THE ECONOMICS OF PRODUCT-LINE RESTRICTIONS
WITH AN APPLICATION TO THE NETWORK NEUTRALITY DEBATE

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1 Introduction

Firms often offer product lines—several variants of the same product. Examples include different programming packages offered by cable television companies and different versions of software (*e.g.,* standard or professional) offered by software manufacturers. Typically, a firm's decision to offer a product line is viewed as unexceptional. In some circumstances, however, there are calls for public policy to limit the range of products offered. At the time we write this paper, for example, there is an intense debate taking place in the halls of Congress regarding "network neutrality" regulation. One of the central issues in this debate is whether providers of "last mile" Internet access services (typically a local telephone company offering DSL service, a cable company offering cable modem service, or a wireless service such as 3G) should be allowed to offer more than one grade of service. Proponents of regulation argue that offering multiple grades is unfair and results in some consumers' being provided unduly low-quality service.[1] For instance, Senator Olympia Snowe warned that, absent regulation, "Consumers will have all the selections of a former Soviet Union supermarket. We are going to create a two-tier Internet, for the haves who can pay the price, and the have nots who will be relegated to the Internet dirt road."[2]

In this paper, we examine the effects of product-line restrictions in markets where, absent any restrictions, any given supplier could offer a continuum of vertically differentiated variants. Our results suggest that product-line restrictions affect welfare through several mechanisms and that the effects are often negative.

We introduce the model in Section 2. In order to shed light on the network neutrality debate, we examine situations corresponding both to traditional

[1]This type of concern arises in other sectors of the economy as well. For example, a somewhat similar set of issues arise with respect to supermarkets and the quality of shelf space that they provide to different vendors; the sale of preferential product locations has been controversial. Turning to publicly provided goods, there is typically strong resistance to having premium lanes on toll bridges or highways that allow travel in a less congested lane in return for payment of a fee. Ironically, Air France has complained about plans by the Marseilles airport to offer lower-price, lower-quality terminal services to other airlines. (John Lichfield and Jen Wainwright, "France takes cheap flights to the next level with launch of budget airport in Marseilles," *The Independent,* 26 October 2006, at 18.)

[2]Ira Teinowitz, "Senate Panel Kills 'Net Neutrality' Proposal: Web Access Providers Free to Charge More for Better Service," TVWeek.com, June 28, 2006, *available at* http://www.tvweek.com/news.cms?newsId=10287, site visited July 24, 2006.

markets and to what have come to be known as two-sided markets.[3] Specifically, we examine a platform provider that offers services that connect applications providers with households. For example, the platform provider may be an Internet service provider (ISP) selling services that connect households with Internet application providers, such as Google or a newspaper's proprietary web site. We explicitly model the platform provider's simultaneous choices of how to price to households and to application providers.

In Section 3, we compare the levels of profits, consumer surplus, and total surplus when a monopoly platform provider can offer a full range of service qualities with the corresponding levels when the monopolist can offer only one product. We examine situations in which applications providers have heterogeneous demands for the qualities of their connections to households. We find that, as a result of the single-product restriction: (a) application providers who would otherwise have purchased low-quality connections are excluded from the market; (b) application providers "in the middle" of the market purchase connection of higher and more efficient qualities; and (c) application providers at the top of the market purchase connections of lower and less efficient qualities. Effects (a) and (c) reduce total surplus, while effect (b) raises it. Although we find that the negative effects frequently dominate, there are situations in which restricting a monopolist raises welfare. That said, it should be observed that application providers at the bottom of the market—the ones that single-product restrictions typically are intended to aid—are almost always harmed by the restriction. Moreover, the restriction reduces the set of applications to households.

In Section 4, we consider several extensions of our baseline model. The most important is to allow for platform competition. The network neutrality debate, for example, concerns the regulation of cable and telephone companies that often are local duopolists in the provision of broadband Internet access services. We examine the effects of product-line restrictions in a Hotelling duopoly. Our principal technical finding is that the equilibrium quality schedules offered by the duopolists are analogous to the schedule offered by a platform monopoly. The central policy implication of this finding is that the welfare results obtained in the monopoly case carry over to this one. Specifically, a single-product restriction excludes application providers at the bottom of the market, has mixed effects on application providers higher up in the quality spectrum, and typically—but not necessarily—lowers overall

[3] In our formal model below, $\sigma = 0$ corresponds to a one-sided market.

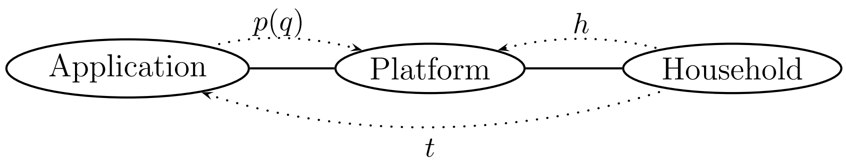

Figure 1: Fee-for-service model. Solid lines indicate service flows (e.g., Internet connections). Dotted curves indicate payments.

welfare.

The paper closes with a brief conclusion. Proofs not given in the text may be found in Appendix A.

2 The Model

We begin by considering a monopoly platform that provides an intermediation service that facilitates exchange between application providers and households. We consider a monopoly platform for two reasons. First, the lack of competition simplifies the analysis and allows us to identify forces that are also at work in settings with imperfect competition. Second, proponents of network neutrality regulation argue that market power lies at the heart of the problem. Thus, this is a useful setting to examine in its own right.

The platform charges a hookup fee of h to households and $p(q)$ to an application provider purchasing connection quality q. Under some public policies, the platform can offer different connection qualities (*e.g.,* an ISP can offer different combinations of bandwidth, latency, and packet loss rate) to different application providers. As a convention, we use $q = 0$ to indicate that an application provider has *not* connected to the platform.

Figures 1 and 2 illustrate two relevant market structures, which differ in terms of the economic relationship between application providers and households. In the first market structure—the fee-for-service model—a household pays a transaction fee t to the application provider for each use. In the second market structure—the advertiser-supported-content model—application providers earn revenues solely from the sale of advertising and offer their applications to households at no charge.

There is a continuum of potential application providers providers, which we normalize to have unit mass. Each application provider has type, $\theta \in$

$[0,\bar{\theta}] \equiv \Theta$, where households value the content from high-θ providers more than from low-θ providers *ceteris paribus*. Let $F(\cdot)$ denote the cumulative distribution function for types, which has a continuous density $f(\cdot)$ such that $0 < f(\theta) < \infty$ for all $\theta \in [0,\bar{\theta}]$.[4] The last assumptions ensure that the inverse hazard rate exists for all θ and is bounded away from 0 for all $\theta < \bar{\theta}$. Each application provider knows its type, while the platform provider knows only the population distribution of application types.

Technology is such that the platform cannot provide quality of service less than some minimum level $\underline{q}$, where $\underline{q} > 0$.[5] Denote the platform's choice space for quality by

$$\mathcal{Q} \equiv \{0\} \cup \{q | q \geq \underline{q}\} \, .$$

The platform incurs a cost of $c(q)$ in providing quality q. We assume $c(\cdot)$ is at least twice differentiable, with $c'(\cdot) > 0$ and $c''(\cdot) > 0$. We also assume that $\underline{q}c'(\underline{q}) > c(\underline{q})$. Given the convexity of $c(\cdot)$, this last assumption implies that $\frac{c(q)}{q}$ is increasing in q (*i.e.,* the average cost of quality rises with the quality level). Lastly, in keeping with our notational convention that $q = 0$ corresponds to non-consumption, $c(0) = 0$.

There is a unit mass of households with identical preferences. Let $u(\frac{x}{\theta q})$ denote the marginal utility derived from consumption of the x^{th} unit of a type-θ application with connection quality q. We assume that $u(\cdot)$ is continuous and decreasing (*i.e.,* there is diminishing marginal utility from consuming the content of any specific provider) with $\lim_{x\to\infty} u(x)x = 0$.

A household's utility from connecting to the platform and consuming the available applications is quasi-linear and given by

$$U \equiv \int_{\Theta} \int_0^{x(\theta)} u\left(\frac{z}{\theta q(\theta)}\right) f(\theta) dz d\theta + y \, ,$$

where $q(\theta)$ is the connection quality chosen by a type-θ application provider, $x(\theta)$ is the household's consumption of each type-θ application, and y is the number of units of the numeraire good consumed.

The quantity a household purchases from a given application provider at

[4] In Appendix B, we discuss how our results can be extended to discrete type spaces.

[5] Alternatively, the platform could provide lower-quality service but no application provider would be willing to use that quality even if it were priced at cost.

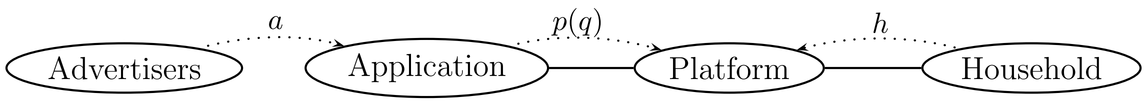

Figure 2: Advertiser-supported-content model. Solid lines indicate service flows (e.g., Internet connections). Dotted curves indicate payments.

price t is the solution to

$$\max_x \int_0^x u\left(\frac{z}{\theta q(\theta)}\right) dz - tx\,.$$

The first-order condition, which is sufficient as well as necessary because $u(\cdot)$ is decreasing, is

$$u\left(\frac{x}{\theta q(\theta)}\right) - t = 0\,.$$

Because $u(\cdot)$ is monotonic, it is invertible. Hence, an application provider faces demand

$$\theta q d(t)$$

from each household subscribed to the platform, where $d(t) \equiv u^{-1}(t)$.

The marginal cost of providing an application is a constant, $k \geq 0$, common to all application providers. To ensure that trade is socially desirable, assume $u(0) > k$. In the fee-for-service case, we assume that any one application provider generates a sufficiently small proportion of household benefits that it ignores the effect of its pricing on households' network connection decisions.[6] Conditional on its choice of quality, each application provider chooses its price to solve

$$\max_t \theta q(t-k)d(t)\,. \tag{1}$$

[6]Were this not the case, the platform would be able to engage in a price squeeze, whereby application providers would be forced to price at cost in order to induce households to connect to the network.

An equivalent assumption to the one made in the text would be to have the content providers set their prices before the platform; this timing, however, seems less realistic than the one used in the text.

Observe that θ and q are irrelevant to the solution of this maximization problem. For expositional convenience, we assume that $u(\cdot)$ gives rise to a unique profit-maximizing application price, t^*.[7] Observe that the profit-maximizing price is common to all content providers.

In the advertising-supported-content model, let $a > k$ be the amount paid by advertisers to the application provider per unit of demand.[8] The profits of an application provider conditional on q are

$$\theta q(a-k)d(0)\,. \tag{2}$$

Observe that, under either business model, an application provider's profits (gross of fees paid the platform) have the form $\theta q\rho$, where

$$\rho = (t^* - k)d(t^*)$$

under the fee-for-service business model and

$$\rho = (a-k)d(0)$$

under the advertiser-supported-content business model.[9] Henceforth, we will express the gross profits of an application provider of type θ that has purchased connection quality q as $\theta q\rho$.

The consumer surplus that a household derives from consumption of an application provider's service is

$$\theta q s(t)\,,$$

where $s(t) \equiv \int_t^\infty d(z)dz$. The surplus a household derives from consumption of a type-θ application whose provider has purchased a connection of quality q can compactly be expressed as $\theta q\sigma$, where $\sigma = s(t^*)$ in the fee-for-service model and $\sigma = s(0)$ in the advertising model.

[7] A sufficient condition for this would be that $xu'(x)$ be decreasing in x; this condition entails that $d(t)$ is log-concave, which is sufficient for (1) to have a unique solution.

[8] One can also think of a as advertisers' expected payment to the content provider given the click-through rate.

[9] There can also be a hybrid business model in which an application collects revenues from both households and advertisers. Assuming that advertising does not affect households' willingness to pay for an application, the hybrid model is equivalent to the fee-for-service model with a cost of $k - a$.

To ensure that some trade across the platform is always socially desirable, assume that, for the highest-value application offered over the least-costly connection, the sum of the application provider's gross profits and households' consumer surplus are greater than the connection cost:

$$(\rho + \sigma)\bar{\theta}\underline{q} > c(\underline{q})\,.$$

Lastly, to rule out the uninteresting scenario in which $\underline{q}$ is the only efficient level of quality to offer, assume that $(\rho + \sigma)\bar{\theta} > c'(\underline{q})$.

Absent any fixed costs, welfare (*i.e.*, total surplus) is

$$W = \int_{\Theta} \Big((\rho + \sigma)\theta q(\theta) - c\big(q(\theta)\big) \Big) f(\theta)d\theta\,, \tag{3}$$

where $q(\theta)$ is the connection quality supplied to type-θ applications.[10] Let $q_w(\cdot)$ denote the quality-consumption schedule that maximizes welfare.

Lemma 1 *Welfare is maximized by: (i) excluding a type-θ application if the cost of the lowest quality service exceeds the gross benefit the application generates (i.e., $c(\underline{q}) > (\rho + \sigma)\theta\underline{q}$); and (ii) otherwise providing access of quality $q_w(\theta)$, where $q_w(\theta)$ is the solution to*

$$\max_{q \geq \underline{q}} (\rho + \sigma)\theta q - c(q)\,.$$

Under the first-best outcome, a positive measure of application types are excluded and a positive measure of types are served.

The proof of Lemma 1 establishes the existence of a marginal type, $\underline{\theta}_w$, such that this and all higher types of application provider operate and lower types shut down. Because the marginal contribution of quality to welfare is increasing in type, it follows that $\theta > \theta'$ implies $q_w(\theta) \geq q_w(\theta')$, with the inequality being strict if $q_w(\theta) > \underline{q}$. Recalling our notational convention, set $q_w(\theta) = 0$ for those application types that do not operate.

Now consider the equilibrium behavior of a profit-maximizing platform provider. The timing of play is as follows.

[10] Observe that this expression does not account for the welfare of advertisers. One can interpret the model as assuming that advertising is provided in a broader, perfectly competitive market, such that changes in the quantity of online advertising do not give rise to changes in advertiser welfare.

- The platform sets its price schedule to application providers, $p(\cdot)$, and hookup fee to households, h.
- Application providers simultaneously choose their connection qualities, $q(\theta)$, and, in the fee-for-service model, their prices to households.
- Households observe the hookup fee, connection qualities, and, in the fee-for-service model, application prices. Households then choose whether to connect to the network and their consumption levels of the various applications.

We assume that the platform quotes its prices to application providers on a per-household basis. That is, the platform commits to a schedule $p(q)$ that is prorated by the number of end-users it attracts, so that an application provider that selects quality q pays the platform provider $\lambda p(q)$, where λ is the proportion of households that subscribe to the platform.

We make this assumption as a means of approximating the fact that, in reality, platforms attract new applications and households over time and that a platform may adjust its price over time as well.[11] In our one-shot (albeit multi-stage) model, having the platform set access prices for application providers that were independent of the eventual number of household subscribers could lead to widely varying equilibrium outcomes as the result of bootstrapping on different expectations. For example, as is well known, the two-sided nature of the market makes the analysis vulnerable to a degenerate equilibrium in which applications choose not to connect to the platform because they anticipate that households will not connect, and—given that no applications are connected—households indeed choose not to connect. This degenerate outcome is not an equilibrium in our model; because the platform's charge is prorated, it is a weakly dominant strategy for a type-θ application provider to sign with the platform as long as

$$\max_q \rho\theta q - p(q) \geq 0\,.$$

In equilibrium, the surplus realized by a household from *all* applications—and hence the household's value of subscribing to the platform—is

$$\sigma \int_\Theta \theta q(\theta) f(\theta) d\theta\,. \tag{4}$$

[11] For an insightful model of adoption dynamics in a two-sided market, see Caillaud and Jullien (2003).

Because households are identical, the platform can capture their entire gross surplus by setting the hookup fee, h, to equal (4). Recalling that there is a unit mass of households, h is also the platform's revenues from households-users. The platform's profits are

$$\begin{aligned}\pi &= h + \int_\Theta \Big(p(\theta) - c\big(q(\theta)\big)\Big) f(\theta) d\theta \\ &= \int_\Theta \Big(\sigma\theta q(\theta) + p(\theta) - c\big(q(\theta)\big)\Big) f(\theta) d\theta \,. \end{aligned} \tag{5}$$

3 The Effects of a Single-Quality Restriction

In this section, we compare equilibrium with and without a single-quality restriction.

3.1 The Unrestricted Monopoly Equilibrium

We first characterize the equilibrium when a profit-maximizing monopoly platform is not subject to legal restrictions on the range of qualities offered. Standard analysis (*e.g.,* Mussa and Rosen, 1978; Caillaud and Hermalin, 2000) demonstrates that the profit-maximizing, incentive-compatible, individually rational price schedule to offer is

$$p\big(q(\theta)\big) = \rho\theta q(\theta) - \int_0^\theta \rho q(z) dz \,. \tag{6}$$

Consequently, the provider's problem is

$$\max_{\{q(\theta)|q(\theta)\in\mathcal{Q}\}} \int_0^{\bar\theta} \Big((\rho+\sigma)\theta q(\theta) - \int_0^\theta \rho q(z) dz - c\big(q(\theta)\big)\Big) f(\theta) d\theta \tag{7}$$

$$\text{subject to } \underline{q} \leq q(\theta') \leq q(\theta) \quad \forall \theta, \theta' \text{ such that } \theta' < \theta \,, \tag{8}$$

where (7) follows from (5) and (6), and (8) is necessary and sufficient for the resulting quality schedule to be incentive compatible. Integration by parts allows us to rewrite (7) as

$$\max_{\{q(\theta)|q(\theta)\in\mathcal{Q}\}} \int_0^{\bar\theta} \Big((\rho+\sigma)\theta q(\theta) - m(\theta)\rho q(\theta) - c\big(q(\theta)\big)\Big) f(\theta) d\theta \,, \tag{7$'$}$$

where

$$m(\theta) \equiv \frac{1 - F(\theta)}{f(\theta)}$$

is the inverse of the hazard rate.[12] Let $q_u(\cdot)$ denote the solution to (7).

As we will show shortly, there exists a lowest type of application provider, $\underline{\theta}_u > \underline{\theta}_w$, that connects (*i.e.*, $q_u(\theta) = 0$ for all $\theta < \underline{\theta}_u$). Equation (6) implies $p_u(\underline{\theta}_u) = \rho\underline{\theta}_u q_u(\underline{\theta}_u)$; as is well known, the lowest type served enjoys no surplus.

The marginal contribution to profits of an increase in $q(\theta)$ is proportional to

$$(\rho + \sigma)\theta - \rho m(\theta) - c'\big(q(\theta)\big) . \tag{9}$$

The presence of $m(\theta)$ captures the fact that increasing the quality level offered to type-θ application providers increases the information rents that have to be given to all higher types to keep them from purchasing the quality intended for type-θ providers.

Because the quality allocated to the top type cannot affect the information rent received by any other type ($m(\bar{\theta}) = 0$), the monopolist has no incentive to distort the quality allocated the top type away from its efficient level; as is well known, $q_u(\bar{\theta}) = q_w(\bar{\theta})$.

Next, consider application types at the bottom of the distribution. Consider the effects of lowering the lowest application type served by an infinitesimal amount. The revenues collected from the incremental application providers and through the increase in the hookup fee charged to households are just equal to the social benefits. However, the information rents of higher application types rise. Hence, the platform's incremental profits are less than the social benefits. Combined with Lemma 1, this logic establishes:

Lemma 2 *The unrestricted platform excludes more application provider types than is socially optimal. Specifically, there exists an application type $\underline{\theta}_u$, $\underline{\theta}_w < \underline{\theta}_u < \bar{\theta}$, such that $q_u(\theta) = 0$ for all $\theta < \underline{\theta}_u$.*

Total surplus under the unrestricted equilibrium is

$$W_u \equiv \int_{\underline{\theta}_u}^{\bar{\theta}} \Big((\rho + \sigma)\theta q_u(\theta) - c\big(q_u(\theta)\big) \Big) f(\theta) d\theta \, . \tag{10}$$

[12] We use m as a mnemonic because the inverse hazard rate is also known as the Mills ratio.

3.2 The Restricted Monopoly Equilibrium

Now suppose the platform is restricted to offering only a single level of quality, and let q_r denote the monopolist's choice.

Clearly, it would be suboptimal for the monopolist to charge a price, p_r, such that $p_r > \rho\bar{\theta}q_r$ or $p_r \leq 0$. It also could never maximize profits to offer a price and quality such that all types of application providers earned positive surplus. Instead, the monopolist chooses a price and quality such that there is a marginal application provider type just indifferent between connecting and not. Rather than view the monopolist's problem as one of choosing an optimal quality and price, we can view it as one of choosing an optimal cutoff type and quality. Note the cutoff type satisfies $\theta = \frac{p_r}{\rho q}$. Writing the platform's problem in this manner, we have

$$\max_{\{\theta\in\Theta,q\in\mathcal{Q}\}} \int_{\theta}^{\bar{\theta}} \big(\rho\theta q + \sigma t q - c(q)\big) f(t)dt\,. \tag{11}$$

Maximization with respect to q is equivalent to

$$\max_{q\in\mathcal{Q}} \big(\rho\theta + \sigma T(\theta)\big)q - c(q)\,, \tag{12}$$

where $T(\theta)$ is the expected value of a content provider's type conditional on that type not being less than θ. Let $\underline{\theta}_r$ be the marginal type (*i.e.*, the solution to (11)).

Proposition 1 *A profit-maximizing platform restricted to offering a single product quality excludes more application providers than would a welfare maximizing platform (i.e., $\underline{\theta}_w < \underline{\theta}_r$). Moreover, the profit-maximizing platform chooses a quality level that is lower than the one that would maximize total surplus conditional on the types of application providers that connect to the platform.*

For the case of traditional, one-sided markets, Spence (1975) showed that the fundamental source of distortions in a single-product firm's choice of quality is that prices and profits are driven by the marginal buyer's valuation of quality, while welfare depends on the average buyer's valuation of quality. In a two-sided market, one must consider the effects of an incremental quality increase on the willingness of both sides to pay. Because all households are identical in our model, the marginal and average valuations of quality are the same. The distortion arises on the application provider side of the market—the marginal application provider values increased connection quality by less than do the inframarginal application providers.

3.3 Comparison of the Equilibria

We now compare the equilibrium in which the monopolist is restricted to a single product to the equilibrium in which the platform is free to offer a full product line. One consequence of imposing a single-product restriction is that low-value application providers can get priced out of the market.

Lemma 3 *Suppose that at least one of the following conditions holds:*

(i) There is a unique solution to the restricted platform's problem;

(ii) There is a unique solution to the unrestricted platform's problem;

(iii) If there are multiple solutions to the unrestricted platform's problem, then the firm chooses the one that maximizes the set of application types served (i.e., that minimizes $\underline{\theta}_u$);

(iv) It is not a solution to the restricted platform's problem to set quality equal to the minimum feasible quality (i.e., $q_r > \underline{q}$);

(v) The hazard rate associated with the distribution over application types is everywhere non-decreasing.

Then a single-product restriction weakly reduces the set of application types offered to households in equilibrium (i.e., $\underline{\theta}_r \geq \underline{\theta}_u$).[13]

Contrary to the wishes of network neutrality proponents, regulation may move the equilibrium outcome closer to "a former Soviet Union supermarket" rather than away from it.

Lemma 3 leaves open the possibility that, if there are multiple equilibria absent the single-product restriction and there are multiple equilibria with the single-product restriction, including one in which the platform provider offers only the lowest technically feasible quality, *and* the hazard rate is decreasing on at least one interval, then the lowest application type served in

[13]This proposition also holds for a discrete type space.

some of the restricted equilibria may be lower than the lowest type served in some—but not all—of the unrestricted equilibria. Given the highly restrictive set of conditions this case would have to satisfy, we strongly doubt that this possibility is of empirical importance.

Lemma 3 also leaves open the possibility that the lowest application type served would be unaffected by the single-product restriction. As we now show, this can happen only if the restricted platform would maximize its profits by offering the lowest technologically feasible quality. This seems an unlikely scenario in many markets and is certainly not the scenario envisioned by proponents of network neutrality, who worry that applications with low types will receive worse service ("the Internet dirt road") absent network neutrality.

Proposition 2 *If the restricted platform does not offer the product with the lowest technically feasible quality level, then a single-product restriction strictly reduces the set of applications offered to households in comparison with the unrestricted equilibrium.*

The intuition underlying this result is the following. When the unrestricted platform connects additional application types at the bottom of the market, it must lower the prices charged to higher-type application providers, but it does *not* have to distort higher types' quality levels further. When the restricted platform serves additional application types at the bottom of the market, it does so by lowering the price and the quality level consumed by all types connecting to the platform. The change in quality is a costly distortion that affects the revenues collected from both applications providers and households.

Restricting the platform to offering a single quality level results in some applications' purchasing lower quality connections than they would have purchased if platform were unrestricted. One response could be to require the platform to offer only the highest level offered prior to imposition of regulation.[14] Recall that the highest quality product offered by the unrestricted platform is the quality that is efficient for the most valuable application, $q_w(\bar{\theta})$.

[14]We are assuming here that regulation is unanticipated, so that the service provider did not strategically reduce the highest quality offered prior to the imposition of regulation.

Proposition 3 *Consider a policy that requires the platform to offer only the highest quality variant that would be offered absent regulation. In equilibrium, strictly fewer application providers connect to the platform than under either of the following policies: (a) the platform is allowed to offer at most one quality level but can choose any technologically feasible quality level, and (b) the platform is free to offer a full range of qualities.*

In other words, a policy that mandates a high quality reduces the set of applications available to households.

Now, return to consideration of a single-product restriction that grants the platform the freedom to choose the quality. As Proposition 2 shows, a likely consequence of this restriction would be greater exclusion of low-value applications in comparison with the unrestricted equilibrium. Consider the applications that are excluded under the single-product restriction, but that would have been provided under the unrestricted equilibrium. These applications, with the possible exception of the lowest type, would generate provider profits plus household surplus strictly greater than the platform's costs of transmitting the content. Hence, the increased exclusion is a reduction in total surplus *ceteris paribus*. We refer to this as the *exclusion effect* of a single-product restriction.

There are two other effects of a single-product restriction. One is that the connection quality for the highest-type application falls from $q_w(\bar{\theta})$ to q_r. It follows that there is a positive measure of application types that enjoy less efficient quality in the sense that

$$(\rho+\sigma)\theta q_r - c(q_r) < (\rho+\sigma)\theta q_u(\theta) - c\big(q_u(\theta)\big)\,.$$

We refer to this reduction in welfare as the *reduced-quality effect* of a single-product restriction.

On the other hand, the connection quality for some application types is greater under the single-product restriction than it would be in the unrestricted equilibrium. In Appendix A (Lemma A.1), we show that there is one type, $\hat{\theta} \in (\underline{\theta}_r, \bar{\theta})$, such that $q_r = q_w(\hat{\theta})$. Because the unrestricted equilibrium entails a downward distortion in quality for all types except the top one, $q_u(\hat{\theta}) < q_w(\hat{\theta})$. By continuity, there must be a positive measure of types for which quality is more efficient under network neutrality than in the unrestricted equilibrium. That is,

$$(\rho+\sigma)\theta q_r - c(q_r) > (\rho+\sigma)\theta q_u(\theta) - c\big(q_u(\theta)\big)$$

for a positive measure of types. We refer to this welfare benefit as the *improved-quality effect* of a single-product restriction.

In Appendix B, we show by example that the net welfare effects of a single-product restriction can be positive or negative.[15] Despite the overall ambiguity about the welfare consequence of imposing product-line restrictions of the sort proposed by proponents of network neutrality, we can derive a condition that must necessarily hold if network neutrality is to increase welfare (correspondingly, the negation of that condition is a sufficient condition for the unrestricted equilibrium to yield greater welfare). Recall that $q_u(\theta) = 0$ for $\theta < \underline{\theta}_u$. Define $q_r(\theta) = q_r$ if $\theta \geq \underline{\theta}_r$ and $= 0$ if $\theta < \underline{\theta}_r$.

Proposition 4 *Suppose the connection quality offered when the platform is subject to a single-product restriction exceeds the lowest technically feasible quality level (i.e., $q_r > \underline{q}$). If $q_r(\cdot)$ dominates $q_u(\cdot)$ in the sense of second-order stochastic dominance, then the single-product restriction lowers welfare.*[16]

The condition identified in the previous result provides the basis of intuition. Because the single-product restriction results in greater exclusion of applications, it follows that

$$\int_0^\theta q_u(t)dt > \int_0^\theta q_r(t)dt = 0$$

for $\theta \in (\underline{\theta}_u, \underline{\theta}_r]$. Because $q_u(\bar{\theta}) = q_w(\bar{\theta}) > q_r$, it follows that, if it is *not* to be the case that

$$\int_0^\theta q_u(t)dt \geq \int_0^\theta q_r(t)dt$$

for *all* θ, then q_r must be considerably larger than $q_u(\underline{\theta}_r)$. In other words, for the single-product restriction to raise welfare, the marginal application type served under the restricted equilibrium must obtain a connection of much higher quality than the the connection it would obtain absent the restriction.

[15] Some of these examples assume a discrete type space. This is done to simplify the exposition. A more general analysis of the discrete-type case is presented in Appendix B.

[16] Recall that $q_r(\cdot)$ dominates $q_u(\cdot)$ in the sense of second-order stochastic dominance if $\int_0^x q_r(t)dt \leq \int_0^x q_u(t)dt$ for all x in the common domain. Because neither $q_r(\cdot)$ nor $q_u(\cdot)$ is a distribution, it would arguably be more appropriate to describe the condition in terms of weak majorization (Pečarić et al., 1992, §12.1). We use the language of stochastic dominance because it is more common in economics.

We next make an observation about the political economy of single-product restrictions, such as those advocated by some proponents of network neutrality regulation. We show by example in Appendix B (Example 2) that there exist situations in which application providers favor network neutrality, the platform (*e.g.,* an ISP) opposes it, and imposition of the restriction would reduce overall welfare. It should be emphasized, however, that there are also examples in which a single-product restriction reduces application providers' aggregate surplus (Appendix B, Example 3).

4 Extensions

We next extend the basic model to consider alternative cost assumptions, technological restrictions on quality choice, and competitive platforms.

4.1 Alternative Cost Assumptions

In this subsection, we briefly discuss three extensions with respect to our modeling of costs. First, our model assumes that there are no product-specific fixed costs. In the presence of fixed costs, variety is costly as well as potentially beneficial. For one-sided markets, Katz (1980) has shown that, in the presence of such fixed costs, a profit-maximizing monopolist may offer more or fewer than the total-surplus-maximizing number of products (quality levels). Depending on the structure of household and application provider preferences, similar effects arise in a two-sided market. Thus, in some cases, restricting the number of products offered by the monopolist would exacerbate the distortion, and in other cases, it would ameliorate it.

Second, we have assumed that there are no economies of scale in production. Hence, marginal-cost pricing would allow a platform to cover its costs. The presence of economies of scale can give rise to an additional social benefit of allowing the platform to offer a variety of qualities: a multi-product firm has a greater ability to cover its costs. Moreover, in a model that required a large, fixed investment in facilities, an unrestricted platform would have greater investment incentives than would a restricted platform.

Third, we have assumed that unit costs rise with quality. Some participants in the network neutrality debate have argued that increased quality is essentially costless, at least up to some point. We doubt the empirical validity of this claim, but it is nonetheless of interest to examine the case in which $c(q) \equiv c$ for all q not exceeding some maximum possible quantity, $\bar{q}$.

Observe that, when $c(q) \equiv c$, there is no social benefit of variety in the following sense. Under the first-best outcome, all content that is worth transmitting (*i.e.,* such that $(\rho + \sigma)\theta \geq c$) is transmitted at the highest possible quality, $\bar{q}$. Now, consider the profit-maximizing outcomes. Recall that the marginal contribution to the unrestricted platform's profits of an increase in $q(\theta)$ is proportional to $(\rho+\sigma)\theta - \rho m(\theta) - c'\big(q(\theta)\big)$ (see equation (9) above). It can be seen by inspection that the platform's problem of picking a quality has a bang-bang solution when $c'(\theta) = 0$: a type-θ application is either connected at quality $\bar{q}$ (if $(\rho + \sigma)\theta > \rho m(\theta)$) or is excluded from the market (if $(\rho + \sigma)\theta < \rho m(\theta)$). Given that the platform offers only a single quality level absent any restriction, imposing a net-neutrality restriction has no effect on the equilibrium outcome.

Summarizing this analysis,

Proposition 5 *Suppose $c(q) \equiv c$ for all $q \in [\underline{q}, \bar{q}]$. Then a single-product restriction has no effect on the set of equilibrium outcomes.*

It should be noted that this result depends on the functional form we have assumed for user benefits. Specifically, let $v(q, \theta)$ denote the gross benefits enjoyed by a type-θ application provider that connects at quality q. The standard screening condition requires that $v_{q\theta}(q, \theta) > 0$ (where subscripts here indicate partial derivatives). Our functional form imposes the stronger condition that $v_{q\theta}(q, \theta)$ is a positive constant. When this cross partial can vary with q, there are cases in which the unrestricted platform would offer quality variety even though there is no marginal cost of higher quality. Although there is no social value to variety, the platform would offer a product line as a screening device (*i.e.,* as a form of second-degree price discrimination). Relative to the first best, this is a privately profitable action that is socially wasteful (some content is transmitted via an inefficiently low quality connection).

Even with a more general $v(q, \theta)$ function, a platform restricted to offering a single quality would offer the efficient quality level, $\bar{q}$. It does not follow, however, that the restriction would raise welfare. One would also have to check whether the restricted platform would serve more or fewer application providers than would the unrestricted platform. This remains a question for future research. Observe that—because the connection quality provided under a single-product restriction is efficient, but those utilized when a variety of qualities are offered are not—a sufficient condition for the restriction to raise welfare is that it not reduce the range of applications offered to households in equilibrium.

The contrapositive of this result is that a necessary condition for the platform's offering a variety of qualities to raise total surplus is that it lead to more applications' being offered. This finding parallels the well-known result that a necessary condition for third-degree price discrimination to raise welfare is that total output rise under discrimination.[17] There are, however, important differences between the effects of quality variety (or second-degree price discrimination) and third-degree discrimination. First, a simple measure of total applications lacks economic meaning when applications are heterogeneous and are delivered via connections of varying qualities. Second, the distributions of the welfare effects across market segments are somewhat different. In the case of third-degree price discrimination, discrimination lowers efficiency in high-value markets (prices rise), raises efficiency in low-value markets (prices fall), and has ambiguous effects in middle-value markets. In contrast, "discrimination" in the form of a product line increases efficiency at both the high and low ends of the market, and efficiency losses occur in the middle.

4.2 Technologically Restricted Quality levels

Heretofore, we have allowed the producer to choose the quality level from a continuum of possibilities. In this section, we assume that there are only two technologically feasible quality levels, high (h) and low (ℓ). This assumption is motivated by the fact that often quality levels (*e.g.,* bandwidth and latency) are limited and, in part, determined by forces beyond a platform's control (*e.g.,* capabilities of routers and computers or the installed base of complementary products).[18]

Let $c > 0$ be the cost of providing a unit of the high-quality good. For convenience, we set the cost of providing a unit of the low-quality good to 0. So that there is a welfare benefit to high quality, we assume $(\rho+\sigma)\bar{\theta}h - c > (\rho+\sigma)\bar{\theta}\ell$. Observe that there is also a welfare benefit from offering the low-quality product (*i.e.,* $(\rho+\sigma)\theta h - c < (\rho+\sigma)\theta\ell$ for θ sufficiently close to zero).

[17] See, *e.g.,* Varian (1985) and references therein.

[18] In practice, a continuum of quality levels may be feasible, but it may be efficient to offer only a finite, discrete set.

At prices p_h and p_ℓ, a type-θ content provider is indifferent between the two qualities if and only if

$$\rho h\theta - p_h = \rho\ell\theta - p_\ell\,.$$

Solving this expression for θ and noting that the left-hand side increases faster in θ than the right-hand side, we can conclude that all types such that

$$\theta \geq \frac{p_h - p_\ell}{\rho(h-\ell)}$$

prefer high to low quality. If the ratio on the right-hand side exceeds 1, then there is no demand for the high-quality product.

An application provider prefers low quality to not connecting at all if and only if

$$\rho\ell\theta - p_\ell \geq 0\,.$$

Provided

$$\frac{p_h - f_\ell}{\rho(h-\ell)} > \frac{p_\ell}{\rho\ell}\,, \tag{13}$$

we have

$$\text{demand of type } \theta = \begin{cases} \text{no connection}\,, \text{ if } \theta < \frac{p_\ell}{\rho\ell} \\ \ell \text{ quality}\,, \text{ if } \frac{p_\ell}{\rho\ell} \leq \theta < \frac{p_h - p_\ell}{\rho(h-\ell)} \\ h \text{ quality}\,, \text{ if } \frac{p_h - p_\ell}{\rho(h-\ell)} \leq \theta \end{cases}\,. \tag{14}$$

We first characterize the unrestricted equilibrium.

Lemma 4 *Suppose only two quality levels are technologically feasible and the distribution of application types has a non-decreasing hazard rate. An unregulated platform will offer both qualities and set prices to application providers defined by*

$$p_\ell = \frac{\rho}{\rho+\sigma}\rho\ell m\left(\frac{p_\ell}{\rho\ell}\right)$$

and

$$p_h = p_\ell + \frac{\rho}{\rho+\sigma}c + \frac{\rho}{\rho+\sigma}\rho(h-\ell)m\left(\frac{p_h - p_\ell}{\rho(h-\ell)}\right)\,. \tag{15}$$

It follows from this result that the marginal application types purchasing low- and high-quality connections satisfy

$$\theta_\ell = \frac{\rho}{\rho+\sigma} m(\theta_\ell) \tag{16}$$

and

$$\theta_h = \frac{\rho}{\rho+\sigma}\left(\frac{c}{\rho(h-\ell)} + m(\theta_h)\right), \tag{17}$$

respectively.

Welfare absent regulation is

$$\int_{\theta_\ell}^{\theta_h} (\rho+\sigma)\ell\theta f(\theta)d\theta + \int_{\theta_h}^{\bar{\theta}} \big((\rho+\sigma)\theta h - c\big) f(\theta)d\theta\,. \tag{18}$$

The derivative of this expression with respect to θ_h is

$$\big(c - (\rho+\sigma)\theta_h(h-\ell)\big) f(\theta_h)\,.$$

By (17), this last expression is equal to $-\rho m(\theta_h)(h-\ell)f(\theta_h) < 0$. Hence, welfare would rise if the platform lowered θ_h while holding θ_ℓ constant. In other words, conditional on the set of active application providers, the platform transmits too little content via the high-quality connection.

Now, suppose that regulation forces the platform to offer at most one of the two possible qualities. A restricted platform chooses price, p_r, to maximize

$$\left(1 - F\big(\frac{p_r}{\rho q_r}\big)\right)\big(p_r - c(q_r)\big) + \int_{\frac{p_r}{\rho q_r}}^{\bar{\theta}} \sigma q_r \theta f(\theta)d\theta\,, \tag{19}$$

where $c(h) = c$ and $c(\ell) = 0$.

Lemma 5 *Suppose that only two quality levels are technologically feasible and the distribution of application types has a non-decreasing hazard rate. If the monopolist offers a single product of quality q_r it maximizes its profits by charging a price, p_r, that satisfies*

$$p_r = \frac{\rho}{\rho+\sigma}\left(c(q_r) + \rho q_r m\big(\frac{p_r}{\rho q_r}\big)\right). \tag{20}$$

The proof is similar to that of Lemma 4 and is omitted.

It follows from this result that

$$\underline{\theta}_r = \frac{\rho}{\rho + \sigma} \left(\frac{c(q_r)}{\rho q_r} + m(\underline{\theta}_r) \right) .$$

When $q_r = \ell$, this expression is identical to (16). Thus, if regulation induces the platform to provide only low-quality service, then the restriction has no effect on the set of applications served in equilibrium. However, the restriction reduces the amount of content transmitted via high-quality connections to zero from a level that itself would have been too low absent the restriction. We can thus conclude:

Proposition 6 *Suppose that only two quality levels are technologically feasible and the distribution of application types has a non-decreasing hazard rate. If the platform chooses low quality service under a single-product restriction, then that restriction lowers total surplus.*

Lastly, suppose that the restricted platform offers only high-quality service. We have found numerous examples (*e.g.,* application types are uniformly distributed) in which this restriction lowers total surplus relative to the unrestricted equilibrium, but we have been unable to construct an example in which it raises total surplus. That said, we have also been unable to construct a proof that such an example does not exist.

4.3 Oligopoly

We now examine a market in which two platforms (*e.g.,* a local telephone company offering DSL service and a cable company offering cable modem service) compete to serve households and applications providers.

We model the platforms as competing *à la* Hotelling. Variants of the Hotelling model have been used by several previous authors to examine competition between single-product platforms.[19] Examples include Anderson and

[19]There have also been many insightful papers examining quality competition in one-sided markets, with several focusing on the issue of whether the firms will choose to compete head to head or offer different products than one another to avoid direct competition. For single-product firms, Hotelling (1929) found that minimum differentiation would result as each firm staked out the middle of the famous Hotelling line. Shaked and Sutton (1982), however, analyzed a multi-stage game in which single-product firms choose

Coate (2005), Armstrong (in press), Gabszewicz et al. (2002), and Rochet and Tirole (2003).[20]

There are two platforms, located on the unit interval. Platform i is located at point i on the line, where $i = 0, 1$. The timing of play in the duopoly game is as follows.

- The platforms simultaneously set their price schedules to application providers, $p_i(\cdot)$, and hookup fees to households, h_i, $i = 0, 1$. As in the monopoly model, $p_i(\cdot)$ is the price per household that platform i charges an application provider for access to the platform's household subscriber base.

- Application providers simultaneously choose their connection qualities on the two platforms, $q_i(\theta)$, and, in the fee-for-service model, their prices to households.

- Households observe the hookup fees, connection qualities, and, in the fee-for-service model, application prices. They then choose, for each platform, whether to connect to it and their consumption levels of the various applications.

We assume households are uniformly distributed along the Hotelling line. A household incurs "transportation costs" equal to τ times the distance between the household and a platform to which it subscribes. By analogy to equation (4), the surplus gross of transportation costs derived by a household from consuming the applications available on platform i is

$$S_i \equiv \sigma \int_{\Theta} \theta q_i(\theta) f(\theta) d\theta - h_i \,.$$

A household at location L prefers platform 0 to platform 1 if and only if

$$S_0 - \tau L > S_1 - \tau(1 - L)\,. \tag{21}$$

to differentiate themselves in the first stage (*i.e.,* choose product locations that are not near one another) in order to relax second-stage price competition. Brander and Eaton (1984), Champsaur and Rochet (1989), and others have extended this analysis to multi-product firms. However, DeFraja (1996) finds that firms offer identical product lines in a single-stage game with competition in quantities and vertically differentiated products. As we will demonstrate below, the platforms in our model compete head to head in any equilibrium in which households do not multi-home.

[20]For a survey of research on two-sided markets, see Rochet and Tirole (in press).

If the righthand side is positive, then location L is a *contested* household. More generally if there are households for whom either platform would provide positive net surplus (including transportation cost), we say there are contested households. If each household can obtain positive surplus from at most one platform, we say there are no contested households.[21]

Up to this point, we ignored the cost incurred by the platform to connect a household because that cost was irrelevant provided that it was not so great that hooking-up a household to the network would yield negative surplus. The hookup cost, denoted K, is now relevant because it affects whether households wish to single-home (connect to at most one platform) or multi-home (connect to both platforms). For the time-being, we will analyze the market under the assumption that household multi-homing is infeasible. Below, however, we will establish a condition on K such that no household multi-homes in equilibrium even when it is otherwise feasible to do so.

Recall that platforms' prices to application providers are quoted on a prorated basis; hence, when households single-home, it is a weakly dominant strategy for a type-θ application provider to sign with any platform for which[22]

$$\max_{q\in\mathcal{Q}_i} \rho\theta q - p_i(q) \geq 0\,,$$

where $\mathcal{Q}_i$ is the range of qualities offered by platform i.

Let $D_i(S_0, S_1)$ denote the number of households that prefer platform i when platforms 0 and 1 offer surplus levels S_0 and S_1, respectively. Observe

$$D_i(S_0,S_1) = \begin{cases} \frac{S_i}{\tau}\,, & \text{if no contested households} \\ \frac{1}{2} + \frac{S_i - S_j}{2\tau}\,, & \text{if there exist contested households} \end{cases}\,,$$

where $j \neq i$.

Platform provider i's problem is to choose h_i and $q_i(\cdot)$ to maximize profits. One can equivalently think of a platform having a choice of both quality schedule and household surplus level. Platform i's profits are

$$\pi_i = \left(\int_\Theta \left(\sigma\theta q_i(\theta) + p_i\big(q_i(\theta)\big) - c\big(q_i(\theta)\big)\right) f(\theta)d\theta - S_i - K\right) D_i(S_0,S_1)\,. \quad (22)$$

[21]In the literature, this situation is often described by saying the market is uncovered. A contested market, then, corresponds to a covered market.

[22]We do not allow a platform to require applications to sign exclusive-dealing contracts. To date, ISPs have not demanded exclusivity for most applications.

Observe that, conditional on the value of S_i, a duopoly platform's optimization problem with respect to $q_i(\cdot)$ is identical to the monopolist's (*i.e.,* expression (7) in the unrestricted case or expression (11) in the restricted case). We have shown,

Lemma 6 *Consider a Hotelling market in which household multi-homing is infeasible. The quality schedule offered by each duopoly platform in equilibrium is the same as the schedule offered by the platform in the monopoly model of Section 3.*

This result is a manifestation of what has come to be known as the "terminating access problem."[23] Although there are competing platforms, an application provider has only one way to connect with a given household that has chosen a particular platform. There is a sense in which, from the application providers' perspective, that platform has a monopoly over access to that household. The result is also a consequence of the fact that the degree of market power a platform has with respect to single-homing households is irrelevant to determining the nature of $q_i(\cdot)$ and affects only h_i.

Define

$$\Pi^M \equiv \int_\Theta \Big(\sigma\theta q_z(\theta) + p_z\big(q_z(\theta)\big) - c\big(q_z(\theta)\big)\Big) f(\theta)d\theta\,,$$

where $z = u$ or r depending on regulatory regime. We can rewrite (22) as

$$\pi_i = (\Pi^M - K - S_i)D_i(S_0, S_1)\,. \tag{23}$$

Observe, from (23), that the market exists only if the hookup cost is less than the monopoly profit per customer (*i.e.,* $\Pi^M \geq K$), a condition we have been assuming implicitly and, now, assume explicitly.

We can now establish the following.

Lemma 7 *The Hotelling duopoly model in which household multi-homing is infeasible has a unique equilibrium, which is symmetric. Each platform offers the same quality schedule and price schedule to the application providers as would the monopolist in the model of Section 3 when subject to the same regulatory regime. Moreover, hookup fees are as follows:*

[23] For a regulatory body's expression of concern with this problem, see Federal Communications Commission (2001), paragraphs 13 and 14. For an early analysis of the problem, see Doyle and Smith (1998).

(i) If $\tau > \Pi^M - K \geq 0$*, then*

$$h = \frac{1}{2}\int_\Theta \Big(\sigma\theta q_z(\theta) + c\big(q_z(\theta)\big) - p_z\big(q_z(\theta)\big) + K\Big) f(\theta)d\theta\,. \tag{24}$$

(ii) If $\Pi^M - K \geq \tau > \frac{2}{3}(\Pi^M - K) \geq 0$*, then*

$$h = \int_\Theta \sigma\theta q_z(\theta) f(\theta)d\theta - \frac{1}{2}\tau\,. \tag{25}$$

(iii) If $\frac{2}{3}(\Pi^M - K) \geq \tau \geq 0$*, then*

$$h = \tau + K - \int_\Theta \Big(p_z\big(q_z(\theta)\big) - c\big(q_z(\theta)\big)\Big) f(\theta)d\theta\,. \tag{26}$$

Observe that we can view the platform's marginal cost of signing up a household as the hookup cost minus the profits the platform earns from application providers wishing to serve the household:

$$K - \int_\Theta \Big(p_z\big(q_z(\theta)\big) - c\big(q_z(\theta)\big)\Big) f(\theta)d\theta\,.$$

In this light, (24) is just the usual result that a monopolist facing linear demand with price intercept $\bar{P}$ charges price $\frac{1}{2}(\bar{P} + C)$, where C is marginal cost. Similarly, (26) is the standard result for Hotelling models with contested consumers in which the equilibrium price is transportation cost plus marginal cost. Observe, too, from (26) that, as $\tau \downarrow 0$, the equilibrium approaches the Bertrand equilibrium in which firms price at marginal cost. Absent platform differentiation, Bertrand competition forces the platforms to bid away the profits they make from the application providers in their efforts to entice households to sign with their platform.

We now return to the question of multi-homing. Suppose

$$K \geq \max_{q(\theta)} \int_0^{\bar{\theta}} \Big(p\big(q(\theta)\big) - c\big(q(\theta)\big)\Big) f(\theta)d\theta\,, \tag{27}$$

where $q(\cdot)$ is required to be technically feasible, incentive compatible, and

$$p\big(q(\theta)\big) = \rho\theta q(\theta) - \int_0^\theta \rho q(t)dt\,.$$

The righthand side of expression (27) is the maximum profit an unrestricted monopoly platform can earn from trade with application providers on a per-household basis. If expression (27) didn't hold, then it would be possible for a platform to charge households negative hookup fees and still earn a positive profit. As we see no evidence of platforms engaging in such extreme subsidization of households, we view (27) as a reasonable assumption. Given this assumption we can establish the following result.

Lemma 8 *Suppose that the platforms' choices of product lines are unrestricted, condition* (27) *holds, and application providers believe that no household will multi-home regardless of the platforms' play. Then the equilibrium identified in Lemma 7 is also an equilibrium when household multi-homing is feasible.*

Next, suppose the public policy limits each platform to offering a single product quality.

Lemma 9 *Suppose that each platform is restricted to offering a single quality, application providers believe that no household will multi-home regardless of the platforms' play, and the following conditions hold:*

$$\max_{\{\theta\in\Theta,\theta\leq\theta_r,q\in\mathcal{Q},q\leq q_r\}}\int_{\theta}^{\theta_r}\big(\rho\theta q+\sigma\theta q-c(q)\big)f(t)dt<K\,, \tag{28}$$

and

$$\max_{\{\theta\in\Theta,\theta\geq\theta_r,q\in\mathcal{Q},q\geq q_r\}}\int_{\theta}^{\bar{\theta}}\big(\rho\theta q+\sigma\theta\{q-q_r\}-c(q)\big)f(t)dt<K\,. \tag{29}$$

Then the equilibrium identified in Lemma 7 is also an equilibrium when household multi-homing is feasible.

The two inequalities stated in the lemma guarantee that there is no quality level and set of application providers served such that the incremental surplus created by multi-homing is greater than or equal to the cost of a household's second network connection. Hence, there is no profitable deviation that will induce multi-homing.

Because welfare is determined by the quality schedule, Lemmas 6 through 9 establish conditions under which all of our earlier welfare results apply to the case of competing platforms:

Proposition 7 *Suppose that conditions* (27) *through* (29) *hold and application providers believe that no household will multi-home regardless of the platforms' play. Then the welfare consequences of imposing a single-quality restriction on Hotelling duopolists are identical to the consequences of applying such a restriction to a monopoly platform.*

We close our discussion of platform competition and product-line restrictions by noting that, in some circumstances when the conditions of Lemma 9 are not satisfied, households may multi-home in equilibrium. A single-product restriction creates an artificial value of platform variety. It is artificial because—absent regulation—a single platform would have the ability to offer a full range of qualities. We observe that, in addition to inefficiently doubling expenditures on network connections, a public policy that forces any given firm to offer at most one quality level may not result in all households' consuming the same quality of service—suppliers can collectively offer a range of products even if each firm offers only one.

5 Conclusion

We have formally modeled the effects of product-line restrictions such as those sought by some proponents of network neutrality regulation. We examined a two-sided market for platform services in which application providers on one side of the market purchase platform services of varying qualities that allow the applications to be provided to the households that subscribe to the relevant platform.

For both a monopoly platform and Hotelling duopolists, we found that a single-product restriction results in: (a) application providers that would otherwise have purchased a low-quality variant being excluded from the market; (b) applications "in the middle" of the market purchasing a higher and more efficient quality; and (c) applications at the top of the market purchasing a lower and less efficient quality. We find that the net welfare effects can be positive or negative, although the analysis suggests to us that harm is the more likely outcome. Moreover, applications at the bottom of the market—the ones that a single-product restriction is typically intended to aid—are almost always harmed by the restriction, and consumers have fewer applications available to them as a consequence.[24] In summary, although our

[24]In an earlier version of this paper, we examined a model of Cournot competition

specific findings must be regarded as tentative given the exploratory nature of our model, we believe that our analysis urges caution. A rigorous case for network neutrality regulation entailing product-line restictions has not been made, and there are sound reasons to expect such policies to harm consumers and economic efficiency.

We close by noting that this type of analysis could be extended to other elements of proposed network neutrality regulation, including the proposal that providers of last-mile Internet access services be allowed to charge households but not applications providers. It is readily shown that—because all application providers would utilize the highest available quality level—the platform would offer application providers only one quality of service. Moreover, because application-provider surplus would be increasing in the service quality, the monopolist in our model would offer a quality level that would be lower than either the socially efficient quality level or the quality level that would be offered if the platform were restricted to offering a single quality but allowed to charge both sides of the market for access services.[25]

Appendix A: Proofs

Proof of Lemma 1: Recall that $\frac{c(q)}{q}$ is increasing. Hence, if

$$(\rho + \sigma)\theta \underline{q} - c(\underline{q}) < 0 \,, \tag{30}$$

then

$$(\rho + \sigma)\theta q - c(q) < 0 \,,$$

for all $q \geq \underline{q}$. Thus, for all feasible q, welfare is greater if these types are excluded than if they are served.

between undifferentiated duopolists in a one-sided market in which each firm could either offer two quality levels or was restricted to offering only one. In that model, imposition of a single-product restriction always reduces welfare. Absent the restriction, the two firms engage in head-to-head competition across full product lines, a result first obtained by DeFraja (1996). In some circumstances, the single-product restriction induces the two firms to offer identical products. The resulting loss of variety reduces welfare. In other circumstances, a restriction on the number of products that each firm is allowed to offer induces the firms offer non-overlapping, or vertically differentiated, products. Here, the resulting loss of competition harms both consumers and economic efficiency.

[25]The finding that the quality level would be less than socially optimal builds on the fact that, in our model, the monopolist is able to appropriate all of the marginal benefits of quality enjoyed by households.

If

$$(\rho+\sigma)\theta\underline{q}-c(\underline{q})\geq 0\,, \tag{31}$$

then it is welfare enhancing (at least weakly) to serve these types. Note that the minimum θ satisfying (31) is the least upper bound of θ satisfying (30). It follows that there is a marginal type, $\underline{\theta}_w$, such that $\theta < \underline{\theta}_w$ should be excluded and $\theta \geq \underline{\theta}_w$ should operate. Clearly, $\underline{\theta}_w > 0$, and the assumption that $\bar{\theta}\underline{q} > c(\underline{q})$ implies $\underline{\theta}_w < \bar{\theta}$. Hence, there is a positive measure of types who should be excluded and a positive measure of types who should be served. If a type is served, then its contribution to welfare is maximized choosing q to maximize

$$(\rho+\sigma)\theta q-c(q)$$

subject to $q \geq \underline{q}$. ■

Proof of Proposition 1: Utilizing the envelope theorem, the first-order condition for $\underline{\theta}_r$ is given by

$$\rho q\big(1-F(\underline{\theta}_r)\big)-\big((\rho+\sigma)\underline{\theta}_r q-c(q)\big)f(\underline{\theta}_r)=0\,. \tag{32}$$

Suppose that $0 \leq \underline{\theta}_r \leq \underline{\theta}_w$. Then

$$\begin{aligned}(\rho+\sigma)\underline{\theta}_r q-c(q) &\leq (\rho+\sigma)\underline{\theta}_w q-c(q)\\ &\leq (\rho+\sigma)\underline{\theta}_w \underline{q}-c(\underline{q}) &&\text{(because } c(q)/q \text{ is increasing in } q)\\ &=0 &&\text{(by definition of } \underline{\theta}_w)\end{aligned}$$

Hence, the first term on the left-hand side of (32) is positive, while the second is non-negative, which contradicts (32). The result that $\underline{\theta}_r > \underline{\theta}_w$ follows.

Conditional on connecting only types $\theta \geq \underline{\theta}_r$, welfare maximization requires maximizing

$$(\rho+\sigma)T(\underline{\theta}_r)q-c(q)\,.$$

Because $T(\underline{\theta}_r) > \underline{\theta}_r$, the q that solves that problem must exceed the q that solves (12). ■

Proof of Lemma 3: Define quality q' such that $\underline{q} \leq q' \leq \min\{q_u(\underline{\theta}_u), q_r\}$. Suppose, counterfactually, that $\underline{\theta}_r < \underline{\theta}_u$. Consider an extension of the *un*restricted platform's equilibrium quality offerings in which $q(\theta)$ remains the same for $\theta \geq \underline{\theta}_u$, but now $q = q'$ for $\theta \in [\underline{\theta}_r, \underline{\theta}_u)$. By construction, this extension satisfies the order restriction and, thus, it would have been feasible for the unrestricted platform to have offered it. By (6), this extension is incentive compatible if and only if the prices charged to all types $\theta \geq \underline{\theta}_u$ are reduced by $\rho(\underline{\theta}_u - \underline{\theta}_r)q'$. Because this extended line was not the line chosen by the unrestricted platform, the change in profits from adopting this extension cannot be positive:

$$\begin{aligned}\int_{\underline{\theta}_r}^{\underline{\theta}_u} \big((\rho+\sigma)\theta q' - m(\theta)\rho q' - c(q')\big) f(\theta) d\theta \\ = \big((\rho+\sigma)\underline{\theta}_r q' - c(q')\big)\big(F(\underline{\theta}_u) - F(\underline{\theta}_r)\big) \\ - \big(1 - F(\underline{\theta}_u)\big)(\rho+\sigma)(\underline{\theta}_u - \underline{\theta}_r)q' \\ + \sigma q' \int_{\underline{\theta}_r}^{\underline{\theta}_u} \big(1 - F(\theta)\big) d\theta \leq 0\,.\end{aligned} \tag{33}$$

Note this inequality is strict if conditions (ii) or (iii) are satisfied.

By revealed preference under the restricted regime,

$$\begin{aligned}&\big(\underline{\theta}_r \rho q_r - c(q_r)\big)\big(F(\underline{\theta}_u) - F(\underline{\theta}_r)\big) - \big(1 - F(\underline{\theta}_u)\big)(\underline{\theta}_u - \underline{\theta}_r)\rho q_r \\ &\qquad + \sigma q_r \int_{\underline{\theta}_r}^{\underline{\theta}_u} \theta f(\theta) d\theta \\ &= \big((\rho+\sigma)\underline{\theta}_r q_r - c(q_r)\big)\big(F(\underline{\theta}_u) - F(\underline{\theta}_r)\big) - \big(1 - F(\underline{\theta}_u)\big)(\rho+\sigma)(\underline{\theta}_u - \underline{\theta}_r)q_r \\ &\qquad + \sigma q_r \int_{\underline{\theta}_r}^{\underline{\theta}_u} \big(1 - F(\theta)\big) d\theta \geq 0\,.\end{aligned} \tag{34}$$

Note this inequality is strict if condition (i) is satisfied.

Let $\Delta \equiv \underline{\theta}_u - \underline{\theta}_r$ and $\Delta F \equiv F(\underline{\theta}_u) - F(\underline{\theta}_r)$. Note both Δ and ΔF are positive given the supposition that $\underline{\theta}_u > \underline{\theta}_r$. Expressions (33) and (34) imply

$$(\rho+\sigma)\underline{\theta}_r \Delta F - \big(1 - F(\underline{\theta}_u)\big)(\rho+\sigma)\Delta + \sigma \int_{\underline{\theta}_r}^{\underline{\theta}_u} \big(1 - F(\theta)\big) d\theta \tag{35}$$

$$\leq \Delta F \frac{c(q')}{q'} \leq \Delta F \frac{c(q_r)}{q_r} \tag{36}$$

$$\leq (\rho+\sigma)\underline{\theta}_r \Delta F - \big(1 - F(\underline{\theta}_u)\big)(\rho+\sigma)\Delta + \sigma \int_{\underline{\theta}_r}^{\underline{\theta}_u} \big(1 - F(\theta)\big) d\theta\,. \tag{37}$$

The first inequality in (36) follows from (33) and the inequality in (37) follows from (34). The second inequality in (36) follows because $q' \leq q_r$ and $c(q)/q$ is an increasing function. Observe that (35) and (37) are the same expression, so this chain of inequalities yields a contradiction if any of the inequalities are strict. However, as noted, under condition (i), the inequality in (37) is strict; while, as noted, under conditions (ii) or (iii) the first inequality in (36) is strict. Hence, by contradiction, we can conclude $\underline{\theta}_r \geq \underline{\theta}_u$ if any of conditions (i)–(iii) hold.

If $q_r > \underline{q}$, then q' can be chosen so $q' < q_r$. Hence, the second inequality in (36) would be strict. So, by contradiction, we can conclude $\underline{\theta}_r \geq \underline{\theta}_u$ if condition (iv) holds.

We now turn to the sufficiency of condition (v). Note the same revealed preference argument used to derive (33) applies to any $\theta \in [\underline{\theta}_r, \underline{\theta}_u]$; hence,

$$\begin{aligned}\int_{\theta}^{\underline{\theta}_u} &\big((\rho+\sigma)tq' - m(t)\rho q' - c(q')\big)f(t)dt \\ &= \big((\rho+\sigma)\theta q' - c(q')\big)\big(F(\underline{\theta}_u) - F(\theta)\big) \\ &\quad - \big(1 - F(\underline{\theta}_u)\big)(\rho+\sigma)(\underline{\theta}_u - \theta)q' \\ &\quad + \sigma q' \int_{\theta}^{\underline{\theta}_u} \big(1 - F(t)\big)dt \leq 0\end{aligned} \tag{38}$$

(*i.e.,* the unrestricted platform would not wish to extend down to θ). Similarly, the same reveal preference argument used to derive (34) applies to any $\theta \in [\underline{\theta}_r, \underline{\theta}_u]$; hence,

$$\begin{aligned}&\big(\underline{\theta}_r \rho q_r - c(q_r)\big)\big(F(\theta) - F(\underline{\theta}_r)\big) - \big(1 - F(\theta)\big)(\theta - \underline{\theta}_r)\rho q_r \\ &\qquad + \sigma q_r \int_{\underline{\theta}_r}^{\theta} tf(t)dt \\ &= \big((\rho+\sigma)\underline{\theta}_r q_r - c(q_r)\big)\big(F(\theta) - F(\underline{\theta}_r)\big) - \big(1 - F(\theta)\big)(\rho+\sigma)(\theta - \underline{\theta}_r)q_r \\ &\qquad + \sigma q_r \int_{\underline{\theta}_r}^{\theta} \big(1 - F(t)\big)dt \geq 0\end{aligned} \tag{39}$$

(*i.e.,* the restricted platform would not wish to cutoff sales at θ). Observe (38) implies

$$\frac{(\rho+\sigma)\theta q' - c(q')}{q'} \leq (\rho+\sigma)\big(1 - F(\underline{\theta}_u)\big)\frac{\underline{\theta}_u - \theta}{F(\underline{\theta}_u) - F(\theta)} - \sigma\frac{\int_{\theta}^{\underline{\theta}_u}\big(1 - F(t)\big)dt}{F(\underline{\theta}_u) - F(\theta)}$$

for all $\theta \in [\underline{\theta}_r, \underline{\theta}_u]$. Hence, it is true in the limit as $\theta \to \underline{\theta}_u$:

$$\begin{aligned}\frac{(\rho+\sigma)\underline{\theta}_u q' - c(q')}{q'} &\leq (\rho+\sigma)\frac{1 - F(\underline{\theta}_u)}{f(\underline{\theta}_u)} - \sigma\frac{1 - F(\underline{\theta}_u)}{f(\underline{\theta}_u)} \\ &= \rho m(\underline{\theta}_u)\,.\end{aligned} \tag{40}$$

Similarly, we can write (39) as

$$\frac{(\rho+\sigma)\underline{\theta}_r q_r - c(q_r)}{q_r} \geq (\rho+\sigma)\big(1-F(\theta)\big)\frac{\theta-\underline{\theta}_r}{F(\theta)-F(\underline{\theta}_r)} - \sigma\frac{\int_{\underline{\theta}_r}^{\theta}\big(1-F(t)\big)dt}{F(\theta)-F(\underline{\theta}_r)}$$

for all $\theta \in [\underline{\theta}_r, \underline{\theta}_u]$. Hence, it is true in the limit as $\theta \to \underline{\theta}_r$:

$$\frac{(\rho+\sigma)\underline{\theta}_r q_r - c(q_r)}{q_r} \geq \rho m(\underline{\theta}_r)\,. \tag{41}$$

Recall that $-c(q)/q$ is a decreasing function of q and $q' \leq q_r$. By supposition $\underline{\theta}_u > \underline{\theta}_r$, hence the left-hand side of (40) is strictly greater than the left-hand side of (41). Hence,

$$m(\underline{\theta}_u) > m(\underline{\theta}_r)\,;$$

but this means the hazard rate evaluated at $\underline{\theta}_u$ is strictly less than it is evaluated at $\underline{\theta}_r$, which contradicts condition (v). *Reductio ad absurdum*, we can conclude that $\underline{\theta}_r \geq \underline{\theta}_u$ if condition (v) holds. ■

Proof of Proposition 2: From Lemma 3, $\underline{\theta}_r \geq \underline{\theta}_u$. Suppose, counterfactually, that $\underline{\theta}_r = \underline{\theta}_u$. Select a q' such that $\underline{q} \leq q' < q_r$ and $q' \leq q_u(\underline{\theta}_u)$. The same revealed preference arguments used to establish that the unrestricted platform would not wish to extend downward (*i.e.*, serve types worse than $\underline{\theta}_u$) and that the restricted platform would not wish to have a higher cutoff than $\underline{\theta}_r$ continue to apply, so expressions (40) and (41) remain valid. By supposition $\underline{\theta}_r = \underline{\theta}_u$, so making that substitution into (40) and utilitizing the fact that $c(q)/q$ is a strictly increasing function of q, we can combine (40) and (41) as

$$\rho m(\underline{\theta}_r) \geq \frac{(\rho+\sigma)\underline{\theta}_r q' - c(q')}{q'} > \frac{(\rho+\sigma)\underline{\theta}_r q_r - c(q_r)}{q_r} \geq \rho m(\underline{\theta}_r)\,,$$

which is impossible. Hence, the supposition $\underline{\theta}_u = \underline{\theta}_r$ must be false *reductio ad absurdum*. Given $\underline{\theta}_u \leq \underline{\theta}_r$, we are left with $\underline{\theta}_u < \underline{\theta}_r$, as was to be shown. ■

Proof of Proposition 3: First, we assemble some facts. Recall that $q_w(\bar{\theta}) > q_w(\theta)$ for all $\theta < \bar{\theta}$, including $\underline{\theta}_r$, where $\underline{\theta}_r$ is the marginal type served under the first policy. By Proposition 1, $q_r < q_w(\underline{\theta}_r)$. Hence, $q_r < q_w(\bar{\theta})$.

Let $\underline{\theta}_R$ denote the marginal type under the second policy. We can establish that $\underline{\theta}_R \geq \underline{\theta}_r$ by mimicking the proof of Lemma 3. Specifically, suppose $\underline{\theta}_R < \underline{\theta}_r$. Let $\underline{\theta}_R$ play the role played by $\underline{\theta}_r$ in the proof of Lemma 3 and let $\underline{\theta}_r$ play the role played by $\underline{\theta}_u$ in that proof. Note, because $q_r < q_w(\bar{\theta})$, the equivalent of condition (iv) holds.

To establish that $\underline{\theta}_R > \underline{\theta}_r$, suppose, counterfactually, that $\underline{\theta}_R = \underline{\theta}_r$. Then $\underline{\theta}_r$ must satisfy the first-order conditions for both of the following programs,

$$\max_{\theta} \big(\rho\theta q_r + \sigma T(\theta) q_r - c(q_r)\big)\big(1 - F(\theta)\big)$$

$$\text{and} \qquad \max_{\theta} \Big(\rho\theta q_w(\bar{\theta}) + \sigma T(\theta) q_w(\bar{\theta}) - c\big(q_w(\bar{\theta})\big)\Big)\big(1 - F(\theta)\big)\,,$$

where, recall, $T(\theta)$ is the expected type conditional on type not being less than θ. Hence,

$$q_r\big(\rho + \sigma T'(\underline{\theta}_r)\big)\big(1 - F(\underline{\theta}_r)\big) - \Big(\big(\rho\underline{\theta}_r + \sigma T(\underline{\theta}_r)\big)q_r - c(q_r)\Big) f(\underline{\theta}_r) = 0$$

and

$$q_w(\bar{\theta})\big(\rho + \sigma T'(\underline{\theta}_r)\big)\big(1 - F(\underline{\theta}_r)\big) - \Big(\big(\rho\underline{\theta}_r + \sigma T(\underline{\theta}_r)\big)q_w(\bar{\theta}) - c\big(q_w(\bar{\theta})\big)\Big) f(\underline{\theta}_r) = 0\,.$$

Rearranging and combining, we have

$$\frac{c(q_r)}{q_r} = \big(\rho\underline{\theta}_r + \sigma T(\underline{\theta}_r)\big) - \big(\rho + \sigma T'(\underline{\theta}_r)\big)m(\underline{\theta}_r) = \frac{c\big(q_w(\bar{\theta})\big)}{q_w(\bar{\theta})}\,. \tag{42}$$

But, as established earlier, $c(q)/q$ is a strictly increasing function of q. Hence, (42) contradicts the fact that $q_w(\bar{\theta}) > q_r$. The result follows *reductio ad absurdum.*

The second part of the proposition follows from the first coupled with Lemma 3. ■

Lemma A.1 *There exists an application type, $\hat{\theta} \in (\underline{\theta}_r, \bar{\theta})$, such that the quality under a single-product restriction (i.e., q_r) is efficient for that type (i.e., $q_r = q_w(\hat{\theta})$).*

Proof: Because some trade is always profitable for the platform, $\underline{\theta}_r < \bar{\theta}$. Hence,

$$\rho\underline{\theta}_r + \sigma T(\underline{\theta}_r) < \rho\bar{\theta} + \sigma T(\bar{\theta}) = \rho\bar{\theta} + \sigma\bar{\theta}\,,$$

which implies that $q_r < q_w(\bar{\theta})$. But, because

$$\rho\underline{\theta}_r + \sigma T(\underline{\theta}_r) > \rho\underline{\theta}_r + \sigma\underline{\theta}_r\,,$$

$q_r > q_w(\underline{\theta}_r)$. The result follows because $q_w(\cdot)$ is a continuous function. ■

Proof of Proposition 4: By revealed preference, the platform's profit cannot be less absent regulation than under the single-product restriction. It follows, therefore, that a sufficient condition for welfare to be greater in the unrestricted equilibrium is that application-provider surplus (AS) be greater in that equilibrium than under regulation.[26] Observe

$$CPS_r = \int_{\underline{\theta}_r}^{\bar{\theta}} \big(\rho\theta q_r - \rho\underline{\theta}_r q_r\big) f(\theta)d\theta = \int_0^{\bar{\theta}} \left(\rho \int_0^{\theta} q_r(t)dt\right) f(\theta)d\theta \tag{43}$$

and

$$CPS_u = \int_{\underline{\theta}_u}^{\bar{\theta}} \left(\rho\theta q_u(\theta) - f\big(q(\theta)\big)\right) f(\theta)d\theta = \int_0^{\bar{\theta}} \left(\rho \int_0^{\theta} q_u(t)dt\right) f(\theta)d\theta, \tag{44}$$

where the second equality in (43) follows from the definition of integration and the fact that $q_r(t) \equiv 0$ for $t < \underline{\theta}_r$, while the second equality in (44) follows from (6) and the fact that $q_u(t) \equiv 0$ for $t < \underline{\theta}_u$. If $q_r(\cdot)$ dominates $q_u(\cdot)$ in the sense of second-order stochastic dominance, then it follows from (43) and (44) and the fact that $\underline{\theta}_u < \underline{\theta}_r$ by Proposition 2 that $CPS_u > CPS_r$. Sufficiency follows. Because the necessary condition is the contrapositive of the sufficiency condition, it follows as well. ■

Proof of Lemma 4: To begin, define $\Delta_f = f_h - f_\ell$ and let $\Delta_q = h - \ell$.

[26] Recall end-users' surplus is fully captured by the platform so is accounted for in the platform's profit.

Consider the platform's maximization problem:

$$\max_{\{f_\ell,\Delta_f\}} \left(1 - F\big(\frac{\Delta_f}{\rho\Delta_q}\big)\right)\left(\Delta_f + f_\ell - c\right) + \int_{\frac{\Delta_f}{\rho\Delta_q}}^{\bar{\theta}} \sigma h\theta f(\theta)d\theta$$
$$+ \left(F\big(\frac{\Delta_f}{\rho\Delta_q}\big) - F\big(\frac{f_\ell}{\rho\ell}\big)\right) f_\ell + \int_{\frac{f_\ell}{\rho\ell}}^{\frac{\Delta_f}{\rho\Delta_q}} \sigma\ell\theta f(\theta)d\theta\,. \quad (45)$$

For the moment, we ignore the constraints (*i.e.,* that (13) hold and that both fractions in (13) be between 0 and $\bar{\theta}$). As we will demonstrate, they are not binding. Observe (45) is equivalent to the following program.

$$\max_{\{f_\ell,\Delta_p\}} \left(1 - F\big(\frac{\Delta_f}{\rho\Delta_q}\big)\right)\left(\Delta_f - c\right) + \int_{\frac{\Delta_f}{\rho\Delta_q}}^{\bar{\theta}} \sigma h\theta f(\theta)d\theta$$
$$+ \left(1 - F\big(\frac{f_\ell}{\rho\ell}\big)\right) f_\ell + \int_{\frac{f_\ell}{\rho\ell}}^{\frac{\Delta_f}{\rho\Delta_q}} \sigma\ell\theta f(\theta)d\theta\,. \quad (46)$$

If the solution to the unconstrained problem satisfies the constraints, then that solution is also the solution to the constrained problem. The first order conditions for (46) can be reëxpressed as

$$m\left(\frac{f_\ell}{\rho\ell}\right) - \frac{f_\ell}{\rho\ell} - \frac{\sigma}{\rho}\frac{f_\ell}{\rho\ell} = 0 \quad (47)$$

and

$$m\left(\frac{\Delta_f}{\rho\Delta_q}\right) - \frac{\Delta_f - c}{\rho\Delta_q} - \frac{\sigma}{\rho}\frac{\Delta_f}{\rho\Delta_q} = 0\,. \quad (48)$$

Because $m(\cdot)$ is non-increasing, it is readily seen (i) that if solutions exist they must be unique and (ii) they satisfy the second-order condition because the derivatives (the left-hand sides of (47) and (48)) are positive to the left of the optimum and negative to the right of the optimum.

Given (i) the continuity of $m(\cdot)$ and (ii) that $m(0) > 0 = m(\bar{\theta})$, (47) must have a solution such that

$$0 < \frac{f_\ell}{\rho\ell} < \bar{\theta}\,.$$

Because $m(\cdot)$ is non-increasing and $c > 0$, it follows that, if (48) has a solution, then it satisfies

$$\frac{\Delta_f}{\rho\Delta_q} > \frac{f_\ell}{\rho\ell};$$

that is, (13). Hence, we're done if we can show that there exists a Δ_f such that $\frac{\Delta_f}{\rho\Delta_q} < \bar{\theta}$. Suppose there weren't. Then

$$\frac{c-\Delta_f}{\rho\Delta_q} - \frac{\sigma}{\rho}\frac{\Delta_f}{\rho\Delta_q} > 0 \quad \text{or, equivalently,} \quad c > \Delta_f\left(1+\frac{\sigma}{\rho}\right)$$

for any $\Delta_f \geq \rho\bar{\theta}\Delta_q$. In particular, this would require that

$$c > \bar{\theta}\rho\Delta_q\left(1+\frac{\sigma}{\rho}\right) = \bar{\theta}(h-\ell)(\rho+\sigma),$$

but that violates the assumption that there is a welfare benefit to high quality. By contradiction, (48) has a solution.

We have shown that the solutions to (47) and (48) maximize (46) and satisfy the relevant constraints. Algebra yields (15). ■

Proof of Lemma 7: Regardless of which definition of $D_i(S_0, S_1)$ applies, (23) is global concave in S_i and, thus, has a unique solution. When, in equilibrium, there are no contested households, then S_j, $j \neq i$, is irrelevant to the optimal S_i. Given the symmetry between the platforms and the concavity of (23), it follows that any equilibrium without contested households must be symmetric. Suppose there are contested households in equilibrium, then S_0 and S_1 must simultaneously satisfy the first-order conditions:

$$\frac{1}{2\tau}\left(\Pi^M - K - 2S_0 - \tau + S_1\right) = 0 \text{ and} \qquad (49)$$
$$\frac{1}{2\tau}\left(\Pi^M - K - 2S_1 - \tau + S_0\right) = 0.$$

Straightforward algebra reveals

$$S_0 = S_1 = \Pi^M - K - \tau. \qquad (50)$$

Hence, we've established uniqueness and symmetry.

In a no-contested-households equilibrium, a platform selects S to maximize

$$(\Pi^M - K - S)\frac{S}{\tau} \tag{51}$$

subject to the constraint that it not "intrude" into the other platform's market. Given symmetry, we can express this constraint as

$$S \leq \frac{\tau}{2}\,. \tag{52}$$

Maximizing (51), we find that the profit-maximizing (and, thus, equilibrium) S satisfies

$$S = \begin{cases} \frac{\Pi^M - K}{2}\,, \text{ if } \tau > \Pi^M - K \text{ (\textit{i.e.}, (52) doesn't bind)} \\ \frac{\tau}{2}\,, \text{ if } \tau \leq \Pi^M - K \text{ (\textit{i.e.}, (52) binds)} \end{cases}. \tag{53}$$

A no-contested-households equilibrium when $\tau \leq \Pi^M - K$ and, thus, $S = \tau/2$ can exist only if the best response to $S_j = \tau/2$ is not $S_i > \tau/2$. Using (49), this requires

$$\pi^M - K - 2S - \frac{\tau}{2} \leq 0 \tag{54}$$

for all $S > \tau/2$. Expression (54) can hold for *all* such S if and only if

$$\tau > \frac{2}{3}(\pi^M - K)\,. \tag{55}$$

If (55) doesn't hold, then the equilibrium results in the platforms contesting for some households, so S is given by (50). Straightforward algebra on expressions (50) and (53) yield expressions (24)–(26). ■

Proof of Lemma 8: First, given their beliefs—which are correct if the Lemma 7 equilibrium is played—application providers will sign with any platform for which

$$\max_{q \in \mathcal{Q}_i} \rho\theta q - p_i(q) \geq 0\,.$$

Second, we verify that no household would multi-home given the equilibrium strategies set forth in Lemma 7. Observe $h > 0$ as a consequence of (27). It follows, therefore, given the symmetric offerings of the two platforms, that a household could only lose by multi-homing.

Third, we verify that neither platform would wish to deviate. Without loss of generality, suppose that network 1 is offering the candidate equilibrium surplus level and quality schedule, and network 0 deviates. By definition of equilibrium in the model when multi-homing is infeasible, there is no deviation that is both profitable and does not induce any multi-homing. Hence, we need only rule out deviations by a platform that induces at least some multi-homing.

For a household at location L that multi-homes, the incremental surplus offered by platform 0 over signing with platform 1 exclusively is

$$\sigma \int_\Theta \theta \max\left\{q_0(\theta) - q_u(\theta)\,,\, 0\right\} f(\theta)d\theta - h_0 - L\tau\,. \tag{56}$$

A multi-homing household will sign with platform 0 only if that surplus is non-negative. Observe that platform 0 will lose money on *every* household it connects if

$$h_0 < K - \int_\Theta \Big(p_0\big(q_0(\theta)\big) - c\big(q_0(\theta)\big)\Big) f(\theta)d\theta\,. \tag{57}$$

Using (57), a necessary condition for (56) to be non-negative is, therefore,

$$\begin{aligned}\sigma \int_\Theta \theta \max\left\{q_0(\theta) - q_u(\theta)\,,\, 0\right\} f(\theta)d\theta \\ + \int_\Theta \Big(p_0\big(q_0(\theta)\big) - c\big(q_0(\theta)\big)\Big) f(\theta)d\theta - K > 0\,.\end{aligned} \tag{58}$$

Clearly, the left-hand side of (58) cannot exceed the value of that expression maximized by the choice of $q_0(\cdot)$. That maximization program can be written as

$$\max_{q(\theta)} \int_0^{\bar\theta} \Big(\sigma\big(q(\theta) - q_u(\theta)\big)^+ + \rho\theta q(\theta) - m(\theta)\rho q(\theta) - c\big(q(\theta)\big)\Big) f(\theta)d\theta \tag{59}$$

subject to the constraint that $q(\cdot)$ be non-decreasing (note $x^+ = x$ if $x > 0$ and $= 0$ otherwise). Suppose we maximized (59) *ignoring* the constraint that $q(\cdot)$ be non-decreasing; then that maximized value must be at least as great as the constrained maximum of (59). Observe that the first-order condition for the unconstrained problem if $q(\theta) \geq q_u(\theta)$ is the same as that of the original unrestricted monopolist's problem, (9); so, in these cases $q(\theta) = q_u(\theta)$. Hence, optimally $q(\theta) \leq q_u(\theta)$ almost everywhere. Consequently, the top line of (58) is zero. But the remainder of (58) cannot be positive by (27). By contradiction, we have shown that Platform 0 can never offer a household enough surplus to induce it to multi-home. ■

Appendix B: Examples

It is useful to illustrate certain points using discrete examples. The following analysis is a straightforward extension of the continuous case.

Assume that there is a finite number, T, of content types indexed so that $s < t$ implies $\theta_s < \theta_t$. Let f_t denote the probability a randomly drawn household is type θ_t. The distribution, correspondingly, can be denoted

$$F_t = \sum_{\tau=1}^{t} f_t \, .$$

Define

$$m_t \equiv \frac{1 - F_t}{f_t} \, .$$

Note $m_T = 0$. Define

$$R_t(q) \equiv \rho\theta_{t+1}q - \rho\theta_t q \, .$$

The fact that $R_T(\cdot)$ is not defined is not, as will become apparent, an issue. The function $R_t(\cdot)$ is positive, strictly increasing, and convex on $\mathbb{R}_+$. As a consequence, we know from Proposition 2 of Caillaud and Hermalin (1993) that the monopolist's profit-maximization problem with respect to the choice of qualities reduces to the following.[27]

$$\max_{\{q_1,\dots,q_T\}} \sum_{t=1}^{T} f_t\big((\rho+\sigma)\theta_t q_t - c(q_t) - m_t R_t(q_t)\big) \tag{60}$$

$$\text{subject to } \underline{q} \le q_1 \le \cdots \le q_T \, . \tag{61}$$

Let $\{\hat{q}_1, \dots, \hat{q}_T\}$ denote the solution. Observe this solution must satisfy, for each t, the following condition.

$$(\rho+\sigma)\theta_t - m_t(\theta_{t+1} - \theta_t)\rho - c'(\hat{q}_t) \gtreqless 0 \, , \tag{62}$$

[27]Caillaud and Hermalin's Proposition 2 is essentially a fairly straightforward extension of standard results in mechanism design for two types or for a continuum of types to an arbitrary, but finite, number of types.

where the expression is an equality if the relevant order restriction, condition (61), doesn't bind, is greater than zero if the upward restriction binds, and is less than if the downward restriction binds.

If we impose the assumption that m_t is non-increasing in t (*i.e.,* a monotone hazard rate), then it can be shown that (61) is not binding except, possibly, at $\underline{q}$. In that case, if $\hat{q}_t = 0$ or $\underline{q}$, then the left-hand side of expression (62) is less than zero.

Welfare when the monopolist is unrestricted is

$$W_u = \sum_{t=1}^{T} \big((\rho+\sigma)\theta_t \hat{q}_t - c(\hat{q}_t)\big) f_t \,. \tag{63}$$

Example 1: Consider a discrete type space with three elements. Let

$$\theta_1 = \frac{1}{10\,000} + \frac{53}{381}(15 + 2\sqrt{2}) \qquad \theta_2 = \frac{15+2\sqrt{2}}{6} \qquad \theta_3 = 3$$
$$f_1 = \frac{7}{60} \qquad f_2 = \frac{13}{60} \qquad f_3 = \frac{40}{60}$$

(the value of θ_1 is set just above the value at which the unrestricted platform would exclude that type). Suppose $\rho = 100$, $\sigma = 50$, and $c(q) = q^2/2$, which implies $q_w(\theta) = 150\theta$, and assume $\underline{q} < 1/20$.

Consider the unrestricted case first. Using expression (62) and the fact that $c'(q) = q$, one finds that all types are served, with

$$q_u(\theta_1) = \frac{127}{1400} \approx .0907$$
$$q_u(\theta_2) = \frac{25}{39}(345 + 238\sqrt{2}) \approx 436.9$$
$$q_u(\theta_3) = q_w(3) = 450\,.$$

Consequently, unrestricted welfare is

$$\begin{aligned} W_u &= \sum_{t=1}^{3} \Big(\theta_t q_u(\theta_t) - c\big(q_u(\theta_t)\big)\Big) f_t \\ &= \left(\frac{61,669,559,167,397}{786,240,000} + \frac{35,014,567}{2340\sqrt{2}}\right) \\ &\approx 89,016.8\,. \end{aligned}$$

Now consider the imposition of a single-product restriction. Calculations reveal

$$\text{profit} = \begin{cases} \frac{(12{,}525{,}381+1{,}190{,}000\sqrt{2})^2}{2{,}592{,}000{,}000} \approx 77,884\,, \text{ if } \underline{\theta}_r = \theta_1 \\ \frac{125(2505+238\sqrt{2})^2}{11{,}448} \approx 88,166\,, \text{ if } \underline{\theta}_r = \theta_2 \\ 67,500\,, \text{ if } \underline{\theta}_r = \theta_3 \end{cases}.$$

Under the single-product restriction, the platform excludes the lowest type. Nevertheless, welfare increases:[28]

$$\begin{aligned} W_r &= \sum_{t=2}^{3} \Big(\theta_t q_r - c\big(q_r\big)\Big) f_t \\ &= \frac{125(7,438,393 + 505,020\sqrt{2})}{11,448} \\ &\approx 89,017.7 > W_u\,. \end{aligned}$$

□

Example 2: Assume that $c(q) = q^2/2$ and θ is distributed on $[0, 1]$ according to the distribution $F(\theta) = \theta^5$. Let $\rho = 2$, $\sigma = 1$, and $\underline{q} = 0$. Calculations reveal:[29]

$$\underline{\theta}_r \approx .741 \qquad W_r \approx 2.77 \qquad AS_r \approx .572$$

$$\underline{\theta}_u = \left(\frac{2}{17}\right)^{1/5} \approx .652 \qquad W_u = 5\left(\frac{31}{42} - \frac{8}{21}\left(\frac{2}{17}\right)^{1/5}\right) \approx 2.88 \qquad AS_u = \frac{5(221 - 2^{\frac{2}{5}}17^{\frac{3}{5}}25)}{357} \approx .566$$

□

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[28]The precise difference between W_r and W_u is

$$W_r - W_u = \frac{115,982,179,127,959 - 81,985,604,568,000\sqrt{2}}{41,670,720,000}\,.$$

[29]The *Mathematica* program used is available from the authors upon request.

Example 3: Suppose $F(\cdot)$ is the uniform distribution, $\bar{\theta} = 1$, $c(q) = q^2/2$, $\rho = 2$, $\sigma = 1$, and, for convenience, set $\underline{q} = 0$. Note that

$$m(\theta) = 1 - \theta\,.$$

Hence, from (9),

$$q_u(\theta) = \max\{0, 3\theta - 2(1-\theta)\} = \max\{0, 5\theta - 2\}\,.$$

Note $q_u(\theta) \geq 0$ for all $\theta \geq 2/5 = \underline{\theta}_u$. Hence,

$$W_u = \int_{2/5}^{1} \left(3\theta\big(5\theta - 2\big) - \frac{1}{2}\big(5\theta - 2\big)^2\right) d\theta = \frac{5}{6}\theta^3 + 2\theta^2 - 2\theta\Big|_{2/5}^{1} = \frac{63}{50}$$

and

$$\begin{aligned} AS_u &= \int_{2/5}^{1} 2\left(\int_0^{\theta} q_u(t)dt\right) d\theta = 2\int_{2/5}^{1}\left(\int_{2/5}^{\theta}\big(5t-2\big)dt\right) d\theta \\ &= 2\int_{2/5}^{1}\left(\frac{5}{2}\theta^2 - 2\theta + \frac{2}{5}\right) d\theta = 2\left(\frac{5}{6}\theta^3 - \theta^2 + \frac{2}{5}\theta\right)\Big|_{2/5}^{1} = \frac{9}{25}\,. \end{aligned}$$

Turning to the case of the single-product restriction, observe

$$T(\theta) = \frac{\int_\theta^1 tdt}{1-\theta} = \frac{\frac{1}{2} - \frac{\theta^2}{2}}{1-\theta} = \frac{\theta+1}{2}\,.$$

Hence, solving (12),

$$q_r = 2\underline{\theta}_r + 1\frac{\underline{\theta}_r + 1}{2} = \frac{5\underline{\theta}_r + 1}{2}\,.$$

The first-order condition for (11), utilizing the Envelope Theorem, is

$$\rho + \sigma T'(\theta)(1-\theta)q_r - \big(\rho\theta q_r + \sigma T(\theta)q_r - c(q_r)\big) =$$
$$(2+\frac{1}{2})(1-\underline{\theta}_r)\frac{5\underline{\theta}_r+1}{2} - \left(2\underline{\theta}_r\frac{5\underline{\theta}_r+1}{2} + \frac{\underline{\theta}_r+1}{2}\frac{5\underline{\theta}_r+1}{2} - \frac{1}{2}\big(\frac{5\underline{\theta}_r+1}{2}\big)^2\right) = 0\,.$$

Solving, $\underline{\theta}_r = 3/5$ and $q_r = 2$. Hence,

$$W_r = \int_{3/5}^{1}\left(3\theta \times 2 - \frac{1}{2}2^2\right) d\theta = 3\theta^2 - 2\theta\Big|_{3/5}^{1} = \frac{56}{50}$$

and

$$AS_r = \int_{3/5}^{1} \big(2 \times 2(\theta - \frac{3}{5})\big) d\theta$$
$$= 2\theta^2 - \frac{12}{5}\theta \Big|_{3/5}^{1} = \frac{8}{25}\,.$$

In this example, the improved-quality effect is positive but is dominated by the negative exclusion and reduced-quality effects. A single-product restriction reduces welfare from $W_u = 63/50$ to $W_r = 56/50$.

□

References

Anderson, Simon P. and Stephen Coate, "Market Provision of Broadcasting: A Welfare Analysis," *Review of Economic Studies*, October 2005, *72* (4), 947–972.

Armstrong, Mark, "Competition in Two-Sided Markets," *RAND Journal of Economics*, in press.

Brander, James A. and Jonathan Eaton, "Product Line Rivalry," *American Economic Review*, June 1984, *74* (3), 323–334.

Caillaud, Bernard and Benjamin E. Hermalin, "The Use of an Agent in a Signalling Model," *Journal of Economic Theory*, June 1993, *60* (1), 83–113.

_ and _ , "Hidden-Information Agency," 2000. http://faculty.haas.berkeley.edu/hermalin/mechread.pdf.

_ and Bruno Jullien, "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, Summer 2003, *34* (2), 309–28.

Champsaur, Paul and Jean-Charles Rochet, "Multiproduct Duopolists," *Econometrica*, May 1989, *57* (3), 533–557.

DeFraja, Giovanni, "Product Line Competition in Vertically Differentiated Markets," *International Journal of Industrial Organization*, May 1996, *14* (3), 389–414.

Doyle, Chris and Jennifer C. Smith, "Market structure in mobile telecoms: qualified indirect access and the receiver pays principle," *Information Economics and Policy*, December 1998, *10* (4), 471–488.

Federal Communications Commission, "Notice of Proposed Rulemaking," *In the Matter of Developing a Unified Intercarrier Compensation Regime*, April 2001.

Gabszewicz, Jean J, Didier Laussel, and Nathalie Sonnac, "Press Advertising and the Political Differentiation of Newspapers," *Journal of Public Economic Theory*, 2002, *4* (3), 317–34.

Hotelling, Harold, "Stability in Competition," *Economic Journal*, March 1929, *39* (153), 41–57.

Katz, Michael L., "Multiplant Monopoly in a Spatial Market," *Bell Journal of Economics*, Autumn 1980, *11* (2), 519–535.

Mussa, Michael and Sherwin Rosen, "Monopoly and Product Quality," *Journal of Economic Theory*, August 1978, *18* (2), 301–317.

Pečarić, Josip E., Frank Proschen, and Yung Liang Tong, *Convex Functions, Partial Orderings, and Statistical Applications*, San Diego: Academic Press, 1992.

Rochet, Jean-Charles and Jean Tirole, "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 06 2003, *1* (4), 990–1029.

_ **and** _ , "Two-Sided Markets: A Progress Report," *RAND Journal of Economics*, in press.

Shaked, Avner and John Sutton, "Relaxing Price Competition Through Product Differentiation," *Review of Economic Studies*, January 1982, *49* (1), 3–13.

Spence, A. Michael, "Monopoly, Quality, and Regulation," *Bell Journal of Economics*, Autumn 1975, *6* (2), 417–429.

Varian, Hal R., "Price Discrimination and Social Welfare," *American Economic Review*, September 1985, *75* (4), 870–875.