).

In stage 5, the intermediary knows that by setting  $A_b = \hat{A}_b(s, b)$ , he induces b buyers to switch given that s sellers have done so before. His problem is thus to choose, for a given s, the value of b that maximizes  $b\hat{A}_b(s,b)$ . This defines the function  $b^*(s)$ . In contrast with the previous game, the form of this function depends on the value of  $\gamma$ . For  $\gamma = 0$ , the expression  $b\hat{A}_b(s,b) - (b-1)\hat{A}_b(s,b-1)$  is proportional to  $2s - S.^{22}$ When  $\gamma = 0$ , there is no competition effect among buyers. As a result, they are attracted towards the marketplace that comprises the largest number of sellers. Therefore, the intermediary attracts all buyers if he has attracted at least half of the sellers beforehand. He attracts a single buyer otherwise.<sup>23</sup>

<sup>&</sup>lt;sup>22</sup>More precisely, the expression is equal to  $-(2s(S-s)+S)(S-2s)/4(s+1)^2(S-s+1)^2$ .

 $<sup>^{23}</sup>$ For s < S/2, a subsidy has to be paid to attract any buyer. To minimize losses at this stage, the intermediary attracts the minimum number of buyers, i.e. one (recall that when deciding to launch the new marketplace, the intermediary commits to make it viable

For  $\gamma = 1$ , the expression  $\hat{A}_b(s, 1) - b\hat{A}_b(s, b)$  is positive.<sup>24</sup> It follows that  $b^*(s) = 1 \ \forall 1 \leq s \leq S$ . The intermediary attracts a single buyer.

In stage 4, sellers base their decision to switch on the membership fee,  $A_s$ , set by the intermediary and on the equilibrium decisions in the next stages. They thus anticipate correctly that  $b^* = B$  if  $\gamma = 0$ , or that  $b^* = 1$  if  $\gamma = 1$ . It is a Nash equilibrium for s sellers to switch if conditions (4) are fulfilled.

In stage 3, the intermediary maximizes  $s\hat{A}_s(s, b^*) + b_b^*\hat{A}(s, b^*)$ . For  $\gamma = 0$ , stage 6 of the game states that the number of buyers depends on the number of sellers that the intermediary attracts at this stage. Three cases must be considered: s < S/2 so that b = 1, s > S/2 or s = S/2 so that b = B. In the technical appendix, we prove that the intermediary attracts all buyers and all sellers.

For  $\gamma = 1$ , the game is analytically more difficult to solve. The full proof of the lemma is left for the technical appendix. The sketch of the proof goes as follows. We first show that when the intermediary selects only one buyer, his profit  $\Pi_I(s, 1)$  is concave in the number of sellers. Hence, there is only one value of the number of sellers that maximizes the intermediary's profit. This value is approximated by the value of s such that  $d\Pi_I(s, 1)/ds = 0$  (it is the integer that is the closest to this value). To prove that this number is non decreasing in the size of the pool of sellers (S) and non increasing in the size of the pool of buyers (B), we compute  $d^2\Pi_I(1,b)/dsdS > 0$  and  $d^2\Pi_I(s,1)/dsdB < 0$ . Finally we show that  $\Pi_I((S+3)/3,1) < \Pi_I(S/3,1)$ . From the concavity of  $\Pi_I(s,1)$ , this implies that  $s^* < (S+3)/3$ .

by letting at least one buyer and one seller access it).

<sup>&</sup>lt;sup>24</sup>More precisely, this expression is equal to  $[s^2 (b-1)^2 / 4 (s+1)^2 (1+b)^2] + \{(b-1) (S-s)^2 [(B+2)^2 - b] / (S-s+1)^2 (B-b+2)^2 (1+B)^2\}.$