Information and the Hold-Up Problem

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Abstract

This article examines the holdup problem that arises when the investing party must purchase a necessary input from a second party. Consistent with many real-world settings, we suppose that the investing party must sink its investment prior to purchasing the necessary input, but before contracting with the seller of this input (*e.g.,* a firm that contracts for necessary distribution access only after developing its new product). We study how the resulting holdup problem is affected by the seller's information about the investing party's likely returns from its investment. We find, contrary to popular intuition, that improving the seller's information can actually benefit the buyer and harm the seller. We also find that other popular intuitions about this problem are either false, incomplete, or require fairly strong assumptions to be valid.

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1 Introduction

The hold-up problem is a central issue in economic analysis.[1] It arises when one party makes a sunk, relationship-specific investment and then engages in bargaining with an economic trading partner. That partner may be able to appropriate some of the gains from the sunk investment, thus distorting investment incentives, either toward too little investment or toward investments that are less subject to appropriation. Examples include a buyer who requires the seller's facility to market the buyer's products (*e.g.,* a coal mine reliant on the local railroad or a web-based application provider reliant on an Internet service provider), a buyer who must invest in complementary assets to be used in conjunction with the seller's product (*e.g.,* a firm undertaking marketing expenditures or investment in specialized facilities in order to distribute a manufacturer's product), investment in R&D or specialized production assets early on in a procurement process, and private investment subject to later government regulation (*e.g.,* construction of a regulated oil or gas pipeline).

Economists have devoted considerable attention to the hold-up problem. Recent work has fallen into several different areas. One strand of the literature examines various means of solving the hold-up problem through clever contracts and mechanisms entered into *prior* to investment. Examples include Demski and Sappington (1991), Nöldeke and Schmidt (1995, 1998), Bernheim and Whinston (1998), and Edlin and Hermalin (2000).[2] Che and Hausch (1999) and others consider *cooperative* investments (*i.e.,* the investment made by one party increases the value of exchange for the other party). Gul (2001), among others, examines situations in which investment occurs prior to any contracting. *Inter alia*, Gul demonstrates how the observability of investment affects the equilibrium outcome. Specifically, he shows that the hold-up problem is solved when the buyer's investment is unobservable, all of the offers are made by the seller, and the time between offers is small.

The present paper further analyzes the effects of the information structure on the hold-up problem when pre-investment contracting is infeasible.[3]

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[1]For classic analyses of the hold-up problem, see Klein (1988) and Williamson (1975, 1976). More recent work is discussed below.

[2]For a more complete survey, see §4.3.2 of Hermalin et al. (in press).

[3]Intellectual property licensing represents an interesting case in which pre-investment contracting is particularly difficult because neither the intellectual property owner nor

Our principal focus is on the effects of the investment's being observable by the non-investing party. In a departure from Gul, we allow for stochastic returns to investment. In this setting, even when the non-investing party observes the investing party's level of investment and the non-investing party has all the bargaining power, the non-investing party is typically unable to appropriate the investing party's surplus fully. Like us, Skrzypacz (2005) allows for investment with noisy returns. However, like Gul, Skrzypacz focuses on the limiting case of a bargaining process in which the degree of *ex post* inefficiency goes to zero. In contrast, we limit ourselves to letting the non-investing party make a single, take-it-or-leave-it offer. Our simpler bargaining process gives rise to the possibility of *ex post* inefficiency, which we believe is an important feature of many settings of interest.

The situation we have in mind is the following. There is an initial stage in which a buyer invests in complementary assets that are necessary to generate value from a seller's product and which have no value in alternative uses.[4] After the results of the buyer's investment have been realized, the seller makes the buyer a take-it-or-leave it offer. In deciding the price to offer, the seller may have information about (*i.e.,* receive a signal of) the buyer's realized value for the seller's product. At one extreme, the signal could be perfect and reveal the buyer's realized value. Then, absent any *ex ante* pricing commitments to do otherwise, the seller will set a price that fully extracts the buyer's surplus. Anticipating such pricing, the buyer expects to earn zero profits *gross* of its investment expenses regardless of its level of investment. Hence, a rational buyer makes no investment. In other words, as is well known, perfect information leads to complete holdup and destroys buyer investment incentives. Given the choice between a regime of symmetric information and seller ignorance, both the buyer and the seller will prefer ignorance if that allows the buyer to earn enough surplus that the buyer makes a positive level of investment in equilibrium.

In our experience, there is a widely held intuition that improving the

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the party producing an infringing product may be aware of the infringement until after the producer has sunk its investment and begun operations. One difference from our formal model below is that with positive probability the producer does not have to obtain a license. Incorporating this feature requires a minor and obvious modification of our model.

[4]As should become evident, our analysis applies equally well to settings in which the seller makes an investment that lowers its costs and the buyer then makes a take-it-or-leave-it offer.

seller's information lowers the buyer's profits and investment incentives even when the improved information is itself imperfect. As we will demonstrate, however, this intuition is incorrect in important circumstances. It should not be surprising that "anything can happen" absent sufficient structure. But suppose one restricts attention to settings in which investment improves the distribution of the buyer's returns in the sense of first-order stochastic dominance and a higher value of the seller's signal leads to an improvement in the conditional distribution of the buyer's returns in the sense of first-order stochastic dominance. With this structure, intuition suggests that the seller's price is increasing in the signal value and that, in comparison with an uninformative signal, an informative signal lowers the equilibrium levels of investment, buyer profits, and joint profits. As we will show, however, all of these claims are false.

Our analysis proceeds as follows. After describing the model and characterizing a baseline case in which the seller is perfectly uninformed about the buyer's investment level and the realized value of trading, we consider two situations, which differ in terms of the seller's information.

We first examine settings in which the seller can observe—and condition its price on—the seller's investment level. We demonstrate that, when the seller cannot commit to a price schedule prior to the buyer's sinking its investment, the observability of investment may in general raise or lower the buyer's equilibrium investment level and the seller's price may be increasing or decreasing in the investment level. We derive broad conditions under which the seller's price is increasing in investment and the additional information reduces equilibrium buyer investment, in accord with the common intuition that additional information allows the seller to appropriate more of the returns to investment and thus reduces the buyer's investment incentives. Even in this case, however, we obtain the surprising result that the additional information results in the *buyer's* equilibrium profits rising and the *seller's* falling *vis-à-vis* the situation when the seller cannot observe investment.

This last result holds quite generally. The result is surprising under the misleading intuition that increased information provides the seller with a greater ability to extract rents from the buyer. But the result is clear once one adopts the correct intuition that the seller's being able to see the buyer's action allows the buyer to act as a Stackelberg leader.

We also show that, because there are two opposing forces at work, the net effect of investment-based pricing on total surplus is ambiguous even when such pricing lowers buyer investment further below the efficient level.

First, investment-based pricing induces the buyer to invest less, which tends to lower welfare. But, second, the seller lowers its price in response to lower investment, which increases the social benefits associated with a given level of investment because the seller is less likely to inefficiently price the buyer out of the market (*i.e.,* to cause the buyer to shut down). We demonstrate that a necessary condition for investment-based pricing to increase welfare is that it raise the equilibrium probability of trade.

We also briefly examine the effects of seller commitment. When it can commit to a price schedule before the buyer makes its investment, the seller can, in some circumstances, impose a "forcing contract" that induces the buyer to choose a specific level of investment. We find that, because of the possibility that a buyer will shut down its operations, the seller generally does not induce the first-best investment level. Indeed, we derive conditions under which the seller induces the buyer to invest more than the level that is efficient given the seller's price.

The second situation we examine is a generalization of the first. We examine markets in which the seller conditions its price on a general, noisy signal of the returns realized from the buyer's investment. We derive conditions under which the seller's price is an increasing function of the signal's value and the buyer's equilibrium investment is less than second-best level. However, we also demonstrate by example that the investment and welfare effects of increased seller information are generally ambiguous even under what appear to be strong regularity conditions.

We close this introduction with two issues with respect to the application and interpretation of the analysis. The first issue is whether the case of observable investment or a noisy signal of returns is the more appropriate model. At one pole, suppose the input is essential to some business activity that is conducted by an organizational unit that makes public reports of its financial performance at the unit level. In this case, the financial reports could be interpreted as a noisy signal of returns. At the other pole, suppose that a firm undertakes many different activities, only one of which requires the input in question, and the firm does not report financial performance broken down by activity. In this case, investment in specialized plant and equipment may be more readily observable and our observable-investment model is more relevant.

The second issue concerns the nature of comparative statics. One interpretation of our analysis is that it demonstrates the effects of exogenously given differences in the information structure (*e.g.,* human capital investments may be harder to observe than investments in specialized machinery). Another interpretation is that the analysis sheds light on the effects of price discrimination and, thus, on public policies that forbid or allow price discrimination.[5]

The investment effects of discrimination are of considerable practical interest. For example, arguments about the effects of price discrimination on investment lie at the heart of much of the current debate over whether Internet access providers should be allowed to engage in price discrimination toward various application web sites or "content providers." Applying the standard intuition described above, much of the policy debate has been framed as a choice between promoting Internet access provider investment by allowing discrimination or promoting application provider investment by banning discrimination. Our analysis demonstrates that this framing could well be incorrect when the seller is imperfectly informed about the returns to the buyer's investment.

Our findings stand in contrast to the main thrust of the relatively small extant literature on price discrimination and investment. Katz (1987, Proposition A.4) showed that, when buyers cannot backward integrate and the seller is perfectly informed about the cost and demand conditions that the buyers face, a discriminating upstream monopolist selling to downstream Cournot competitors charges a higher price to those firms that have otherwise lower production costs. Several authors then showed that this pattern of input pricing dampens the downstream firms' incentives to make cost-reducing investments when the results of those investments are observable. DeGraba (1990) examines an input producer, Haucap and Wey (2004) consider a labor union, and Choi (1995) examines tariff setting, where the government can be interpreted as a monopoly seller of sales licenses.[6] Inderst and Valletti (2006) generalize Katz's analysis of the threat of buyer integration. They examine settings in which the seller's pricing is constrained by the buyers' access to an alternative, low-cost source of supply subject to a fixed switching cost.

[5]At the federal level, the Robinson-Patman Act prohibits a supplier from engaging in price discrimination that harms competition. There are also numerous state laws that limit the franchisors, manufacturers, or wholesalers of various products from discriminating among their retail distributors or franchisees.

[6]Kim and Nahm (in press) also examine the effects of price discrimination on investment incentives. They examine a different model of downstream competition and find that discrimination raises downstream investment incentives in some circumstances. We discuss their analysis and its relationship to ours in the concluding section below.

Inderst and Valletti find that, as a buyer reduces its costs, a discriminating upstream supplier reduces the priced charged to that buyer and raises the price charged to a rival buyer. Hence, in this setting, price discrimination raises buyer investment incentives.

Pricing conditional on a signal of investment or the returns to that investment can be viewed as discriminatory pricing and, indeed, we use this terminology below. The modeling of non-discrimination is more delicate. A natural interpretation of non-discrimination is that the seller can observe the value of each buyer's signal but must charge the same price to all buyers. Below, we model non-discrimination as a perfectly uninformative signal. When discrimination corresponds to basing price on the buyer's investment level and there is a continuum of *ex ante* identical buyers, the two approaches are equivalent. However, differences arise when discrimination is based on a general, noisy signal of the returns to investment. In this case, the seller's price under non-discrimination typically will be a random variable that is dependent on the sample distribution of the various buyers' signals. In contrast, when the signal is perfectly uninformative, the seller's price will be a constant. For this reason, we describe the analysis absent a signal as non-discrimination only in the case in which the signal is the buyer's investment.

2 The Model

We examine a setting in which there is a single buyer that requires the output of a monopoly seller to generate value by selling a downstream product. For example, the monopolist might control a bottleneck facility through which the buyer reaches its market. Alternatively, the buyer might be a distributor of a monopoly manufacturer's product. Or the buyer might need to license the seller's intellectual property. We assume the buyer is a monopoly provider of its downstream product. This assumption avoids complications that arise when there are multiple buyers that are downstream competitors and, consequently, have interdependent demands.

The timing of the baseline game is as follows:

- The buyer chooses and sinks its investment, I, in its product.
- The seller observes a signal s, which may contain information about the buyer's benefit of trade, r (its return on investment). The seller then makes a take-it-or-leave offer to sell one unit of its output for $p(s)$.

- After observing the realized values of r and $p(s)$, the buyer chooses whether to shut down or continue. If the buyer shuts down, it loses its investment, I, earns no returns, and makes no payment to the seller. If the buyer continues operation, it earns profits of $r - p(s) - I$ and the seller receives payment $p(s)$. For simplicity, we assume the seller incurs no marginal costs to produce output.[7]

The buyer's investment yields a conditional distribution of product-market quasi-rents (*i.e.,* buyer profits gross of the investment cost and any payments to the seller). As a shorthand, we will refer to the product-market quasi-rents as the buyer's returns. Formally, the buyer earns return $r \in [0, \bar{r}]$, $\bar{r}$ finite. Returns have the conditional distribution $F(r, s|I)$, where s is a signal whose properties will be discussed below. Let $f(r, s|I)$ denote the corresponding density function.[8] We assume $F(r, s|I)$ is at least twice differentiable in I for all $I \in (0, \infty)$, r, and s. Let $F_r(r|I)$ denote the corresponding marginal distribution and $f_r(r|I)$ its corresponding density. The latter is assumed to exist for all $r \in (0, \bar{r})$. Let

$$h(r|I) = \frac{f_r(r|I)}{1 - F_r(r|I)}$$

denote the corresponding hazard rate.

We assume that $F_r(0|0) = 1$; that is, the buyer earns no returns if it makes no investment. To ensure that the problem is nontrivial, we also assume that there exists at least one positive price at which the buyer is willing to invest:

Assumption 1 *There exists some $p > 0$ and $I > 0$ such that*

$$\int_p^{\bar{r}} \big(1 - F_r(r|I)\big) dr > I \text{ .}^9 \tag{1}$$

[7]One could allow for production costs with no effects on the qualitative results as long as the seller incurred costs only at the time that the buyer placed a firm order for the good.

[8]For much of our analysis, the assumption that $F(\cdot, \cdot|I)$ is a continuous distribution is not necessary. Generally, $f(r, s|I)$ could be interpreted as a probability mass on the point (r, s). Because they are more tractable, at points in the text we consider examples in which r and s have discrete conditional distributions.

The signal s is publicly observable and is drawn from the interval $[0, \bar{s}]$. In cases where the seller commits to setting its price conditional on s, s is also assumed to be verifiable in court. Throughout, we assume that r is the private information of the buyer. Let $F_s(s|I)$ denote the marginal distribution of the signal conditional on investment, and let $f_s(s|I)$ denote the corresponding density function.

Our solution concept is perfect Bayesian equilibrium and, as usual, we solve the game working backwards.

In what follows, the relationship between investment and the distribution of returns plays a critical role. We consider three assumptions:

- *Productive Investment:* An increase in I raises the expected value of r.
- *FOSD Improvement:* An increase in I improves the distribution of r in the sense of first-order stochastic dominance.[10]
- *Monotone Hazard:* For any $I > 0$ and $r < \bar{r}$, the hazard rate $h(r|I)$ is non-decreasing in r and decreasing in I.

The Monotone Hazard Condition has two components. First, it requires that the demand for the seller's product (*i.e.,* $1 - F_r(p|I)$) cannot flatten too quickly as price, p, falls. Indeed, it implies that the price elasticity of demand, $ph(p|I)$, must fall as p falls. Second, the condition implies that, for a given p, an increase in I reduces the price elasticity of demand.

As is well known, the FOSD Improvement Condition implies the Productive Investment Condition. The following lemma demonstrates that the Monotone Hazard Condition implies the FOSD Improvement Condition (the converse, however, does not hold).

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[9]The derivation of this condition will become clear below. A sufficient condition for it to hold is there exist an $\varepsilon > 0$ such that

$$\lim_{I \downarrow 0} \int_0^{\bar{r}} \frac{\partial F_r(r|I)}{\partial I} dr < -1 - \varepsilon\,.$$

[10]Throughout, when we refer to first-order stochastic dominance, we mean it in the following strict sense: If $I > I'$, then $F_r(r|I) < F_r(r|I')$ for any r such that $F_r(r|I) < 1$. Because $F_r(r|\cdot)$ is differentiable, it is equivalent to express the first-order stochastic dominance ordering as $\partial F_r(r|I)/\partial I < 0$ for any r and I such that $F_r(r|I) < 1$.

Lemma 1 *If $h(r|\cdot)$ is a decreasing function for all r and if $I > I'$, then $F_r(\cdot|I)$ dominates $F_r(\cdot|I')$ in the sense of first-order stochastic dominance.*

Lemma 1 is a special case of Lemma A.1, which is proved in the Appendix.

In addition to examining the effects of the information structure on equilibrium investment and profits, we examine the effects on equilibrium welfare. We take expected total surplus as our welfare measure. We assume that the buyer's customers (if any) derive zero consumer surplus from consumption of the buyer's output. We do this for expositional convenience and because it is well known that a supplier (here, "the buyer") tends to underinvest when an increase in investment generates consumer benefits that the supplier is unable to appropriate. Our interest is in the new phenomena that arise directly as a consequence of the upstream seller's pricing conditional on its signal.

Given the assumption that r captures the full social benefits derived from the production and consumption of the seller's output, expected total surplus is

$$W\big(p(\cdot), I\big) \equiv \int_0^{\bar{s}} \int_{p(s)}^{\bar{r}} r f(r, s|I) dr ds - I \,. \tag{2}$$

As long as $p(\cdot)$ is Lebesgue integrable, $W\big(p(\cdot), I\big)$ is continuous in I. Because the expected social benefit of investment is bounded above by $\bar{r}$, there is no loss of generality in assuming that I is chosen from the compact interval $[0, \bar{r}]$. Hence, there exists at least one welfare-maximizing investment level given $p(\cdot)$.[11]

It is apparent that $p(s) \equiv 0$ is the welfare-maximizing (first-best) price schedule. For $p(s) \equiv 0$, welfare is

$$W(0, I) = \int_0^{\bar{r}} r f_r(r|I) dr - I = \int_0^{\bar{r}} \big(1 - F_r(r|I)\big) dr - I \,, \tag{3}$$

where the second equality follows from integration by parts. Note that Assumption 1 implies there exists an $I > 0$ such that $W(0, I) > 0$; that is, the welfare-maximizing investment level is positive.

In the remainder of the paper, we consider three cases:

1. The seller's signal is completely uninformative about I and the realized value of r. We refer to this as the "uninformed-seller" case.

[11] This analysis is easily extended to include mixed strategies by the buyer, as will become clear in our analysis of the buyer's privately optimal investment level.

2. The seller's signal is perfectly informative about I, but provides no information about the realized value of r beyond that contained in knowledge of the value of I. We refer to this as the "observable-investment" case.

3. s is a noisy signal of r. We refer to this as the "noisy-signal-of-returns" case.

3 An Uninformed Seller

We begin by characterizing the equilibrium outcome when the seller's signal is perfectly uninformative. In this case, the seller bases its pricing on its inference of the equilibrium value of investment and the corresponding distribution of returns.

3.1 Best-Response Functions

Suppose that the seller plays the (possibly mixed) pricing strategy $\sigma(p)$, where the mixing (if any) occurs over $\mathcal{P} \subset [0, \bar{r}]$. Any investment level that the buyer chooses with positive probability must maximize

$$\begin{aligned}
\pi^B(\sigma(p), I) - I &\equiv \int_{\mathcal{P}} \left(\int_0^{\bar{r}} \max\{0, r - p\} f_r(r|I) dr \right) d\sigma(p) - I \\
&= \int_{\mathcal{P}} \left(\int_p^{\bar{r}} 1 - F_r(r|I) dr \right) d\sigma(p) - I \, . \qquad (4)
\end{aligned}$$

Because (4) is bounded above and continuous in I, at least one maximum must exist.

We assume:

Assumption 2 *For any price in the interval $[0, \bar{r})$, the buyer's expected surplus is concave in investment for any level of investment; specifically, for all I and all $p \in [0, \bar{r})$,*

$$\int_p^{\bar{r}} \big(1 - F_r(r|I)\big) dr$$

is strictly concave in I.

This assumption corresponds to assuming that there are diminishing marginal returns to increased investment given the restriction that the buyer will shut down if $r < p$. Because the set of concave functions is closed under scalar multiplication and addition, Assumption 2 implies that the buyer's objective function, expression (4), is strictly concave in I and, thus, the maximum is unique. Denote the buyer's unique best response by $I^*(\sigma(p))$.

Now, consider the seller's best response to the buyer's choice of investment level. If the buyer invests I and the seller charges price p, then the seller's profits are

$$\pi^S(p, I) \equiv p\big(1 - F_r(p|I)\big) . \tag{5}$$

If $I = 0$, then expression (5) is identically zero and any p is a best response. Because both a zero price and a price of $\bar{r}$ yield zero profits, any maximizer of (5) is an element of the interior of the support of $F(\cdot|I)$ when $I > 0$.

We assume that (5) is strictly concave for $p \in [0, \bar{r}]$ for all $I > 0$. It is worth noting that the Monotone Hazard Condition implies $1 - F_r(r|I)$ is log concave in r for $I > 0$, which in turn is a sufficient condition for (5) to be strictly concave.[12] Because (5) is strictly concave, the seller has a unique best response to any $I > 0$, which we denote by $p^*(I)$.

We have shown:

Lemma 2 *If the seller's signal is perfectly uninformative, any equilibrium in which the buyer invests a positive amount is a pure-strategy equilibrium.*

As a consequence, we can express the buyer's best response as $I^*(p)$. Because the buyer's optimization problem is concave in I for all p and continuous in both p and I, it follows that $I^*(p)$ is continuous in p.

The first-order condition for the seller's optimization problem given $I > 0$ is

$$1 - F(p|I) - pf(p|I) = 0 ,$$

which can be written as

$$p = \frac{1}{h(p|I)} . \tag{6}$$

One question of interest is how the value of p that maximizes (5) varies with I. It might appear that, if investment is productive (*i.e.,* the expected

[12] We can also state a sufficient condition in terms of the density function. By a theorem of Prékopa (1971) (Theorem 13.20 of Pečarić et al., 1992) if $f(\cdot|I)$ is log-concave, then $1 - F(\cdot|I)$ is as well.

value of r is increasing in I), then p should be increasing in I. This is not a valid inference, however, as the following example demonstrates. Suppose that $I \in \{0, 1, 2\}$, where $I = 1$ corresponds to undertaking a safe project, while $I = 2$ corresponds to investing in a risky project. The safe project yields return r_M with certainty. The risky project yields return r_H with probability ρ and return r_L with probability $1 - \rho$. Assume

$$\rho r_H + (1 - \rho) r_L > r_M > r_L > \rho r_H > 0$$

(*e.g.,* $r_H = 32$, $r_M = 20$, $r_L = 18$, and $\rho = 1/4$).[13] Then the seller's best response to $I = 1$ is $p = r_M$, but its best response to $I = 2$ is $p = r_L$. In other words, what determines the profit-maximizing price is not merely the buyer's expected return, but the distribution of its returns.

As Figure 1 illustrates, even if investment satisfies the FOSD Improvement Condition, the seller's optimal price may decline as the buyer's investment rises. In this figure and throughout the paper, $D(\cdot|\cdot)$ is the conditional survival function (*e.g.,* $D(p|I_1) = 1 - F_r(p|I_1)$ in Figure 1). Viewing the probability of trade as the quantity, $D(\cdot|I)$ is the demand function conditional on I.

To say something more definitive about how the seller's best response function varies with investment, assume that $h(r|I)$ is non-decreasing in r for all I and decreasing in I for all r (*i.e.,* the Monotone Hazard Condition is satisfied). The relevance of this assumption can be seen by examining the first-order condition for the seller's choice of price. Consider an I and p that solve Equation (6). If I is increased, then the right-hand side of (6) increases. Raising p in response increases the left-hand side and also reduces the right-hand side, restoring equality. We have established:

Proposition 1 *If the Monotone Hazard Condition is satisfied, then the seller's profit-maximizing price is increasing in the buyer's investment level.*

Next, consider the buyer's best response to the seller's price. Intuitively, one might expect that $I^*(p)$ is everywhere non-increasing as long as the Productive Investment Condition is satisfied because, as the price rises, the returns to investment are realized in a smaller set of states (*i.e.,* when $r > p$) and the buyer earns less in those states (*i.e.,* $r - p$). However, this intuition is incomplete and, consequently, misleading. Figure 2 illustrates why.

[13] Note these distributions trivially exhibit non-decreasing hazard rates.

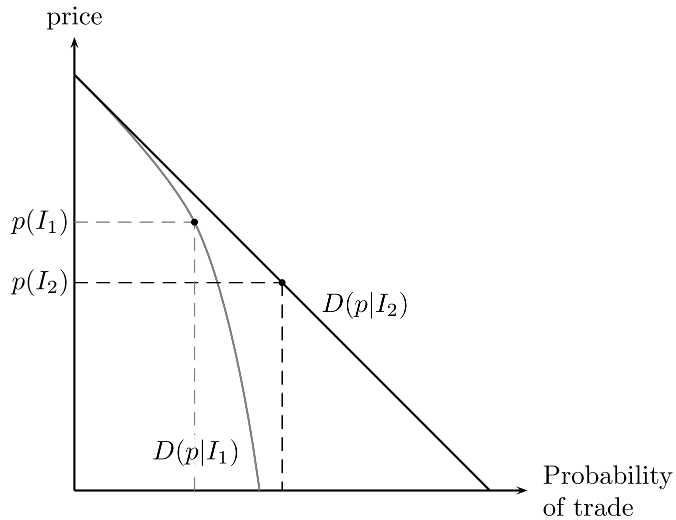

Figure 1: An example in which the FOSD Improvement Condition is satisfied and the seller's best response is to price higher for low investment (I_1) than for high investment (I_2).

Demand (the survival function) given low investment, I', is the straight line running from $(0, \bar{r})$ to $(1, 0)$. The demand given high investment, I, is the parabolic-shaped curve. As drawn, these curves satisfy the Productive Investment Condition.[14] Consider a price increase from p_1 to p_2. The gray region illustrates the change in the returns to investing when the price rises. Because, for prices less than p_2, the low-investment demand is greater than the high-investment demand, losing this gray area means the returns to investing have *increased*. Hence, *ceteris paribus* investing a high amount is relatively more attractive when $p = p_2$ than when $p = p_1$, which is why the buyer's best response can be *increasing* in price for some region of prices.[15] Another way to view the forces at work is in terms of the riskiness of the buyer's investment project. In this example, the higher level of investment corresponds to a riskier project: it has a greater chance of performing very

[14]For instance, if $1 - F_r(r|I') = 1 - \frac{p}{100}$ and $1 - F_r(r|I) = \frac{5}{6}\sqrt{1 - \frac{p}{100}}$, then the curves resemble those in Figure 2 and $\mathbb{E}\{r|I'\} = 50$ and $\mathbb{E}\{r|I\} = 500/9 \approx 55.56$.

[15]In the Appendix, we provide a completely worked out example showing that the buyer's best response can be increasing in price.

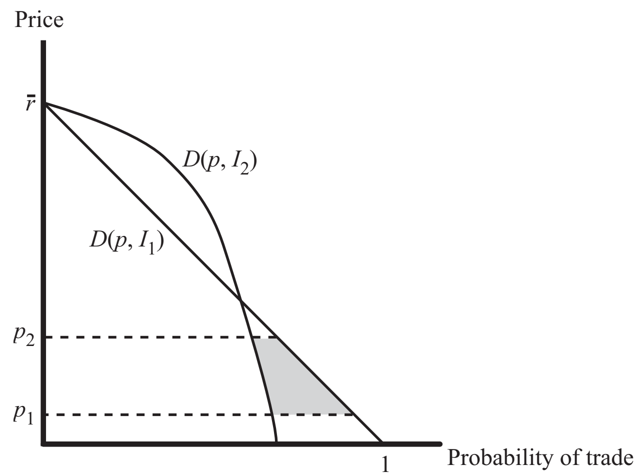

Figure 2: When the FOSD Improvement Condition is not satisfied, a price increase can raise the buyer's investment incentives.

well, but also a greater chance of performing very poorly. The seller's price acts as a hurdle, where only those returns that clear the hurdle are realized. Setting a higher hurdle encourages the buyer to adopt a riskier project because the returns from a safer project are unlikely to clear the hurdle.

This example relies on the fact that the demand curves cross. These curves cannot cross when the FOSD Improvement Condition is satisfied. Hence, although it is not strong enough to guarantee that the seller's profit-maximizing price is increasing in the buyer's investment level, the FOSD Improvement Condition is strong enough to insure that the buyer's best-response investment level falls as the seller's price rises:

Lemma 3 *If the FOSD Improvement Condition is satisfied, then the buyer's best-response investment level is decreasing in the seller's price whenever the investment level is positive.*[16]

Proof: Let $p_1 < p_2$ be two prices and let I_1 and I_2 be the corresponding best responses. Suppose, contrary to the proposition, that $I_2 \geq I_1 > 0$. Observe

[16]Under weaker conditions than assumed here, it can be shown that the buyer's best-response investment level is non-increasing in the seller's price. Specifically, this can be shown when first-order stochastic dominance is interpreted weakly (*i.e.*, $F_r(r|I) \leq F_r(r|I')$ for all $I > I'$ with the inequality being strict on a measurable subset of $[0, \bar{r})$). The proof of this weaker result is by revealed preference.

the marginal return to investment,

$$\int_{p}^{\bar{r}} \frac{-\partial F_r(r|I)}{\partial I} dr\,,$$

is greater for a given I when $p = p_1$ than when $p = p_2$ because FOSD implies $\partial F_r(r|I)/\partial I < 0$. By the concavity of the buyer's problem (Assumption 2),

$$\int_{p}^{\bar{r}} \frac{-\partial F_r(r|I_1)}{\partial I} dr \geq \int_{p}^{\bar{r}} \frac{-\partial F_r(r|I_2)}{\partial I} dr$$

if $I_2 \geq I_1$. Hence,

$$\int_{p_1}^{\bar{r}} \frac{-\partial F_r(r|I_1)}{\partial I} dr \geq \int_{p_1}^{\bar{r}} \frac{-\partial F_r(r|I_2)}{\partial I} dr > \int_{p_2}^{\bar{r}} \frac{-\partial F_r(r|I_2)}{\partial I} dr\,.$$

This inequality violates the following first-order conditions for I_1 and I_2 to be best responses:

$$\int_{p_1}^{\bar{r}} \frac{-\partial F_r(r|I_1)}{\partial I} dr = 1 = \int_{p_2}^{\bar{r}} \frac{-\partial F_r(r|I_2)}{\partial I} dr\,.$$

By contradiction, $I_2 < I_1$. ■

Another property of interest is how the buyer's choice of investment level compares with the welfare-maximizing one. Assumption 2 implies that (3) is concave in I and, thus, there is a unique socially optimal level of investment, $I^w(p)$ for any given price, p. We refer to $I^w(p)$ as the second-best investment level conditional on p and $I^w(0)$ as the first-best investment level. When the FOSD Improvement Condition is satisfied, the buyer invests too little from a welfare perspective:

Proposition 2 *Suppose that FOSD Improvement Condition is satisfied. Then, given any price, $p \in (0, \bar{r})$, the buyer's best-response investment level is less than the second-best amount unless the latter is 0.*

Proof: Consider the program

$$\max_{I} \int_{p}^{\bar{r}} (r - q) f_r(r|I) dr - I\,. \tag{7}$$

Observe that (7) is the buyer's optimization program if $q = p$ and is the social planner's second-best program if $q = 0$. The derivative of the marginal return to investment with respect to q is

$$\frac{\partial F_r(p|I)}{\partial I} < 0\,, \tag{8}$$

where the inequality follows from first-order stochastic dominance. It follows that $I^*(p) < I^w(p)$ (unless the latter is zero, in which case they both equal zero). ■

The underlying intuition is clear. When the FOSD Improvement Condition holds, an increase in I, holding price fixed, raises the probability a trade and, thus, the seller's profits. The buyer does not take the increase in seller profits into account in choosing its investment level. Observe that, when FOSD Improvement Condition does not hold, an increase in I could lower the probability of trade, creating an incentive wedge in the other direction.

3.2 Equilibrium

Having characterized the buyer and seller's best-response functions, we now examine equilibrium. A first observation is that there are generally multiple equilibria, even if there is a unique interior equilibrium. Specifically, there exists a degenerate equilibrium in which the buyer believes the seller will charge such a high price (*e.g.,* $p = \bar{r}$) that the buyer's best response is to invest nothing. As discussed, if the seller believes $I = 0$, then the seller is indifferent as to the price it quotes and, so, it is a weak best response for it to charge a high price.[17]

Under some circumstances, there also exists a non-degenerate equilibrium in which the buyer invests a positive amount. In the Appendix, we prove:

Lemma 4 *Suppose the seller is perfectly uninformed and cannot commit to a price. If*

$$\lim_{I \downarrow 0} \frac{1 - F_r(\bar{p}|I)}{f_r(\bar{p}|I)} < \bar{p}\,, \tag{9}$$

where $\bar{p} \equiv \min\{p|I^*(p) = 0\}$*, then there exists at least one pure-strategy equilibrium in which the buyer invests a positive amount.*

[17] The zero-investment equilibrium can survive trembles if there exists an investment level, I_0, such that $\max_I \pi^B(p^*(I_0), I) - I < 0$.

To see that condition (9) is not vacuous, consider, as an example, $F_r(r|I) = r^{10I}$ and $\bar{r} = 1$.[18] Calculations reveal $\bar{p} \approx .58754$. Observe that the limit in (9) is $-\bar{p}\log(\bar{p}) \approx .31246$; (9) holds. It follows from Lemma 4 that an equilibrium with a positive level of investment exists.[19]

We close this subsection with two results that will be useful in our later analysis. One question of interest is how the seller's profits vary with the investment level. One can readily construct examples in which the unconditional expected value of r is increasing in I but the seller's profits are not. Figure 3 illustrates this possibility graphically. Even though an increase in I may raise unconditional expected return, it could, as illustrated, reduce the share of those returns that the seller is able to appropriate by so much that the seller's profit falls. The next result demonstrates that the share effect cannot dominate when when increased investment leads to an increase in returns in the sense of first-order stochastic dominance (*i.e.,* when greater investment leads to everywhere greater demand).

Lemma 5 *If the FOSD Improvement Condition is satisfied, then the seller's profit, $\pi^S(p, I)$, is increasing in I for any $p \in (0, \bar{r})$.*

Proof: The seller's profits are $\pi^S(p, I) = p(1 - F_r(p|I))$, and first-order stochastic dominance implies that $F_r(p|I)$ falls as I rises. ■

The final result of this subsection demonstrates that, even when the buyer's investment is unobservable, the stochastic nature of r conditional on I affects the degree of hold up.

Lemma 6 *Suppose that investment I yields $r(I)$ with certainty, where $r(0) = 0$ and $r(\cdot)$ is an increasing continuous function. Then, in equilibrium, the buyer's expected investment level is zero.*

This result is an implication of Gul (2001), Proposition 1. For completeness, we provide a proof in the Appendix. Intuitively, the seller will always set its price at least as high as the return generated by the lowest positive level of investment that the buyer chooses in equilibrium. But then the buyer would not recover the costs of that investment. Hence, the only equilibrium investment level is zero.

[18] It can be shown that $F_r(r|I)$ satisfies Assumption 2. The function $p\big(1 - F_r(p|I)\big)$ is readily shown to be strictly concave in p.

[19] Calculations for this example reveal that the equilibrium price is approximately .501116 and equilibrium investment is approximately .101159.

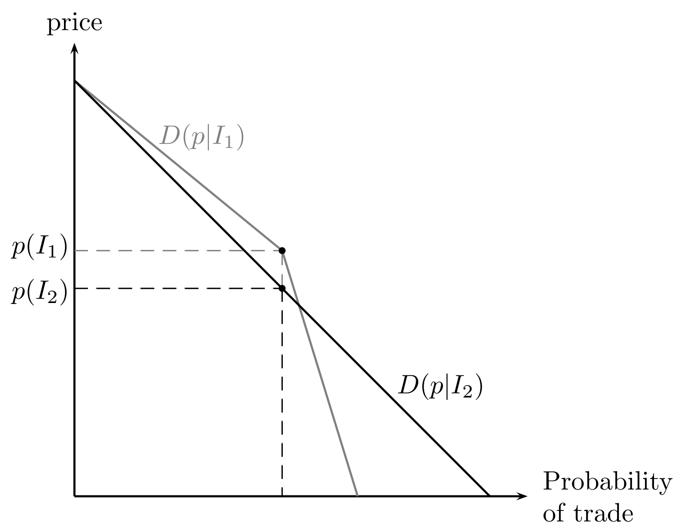

Figure 3: The area under $D(\cdot|I)$ equals $\mathbb{E}\{r|I\}$ and, as drawn, $\mathbb{E}\{r|I_2\} > \mathbb{E}\{r|I_1\}$, where $I_2 > I_1$. Expected returns increase in the level of buyer investment and the seller's profits are greater given low investment than high.

3.3 Seller Commitment

In order to examine the effects of holdup, we now briefly characterize the equilibrium when the seller can commit to a price before the buyer makes its investment decision. When the seller cannot commit to its price, there is no difference in our model between a two-part tariff and a single price. However, there is a difference when the seller can commit to its price schedule. If two-part tariffs were feasible *and* the buyer could commit to a contract before it learned the realized value of r, then the seller would offer the following contract: prior to making its investment decision, the buyer would pay the seller a flat amount for the right to later purchase the input at a price of 0. By making the buyer a residual claimant, this pricing mechanism would support the first-best outcome in our simple setting. The seller would set the flat payment to extract all of the buyer's expected surplus. Manifestly, the seller's profits and social welfare would be greater than when the seller cannot commit to its price.

To explore the effects of holdup further and to facilitate comparison with our earlier results, suppose that the seller can commit only to single-part pricing.[20] Our next result indicates that, in contrast to the no-commitment case, there is no degenerate equilibrium in which $I = 0$.

Lemma 7 *Suppose the seller's signal is perfectly uninformative and the seller can commit to its pricing strategy. Then at least one equilibrium exists. Moreover, in any equilibrium, the buyer's investment level is positive.*

Proof: By Assumption 2, the buyer has a unique best response to any pricing strategy to which the seller commits; moreover, $I^*(\sigma(\cdot))$ is continuous in $\sigma(\cdot)$. Therefore, the seller chooses $\sigma(\cdot)$ to maximize

$$\int_{\mathcal{P}} p\Big(1 - F_r\big(p|I^*(\sigma(\cdot))\big)\Big)d\sigma(p)\,, \tag{10}$$

which is a continuous function of $\sigma(\cdot)$. Because we can—without loss of generality—restrict the seller to choosing prices in the finite interval $p \in [0, \bar{r}]$

[20] Alternatively, the buyer cannot commit to waiting until after signing a contract with the seller before learning the value of r. When the buyer learns r before signing a contract, single-part and two-part tariffs are equivalent. We are also ruling out more general mechanisms. The introduction of more sophisticated contracts is not necessary to establish that a hold-up problem arises if the seller is unable to commit *ex ante*.

and the set of probability measures on $[0, \bar{r}]$ is compact under the topology of weak convergence, it follows that (10) has a solution (*i.e.,* an equilibrium exists).

Now, consider the buyer's equilibrium investment level. Recall that, by Assumption 1, there exists some $p > 0$ such that $\pi^B(p, I^*(p)) > 0$. This price would yield the seller positive profits. Therefore, it cannot be privately optimal for the seller to set a pricing strategy that induces the buyer to set $I = 0$ and thus yields the seller no profit. ∎

Our primary interest in the commitment case is to serve as comparison for the no-commitment case. As expected, the absence of commitment gives rise to a hold-up problem.

Proposition 3 *Suppose that the seller's signal is perfectly uninformative and the FOSD Improvement Condition is satisfied. Then, in equilibrium, the buyer's investment is weakly higher when the seller can commit to a price than when it cannot.*

Proof: Suppose, counterfactually, that $I^c < I^h$, where c denotes equilibrium value under commitment and h denotes equilibrium value given no commitment (holdup). Then, for any $p \in (0, \bar{r})$,

$$\pi^S(p, I^c) < \pi^S(p, I^h)$$

by Lemma 5. Hence,

$$\int_0^{\bar{r}} \pi^S(p, I^c) d\sigma^c(p) < \int_0^{\bar{r}} \pi^S(p, I^h) d\sigma^c(p) \leq \pi^S(p^*(I^h), I^h) \,, \tag{11}$$

where the weak inequality follows from the definition of the seller's best-response function. When commitment is feasible, the seller can commit to $p = p^*(I^h)$ and earn $\pi^S(p^*(I^h), I^h)$. By (11), doing so would raise profits, a contradiction. ∎

4 Observable Investment

With the uninformed-seller case as a benchmark, we now examine the equilibrium outcome when the seller can observe the buyer's investment level and condition its price on it.[21]

4.1 Equilibrium

If the buyer invests I, then perfection requires that the seller charge price $p^*(I)$. The buyer chooses I to maximize

$$\begin{aligned}\pi^B(p^*(I), I) - I &= \int_0^{\bar{r}} \max\{0, r - p^*(I)\} f(r|I) dr - I \\ &= \int_{p^*(I)}^{\bar{r}} \big(1 - F_r(r|I)\big) dr - I \,.\end{aligned}$$

The seller's optimization problem is concave in p for all I and continuous in both p and I, which implies that $p^*(I)$ is continuous in I. Given the assumed properties of $F_r(r|I)$, it follows that $\pi^B(p^*(I), I) - I$ is continuous in I and thus achieves a maximum over the compact interval $[0, \bar{r}]$. Therefore, at least one perfect equilibrium exists.

Recall that when the seller is perfectly uninformed, there exists a degenerate equilibrium with $I = 0$. When the seller can observe the buyer's investment level, there is a Nash equilibrium with $I = 0$. But this outcome cannot be a subgame perfect equilibrium if there exists any I such that $\pi^B(p^*(I), I) > 0$.

As when buyer investment is unobservable, the buyer invests less than the socially optimal amount given the seller's pricing strategy:

Proposition 4 *If the seller can observe the buyer's investment level and the FOSD Improvement Condition is satisfied, then, in any equilibrium, the buyer's investment level is less than the second-best amount unless the latter is zero.*

Proof: The buyer chooses I to maximize the buyer's profits. The second-best program seeks to maximize the sum of the buyer's profits and the seller's profits. By the envelope theorem, $d\pi^S(p^*(I), I)/dI = \partial\pi^S(p^*(I), I)/\partial I$,

[21] It is readily shown that—as long as the support of the signal is independent of the value of I—allowing the seller to observe a noisy measure of I, but not I itself, is equivalent to the uninformed-seller case previously analyzed. Observe, too, that one consequence of being a noisy measure of I is that the signal has no value in predicting the value of r conditional on knowing I.

which is positive by Lemma 5. Therefore, for any value of I that maximizes $\pi^B(p^*(I), I)$, there is a larger value of I that maximizes $\pi^B(p^*(I), I) + \pi^S(p^*(I), I)$. ■

Although our welfare measure is total surplus, it is informative to compare the equilibrium investment levels with observable and unobservable investment. We do this because, outside of our formal model, one might expect buyer investment to generate positive externalities, either real (*e.g.,* technological spillovers) or pecuniary (*e.g.,* consumer surplus enjoyed by the buyer's customers).

Proposition 5 *If the Monotone Hazard Condition is satisfied, then the buyer's equilibrium investment level is lower when the seller can observe investment than when the seller's signal is perfectly uninformative, unless both investment levels are zero.*

Proof: Let a "d" or "n" superscript denote the equilibrium value of a variable when the seller can discriminate based on I or not, respectively. By revealed preference,

$$\pi^B\big(p^*(I^d), I^d\big) - I^d \geq \pi^B\big(p^*(I^n), I^n\big) - I^n \geq \pi^B\big(p^*(I^n), I^d\big) - I^d\,. \tag{12}$$

Suppose $I^d > I^n$. Then, by Proposition 1, $p(I^d) > p(I^n)$. But then

$$\pi^B\big(p^*(I^n), I^d\big) > \pi^B\big(p^*(I^d), I^d\big)\,,$$

which contradicts (12). Hence $I^d \leq I^n$.

To establish $I^d \neq I^n$ when $I^n > 0$, observe that such an I^n would satisfy the first-order condition

$$\int_{p^*(I^n)}^{\bar{r}} \frac{-\partial F_r(r|I^n)}{\partial I} dr - 1 = 0\,. \tag{13}$$

In contrast, I^d satisfies the first-order condition

$$\int_{p^*(I^d)}^{\bar{r}} \frac{-\partial F_r(r|I^d)}{\partial I} dr - p^{*\prime}(I^d)\Big(1 - F_r\big(p^*(I^d)|I^d\big)\Big) - 1 = 0\,. \tag{14}$$

If $I^d = I^n$, then $p^*(I^d) = p^*(I^n)$. Making those substitutions into (14) and using (13) implies $p^{*\prime}(I^d) = 0$, which contradicts Proposition 1. Hence

$I^d < I^n$. ■

As our next example illustrates, the conclusion of Proposition 5 depends critically on the assumptions made about the distribution of returns given investment. In particular, absent such assumptions, investment-based pricing can increase equilibrium investment level.

Suppose that the buyer chooses $I \in \{0, \underline{I}, \bar{I}\}$ where $0 < \underline{I} < \bar{I}$. If the buyer invests nothing, then $r = 0$ with probability 1. An investment of $\underline{I}$ yields return r_M with certainty, and an investment of $\bar{I}$ yields r_H with probability ρ and r_L with probability $1 - \rho$. Assume:

- $\rho r_H + (1 - \rho) r_L - \bar{I} < r_M - \underline{I}$, which implies that $\underline{I}$ is the first-best investment level;
- $\rho r_H < r_L$, which implies that r_L is the seller's profit-maximizing price conditional on $I = \bar{I}$; and
- $\rho(r_H - r_L) - \bar{I} > 0$, which implies that the buyer would earn positive expected profits if it invested $\bar{I}$ and the seller set $p = r_L$.[22]

Observe that, under discrimination, it cannot be an equilibrium for the buyer to invest $\underline{I}$ because the seller would fully appropriate the resulting return (*i.e.,* $p^*(\underline{I}) = r_M$). Similarly, when discrimination is infeasible (*i.e.,* investment is unobservable), there no equilibrium in which the seller invests $\underline{I}$ with certainty.

Suppose the buyer invests $\bar{I}$. Under discrimination, the seller sets $p^*(\bar{I}) = r_L$, and the buyer earns profits of $\rho(r_H - r_L) - \bar{I}$, which are positive by hypothesis. Hence, the buyer invests $\bar{I}$ under discrimination. There is no pure-strategy equilibrium in which the buyer invests $\bar{I}$ absent discrimination. To see why, assume to the contrary. Then the seller must set $p = r_L$. But anticipating this price, the buyer would maximize its profits by choosing $I = \underline{I}$, in which case the seller would not set $p = r_L$. The only pure-strategy equilibrium investment level absent discrimination is 0, which can be supported by having the seller set $p = r_H$.

To summarize this example, the buyer invests more than the first-best amount when the buyer's investment is observable and less than the first-best amount when the seller's signal is perfectly uninformative. Moreover,

[22] The following numerical values satisfy all of the assumed conditions: $\underline{I} = 1$, $\bar{I} = 2$, $r_L = 5$, $r_M = 7$, $r_H = 10$, and $\rho = .45$.

equilibrium welfare is positive when investment is observable, but zero when it is not.

The intuition underlying this example is as follows. If the buyer efficiently chooses to invest $\underline{I}$, it earns a deterministic level of returns, which the seller can then fully appropriate. Absent discrimination, it is never an equilibrium to choose $\bar{I}$ with probability one because then the seller would set a low price and, knowing this, the buyer would deviate by setting $I = \underline{I}$.[23] When its investment level is observable, however, the buyer can credibly "show" the seller that its investment level is $\bar{I}$, which leads to noisy and, thus, less-than-fully appropriable returns.

This example illustrates a broader and well-known phenomenon with respect to investment and the hold-up problem: in some situations, the buyer's investment choice will affect the *share* of the returns that the buyer will appropriate, and the buyer's incentives are biased toward investments that increase the buyer's share of the total. Actions to affect the buyer's share include: randomization of the choice of I (see Gul, 2001); investment in projects with noisy returns (see, *e.g.,* Skrzypacz, 2005); adoption of flexible technologies, which improve the buyer's bargaining threat point; and second-sourcing, which is costly when the seller's costs are subadditive (see, *e.g.,* Farrell and Gallini, 1988).

The next example shows that, even if investment-based pricing lowers equilibrium investment, it may raise welfare. Suppose that the buyer chooses $I \in \{0, \underline{I}, \bar{I}\}$ where $0 < \underline{I} < \bar{I}$ and $\bar{I} - \underline{I} < .02$. Assume that $r \in \{0, 4, 6, 8\}$. As above, $r = 0$ with probability 1 when $I = 0$. In the present example, both of the positive investment levels yield noisy returns. Assume that

$$\begin{array}{ccc} & F(4|\underline{I}) = .40 & F(6|\underline{I}) = .75 \\ \text{and} & & \\ & F(4|\bar{I}) = .33 & F(6|\bar{I}) = .74 \end{array}.$$

Observe that $F(\cdot|\bar{I}) \underset{\text{FOSD}}{\succeq} F(\cdot|\underline{I})$. Clearly, the only possible price points are 4, 6, and 8. Straightforward calculations reveal that $p^*(\underline{I}) = 4$ and $p^*(\bar{I}) = 6$.[24] The buyer's expected profits under price discrimination are

$$.35 \times 2 + .25 \times 4 - \underline{I} = 1.7 - \underline{I}$$

[23]There are parameter values under which it is profitable for the buyer to play a mixed strategy that entails a positive probability of investing $\bar{I}$ but the probability is less than 1 in order to avoid full return appropriation.

[24]Respectively, $4 > \max\{.6 \times 6, .25 \times 8\}$ and $.67 \times 6 > \max\{4, .26 \times 8\}$.

if $I = \underline{I}$ and

$$.26 \times 2 - \bar{I} = .52 - \bar{I}$$

if $I = \bar{I}$. Because the former exceeds the latter, the buyer invests $\underline{I}$ in equilibrium and the seller's price on the equilibrium path is 4. Absent price discrimination, there is a unique equilibrium in which the buyer invests $\bar{I}$ and $p = 6$.[25] We can now compare expected welfare:

$$W(4, \underline{I}) = .40 \times 4 + .35 \times 6 + .25 \times 8 - \underline{I} = 5.70 - \underline{I} \tag{15}$$

and

$$W(6, \bar{I}) = .41 \times 6 + .26 \times 8 - \bar{I} = 4.54 - \bar{I}\,. \tag{16}$$

In this example, investment-based pricing raises welfare even though investment falls. Intuitively, price also falls by a sufficient amount that the probability of trade is higher at the low-investment/low-price outcome than at the high-investment/high-price outcome. In other words, there is an *ex post* efficiency improvement associated with the outcome under investment-based pricing.

The previous examples demonstrate that the sign of the welfare effects of investment-based pricing is ambiguous and depends on specific market characteristics. The following result characterizes one set of markets in which the sign is unambiguous:

Proposition 6 *Suppose that the Monotone Hazard Condition is satisfied. If the equilibrium probability of trade is lower when the seller can observe the buyer's investment than when its signal is perfectly uninformative, then the improvement in the seller's information lowers equilibrium welfare.*[26]

[25] Given $p = 6$, the buyer's expected profits are $.50-\underline{I}$ and $.52-\bar{I}$ under its two investment options. The latter exceeds the former, so $I^*(6) = \bar{I}$. We earlier showed $p^*(\bar{I}) = 6$. Simple calculations reveal that $\bar{I} = I^*(4)$, but, as noted above, $p^*(\bar{I}) \neq 4$. Hence, the equilibrium is unique.

[26] This result is suggestive of the well-known result that third-degree price discrimination lowers welfare if its lowers equilibrium output. The mechanisms at work are, however, different.

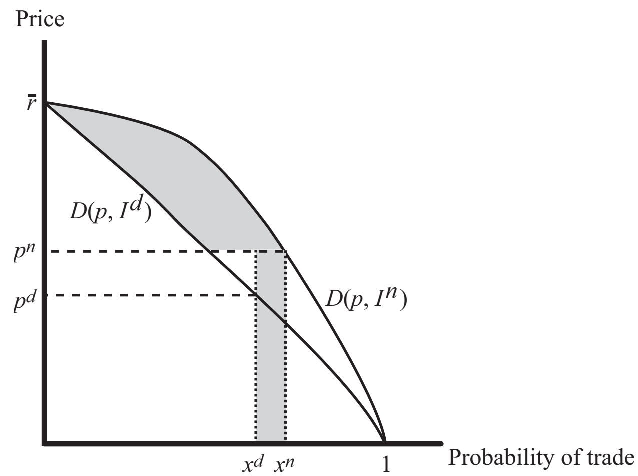

Figure 4: The welfare effects of investment-based pricing.

Proof: Let x^d denote the equilibrium probability of trade when the seller can observe the buyer's investment level, and let x^n denote the corresponding probability when it cannot. By the previous proposition, $I^d < I^n$. Proposition 1 then implies $p^d \equiv p^*(I^d) < p^*(I^n) \equiv p^n$. Figure 4 illustrates the change in total surplus gross of the buyer's investment cost. The relative positions of the demand curves follow because the Monotone Hazard Condition implies the FOSD Improvement Condition. Observe that the change in total surplus gross of investment costs exceeds the two shaded regions in Figure 4. The area of these regions are

$$\pi^B(p^n, I^n) - \pi^B(p^n, I^d) + p^n(x^n - x^d) \tag{17}$$

The result follows if (17) exceeds the incremental cost of investment, $I^n - I^d$. That, in turn, follows if

$$\left(\pi^B(p^n, I^n) - I^n\right) - \left(\pi^B(p^n, I^d) - I^d\right) + (x^n - x^d)p^n > 0\,.$$

By revealed preference, the difference in the first two terms is positive. And the third term is positive by hypothesis. Therefore, total surplus must be higher when the seller cannot observe the buyer's investment than when it can. ■

Corollary 1 *Suppose that the Monotone Hazard Condition is satisfied and the elasticity of demand at any probability of trade is increasing in investment (i.e., demand curves get "flatter" as investment increases). Then equilibrium welfare is lower when the seller can observe I than when its signal is perfectly uninformative.*

Proof: Given Proposition 6, it is sufficient to show that $x^n > x^d$. To do so, consider the seller's problem as one of choosing the profit-maximizing quantity (probability of sale) given the buyer's choice of I. Observe that the result follows if we can show that the seller's marginal revenue as a function of x is increasing in I. To this end, let $P(x, I)$ denote the inverse demand curve. By Proposition 5, $I^d < I^n$ and, thus, $P(x, I^n) > P(x, I^d)$ for all $x \in (0, 1)$. We need to show that

$$P(x, I^n) + x\frac{\partial P(x, I^n)}{\partial x} > P(x, I^d) + x\frac{\partial P(x, I^d)}{\partial x}\,, \tag{18}$$

which is equivalent to

$$P(x, I^n)\left(1 - \frac{1}{\epsilon(x, I^n)}\right) > P(x, I^d)\left(1 - \frac{1}{\epsilon(x, I^d)}\right)\,, \tag{19}$$

where $\epsilon(x, I)$ is the elasticity of demand at x given I. Because, by assumption, $\epsilon(x, I^n) > \epsilon(x, I^d)$ and we showed $P(x, I^n) > P(x, I^d)$, it follows that (19) holds. ■

Lastly, we examine the distributional effects of the observability of the buyer's investment level. Somewhat surprisingly, the improvement in the seller's information raises the buyer's equilibrium profits under very general conditions.

Proposition 7 *The buyer's equilibrium expected profits are weakly greater when the seller can observe the buyer's investment than when the seller's signal is perfectly uninformative.*

Proof: The result follows by revealed preference:

$$\pi^B\big(p^*(I^d), I^d\big) - I^d \geq \pi^B\big(p^*(I^n), I^n\big) - I^n = \pi^B(p^n, I^n) - I^n\,.$$

■

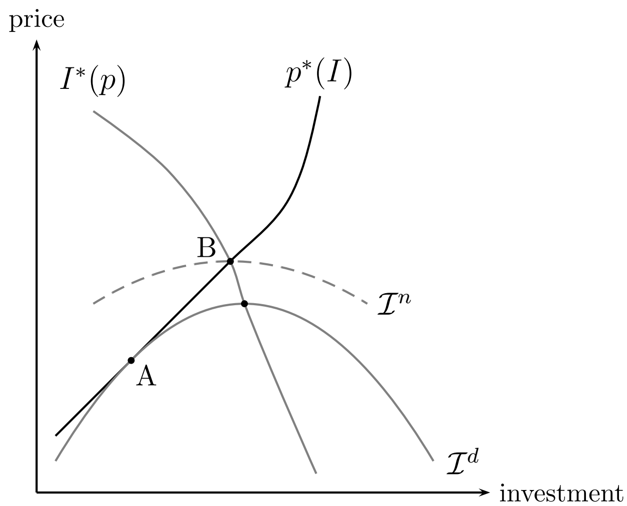

Figure 5: The Stackelberg effect.

Intuitively, the observability of I allows the buyer to choose from among a wider range of options and behave as a Stackelberg leader. Figure 5 illustrates. In the figure, $\mathcal{I}^n$ and $\mathcal{I}^d$ denote two iso-profit curves for the buyer. For a given price p, the buyer's profit is maximized at $I^*(p)$; hence, the summits of the iso-profit curves coincide with best-response function $I^*(\cdot)$. Point B denotes the equilibrium point absent discrimination. When the seller can observe I, the buyer chooses its desired point on $p^*(I)$. As illustrated, point A is its most desired point and the buyer's profit is increased *vis-à-vis* point B.

This result has a simple but powerful implication: if the seller prefers to engage in investment-based pricing, it is socially optimal for the seller to do so. Hence, banning a seller from using such information could be welfare improving only in those circumstances in which the seller cannot commit to ignoring the information.[27] Of course, such commitment is difficult in some settings, but even here this result provides a very different perspective on why regulation could be valuable than does the prevailing intuition.

This result also highlights the difference between marginal and total profit effects on investment incentives. Even though the buyer earns higher profits

[27]Many states, for example, have laws that prohibit the manufacturers or wholesales of certain products from discriminating among their retail distributors.

when the seller is better informed, we have seen that the buyer may also invest less.[28]

Although the buyer gains from the improvement in the seller's information, the seller loses, at least under the Monotone Hazard Condition:

Proposition 8 *If the Monotone Hazard Condition is satisfied, then the seller's equilibrium expected profits are lower when the seller can observe the buyer's investment than when the seller's signal is perfectly uninformative.*

Proof: By Proposition 5, $I^n > I^d$. Using the fact that the Monotone Hazard Condition implies the FOSD Improvement Condition, the result follows from application of Lemma 5. ■

Under the conditions of Proposition 8, the seller would wish to commit *ex ante* not to price discriminate on the basis of investment. Observe that an *ex ante* contractual agreement with the buyer would be insufficient if renegotiation were possible. A contractual agreement could prevent the seller from unilaterally raising the price. But suppose the buyer invested I', where $I' < I^n$, and proposed to the seller that it lower its price from p^n to $p^*(I')$. It would be in both the buyer and seller's interest to renegotiate the contract in this way rather than maintain $p = p^n$. Anticipating renegotiation, the buyer would solve the program

$$\max_I \begin{cases} \int_{p^*(I)}^{\bar{r}} \big(1 - F_r(r|I)\big)dr - I\,, \text{ if } I \leq I^n \\ \int_{p^n}^{\bar{r}} \big(1 - F_r(r|I)\big)dr - I\,, \text{ if } I > I^n \end{cases} .$$

When the Monotone Hazard Condition is satisfied, the solution to this program is $I^d < I^n$. In other words, it is not necessarily enough for the seller to commit not to price opportunistically, it could also be necessary for the seller to commit not to negotiate discounts off its posted price.

In some settings, the seller may be able to establish a reputation for neither making use of the available information nor engaging in holdup. Lafontaine and Shaw (1999) examined a large panel of franchise contracts

[28]In a different context, Inderst and Wey (2006) also find that a change that raises the investor's overall level of returns can lower investment incentives. Specifically, they find that an decrease in buyer power, which raises the seller's profits, can decrease the seller's incentives to invest in reducing its costs.

(specifically, royalty rates and franchise fees) over 13 years. The authors found large differences across franchise systems but that the franchise fee and royalty rate are generally the same for all franchisees joining a given system at a given time (*i.e.,* there is no customization of contracts to idiosyncratic conditions). Moreover, they found that renewals occur at the then-current terms for new franchisees. Although there are other explanations for this behavior, we observe that it is consistent with the implications of the present analysis.

4.2 Seller Commitment

We close this section by examining the outcome when the seller can commit to its price schedule prior to the buyer's making its investment.[29] By setting $p(I)$ sufficiently large for any $I \neq I_0$, the seller can force the buyer either to shut down or pick I_0. The seller will choose I_0 to maximize the value of the following program:

$$\max_p p\big(1 - F_r(p|I_0)\big)$$

subject to

$$\int_p^{\bar{r}} \big(1 - F_r(r|I_0)\big) dr \geq I_0 \,.$$

Forming the Lagrangian and differentiating it with respect to I yields the first-order condition

$$0 = -\frac{p\partial F_r(p|I)}{\partial I} + \lambda\left(\int_p^{\bar{r}} \frac{-\partial F_r(r|I)}{\partial I} dr - 1\right).$$

Suppose that the FOSD Improvement Condition holds. Then the seller increases I until the constraint binds. Hence, $\lambda > 0$ and we can write the first-order condition as

$$\frac{p}{\lambda} \times \frac{\partial F_r(p|I)}{\partial I} = \int_p^{\bar{r}} -\frac{\partial F_r(r|I)}{\partial I} dr - 1 \,.$$

By first-order stochastic dominance, the left-hand side is negative. The right-hand side is the social marginal return to investment when the seller sets a

[29] As in the case of a perfectly uninformative signal, we assume that it is infeasible for the seller to offer the buyer a contract that provides an option to buy the good at $p = 0$ in return for a payment made prior to the buyer's sinking its investment.

price of p. Hence, the seller induces the buyer to make more than the second-best level of investment given the seller's price.

Summarizing this analysis,

Proposition 9 *Suppose that the seller can observe the buyer's investment level and can commit to its price schedule before the buyer makes its investment decision. If the FOSD Improvement Condition holds, then the seller induces the buyer to invest more than the second-best level given the seller's price.*

The intuition is as follows. Holding its price fixed, the seller's profits rise with I because the probability of making a sale increases. As long as the buyer's individual rationality constraint is not binding, the seller increases I without regard for the additional investment costs. Observe that if the buyer could commit to paying the seller regardless of the realization of r, then this effect would not arise. Indeed, it is easily seen that, in this alternative game, the seller would force the buyer to choose the first-best level of investment and then extract all of the expected surplus through the choice of p.

Absent commitment, the buyer invests less than the second-best amount. With commitment, the buyer is induced to invest more than the second-best amount. It should be noted, however, that the equilibrium prices may be different, so that these results do not provide a direct comparison of the two investment levels.

5 A Noisy Signal of Returns

We now allow s to be an arbitrary signal that is informative with respect to return, r. Conditional on the signal s and its anticipated value of I, I_0, the seller seeks to maximize

$$p\big(1 - G(p|s, I_0)\big) \,, \tag{20}$$

where $G(r|s, I)$ is the distribution of r conditional on s and I.

Given a price schedule $p^*(s, I_0)$, the buyer's investment problem is

$$\begin{aligned}\max_I \int_0^{\bar{s}} \int_{p^*(s,I_0)}^{\bar{r}} \big(r - p^*(s, I_0)\big) f(r, s|I) dr\, ds - I \\ = \max_I \int_0^{\bar{s}} \left(\int_{p^*(s,I_0)}^{\bar{r}} \big(1 - G(r|s, I)\big) dr \right) f_s(s|I) ds - I \,. \end{aligned} \tag{21}$$

As we saw in our examination of $s \equiv I$, it is necessary to put considerable structure on the problem in order to obtain definitive results. To that end, we make several assumptions:

- First, as discussed in the Introduction, a perfectly informative signal destroys buyer investment incentives when the seller cannot commit to its price schedule prior to the buyer's investing. Hence, we assume that s is a noisy signal of r.
- Second, we assume that higher values of investment tend to give rise to higher values of the signal. Specifically, an increase in I leads to an improvement in the distribution of s in the sense of first-order stochastic dominance. We also assume that $s = 0$ with probability 1 if $I = 0$.
- Lastly, we extend the Monotone Hazard Condition to include s:

Assumption 3 (Extended Monotone Hazard Condition) *For any $I > 0$, the hazard rate associated with the distribution of the buyer's returns conditional on its investment and the signal is non-decreasing in return and decreasing in both the signal and investment:*

Formally, the hazard rate is

$$h_G(p|s, I) \equiv \frac{g(p|s, I)}{1 - G(p|s, I)}\,,$$

where $g(r|s, I)$ is the density function corresponding to $G(r|s, I)$. Let $\mathbf{a}$ denote the vector (s, I). Employing the usual ordering over vectors, we assume

$$h_G(p|\mathbf{a}) \geq h_G(p'|\mathbf{a}) \text{ if } p > p' \tag{22}$$

and

$$h_G(p|\mathbf{a}) < h_G(p|\mathbf{a}') \text{ if } \mathbf{a} \succ \mathbf{a}' \tag{23}$$

for p and $p' \in (0, \bar{r})$ and $\mathbf{a}$ and $\mathbf{a}' \in [0, \bar{s}] \times (0, \infty)$.

A well-known consequence of assuming the hazard rate is non-decreasing in price is that conditional demand, $1 - G(p|s, I)$, is log concave in p for all s and all $I > 0$. The log concavity of $1 - G(p|s, I)$ is sufficient for (20) to be strictly concave in p and, hence, for $p^*(s, I_0)$ to be unique.

In the Appendix, we extend several results derived for the case of $s \equiv I$ to this more general setting:

Proposition 10 *If the Extended Monotone Hazard Condition holds, then the seller's profit-maximizing price increases with both the signal and the anticipated value of investment; that is,* $p^*(s, I)$ *is increasing in both its arguments.*

Proposition 11 *Suppose that an increase in investment leads to an improvement in the distribution of* s *in the sense of first-order stochastic dominance and that the Extended Monotone Hazard Condition is satisfied. Then an increase in investment by the buyer raises the seller's profits and, given the equilibrium price schedule chosen by the seller, the buyer's equilibrium investment level is less than the second-best amount unless the latter is zero.*

Not all of the results from the observable-investment case extend to a general signal of r. For instance, with a general s, the buyer could be worse off than when the seller's signal is perfectly uninformative (*e.g.,* when s is a near-perfect signal of r and, thus, allows near-perfect rent extraction by the seller). This result stands in contrast with Proposition 7, which applied to the case of $s \equiv I$.

An obvious question is whether the seller's ability to condition its price on the signal reduces the buyer's investment incentives *vis-à-vis* the situation in which the seller is incapable of conditioning price on s. At this level of generality, there is no definitive answer:

Proposition 12 *Depending on the parameter values, pricing based on a noisy signal of return may raise or lower the equilibrium level of buyer investment. This ambiguity arises even when an increase in investment leads to an improvement in the distribution of* s *in the sense of first-order stochastic dominance and the Extended Monotone Hazard Condition is satisfied.*

We prove the proposition by example.

Suppose $s \in \{0, 1, 2\}$, and let $f_s(s|I)$ denote the probability mass function for s conditional on the value of I. Assume that an increase in I leads to a improvement in the distribution of s in the sense of first-order stochastic dominance (*i.e.,* $f_s(0|I)$ and $f_s(0|I) + f_s(1|I)$ are decreasing in I).

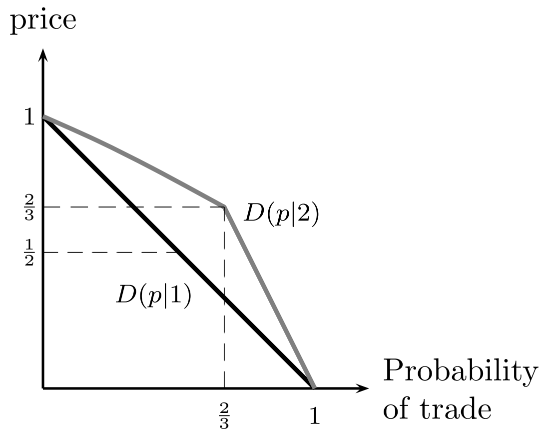

Figure 6: $D(p|s)$ is the probability of trade conditional on the price and signal value. The point $(2/3,\ 2/3)$ is a kink in $D(\cdot|2)$.

Let $\bar{r} = 1$. Assume

$$1 - G(r|s,I) = \begin{cases} 0\,, \text{ if } s = 0 \\ 1 - r\,, \text{ if } s = 1 \\ \left\{ \begin{array}{l} 1 - \frac{1}{2}r\,, \text{ if } r \leq \frac{2}{3} \\ 2(1-r) + \frac{1}{20}\left(r - \frac{2}{3}\right)\left(1 - r\right)\,, \text{ if } r > \frac{2}{3} \end{array} \right\}, \text{ if } s = 2 \end{cases}.$$

Observe that s is a sufficient statistic for r and that $r = 0$ with probability 1 if $s = 0$. The conditional demand curves $1 - G(r|1,I)$ and $1 - G(r|2,I)$ are illustrated in Figure 6.[30]

Lemma 8 *The distributions $G(r|s,I)$ given above satisfy the Extended Monotone Hazard Condition.*

If the seller can condition its price on the signal, then it is readily shown that $p(1) = 1/2$ and $p(2) = 2/3$.

[30]The slight degree of curvature introduced in the last line of the definition of $1 - G(r|s,I)$ (the term beginning $1/20$) is needed to ensure that the Extended Monotone Hazard Condition is strictly satisfied for all r; were that term deleted, then the hazard rate would be the same regardless of signal for $r > 2/3$.

Define

$$\hat{\pi}^B(p,s) = \int_p^{\bar{r}} \big(1 - G(r|s,I)\big)dr\,.$$

$\hat{\pi}^B(p,s)$ is the area beneath $1 - G(r|s,I)$ from p to 1 (*e.g.*, $\hat{\pi}^B(2/3,2)$ is the triangular area in Figure 6 defined by $(0,2/3)$, $(0,1)$, and $(2/3,2/3)$).

When the seller does not observe the signal and, thus, cannot condition its price on it, the seller chooses its price to maximize

$$f_s(1|I)p(1-p) + f_s(2|I)p\big(1 - G(p|2,I)\big)\,. \tag{24}$$

Because the seller would *not* set $p > 2/3$ if it knew $s = 2$, the solution to (24) cannot exceed 2/3. Consequently, we can rewrite (24) as

$$f_s(1|I)p(1-p) + f_s(2|I)p(1 - \tfrac{1}{2}p)\,.$$

It follows that, if

$$\frac{f_s(1|I) + f_s(2|I)}{2f_s(1|I) + f_s(2|I)} \geq \frac{2}{3}\,, \tag{25}$$

then the solution to maximizing (24) is $p = 2/3$.

Consequently, the buyer's problem when the seller can observe s is

$$\max_I f_s(1|I)\hat{\pi}^B\big(\tfrac{1}{2},1\big) + f_s(2|I)\hat{\pi}^B\big(\tfrac{2}{3},2\big) - I\,. \tag{26}$$

If (25) holds, then the buyer's problem when the seller cannot observe the signal is

$$\max_I f_s(1|I)\hat{\pi}^B\big(\tfrac{2}{3},1\big) + f_s(2|I)\hat{\pi}^B\big(\tfrac{2}{3},2\big) - I\,. \tag{27}$$

Conditional on the $s = 2$, the seller's price and the buyer's surplus are the same whether or not the seller can observe s. The observability of s matters, however, when $s = 1$. Conditional on $s = 1$, the seller charges a lower price and the buyer derives greater surplus when s is observable.

Because $\hat{\pi}^B(2/3,1) < \hat{\pi}^B(1/2,\,1)$, it follows that which regime leads the buyer to invest more is determined by whether $f_s(1|I)$ is increasing or decreasing in a neighborhood of the optimal I. If the former, then the buyer invests more when the seller can condition its price on the signal. If the latter, then the buyer invests more when the seller cannot observe the signal.

Either case is possible. For instance, suppose that

$$f_s(s|I) = \begin{cases} e^{-20I}\,, & \text{if } s = 0 \\ \frac{1}{2}\big(1 - e^{-20I}\big)\,, & \text{if } s = 1 \text{ or } 2 \end{cases}\,.$$

It is readily checked that condition (25) is met. Observe the sum $f(0|I) + f(1|I)$ is decreasing in I; however, because $f(1|I)$ is increasing in I everywhere, it follows that buyer investment is greater when the seller can condition on the signal than when the seller cannot.[31]

Alternatively, suppose

$$f_s(s|I) = \begin{cases} e^{-20I} - Ie^{-40I}\,, \text{ if } s = 0 \\ Ie^{-40I}\,, \text{ if } s = 1 \\ 1 - e^{-20I}\,, \text{ if } s = 2 \end{cases} . \tag{28}$$

Condition (25) is met because

$$\frac{f_s(1|I) + f_s(2|I)}{2f_s(1|I) + f_s(2|I)}$$

is increasing in I and

$$\lim_{I \downarrow 0} \frac{f_s(1|I) + f_s(2|I)}{2f_s(1|I) + f_s(2|I)} = \frac{21}{22} > \frac{2}{3} .$$

Numerical calculations reveal that the solution to (26) is $I^* \approx .039$, whereas the solution to (27) is $I^* \approx .040$. The seller's ability to condition price on the signal reduces investment in this case because

$$\frac{\partial f(1|I)}{\partial I} = e^{-40I}(1 - 40I) < 0$$

for $I > .025$. One can interpret this result in terms of opportunity cost: a cost of increasing I given this $f_s(s|I)$ is that the buyer lowers its probability of getting the expected surplus associated with $s = 1$. This cost is less when that expected surplus is lower (*i.e.,* when the seller will price at 2/3 and not 1/2). The buyer invests more when the cost is lower and, thus, invests more when price "conditional" on $s = 1$ is 2/3 and not 1/2; that is, when the seller cannot condition its price on the signal than when it can.

This last example (*i.e.,* $f_s(s|I)$ given by Equation (28)) provides another illustration of the importance of marginal—as opposed to average—effects. Specifically, note that, for a given level of investment, the equilibrium price when the seller cannot observe the signal is always at least as high and

[31] Numerical calculations reveal $I^* \approx .043$ when the seller can so condition and $I^* \approx .026$ when it cannot.

sometimes strictly higher than the price charged when the seller can observe the signal. Yet, the buyer's investment incentives are higher when the seller cannot observe the signal. The reason is that the slope, as well as the overall level, of the price function matters. When the seller cannot condition its price on the signal, the buyer's decision to increase I does not lead to a higher price. In contrast, when the seller can condition on s, an increase in I triggers a shift in the distribution of s and, thus, the price.

The reason for no definitive results despite strong assumptions about the distributions (*e.g.,* the Extended Monotone Hazard Condition) is that there are a number of effects at work. Discrimination typically affects both the level and slope of the seller's price as a function of s. When s is unobservable, this function is flat. When the Extended Monotone Hazard Condition is satisfied, the seller tends to charge the buyer more when the buyer's revenues are high than when they are low.[32] Although it might seem that the upward slope would tend to discourage investment, as the previous example shows, reality is much more complicated and the effects depend on the the full set of changes in the distribution of s conditional on I. Moreover, the effects of discrimination depend both on whether the price rises faster or slower than the buyer's returns conditional on s.

This last point raises the issue of *ex post* efficiency. When the signal is perfectly informative, discrimination leads to perfect *ex post* efficiency, while an uninformative signal does not. There are other cases, however, in which discrimination reduces *ex post* efficiency. As an example, suppose there are only two levels of investment, $\{0, \hat{I}\}$, and that, in equilibrium, the buyer invests $\hat{I}$ both under a regime in which the seller cannot observe s and under a regime when the seller can condition price on $s \in \{0, 1, 2\}$. Suppose, further that

$$1 - G(r|s, I) = \begin{cases} 0\,, \text{ if } s = 0 \\ \left\{ \begin{array}{l} 1 - \frac{2}{3}r\,, \text{ if } r \leq \frac{3}{5} \\ \frac{3}{2}(1-r)\,, \text{ if } r > \frac{3}{5} \end{array} \right\}, \text{ if } s = 1 \\ \left\{ \begin{array}{l} 1 - \frac{3}{7}r\,, \text{ if } r \leq \frac{7}{10} \\ \frac{7}{3}(1-r)\,, \text{ if } r > \frac{7}{10} \end{array} \right\}, \text{ if } s = 2 \end{cases}.$$

[32] Although it is outside of our model, observe that, if the buyer were risk averse, pricing based on s could allow the seller to provide a form of insurance in those cases where a low realization of r was likely to be correlated with a low realization of s.

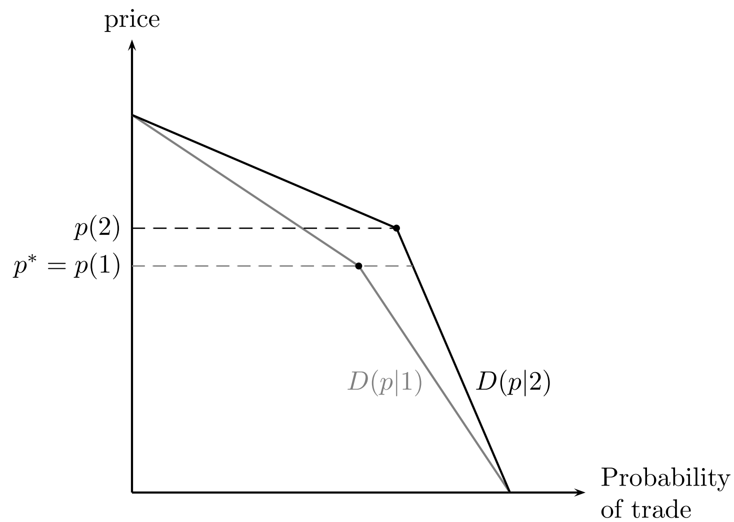

Figure 7: Because the unconditional price, p^*, is the price if $s = 1$, *ex post* efficiency is greater when the seller cannot condition price on the signal than when it can.

This $G(r|s, I)$ function is readily shown to satisfy the Extended Monotone Hazard condition.[33] Straightforward calculations reveal that $p(1) = 3/5$ and $p(2) = 7/10$; that is, the seller prices at the respective kink points as shown in Figure 7. It can be shown that if $f_s(1|\hat{I}) \geq 2/3$, then a seller who could *not* observe s would charge $p^* = 3/5$. Hence, *ex post* efficiency is greater when the seller cannot condition price on s than when it can condition price on s.

6 Concluding Remarks

In our baseline model, only the buyer makes an investment decision. We now briefly examine the seller's investment incentives. Suppose that the seller sinks its investment before the buyer arrives. If the seller's potential invest-

[33] To be precise, $h_G(r|2, I) = h_G(r|1, I)$ for $r > 7/10$. We could, however, make the latter strictly greater, as required by the EMH condition, by adding a slight bit of curvature to $G(r|2, I)$ for $r > 7/10$ similar to what we did above. Doing so would not, however, change the conclusions of *this* example, but would serve to unnecessarily complicate the analysis.

ment projects have the following pattern of returns, then there is a simple mapping from the seller's expected profits in the continuation game to the investment levels. Suppose that all projects have only two possible states, *success* and *failure,* and the seller cannot operate absent success. Moreover, suppose that increased amounts of seller investment raise the probability of success. Then, the seller's equilibrium investment level is a weakly increasing function of its profits in the continuation game. The analysis in the text thus demonstrates, for example, that, if the buyer's investment returns satisfy the Monotone Hazard Condition, then the seller's investment incentives are lower when it can later observe the buyer's investment level than when the seller's signal is perfectly uninformative. This finding is the opposite of that suggested by intuition. Of course, in a more complex model of seller investment or buyer-seller bargaining, additional effects could arise.[34]

As we have shown, in general "anything can happen" for a noisy signal of return or when the signal is the buyer's investment level but the only regularity condition imposed is that an increase in buyer investment raises the unconditional expected value of the buyer's gross value of the good. The reason, in part, is that the shape of the distribution of gross buyer value influences the share of the gross value that the seller can appropriate. An increase in buyer investment can change the distribution in ways such that this share may rise or fall. When the seller's share falls sufficiently fast, the buyer can face socially excessive investment incentives. In other cases, however, the distortion runs in the opposite direction. As we showed above, when the buyer's investment level is the signal, one can say much more under the assumption that an increase in investment leads to an improvement in the distribution of gross value in the sense of first-order stochastic dominance, and still more under the Monotone Hazard Condition.

So what should one make of the noisy-signal-of-returns case and the finding that definitive results are scarce? One implication, is that it is necessary to look in great detail at the buyer's technology and the market's information structure before reaching conclusions about the investment effects of increased seller information or an ability to engage in price discrimination. Although we cannot prove it, we suspect that participants in the network

[34]When both buyer and seller can invest, the problem is similar to a two-sided agency problem (see, *e.g.,* Demski and Sappington, 1991). In addition to Demski and Sappington, other analyses of that problem include Nöldeke and Schmidt (1995, 1998) and Edlin and Hermalin (2000).

neutrality debate have not conducted such an analysis.

There are several directions in which to extend this work, including: (a) allow for a more general bargaining game between the buyer and seller; (b) allow for exogenously given, *ex ante* buyer heterogeneity; and (c) consider a set of buyers that have interdependent demand functions (*e.g.,* the buyers compete with one another in the downstream market).

With respect to this last direction, it is worth noting that the models of price discrimination analyzed by Choi (1995), DeGraba (1990), Haucap and Wey (2004), and Inderst and Valletti (2006) examine the investment effects of an upstream monopolist's ability to price discriminate when selling to downstream firms that are product-market rivals and, thus, have interdependent demands. Their results, however, also hold for the case of downstream buyers that are not product-market competitors. In contrast, buyer interdependency is essential to the results of Kim and Nahm (in press). Kim and Nahm find that discrimination lowers downstream R&D investment incentives when the buyers are local monopolists but can increase investment incentives when the downstream firms compete and are not very differentiated. Kim and Nahm examine a model in which the buyers are Hotelling duopolists that charge two-part tariffs to final customers. Under this structure, the downstream firm with lower costs sells more units of output per customer than does the higher cost firm. Hence, the upstream supplier has an incentive to steer downstream customers to the lower-cost firm and does so by raising the input price charged to the higher-cost firm. This penalty on having higher costs strengthens the downstream incentives to innovate. Combining this richer structure of buyer demands with a more general model of the seller's signal may yield interesting new results.

Appendix A: Proofs

Example 1 (The buyer's best response can be increasing in price): Suppose $\bar{r} = 100$ and

$$1 - F_r(r|I) = \frac{I}{I+1}\left(\frac{5}{6}\sqrt{1 - \frac{r}{100}}\right) + \frac{1}{I+1}\left(1 - \frac{r}{100}\right)$$

for $I > 0$. If $1 - F_r(r) = \frac{5}{6}\sqrt{1 - \frac{r}{100}}$, then $\mathbb{E}\{r\} = 500/9 \approx 55.56$; and if $1 - F_r(r) = 1 - \frac{r}{100}$, then $\mathbb{E}\{r\} = 50$. Hence, $\mathbb{E}\{r|I\}$ is increasing in I. The buyer's problem is concave in I.[35] It can be shown that buyer's best response is *increasing* in price for all prices between $[0, 275/9]$.[36] □

Proof of Lemma 4: First observe that, because $I^*(0) > 0$ and $I^*(\cdot)$ is continuous, $\bar{p}$ is well defined and $\bar{p} > 0$. Next, observe that the first-order condition for maximizing the seller's profits is

$$1 - F(p|I^{\mathrm{E}}) - pf(p|I^{\mathrm{E}}) = 0\,.$$

Given the strict concavity of the seller's pricing problem, this condition is sufficient. For an equilibrium to exist in which the buyer invests, it follows that there must be a p that solves the following equation

$$1 - F\big(p|I^*(p)\big) - pf\big(p|I^*(p)\big) = 0 \tag{29}$$

such that $I^*(p) > 0$. To establish this, define

$$\xi(p, I) \equiv \frac{1 - F(p|I)}{f(p|I)}\,.$$

Observe that $\xi\big(p, I^*(p)\big)$ is continuous in p. Observe that $\xi\big(0, I^*(0)\big) > 0$ and, by (9),

$$\lim_{p \uparrow \bar{p}} \xi\big(p, I^*(p)\big) < \bar{p}\,.$$

---

[35]Proof: consumer surplus given $I \downarrow 0$ is $50 - p + p^2/200 \equiv \underline{\mathrm{CS}}(p)$. Consumer surplus given $I \uparrow \infty$ is $(100-p)^{3/2}/18 \equiv \overline{\mathrm{CS}}(p)$. The latter always exceeds the former for $p \in [0, \bar{r})$. It is readily shown that $\big(\overline{\mathrm{CS}}(p)I + \underline{\mathrm{CS}}(p)\big)/(I+1)$ is concave in I given $\overline{\mathrm{CS}}(p) > \underline{\mathrm{CS}}(p)$.

[36]Straightforward calculations reveal that

$$I^*(p) = -1 + \sqrt{\overline{\mathrm{CS}}(p) - \underline{\mathrm{CS}}(p)}\,,$$

where $\underline{\mathrm{CS}}(p)$ and $\overline{\mathrm{CS}}(p)$ are as defined in the preceding footnote. It is readily shown that $\overline{\mathrm{CS}}(p) - \underline{\mathrm{CS}}(p)$ is concave in p and reaches a maximum at $p = 275/9$.

By continuity, it follows that there exists a $\hat{p} \in (0, \bar{p})$ such that $\xi\big(\hat{p}, I^*(\hat{p})\big) = \hat{p}$. Observe, therefore, that $\hat{p}$ is a solution to (29). Because $\hat{p} \in (0, \bar{p})$, $I^*(\hat{p}) > 0$. ■

Proof of Lemma 6: If the the buyer played a pure strategy, then the seller's best response would be $p^*(I) = r(I)$ and the buyer's payoff would be $-I \leq 0$. Hence, the only pure-strategy equilibria are of the form $I = 0$ and $p \geq \bar{r}$.

Now, suppose the buyer mixed and its expected investment level was positive. Then there would exist some $I_0 > 0$ such that $prob(I \geq I_0) > 0$. Any price, p, that the seller charges with positive probability in equilibrium must satisfy $p \geq prob(I \geq I_0) r(I_0) \equiv \underline{p} > 0$.

Let $\mathcal{I}$ denote the support of the buyer's strategy. Any $I \in \mathcal{I}$ must satisfy $r(I) - I - \underline{p} \geq 0$, which implies $\underline{I} \equiv \inf \mathcal{I} > 0$. Clearly, the seller will always charge at least $r(\underline{I})$. Hence, $r(\underline{I}) - \underline{I} - p < 0$ for any p charged with positive probability. By the definition of $\underline{I}$ as the greatest lower bound on the buyer's investment, and the continuity of $r(\cdot)$, this is a contradiction. ■

Proof of Proposition 10: The seller's first-order condition, which is sufficient as well as necessary, is

$$1 - G(p|s, I) - pg(p|s, I) = 0\,.$$

Rearranging, we have

$$p = \frac{1}{h_G(p|s, I)}\,. \tag{30}$$

An increase in either s or I requires an increase in p to maintain (30) as an equality. ■

Lemma A.1 *Condition* (23) *implies that* $G(r|\mathbf{a})$ *dominates* $G(r|\mathbf{a}')$ *in the sense of first-order stochastic dominance.*

Proof: Inequality (23) implies that

$$\int_0^r h_G(x|\mathbf{a})dx < \int_0^r h_G(x|\mathbf{a})dx\,.$$

Using the fact that

$$G(r|\mathbf{a}) \equiv 1 - e^{-\int_0^r h_G(x|\mathbf{a})dx}\,,$$

it follows that

$$G(r|\mathbf{a}) = 1 - e^{-\int_0^r h_G(x|\mathbf{a})dx} < 1 - e^{-\int_0^r h_G(x|\mathbf{a}')dx} = G(r|\mathbf{a}')\,.$$

■

Proof of Proposition 11: Fix an investment level $I_0 > 0$ (if $I_0 = 0$, then the result is immediate). Consider $I_1 > I_0$. Observe that the seller's profit given I_1 is

$$\max_{p(\cdot)} \int_0^{\bar{s}} p(s)\big(1 - G(p(s)|s, I_1)\big) f_s(s|I_1)ds$$

$$\geq \int_0^{\bar{s}} p^*(s, I_0)\big(1 - G(p^*(s, I_0)|s, I_1)\big) f_s(s|I_1)ds \qquad (31)$$

$$> \int_0^{\bar{s}} p^*(s, I_0)\big(1 - G(p^*(s, I_0)|s, I_0)\big) f_s(s|I_1)ds \qquad (32)$$

$$\geq \int_0^{\bar{s}} p^*(s, I_0)\big(1 - G(p^*(s, I_0)|s, I_0)\big) f_s(s|I_0)ds\,, \qquad (33)$$

where the inequality in (32) follows because an increase in I improves $G(p|s, I)$ in the sense of first-order stochastic dominance. Given the first-order stochastic dominance assumption on $F_s(s|I)$, the inequality in (33) follows if $p^*(s, I_0) \times \big(1 - G(p^*(s, I_0)|s, I_0)\big)$ is a non-decreasing function of s. That it is can be seen by employing the envelope theorem and recalling that an increase in s improves $G(p|s, I)$ in the sense of first-order stochastic dominance. Because (33) is the seller's profit given I_0, an increase in the buyer's investment would raise the seller's profits.

Now, suppose that, contrary to the final part of the statement of the proposition, the buyer chooses the investment level that maximizes $W\big(p^*(\cdot), I\big)$. From the necessary conditions for an optimum, the partial derivative of $W\big(p^*(\cdot), I\big)$ with respect to I is zero. Because that derivative is the sum of the derivatives of the seller's and buyer's profits with respect to I, and the first part of the proof showed that the former is positive, the latter must be negative. But, then, the buyer is not playing a best response to the seller's equilibrium strategy, a contradiction. The result follows *reductio ad absurdum.* ■

Proof of Lemma 8: Straightforward calculations reveal[37]

$$h_G(r|s,I) = \begin{cases} \frac{1}{1-r}\,, \text{ if } s=1 \\ \left\{ \begin{array}{l} \frac{1}{2-r}\,, \text{ if } r < \frac{2}{3} \\ \frac{1}{1-r}\frac{115+6r}{118+3r}\,, \text{ if } r > \frac{2}{3} \end{array} \right\}, \text{ if } s=2 \end{cases}$$

Observation readily confirms that $h_G(r|2,I) < h_G(r|1,I)$. In the first two cases, $h_G(\cdot|s,I)$ clearly is an increasing function. Observe the derivative of the last line is

$$\frac{13,933 + 690r + 18r^2}{(1-r)^2(118+3r)^2} > 0\,.$$

Hence, the example satisfies the Extended Monotone Hazard Condition. ■

[37] Note $h_G(r|2,I)$ is not defined at $r = (\nu - 1)/(\nu - \mu)$.

References

Bernheim, B. Douglas and Michael D. Whinston, "Incomplete Contracts and Strategic Ambiguity," *American Economic Review*, 1998, *88* (4), 902–932.

Che, Yeon-Koo and Donald B. Hausch, "Cooperative Investments and the Value of Contracting," *American Economic Review*, March 1999, *89* (1), 125–147.

Choi, Jay Pil, "Optimal tariffs and the choice of technology Discriminatory tariffs vs. the 'Most Favored Nation'," *Journal of International Economics*, February 1995, *38* (1–2), 143–160.

DeGraba, Patrick, "Input Market Price Discrimination and the Choice of Technology," *American Economic Review*, December 1990, *80* (5), 1246–1253.

Demski, Joel S. and David E.M. Sappington, "Resolving Double Moral Hazard Problems with Buyout Agreements," *RAND Journal of Economics*, 1991, *22* (2), 232–240.

Edlin, Aaron S. and Benjamin E. Hermalin, "Contract Renegotiation and Options in Agency Problems," *Journal of Law, Economics, and Organization*, 2000, *16* (2), 395–423.

Farrell, Joseph and Nancy T. Gallini, "Second-Sourcing as a Commitment: Monopoly Incentives to Attract Competition," *The Quarterly Journal of Economics*, nov 1988, *103* (4), 673–694.

Gul, Faruk, "Unobservable Investment and the Hold-Up Problem," *Econometrica*, March 2001, *69* (2), 343–376.

Haucap, Justus and Christian Wey, "Unionisation Structures and Innovation Incentives," *The Economic Journal*, March 2004, *114*, C149–C165.

Hermalin, Benjamin E., Avery W. Katz, and Richard Craswell, "Law of Contracts," in A. Mitchell Polinsky and Steven Shavell, eds., *Handbook of Law and Economics*, Vol. 1, Amsterdam: North-Holland, in press.

Inderst, Roman and Christian Wey, "Countervailing Power in Upstream Innovation," October 2006. unpublished manuscript, London School of Economics.

— **and Tommaso M. Valletti**, "Price Discrimination in Input Markets," March 2006. unpublished manuscript, London School of Economics.

Katz, Michael L., "The Welfare Effects of Third-Degree Price Discrimination in Intermediate Good Markets," *American Economic Review*, March 1987, *77* (1), 154–167.

Kim, Jeong-Yoo and Jae-hyon Nahm, "Reexamining the Effects of the Most-Favored-Nation (MFN) Provision in Input Prices on R&D Incentives: The Effect of the MFN on Access Charge Agreements," *International Journal of Industrial Organization*, in press.

Klein, Benjamin, "Vertical Integration as Organizational Ownership: The Fisher Body-General Motors Relationship Revisited," *Journal of Law, Economics, and Organization*, Spring 1988, *4* (1), 199–213.

Lafontaine, Francine and Kathryn L. Shaw, "The Dynamics of Franchise Contracting: Evidence from Panel Data," *Journal of Political Economy*, 1999, *107* (5), 1041–1080.

Nöldeke, Georg and Klaus M. Schmidt, "Option Contracts and Renegotiation: A Solution to the Hold-up Problem," *RAND Journal of Economics*, 1995, *26* (2), 163–179.

— **and** — , "Sequential Investments and Options to Own," *RAND Journal of Economics*, 1998, *29* (4), 633–653.

Pečarić, Josip E., Frank Proschen, and Yung Liang Tong, *Convex Functions, Partial Orderings, and Statistical Applications*, San Diego: Academic Press, 1992.

Prékopa, Andras, "Logarithmic Concave Measures with Applications," *Acta Scientiarum Mathematicarum*, 1971, *32*, 302–316.

Skrzypacz, Andrzej, "Bargaining Under Asymmetric Information and the Hold-Up Problem," January 2005. Unpublished manuscript, Stanford University.

Williamson, Oliver E., *Markets and Hierarchies, Analysis and Antitrust Implications: A Study in the Economics of Internal Organization*, New York: The Free Press, 1975.

_ , "Franchise Bidding for Natural Monopolies—in General and with Respect to CATV," *The Bell Journal of Economics*, Spring 1976, *7* (1), 73–104.