COMPETING SEARCH ENGINES WITH UTILITY MAXIMISING CONSUMERS AND ORDINARY SEARCH RESULTS

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Summary: I model strategic interaction amongst search engines that compete to serve consumer needs. Search engines generate revenue from advertisers, but also provide free organic search results. I demonstrate that, in an attempt to win market share, the search engines compete not only against each other, but also against themselves: providing high-quality free links that compete for clicks with their own advertisements—thus cannibalising their advertising revenues. In particular, I find that in equilibrium consumers always (at least weakly) prefer to click on at least one non-paid-for link before clicking on a revenue generating advertisement link so that some consumers never click an advertisement. I also show that reductions in the strength of competition in the search industry can facilitate a transition to new equilibria with lower link quality, which may be an important consideration for regulators.

Keywords: search engines; vertical differentiation; organic links; advertising; sponsored search.

JEL classification: D43; L13; L15

I INTRODUCTION

In this paper I build a simple model of the competitive environment within which search engines operate and, in particular, examine the interplay between paid-for advertisements (or A-links) and free non-advertisement search results (organic links, or O-links) when there are consumers that search optimally.

A search engine ostensibly competes for A-link clicks, and thus has an incentive to provide a high quality of service in order to win market share. Intuitively, by

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providing high quality O-links, a search engine attracts consumers to visit its site first. This is beneficial if the same consumers, in an attempt to minimise search costs, stay to also click on advertisements, rather than continuing their search elsewhere. However, there exists a countervailing effect since search engines face competition for A-link clicks not only from links at rival search engines, but also from their own O-links. Thus, the market is characterised by a kind of revenue cannibalisation that results in a delicate trade-off between the complementary effects of O-links (the incentive to compete for market share) and the desire to minimise the extent to which revenues are cannibalised.

The extent of equilibrium cannibalisation is variable. Sometimes, global free-link quality can get 'too good' so that consumers find it profitable to switch search engines and continue clicking free links on another site, rather than clicking advertisements. In this case, there is a natural ceiling to how good the free links can get in equilibrium. In other cases, search engines set their quality to the maximum possible so that competition is, in some sense, maximal. The cannibalisation effect also engenders a class of equilibrium in which quality is comparatively low, even when the provision of quality is costless. What all oligopoly equilibria have in common is that the quality of non-paid-for links is at least as good as that of advertisements, so that consumers (at least weakly) prefer to click on a non-paid for link first. If the market exists as a monopoly then the 'competition for market share' effect disappears and revenue cannibalisation compels the monopolist search engine to set a low quality.

I also analyse competition when consumers have heterogeneous visit costs for each search engine. This gives each search engine a degree of monopoly power over those consumers located nearby which weakens the competition effect relative to that of revenue cannibalisation so that equilibrium qualities are often lower. Nevertheless, I find that the result that organic links are at least as good as their advertising counterparts in equilibrium is robust to this relaxation of the model's assumptions.

I.1 Literature

Gandal (2001) conducts an empirical study of competition within the Internet search engine market. Two results are of particular interest for the current work: firstly, Gandal finds that (on average) consumers use more than one search engine, so that consumers are demonstrably willing to switch between engines in order to find what they are looking for. Secondly, it is shown that the relevance of search results is by far the most important determinant of search engine ranking (by number of searches conducted). This suggests that search engines have a strong incentive to compete on result quality. Taken together with the obvious importance of search engines in modern society, these results motivate and justify the model developed below.

Most closely related to this work is the paper of White (2008), who obtains results similar in spirit to those presented here using a monopolist search engine that offers both advertisement and non-advertisement links. The non-advertising links reduce consumers' search costs and induce more consumers to search—increasing the demand faced by advertising merchants—but simultaneously provide competition for those consumers' business, thus reducing price in the final goods market. This competition amongst the sellers reduces the amount that they are willing to pay for advertisements. The search engine trades-off these two factors to maximise the profits it is able to generate when charging a fixed fee to advertisers.

Telang, Rajan, and Mukhopadhyay (2004), and Pollock (2008) have theoretical models of the search industry with competing search engines. Telang, Rajan, and Mukhopadhyay (2004) model entry in the Internet search industry and attempt to explain the existence of low quality firms when consumers pay a price of zero. Pollock (2008) demonstrates a tendency for concentration in the internet search industry and then explores a number of welfare and regulation issues with a monopolist search provider. In both papers, search engine revenues are treated in a reduced form fashion. In this paper, by contrast, I explicitly allow the consumer to choose the *type* of link that he clicks, which makes the value of a visit endogenous to the search engine's choice of quality. This introduces a number of new equilibrium considerations which lead to the cannibalisation discussed above.

There is also a large literature on the relationship between advertising and other media content—for a summary see Bagwell (2007) section 10. Examples include Anderson and Coate (2005), who analyse the provision of programs and advertisements by radio and television broadcasters, and Gabszewicz, Laussel, and Sonnac (2001), who study advertising in the newspaper press. In both cases, much as for search engines, advertisements are provided alongside additional content that is attractive to consumers but does not generate revenue directly. The Internet search advertising market differs, however, in the fact that organic search results compete directly for clicks with the revenue generating advertisements on the same site.

This work also has links to other branches of the literature. In particular, Varian (2007); Edelman, Ostrovsky, and Schwarz (2007); and Athey and Ellison (2007) model the 'position' auction framework that search engines typically use to sell advertisements, whilst Caillaud and Jullien (2001, 2003); and Rochet and Tirole (2003), amongst others model price competition in two-sided markets of which Internet search is often used as an archetypal example.

The remainder of this paper is organised as follows: In section II, I establish a duopoly model of search engine competition in an environment with a mass of identical consumers and then, in section III, proceed to characterise equilibrium behaviour within this model. I generalise the duopoly model to the monopoly and oligopoly cases in section IV and, In section V, I relax the assumption of consumer homogeneity and analyse competition when consumers have heterogeneous costs for visiting each search engine. Section VI concludes.

II Simple Model

A mass 1 continuum of homogeneous risk-neutral consumers in this model have a particular need or desire that they seek to satisfy by searching the Internet. Each time a consumer visits a search engine they must pay a visit cost, $S > 0$. When the search results are returned, the consumer may click on them as he or she pleases, but must pay a further cost, $s > 0$, for each link that is clicked. If a clicked link matches the consumer's need then the consumer receives a surplus, which I normalise to 1. In the case of internet search, the switching cost, S, may be small. However, I demonstrate below that it is that size of this switching cost relative to the cost of clicking a further link at the current site that is of greatest importance.

Two search engines, g and y, provide search results to consumers at a marginal cost of zero. A search at a given search engine returns two results: one organic (non-advertising) search result (O-link), and one advertising search result (A-link). Let A_i and O_i respectively denote the A-link and O-link at site i. The search engines simultaneously choose an algorithm which results in a quality for their O-links, denoted by $p_i \in [0, p^{max}], i = g, y$. This quality is the probability that the O-link at site i is able to satisfy a given consumer's need. I assume that satisfaction is statistically independent across consumers. I wish to focus attention on the pure incentives for quality choice in search competition, rather than on the broader competition in R&D which, although frantic, occurs over longer time scales. In particular, I wish to clearly identify the extent to which limitation of result quality is due to the cannibalisation effect. I thus assume that a commonly, and freely accessible search technology already exists, and permits qualities up to p^{max}, which can be thought of as the maximum technologically feasible algorithm quality.

The A-link also has a quality, q. I make the assumption that the same link appears in the A-link slot at both search engines, so that both search engines also share the same q.[2] I assume that the A-link and O-link point to websites each drawn from separate pools of firms so that the two are always different.

I assume that the consumers are able to observe the match probabilities p_g, p_y, and q. When the A-link at a search engine is clicked, that search engine receives a per-click price, which I denote by b. The search engine receives nothing when its O-link is clicked.

Let μ_m denote the match probability of the m^{th} link that a consumer clicks, and denote by C_m the total number of times that the consumer has paid the visit cost, S, by the time that he has clicked on the m^{th} link.[3] The consumer's expected utility from clicking M links is of the form

$$(1) \quad U = \sum_{m=1}^{M} \left[\left(\prod_{k=1}^{m-1} (1-\mu_k) \right) \mu_m \left(1 - C_m S - ms\right) \right] + \left[\left(\prod_{k=1}^{M} (1-\mu_k) \right) \left(-C_M S - Ms\right) \right].$$

To summarise: search engines move first and simultaneously select a quality p_i. Consumers observe p_g, p_y, q, S and s, and select whether, and in which order to click each link. The game ends when all consumers have either had their need met or do not wish to click any further links.

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[2]This assumption could, for example, be justified by observing that, with a single A-link at each site, advertiser-firms bidding per-click prices in a position auction have a dominant strategy to bid up to their value—which is given by the expected profit per click. When multiple search engines sell a single slot simultaneously, it remains a dominant strategy for each firm to bid up to its valuation in each auction. If there is no systematic difference in consumption habits of the users of each search engine then the result should be the same firm winning each auction.

[3]Thus, $C_1 = 1$. If the consumer clicks the first two links at the same site then $C_2 = 1$, otherwise $C_2 = 2$. $C_3 \in \{2, 3\}$.

III Equilibrium Behaviour in the Simple Model

III.1 Optimal Consumer Behaviour

The problem faced by each consumer in this model is to determine whether to click on each link and in which order to do so. A strategy for a consumer specifies these actions as a function of the choices of p_g and p_y, as well as the model parameters S, s, and q. Since the consumers are aware that both sites will display the same A-link in equilibrium, they only ever click the A-link at one of the two sites. Once a consumer's need is met, that consumer always stops clicking on links. Note that since all consumers are assumed homogeneous they will agree on a preference ordering over the set of possible strategies.

If $q < s$ then no consumer ever clicks an A-link and search engines, which make zero profits, are indifferent across all choices of p. When $s \leq q < S + s$, consumers click on an A-link if and only if $max\,\{p_g, p_y\} \geq S + s$ (which will always be true in equilibrium), in which case the below analysis remains fundamentally unchanged. Throughout the remainder of this and the following section, I assume that $q \in (S + s, 1)$, so that consumers are always willing to click on A-links.[4]

Given the form of (1), it is possible to characterise the best response strategy for a consumer to each possible $\{p_g, p_y, q\}$, and this is done in strategy 1. First, let

$$e \equiv \begin{cases} g \text{ if } p_g > p_y \text{ and } p_g \geq s \\ y \text{ if } p_g < p_y \text{ and } p_y \geq s \\ g \text{ w.p. } \alpha, y \text{ w.p. } 1-\alpha \text{ if } p_g = p_y \text{ or } \max\{p_g, p_y\} < s, \end{cases}$$

$$-e = \{g, y\} - \{e\}.$$

Strategy 1 *Suppose that $q \geq S + s$. A consumer's best response strategy maps the link qualities $\{p_g, p_y, q\}$ into a click order $\{a_1, a_2, a_3\}$ as follows. Begin by clicking a_1 thus:*

$$a_1 = \begin{cases} O_e \textit{ if } p_e > q \\ A_e \textit{ if } p_e < q \\ A_e \textit{ w.p. } \lambda_e, O_e \textit{ w.p. } 1-\lambda_e \textit{ if } p_e = q, \end{cases}$$

[4] In order to simplify the exposition, I am also ruling out the trivial case in which $q = 1$. Briefly, $q = 1$ gives rise to a continuum of uninteresting equilibria in which, search engines choose an arbitrary $p \in [0, 1)$, and consumers click an A-link at an arbitrarily chosen search engine resulting in immediate satisfaction.

If the consumer's need was met by a_1 then stop clicking (i.e. $a_2 = a_3 = \emptyset$), otherwise click a_2 as follows:

$$a_2 = \begin{cases} A_e \textit{ if } a_1 = O_e \textit{ and } p_{-e} < \gamma \\ O_{-e} \textit{ if } a_1 = O_e \textit{ and } p_{-e} > \gamma \\ A_e \textit{ w.p. } \phi_e, O_{-e} \textit{ w.p. } 1-\phi_e \textit{ if } a_1 = O_e \textit{ and } p_{-e} = \gamma \\ O_e \textit{ if } a_1 = A_e \textit{ and } p_e \geq s \\ \emptyset \textit{ if } p_e < s \end{cases}$$

If the consumer's need was met by a_1 or a_2 then stop clicking (i.e. $a_3 = \emptyset$), otherwise click a_3 as follows:

$$a_3 = \begin{cases} O_{-e} \textit{ if } \{a_1, a_2\} = \{O_e, A_e\}, \textit{ and } p_{-e} \geq S + s \\ A_{-e} \textit{ if } a_2 = O_{-e} \\ \emptyset \textit{ if } p_{-e} < S + s. \end{cases}$$

Any best response strategy for the consumer must take the form of strategy 1 with some choice of $\alpha, \phi_g, \phi_y, \lambda_g, \lambda_y \in [0,1]$.[5] Since all consumers' behaviour must be described by strategy 1 in equilibrium, the probability parameters may, without loss of generality, be interpreted as proportions of the population undertaking any given action.

Action a_2 makes reference to a parameter, γ. When $p_g, p_y > q$, this value represents the threshold for $\min\{p_g, p_y\}$ above which consumers prefer to switch from one search engine to the other—clicking the O-link at both—before finally clicking the A-link at the second site he visits. In contrast, if $\min\{p_g, p_y\} < \gamma$ then the consumer prefers to click both the O-link and A-link at the first site he visits before switching over to the second site. I shall refer to these behaviours respectively as 'switching' and 'sticking'.

Calculation of the parameter γ is as follows: Suppose, for concreteness that $q < p_g \leq p_y$. Consumers find it optimal to click on O_y first. The total expected utility that each consumer gets from 'switching' to next click on O_g is

$$\textbf{(2)} \quad U = p_y(1-S-s) + (1-p_y)p_g(1-2S-2s) + (1-p_y)(1-p_g)q(1-2S-3s) + (1-p_y)(1-p_g)(1-q)(-2S-3s).$$

[5] For the sake of notational (and, occasionally, algebraic) simplicity, I have assumed that $\alpha, \phi_g, \phi_y, \lambda_g$, and λ_y are constant. In principle, these variables could be functions of p_g and p_y—permitting such a functional relationship has no substantive effect on the results presented below.

Similarly, the utility from 'sticking' is

$$(3) \quad U = p_y(1-S-s) + (1-p_y)q(1-S-2s) + \\ (1-p_y)(1-q)p_g(1-2S-3s) + (1-p_y)(1-q)(1-p_g)(-2S-3s).$$

Setting (2) and (3) equal to one another, and calculating γ as the p_g that yields indifference gives:

$$p_g = \frac{S+s}{s}q \equiv \gamma.$$

This simply says that the utility per unit of expenditure for clicking O_g and A_y should be equal. An increase in p_g to some $p'_g \in (\gamma, p_y)$ makes clicking $\{O_y, O_g, A_g\}$ strictly more attractive than $\{O_y, A_y, O_g\}$, and so consumers switch. Conversely, a reduction in p_g leaves consumers preferring to stick. A symmetric argument applies to the case of $p_y < p_g$.

III.2 Equilibrium Characterisation

I now proceed to characterise the equilibria of this game, focusing on equilibria that are in pure strategies for the search engines. I refer to such equilibria as SE-pure. The first result establishes that, when search engines are constrained to use particularly poor algorithms, quality competition will generally be maximal.

Lemma 1 *If $p^{max} < q$ then at least one $i \in \{g, y\}$ sets $p_i = p^{max}$ in equilibrium. If, in addition, $\alpha \in (0,1)$ then $p_g = p_y = p^{max}$ is the unique equilibrium search engine behaviour.*

The proof for this and other results can be found in Appendix A.

Note that cannibalisation is not in effect for values of p less than q since all consumers then prefer to click the A-link first. Thus, there is no offsetting force for the 'competition for market share' incentive, which is why the intensity of competition described in lemma 1 obtains. For the remainder of the paper, I shall make the assumption that $p^{max} \geq q$. Moreover, since allowing general values for α provides no significant further insights, I shall henceforth assume that search engines are treated symmetrically (at least in aggregate) in the sense that $\alpha = 1/2$. I next show that all SE-pure equilibria are necessarily symmetric in search engine strategies.

Lemma 2 *All SE-pure equilibria have* $p_g = p_y$.

Now, in lemma 3, I foreshadow what is to come with a result demonstrating that search engines have a strong incentive to provide O-links that compete fiercely against their own A-links.

Lemma 3 *There is no SE-pure equilibrium in which* $\min\{p_g, p_y\} < q$.

The intuition here is similar to that for lemma 1: There is no cannibalisation incentive for quality limitation so long as consumers prefer to click an A-link first, which is true whenever O-link qualities are less than q. Thus, so long as both search engines offer a $p < q$, there is no disincentive to increasing p in order to capture market share. When $p^{max} > q$, lemma 3 implies that the lowest $\{p_g, p_y\}$ pair that could ever be consistent with an SE-pure equilibrium is $p_g = p_y = q$. In equilibrium 1, I demonstrate that this is indeed an equilibrium, albeit under a restricted set of circumstances.

Equilibrium 1 *The search engine strategy* $p_g = p_y = q$*, and the consumer strategy detailed in strategy 1, form a subgame perfect equilibrium of the above game whenever*

(4) $$q \geq \frac{1}{1+\lambda_i} \; \forall \, i.$$

The higher is $\min\{\lambda_g, \lambda_y\}$, the lower is the minimum value of q consistent with the above equilibrium. For $\lambda_g = \lambda_y = 1$, the equilibrium in proposition 1 is sustained by any $q \geq 1/2$—if $q < 1/2$ then there are no $\{\lambda_g, \lambda_y\}$ such that neither g nor y wishes to deviate. The more natural assumption is $\lambda_g = \lambda_y = 1/2$, in which case the condition for the proposed equilibrium to exist is $q \geq 2/3$.[6]

In equilibrium, search engines do not wish to reduce O-link quality since their competitor then captures the entire market.[7] Each search engine can, in fact, capture the entire market for themselves with a (potentially small) increase in quality to

[6]Were $S + s > q$ for a large enough proportion of consumers then the existence condition for equilibrium 1 would become more demanding: search engines would have an additional incentive to increase quality in order to induce such consumers to search.

[7]Switching costs in the Internet Search industry are low so that, as Gandal (2001) has shown, consumers are likely to switch to the highest quality search engine, even when the quality difference is small.

some $q + \epsilon$. This, though, carries a cost: at the higher quality, consumers are induced to click the deviator's O-link first so that only those left unsatisfied by the O-link ever click on the A-link. When consumers play in such a way that 4 is satisfied, the loss in clicks that this cannibalisation effect generates outweighs the gain in market share so that the search engine prefers not to deviate.

If q is low, so that 4 can not be satisfied, then any SE-pure equilibrium must have $p_g, p_y > q$. The result of the next equilibrium makes this statement more stark. Under certain conditions on the parameters of the model, there is an equilibrium in which the intensity of competition is maximal in the sense that both p_g and p_y are set to p^{max}. This result is formalised in equilibrium 2.

Equilibrium 2 *If $\gamma > p^{max}$ then the search engine strategy $p_g = p_y = p^{max}$, and the consumer strategy detailed in strategy 1, form a subgame perfect equilibrium of the above game.*

The condition for existence of equilibrium 2, *viz.* $\gamma > p^{max}$, is more likely satisfied if S is large relative to s: a search engine's power over its captive audience increases when sticking to click more links is much cheaper than continuing the search elsewhere—and hence so does the incentive to compete for visitors. More specifically, equilibrium 2 comes about because, when $\min\{p_g, p_y\} \in [0, \gamma)$, the consumer always prefers to play 'stick' rather than 'switch'. If both search engines choose some $p < p^{max}$ then there is an incentive for each engine to offer a slightly higher p than its rival in order to capture all of the A-link clicks. In this manner, large portions of search engine profits are dissipated. This result is clearly reminiscent of Bertrand price competition. When $p^{max} = 1$, equilibrium 2 implies zero profits for search engines. In fact, it turns out that if $p^{max} = 1$, $p_g = p_y = 1$ is always an equilibrium.[8]

In equilibrium 2, search engines have an incentive to increase their p to the greatest extent possible. This is true whenever $p_g, p_y < \gamma$. In contrast, if $\min\{p_g, p_y\} > \gamma$, the consumer always switches and there is thus an incentive for each search engine to offer a p that is *lower* than that of its rival. It might seem natural, then, to look for an equilibrium in which $p_g = p_y = \gamma$. This is the business of equilibrium 3.

[8]By assumption, $q < 1$. Thus, with $p_g = p_y = 1$, the consumer clicks one of O_g or O_y, and is immediately satisfied. A-links are never clicked, and search engine profits are thus zero. A deviation by i, namely setting $p_i < p_{-i} = 1$, results in the consumer clicking on O_{-i} first with probability one and thus i's profits remain at zero. Thus i has no profitable deviation.

Equilibrium 3 *If* $\gamma \leq p^{max}$ *then the search engine strategy* $p_g = p_y = \gamma$*, and the consumer strategy detailed in strategy 1, with*

$$(5) \quad \frac{\phi_i}{1 - \phi_{-i}} \leq 1 - \gamma \; \forall i.$$

form a subgame perfect equilibrium of the above game.

Intuitively, equilibrium 3 works as follows: any reduction in O-link quality by a search engine is unprofitable because consumers do not have sufficient incentive to visit until its rival's links have been exhausted. Positive deviations are also unprofitable for the search engines since O-links are then 'too good'—when (5) is satisfied, consumers that are attracted by a high p are too likely to switch sites and continue clicking O-links, rather than stick around to try the A-link at the deviator's site.

In summary, the simple model has produced three classes of SE-pure equilibrium, with up to two existing at any one time. Firstly, if $\gamma \geq p^{max}$ then there exists an equilibrium of the form $p_g = p_y = p^{max}$. When $\gamma < p^{max}$, this equilibrium can no longer be sustained, and is replaced by a $p_g = p_y = \gamma$ equilibrium with appropriate choice of ϕ by the consumer. Precisely one of these two classes of equilibria is guaranteed to exist. In addition, if $q \geq 1/2$ then there exists a second equilibrium in which search engines set $p_g = p_y = q$ and the consumer chooses λ to satisfy (4). Note that when there exist two equilibria, the consumers would prefer to steer search engines into the higher quality equilibrium by setting a low λ. This type of coordination, though, is likely to be difficult when there are many 'small' consumers.

IV MARKET STRUCTURE

In this section, I generalise the simple model to the oligopoly and monopoly search provider cases, and perform comparative statics analysis on the existence conditions for equilibria under these circumstances.

IV.1 n-Search Engine Oligopoly

With n search engines, optimal consumer behaviour is somewhat analogous to the duopoly case. In order to simplify the notation, I drop the search engine subscripts for λ and ϕ. Moreover, I assume that, whenever the consumer is indifferent between visiting two or more search engines, he visits each with equal probability, so that the consumer treats search engines symmetrically. The best response of the consumer is, then, given in strategy 2 below.

Strategy 2 *Label the search engines so that $p_1 \geq p_2 \geq \ldots \geq p_n$. A consumer's best response strategy maps the link qualities $\{p_1, p_2, \ldots, p_n, q\}$ into a click order $\{a_1, a_2, \ldots, a_{n+1}\}$ as follows. Begin by clicking a_1 thus:*

$$a_1 = \begin{cases} NA_1 \textit{ if } p_1 > q \\ A_1 \textit{ if } p_1 < q \\ A_1 \textit{ w.p. } \lambda, NA_1 \textit{ w.p. } 1-\lambda \textit{ if } p_1 = q, \end{cases}$$

Now, for $1 \leq i \leq n$, if the consumer's need was met by a_i then stop clicking (i.e. $a_{i+1} = \ldots = a_{n+1} = \emptyset$), otherwise click a_{i+1} as follows:

$$a_{i+1} = \begin{cases} A_i \textit{ if } a_i = O_i \textit{ and } p_{i+1} < \gamma, \textit{ or if } a_i = O_i \textit{ and } i = N \\ NA_{i+1} \textit{ if } a_i = O_i \textit{ and } p_{i+1} > \gamma \\ A_i \textit{ w.p. } \phi, NA_{i+1} \textit{ w.p. } 1-\phi \textit{ if } a_i = O_i \textit{ and } p_{i+1} = \gamma \\ O_i \textit{ if } a_i \neq O_i \textit{ and } p_i \geq s \\ \emptyset \textit{ if } p_i < s. \end{cases}$$

Strategy 2 again makes reference to γ. Suppose that the consumer is weighing the click orders $\{\ldots, NA_{i-1}, NA_i, A_i, NA_{i+1}, \ldots\}$ and $\{\ldots, NA_{i-1}, A_{i-1}, NA_i, NA_{i+1}, \ldots\}$, *viz.* the consumer is comparing the utility from clicking the A-link at the i^{th} search engine he visits, to that from doing so at the $i-1^{th}$ site. The respective utilities are

(6)
$$\sum_{e=1}^{i-1}\left[p_e(1-eS-es)\prod_{k=1}^{e-1}(1-p_k)\right]+\left[p_i(1-iS-is)\prod_{k=1}^{i-1}(1-p_k)\right]+$$
$$\left[q(1-iS-(i+1)s)\prod_{k=1}^{i}(1-p_k)\right]+\sum_{e=i+1}^{n}\left[p_e[1-eS-(e+1)s](1-q)\prod_{k=1}^{e-1}(1-p_k)\right]-$$
$$\left[[nS+(n+1)s](1-q)\prod_{k=1}^{n}(1-p_k)\right],$$

and

(7)
$$\sum_{e=1}^{i-1}\left[p_e(1-eS-es)\prod_{k=1}^{e-1}(1-p_k)\right]+\left[q(1-(i-1)S-is)\prod_{k=1}^{i-1}(1-p_k)\right]+$$
$$\left[p_i[1-iS-(i+1)s](1-q)\prod_{k=1}^{i-1}(1-p_k)\right]+\sum_{e=i+1}^{n}\left[p_e[1-eS-(e+1)s](1-q)\prod_{k=1}^{e-1}(1-p_k)\right]-$$
$$\left[[nS+(n+1)s](1-q)\prod_{k=1}^{n}(1-p_k)\right].$$

The first, fourth and fifth terms in (6) and (7) are identical so that comparison of the two amounts to comparing the second and third terms. It transpires that the two expressions are equal precisely when $p_i = \gamma$. Thus, when $p_i = \gamma$, the consumer is indifferent between the two considered click orders.

Given this consumer strategy, I am ready to examine equilibrium search engine behaviour when each search engine faces $n-1 \geq 1$ competitors. It is fairly straightforward to transfer lemmas 2 and 3 to the n-search engine case. The proofs of lemmas 4 and 5 are easily adapted from those of lemmas 2 and 3, and are consequently omitted.

Lemma 4 *All SE-pure equilibria in the n-search engine version of the simple model are symmetric in search engine strategies.*

Lemma 5 *There is no SE-pure equilibrium with $p_i < q$ for some i in the n-search engine version of the simple model.*

It again must be the case, then, that any SE-pure equilibrium has $p_i \geq q\ \forall\ i$. The existence of equilibrium 1 rests upon the requirement that the consumer clicks A-links first with a probability sufficient to ensure that no search engine can obtain a big enough increase in (expected) A-link clicks from an increase in its p to make such a deviation profitable. When there are $n \geq 2$ search engines, the probability with which each of the search engines is visited first is given by $\frac{1}{n}$. Recalculating (4) using this probability reveals the following:

Equilibrium 4 *The search engine strategy $p_i = q \forall i$, and the consumer strategy detailed in strategy 2 form a subgame perfect equilibrium of the n-search engine oligopoly game if*

(8) $$q \geq \frac{n-1}{n-1+\lambda}.$$

The proof is identical to that of equilibrium 1, with $1/n$ substituted in place of $1/2$. It is immediately apparent from (8) that the condition for existence of the 'low p' equilibrium becomes less demanding as the number of competing search engines in the industry is reduced.

In contrast, it turns out that the condition for existence of the analogue of equilibrium 2 remains unchanged for any $n > 1$. This is formalised in equilibrium 5.

Equilibrium 5 *When $\gamma \geq p^{max}$, the search engine strategies $p_1 = p_2 = \ldots = p_n = p^{max}$, and the consumer behaviour detailed in strategy 2 form a subgame perfect equilibrium of the n-search engine oligopoly game.*

What, then, of the case with $\gamma < p^{max}$? It has already been demonstrated that, when $p_i = \gamma$, the consumer is indifferent between clicking the A-link at the i^{th} search engine he visits and doing so at the $i - 1^{th}$ site. Moreover, if it is also the case that $p_{i+1} = \gamma$ then the consumer is indifferent between the click orders $\{\ldots, NA_{i-1}, NA_i, A_i, NA_{i+1}, \ldots\}$ and $\{\ldots, NA_i, NA_{i+1}, A_{i+1}, NA_{i+2}, \ldots\}$. Thus, if $p_i = \gamma \; \forall i$, it can be established by transitivity that the consumer is indifferent between any two click orders which do not have him click on an A-link first.

Now, let us look for an equilibrium with $p_i = \gamma \forall i$. When i deviates by setting some $p_i' > \gamma$, he is visited first with probability 1. Thus, profits from deviation to some $p_i' > \gamma$ are given by

(9) $$\pi_i' = (1 - p_i')\phi b,$$

which is maximised when p_i' is arbitrarily close to γ. Suppose, instead, that i complies with the suggested equilibrium. Profits then are given by[9]

$$\pi_i = \left[\frac{\phi}{n}\sum_{k=1}^{n-1}(1-\gamma)^k(1-\phi)^{k-1}\right]b + \left[\frac{1}{n}(1-\gamma)^n(1-\phi)^{n-1}\right]b. \tag{10}$$

Note that (9) approaches zero with ϕ, whilst the second term in (10) remains positive since $\gamma < p^{max}$ implies $\gamma < 1$. Thus a $p = \gamma$ equilibrium can always be sustained by setting a low enough ϕ.

Proposition 6 *There exists a $\bar{\phi}$ such that, for $\phi < \bar{\phi}$, (9) is less than (10). Strategy 2 with any such ϕ sustains a sub-game perfect equilibrium in which $p_i = \gamma \forall i$.*

IV.2 Monopoly Search Provider

I now extend the simple duopoly model of II to examine the equilibrium considerations induced by a monopolist search engine. In the results above, competition for visits prompts search engines to cannibalise their revenues from A-link clicks. When this competition is taken away so is the incentive to provide O-links of a high quality. Only the cannibalisation effect remains, and thus monopolists will generally set a low quality.[10]

Proposition 7 *With a monopoly search provider, any equilibrium must involve some $p \leq q$. If $\lambda < 1$ then $p < q$ must hold for equilibrium to be sustained.*

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[9] The probability that site i is the k^{th} site to be visited is $1/n$. Conditional on being the k^{th} site (for $k < n$), i receives a profit of b if and only if all of the first $k-1$ O-links, as well as i's own O-link fail to match the consumer's need (each link failing with probability $1-\gamma$), and if the consumer chooses to switch at the first $k-1$ sites and stick at i's site. This gives rise to the first term in (10). The second term comes from the fact that if i is the n^{th} site to be visited, and the consumer has switched at the first $n-1$ sites, then the consumer sticks at i with probability 1 since there are no more search engines to switch to.

[10] If $S + s > q$ for a large enough proportion of consumers then the monopolist may still wish to set a $p > q$ in order to induce those consumers to search.

IV.3 Comment

One of the issues in the regulation of Internet search has been to fully understand the effects of reduced competition in the industry. This is especially true since advertisers can easily substitute to other mediums, and the ability of search engines to exercise any market power over them is thus limited. However, the above results demonstrate that reduced competition may also spill-over into the quality of search services enjoyed by consumers: In the oligopoly case, firms may have an incentive to consolidate or collude since this can create new equilibria with higher total industry profits, but lower O-link quality. This is even more true if the consolidated firm is a monopolist—in which case the industry profits are maximised and quality is particularly low. This may prove to be an important consideration if the consumer's search experience is part of the regulator's objective.

V HETEROGENEOUS VISIT COSTS

The results from section III are stark, but the Bertrand-like assumption of all-or-nothing profits is equally so. One question that naturally arises is whether or not the above results remain valid when there is some degree of continuity to the demand faced by each search engine. To this end, I invoke a standard Hotelling (1929) linear city type model in which consumers have heterogeneous visit costs distributed along a segment of the real line. As well as being of interest in its own right, this serves as a robustness check for the model of section II.

More concretely, suppose that a mass 1 of consumers are uniformly distributed along a line of unit length, with g located at point 0, and y at point 1. A consumer located at point x must pay a cost $S_g(x) = tx$ to visit g and $S_y(x) = t(1-x)$ to visit y, where $t \in (0,1)$ is a parameter that scales costs. The model is otherwise as above. One possible intuition for this model is the observation that some consumers have a search engine bookmarked, set as their browser's homepage, installed as a browser tool bar, or else use a particular search engine provider for other services such as email and calendaring—making the use of such a search engine relatively less costly vis-à-vis its competitors.

Whereas, in section III, I maintained the condition that $1 > q > S+s$, I now now use the slightly stronger assumption that $1 > q > t+s$.[11] Moreover, in order to simplify

[11] This is consistent with the consensus that switching costs in the Internet search industry are

the analysis, I focus on the limiting case of s positive and tending to zero.[12] In the first instance, this rules out the potentially complicated switching behaviour (which, in general, will now be location dependent). The assumption of small s also admits the tractability of the analytic solutions that follow.

Given the structure of visit costs, consumers now determine which search engine to visit first in consideration not only of p_g and p_y, but also their own personal location, x. In particular, the utility from any click order involving clicking both links at g first is decreasing in x, whilst that from clicking both links at y first is increasing in x. Thus, for some x^*, the best response for a consumer is to visit g first whenever $x \leq x^*$ (otherwise begin at y), and once there to click on the O-link first if its quality exceeds that of the A-link, and *vice-versa*. As in section III, consumers stop clicking if their need is satisfied, or if the quality of the O-link at the second site is less than S. More formally, let

$$e(x) \equiv \begin{cases} g \text{ if } x \leq x^* \\ y \text{ if } x > x^* \end{cases}, -e(x) = \{g, y\} - \{e(x)\}.$$

Strategy 3 *A consumer's best response strategy, then, maps his position and the qualities into a click order* $\{a_1, a_2, a_3\}$ *thus:*

$$a_1 = \begin{cases} A_{e(x)} \textit{ if } q > p_{e(x)} \\ O_{e(x)} \textit{ if } q < p_{e(x)} \\ A_{e(x)} \textit{ with probability } \lambda_{e(x)}, NA_{e(x)} \textit{ with probability } 1 - \lambda_{e(x)} \textit{ if } q = p_{e(x)}, \end{cases}$$

$$a_2 = \begin{cases} A_{e(x)} \textit{ if } a_1 = NA_{e(x)} \textit{ and consumer's need remains unmet} \\ O_{e(x)} \textit{ if } a_1 = A_{e(x)} \textit{ and consumer's need remains unmet} \\ \emptyset \textit{ if consumer's need was met by } a_1, \end{cases}$$

and

$$a_3 = \begin{cases} O_{-e(x)} \textit{ if } p_{-e(x)} \geq S_{-e(x)} \textit{ and consumer's need remains unmet} \\ \emptyset \textit{ if } p_{-e(x)} < S_{-e(x)} \textit{ or if consumer's need was met by } a_1 \textit{ or } a_2. \end{cases}$$

An immediate question is whether it is ever optimal for search engine i to choose some $p_i < t$—thus inducing some consumers to click only the two links at its rival.

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low.

[12] This is equivalent to assuming that consumers prefer to click the higher quality of the two links at a site first, but pay no cost to do so.

In fact, in a result analogous to lemma 3, I am able to show that, when consumers behave as defined in strategy 3, no $p_i < q$ is ever optimal for search engine i.

Proposition 8 *When consumers play a best response, choosing some $p_i < q$ with positive probability is never optimal for search engine $i \in \{g, y\}$.*

Intuitively, increasing p_i causes more consumers to visit i first. As long as p_i remains below q, the proportion of consumers visiting i who click A_i does not decrease (there is no cannibalisation effect). Thus, when $p_i < q$, i can always do better by finding some $p_i' \in (p_i, q)$, and the existence of such a p_i' is assured since $[0, q)$ has no largest element.

Although proposition 8 exists in the same spirit as does proposition 3, the former is a stronger result: In addition to ruling out equilibria with some $p_i < q$, proposition 8 demonstrates that—conditional on the consumer behaving rationally—any $p_i < q$ is strictly dominated.

Given that consumers will always be induced to click all three links (so long as their need remains unsatisfied), one can use utility functions of the general form specified in 1 to identify the location of the consumer indifferent between first visiting g and y as approaching

(11) $$x_I^* = \frac{p_g + q - p_g q}{p_g + p_y + 2q - p_g q - p_y q}$$

in the $s \to 0$ limit, when $p_g, p_y \geq t$. The assumption that s is small is of immediate assistance here, since it ensures that the above calculated indifference point is valid irrespective of the relative size of q vis-à-vis p_g and p_y, *viz.* (11) is valid for all $p_g, p_y \in [t, 1]$.

Thus, for $p_g, p_y > t$, given that consumers do not switch, and given that they click on links at the first site in declining order of quality, g's profits are given by

(12) $$\pi_g = x_I^*(1 - p_g)b$$

when $p_g > q$,

(13) $$\pi_g = x_I^* b$$

when $t \leq p_g < q$, and

(14) $\pi_g = x_I^* \left[(1-\lambda_g)(1-p_g)b + \lambda_g b\right]$

when $p_g = q$. The analogous profits for y are $\pi_y = (1-x_I^*)(1-p_y)b$, $\pi_y = (1-x_I^*)b$, and $\pi_y = (1-x_I^*)\left[(1-\lambda_y)(1-p_y)b + \lambda_y b\right]$ respectively.

From proposition 8, there can clearly be no equilibrium with some $p_i < q$. I now turn my attention to the possibility of equilibria with $p_g, p_y \geq q$, focusing again on SE-pure equilibria. It is immediately apparent that any equilibrium with some $p_i = q$ must have $\lambda_i = 1$, otherwise (13) is strictly greater than (14) for p_g less than, but sufficiently close to q (and likewise for y's profit functions). I am now able to establish the following equilibria:

Equilibrium 6 *There exists a* $\underline{q} = 0.0836$ *such that the search engine strategies* $p_g = p_y = q$*, and the consumer strategy detailed in strategy 3 with* $\lambda_i = 1 \forall i$ *form a sub-game perfect equilibrium of the heterogeneous visit costs game whenever* $q \geq \underline{q}$*.*

Equilibrium 7 *There exists a* $\bar{q} = 0.1042$ *such that, whenever* $q \leq \bar{q}$*, the search engine strategies*

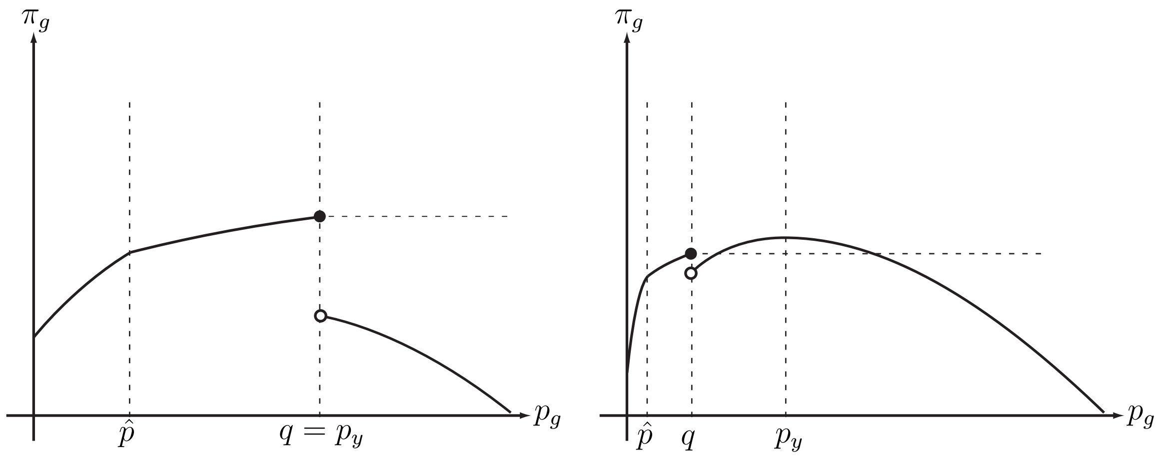
(15) $p_g = p_y = \dfrac{1-3q}{3-3q} \ (> q),$

and the consumer strategy detailed in strategy 3 form a sub-game perfect equilibrium of the heterogeneous visit costs game. Moreover, this is the only SE-pure equilibrium with $p_g, p_y > q$*.*

These two classes of equilibria are shown diagrammatically in figure 1. The discontinuity in g's profits at $p_g = q$ originates from the fact that, when p_g is increased from $p_g = q - \epsilon$ to $p_g' = q + \epsilon$ (ϵ small), the entire mass of consumers who click g's A-link switch from clicking A_g first to clicking O_g first so that g's profits jump discontinuously from (13) to (12). The profit function also has a kink at $p_g = \hat{p}$, where

$$\hat{p} = \frac{p_y q - p_y - 2q + t(1-q) + \sqrt{4(1-q)tq + (p_y q - p_y - 2q + t(1-q))^2}}{2(1-q)}.$$

For very low values of p_g ($< \hat{p}$), consumers close to g use the click order $\{A_g, O_g, O_y\}$, whilst all others click both links at y but never visit g. When p_g is increased slightly,



(a) $q \geq \underline{q}$, $p_g = p_y = q$, $\lambda_g = \lambda_y = 1$ (b) $q \leq \bar{q}$, $p_g = p_y = (1-3q)(3-3q)^{-1} > q$

FIGURE 1 Two classes of equilibria in the heterogeneous visit costs game.

the utility from the former click order increases, whilst that of the latter does not—consumers who never click O_g do not gain from an increase in its quality. It is thus fairly easy for g to attract the marginal consumer, and the market share of g thus increases rapidly in p_g below $\hat{p}$.

Once p_g exceeds $\hat{p}$, however, some consumers, having clicked both links at y, subsequently find it worth while to visit g and click O_g. The relevant indifference point for determining g's demand is now the point at which the consumer is indifferent between this behaviour and the click order $\{A_g, O_g, O_y\}$. Demand, then, becomes less responsive to changes in p_g above $\hat{p}$, since any increase in p_g not only increases the utility of visiting g first, but also that of the relevant alternative—namely starting the search at y.

Further insight into the mechanics of these equilibria may be obtained by examining the reaction functions that give rise to them. Taking a first order condition reveals the choice of p_g that maximises (12) to be

$$\text{(16)} \quad p_g = \frac{2q^2 - 2q - p_y\,(q-1)^2 + \sqrt{(q-1)\left(p_y^2(1-q)^2 + q(1-q) + p_y\,(1+q-2q^2)\right)}}{(q-1)^2}.$$

A symmetric function obtains for y. Provided this function evaluates to some $p_g > q$, which is the case whenever

$$p_y > \frac{q - 3q^2}{(1-q)(2q-1)},$$

(16) represents the p_g that maximises (12).[13] Alternatively, g can choose $p_g = q$, in which case, profits are given by (14). Thus g's complete reaction function is formed by choosing between these two values of p_g to maximise profits. Such composite reaction functions are shown in figure 2 for various values of q, and for $\lambda_g = \lambda_y = 1$ (recall that $\lambda_g = \lambda_y = 1$ is necessary to sustain a $p_g = p_y = q$ equilibrium).

Firstly, there exists a $q_0 = 0.0573$ such that for any $q \leq q_0$, g's best response is to select some $p_g > q$, and play in accordance with (16)—irrespective of the value of p_y. In equilibrium 7, it was shown that (15), which lies in the range $[0, 1]$ whenever $q \leq 1/3$, is a fixed point of (16). Symmetry of the reaction functions ensures that this is a point of intersection between the two.

Increasing p_g above q has both a benefit and a cost for g. The benefit arises when the higher p attracts more consumers to visit g first, so that x^* increases. The manifestation of the cost is that consumers are induced to click O_g first, and thus g's A-link is clicked with probability $1 - p_g$, rather than with probability 1. When q is increased, so is the smallest p_g such that $p_g > q$. Thus, each extra consumer attracted by the higher p_g clicks on the A-link with an ever decreasing probability. As q becomes particularly high, the number of additional visitors necessary to compensate g for this fall soon exceeds the actual increase in x^* brought about by the original increase in p_g.

Thus, as q rises above q_0, it becomes optimal for g to respond to low values of p_y with $p_g = q$, rather than to use (16), and likewise for y to play $p_y = q$ when p_g is low. Moreover, as q is increased further, the size of the interval over which search engines wish to play in this manner increases, so that, by the time $q \geq \underline{q}$, $p_g = p_y = q$ is a mutual best response (provided that $\lambda_g = \lambda_y = 1$) giving rise to a new equilibrium (see figure 2(c)). For yet higher values of q this effect is so strong that the $p_g, p_y > q$ equilibrium is undermined (see figure 2(e)) so that $p_g = p_y = q$ is the only remaining equilibrium behaviour for the search engines.

Reaction functions depend only on q, and are symmetric and (weakly) increasing *viz.* p_g and p_y are strategic complements. This implies that there are no asymmetric equilibria.

Proposition 9 *There are no SE-pure equilibria with asymmetric search engine strategies in the heterogeneous visit costs game.*

[13] If (16) yields a $p_g \leq q$ then, since (12) is concave, profits are decreasing in p_g everywhere above q.

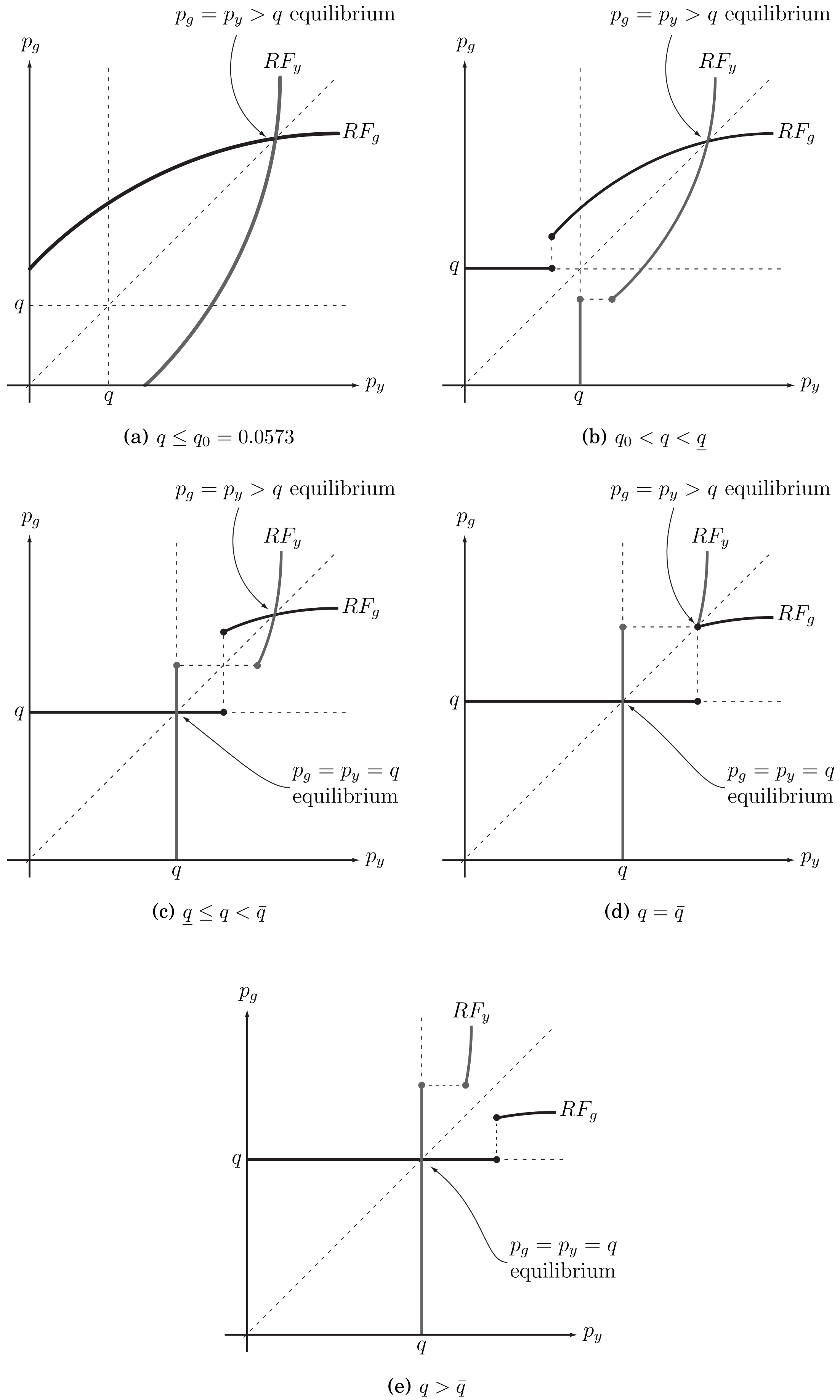


FIGURE 2 Reaction functions and equilibrium points with $\lambda_g = \lambda_y = 1$, for various values of q.

Taken together, proposition 8, proposition 9, and equilibrium 7 imply that equilibria 6 and 7 are the only sub-game perfect SE-pure equilibria of the heterogeneous visit costs game.

VI Discussion

In this paper I have examined equilibrium behaviour in a simple model of the Internet search market. My main finding is that if search engines compete on result quality and consumers select search engines according to link relevance—as evidenced by Gandal (2001), equilibrium quality competition is strong in the sense that organic search results are at least as good as profit-yielding advertising links. Search engines provide such high quality links in an attempt to attract consumers who may stay to click on advertisements. It is in this fashion that search engines ‘cannibalise’ their own revenue streams, since the organic links that a search engine provides compete for clicks with its own advertisement links. In fact, unless the expected quality of advertising links is high (in the sense of equation (4)), and the number of competing firms small, the only equilibria in pure strategies for the search engines involve non-advertising link qualities that strictly exceed the quality of the advertising links. I have also found that reductions in the strength of competition in the search industry may create new equilibria with lower link quality, which may be an important consideration for regulators.

Introducing heterogeneity in visit costs moderates the incentive to compete for consumer visits since search engines have a degree of monopoly power over consumers located nearby. This notwithstanding, I again find that non-advertising links must be at least as good as (and, for low advertising link quality, strictly better than) their advertising counterparts in equilibrium.

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Appendix A Omitted Proofs

A.1 Proofs from Section III

Proof of Lemma 1. In equilibrium, the consumer uses a strategy of the form given in strategy 1. For concreteness, suppose that $p_i \leq p_{-i}$. If $p_i, p_{-i} < p^{max}$ then at least one search engine is visited with probability less than 1, and can profit by setting $p = p^{max}$, which results in that search engine being visited and receiving an A-link click with probability 1. Thus, equilibrium requires that at least one search engine sets $p = p^{max}$ with probability 1.

If $p_i < p_{-i} = p^{max}$ and $\alpha \in (0,1)$ then i makes zero profits, and can make positive expected profits by deviating to $p_i = p^{max}$. It follows that playing any $p_i < p^{max}$ with positive probability is not consistent with equilibrium when $\alpha \in (0,1)$. ■

Proof of Lemma 2. In equilibrium, the consumers' strategies must have the form of strategy 1. I now proceed by establishing a proof by contradiction. For concreteness, suppose that $p_i > p_{-i}$. If $p_i \leq q$ then $-i$ makes zero profits, but can make positive profits by setting $p'_{-i} > \max\{s, p_i\}$. When $p_i \in (q, \gamma]$, i receives a profit of $(1 - p_i)b$—which is decreasing in p_i—and therefore prefers to reduce p_i slightly. The same is true when $p_i > \gamma \geq p_{-i}$. Finally, if $p_i > p_{-i} > \gamma$ then the consumer uses click order $\{O_i, O_{-i}, A_{-i}\}$ and i's profits are zero. There is thus a profitable deviation for i which has it set $p'_i \in (\gamma, p_{-i})$. ■

Proof of Lemma 3. By lemma 2, all SE-pure equilibria are symmetric in search engine strategies. Thus, $\min\{p_g, p_y\} < q$ implies that $p_g = p_y = p < q$. At least one $i \in \{g, y\}$ must, then have its A-link clicked with probability less than 1. It follows that both g and y have a profitable deviation, namely to set some $p'_i \in (p, q)$. ■

Proof of Equilibrium 1. When $p_g = p_y = q$, the consumer is indifferent about which site he visits first, and also about which link he clicks first. Suppose that the consumer visits each search engine first with probability $1/2$ and, having visited site i first, clicks on A_i first with probability λ_i. Search engine expected profits are then

(A-17) $$\pi_i = \frac{1}{2}\left(\lambda_i + (1 - \lambda_i)(1 - p_i)\right) b,$$

Consider a deviation by search engine i to $p_i < p_{-i} = q$. The optimal behaviour for the consumer now involves clicking on both A_{-i} and O_{-i} before visiting i to click on

O_i. Search engine i's profits are zero and the deviation is not profitable.

Suppose, instead, that i sets $p'_i > p_{-i} = q$. The optimal click-order for the consumer is now $\{O_i, A_i, O_{-i}\}$, which gives an expected profit for i of

(A-18) $\pi'_i = (1 - p'_i)\, b.$

Since (A-18) is decreasing in p'_i, it suffices to consider the limiting case with p'_i arbitrarily close to q. For the deviation to be non-profitable, (A-17) must be greater than (A-18), which gives (4) when q has been substituted in place of p_i and p'_i.

Since the consumer is indifferent about click order when $p_g = p_y = q$, any $\{\lambda_g, \lambda_y\}$ constitute a best response so that the proposed strategies form an equilibrium. Moreover, since strategy 1 details a best response for any search engine actions, the equilibrium is subgame perfect. ■

Proof of Equilibrium 2. Values of p_i greater than p^{max} are not possible. Consider a deviation in which i sets $p_i < p_{-i} = p^{max}$. From strategy 1, $p_i < min\{p_{-i}, \gamma\}$ implies that the consumer never clicks A_i, and i's profits are thus zero.

Noting that strategy 1 details a best response for any search engine actions completes the proof. ■

Proof of Equilibrium 3. With $p_g = p_y = \gamma > q$, the expected profit for i is

$$\pi_i = \frac{1}{2}\left[\phi_i(1 - p_i) + (1 - \phi_{-i})(1 - p_{-i})(1 - p_i)\right] b.$$

A deviation by i that has it set $p_i < p_{-i} = \gamma$ leaves it with a profit of zero since the consumer never clicks on A_i if $p_i < \min\{\gamma, p_{-i}\}$. Suppose instead that i deviates with $p'_i > p_{-i} = \gamma$. The expected pay-off for i becomes

$$\pi'_i = \phi_i(1 - p'_i)b.$$

Since this payoff is decreasing in p'_i, it suffices to consider the limiting case of $p'_i = p_i = \gamma$. The deviation is not profitable so long as $\pi_i \geq \pi'_i$. Substituting γ for p'_i, p_i, and p_{-i} in this expression and rearranging yields (5). Thus neither search engine has a profitable deviation so long as (5) holds. Given that the consumer is, by definition, indifferent over all $\phi_g, \phi_y \in [0, 1]$ when $p_g = p_y = \gamma$, satisfaction of (5) is consistent with equilibrium. Moreover, since strategy 1 details a best response for any search engine actions, the proposal describes a subgame perfect equilibrium. ■

A.2 Proofs from Section IV

Proof of Equilibrium 5. Expected profits from compliance with the equilibrium are given by $\pi_i = (1/n)(1 - p^{max})b \geq 0$. Values of p greater than p^{max} are not possible. Consider a deviation in which i sets $p_i < p^{max}$. From strategy 2, $p_i < min\{p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, \gamma\}$ implies that the consumer never clicks A_i, and i's profits are zero. Thus, i has no profitable deviation.

Noting that strategy 2 details a best response for any search engine actions completes the proof. ■

Proof of Proposition 7. The best response for a consumer is now to click the O-link first if $p > q$, to click the A-link first with probability 1 if $p < q$, and to click the A-link first with some probability λ if $p = q$. Given this behaviour, the monopoly search engine's best response is to select a p that ensures the A-link is clicked first—this implies $p < q$ (or $p \leq q$ if $\lambda = 1$). ■

A.3 Proofs from Section V

Proof of Proposition 8.

Suppose that $p_g < q$. Since s is small, consumers click on A_g if and only if they visit g first. Denote by x^* the mass of all such consumers. If p_g is a best response, then it must be the case that $x^* > 0$, since g can always induce nearby consumers to visit it first by setting a p_g close enough to p_y. Similarly, $x^* = 1$ implies that $p_y < p_g$, and in this case a symmetric argument for y establishes that p_y is not a best response.

Consider, then, the interior case with $0 < x^* < 1$. Since $p_g < q$, rational consumers always click on A_g before they do O_g. Thus g's profits are given by the mass of consumers that visit g first, x^*, multiplied by b. A consumer that clicks A_g first must either use the click order $\{A_g, O_g, O_y\}$ or else use $\{A_g, O_g, \emptyset\}$, which implies utility functions

(A-19)
$$U(A_g, O_g, O_y) = q(1 - tx - s) + (1 - q)p_g(1 - tx - 2s) + (1 - q)(1 - p_g)p_y(1 - t - 3s) + (1 - q)(1 - p_g)(1 - p_y)(-t - 3s),$$

and

$$\textbf{(A-20)}\quad U(A_g, O_g, \emptyset) = q(1 - tx - s) + (1 - q)p_g(1 - tx - 2s) + (1 - q)(1 - p_g)(-t - 2s).$$

Any rational consumer who does not find it optimal to use either of the above two click orders must visit y first. The possible click orders in use by such consumers are $\{A_y, O_y, O_g\}$; $\{A_y, O_y, \emptyset\}$; $\{O_y, A_y, O_g\}$; and $\{O_y, A_y, \emptyset\}$. These are respectively associated with the following utility functions.

$$\textbf{(A-21)}\quad U(A_y, O_y, O_g) = q(1 - t(1 - x) - s) + (1 - q)p_y(1 - t(1 - x) - 2s) + (1 - q)(1 - p_y)p_g(1 - t - 3s) + (1 - q)(1 - p_y)(1 - p_g)(-t - 3s),$$

$$\textbf{(A-22)}\quad U(A_y, O_y, \emptyset) = q(1 - t(1 - x) - s) + (1 - q)p_y(1 - t(1 - x) - 2s) + (1 - q)(1 - p_y)(-t(1 - x) - 2s).$$

$$\textbf{(A-23)}\quad U(O_y, A_y, O_g) = p_y(1 - t(1 - x) - s) + (1 - p_y)q(1 - t(1 - x) - 2s) + (1 - p_y)(1 - q)p_g(1 - t - 3s) + (1 - p_y)(1 - q)(1 - p_g)(-t - 3s),$$

and

$$\textbf{(A-24)}\quad U(O_y, A_y, \emptyset) = p_y(1 - t(1 - x) - s) + (1 - p_y)q(1 - t(1 - x) - 2s) + (1 - p_y)(1 - q)(1 - t(1 - x) - 2s).$$

Now, given that the consumer's cost for visiting g (y) is increasing (decreasing) in x, the set of x for which consumers click A_g first must be a connected interval, and must include point $x = 0$. Thus, every consumer with an $x < x^*$ must be visiting g first, and every consumer for whom $x > x^*$ must be using some click order that has him visit y first. By the continuity of x (and since utility varies continuously with x), there must exist a marginal consumer at x^* who is just indifferent between the click orders in use by those consumers at $x^* + \epsilon$ and $x^* - \epsilon$, with ϵ small. That is to say, the maximum of (A-19) and (A-20) must be equal to the maximum of (A-21), (A-22), (A-23), and (A-24) at $x = x^*$.

Consider a small increase in p_g to $p_g' \in (p_g, q)$. The derivatives of (A-19) and (A-20) with respect to p_g are positive, and are in every case both greater than those of (A-21), (A-22), (A-23), and (A-24). Thus, the increase in p_g causes the marginal consumer to strictly prefer some click order that has him click A_g first to all others: the mass of consumers clicking A_g, (and hence g's profits) is thus increased. By the continuity of p_g, there exists such a p_g' for all $p_g < q$. Choosing a $p_g < q$ with positive probability is thus not optimal.

A symmetric argument holds for y. ■

Proof of equilibrium 6. The consumer's strategy specifies a best response to any combination of p_g and p_y. It remains to show that the search engine strategies are optimal responses to one another, given this subsequent consumer behaviour. I show that there is no profitable deviation for g, and appeal to symmetry to complete the argument for y.

With $\lambda_g = 1$, (14) collapses to (13). Since, by lemma **??**, profit from any $p_g < q$ is increasing in p_g it suffices to consider deviations to some $p_g > q$. Such a deviation yields profits for g given by (12). Substituting $p_y = q$ and calculating the first order condition gives g's reaction function:

$$\text{(A-25)} \quad p_g = \frac{4q^2 - 3q - q^3 + Z}{(q-1)^2},$$

where

$$Z = \sqrt{(q-1)^2 q(2 + 3q - 4q^2 + q^3)}.$$

Now, substituting (A-25) into (12) yields g's deviation profits thus:

$$\text{(A-26)} \quad \pi_g = \frac{(2q - 3q^2 + q^3 - Z)\,(3q^2 - q^3 - q - 1 + Z)}{Z(q-1)^2} b.$$

These are to be compared with the profits from compliance with the candidate equilibrium, which are given by (14) with $p_g = p_y = q$, and $\lambda_g = 1$, which gives

$$\text{(A-27)} \quad \pi_g = \frac{b}{2}.$$

Finding the values of $q \in [0,1]$ that equate (A-26) and (A-27) is equivalent to finding the roots of the following cubic (see figure A-1):

$$8q^3 - 25q^2 + 14q - 1,$$

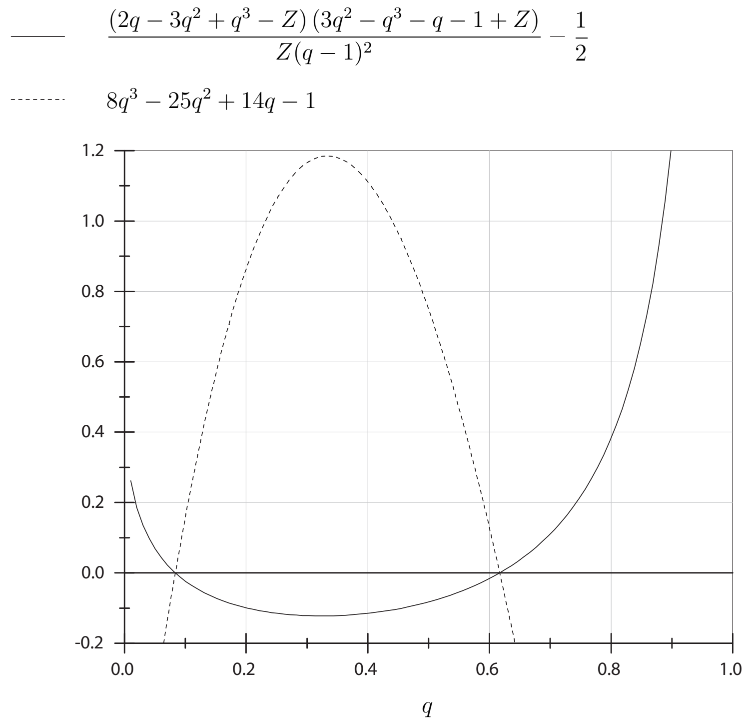

FIGURE A-1 The q in the range $0 \leq q \leq 1$ for which deviation yields profits equal to those from compliance are given by the roots of the cubic $8q^3 - 25q^2 + 14q - 1$.

which can be achieved using, for example, the method of Nickalls (1993). The two solutions to this equation which lie within the $[0, 1]$ interval are given by

(A-28) $$q = \frac{25}{24} - \frac{17}{12}\sin\left[\frac{\pi}{6} \pm \frac{1}{3}\cos^{-1}\left(\frac{3889}{4913}\right)\right] \approx \{0.08356, 0.616981\},$$

However, for any $q > 1/3\left(3 - \sqrt{6}\right) \approx 0.1835$, (A-25) demands a value of $p_g < q$. Moreover, twice differentiating (12) with respect to p_g yields the following (having substituted $p_y = q$):

$$\frac{\partial^2 \pi_g}{(\partial p_g)^2} = \frac{2q\left(2 + q - 7q^2 + 5q^3 - 4q^4\right)}{\left(p_g(q-1) + (q-3)q\right)^3} b,$$

which is negative for $0 \leq q \leq 1$. The final piece of the puzzle is to note that, for any p_y, (12) is less than (13) when both are evaluated at $p_g = q$. Taken together, these facts imply that the second root in (A-28) can be ignored, and that, for $q \geq 0.08356$, deviation to some $p_g > q$ is not profitable. This argument is is summarised in figure A-2. ■

Proof of Equilibrium 7. The consumer's strategy specifies a best response to

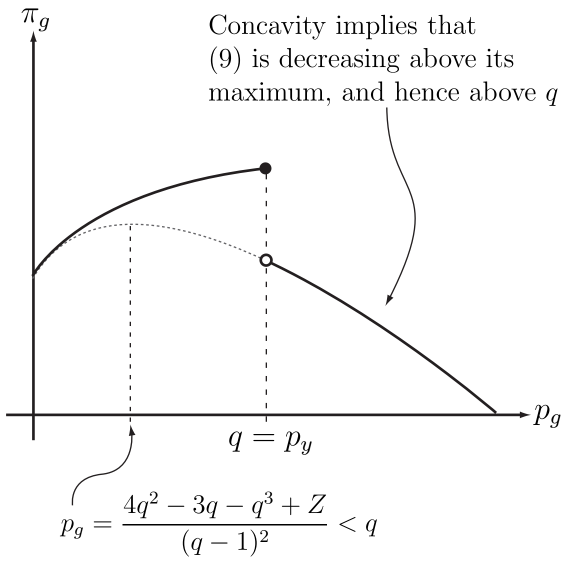

FIGURE A-2 For $q > 1/3\left(3 - \sqrt{6}\right) \approx 0.1835$, (A-25) demands a value of $p_g < q$.

any combination of p_g and p_y. It remains to show that the search engine strategies are optimal responses to one another, given this subsequent consumer behaviour. I show that there is no profitable deviation for g, and appeal to symmetry to complete the argument for y.

Suppose that the proposed equilibrium is valid. $(1-3q)/(3-3q)$ is greater than q for $q < 1/3\left(3-\sqrt{6}\right) \approx 0.1835$, so that the appropriate profit function for g is (12). Taking the derivative of (12) with respect to p_g, yields the quasi-reaction function

$$p_g^* = \frac{2q^2 - 2q - p_y\,(q-1)^2 + \sqrt{(q-1)\left(p_y^2(1-q)^2 + q(1-q) + p_y\,(1+q-2q^2)\right)}}{(q-1)^2},$$

which is valid for $p_g > q$. The corresponding function for y is symmetric. Differentiating g's quasi-reaction function with respect to p_y, and substituting $p_y = 1$ yields

$$\left.\frac{\partial p_g^*}{\partial p_y}\right|_{p_y=1} = \frac{3\sqrt{2}(1-q) - 4\sqrt{(1-q)^2}}{4\sqrt{(1-q)^2}},$$

which is constant (with a value of around 0.06066), and positive for $q \in [0,1)$. Now, taking the second derivative of p_g^* gives

$$\frac{\partial^2 p_g^*}{(\partial p_y)^2} = -\frac{(1-q)^4}{4\left[(1-q)^2\left((p_y)^2(1-q)^2 + q(1+q) + py(1+q-2q^2)\right)\right]^{3/2}} < 0.$$

Since the second derivative is negative, and the first derivative is positive at $p_y = 1$, the first derivative of p_g^* must be positive for all $p_y \in [0, 1]$. That the quasi-reaction functions are symmetric, increasing and concave implies that there can be at most two points of intersection, and that both of these must have $p_y = p_g$.

Imposing $p_y = p_g$ for symmetry and solving the quasi-reaction function gives

(A-29) $$p_g = p_y = \frac{1 - 3q}{3 - 3q}, .$$

and

$$p_g = p_y = \frac{q}{q - 1}.$$

The second solution is non-positive for all $0 < q < 1$. Since these are the only two points of intersection of the two $p > q$ quasi-reaction functions, the only possible equilibrium behaviour in which $p_g, p_y > q$ is given by (A-29).

By construction, when p_y plays according to (A-29), no $p_g > q$ can yield a higher profit for g than will compliance with (A-29). Moreover, by lemma **??**, any deviation to $p_g < q$ is unprofitable. It suffices, then, to show that the limit of (13) as $p_g \to q$ can not be higher than the profit from compliance with the proposed equilibrium.

Substituting (A-29) into (12) gives profits for compliance with the candidate equilibrium thus:

(A-30) $$\pi_g = \pi_y = \frac{1}{3 - 3q} b.$$

Substituting $p_g = q$, and $p_y = (1 - 3q)/(3 - 3q)$ into (13) gives an expression for the maximal deviation profits:

(A-31) $$\pi_g = \frac{3q^2 - 6q}{3q^2 - 6q - 1} b.$$

Equating (A-30) and (A-31) yields the cubic

$$-9q^3 + 24q^2 - 12q + 1 = 0,$$

which can, again, be solved using the method of Nickalls (1993). There are two

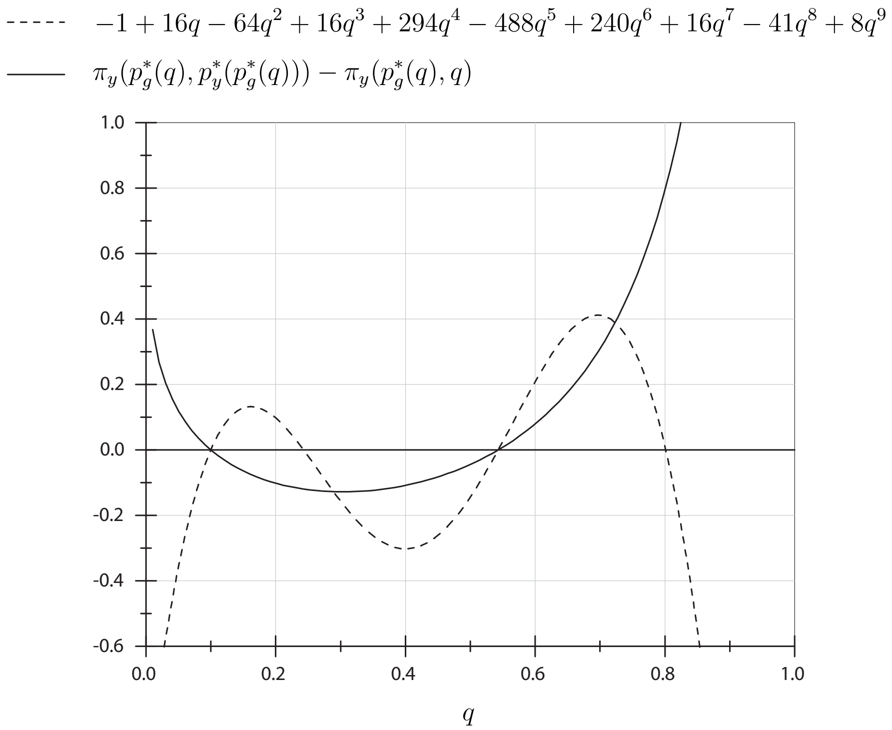

FIGURE A-3 The q in the range $0 \leq q \leq 1$ for which $\pi_y(p_g^*(q), p_y^*(p_g^*(q))) - \pi_y(p_g^*(q), q)$ is negative are given by roots of a polynomial.

roots that lie in the interval $[0, 1]$, namely

$$\frac{8}{9} - \frac{4}{9}\sqrt{7}\sin\left[\frac{\pi}{6} \pm \frac{1}{3}\cos^{-1}\left(\frac{241}{112\sqrt{7}}\right)\right] \approx \{0.1042, 0.5228\}.$$

For $0 < q \leq 0.1042$ profits from compliance exceed those from deviation; for $0.1042 < q < 0.5228$ deviation appears strictly profitable, and for $0.5228 \leq q$ compliance is again optimal. However, for $q \geq {}^1\!/_3\left(3 - \sqrt{6}\right) \approx 0.1835$, (A-29) demands a $p_g \leq q$. Thus, (A-29) constitutes a valid equilibrium strategy only when $0 < q \leq 0.1042$. ■

Proof of Proposition 9. By lemma **??**, $p_e < q$ is never consistent with equilibrium. By equilibrium 7, there are no asymmetric equilibria with $p_g, p_y > q$. It follows that, if there exists an asymmetric equilibrium, it must have $p_e = q, p_{-e} > q$. Since a condition for any equilibrium with $p_e = q$ existing is $\lambda_g = \lambda_y = 1$, I suppose that this is the behaviour used by consumers.

Now, inverting the logic of equilibrium 6, a condition for g wanting to choose some $p_g > q$, when $p_y = q$ is that $q \leq 0.0836$. Demonstrating that $p_y = q$ being a best response to this optimal p_g requires some $q > 0.0836$ will thus suffice to complete the proof. Denote by $p_e^*(p_{-e})$ e's best response in the range $(q, p^{max}]$ to the specified p_{-e}, and define $\pi_e(p_g, p_y)$ as e's profits when g plays p_g and y plays p_y. Thus, solving

$$\frac{\partial \pi_g(p_g > q, q)}{\partial p_g} = 0$$

for p_g yields

$$p_g^*(q) = \frac{4q^2 - 3q - q^3 + Z}{(q-1)^2},$$

where

$$Z = \sqrt{(q-1)^2 q(2 + 3q - 4q^2 + q^3)}.$$

Substituting $p_g^*(q)$ into $p_y^*(p_g)$ gives

$$p_y^*(p_g^*(q)) = \frac{q - 2q^2 + q^3 + 8q^2 - Z + \sqrt{(1-q)\left(10q^4 - 2q^5 - 16q^3 + Z - 4qZ + 2q^2 Z\right)}}{(1-q)^2}.$$

Substituting $p_y = p_y^*(p_g^*(q)), p_g^*(q)$ into $\pi_y = (1 - x_I^*)(1 - p_y)b$, and $p_y = q, p_g = p_g^*(q)$ into $\pi_y = (1 - x_I^*)b$ gives expressions for $\pi_y(p_g^*(q), p_y^*(p_g^*(q)))$ and $\pi_y(p_g^*(q), q)$. Finding the values of $q \in [0, 1]$ for which the former is less than the latter can be achieved by identifying roots of the following polynomial (see figure A-3)

$$-1 + 16q - 64q^2 + 16q^3 + 294q^4 - 488q^5 + 240q^6 + 16q^7 - 41q^8 + 8q^9 = 0.$$

In particular, there are two roots of $\pi_y(p_g^*(q), p_y^*(p_g^*(q))) - \pi_y(p_g^*(q), q)$ for q in the interval $[0, 1]$, namely $\{0.0998, 0.5432\}$. Since $\pi_y(p_g^*(q), p_y^*(p_g^*(q))) - \pi_y(p_g^*(q), q)$ is positive when $q < 0.0998$, y strictly prefers to deviate to some $p_y > q$ in this range.

As required, it is thus proven that $p_y = q, p_g > q$ cannot simultaneously be best response strategies. A symmetric argument applies to the case of $p_g = q, p_y > q$. ■

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