Mixing Goods with Two-Part Tariffs*

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Abstract

We consider a market where consumers mix goods offered by two firms differentiated a la Hotelling, and show how tariff structures affect consumers, profits, and location decisions. As compared to linear pricing, when firms charge two-part tariffs they make higher profits while consumers are worse off. The resulting allocation is not efficient since firms choose extreme locations and too little mixing occurs. Still, under competition in flat subscription fees only there is no mixing at all, and the outcome is Pareto-dominated by competition in the other types of tariffs. Results are discussed with a particular emphasis on the media industry.

Keywords: Two-part tariffs, flat fees, combinable products, media economics, pay-per-view

JEL: L13, L82

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1 Introduction

The Hotelling location model is widely used in Industrial Organization to describe imperfect competition among suppliers that are located in some product or characteristic space. A feature of the standard model is that each consumer buys exclusively from one single firm. However, in many market situations, consumers are free to mix the characteristics embodied in different goods. Anderson and Neven (1989) allow consumers to combine various products (e.g., flavored coffee beans), instead of buying exclusively from a single supplier. They study a location-price game and find the remarkable result that first-best allocations are reached in equilibrium. This is in contrast with the Hotelling location model that typically delivers inefficient (excessive) differentiation (see d'Aspremont *et al.*, 1979).

In this paper we reconsider the problem of combinable goods, allowing for more general pricing strategies than Anderson and Neven (1989) who only look at the case of linear pricing. Our study is motivated by a recent and growing literature on media markets.[1] This literature, among other topics, models the behavior of viewers/listeners of media programmes as a main ingredient of the analysis. In most models it is assumed that audiences make a discrete choice of which broadcaster to watch; typically, the standard Hotelling model with exclusivity is adopted. This assumption fits quite well, e.g., for newspapers. However, there are some media markets where mixing is more appropriate. For instance, a listener may want to spend some time listening to classical music and some time listening to jazz. Similarly, a viewer may want to mix between a sport channel and a movie channel over a particular time period, say a month. In this case, an alternative approach would be to follow the mixing model of Anderson and Neven (1989). This is done for instance in Gal-Or and Dukes (2003) and Gabszewicz *et al.* (2004), where viewers/listeners are able to diversify their viewing/listening experience, obtaining a mix of programs to match their preferences. The resulting aggregate demand in the mixing model is the same one as the demand derived in the Hotelling model if audiences, under the mixing model, pay only for the proportion of time they spend with a broadcaster (pay-per-view).

Linear pricing (pay-per-view) seems to be rather restrictive and is not widely adopted in practice. Instead, media stations typically charge subscription fees as well as extra charges if a viewer wants to watch particular movies or programmes. Still, viewers can mix among different broadcasters if they want to, i.e., exclusivity cannot be imposed. In our model all viewers can mix in principle, while firms charge two-part tariffs (e.g., both a subscrip-

[1] See Anderson and Gabszewicz (2006) for a survey.

tion fee and a pay-per-view fee in the media example). We show how this leads to significant departures from the linear pricing setting of Anderson and Neven (1989). The introduction of subscription fees induces some viewers to buy from one firm only, implying that exclusivity arises endogenously and that sub-optimal mixing occurs in equilibrium. We also show, contrary to the mixing model with linear pricing, which always produces equilibrium profits identical to the Hotelling profits, that when firms compete in two-part tariffs they make more profits in equilibrium. This increase in profits does not come from exclusive customers but from the mixing ones who pay two subscription fees.

Our equilibria with two-part tariffs are quite robust to alternative pricing structures. We consider two variations of the initial two-part tariff set-up: a change in timing, where firms simultaneously choose the form of tariffs (linear or two-part) before actually setting prices, and the setup where both firms offer a menu of a fixed fee and a linear tariff. In both cases the equilibrium outcome results in the equilibrium two-part tariff derived before.

We also study a version of the model with flat fees only. Again, this is motivated by the media industry where flat subscription pricing is quite common. This could be due, for instance, to costly monitoring technologies of usage, which makes it too expensive to charge per viewing time. A true pay-per-view system, such as video-on-demand, necessitates sophisticated (two-way) broadcasting technologies, where a viewer can download a particular movie when she wants it. This may not be possible, and simpler (one-way) standard broadcasting could be the only option available. This makes it much more difficult (or even irrelevant) to charge for usage, while subscription fees are easier to administer. We show that competition in flat fees produces a very striking inefficiency result where no single customer mixes but everybody buys from a single supplier instead. Making two-part tariffs possible leads to a Pareto improvement where consumers who start mixing increase their utility, while at the same time firms increase their profits. In our view, this result on the comparison between flat fees and two-part tariffs is quite relevant when applied to recent trends in the broadcasting industry which make two-part tariffs increasingly available to consumers.

Our study is related to the more general theme of price discrimination under oligopoly (see Armstrong (2006) for a recent overview). An issue in this literature is to investigate what happens to firms' profits and consumer surplus when firms can use more instruments in their tariff design (in the language of Armstrong (2006), "more ornate" tariffs). Examples of more ornate tariffs are two-part tariffs instead of linear prices, or two-part tariffs instead of simple flat fees, which is what we study. With monopoly, the effects of having more instruments are clear. Ignoring issues of commitment, monopoly profits generally increase, while with competition results are much less clear cut. In Armstrong's model, profits go up when there is "one-stop" shopping (exclusivity is exogenously imposed) and the problem under competition becomes a monopoly-adjusted problem. This does not arise in our context, where exclusivity may arise as an equilibrium phenomenon. We show that profits increase with two-part tariffs, with some customers mixing. These profits are strictly higher compared both to the case where no one mixes as a consequence of a flat fee, and to the case when everybody mixes because of a linear fee.

Our model with mixing is similar in spirit to models with mix-and-match, which have been employed in the literature on mixed bundling. Matutes and Regibeau (1992) show that profits are lower with more instruments, in particular when firms practice mixed bundling compared to when products are sold separately. In our model, customers can also mix-and-match between products. Contrary to Matutes and Regibeau (1992), profits go up with more instruments. One result that we do share with Matutes and Regibeau (1992) is that there can be excessive "one-stop" shopping in equilibrium. Too many customers end up buying exclusively from one firm when there are more ornate tariffs, losing the social benefits from mixing.

Our contribution to the literature on competitive price discrimination comes from the understanding of consumers' choices between goods. In our model, combinable goods are substitutes for customers that decide to buy exclusively, but are complements for customers that decide to mix, as their combination matches more closely consumers' ideal goods. It is this complementarity that is the source of extra revenues when more "ornate" tariffs are available.

The rest of the paper is organized as follows. Section 2 sets out the model and derives the demand system. Section 3 derives the equilibrium two-part tariffs and location choice, while Section 4 considers flat rates. Section 5 discusses alternative pricing strategies and the issue of prior commitment to specific types of tariffs. Finally, Section 6 concludes.

2 A mixing model with two-part tariffs

2.1 The model

There are two firms $i = 1, 2$, located along the Hotelling unit line at locations $0 \leq a_1 \leq a_2 \leq 1$. Firm i charges a two-part tariff $T_i(q_i) = F_i + p_i q_i$. In the media example, F_i is the fixed subscription fee, while p_i is price per-view of a quantity q_i of programs. This formulation allows to consider the extreme cases of pure pay-per-view ($F_i = 0$; this is the case analyzed by Anderson and Neven, 1989), and pure subscription fee for unlimited viewing ($p_i = 0$).

Each firm incurs a constant marginal cost $c \geq 0$ per unit supplied. Firms play a two-stage game. First, they choose their locations simultaneously, then they compete in two-part tariffs. We will consider subgame-perfect equilibria of this game.

Consumers buy a total quantity normalized to 1 and are uniformly located between locations 0 and 1. They can decide whether to buy only from firm 1, only from firm 2, or to combine products to obtain a mix of their characteristics. As in Anderson and Neven (1989), a consumer located at x who combines the two products with a share $0 \leq \lambda \leq 1$ of product 1 and a share $(1-\lambda)$ of product 2 incurs a quadratic transport cost $t\left(\lambda a_1 + (1-\lambda)\, a_2 - x\right)^2$, where t is the unit transportation cost.[2] Consumers also derive a fixed utility v from buying from any firm, which is assumed to be high enough such that the market is always "covered" in equilibrium. This fixed utility represents the utility of simply being able to consume, while the gain in utility due to more variety under mixing is represented by the transport cost specification.

The interpretation of the utility function in the context of our leading example, the media industry, is as follows. Think of a fixed total demand for movies by possible viewers, e.g., two movies a week for the whole year. Also imagine these movies are offered by two dedicated channels, one offering comedies and the other one sci-fi movies. Potential viewers have different preferences over these genres, and typically want to mix them in different proportions over the year. In order to do this, they have to pay subscription fees, pay-per-view fees, etc., according to the pricing structure offered by each channel. Our setting can also find applications to other industries with subscription fees where mixing can occur. One example are magazines, many others can be found in digital markets. Traditional industries can fit in our framework too. For instance, imagine two health clubs with differentiated features (e.g., one has tennis courts and the other one has swimming pools). A potential customer that is interested in going to a health club, say, twice a week for a whole year, may mix between the two health clubs according to her preferences and to the pricing structures offered by the clubs. One option could be to pay two membership fees, and then split her visits to

[2] We emphasize that the transport cost resulting from mixing is very different from the transport cost $\lambda t(a_1 - x)^2 + (1-\lambda)\, t(a_2 - x)^2$ that would arise in a standard Hotelling model if a customer purchases from different firms. In our model, an ideal mix can be created from existing distant products by combining them, while in the standard Hotelling model an ideal mix never arises as the customer pays no transport cost only when his ideal variety coincides with the firm's location. Therefore mixing never occurs.

the clubs according to her relative preferences over tennis and swimming. Alternatively, if the clubs allow this, she may pay per visit, and so on.[3]

To summarize, the utility of a customer who buys only good 1 is:

$$U_1(x) = v - F_1 - p_1 - t(a_1 - x)^2 . \tag{1}$$

Similarly, the utility if only good 2 is bought is:

$$U_2(x) = v - F_2 - p_2 - t(a_2 - x)^2 . \tag{2}$$

Finally, the utility if a customer buys some mixture λ of both goods is:

$$V(x,\lambda) = v - F_1 - F_2 - \lambda p_1 - (1-\lambda)p_2 - t(\lambda a_1 + (1-\lambda)a_2 - x)^2 . \tag{3}$$

In the latter case, the optimal mixture is endogenous and is found minimizing total cost (pay-per-view plus transportation cost):

$$\min_{\lambda} \lambda p_1 + (1-\lambda)p_2 + t(\lambda a_1 + (1-\lambda)a_2 - x)^2 . \tag{4}$$

The first-order condition can be rewritten as:

$$\lambda(x) = \frac{a_2 - x - R}{a_2 - a_1}, \tag{5}$$

with $R = \frac{p_1 - p_2}{2t(a_2 - a_1)}$. Notice that, for mixing to occur, we must have $0 \leq \lambda \leq 1$, which can be restated as:

$$a_1 - R \leq x \leq a_2 - R. \tag{6}$$

The previous inequalities represent a necessary condition for an interior choice of λ. The actual interval of mixing consumers will be smaller in the presence of fixed payments, as some customers may decide to buy only from one firm and avoid paying two subscription fees. To determine this, we first need to consider their net utility, conditional on buying both goods and mixing them:

$$\begin{aligned} U_{12}(x) &= V(x,\lambda(x)) \\ &= v - F_1 - F_2 - \lambda(x)p_1 - (1-\lambda(x))p_2 - tR^2 \\ &= v - F_1 - F_2 - p_2 - 2t(a_2 - x)R + tR^2 \\ &= v - F_1 - F_2 - p_1 + 2t(x - a_1)R + tR^2. \end{aligned} \tag{7}$$

[3] This also occurs with other forms of entertainment. Think of concert and theatre subscriptions: there is a demand for cultural entertainment, where people, say, want to go out once a week over a time period, and they typically can decide to mix among various forms of cultural entertainment. In this context a “flat rate” corresponds to a season ticket, and a linear tariff to events being sold separately.

A consumer located at x chooses to buy only from firm 1 either if $x < a_1 - R$ (thus $\lambda = 1$), or if $x \geq a_1 - R$ and $U_1(x) > U_{12}(x)$, which leads to $F_2 > t(x - (a_1 - R))^2$ or

$$x < x_l = (a_1 - R) + \sqrt{F_2/t}. \tag{8}$$

In a similar fashion, the consumer chooses to buy only from firm 2 if

$$x > x_h = (a_2 - R) - \sqrt{F_1/t}. \tag{9}$$

It is clear that the last two inequalities (8) and (9) for mixing are more stringent than (6). They boil down to the same inequalities only when there are no fixed subscription fees imposed by either firm. Only consumers in (x_l, x_h) mix, that is to say, only customers located in the middle between the two firms eventually combine the two products. These are the customers who benefit most from mixing, as they can "create" their ideal product by combining the two products that are more or less equally distant. On the other hand, a customer who is already located close to one of the two firms is already enjoying a product that is very close to her ideal, and thus has less benefits from mixing. Consumers in $[0, x_l]$ buy exclusively from firm 1, and consumers in $[x_h, 1]$ buy exclusively from firm 2.

Note that the intervals of exclusive customers widen with the level of the *rival's* subscription fee. The own subscription fee does not matter since it is paid either way, while the benefits from mixing are reduced if a customer has to pay the competitor's fixed fee. In the presence of positive fixed fees, $\lambda(x_h) > 0$ and $\lambda(x_l) < 1$. This means that firm 1's marginal consumer x_h buys a strictly positive amount from firm 1, while consumers slightly further to the right completely forgo the benefits of mixing in order to save on the fixed fee they would have to pay to firm 1. Also note that $x_h - x_l = a_2 - a_1 - \sqrt{F_1/t} - \sqrt{F_2/t} \leq a_2 - a_1 \leq 1$, i.e., in the presence of fixed subscription fees there will always be customers who prefer to buy from a single firm even if these have chosen extreme differentiation ($a_1 = 0$ and $a_2 = 1$).

2.2 Definition of demands

Let D_1 denote the quantity demand of firm 1. Since we assumed the market is always covered, the corresponding demand of firm 2 is $D_2 = 1 - D_1$. If customers were buying exclusively from one firm only, demand for each firm would be equivalent to the number of its buyers. However, because of the possibility of mixing, it is indeed possible that a buyer is a customer of both firms. Thus we need to determine subscription demands B_i as well, with $B_1 + B_2 > 1$ whenever there is mixing (see Figure 1).

Several cases should be considered, according to the values taken by x_l and x_h. These are relegated to the Appendix. We report here only the case with $0 < x_l < x_h < 1$, which is a natural candidate to emerge in equilibrium. In this case customers located close to the extremities buy exclusively from one firm, while the customers close to the middle buy both products and obtain a mix.

$$\begin{aligned} D_1(p_1, F_1, p_2, F_2) &= x_l + \int_{x_l}^{x_h} \lambda(x)\, dx \\ &= \frac{a_1 + a_2}{2} + \frac{(F_2 + p_2) - (F_1 + p_1)}{2t(a_2 - a_1)}, \\ B_1(p_1, F_1, p_2, F_2) &= x_h, \quad B_2(p_1, F_1, p_2, F_2) = 1 - x_l. \end{aligned} \tag{10}$$

Aggregate quantity demand D_i for each firm (10) coincides with aggregate demand in the classic Hotelling model, where buyers cannot mix but buy products exclusively from a single firm (given unit demand, what matters in this case is only the total price paid to the producer, $F_i + p_i$). This important remark explains the results of Anderson and Neven (1989). In their model, buyers pay only a fee linear in the amount they purchase, thus $p_i > 0$ and $F_i = 0$ in our notation. As revenues come only from D_i and not from B_i, and D_i is the same as in the Hotelling model, indeed they find that prices and locations in equilibrium are the same ones as in the Hotelling model. However, once fixed subscription fees are introduced and can be charged to buyers, and $B_i \neq D_i$, the formal equivalence between these models breaks down.

<< Please put Figure 1 here >>

3 Price equilibrium with two-part tariffs

3.1 Tariff Choice

Given locations (a_1, a_2), profits for firm 1 are:

$$\begin{aligned} \pi_1(p_1, F_1, p_2, F_2) &= (p_1 - c)\, D_1(p_1, F_1, p_2, F_2) + F_1 B_1(p_1, F_1, p_2, F_2) \\ &= (p_1 - c)\left(\frac{a_1 + a_2}{2} + \frac{F_2 + p_2 - F_1 - p_1}{2t(a_2 - a_1)}\right) \\ &\quad + F_1\left(a_2 - \frac{p_1 - p_2}{2t(a_2 - a_1)} - \sqrt{\frac{F_1}{t}}\right). \end{aligned} \tag{11}$$

Proposition 1 *In the second stage of the game, there is a unique Nash equilibrium in two-part tariffs for each pair of locations* $0 \leq a_1 \leq a_2 \leq 1$. *Firms choose the following subscription fees and variable prices:*

$$\begin{aligned} F_1 &= F_2 = \frac{7-3\sqrt{5}}{2} t \left(a_2 - a_1\right)^2, \\ p_1 &= c + \frac{t}{3}\left(2 + a_1 + a_2\right)\left(a_2 - a_1\right) - F_1, \\ p_2 &= c + \frac{t}{3}\left(4 - a_1 - a_2\right)\left(a_2 - a_1\right) - F_2. \end{aligned} \tag{12}$$

Proof. See the Appendix.

The equilibrium subscription fee is always positive. This means that it is not profit-maximizing for firms to charge based on usage only, nor that in equilibrium they would subsidize subscription. That firms use both components of their two-part tariffs in equilibrium should come as no surprise. Imagine F_i was zero instead. By decreasing p_i and increasing F_i by the same amount, a firm does not change its profits from exclusive customers, but receives more from each infra-marginal mixing customer. This is so because the increase in the fixed fee is paid in full by mixing customers, while the decrease in the linear price is "diluted" by the mixing share $\lambda < 1$.

It can also be shown that variable prices do not fall below marginal cost, $p_i \geq c$ for $i = 1, 2$. Therefore flat fees would not be chosen, either. Finally, we have $p_i = c$ if and only if both firms are located at the same spot, $a_1 = a_2$. In this case competition brings variable prices down to marginal costs, and subscription fees fall to zero.

Note that the sum of the prices paid to firm 1 is

$$p_{Tot} = p_1 + F_1 = c + \frac{1}{3} t \left(a_2 - a_1\right)\left(2 + a_1 + a_2\right), \tag{13}$$

which is the *same* expression as the one that emerges in a standard Hotelling model under exclusivity. Hence customers who still prefer to buy exclusively from a firm, despite having the opportunity to mix, pay the same price, and firms make the same profits from these customers, as in the Hotelling model. However, there is a group of customers who do mix. These pay the fixed fee F_i to each firm while the price p_i is only paid for the amount bought. Thus it must be the case that each firm earns less per mixing customer with two-part pricing than in a Hotelling model with exclusivity. Still, this does not imply that profitability is decreased with two-part pricing since there is also a demand effect: Each firm sells to more customers than under exclusivity. The overall effect is analyzed in the next proposition.

Proposition 2 *For any given locations $0 \leq a_1 < a_2 \leq 1$, Nash equilibrium profits are higher with two-part tariffs than in the classic Hotelling duopoly.*

Proof. In the standard Hotelling duopoly, firm 1's profits are (the corresponding result holds with firm 2):

$$\pi_1(Hotelling) = \frac{t}{18}\left(a_2 - a_1\right)\left(2 + a_1 + a_2\right)^2.$$

After substituting the equilibrium prices, profits with two-part tariffs are:

$$\begin{aligned} \pi_1(2\ part) &= \left(p_1 - c\right) D_1 + F_1 B_1 \\ &= \frac{t}{18}\left(a_2 - a_1\right)\left(2 + a_1 + a_2\right)^2 + \frac{13\sqrt{5} - 29}{4} t \left(a_2 - a_1\right)^3. \end{aligned}$$

Taking the difference, one obtains:

$$\begin{aligned} \pi_1(2\ part) - \pi_1(Hotelling) &= \frac{13\sqrt{5} - 29}{4} t \left(a_2 - a_1\right)^3 \\ &\simeq 0.017t \left(a_2 - a_1\right)^3 > 0. \text{ QED} \end{aligned}$$

While we have seen that it is intuitive that in equilibrium each firm uses its two-part tariff, it is perhaps more intriguing that the net effect results in *higher* profits than in the Hotelling model (or, equivalently, the model with mixing and linear tariffs only). Clearly, customers who decide to buy exclusively from one firm still pay the same amount. Note that these customers now incur higher transport costs because they would have been mixing under linear tariffs. The increase in profits comes from the mixing customers. Their mixing choices are dictated only by the linear prices, not by the fixed fees. Without fixed fees, they would have paid p_{Tot} in total, λp_{Tot} accruing to firm 1 and the remaining to firm 2. With fixed fees, a customer pays $\lambda p_1 + F_1$ to firm 1 and $\left(1 - \lambda\right) p_2 + F_2$ to firm 2. The total payment $p_{Tot} + F_1$ exceeds p_{Tot} because this customer pays two subscription fees. Compared to linear pricing, mixing customers still mix optimally but are worse off because of the price effect. There is no welfare loss from these customers, but a redistribution of rents occurs.

3.2 Location choice

In the first stage, firms choose their locations. Given the pricing equilibrium in the second stage, the market shares that delimit the mixing area are:

$$x_l = \frac{2}{3}a_1 + \frac{1}{3}\left(1 - a_2\right) + \frac{3 - \sqrt{5}}{2}\left(a_2 - a_1\right), \tag{14}$$

$$x_h = \frac{2}{3}a_2 + \frac{1}{3}\left(1 - a_1\right) - \frac{3 - \sqrt{5}}{2}\left(a_2 - a_1\right). \tag{15}$$

Proposition 3 *Firms locate at the end points of the Hotelling line.*

Proof. Profits for firm 1 are made of two additive terms as expressed in the proof of Proposition 2:

$$\pi_1 = \frac{t}{18}\left(a_2 - a_1\right)\left(2 + a_1 + a_2\right)^2 + \frac{13\sqrt{5} - 29}{4} t \left(a_2 - a_1\right)^3 .$$

The first term is the same as in the standard Hotelling model with quadratic transportation costs. As shown by d'Aspremont et al. (1979), firm 1 would choose to locate at the extreme $a_1 = 0$ in this case. The second term is strictly decreasing in a_1. Thus the maximum of the overall profit π_1 is at $a_1 = 0$. Similarly, the maximum for firm 2 is at $a_2 = 1$. QED

Given the extreme location equilibrium, $a_1 = 0$, $a_2 = 1$, prices in equilibrium simplify to:

$$p_1 = p_2 = p^* = c + \frac{3\sqrt{5} - 5}{2} t \simeq c + 0.854t, \tag{16}$$

$$F_1 = F_2 = F^* = \frac{7 - 3\sqrt{5}}{2} t \simeq 0.146t. \tag{17}$$

The mixing area is delimited by

$$x_l^* = \frac{3 - \sqrt{5}}{2} \simeq 0.382,\ x_h^* = \frac{\sqrt{5} - 1}{2} \simeq 0.618, \tag{18}$$

that is, a fraction $x_h - x_l = \sqrt{5} - 2 \simeq 0.236$ of the customers mix. Profits are:

$$\pi_1 = \pi_2 = \frac{13\sqrt{5} - 27}{4} t \simeq 0.5172t. \tag{19}$$

Turning to consumers, a mixing consumer in the area $x_l \leq x \leq x_h$ has a net utility of

$$U_{12}(x) = \bar{U}_{12} = v - c - \frac{9 - 3\sqrt{5}}{2} t \simeq v - c - 1.146t, \tag{20}$$

while a non-mixing customer in $0 \leq x < x_l$ who buys exclusively from firm 1 derives a net utility:

$$U_1(x) = v - c - \left(1 + x^2\right) t. \tag{21}$$

These are equal at $x = \frac{3-\sqrt{5}}{2} = x_l$, as it should be. Total consumer surplus is

$$\begin{aligned} CS &= \int_0^{x_l} U_1(x)\,dx + (x_h - x_l)\,\bar{U}_{12} + \int_{x_h}^{1} U_2(x)\,dx \\ &= v - c - \frac{23\sqrt{5}-45}{6}t \simeq v - c - 1.072t. \end{aligned}$$

We are now able to compare our results with the model of Anderson and Neven (1989). This is the only legitimate comparison once locations are endogenized and consumers can mix, as the standard Hotelling model would have different implications in terms of socially optimal locations. While the equilibrium in Anderson and Neven with linear prices achieves the first-best allocation where *all* consumers mix, this is no longer true once two-part tariffs are allowed. This indicates that the first-best efficiency result of Anderson and Neven holds only conditional on the assumption that no subscription fees can be charged.

Proposition 4 *Competition with mixing and two-part tariffs leads to inefficient outcomes. As compared to linear pricing both firms benefit, while consumers fare worse (both mixing and non mixing) and overall welfare decreases.*

Proof. Under linear pricing, firms choose extreme locations. All consumers mix optimally and pay a price $p_{Tot} = c + t$, thus their net utility is $v - c - t$, which is always greater than either $U_1(x)$ or $U_{12}(x)$ from (20) and (21) (unless $x = 0$ or $x = 1$). The result on profits is a re-statement of Proposition 2. Finally, welfare must decrease because customers in $0 \leq x < x_l$ and $x_h < x \leq 1$ buy exclusively, rather than mix optimally. They incur in higher transportation costs, which is socially wasteful. QED

The two models produce no difference with respect to location choices: there is always extreme differentiation, which is efficient when consumers can mix. In Anderson and Neven (1989), the social costs of transport are zero because every consumer can obtain her ideal mix. We have illustrated, however, that when firms have the ability to offer two-part tariffs, inefficiencies reemerge. Because mixing raises the amount of surplus available to consumers, it is optimal for each firm to set a positive fixed fee (subscription) in order to extract a portion of this surplus. The inefficiency arises because the positive fixed fee causes extreme consumers to avoid paying the fees twice. These consumers do not mix and thus incur transport costs, as a result.

We end this section by briefly discussing what happens when firms are able to locate outside the unit interval where the consumers are. In our analysis we have assumed that the location space is $[0,1]$. However, it is conceivable in media applications that media platforms can offer more extreme content, outside the $[0,1]$-interval. This corresponds to polarization of content that is not matched by the heterogeneity of viewers' tastes. If we allow for these extreme locations, the equilibrium locations would become $a_1 = -0.686$ and $a_2 = 1.686$, while in Anderson and Neven these values were -0.25 and 1.25, respectively. That is, with two-part tariffs platforms would seek *more* differentiation. In terms of our leading media example, this can be interpreted as an extreme polarization of content, which goes beyond the taste of any viewer.

4 Flat Rates

We have shown above that tariffs consisting only of a subscription payment (flat rates) will not arise in equilibrium if firms have the possibility to charge for usage. As mentioned in the Introduction, there may be situations where charging for usage is not possible. In this case firms must compete in subscriptions only.

Proposition 5 *If firms can only compete in flat rates, then for any pair of locations* $0 \leq a_1 \leq a_2 \leq 1$ *there is a unique Nash equilibrium, with subscription fees*

$$F_1^* = c + \frac{1}{3}t\left(2 + a_1 + a_2\right)\left(a_2 - a_1\right), \tag{22}$$

$$F_2^* = c + \frac{1}{3}t\left(4 - a_1 - a_2\right)\left(a_2 - a_1\right). \tag{23}$$

In equilibrium no consumer mixes.

Proof: See the Appendix.

The remarkable feature of this equilibrium is that the outcome is identical to the standard Hotelling model with unit demands. No consumer mixes, even though all of them could do so. The intuition is simple: Since there are no usage charges, both firms have an incentive to charge higher subscription fees, which makes it too expensive to buy from both firms simultaneously. This result highlights therefore the role of usage charges in limiting the level of subscription fees.[4]

[4] In a media model, Peitz and Valletti (2005) conjecture that no mixing might occur with flat tariffs if media content is sufficiently differentiated (i.e., transportation costs are sufficiently high). We have shown here that this result is more general as it does not depend on the magnitude of transportation costs.

Given the above result, the firms' choice of locations leads to the familiar maximum differentiation result of $a_1 = 0$ and $a_2 = 1$. Equilibrium subscription fees are $F_1 = F_2 = c + t$, and profits are equal to $\pi_1 = \pi_2 = t/2$. These profits are lower than under two-part tariffs. Consumer surplus of firm 1's subscribers is $U_1(x) = v - c - (1 + x^2)\, t$, equal to the non-mixing consumers' utility (21) under two-part tariffs (a corresponding results holds for subscribers to firm 2). Therefore all consumers in (x_l, x_h) are strictly worse off under competition in flat rates, and total consumer surplus $CS = 2\int_0^{1/2} U_1(x)\, dx = v - c - \frac{13}{12}t \simeq v - c - 1.083t$ is lower.

The outcome is inefficient in a circular sense: First, given that consumers do not mix, firms are too far apart: the socially optimal locations would be $a_1 = \frac{1}{4}$ and $a_2 = \frac{3}{4}$; second, given firms' locations consumers should be mixing.

Let us summarize:

Proposition 6 *If firms compete in flat rates then the equilibrium market outcome is as if consumers were not able to mix at all. Consumer surplus and profits are identical to the standard Hotelling model, and lower than under two-part tariffs. The outcome is inefficient because no mixing occurs.*

5 Extensions and discussion

In this section we check the robustness of our findings. We first consider what happens when firms offer menus of contracts. We then discuss if results would change if firms choose the type of tariffs before actually setting the prices.

5.1 A simple menu

Imagine both firms offer menus of relatively "simple" tariffs: a flat fee with unlimited usage and a linear tariff. This type of menu is often observed in reality. One can show that the *same* equilibrium outcome as under two-part tariffs is reproduced.[5] Define the menu (F^{**}, p^{**}) by $F^{**} = p^* + F^*$ and $p^{**} = p^* + 2F^*$, where p^* and F^* are the equilibrium prices with two-part tariffs in (16) and (17). At these prices the consumers to the left of x_l^* and to the right of x_h^*, adhere to the flat fee of the respective operator and do not mix, while the consumers in (x_l^*, x_h^*) mix using the linear tariffs of both firms. Consumers have an additional choice, though: They could mix using

[5] For simplicity we only consider locations $a_1 = 0$ and $a_2 = 1$. Calculations are available from the authors on request.

one operator's flat rate and the other operator's linear tariff. It can be shown that the latter does not happen at the equilibrium tariff (F^{**}, p^{**}): Given that the per-unit price of good 1 is zero under the flat rate, the consumer at x_l^*, who is indifferent between choosing firm 1's flat rate or the two linear tariffs, would still want to buy only from firm 1. Thus he does not make use of firm 2's linear tariff and effectively selects the flat rate only. Last but not least, no firm can increase its profits by changing its tariff from (F^{**}, p^{**}), therefore this is a Nash equilibrium (and the only one).

With this menu, all consumers receive exactly the same utility as under the equilibrium two-part tariffs, and firms obtain the same profits. Thus, in our model a two-part tariff is analogous to a menu of two simple tariffs. We thus have found an additional rationale for observing, for example, in real life, season tickets for a concert series coexisting with tickets sold for each single concert.[6] In our model, it is mixing rather than screening between high- and low-users that drives this result.

5.2 *Ex ante* tariff choice

Implicit in our results is the firms' simultaneous and non-cooperative choice between linear tariffs, flat rates and two-part tariffs, with the latter individually dominating the former two. For a given tariff structure of the rival, it is always in the interest of a firm to adopt a more ornate tariff when available. Our model has this in common with related results in the literature on mixed bundling, where consumers decide whether to buy complementary goods from one or several firms.

Still, in these models equilibrium profits may decrease with price discrimination, creating a prisoner's dilemma. Firms would therefore like to commit not to price discriminate. Furthermore, as shown in Matutes and Regibeau (1992) and Armstrong (2006, p. 27), in a two-stage game where firms first choose whether to price discriminate (bundle) or not, and then compete given their choices, in equilibrium no price discrimination may arise. Thus the price discrimination game is not robust to changes in timing.

If we introduce this two-stage structure in our game then the choice of two-part tariffs remains dominant, and the equilibrium outcome is identical to the one-stage game of simultaneous choice of tariff. This means that our

[6] Another example can be found in mobile telephony. Consumers who subscribe to post-paid tariffs (also called contracts; they often come in the form of a bundle with a certain amount of minutes included) typically buy from a single network operator. This contrasts with the choice of pre-paid cards (pay-as-you-go), where many consumers hold several phones, from different operators, at the same time.

model is robust to changes in timing or commitments concerning the tariff type.

5.3 Comparison between tariffs

We can summarize our results and compare them to Anderson and Neven (1989, AN) in the following table:

Pricing	Mixing	Consumers	Profits	Welfare
Linear (AN)	$[0, 1]$	$v - c - 1.000t$	$0.5000t$	$v - c$
Two-part, or menu	$[0.382, 0.618]$	$v - c - 1.072t$	$0.5172t$	$v - c - 0.03715t$
Flat rate	$\emptyset$	$v - c - 1.083t$	$0.5000t$	$v - c - 0.08333t$

Firms prefer two-part tariffs (or, equivalently, menus of contracts) over both linear tariffs and flat rates. Consumers, on the other hand, have the opposite preference ordering, with the exception of flat rates which again fare worst. Indeed, flat rates are Pareto-dominated by the other types of tariffs, which occurs due to the total absence of mixing under this tariff.

Total welfare, as given by the sum of consumer surplus and profits, is highest for linear tariffs, second-highest for two-part tariffs, and lowest for flat rates. Given that total consumption is fixed this ordering simply mirrors the extent of equilibrium mixing: With all tariffs, apart from the linear tariff, inefficient exclusivity arises endogenously in equilibrium.

Our main and robust result is that firms' profits raise as the number of instruments is increased starting from either a flat fee, or a linear tariff, respectively.[7] This result is contrary to some related results in the literature on price discrimination, where the availability of more ornate tariffs can decrease equilibrium profits if consumers buy from more than one firm (see Armstrong, 2006). At the same time, the effect on exclusivity is ambiguous: The number of customers buying exclusively can decrease (starting from a flat fee) or increase (starting from a linear tariff). Thus total welfare increases in the former case, but decreases in the latter.

The intuition for our results on profits comes from the fact that combinable goods are substitutes for customers who decide to buy exclusively, but are complements for customers that decide to mix, as their combination allows customers to reduce transportation costs. This extra surplus makes

[7] We also solved the model for general nonlinear tariffs, which represents an even more ornate tariff structure. In line with our results, profits increase further compared to two-part tariffs, though the absolute increase turns out to be negligible.

mixing consumers willing to pay two fixed fees, which in turn increases their total value for firms.

6 Conclusions

This paper has considered the problem of location choice with combinable goods. We have extended the model of Anderson and Neven (1989) with linear pricing to the cases of two-part tariffs, flat rates, and menus of contracts. We have shown how tariff structures have an impact on both firms' profits and efficiency and, at times, generate unexpected results.

Two-part tariffs and flat rates have practical relevance for example in media markets. For these tariffs, firms always choose extreme differentiation in all the versions studied. Notwithstanding this robust result, the change in pricing produces significant changes on a different type of allocation, namely on the choice of subscribers whether to mix or buy exclusively instead.

In the media industry, viewers typically have had to pay a flat subscription fee in the past to gain access to commercial TV stations (except for channels that are solely financed by advertising revenues). Because of technological progress, pay-per-view fees, in addition to fixed fees, have become increasingly more important. It has been argued that this trend is negative from the consumers' point of view, since it allows media firms to capture a larger share of the consumer surplus. Our Pareto dominance result shows that this view is too simplistic.

Even though in our model each consumer may buy a variable quantity from each firm, we have assumed inelastic *total* demand, as is common in the literature. Future research will relax this assumption and allow for elastic total demand. Our intuition is that, with elastic demand, two-part tariffs may lead to higher total welfare as compared to linear tariffs, because the lower marginal price may well induce sufficiently more additional demand as to outweigh the loss due to fewer mixing consumers.

Furthermore, in this paper we have concentrated the analysis on competition for viewers only, while we have neglected other sources of revenues for broadcasters, such as revenues from advertisers that want to reach viewers. As a simple extension of our model, consider the case where the number of commercials aired does not impose any nuisance on viewers, and broadcasters are able to earn a fixed amount per customer from advertisers. If advertisers pay broadcasters only per minute of viewing time, then our model with two-part tariffs would be solved in a straightforward way. All advertising revenues would be passed on to customers via a reduction in the pay-per-view price, while the subscription fee would be unaffected. In equilibrium, profits would be neutral to the presence of advertising revenues. On the contrary, if advertisers pay broadcasters per subscriber (independently of viewing time), then there would not be a full pass-through of advertising revenues anymore. Advertising revenues would lower also the subscription fee, which implies that more viewers would mix than without advertising revenues. It would be interesting to analyze in detail how our results with mixing viewers interact with the two-sidedness of these markets.

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Appendix

Proof of Proposition 1.

We first start with all the possible cases of demand.

Case I: $x_l \geq x_h$. There will be no mixing if

$$x_l = (a_1 - R) + \sqrt{F_2/t} \geq x_h = (a_2 - R) - \sqrt{F_1/t}$$

or

$$a_2 - a_1 \leq \sqrt{F_1/t} + \sqrt{F_2/t}.$$

In this case, demands will be defined by the choice between the two varieties, with indifferent consumer at

$$v - F_1 - p_1 - t\,(a_1 - x)^2 = v - F_2 - p_2 - t\,(a_2 - x)^2$$

or

$$x_m = \frac{a_1 + a_2}{2} + \frac{(F_2 + p_2) - (F_1 + p_1)}{2t\,(a_2 - a_1)}$$

This corresponds to the standard Hotelling model, and demands are

$$\begin{aligned} D_1\,(p_1, F_1, p_2, F_2) &= \begin{cases} 0 & if \quad x_m < 0 \\ x_m & if \quad 0 \leq x_m \leq 1 \\ 1 & if \quad x_m > 1 \end{cases} \\ B_1\,(p_1, F_1, p_2, F_2) &= D_1\,(p_1, F_1, p_2, F_2) \end{aligned}$$

Case II: $0 < x_h - x_l$ or $0 < a_2 - a_1 - \sqrt{F_1/t} - \sqrt{F_2/t}$

IIa) $1 \leq x_l < x_h$: all consumers prefer variety 1 to mixing or variety 2.

$$D_1\,(p_1, F_1, p_2, F_2) = B_1\,(p_1, F_1, p_2, F_2) = 1, B_2\,(p_1, F_1, p_2, F_2) = 0.$$

IIb) $0 < x_l < 1 \leq x_h$: variety 2 is only bought by mixers.

$$\begin{aligned} D_1\,(p_1, F_1, p_2, F_2) &= x_l + \int_{x_l}^{1} \lambda\,(x)\,dx \\ &= 1 - \frac{1}{2\,(a_2 - a_1)} \left(1 - a_1 + \frac{p_1 - p_2}{2t\,(a_2 - a_1)}\right)^2 + \frac{1}{2}\frac{F_2}{t\,(a_2 - a_1)} \end{aligned}$$

and

$$B_1\,(p_1, F_1, p_2, F_2) = 1, B_2\,(p_1, F_1, p_2, F_2) = 1 - x_l.$$

IIc) $0 < x_l < x_h < 1$: both varieties are bought alone and mixed.

$$\begin{aligned} D_1(p_1, F_1, p_2, F_2) &= x_l + \int_{x_l}^{x_h} \lambda(x)\, dx \\ &= \frac{a_1 + a_2}{2} + \frac{(F_2 + p_2) - (F_1 + p_1)}{2t(a_2 - a_1)} \\ B_1(p_1, F_1, p_2, F_2) &= x_h, B_2(p_1, F_1, p_2, F_2) = 1 - x_l. \end{aligned}$$

From (11) the first-order condition with respect to price p_1 is:

$$\frac{\partial \pi_1}{\partial p_1} = \frac{a_1 + a_2}{2} + \frac{F_2 + p_2 - 2(F_1 + p_1) + c}{2t(a_2 - a_1)} = 0,$$

which can be re-written as:

$$F_1 + p_1 = \frac{1}{2}\left(t\left(a_2^2 - a_1^2\right) + c + (F_2 + p_2)\right) \tag{24}$$

The derivative with respect to the fixed fee F_1 is:

$$\begin{aligned} \frac{\partial \pi_1}{\partial F_1} &= a_2 + \frac{p_2 - 2p_1 + c}{2t(a_2 - a_1)} - \frac{3}{2}\sqrt{F_1/t} = 0 \\ F_1 &= t\left(\frac{2}{3}a_2 + \frac{p_2 - 2p_1 + c}{3t(a_2 - a_1)}\right)^2 \end{aligned}$$

The Hessian is negative semidefinite iff $F_1 \leq \frac{9t(a_2-a_1)^2}{16}$. Similar expressions can be obtained for firm 2, leading to a system of equations in p_1, F_1, p_2, F_2, which can be solved to obtain the candidate solution (12). It is easy to check that the conditions $0 < x_l < x_h < 1$ are satisfied by the candidate solution, which also satisfies the condition for the Hessian.

IId) $x_l \leq 0 < x_h < 1$): variety 1 is only bought by mixers.

$$\begin{aligned} D_1(p_1, F_1, p_2, F_2) &= \int_0^{x_h} \lambda(x)\, dx \\ &= \frac{1}{2(a_2 - a_1)}\left(\frac{p_1 - p_2}{2t(a_2 - a_1)} - a_2\right)^2 - \frac{1}{2}\frac{F_1}{t(a_2 - a_1)} \\ B_1(p_1, F_1, p_2, F_2) &= x_h, B_2(p_1, F_1, p_2, F_2) = 1. \end{aligned}$$

IIe) $x_l < x_h \leq 0$: all consumers only buy variety 2.

$$D_1(p_1, F_1, p_2, F_2) = B_1(p_1, F_1, p_2, F_2) = 0, B_2(p_1, F_1, p_2, F_2) = 1.$$

Both D_i and B_i are not differentiable in F_i at the borders, and B_i is not differentiable in p_i at these points.

Given the candidate solution (12), we now show that possible deviations to any other branch of the demand system cannot upset this equilibrium candidate, therefore it is a Nash equilibrium.

A lower pair of prices (F_1, p_1) could violate $[x_h < 1]$, so demand would fall under case IIb) $0 < x_l < 1 \leq x_h$. Profits would be:

$$\pi_1 = (p_1 - c)\left(1 - \frac{1}{2(a_2 - a_1)}\left(1 - a_1 + \frac{p_1 - p_2}{2t(a_2 - a_1)}\right)^2 + \frac{1}{2}\frac{F_2}{t(a_2 - a_1)}\right) + F_1.$$

This expression is increasing in F_1, thus it would always collapse either to case IIc) or to case I.

Take now case IIa): $D_1(p_1, F_1, p_2, F_2) = B_1(p_1, F_1, p_2, F_2) = 1$. Profits are simply

$$\pi_1 = (p_1 - c) + F_1$$

which always collapses to case IIb).

A higher p_1 would violate $x_l > 0$, so demand would move to case IId), with $x_l \leq 0 < x_h < 1$. Profits would be:

$$\begin{aligned}\pi_1 &= (p_1 - c)\left(\frac{1}{2(a_2 - a_1)}\left(\frac{p_1 - p_2}{2t(a_2 - a_1)} - a_2\right)^2 - \frac{F_1}{2t(a_2 - a_1)}\right) \\ &\quad + F_1\left(a_2 - \frac{p_1 - p_2}{2t(a_2 - a_1)} - \sqrt{\frac{F_1}{t}}\right)\end{aligned}$$

(which becomes zero at $x_h = 0$ or $p_1 = p_2 + 2t(a_2 - a_1)\left(a_2 - \sqrt{F_1/t}\right)$). This is the same function of F_1 as in case IIc), thus we need only look at p_1. A complication arises as D_1 is not differentiable in p_1 at the border point if $F_2 > 0$. The border price is: $p_1 = p_2 + 2t(a_2 - a_1)\left(a_1 + \sqrt{F_2/t}\right)$, $\frac{\partial \pi_1}{\partial p_1}$ is negative at border price with (p_2^*, F_2^*), independently of the value of F_1:

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_1} &= \frac{1}{2(a_2 - a_1)}\left(\frac{3p_1 - p_2 - 2c}{2t(a_2 - a_1)} - a_2\right)\left(\frac{p_1 - p_2}{2t(a_2 - a_1)} - a_2\right) - \frac{F_1}{t(a_2 - a_1)} \\ &= -\frac{\sqrt{5} - 1}{12}(5a_1 - a_2 + 4) - \frac{F_1}{t(a_2 - a_1)} < 0.\end{aligned}$$

Since π_1 is third order in p_1 with positive coefficient on p_1^3, the border point is on the falling middle segment, and the local maximum is outside interval IId). The upper limit at $x_h = 0$ has value zero, therefore there is no profitable deviation in p_1.

Finally, consider case I. A higher F_1 would move to case I, $x_h \leq x_l$. Profits would be

$$\pi_1 = (p_1 + F_1 - c)\, x_m = (p_1 + F_1 - c)\left(\frac{a_1 + a_2}{2} + \frac{(F_2 + p_2) - (F_1 + p_1)}{2t\,(a_2 - a_1)}\right).$$

The best response in $P_1 = p_1 + F_1$, given $P_2 = p_2 + F_2$, is $P_1 = \frac{1}{2}t\,(a_2^2 - a_1^2) + \frac{1}{2}(P_2 + c)$. However, this is already true at the equilibrium candidate in IIc), therefore there is no profitable deviation to case I.

Let us now consider uniqueness. First of all, there is at most one equilibrium in case IIc). Furthermore, with the above arguments we can rule out equilibria in cases I), IIa) and IIb), and therefore also in cases IId) and IIe). Thus the Nash equilibrium is unique. QED

Proof of Proposition 5.

The relevant demand system is identical to the one described in the proof of Proposition 1, with $p_1 = p_2 = 0$. First we show that the case with $0 < x_l < x_h < 1$, where some consumers mix, does not contain a Nash equilibrium (this is denoted as case IIc). Firm 1's profits are

$$\begin{aligned}\pi_1(F_1, F_2) &= F_1 B_1(F_1, F_2) - cD_1(F_1, F_2)\\ &= F_1\left(a_2 - \sqrt{F_1/t}\right) - c\left(\frac{a_1 + a_2}{2} + \frac{F_2 - F_1}{2t\,(a_2 - a_1)}\right),\end{aligned}$$

with equilibrium candidate $F_1 = t\left(\frac{2}{3}a_2 + \frac{c}{3t(a_2 - a_1)}\right)^2$. In the same manner one finds $F_2 = t\left(\frac{2}{3}(1 - a_1) + \frac{c}{3t(a_2 - a_1)}\right)^2$. It is straightforward to show that these values violate the condition delimiting case IIc).

Our candidate case is the case where $x_l \geq x_h$, i.e., no consumer mixes (this is denoted as case I; the other subcases of case II lead to deviations as before). Demands are

$$B_1(F_1, F_2) = D_1(F_1, F_2) = \frac{a_1 + a_2}{2} + \frac{F_2 - F_1}{2t\,(a_2 - a_1)},$$

leading to standard Hotelling payoffs for both firms:

$$\begin{aligned}\pi_1 &= (F_1 - c)\left(\frac{a_1 + a_2}{2} + \frac{F_2 - F_1}{2t\,(a_2 - a_1)}\right)\\ \pi_2 &= (F_2 - c)\left(1 - \frac{a_1 + a_2}{2} - \frac{F_2 - F_1}{2t\,(a_2 - a_1)}\right)\end{aligned}$$

These payoffs are concave, and the equilibrium candidate is given by (21) and (22) stated in the proposition. These values are contained in case I since

$$\begin{aligned} F_1^*/t &\geq \frac{1}{3}(2+a_1+a_2)(a_2-a_1) \geq (a_2-a_1)^2, \\ F_2^*/t &\geq \frac{1}{3}(4-a_1-a_2)(a_2-a_1) \geq (a_2-a_1)^2, \end{aligned}$$

and therefore $\sqrt{F_1^*/t} + \sqrt{F_2^*/t} \geq 2(a_2 - a_1)$. Given F_2^*, any deviation of firm 1 to one of the subcases of case II involves the choice of F_1 such that $\sqrt{F_1/t} < a_2 - a_1 - \sqrt{F_2^*/t}$. Since the right-hand side is negative there is no profitable deviation by firm 1 that leads to case II. Therefore our candidate is a Nash equilibrium, and there is no other Nash equilibrium in case I (nor in the other subcases of case II). QED

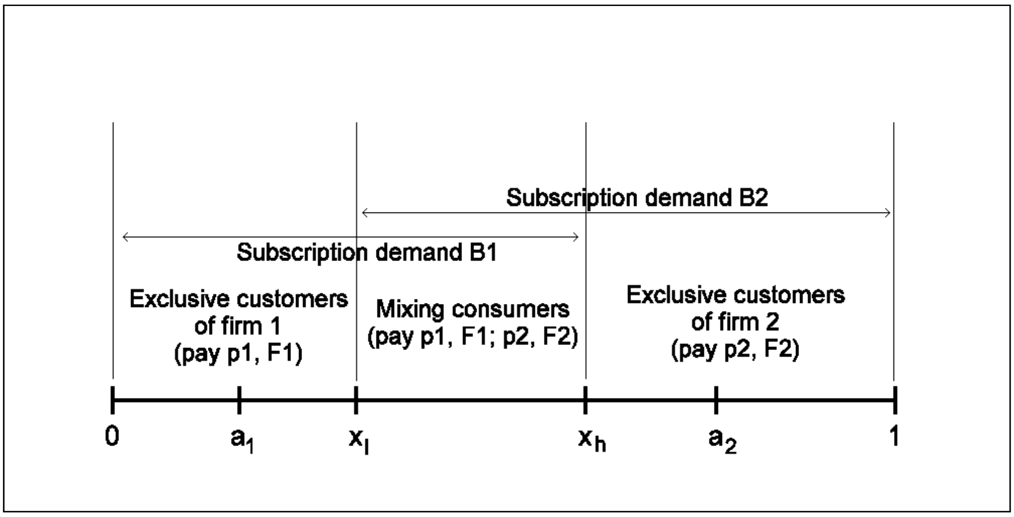

Figure 1: Consumers' choices and firms' subscription demands under two-part tariffs.