Vertical Integration in Two-Sided Markets

Thomas Trégouët*

Preliminary version - Comments welcome

Abstract

This paper assesses whether vertical integration can help solving the chicken-and-egg coordination problem which arises in two-sided markets. The analysis builds on a two-sided market model with the following characteristics: i) a monopoly platform is an essential input that allows buyers to access the products offered by sellers; ii) there is a finite number of sellers which compete for buyers; iii) buyers are heterogenous: "core" buyers have a demand for each product while "casual" buyers purchase only the cheapest product; iv) the platform can be vertically integrated with one seller. We obtain two results of interest. First, vertical integration may have no coordination value *by itself*, i.e., vertical integration may not help solving the chicken-and-egg problem. Second, the coordination value of vertical integration in two-sided markets is closely related to the ability of a platform to commit to its downstream price.

Journal of Economic Literature Classification Number: L12, L14, L86.
Keywords: Two-sided markets, platform, vertical integration.

*Ecole Polytechnique. Address: Department of Economics, Ecole Polytechnique, 91128 Palaiseau Cedex, France. Phone: +33(0)1.69.33.34.17. Email: thomas.tregouet@polytechnique.edu

1 Introduction

There are many markets where two or more groups of users interact through a platform. When cross-group network effects are present, these markets are also known as two-sided markets. These industries include payment systems, media markets, hardware-software markets, auction websites (eBay), etc. A major issue in two-sided markets is the so-called "chicken-and-egg" coordination problem: in order to attract the users on one side of the market, the intermediary should have a large base of users on the other side, but these will be willing to join the platform only if they expect many users of the first side to join the platform. In order to deal with the chicken-and-egg problem, platforms have adopted "divide-and-conquer" pricing strategies. The idea is to subsidize one side of the market to get it on board and then to use the presence of the subsidized side to attract the other side. It has also been argued that vertical integration on one side of the market could help solving the chicken-and-egg problem. Intuitively, a platform vertically that is integrated on one side of the market may actually attract users on the other side since, by joining the platform, these will at least benefit from interacting with the vertically integrated users.

This article proposes an analysis of the coordination role of vertical integration in two-sided market. The article makes two contributions. First, it shows that, depending on the economic environment, vertical integration may have no coordination value *by itself*, i.e. vertical integration may not be a relevant strategy to solve the chicken-and-egg problem. It also underlines that the coordination value of vertical integration in two-sided markets is closely related to the ability of a platform to commit to the price of its vertically integrated division. Second, the article makes a theoretical contribution to the literature on two-sided market. It proposes a tractable model of two-sided market that allows for imperfect competition on one side of the market and for a vertically integrated platform.

In this paper, we focus on a monopoly platform that is vertically integrated with a subset of firms on one side of the market. The analysis of this framework is motivated by the two following distinctions. First of all, we make a distinction between platforms that are vertically integrated with one or a small number of sellers and industries that are fully integrated on one side of the market. Examples of a platform that is vertically integrated with one or more sellers are numerous: in the videogame market, console manufacturers (Nintendo, Microsoft, Sony) have their own game developers; internet portals (Yahoo, MSN) offer both in-house services (e-mail service,...) and outsourced services (news, weather forecast,...); etc. An example of fully integrated platform is the so-called "Palm economy".[1] Palm was initially an integrated applications/software platform and then disintegrated to become a "true" two-sided platform. Since it is obvious that being fully integrated with one side of the market

[1]See Chapter 6 in Evans, Hagiu, and Schmalensee (2006).

solves the chicken-and-egg problem – consumers face a single "one-sided" firm –, this article focuses on platforms that are partially vertically integrated.

Second, we distinguish the role played by vertical integration under monopoly and competition. In both cases, vertical integration raises the intrinsic value of a platform: buyers know that there will be at least one seller supporting the platform. Under competition, a platform can also gain a competitive advantage from being vertically integrated, since vertically integrated sellers do not support another platform. For instance, in the videogame industry, getting exclusivity on blockbuster games has proven to boost consoles' sales substantially.[2] Analysts argue that, at the very beginning of the next-generation DVD format war (Blu-ray vs. HD-DVD), Sony's ownership of its own movie studio (Sony Picture) was a key advantage against Toshiba.[3] It is then striking that, in February 2008, Toshiba decided that it would no longer develop and manufacture HD DVD players and recorders.[4] How this competitive effect interact with the coordination role of vertical integration is an interesting topic which deserves a separate analysis and is left for further research. Yet, this article focuses on vertical integration by a monopoly platform.

People often argue that vertical integration may be effective in solving the chicken-and-egg problem. Before assessing this statement, we need a criterion to appreciate the "effectiveness" of vertical integration. As pointed out above, a platform can overcome the chicken-and-egg problem with divide-and-conquer (DC) pricing strategies. However, these strategies have an opportunity cost: platforms make no profits from users on the subsidized side of the market. Therefore, we will say that vertical integration is effective in solving the chicken-and-egg problem – or, that vertical integration has a *coordination value* – if it lowers the opportunity costs of DC strategies.

With this in mind, we build a model with the following characteristics: i) a monopoly platform is an essential bottleneck input for buyers to access the products offered by sellers ; ii) there is a finite number of sellers which compete imperfectly for buyers; iii) buyers are heterogenous: "core" buyers purchase all product while "casual" buyers purchase only the cheapest one; iv) the platform can be vertically integrated with one seller.

The impact of vertical integration on the opportunity cost of a DC strategy depends on

[2] For instance, in April 2008, analysts estimated that "the Wii [Nintendo's console] seemed to benefit from the launch of Super Smash Bros.: Brawl, a fighting game exclusive to the console that sold a whopping 2.7 million units during the month in North America. Sales of the Wii console hit 721,000 units in March compared to 432 units the month before". (marketwatch.com)

[3] For instance, Norihiro Fujito, senior investment strategist at Mitsubishi Securities in Tokyo, said in 2004: "Although the four major studios picked Toshiba, Toshiba and NEC are to some extent a minor presence in the industry. Sony has the movie studio itself, and it will mount an aggressive strategy, along with Matsushita." (quoted by marketwatch.com)

[4] The Economist draws the conclusions that: "Sony had two advantages: it now owns one of Hollywood's biggest studios, and it built a Blu-ray drive into its PlayStation 3 games console, thus seeding the market with millions of players". ("Everything's gone Blu", *The Economist*, January 10th, 2008)

which side of the market is subsidized. If sellers are subsidized, then, the vertically integrated platform still get the profits of its "downstream" division.[5] On the other hand, if buyers are subsidized, the platform can still extract the surplus buyers would have obtained if no other seller than the vertically integrated one had joined the platform. The key point is that this surplus can be very small, since this is the surplus obtained by buyers when they purchase the product of a monopoly. This is particularly striking when the demand is very inelastic since, in this case, a monopoly seller extracts almost all buyers' surplus. If the platform finds it optimal to subsidize buyers, then, the small surplus it can extract from buyers may be smaller than the cost of being vertically integrated. It may therefore be the case that vertical integration does not reduce the opportunity cost of DC strategies.

The surplus that can be extracted from buyers when they are subsidized depends on the ability of the platform to credibly commit to its downstream price, i.e., the price of the vertically integrated seller. If the platform is not able to credibly commit to its downstream price, buyers should anticipate that, if no other sellers register with the platform, then, they will face a monopoly seller that will set its monopoly price. In other words, they will end up with a small surplus. On the other hand, if the platform is able to credibly commit to a low downstream price, e.g. close to marginal cost, then, buyers should expect a quite large surplus from joining the platform even if there are no other sellers. There is thus a strong connection between the coordination value of vertical integration and a platform's ability to commit to its downstream price.

This commitment issue gives rise to new questions about vertically integrated platforms' business models. Consider a situation where a vertically integrated platform has committed to a low downstream price. Then, in the downstream market, sellers either choose to compete for casual users with the platform's downstream division or to serve only core users. If sellers focus on core users, they set their monopoly prices since there is no competition for these consumers. Therefore, committing to a low downstream price may actually raise prices of all other products. This may lower buyers' surplus and, therefore, platform's profits made on the buyers' side of the market. However, the platform sells its product to all users at a low price. Hence, for these transactions, the joint surplus of buyers and platform's downstream division is high. This joint surplus can be entirely captured by the platform since it controls its downstream division and buyers pay for participation. In the end, committing to a low downstream price has two conflicting effects on platform's profits. First, it lowers the surplus captured on transactions performed by pure downstream sellers. Second, it increases the surplus captured on transactions made by the platform's downstream division. These two

[5] In the literature on vertical integration, "downstream" firms refers to firms that are closer to the consumer, while "upstream" firms refer to firms that are further away from consumers. In a two-sided market, it is not obvious to state whether platforms or sellers are closer to the consumer. With this in mind, in our model, we arbitrarily choose that "downstream" firms refer to the sellers side of the market.

effects go in opposite directions when the platform commits to a high downstream price. A vertically integrated platform therefore faces an interesting trade-off between committing to a low or a high downstream price.

The remainder of this paper is organized as follow. In section 2, we lay out the framework of our model. Section 3 is devoted to the analysis of the monopoly platform without vertical integration. Section 4 analyzes the vertically integrated platform. Section 5 concludes.

Related literature. Our paper belongs to the recent literature on two-sided markets, pioneered by Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2003 and 2006). Armstrong (2006) and Rochet and Tirole (2003 and 2006)'s canonical models of two-sided markets provide an analysis of platform pricing structures. Their focus is on the role of relative demand elasticities in explaining the unbalanced pricing structure usually observed in two-sided markets. However they assume that agents on both sides of the market join the platforms as soon as it is an equilibrium to do so, thereby abstracting away from the chicken-and-egg coordination problem. Caillaud and Jullien (2003) study how competition between two intermediaries may lead to different market structures in equilibrium (dominant platform or market sharing equilibria). They also show that divide-and-conquer pricing strategies may solve the chicken-and-egg coordination problem, thus providing an explanation for the unbalanced pricing structure. We borrow from Caillaud and Jullien (2003) their equilibrium concept and their terminology for pricing strategies. Hagiu (2006) studies the commitment problem faced by a platform when the two sides of the market arrive sequentially. We share with Hagiu (2006) a particular interest in the business models of software and game console platforms. Belleflamme and Toulemonde (2007) propose a model of two-sided markets with negative intra-group externalities. While their motivation is different from ours, their paper is still related to ours since we consider imperfect competition on one side of the market, i.e. negative intra-group externalities on this side. Lee (2007) assesses empirically whether exclusivity and vertical integration may have anti-competitive dynamic effects in two-sided market. His results seems to indicate that vertical integration and exclusive dealing benefits to the entrant platforms. However, his paper does not provide a theoretical model of the platforms' decisions to integrate vertically or to sign exclusive dealing contracts.

2 The model

Participants. There are two sides of the market: buyers and sellers, denoted by B and S respectively, and a monopoly platform. Sellers are firms which sell their products to buyers through the platform. For sake of concreteness, we will sometimes refer to firms as developers which sell applications to consumers.

Platform. End-users on side k ($k \in \{B, S\}$) pay to the platform a^k for membership. We assume that the platform cannot discriminate among sellers. Let $a = (a^B, a^S)$. The platform incurs no cost to serve firms and consumers.

Sellers. There are $N \geq 3$ symmetric firms on side S of the market. Each firm i ($i = 1, \ldots, N$) sets the price p_i at which it sells its application. Denote by $P = (p_i)_{i=1,\ldots,N}$ the applications' prices. Firms incur neither fixed nor marginal cost to produce their applications.[6] Besides, they cannot sell their applications without registering with the platform.

Buyers. There is a mass one of ex-ante homogenous consumers. Consumers cannot purchase developers' applications without joining the platform. Consumers are ex-post heterogeneous. After he subscribes to the platform, a consumer learns his type: with probability α ($0 < \alpha < 1$), he is a "core user" and, with probability $1 - \alpha$, he is a "casual user".[7] A core user has a demand $d(p_i)$ for each application i ($i = 1, \ldots, N$), where $d(.)$ is decreasing in p_i. A casual user purchases only the cheapest application: he has a demand $d(p_i)$ for application i where $p_i = \min_j p_j$.

If $n \geq 2$ firms set the same minimum price, we assume that each firm has a probability $1/n$ for selling its application to a casual user.

We would like to make two comments on these assumptions. First, these assumptions means that applications may be more or less substitutable for different users. The simplest way to capture this heterogeneity among users is to assume that applications are not substitutes for some users (the "core" users), while they are perfect substitutes for the others (the "casual" users). A more general model would allow for "intermediate" degree of substitutability among applications. Yet, this would make the analysis much more intricate, though not yielding any additional insights.

Second, this set of assumptions implies that developers will compete to sell their applications to casual users. We are fully aware that there are many different ways to model imperfect competition between developers. However, we think that none of the "standard" ways to model imperfect competition fit the three ingredients we would like to have in our framework. First, some users have to purchase several applications so that positive indirect network externalities arise in our model. Second, we need a model where firms' and consumers' surplus can be written in a simple way because the whole point of the paper will be to know whether the platform captures firms' or buyers' surplus. Third, the framework must

[6] This is wlog in our framework. Things would be different if there were another platform. In such a context, Hagiu (2006) assumes that developers incur a sunk cost F to make its application "work" on a platform. Hagiu allows for economies of scale in that the fixed cost to make the application available on another platform is smaller than F.

[7] We borrow the terminology core/casual users from the videogame industry. In this industry, "hardcore" refers to an audience of users who play videogame several hours per day and purchase a lot of games, while "casual" refers to an audience of users who play videogame only occasionally and buy few games.

be sufficiently flexible to allow for vertical integration. To the best of our knowledge, we see no standard model that has these three ingredients.[8]

Agents' payoffs. Let $\mathcal{N} = (n^S, n^B)$ denote the number of firms and consumers who join the platform. With a slight abuse of notations, we will sometimes refer to n^S as the set of firms which join the platform. We assume that users have quasi-linear preferences, so that, given applications' prices p_i, $i \in n^S$, the net ex-ante expected utility of a consumer who joins the platform is:

$$U^B(a, P, \mathcal{N}) = \alpha \sum_{i \in n^S} u^B(p_i) + (1-\alpha) u^B(\min_{i \in n^S} p_i) - a^B, \tag{1}$$

where $u^B(p_i) = \int_{p_i}^{+\infty} d(p)dp$ is consumer's surplus from buying application i at price p_i.

On the other side of the market, firm i's profits are given by:

$$\pi_i(a, P, \mathcal{N}) = \alpha n^B p_i d(p_i) + (1-\alpha) n^B \mathbb{1}_{\{p_i = \min_j p_j\}} \frac{1}{n} p_i d(p_i) - a_S, \tag{2}$$

where n is the number of firms which set the same price as firm i if $p_i = \min_j p_j$. We assume that function $\pi(p) = pd(p)$ is strictly concave in p. Denote $p^m = \arg\max_p \pi(p)$ and $\pi^m = \pi(p^m)$. Let $w(p) = \pi(p) + u^B(p)$ denote the total surplus of a transaction between a firm and a consumer. Notice that $w(.)$ is decreasing in p ($p \geq 0$) and is maximal when price equals marginal cost, i.e., when $p = 0$.

The platform may be vertically integrated with firm 1. In this case, we assume that the platform incurs a positive sunk cost f for being vertically integrated. If the platform is vertically integrated, we consider two scenarios, depending on whether the platform is able to credibly commit to its downstream price p_1. Platform's profits are given by:

$$\Pi^P(a, \mathcal{N}) = n^B a^B + n^S a^S, \tag{3}$$

if it is not vertically integrated and by

$$\Pi^P(a, p_1, \mathcal{N}) = n^B a^B + (n^S - 1) a^S + \pi_1 - f, \tag{4}$$

when it is vertically integrated, where π_1 are platform's downstream profits.

Timing and equilibrium. The chronology of events runs as follows:

[8] For instance, a Hotelling-Salop representation of the market fits the second requirement but fails to achieve the first one. Besides, in such a framework, vertical integration would introduce strong asymmetries among downstream competitors and this is not desirable.

1. The platform sets subscription fees a^B and a^S. If the platform is vertically integrated with firm 1, it also commits to p_1 if it is able to do so.

2. Firms and consumers observe a^B, a^S and, possibly, p_1 and, then, decide simultaneously and non cooperatively whether to join the platform. Consumers then learn their types. We will refer to this stage as the participation game.

3. Firms set their prices p_i. Consumers observe firms' prices and, then, purchase applications. We will refer to this stage as the pricing game.

We assume full information at every stage of the game.

There may be multiple equilibria in stage 2. To grasp the intuition, consider a situation where both a^B and a^S are positive and not too high. Then, there is an equilibrium in which all firms and consumers join the platform. Yet, notice that there also exists an equilibrium in which neither firms nor consumers register with the platform. In this equilibrium, each agent anticipates that nobody will register with the platform, so that a seller or a buyer prefers staying outside.

Let $\mathcal{N}(a^B, a^S) = (n^B, n^S)$ ($\mathcal{N}(a^B, a^S, p_1)$ when the platform is vertically integrated and can credibly commit to p_1) be the demand of firms and consumers for the platform given membership fees a^B and a^S (and if applicable p_1). For a given pair (a^B, a^S) (or if applicable for a given triple (a^B, a^S, p_1)), $\mathcal{N}(a^B, a^S)$ is an equilibrium if and only if the firms and consumers who join the platform are better off joining the platform rather than staying outside, and if firms and consumers who stay outside are strictly better off staying outside rather than joining the platform.

An equilibrium of the pricing game in stage 3 is a vector of prices P. It is a function of a and $\mathcal{N}(a)$. Thereafter, if there is no ambiguity, we will denote by P an equilibrium of the pricing game in stage 3, rather than $P(a, \mathcal{N}(a))$.

Definition 1. *An equilibrium is a triple $(a^*, \mathcal{N}^*(.), P^*)$, where (i) P^* is an equilibrium of the pricing game played by firms in stage 3 with profits $\pi_i(p_i, p_{-i})$, (ii) $\mathcal{N}^*(a^*)$ (or $\mathcal{N}(a^*, p_1^*)$) is an equilibrium for the system of prices a^* (or (a^*, p_1^*)) of the participation game played by firms and consumers in stage 2, (iii) a^* (or (a^*, p_1^*)) maximizes platform's profits induced by $\mathcal{N}^*(.)$ and P^*.*

Since our focus is on the coordination role of vertical integration, we will consider two polar scenarios, according to whether firms and consumers are optimistic or pessimistic on each others participation decisions.

Definition 2. *Let $\Sigma(a)$ denote the set of subgame perfect equilibria of the game induced by tariffs a. Define $\Sigma^+(a)$ and $\Sigma^-(a)$ the subsets of $\Sigma(a)$ such that:*

- *for all* $(\mathcal{N}, P) \in \Sigma^+(a)$, $\Pi^P(a, \mathcal{N}, P) = \max_{(\mathcal{N}', P') \in \Sigma(a)} \Pi^P(a, \mathcal{N}', P')$,
- *for all* $(\mathcal{N}, P) \in \Sigma^-(a)$, $\Pi^P(a, \mathcal{N}, P) = \min_{(\mathcal{N}', P') \in \Sigma(a)} \Pi^P(a, \mathcal{N}', P')$.

Definition 3. *Let* $(a^*, \mathcal{N}^*(.), P^*)$ *be an equilibrium. We say that* $(a^*, \mathcal{N}^*(.), P^*)$ *is an optimistic (pessimistic) equilibrium if, for all* a, $(\mathcal{N}^*(.), P^*) \in \Sigma^+(a)$ $((\mathcal{N}^*(.), P^*) \in \Sigma^-(a))$.

In words, in an *optimistic* (*pessimistic*) equilibrium, in each subgame, users coordinate on the equilibrium that yields maximal (minimal) profits for the platform. In an optimistic (pessimistic) equilibrium, we will say that users are optimistic (pessimistic) on each others participation decisions. Put differently, optimistic users all join the platform as long as this is consistent with the price announced, while pessimistic users expect no user will support the platform as long as this is consistent with the price announced.

3 Unintegrated platform

In this section, we derive the unintegrated platform's profits, when it faces optimistic and pessimistic users.

3.1 Stage 3's pricing game

Different scenarios can arise in stage 3, depending on which agents have registered with the platform in stage 2. If at least one group of agents did not join the platform in stage 2, then, there is no need to consider the pricing game, since no transactions take place between firms and consumers.

Consider now the more interesting scenario in which all consumers and some firms have joined the platform in stage 2. Let $n \geq 1$ denote the number of developers which joined the platform in stage 2. If $n = 1$, then, consumers face a monopoly developer which sets the monopoly price p^m. In this case, consumers' and developer's surpluses from joining the platform are given by $u^B(p^m)$ and π^m respectively. If $n \geq 2$, developers compete for users. More precisely, competition for casual users is harsh, since these consumers purchase only the cheapest application. This Bertrand-competition effect pushes prices downward. On the other hand, firms anticipate that core users will buy their applications at any price. This provides firms with incentives to set high prices.

The following lemmas describe the outcome of competition between developers.[9]

Lemma 1. *Assume that* $n \geq 2$ *firms registered with the platform in stage 2. Then, there exists no pure strategy equilibrium in the pricing game played in stage 3.*

[9] It is a well known phenomenon that discontinuous demand functions in Bertrand-competition game leads to the existence of mixed strategy equilibria. See Dasgupta and Maskin (1986) for a general argument.

Proof. See Appendix A.1 □

Lemma 2. *Assume that $n \geq 2$ firms registered with the platform in stage 2. Then, there exists a unique symmetric mixed strategy equilibrium in the pricing game played in stage 3. Each firm plays a mixed strategy on the interval $[\underline{p}, p^m]$ according to the cumulative distribution $\Phi_n(.)$, where: (i) $\underline{p}$ is such that $\pi(\underline{p}) = \alpha\pi^m$ and $\underline{p} < p^m$, and (ii) for all $p \in [\underline{p}, p^m]$,*

$$\Phi_n(p) = 1 - \left(\frac{\alpha}{1-\alpha}\frac{\pi^m - \pi(p)}{\pi(p)}\right)^{\frac{1}{n-1}}. \tag{5}$$

Besides, each firm makes profits equal to $\alpha\pi^m$.

Proof. See Appendix A.2. □

Notice that $\Phi_n(.)$ has an increasing density, so that firms put more weight on higher prices. Notice also that for a given p, $\Phi_n(p)$ is decreasing in n. In other words, for all $n \geq 2$, cumulative distribution $\Phi_{n+1}(.)$ first-order stochastically dominates $\Phi_n(.)$. This means that the fewer firms compete for casual users, the lower the prices are on average. Intuitively, if there are fewer firms, then, each developer is more likely to serve casual users when it sets a low price.

By Lemma 2, consumers' ex-ante surplus is given by:

$$S_n^B = \alpha n \mathbb{E}_{\Phi_n}[u^B(p)] + (1-\alpha)\mathbb{E}_{\Phi_n^{\min}}[u^B(p)], \tag{6}$$

where $\Phi_n^{\min}(.)$ is the cumulative distribution of the random variable $p_{(1)} = \min_{i=1,\dots,n} p_i$:

$$\Phi_n^{\min}(p) = 1 - (1 - \Phi_n(p))^n, \text{ for all } p \in [\underline{p}, p^m].$$

The impact of an increase in n on consumers' surplus is ambiguous: on the one hand, core users buy more applications, which raises the surplus; on the other hand, as stated above, the average applications' price increases which is harmful for surplus. The impact on casual users' surplus is also ambiguous: while the average applications' price increases, the minimum price is likely to be lower since there is more firms.

3.2 Optimistic users

Consider now the participation game in stage 2. Assume first that users are optimistic. On the buyers' side of the market, either all users or none of them register with the platform, depending on a^B and a^S. If a^S is too high, namely, $a^S > \pi^m$, then, consumers anticipate that participation is not profitable for developers. Therefore, they join the platform only if a^B is negative. If a^S is sufficiently low, namely, $a^S \leq \alpha\pi^m$, then, consumers anticipate that all

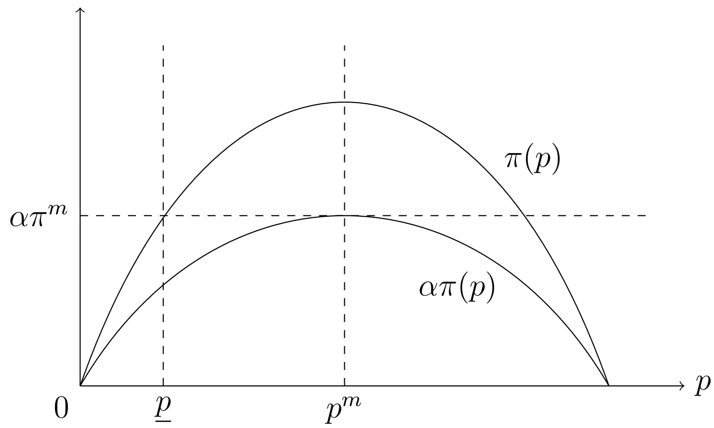

Figure 1: Developer's profits when it serves all users ($\pi(.)$) and when it serves only core users ($\alpha\pi(.)$)

developers will join the platform if a^B is not too high. Indeed, by Lemma 2, developer' entry is profitable whenever $a^S \leq \alpha\pi^m$ and consumers join the platform. Buyers obtain $S_N^B - a^B$ in the platform when all developers have registered. Therefore, if $a^S \leq \alpha\pi^m$, all consumers register with the platform when $a^B \leq S_N^B$ and they stay out otherwise. If a^S is intermediate, namely, $\alpha\pi^m < a^S \leq \pi^m$, then, consumers anticipate that participation is profitable for only one developer. Put differently, if they join the platform, they face a monopolist developer and obtain $u^B(p^m) - a^B$. Therefore, if $\alpha\pi^m < a^S \leq \pi^m$, consumers subscribe to the platform if $a^B \leq u^B(p^m)$. In the end, depending on a^B and a^S, either all consumers or none of them register with the platform, and either all developers, only one of them, or none of them join the platform.

The following proposition states that, facing optimistic users, the platform chooses subscription fees such that all users participate.

Proposition 1. *Assume that users are optimistic. Then, an unintegrated platform sets $a^B = S_N^B$ and $a^S = \alpha\pi^m$, and makes profits $N\alpha\pi^m + S_N^B$. Besides, all users join the platform.*

Proof. See Appendix A.3. □

3.3 Pessimistic users

Suppose now that users are pessimistic. Consider, for instance, the participation decision of a consumer. If a^S is positive, he anticipates that no seller will register with the platform. Hence, he joins the platform only if a^B is negative. Indeed, in this case, he obtains a strictly positive utility, even if he purchases no application. Then, suppose that a^B is negative. As just stated, all buyers join the platform, whatever their expectations on developers' participation decision. Therefore, having observed a^B, a given seller register with the platform if a^S is not too high. More precisely, if $a^S \leq \alpha\pi^m$, participation is profitable for each seller and, if $\alpha\pi^m < a^S \leq \pi^m$, participation is profitable for only one seller. In this situation, buyers are subsidized, while the platform captures part of sellers' profits. By analogy to Caillaud and Jullien (2003), this strategy is called a divide buyers - conquer sellers strategy (hereafter $D_B C_S$).

Similarly, consider buyers' participation decision and assume that a^S is negative. In this case, each consumer anticipates that all sellers will register with the platform, whatever their expectations on buyers' participation. Therefore, consumers join the platform if a^B is lower than S_N^B. In this situation, sellers are subsidized, while the platform captures part of buyers' surplus. This strategy is called a divide sellers - conquer buyers strategy (hereafter $D_S C_B$).

Facing pessimistic users, the platform adopts a divide and conquer strategy. Besides, it chooses to capture either buyers' or sellers' surplus. If it chooses to capture buyers' surplus, it adopts a $D_S C_B$ strategy, sets $a^S = 0$ and $a^B = S_N^B$ and makes profits S_N^B. On the other hand, if it chooses to captures sellers' profits, it adopts a $D_B C_S$ strategy and sets $a^B = 0$ and $a^S = \alpha\pi^m$ or π^m. If the platform sets $a^S = \alpha\pi^m$, then, it attracts all sellers, while only one developer registers if $a^S = \pi^m$. Platform's profits in these two cases are given by $N\alpha\pi^m$ and π^m respectively. In order to rule out undesirable situations where only one developer subscribe to the agency, we make the following assumption:

Assumption 1. $N\alpha > 1$.

Proposition 2. *Under Assumption 1, if the platform faces pessimistic users, then, it adopts a $D_B C_S$ strategy if $N\alpha\pi^m > S_n^B$, and a $D_S C_B$ strategy otherwise.*
More precisely:

- *A sufficient condition for the platform to adopt a $D_S C_B$ strategy is given by:*

$$\pi^m < \left(1 + \frac{1-\alpha}{N\alpha}\right) u^B(p^m). \tag{7}$$

- *A sufficient condition for the platform to adopt a $D_B C_S$ strategy is given by:*

$$\pi^m > \left(1 + \frac{1-\alpha}{N\alpha}\right) u^B(\underline{p}). \tag{8}$$

Proof. Notice that, by equation (6), $(N\alpha + 1 - \alpha)u^B(p^m) \leq S_N^B \leq (N\alpha + 1 - \alpha)u^B(\underline{p})$, which immediately yields the announced result. □

Proposition 2, illustrated by Figure 2, states that, when it faces pessimistic users, the platform chooses to capture the surplus from the users who benefit more from participating. Proposition 2 also reveal the two forces that drive platform's choice between a $D_S C_B$ or a D_BC_S strategy. The first one stems from developers' ability to capture a large share of users' surplus. When the platform faces pessimistic users, it had to choose between capturing developers' or users' surplus. Then, if, for instance, users obtain a large share of the trade surplus, the platform should choose to capture users' surplus through a D_SC_B strategy. This is emphasized by equation (7) which shows in particular that, if $\pi^m < u^B(p^m)$, then, the platform always choose to capture users' surplus. Notice also that the platform may choose a D_BC_S only if $\pi^m > u^B(p^m)$, whatever the proportion of core users and the number of developers.

The second force that drives platform's choice is related to the proportion of core users in the population. To grasp the idea, suppose for instance that there are few core users. Then, competition for casual users is harsh and developers' profits are small. Therefore, the platform is likely to adopt a D_SC_B strategy in order to capture consumers' surplus. When the proportion of core users is high, say close to 1, the trade-off between the two divide-and-conquer strategies depends mainly on the first effect mentioned above. Indeed, in this case, competition for casual users is weak so that developers sell their applications at a price close to the monopoly price. Therefore, the platform is likely to choose a D_BC_S strategy when $\pi^m > u^B(p^m)$ and a D_SC_B strategy otherwise.

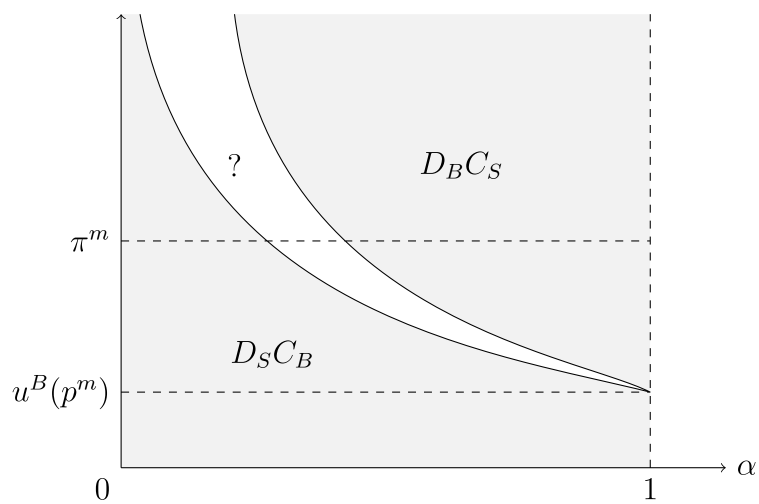

Figure 2: The choice between a D_BC_S and a D_SC_B strategy. The lower and upper curve represents pairs (α, π^m) such that $\pi^m = \left(1 + \frac{1-\alpha}{N\alpha}\right) u^B(p^m)$ and $\pi^m = \left(1 + \frac{1-\alpha}{N\alpha}\right) u^B(\underline{p})$ respectively.

4 Vertically integrated platform

We now turn to the study of a vertically integrated platform.

We first show that the coordination value of vertical integration in two-sided market depends on the ability of the vertically integrated platform to credibly commit to a downstream price at stage 1. Then when commitment is feasible, we analyze the platform's incentives to commit to a high or a low downstream price.

4.1 The platform is unable to credibly commit to its downstream price

In this section, the platform is unable to commit to its downstream price p_1.

Suppose that all consumers join the platform in stage 2. There are two scenarios to consider in stage 3, according to the number of developers which have registered with the platform in stage 2. In the first scenario, the $N-1$ "pure" downstream firms have joined the platform, so that N firms – the vertically integrated one plus the $N-1$ others – compete for casual users. In other words, we are in the situation described in lemma 2. In the second scenario, developers have not joined the platform in stage 2, so that only firm 1's application is available on the platform. In this case, since the platform did not commit to its downstream price, consumers face a monopolist developer that sets the monopoly price p^m.

Optimistic users. Assume first that users are optimistic. In this case, the platform is able to extract the full surplus from developers and consumers. In particular, it sets a^S so that all firms participate: $a^S = \alpha\pi^m$. On the other side of the market, a given consumer rationally expects that he obtains $S_N^B - a^B$ by joining the platform if $a^S \leq \alpha\pi^m$. Therefore, the platform attracts all consumers by charging $a^B \leq S_N^B$. Total platform profits are then $N\alpha\pi^m + S_N^B - f$, where $N\alpha\pi^m$ is the sum of platform downstream division's profits and the $N-1$ subscription fees paid by "pure" downstream developers.

If it faces optimistic users and if it cannot commit to its downstream price, a vertically platform makes strictly less profits than an unintegrated platform. Intuitively, in this case, vertical integration has no impact on the outcome of downstream competition, so that the platform cannot extract more surplus from users. However, it must pay the sunk cost f.

Proposition 3. *Vertical integration is not profitable if the platform faces optimistic users and if it is unable to commit to its downstream price.*

Pessimistic users. Suppose now that users are pessimistic. Pessimistic consumers anticipate that at least one firm will register with the platform, namely, the vertically integrated one. Besides, they anticipate that, in the worst case, they will face a monopoly developer which will charge its monopoly price. In other words, there is an intrinsic benefit $u^B(p^m)$

from joining the platform: consumers are sure that there will be at least one expensive application. This implies that pessimistic consumers always join the platform if $a^B \leq u^B(p^m)$. Therefore, in a D_BC_S strategy, the platform can extract a positive surplus from consumers by charging $a^B = u^B(p^m)$. On the other side of the market, vertical integration does not raise developers' intrinsic benefit from registering with the platform, so that D_SC_B strategies remain unchanged. Yet, the platform always makes some profits on the developers' side through its downstream division.

Proposition 4. *If the vertically integrated platform faces pessimistic users and is unable to commit to its downstream price, then, it adopts a D_BC_S strategy if $(N-1)\alpha\pi^m > S_N^B - u^B(p^m)$, and a D_SC_B strategy otherwise.*
More precisely:

- *A sufficient condition for the platform to adopt a D_SC_B strategy is given by:*

$$\pi^m < u^B(p^m). \tag{9}$$

- *A sufficient condition for the platform to adopt a D_BC_S strategy is given by:*

$$\pi^m > u^B(\underline{p}) + \frac{1}{(N-1)\alpha}\left(u^B(\underline{p}) - u^B(p^m)\right). \tag{10}$$

Proof. Notice that $S_N^B - u^B(p^m)$ lies between $(N-1)\alpha u^B(p^m)$ and $(N\alpha+1-\alpha)u^B(\underline{p}) - u^B(p^m)$. This immediately yields the announced sufficient conditions. □

The same logic as in the Proposition 2 is at work: when developers capture most of consumers' surplus, the platform is more likely to choose a D_BC_S strategy; when there are few core users, competition between developers is harsh, so that the platform is likely to choose a D_SC_B strategy.

Now, we would like to find situations where we are able to compare platform's profits when the platform is unintegrated and vertically integrated, i.e. situations where we can state whether vertical integration has a coordination value. To do so, we identify situations where an unintegrated platform and a vertically integrated platform choose to "divide" the same side of the market. Formally, we compare platform's profits without vertical integration and under vertical integration when both inequalities (7) and (9) hold and when both (8) and (10) hold.

Proposition 5. *Assume that $\pi^m < u^B(p^m)$. Then, vertical integration has a coordination value iff $\alpha\pi^m > f$.*
Assume that $\pi^m > u^B(p^m)$. Then, there exists $\hat{\alpha} \in (0,1)$ such that, for all $\alpha > \hat{\alpha}$, vertical integration has a coordination value iff $u^B(p^m) > f$.

Proof. See Appendix A.4. □

Unsurprisingly, Proposition 5 states that the profitability of vertical integration depends on the fixed cost f: if f is too high, vertical integration is never profitable. This is not surprising, so that we will not put much emphasis on this result. The important point in Proposition 5 is that there is a tension between the coordination value of vertical integration and the nature of divide-and-conquer strategies. Consider for instance a situation where $\pi^m > u^B(p^m)$ and where α is sufficiently high so that both an unintegrated platform and a vertically integrated platform choose to capture developers' profits through a D_BC_S strategy. In this case, the platform's benefits from being vertically integrated are the additional surplus $u^B(p^m)$ that can be extracted from consumers minus the fixed cost f. The interesting point here is that a situation where the platform chooses a D_BC_S strategy under both regimes is likely to arise when $u^B(p^m)$ is small, i.e. when the benefits from being vertically integrated are small.[10]

Let us illustrate this tension with an example. Suppose that the demand for applications is given by $d(p) = \mathbb{1}_{\{p \leq v\}}$, where $v > 0$ is consumers' valuation for one application. The demand for applications is thus completely inelastic for prices below v. It is immediate that $p^m = v$, so that $\pi^m = v$ and $u^B(p^m) = 0$. Assume that α is sufficiently high, so that the platform chooses to capture developers' profits under both regime. Then, the unintegrated platform makes profits $N\alpha v$, while the vertically integrated platform makes profits $N\alpha v - f$. Therefore, in this example, vertical integration has no coordination value.

4.2 The platform can credibly commit to its downstream price

In this section, we make a connection between the coordination value of vertical integration and the ability of the platform to credibly commit to its downstream price. We also analyze under which conditions the platform commits to a low or a high downstream price.

Suppose that the $N-1$ pure downstream firms joined the platform in stage 2. Suppose also that the platform committed to p_1 in stage 1. The level of p_1 has a strong impact on the outcome of competition. To grasp the idea, suppose for instance that p_1 is low, say $p_1 \leq \underline{p}$. In this case, pure downstream firms will not compete for casual users in stage 3. Indeed, suppose that one pure downstream firm sets a price p below p_1. This firm makes profits $\pi(p)$ in the most favorable case, i.e. if p is the lowest downstream price. But then, notice that $\pi(p) < \pi(p_1) \leq \pi(\underline{p}) = \alpha\pi^m$, so that the downstream firm would be strictly better off charging p^m and serving only core users. Conversely, when p_1 is high, say $p_1 \geq p^m$, the vertically integrated developer does not compete for casual users. We are thus exactly

[10] Notice that we could the same point in a situation where the platform chooses a D_SC_B strategy under both regimes. Indeed, this situation is likely to arise when π^m is small compared to $u^B(p^m)$, i.e. when the benefits from being vertically integrated under a D_SC_B strategy are small.

in the situation described in lemma 2, with the only difference that there are only $N-1$ downstream firms competing for casual users.

Lemma 3. *When the platform commits to a downstream price p_1 in stage 1, there exists a unique symmetric pure or mixed strategy equilibrium of the pricing game in stage 3 among firms $2, \ldots, N$:*

- *If $p_1 \leq \underline{p}$, each firm sets $p_i = p^m$, $i = 2, \ldots, N$.*
- *If $\underline{p} < p_1 < p^m$, each firm plays a mixed strategy: with probability $q(p_1)$, each firm plays a mixed strategy on the interval $[\underline{p}, p_1]$ according to the distribution $\xi(., p_1)$ and with probability $1 - q(p_1)$, each firm plays p^m, where:*

  $$q(p_1) = \Phi_{N-1}(p_1),$$

  and for all $p \in [\underline{p}, p_1]$

  $$\xi(p, p_1) = \frac{\Phi_{N-1}(p)}{\Phi_{N-1}(p_1)}.$$

- *If $p_1 \geq p^m$, each firm plays a mixed strategy on the interval $[\underline{p}, p^m]$ according to the cumulative distribution $\Phi_{N-1}(.)$.*

In any case, downstream firms make profits $\alpha\pi^m$.

Proof. See Appendix A.5 □

Optimistic users. When users are optimistic, the platform can extract all firms' profits and consumers' surplus. On the developers's side of the market, this is done by charging $a^S = \alpha\pi^m$, since developers' profits do not depend on p_1 (see Lemma 3).

On the other side of the market, the surplus that can be extracted from consumers depends on the level of p_1. If $p_1 \leq \underline{p}$, both core and casual users purchase the application developed by the vertically integrated firm. Since p_1 is low, this generates a large surplus $u^B(p_1)$. But then, the core users purchase the $N-1$ other applications at the monopoly price, which creates a small surplus $u^B(p^m)$ per application. All in all, by committing to a low downstream price, the platform is able to extract a large surplus from a small number of transactions realized by its downstream division and a smaller one from a large number of transactions realized by the pure downstream firms.

Suppose now that p_1 is high, say $p_1 = p^m$ to make things more stringent. The vertically integrated developer sells its application only to core users, therefore generating a small surplus from a small number of transactions. On the other hand, the $N-1$ pure downstream firms now compete harshly for casual users, which in turn benefits to all core users. In other words, by committing to a high downstream price, the platform is able to extract a large surplus from a large number of transactions realized by the pure downstream firms and a small one from a small number of transactions realized by its downstream division.

Since the platform captures the full consumers surplus through membership fees, it sets p_1 to maximize the sum of consumers' surplus and its downstream profits, denoted by $R^B(p_1)$. Formally, platform's profits are given by $\Pi^P = R^B(p_1) + (N-1)\alpha\pi^m - f$.

When $p_1 \leq \underline{p}$, function $R^B(.)$ is given by:

$$R^B(p_1) = w(p_1) + \alpha(N-1)u^B(p^m). \tag{11}$$

For low values of p_1, all users buy firm 1's application. This generates a surplus $w(p_1) = \pi(p_1) + u^B(p_1)$. The other developers sell their applications to core users only at their monopoly prices, therefore generating a surplus $\alpha(N-1)u^B(p^m)$.

Things are more complicated when p_1 lies between $\underline{p}$ and p^m. In this case, $R(p_1)$ is given by:[11]

$$\begin{aligned} R^B(p_1) \quad = \quad & \left\{\alpha + (1-\alpha)(1-\Phi^{\min}_{N-1}(p_1))\right\}\pi(p_1) \\ + \quad & \alpha\left\{u^B(p_1) + (N-1)\left(\Phi_{N-1}(p_1)\mathbb{E}_{\Phi_{N-1}}[u^B(p)|\underline{p} \leq p \leq p_1] + (1-\Phi_{N-1}(p_1))u^B(p^m)\right)\right\} \\ + \quad & (1-\alpha)\left\{(1-\Phi^{\min}_{N-1}(p_1))u^B(p_1) + \Phi^{\min}_{N-1}(p_1)\mathbb{E}_{\Phi^{min}_{N-1}}[u^B(p)|\underline{p} \leq p \leq p_1]\right\}. \end{aligned} \tag{12}$$

There are three terms in the right hand side of equation (12). The first term is firm 1's profits: firm 1 always sells its applications to core users and only sometimes to casual users. The second term is the core users' surplus: they buy firm 1's application at price p_1 and the other applications at a price lower than p_1 or equal to p^m. The third term is the casual users' surplus: they buy firm 1's application when it is the cheapest one or another application otherwise. If the platform commits to a high price, it chooses the price p_1 to balance various effects. Indeed, taking the derivative of $R^B(.)$ w.r.t. p_1 in equation (12), we find:

$$\begin{aligned} (R^B)'(p_1) \quad = \quad & \left\{\alpha + (1-\alpha)(1-\Phi^{\min}_{N-1}(p_1))\right\}w'(p_1) - (1-\alpha)\left(\Phi^{\min}_{N-1}\right)'(p_1)\pi(p_1) \\ & +\alpha(N-1)\left(\Phi_{N-1}\right)'(p_1)\left(u^B(p_1) - u^B(p^m)\right). \end{aligned} \tag{13}$$

Equation (13) shows that an increase in p_1 has three effects on $R^B(p_1)$: two negative effects and one positive effect. First, following an increase in p_1, the surplus per transaction performed by firm 1 decreases ($w'(p_1) < 0$). Second, casual users buy firm 1's application less often so that platform's revenue decreases. Third, core users benefit from a more intense competition for casual users between developers: they purchase applications at a lower price on average. The overall effect is ambiguous. Let $\hat{p}$ denote the price that maximizes $R^B(.)$ on

[11] The algebra yielding this formula are cumbersome and can be found in appendix.

the interval $[\underline{p}, p^m]$.

Now, notice that $R^B(.)$ is decreasing on $[0, \underline{p}]$ (see equation (11)). Therefore, the platform either commits to $p_1 = 0$ or $p_1 = \hat{p}$.

Proposition 6. *When users are optimistic, the vertically integrated platform either commits to commits to $p_1 = 0$ if $R^B(0) > R^B(\hat{p})$ and to $p_1 = \hat{p}$ otherwise.*

By committing to a low downstream price, the platform sells its application to all users but at the cost that the other developers do not compete for casual users, which is harmful for core users. On the other hand, if the platform commits to a high downstream price, core users may obtain a higher surplus since they benefit from intense competition between developers for serving casual users. However, in this case, the platform sells fewer applications.

An interesting situation arises when the proportion of core users is small. When there are few core users, the fraction of users who benefit from an increase in p_1 is small so that the positive effect is dominated by the two negative effects. Put differently, the platform commits to $p_1 = 0$ and let the other developers sell their applications to core users only. This may explain for instance the business model of the Wii, the latest videogame console of Nintendo. Launched by the end of 2006, the Wii came with Wii Sports, a game developed by Nintendo and given for free with the console. The game is a collection of sport simulations which rules have been simplified to be accessible to new players. This was part of Nintendo's (successful) strategy to reach the audience of casual gamers.[12]

Pessimistic users. When users are pessimistic, there are essentially two decisions to be made by the platform: first, it chooses either a $D_B C_S$ or a $D_S C_B$ strategy; second, it commits either to 0 or to $\hat{p}$ in stage 1.

The main difference with the previous analysis is the additional role played by p_1 when users are pessimistic. As stated above, the first role of p_1 is to modify the outcome of downstream competition and, therefore, the surplus that can be extracted from firms and consumers. The additional role of p_1 is to modify consumers' expectations in stage 2 in the worst scenario, i.e. when they expect that no other developer than the vertically integrated one will support the platform. In this scenario, consumers anticipate that they will face a monopolist which charges p_1. As pointed out in section 4.1, this determines the maximum surplus that can be extracted from consumers in a $D_B C_S$ strategy. More precisely, in a $D_B C_S$

[12] Explaining the success of the Wii, The New York Times explains that the console appeals to "a broader audience than the traditional market of young male hard-core gamers. Younger children, women and older consumers, who historically have not been sought by the video-game industry, have discovered video games through the Wii – just not that many of them. These new gamers are content with the games they have, often going no further than the Wii Sports game that comes with the machine. They don't buy new games with the fervor of a traditional gamer who is constantly seeking new stimulation" ("New Wii Games Find a Big (but Stingy) Audience", *New York Times*, April 21, 2008).

strategy, the platform can charge consumers $a^B = u^B(p_1)$, i.e. the surplus they obtain when they face only the vertically integrated developer. Therefore, charging a low p_1 allows the platform to extract a quite large surplus from consumers even in a $D_B C_S$ strategy.

Let us first analyze the pricing commitment in a $D_B C_S$ strategy, i.e. when the platform decides to extract firms' surplus. Note first that, by Lemma 3, the surplus that the platform can extract from each firm does not depend on p_1, it is always equal to $\alpha\pi^m$. On the other side of the market, as already explained, the platform charges consumers $a^B = u^B(p_1)$ for participation. Besides, the platform also makes profits via its downstream division. First, it makes profits $\alpha\pi(p_1)$ on core users. Second, depending on p_1, it may also makes some profits on casual users: if $p_1 \leq \underline{p}$, all casual users purchase its application; if $p_1 > \underline{p}$, a casual user purchases its application with probability $1 - \Phi_{N-1}^{\min}(p_1)$. Hence, if $p_1 \leq \underline{p}$, the surplus that can be extracted from consumers is given by:

$$u^B(p_1) + \pi(p_1) = w(p_1)$$

and, if $p_1 > \underline{p}$, it is given by:

$$u^B(p_1) + (1 - \Phi_{N-1}^{\min}(p_1))\pi(p_1) = w(p_1) - (1-\alpha)\Phi_{N-1}^{\min}(p_1)\pi(p_1).$$

This surplus is decreasing in p_1, so that the platform commits to $p_1 = 0$ in a $D_B C_S$ strategy.

Consider now the pricing commitment in a $D_S C_B$ strategy. Notice that, the trade-off between charging a low or a high price in a $D_S C_B$ strategy is exactly the same as when users are optimistic. The intuition is rather simple. Since the level of p_1 affects the consumers' surplus but has no impact on developers' profits and since, in a $D_S C_B$ strategy, the platform captures only the consumers' surplus, the platform faces exactly the same trade-off as when users were optimistic.

This discussion shows that, when users are pessimistic, the platform chooses one of the three following strategies:

- $D_B C_S$ and $p_1 = 0$: $\Pi^P = w(0) + \alpha(N-1)\pi^m - f$.
- $D_S C_B$ and $p_1 = 0$: $\Pi^P = R^B(0) - f$.
- $D_S C_B$ and $p_1 = \hat{p}$: $\Pi^P = R^B(\hat{p}) - f$.

Because of the additional role played by p_1 when users are pessimistic, the integrated platform is more likely to commit to a low downstream price. Indeed, the platform may commit to $p_1 = 0$ even if $R^B(0) < R^B(\hat{p})$. This arises when $w(0) + \alpha(N-1)\pi^m > R^B(\hat{p})$.

We can now state whether the ability to credibly commit to a downstream price is sufficient to alleviate the negative result of Proposition 5. In the following, assume that

$\pi^m > u^B(p^m)$ and $\alpha > \hat{\alpha}$. By Proposition 5, in this case, vertical integration has a coordination value if $u^B(p^m) > f$. If a vertically integrated can credibly commit to its downstream price, it can at least make profits $w(0) + (N-1)\pi^m - f$, while an unintegrated platform makes profits $N\alpha\pi^m$. Therefore, a vertically integrated platform makes higher profits if $w(0) - \alpha\pi^m > f$. Now, notice that:

$$(w(0) - \alpha\pi^m) - u^B(p^m) \geq w(0) - w(p^m) > 0.$$

In words, vertical integration is more likely to be profitable when the platform is able to commit to its downstream price. The proposition below follows immediately.

Proposition 7. *Assume that $\pi^m > u^B(p^m)$ and $\alpha > \hat{\alpha}$. Then, a sufficient condition for vertical integration to have a coordination value when the platform can commit to its downstream price is: $w(0) - \alpha\pi^m > f$. Besides, this condition is less restrictive than $u^B(p^m) > f$.*

Proposition 7 illustrates the connection between the coordination value of vertical integration and the ability of a platform to commit to its downstream price. This connection seems relevant to explain why vertically integrated platforms often offer free services that raises the intrinsic value of the platform. For instance, operating systems like Windows or MacOS come with internet browsers (Internet Explorer, Safari), multimedia players (Media Player, Quicktime),... developed by in-house developers. Our analysis shows that this strategy can be part of a complex divide-and-conquer strategy in which the platform both subsidizes participation of users on one side of the market and raises the intrinsic value of the platform for those users by committing that there will be at least one cheap application available on the platform.

5 Conclusion

This paper has shown that vertical integration may not be effective in solving the chicken-and-egg coordination problem which arises in two-sided market. It also emphasizes that the coordination value of vertical integration in two-sided markets is closely related to the ability of a platform to commit to its downstream price. Finally, it makes a theoretical contribution to the literature on two-sided markets. The framework developed in this paper seems convenient to analyze the competition between vertically integrated platforms. This is left for further research.

A Appendix

A.1 Proof of Lemma 1

Proof. Assume that there exists a pure strategy equilibrium $P = (p_i)_{i=1,\dots,n}$ in stage 3. Let $p_{(1)} = \min_{i=1,\dots,n} p_i$ and denote by $\sigma \subset \{1,\dots,n\}$ the set of firms which set $p_{(1)}$ in equilibrium. Let n_σ denotes the cardinality of σ. Denote by $\underline{p}$ the unique price such that $\pi(\underline{p}) = \alpha\pi^m$ and $\underline{p} < p^m$.

Case 1: $p_{(1)} < \underline{p}$. Notice first that, if there is only one firm which sets $p_{(1)}$, then, its profits are given by $\pi(p_{(1)})$. Therefore, for all $i \in \sigma$, firm i's profits are bounded above by $\pi(p_{(1)})$. Notice now that $\pi(p_{(1)})$ is lower than $\pi(\underline{p})$, since function $\pi(.)$ is increasing on the interval $[0, p^m]$. Therefore, each firm $i \in \sigma$ can make strictly higher profits by setting its monopoly price. Indeed, if it sets p^m, then, it sells its application to core users only and it makes profits $\pi_i = \alpha\pi^m = \pi(\underline{p}) > \pi(p_{(1)})$. This is a contradiction.

Case 2: $p_{(1)} = \underline{p}$. Suppose first that $n_\sigma \geq 2$, i.e. there are at least two firms which set $p_{(1)}$. For all $i \in \sigma$, firm i's profits are given by:

$$\begin{aligned}
\pi_i &= \alpha\pi(\underline{p}) + (1-\alpha)\tfrac{1}{n_\sigma}\pi(\underline{p}), \\
&< \alpha\pi(\underline{p}) + (1-\alpha)\pi(\underline{p}), \\
&< \pi(\underline{p}) = \alpha\pi^m.
\end{aligned}$$

The last inequality shows that each firm $i \in \sigma$ is strictly better off setting the monopoly price. This is a contradiction.

Suppose now that $n_\sigma = 1$. Let $p_{(2)}$ denote the second lowest price, i.e. $p_{(2)} = \min_{i\in\{1,\dots,n\}\setminus\sigma} p_i$. Let i denote the index of the developer which set $p_{(1)}$. Firm i's profits are given by $\pi(p_{(1)})$, since it sells its application to all users. Notice that firm i can raise its price while still having the lowest price since $p_{(1)} < p_{(2)}$. In doing so, it strictly raises its profits, since function $\pi(.)$ is increasing on the interval $[0, p^m]$. This is a contradiction.

Case 3: $p_{(1)} > \underline{p}$. It is immediate that each firm $i \in \sigma$ is strictly better off setting its price slightly below $p_{(1)}$. This is a contradiction. □

A.2 Proof of Lemma 2

Proof. Notice first that firms never charge a price higher than p^m in equilibrium, since they can obtain higher profits by charging their monopoly price p^m. Denote by $\underline{p}$ the unique price such that $\pi(\underline{p}) = \alpha\pi^m$ and $\underline{p} < p^m$. In a mixed strategy equilibrium, if it exists, a firm never sets a price below $\underline{p}$, since it can obtain strictly higher profits by setting p^m. This shows that the support of a mixed strategy equilibrium, if it exists, must be a subset of $[\underline{p}, p^m]$.

Let us first prove that there exists a unique symmetric mixed strategy equilibrium with support $[\underline{p}, p^m]$. Suppose that each firm i, $i \in \{1, \ldots, n\}$, plays a symmetric mixed strategy $\Phi_n(.)$ on the interval $[\underline{p}, p^m]$ and assume that $\Phi_n(.)$ is atomless. Note first that each firm i can obtain $\alpha\pi^m$ by playing the pure strategy $p_i = p^m$. Since p^m is in the support of $\Phi_n(.)$, firm i should obtain $\alpha\pi^m$ for all $p \in [\underline{p}, p^m]$, so that:

$$(1-\Phi_n(p))^{n-1}\pi(p) + (1-(1-\Phi_n(p))^{n-1})\alpha\pi(p) = \alpha\pi^m,$$

which immediately yields:

$$\Phi_n(p) = 1 - \left(\frac{\alpha}{1-\alpha}\frac{\pi^m - \pi(p)}{\pi(p)}\right)^{\frac{1}{n-1}}. \tag{14}$$

Clearly, equation (14) defines a unique $\Phi_n(p)$ for all $p \in [\underline{p}, p^m]$. Put differently, there exists a unique symmetric mixed strategy equilibrium with support $[\underline{p}, p^m]$.

Then, let us prove that there exists no symmetric mixed strategy equilibrium with support $S \subset [\underline{p}, p^m]$ and atom points. Assume the contrary, i.e. there exists a symmetric mixed strategy equilibrium where each firm put some positive weight λ on $\tilde{p} \in S$. In this equilibrium, denote by $q_{inf} = Pr\{p_i < \tilde{p}\}$ and $q_{sup} = Pr\{p_i > \tilde{p}\}$, where p_i is firm i's price. In equilibrium, since $\tilde{p} \in S$, firm i should obtain the profits it would obtain by playing the pure strategy $\tilde{p}$. These profits are given by:

$$\begin{aligned}\tilde{\pi} = \quad & Pr\{\exists j \neq i / p_j < \tilde{p}\}\,\alpha\pi(\tilde{p}) \\ + \quad & \textstyle\sum_{k=0}^{n-1} Pr\{k \text{ firms set } p_j = \tilde{p} \text{ while the others set } p_j > \tilde{p}\}\left(\alpha\pi(\tilde{p}) + (1-\alpha)\frac{1}{k+1}\pi(\tilde{p})\right),\end{aligned}$$

or equivalently by:

$$\tilde{\pi} = \left(1-(\lambda+q_{sup})^{n-1}\right)\alpha\pi(\tilde{p}) + \sum_{k=0}^{n-1} C_{n-1}^k \lambda^k q_{sup}^{n-1-k}\left(\alpha\pi(\tilde{p}) + (1-\alpha)\frac{1}{k+1}\pi(\tilde{p})\right), \tag{15}$$

where $C_{n-1}^k = \frac{(n-1)!}{k!(n-1-k)!}$. Notice first that, by the binomial theorem, we have:

$$\sum_{k=0}^{n-1} C_{n-1}^k \lambda^k q_{sup}^{n-1-k}\alpha\pi(\tilde{p}) = (\lambda+q_{sup})^{n-1}\alpha\pi(\tilde{p}).$$

There is still one term to calculate in equation (15): $A = \sum_{k=0}^{n-1} C_{n-1}^k \lambda^k q_{sup}^{n-1-k}(1-\alpha)\frac{1}{k+1}\pi(\tilde{p})$.

Define the polynomial $P(X)$ by:

$$P(X) = \sum_{k=0}^{n-1} C_{n-1}^{k} \lambda^{k} q_{sup}^{n-1-k} \frac{1}{k+1} X^{k+1}.$$

Notice that $A = P(1)(1-\alpha)\pi(\tilde{p})$. Then, notice that the derivative of P is given by:

$$P'(X) = \sum_{k=0}^{n-1} C_{n-1}^{k} \lambda^{k} q_{sup}^{n-1-k} X^{k} = (\lambda X + q_{sup})^{n-1},$$

where the second equality stems from the binomial theorem. Therefore, polynomial P is given by:

$$P(X) = \frac{1}{n\lambda}\left((\lambda X + q_{sup})^{n} - q_{sup}^{n}\right).$$

In particular, $A = \frac{1}{n\lambda}\left((\lambda + q_{sup})^{n} - q_{sup}^{n}\right)(1-\alpha)\pi(\tilde{p})$ and we therefore have:

$$\tilde{\pi} = \alpha\pi(\tilde{p}) + \frac{1}{n\lambda}\left((\lambda + q_{sup})^{n} - q_{sup}^{n}\right)(1-\alpha)\pi(\tilde{p}). \tag{16}$$

Now, assume that firm i deviates and sets p_i slightly below $\tilde{p}$. Then, firm i's profits are approximatively given by:

$$\alpha\pi(\tilde{p}) + (\lambda + q_{sup})^{n-1}(1-\alpha)\pi(\tilde{p}). \tag{17}$$

Then, define function $g(.)$ by:

$$g(\lambda) = n\lambda(\lambda + q_{sup})^{n-1} - \left((\lambda + q_{sup})^{n} - q_{sup}^{n}\right).$$

By equation (16) and (17), it is immediate that firm i's deviation is profitable if $g(\lambda) > 0$. Let us prove that, for all $\lambda > 0$, $g(\lambda) > 0$. Notice first that $g(0) = 0$. Then, taking the derivative of $g(.)$ w.r.t. λ, we obtain:

$$g'(\lambda) = (n-1)(\lambda + q_{sup})^{n-2}\left((n+1)\lambda + q_{sup}\right) > 0.$$

Therefore, function $g(.)$ is strictly increasing in λ. Then, in particular, since $g(0) = 0$, for all $\lambda > 0$, $g(\lambda) > 0$. Firm i is therefore strictly better off setting a price slightly below $\tilde{p}$, a contradiction.

Last, let us now prove that there exists no symmetric mixed strategy equilibrium which support is a strict subset of $[\underline{p}, p^m]$. Assume the contrary, i.e. there exists a symmetric mixed strategy equilibrium where each firm plays according to a cumulative distribution $\xi(.)$ on

$S \subset [\underline{p}, p^m]$, where $S \neq [\underline{p}, p^m]$. Denote by p_{inf} and p_{sup} the infimum and supremum of S. Notice first that $p_{sup} = p^m$. Indeed, if a given firm i sets $p_i = p_{sup}$, then, it sells its application to core users only. Then, it makes profits $\alpha\pi(p_{sup})$. Since $\xi(.)$ is an equilibrium, we must have $\alpha\pi(p_{sup}) \geq \alpha\pi(p^m)$. Since $p_m = \arg\max_p \pi(p)$, we thus have $p_{sup} = p^m$. It immediately follows that $p_{inf} = \underline{p}$. Indeed, if a given developer sets $p_i = p_{inf}$, then, it sells its applications to all users and makes profits $\pi(p_{inf})$. Since $\xi(.)$ is a mixed strategy equilibrium, these profits must be equal to $\alpha\pi^m$. By definition of $\underline{p}$, this implies that $p_{inf} = \underline{p}$.

Since S is strictly included in $[\underline{p}, p^m]$, $p_{inf} = \underline{p}$ and $p_{sup} = p^m$, there exists an interval $I \subset S$ such that firms put no weight on each price in I. Denote by p^I_{inf} and p^I_{sup} the infimum and supremum of I. We can choose p^I_{inf} such that $p^I_{inf} \in S$. Let $p \in I \backslash \{p^I_{inf}, p^I_{sup}\}$. Denote by q the probability that $n-1$ firms set their prices below p^I_{inf}:

$$q = \xi(p^I_{inf})^{n-1}.$$

Notice that q only depends on $\xi(.)$ and p^I_{inf}. If firm i, $i \in \{1, \ldots, n\}$, sets $p_i = p$, then, it makes profits:

$$\begin{aligned}
\pi_i(p) &= q \cdot \alpha\pi(p) + (1-q) \cdot \pi(p), \\
&> q \cdot \alpha\pi(p^I_{inf}) + (1-q) \cdot \pi(p^I_{inf}),
\end{aligned}$$

because function $\pi(.)$ is increasing on the interval $[\underline{p}, p^m]$. Since $p^I_{inf} \in S$, this is a contradiction.

□

A.3 Proof of Proposition 1

Proof. Depending on a^B and a^S, either all developers, only one of them or none of them join the platform. If the platform sets a_S between $\alpha\pi^m$ and π^m and a^B below $u^B(p^m)$, then, one developer and all consumers join the platform. Hence, if it captures all users' surplus, it obtains profits $\pi^m + u^B(p^m) = w(p^m)$. On the other hand, if the platform sets a^S below $\alpha\pi^m$ and a^B below S^B_N, then, all firms and consumers join the platform. Therefore, it can obtain profits $N\alpha\pi^m + S^B_N$. Now, notice that:

$$\begin{aligned}
N\alpha\pi^m + S^B_N &= N\alpha\pi^m + \alpha N \mathbb{E}_{\Phi_N}[u^B(p)] + (1-\alpha)\mathbb{E}_{\Phi^{\min}_N}[u^B(p)], \\
&\geq N\alpha\pi^m + \alpha(N-1)\mathbb{E}_{\Phi_N}[u^B(p)] + \mathbb{E}_{\Phi_N}[u^B(p)], \\
&\geq \mathbb{E}_{\Phi_N}[\pi(p) + u^B(p)] + (N-1)\alpha\left(\pi^m + \mathbb{E}_{\Phi_N}[u^B(p)]\right), \\
&\geq w(p^m) + (N-1)\alpha\left(\pi^m + \mathbb{E}_{\Phi_N}[u^B(p)]\right),
\end{aligned}$$

where the first inequality stems from the fact that $\Phi_N(.)$ first-order stochastically dominates $\Phi^{\min}_N(.)$, so that $\mathbb{E}_{\Phi^{\min}_N}[u^B(p)] \geq \mathbb{E}_{\Phi_N}[u^B(p)]$. Since $(N-1)\alpha\left(\pi^m + \mathbb{E}_{\Phi_N}[u^B(p)]\right)$ is positive, the platform prefers that all users participate.

□

A.4 Proof of Proposition 5

Proof. Hereafter, denote by Π_u^P and Π_{vi}^P the platform's profits when it is unintegrated and vertically integrated respectively.

Assume that $\pi^m < u^B(p^m)$. In this case, equations (7) and (9) together show that both an unintegrated and a vertically integrated platforms choose a $D_S C_B$ strategy. Therefore, Π_u^P and Π_{vi}^P are given by:

$$\begin{aligned}\Pi_u^P &= S_N^B,\\ \Pi_{vi}^P &= S_N^B + \alpha\pi^m - f.\end{aligned}$$

Hence, vertical integration has a coordination value iff $\pi^m > f$.

Assume now that $\pi^m > u^B(p^m)$. By equations (8) and (10), both an unintegrated and a vertically integrated platform chooses a $D_B C_S$ strategy if:

$$\pi^m > \max\left\{\left(1+\frac{1-\alpha}{N\alpha}\right)u^B(\underline{p}), u^B(\underline{p}) + \frac{1}{(N-1)\alpha}\left(u^B(\underline{p}) - u^B(p^m)\right)\right\}. \qquad (18)$$

Then, notice that:

$$u^B(\underline{p}) + \frac{1}{(N-1)\alpha}\left(u^B(\underline{p}) - u^B(p^m)\right) - \left(1+\frac{1-\alpha}{N\alpha}\right)u^B(\underline{p}) = u^B(\underline{p})\frac{N\alpha+1-\alpha}{N\alpha} - u^B(p^m),$$

which is positive since $u^B(\underline{p}) \geq u^B(p^m)$ and $\frac{N\alpha+1-\alpha}{N\alpha} > 1$. Hence, equation (18) rewrites:

$$\pi^m > u^B(\underline{p}) + \frac{1}{(N-1)\alpha}\left(u^B(\underline{p}) - u^B(p^m)\right) = r(\alpha). \qquad (19)$$

Taking the derivative of $r(.)$ wrt to α, we obtain:

$$r'(\alpha) = (1+\frac{1}{(N-1)\alpha})\frac{\partial \underline{p}}{\partial\alpha}(u^B)'(\underline{p}) - \frac{1}{(N-1)\alpha^2}\left(u^B(\underline{p}) - u^B(p^m)\right).$$

Then, notice that $\partial\underline{p}/\partial\alpha < 0$, $(u^B)'(.) < 0$ and $u^B(\underline{p}) - u^B(p^m) \geq 0$ and conclude that $r'(\alpha) < 0$, so that function $r(.)$ is strictly decreasing in α. Now, observe that $\lim_{\alpha\to 0} r(\alpha) = \infty$ and $r(1) = u^B(p^m)$. Therefore, since function is decreasing and continuous in α and $\pi^m > u^b(p^m)$, there exists $\hat{\alpha} \in (0,1)$ such that, for all $\alpha > \hat{\alpha}$, inequality (19) holds. Besides, $\hat{\alpha}$ is given implicitly by the following equation:

$$\pi^m = u^B(\underline{p}) + \frac{1}{(N-1)\hat{\alpha}}\left(u^B(\underline{p}) - u^B(p^m)\right).$$

Assume that $\alpha > \hat{\alpha}$. Then, by definition of $\hat{\alpha}$, both an unintegrated and a vertically integrated platforms choose a $D_B C_S$ strategy. Therefore, Π_u^P and Π_{vi}^P are given by:

$$\begin{aligned} \Pi_u^P &= N\alpha\pi^m, \\ \Pi_{vi}^P &= N\alpha\pi^m + u^B(p^m) - f. \end{aligned}$$

Hence, vertical integration has a coordination value iff $u^B(p^m) > f$.

□

A.5 Proof of Lemma 3

Proof. Hereafter, we only establish existence of a symmetric equilibria among firms $2, \ldots, N$. The uniqueness of symmetric equilibrium can be prove by using the same techniques as in Lemma 2.

There are three cases:

First case: $p_1 \leq \underline{p}$. When $p_1 \leq \underline{p}$, the other $N-1$ firms do not compete for casual users. Indeed, a firm willing to sell its application to casual users should charge a price below p_1. By doing so, the maximum expected profit it could make is $\pi(p_1) \leq \alpha\pi^m$. Hence, each firm has a dominant strategy consisting in charging the monopoly price p^m.

Second case: $\underline{p} < p_1 < p^m$. By the same arguments as in Lemma 2, firms play mixed strategies on the interval $[\underline{p}, p^m]$. Notice first that a firm never puts any weight on a price $p \in]p_1, p^m[$. Indeed, if it does so, a firm does not increase its chance to serve casual users since $p_1 < p$ and, besides, it can make higher profits on core users by charging the monopoly price. This explains why firms put some positive weight on p_1 and play a mixed strategy on the interval $[\underline{p}, p_1]$. We first calculate $q(p_1)$. Given that the other $N-2$ firms set prices according to $(q(p_1), \xi(., p_1))$, firm i should obtain $\alpha\pi^m$ by setting p_i slightly below p_1 so that:

$$(1 - q(p_1))^{N-2}\pi(p_1) + (1 - (1 - q(p_1))^{N-2})\alpha\pi(p_1) = \alpha\pi^m.$$

We then obtain:

$$q(p_1) = 1 - \left(\frac{\alpha}{1-\alpha}\frac{\pi^m - \pi(p_1)}{\pi(p_1)}\right)^{\frac{1}{N-2}} = \Phi_{N-1}(p_1). \tag{20}$$

By setting $p \in [\underline{p}, p_1[$, a firm should obtain $\alpha\pi^m$. If it sets $p \in [\underline{p}, p_1[$, the expected profit of firm i is thus given by:

$$\begin{aligned}
& q^0(1-q)^{N-2}\pi(p) \\
+ \;& C_{N-2}^1 q^1(1-q)^{N-2-1}\left\{\xi(p,p_1)\alpha\pi(p) + (1-\xi(p,p_1))\pi(p)\right\} \\
+ \;& C_{N-2}^2 q^2(1-q)^{N-2-2}\left\{(1-(1-\xi(p,p_1))^2)\alpha\pi(p) + (1-\xi(p,p_1))^2\pi(p)\right\} \\
\vdots \; & \\
+ \;& C_{N-2}^k q^k(1-q)^{N-2-k}\left\{(1-(1-\xi(p,p_1))^k)\alpha\pi(p) + (1-\xi(p,p_1))^k\pi(p)\right\} \\
\vdots \; & \\
+ \;& C_{N-2}^{N-2} q^{N-2}(1-q)^{N-2-(N-2)}\left\{(1-(1-\xi(p,p_1))^{N-2})\alpha\pi(p) + (1-\xi(p,p_1))^{N-2}\pi(p)\right\} \\
= \;& \pi(p)\sum_{k=0}^{N-2} C_{N-2}^k q^k(1-q)^{N-2-k}\left\{\alpha + (1-\alpha)(1-\xi(p,p_1))^k\right\},
\end{aligned}$$

where $C_{N-2}^k = \frac{(N-2)!}{k!(N-2-k)!}$. Separating terms and applying the binomial theorem, we obtain:

$$= \quad \pi(p)\left\{\alpha + (1-\alpha)(1-q\xi(p,p_1))^{N-2}\right\}.$$

By setting $p \in [\underline{p}, p_1]$, a firm should obtain $\alpha\pi^m$. Hence, equalizing the previous expression to $\alpha\pi^m$, we get:

$$q(p_1)\xi(p,p_1) = 1 - \left(\frac{\alpha}{1-\alpha}\frac{\pi^m - \pi(p)}{\pi(p)}\right)^{\frac{1}{N-2}} = \Phi_{N-1}(p).$$

Then, by equation (20), we immediately have that, for all $p \in [\underline{p}, p_1]$:

$$\xi(p,p_1) = \frac{\Phi_{N-1}(p)}{\Phi_{N-1}(p_1)}. \tag{21}$$

Third case: $p_1 \geq p^m$. When $p_1 \geq p^m$, the vertically integrated firm does not compete for casual users. Therefore, everything is such that the $N-1$ other firms compete for casual users. By lemma 2, they thus play a mixed strategy on the interval $[\underline{p}, p^m]$ according to the cumulative distribution $\Phi_{N-1}(.)$.

□

A.6 Proof of equation 12

Proof. We establish here the formula given in equation (12):

$$\begin{aligned}
R^B(p_1) \quad = \quad & \left\{\alpha + (1-\alpha)(1-\Phi_{N-1}^{\min}(p_1))\right\}\pi(p_1) \\
& +\alpha\left\{u^B(p_1) + (N-1)\left(\Phi_{N-1}(p_1)\mathbb{E}_{\Phi_{N-1}}[u^B(p)|\underline{p} \leq p \leq p_1] + (1-\Phi_{N-1}(p_1))u^B(p^m)\right)\right\} \\
& +(1-\alpha)\left\{(1-\Phi_{N-1}^{\min}(p_1))u^B(p_1) + \Phi_{N-1}^{\min}(p_1)\mathbb{E}_{\Phi_{N-1}^{min}}[u^B(p)|\underline{p} \leq p \leq p_1]\right\}.
\end{aligned}$$

$R(p_1)$ is the sum of three terms: platform's downstream profits, core users' surplus and casual users' surplus.

1. Platform's downstream profits. When $p_1 \in]\underline{p}, p^m]$, the platform sells its application to all core users. However it sells its application to a casual user with probability $1-\Phi^{\min}_{N-1}(p_1)$. Platform's downstream profits are thus given by:

$$\left(\alpha + (1-\alpha)(1-\Phi^{\min}_{N-1}(p_1))\right)\pi(p_1). \tag{22}$$

2. Core users' surplus. A core user purchases the platform's application at price p_1 and the others at a random price (see Lemma 3). The core users' surplus is thus given by:

$$u^B(p_1) + (N-1)\left\{\Phi_{N-1}(p_1)\mathbb{E}_{\Phi_{N-1}}[u^B(p)|\underline{p} \le p \le p_1] + (1-\Phi_{N-1}(p_1))u^B(p^m)\right\}. \tag{23}$$

3. Casual users' surplus. The price at which a casual user purchases his application depends on the number of developers which sell their applications below p_1. For instance, with probability $(1-q(p_1))^{N-1}$ (where $q(p_1)$ is defined in Lemma 3), a casual user purchases the platform's application. The casual users' surplus is given by:

$$\begin{aligned}
& (1-q(p_1))^{N-1}u^B(p_1) \\
+ \quad & C^1_{N-1}q(p_1)^1(1-q(p_1))^{N-1-1}\textstyle\int_{\underline{p}}^{p_1} u^B(p)d\left(1-(1-\xi(p,p_1))^1\right) \\
\vdots \quad & \\
+ \quad & C^k_{N-1}q(p_1)^k(1-q(p_1))^{N-1-k}\textstyle\int_{\underline{p}}^{p_1} u^B(p)d\left(1-(1-\xi(p,p_1))^k\right) \\
\vdots \quad & \\
+ \quad & C^{N-1}_{N-1}q(p_1)^{N-1}(1-q(p_1))^{N-1-(N-1)}\textstyle\int_{\underline{p}}^{p_1} u^B(p)d\left(1-(1-\xi(p,p_1))^{N-1}\right) \\
= \quad & (1-q(p_1))^{N-1}u^B(p_1) + \textstyle\sum_{k=1}^{N-1} C^k_{N-1}q(p_1)^k(1-q(p_1))^{N-1-k}\int_{\underline{p}}^{p_1} u^B(p)d\left(1-(1-\xi(p,p_1))^k\right),
\end{aligned}$$

or equivalently by

$$= \quad (1-q(p_1))^{N-1}u^B(p_1) + \textstyle\int_{\underline{p}}^{p_1} u^B(p)d\left(\sum_{k=1}^{N-1} C^k_{N-1}q(p_1)^k(1-q(p_1))^{N-1-k}(1-(1-\xi(p,p_1))^k)\right).$$

Notice that there is no term of index $k=0$ in the sum. Yet, this term does not depend on p so that its derivative wrt to p is equal to 0. We can thus add it into the above formula without modifying the equality. Separating terms and applying the binomial theorem twice, we get:

$$(1-q(p_1))^{N-1}u^B(p_1) + \int_{\underline{p}}^{p_1} u^B(p)d\left(1-(1-q(p_1)\xi(p,p_1))^{N-1}\right).$$

Replacing $q(p_1)$ and $\xi(p, p_1)$ by their expressions (see Lemma 3), we finally obtain:

$$(1 - \Phi_{N-1}^{\min}(p_1))u^B(p_1) + \int_{\underline{p}}^{p_1} u^B(p)d\Phi_{N-1}^{\min}(p). \tag{24}$$

Summing (22), (23) times α and (24) times $1 - \alpha$ finally yields the announced result.

□

References

Armstrong, M. (2006): "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37(3).

Belleflamme, P., and E. Toulemonde (2007): "Negative Intra-Group Externalities in Two-Sided Markets," Cesifo working paper series, CESifo GmbH.

Caillaud, B., and B. Jullien (2003): "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, 34(2), 309–28.

Dasgupta, P., and E. Maskin (1986): "The Existence of Equilibrium in Discontinuous Economic Games, I: Theory," *Review of Economic Studies*, 53(1), 1–26.

Evans, D. S., A. Hagiu, and R. Schmalensee (2006): *Invisible Engines: How Software Platforms Drive Innovation and Transform Industries*. MIT Press, Cambridge, MA.

Hagiu, A. (2006): "Pricing and Commitment by Two-Sided Platforms," *RAND Journal of Economics*, 37(3), 720–737.

Lee, R. S. (2007): "Vertical Integration and Exclusivity in Two-Sided Markets," Working Papers 07-39, NET Institute.

Rochet, J.-C., and J. Tirole (2003): "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 1(4), 990–1029.

——— (2006): "Two-Sided Markets: A Progress Report," *RAND Journal of Economics*, 37(3).