Industry Equilibrium with Open Source and Proprietary Firms

Gastón Llanes
Pontificia Universidad Católica de Chile

Ramiro de Elejalde
Universidad de Chile

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Introduction

- Open Source (OS): freedom to use, modify and copy source code.
- Important participation of profit-maximizing firms in OS.
- Coexistence of OS and Proprietary (P) firms.

Questions:
- What motivates firms to participate in OS?
- What are the implications of competition?
- Will OS or P have higher quality?
- What are the limits to OS?
Some Important Characteristics of OS

General Public License and code sharing.

Firms profit by selling complementary goods.

- **Red Hat**: $650 million in training and support services in 2008.
- **IBM**: Invested $1 billion in Linux in 2001.
  - Support for over 500 software products on Linux.
  - Over 15,000 Linux-related customers.

Complementary goods are differentiated:

- Firms specialize in different technologies.
  - *Oracle*: *Linux support integrated with support for 11g database*.
- Building a strong brand.
  - *Red Hat*: *strong trademarks policy*. 
## Coexistence of OS and P

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Linux Market shares:

- Desktop: Linux below 10%.
- Servers: Linux + Unix $\geq 45\%$.
- Embedded: Linux 50% of cell phones.

Asymmetric Market Structure:

- Large P, small OS.
Related Literature

- Motivations of individual developers.

- Competition between OS and P.

- Contributions:
  - Endogenous decision to be OS.
  - Endogenous market structure.
  - Competition between profit-maximizing OS and P firms.

- Cooperation in R&D.
  (Kamien, Muller and Zhang 1992; Amir, Evstigneev and Wooders 2003)
Overview of the model

- Game: Two-stage non-cooperative game, $n$ firms, continuum of consumers.

- Firms decide:
  1. To be OS or P.
  2. How much to invest in R&D and price.

- Difference: OS share R&D, P do not.

- Firms sell packages:
  1. Primary good (software), potentially OS.
  2. Complementary good (support and training).

- Vertical and horizontal differentiation.
Main Findings

- Equilibrium with both kinds.
  - Asymmetric market structure.
  - Few large P, many small OS.
  - P have higher quality.

- Other results:
  - Equilibrium with all OS: may have high or low quality.
  - Welfare analysis and government policy.
We consider the following extensions:

- OS products are less differentiated than P products.
- Direct investment in the complementary good.
- Initial asymmetries in firm size.
Summary

Main ingredients:

- Industry equilibrium with OS and P firms.
- Firms sell packages with complementary good.
- Decision to be OS is endogenous.

Main results:

- Co-existence can arise as an equilibrium outcome.
- Forces leading to an asymmetric market structure.
- Complementarities may lead to high quality OS products.
Technology

- Fixed number of firms: \( n \).
- Investment in R&D: \( x_i \).
- Fixed cost: \( F = c x_i \). Zero marginal cost.
- Packages: primary good and complementary good.

- Quality of primary good:
  \[
  a_{os} = \ln(\sum_{i \in \text{os}} x_i) \quad \text{for OS firms}
  \]
  \[
  a_i = \ln(x_i) \quad \text{for P firms}
  \]

- Quality of complementary good:
  \[
  b_i = \ln(x_i) \quad \text{for all firms}
  \]
Preferences

- Continuum of consumers. Each consumer buys one package.

- Consumer $j$’s indirect utility from consuming good $i$:

  $$ v_{ij} = \alpha a_i + \beta b_i - p_i + \varepsilon_{ij} $$

- Vertical and horizontal differentiation.

- Taste shocks ($\varepsilon_{ij}$):
  - Each consumer has $n$ shocks (one for each good).
  - $\varepsilon_{ij}$ have double exponential distribution (logit model).
  - Variance: $\mu$ (degree of horizontal differentiation)
Parameters of interest

\[ v_{ij} = \alpha a_i + \beta b_i - p_i + \varepsilon_{ij} \]

\[ \delta = \frac{\alpha + \beta}{\mu} \quad \gamma = \frac{\alpha}{\alpha + \beta} \]

\( \delta \): Importance of Vertical vs. Horizontal differentiation.

\( \gamma \): Importance of Primary vs. Complementary good.

Assumption: \( \mu > \alpha + \beta \implies \delta, \gamma \in [0, 1] \)
Demands

- **Consumer Problem:**
  Observe $a, b, p$ and choose package with highest indirect utility.

- **Interested in Aggregate Consumer Demands**
  (solve Consumer Problem and integrate across consumers)

- **Demand for good $i$ (market share):**

\[
S_i = \frac{\exp \left( \frac{\alpha a_i + \beta b_i - p_i}{\mu} \right)}{\sum \exp \left( \frac{\alpha a_i + \beta b_i - p_i}{\mu} \right)}
\]
Game and Equilibrium Concept

- **Players:** \( n \) firms.

- **Two-stage non-cooperative game:**
  1. Firms decide to become OS or P.
  2. Firms decide investment in R&D and price.

- **Subgame Perfect Equilibrium.**

- **Symmetric Equilibrium in Second Stage:**
  All firms of the same type play the same equilibrium strategy.
Solution of the Second Stage I

- Recursive solution.

- $n_{os}$: number of firms deciding to be OS (given for 2nd stage).

- Second stage problem:

\[
\pi_i = \max_{p_i, x_i \geq 0} s_i(p, x) p_i - c x_i
\]

- Solution: $p_{os}, x_{os}, s_{os}, \pi_{os}$ and $p_p, x_p, s_p, \pi_p$ as functions of $n_{os}$. 
Solution of the Second Stage II

From FOC and imposing symmetry:

Profit maximizing price:

\[ p_{os} = \mu \frac{1}{1 - s_{os}} \quad p_p = \mu \frac{1}{1 - s_p} \]

Investment in R&D:

\[ x_{os} = \frac{\alpha + \beta}{c} s_{os} \left( 1 - \gamma \frac{n_{os} - 1}{n_{os}(1 - s_{os})} \right) \]
\[ x_p = \frac{\alpha + \beta}{c} s_p \]

Substituting \( x \) (which determines \( a \) and \( b \)) and \( p \) into demands \( s_i(p, x) \), we get a system of equations determining \( s_{os}(n_{os}) \) and \( s_p(n_{os}) \).
Second Stage Equilibrium

Proposition:
Second-stage equilibrium exists and is unique.
Given \( n_{os} \), market shares solve \((n - n_{os}) s_p + n_{os} s_{os} = 1\) and:

\[
(1 - \delta) \ln \left( \frac{s_{os}}{s_p} \right) + \frac{1}{1 - s_{os}} - \frac{1}{1 - s_p} = \delta \ln \left( 1 - \gamma \frac{n_{os} - 1}{n_{os} (1 - s_{os})} \right) + \delta \gamma \ln(n_{os})
\]

Interpretation:

\[
g(s_{os}, s_p) = \ln \left( 1 - \gamma \frac{n_{os} - 1}{n_{os} (1 - s_{os})} \right) + \gamma \ln(n_{os})
\]

Free-riding + Collaboration
Second Stage: Bottom line

- Existence and uniqueness.
- \( s_{os} \) and \( s_p \) as functions of \( n_{os} \)
- Free-riding vs. Collaboration.
- \( \pi_{os} \) and \( \pi_p \) as functions of \( n_{os} \)
Solution of the First Stage

Profits:

\[ \pi_{os}(n_{os}) = \mu \frac{s_{os}}{1 - s_{os}} \left( 1 - \delta(1 - s_{os}) + \delta \gamma \frac{n_{os} - 1}{n_{os}} \right) \]

\[ \pi_p(n_{os}) = \mu \frac{s_p}{1 - s_p} (1 - \delta(1 - s_p)) \]

Equilibrium:

\[ \pi_{os}(n_{os}) \geq \pi_p(n_{os} - 1) \quad \pi_p(n_{os}) \geq \pi_{os}(n_{os} + 1) \]

In terms of \( f(n_{os}) = \pi_{os}(n_{os}) - \pi_p(n_{os} - 1) \):

\[ f(n_{os}) \geq 0 \quad f(n_{os} + 1) \leq 0 \]
Co-existence Equilibrium.

\[ \gamma = 1, \ \delta = 1, \ \text{n=10} \]
All OS Equilibrium.
Solution of the First Stage

Proposition:
Given $n > 3$ and $\delta$, there are values $0 < \bar{\gamma} < \hat{\gamma} < 1$ such that:

- $\gamma > \hat{\gamma}$ \quad \{ Both kinds of firms co-exist in equilibrium. \\
P have higher quality and market share than OS. \\

- $\bar{\gamma} < \gamma < \hat{\gamma}$ \quad \{ All firms are OS. \\
P would have higher quality and market share. \\

- $\gamma < \bar{\gamma}$ \quad \{ All firms are OS. \\
P would have lower quality and market share. \\

Llanes (PUC) – de Elejalde (CEA)
Open Source vs Proprietary Firms
Dec 14, 2010
Equilibrium Regions

- Co-existence
- All OS, lower quality
- All OS, higher quality
Conclusion

Main ingredients:

- Model of industry equilibrium with OS and P firms.
- OS profit from selling a complementary good.
- Decision to be OS is endogenous.

Main results:

- Co-existence can arise as an equilibrium outcome.
- Decision to be OS: optimal business strategy.
- Forces leading to an asymmetric market structure.
- Complementarities may lead to high quality OS products.
- Testable implications.
Thank you!
Importance of commercial firms in OS

- Embedded Linux: 73.5% of developers work for commercial firms, and contribute 90% of code (Henkel and Tins 2004).

- 55% of OS developers contribute code at work, and contribute 50% more hours than the rest (Lakhani and Wolf 2005).

- 30% of OS developers work for commercial firms, and these firms are associated with larger and more dynamic OS projects (Lerner, Pathak and Tirole 2006).
Complementarities in OS

- Embedded Linux: 51.1% of developers work for manufacturers of devices, chips or boards and 22.4% work for specialized software companies (Henkel and Tins 2004).

- The dominant trend for appropriating the returns of innovation in OS is the sale of a complementary service (Dahlander 2004).
Derivation of Demand

Assumption

\(\epsilon_{ij}\) are i.i.d. and follow a double exponential distribution:

\[
\Pr(\epsilon_i < \omega) = \exp \left( - \exp \left( \frac{\omega}{\mu} + \nu \right) \right)
\]

where \(\nu\) is Euler’s constant and \(\mu\) is a non-negative constant.

\[s_i = \text{measure of consumers for which } v_{ij} = \max\{v_1, \ldots, v_n\}\]

\[s_i = \Pr \left( v_{ij} = \max\{v_1, \ldots, v_n\} \right)\]
OS are less differentiated than P.

- New dimension: Endogenous product differentiation.

- Two consumer shocks:
  1. Primary good shock.
  2. Complementary good shock (same as before).

\[ v_{ij} = \alpha a_k + \beta b_i - p_i + \sigma \eta_{kj} + (1 - \sigma) \varepsilon_{ij} \]

\( \sigma \): horizontal differentiation of primary vs. complementary good.
Effect of an increase in $\sigma$

Graph showing the effect of an increase in $\sigma$ on the variable $f$ with parameters $\gamma = 0.9$, $\delta = 0.7$, and $n = 10$. Two curves are shown, one for $\sigma = 0.2$ and another for $\sigma = 0.25$. The $x$-axis represents $n_{OS}$ and the $y$-axis represents $f$. The graph illustrates how $f$ changes as $n_{OS}$ increases for different values of $\sigma$. 
OS are less differentiated than P.

All P Equilibrium

\[ f = \gamma = 0.6, \ \delta = 0.6, \ \sigma = 0.3, \ n = 10 \]
OS are less differentiated than P.

Equilibrium regions

- All P
- All OS
- Coexistence
Firms can increase the value of the complementary good, without increasing the value of the primary good.

Quality of complementary good:

\[ b_i = \omega \ln(x_i) + (1 - \omega) \ln(z_i) \]

\( z_i \): direct investment in the complementary good.

As \( \omega \) decreases, investment of OS firms decreases.
Effect of an increase in $\omega$

For $\omega = 0.9$, $0.5$, and $0.1$, the function $f$ behaves differently with $n_{os}$. At $\gamma = 0.9$, $\delta = 0.7$, and $n = 10$, the curves show how $f$ changes with $n_{os}$.