Exclusionary Pricing and Rebates in a Network Industry*

Liliane Karlinger[†] Massimo Motta[‡]

January 3, 2007

Abstract

We consider an incumbent firm and a more efficient entrant, both offering a network good to several asymmetric buyers. The incumbent disposes of an installed base, while the entrant has a network of size zero at the outset, and needs to attract a critical mass of buyers to operate. We analyze different schemes of price discrimination (from uniform to implicit price discrimination - or rebates -, to explicit discrimination) and show that the schemes which - for given market structure - induce a higher level of welfare are also those under which the incumbent is more likely to exclude the rival.

JEL classification: L11, L14, L41.

---

*This is work in progress. We are indebted to Patrick Rey, Jeanine Thal, and Daniel Cerquera, for helpful comments and suggestions. We are also grateful to seminar participants at Hitotsubashi University, Tokyo; CORE, Université Catholique de Louvain; CEPR Conference on Applied Industrial Organisation, Madeira; 2nd CSEF Symposium on Economics and Institutions, Anacapri; 1st CRESSE Conference on 'Abuse of Dominance', Corfu (07/2006); CCP Workshop 2005, University of East Anglia (Norwich, UK); Kiel-Munich Workshop on Economics of Internet and Network Industries (08/2005); EMNet Conference 2005, Budapest; 2nd Workshop on Empirical IO, University of Innsbruck; University of Vienna; European Economic Association Meeting 2006, Vienna; University of Linz. The usual disclaimer applies.

†University of Vienna, Economics Department at BWZ, Brünner Strasse 72, 1210 Vienna, Austria, email: liliane.karlinger@univie.ac.at

‡European University Institute and Universitat Pompeu Fabra. Address: European University Institute, Economics Department, Villa San Paolo, Via della Piazzuola 43, 50133 Florence, Italy, email: massimo.motta@iue.it

1 Introduction

This paper deals with exclusionary pricing practices, that is anti-competitive pricing behavior by a firm endowed with a “dominant position” (as it is called in the EU), or with “monopoly power” (as it is called in the US). One such practice which has recently received renewed attention is *rebates*, i.e. discounts applicable where a customer exceeds a specified target for sales in a defined period.[1]

In the US, for instance, after a long period where rebates received a very favorable treatment by the courts,[2] the recent *LePage* (2003) decision - in which the Appeal Court reversed an earlier judgment and found 3M guilty of attempted monopolization for having used (bundled) rebates - may signal the willingness of the judges to use lower standards of proof for the finding of anticompetitive rebates.

In the EU, rebates have long been looked at with suspicion by the European Commission (which is the EU Competition Authority) and the Community Courts, which have systematically imposed large fines on dominant firms applying different forms of rebates.[3] But until the recent *Michelin II* judgment, dominant firms were at least allowed to grant pure quantity discounts, that is standardized rebates given to any buyer whose purchases exceed a predetermined number of units; *Michelin II*, instead, has established that even pure quantity discounts are anticompetitive if used by a dominant firm.[4,5]

One of the objectives of this paper is to take seriously the Community Court’s assessment, and study whether rebates, in the form of pure quantity discounts, can have an anticompetitive effect. We study an industry exhibiting network effects, and indeed we find that if rebates are allowed, an incumbent firm having a critical customer size is more likely to exclude a more efficient entrant that can use the same rebate schemes but does not have a customer base yet. Rebates are a form of implicit discrimination, and the incumbent can use them to make more attractive offers to some crucial group of consumers, thereby depriving

[1] There are different types of rebates, or discounts. They can be made contingent on the buyer making most or all of its purchases from the same supplier (”fidelity” or ”loyalty” rebates), on increasing its purchases relative to previous years, or on purchasing certain quantity thresholds specified in absolute terms. It is on this last category of rebates that we focus here.

[2] Under US case law (see e.g. the Virgin v. British Airways (2001) case), loyalty rebates were said to promote competition on the merits as a rule, and it was for the plaintiff to demonstrate their anticompetitive effect. See Kobayashi (2005) for a review of the US case law.

[3] For a review of the EU case law on rebates, see e.g. Gyselen (2003).

[4] Unless they are ’objectively’ justified by scale economies, that is unless the dominant firm can prove that the discount matches savings from transaction costs.

[5] The (almost) per se illegal status of exclusive contracts, rebates and discriminatory prices by dominant firms in the EU, as well as the difference relative to their treatment in the US (at least until recently), has led to a hot debate on the EU policy towards abuse of dominance. See Rey et al. (2005) for a contribution to the debate.

the entrant of the critical mass of consumers it needs (in our model, network externalities imply that consumers will want to consume a network product only if demand has reached a critical threshold).

Now, discrimination (implicit and even more so explicit discrimination) will allow the incumbent to play off the different groups of consumers against each other. This strategic use of price discrimination will exacerbate the coordination problems that buyers face, which in turn makes entry even more difficult for the new rival. Only very efficient entrants will be able to overcome the entry barriers that incumbents can raise in this manner.

To give an example of the type of industry that we have in mind, let us briefly review the *Microsoft Licensing Case* of 1994-95 (Civil Action No. 94-1564). Microsoft markets its PC operating systems (Windows and MS-DOS) primarily through original equipment manufacturers ("OEMs"), which manufacture PCs, and has agreements with virtually all of the major microcomputer OEMs. When discussing the substantial barriers to entry for potential rivals of Microsoft, the Complaint explicitly mentions "the difficulty in convincing OEMs to offer and promote a non-Microsoft PC operating system, particularly one with a small installed base". Moreover, "it would be virtually impossible for a new entrant to achieve commercial success solely through license agreements with small OEMs that are not covered by Microsoft's (...) agreements."

The US Department of Justice alleges that Microsoft designed its pricing policy "to deter OEMs from entering into licensing agreements with competing operating system providers", thereby reinforcing the entry barriers raised by the network effects that are inherent in this industry. In particular, the use of two-part tariffs, with high fixed fees and zero per-copy price, was considered strongly anti-competitive. Interestingly, though, the Final Judgment explicitly allows Microsoft to continue granting "volume discounts" (i.e. rebates), as long as Microsoft would use linear prices rather than two-part tariffs.

Although rebates may have exclusionary effects, it is far from clear that they should be presumed to be welfare-detrimental, even if used by a dominant firm. As John Vickers, then Chairman of the UK Office of Fair Trading, put it:

> These cases about discounts and rebates, on both sides of the Atlantic, illustrate sharply a fundamental dilemma for the competition law treatment of abuse of market power. A firm with market power that offers discount or rebate schemes to dealers is likely to sell more, and its rivals less, than in the absence of the incentives. But that is equally true of low pricing generally." (Vickers, 2005: F252)

Discriminatory pricing has similar contrasting effects. Consider for instance an oligopolistic industry. On the procompetitive side, it allows firms to decrease prices to particular customers, thereby intensifying competition: each firm can be more aggressive in the rival's customer segments while maintaining higher prices with the own customer base, but since each firm will do the same, discriminatory pricing will result in fiercer competition than uniform pricing, and consumers will benefit from it.[6] On the anticompetitive side, though, in asymmetric situations discriminatory pricing may allow a dominant firm to achieve cheaper exclusion of a weaker rival: prices do not need to be decreased for all customers but only for the marginal customers.[7]

This fundamental dilemma between, on the one hand, the efficiency effects (consumers would buy more and pay less) created by rebates and discriminatory pricing and, on the other hand, their potential exclusionary effects (rival firms would be hurt by such practices, and may be driven out of the market), is possibly the main theme of the paper. Indeed, we shall study here different pricing schemes that both an incumbent and a rival firm can adopt, and show that the schemes which - for given market structure - induce a higher level of welfare are also those under which the incumbent is more likely to exclude the rival. More specifically, we show that explicit price discrimination is the pricing scheme with the highest exclusionary potential (and hence the worst welfare outcomes), followed by implicit price discrimination (i.e., rebates, or pure quantity discounts) and then uniform pricing. However, for given market structure (i.e., when we look at equilibria where entry does occur), the welfare ranking is exactly reversed: the more aggressive the pricing scheme the lower the prices (and thus the higher the surplus) at equilibrium. This trade-off between maximizing the entrant's chances to enter and minimizing welfare losses for given market structure, illustrates the difficulties that antitrust agencies and courts find in practice: a tough stance against discounts and other aggressive pricing strategies may well increase the likelihood that monopolies or dominant positions are successfully contested, but may also deprive consumers of the possibility to enjoy lower prices, if entry did occur.

Although it deals with pricing schemes rather than contracts, our paper is closely related to the literature on anticompetitive exclusive dealing. However, in our model exclusion will arise although the incumbent and the rival firm *simultaneously* set prices and all the buyers can purchase at the same time.[8]

Since Segal and Whinston (2000) is probably the closest work to ours, let us be more specific on the differences with their work. Building on Rasmusen et al. (1991), they show the exclusionary potential of exclusive contracts when the incumbent can discriminate on the compensatory offers it makes to buyers. Our study differs from theirs in several respects: (i) in their game the incumbent has

[6]See Thisse and Vives (1998). For a recent survey on discriminatory pricing, see e.g. Stole (2005).

[7]See e.g. Armstrong and Vickers (1993).

[8]Bernheim and Whinston (1998) analyze the possible exclusionary effects of exclusive dealing when firms make simultaneous offers, but in *non-coincident markets*: first, exclusivity is offered to a buyer in a first market; afterwards, offers are made to a buyer in a second market. In their terminology, our paper is looking at *coincident market* effects, which makes our analysis closer to Aghion and Bolton (1985), Rasmusen et al. (1991), Segal and Whinston (2000) and Fumagalli and Motta (2006). All these papers, however, study only exclusive dealing arrangements and assume that the entrant can enter the market (if at all) only *after* the incumbent and the buyers have negotiated an exclusive contract. Relative to the models of entry deterrence through price discrimination, the difference is that in our model the entrant is not prevented from selling to some buyers (in Armstrong and Vickers, 1993, for instance, the entrant can enter in only one segment of the market).

a (first-mover) strategic advantage in that it is allowed to contract with buyers before entry occurs; (ii) if buyers accept the exclusivity offer of the incumbent, they commit to it and cannot renegotiate it even if entry occurs; (iii) buyers are symmetric and only linear pricing is considered. In our paper, instead, (i) the incumbent and the entrant choose price schedules simultaneously, (ii) buyers simply observe prices and decide which firm to buy from (therefore avoiding any problems related to assumptions on commitment and renegotiation); (iii) we explore the role of rebates and quantity discounts in a world where buyers differ in size, and also consider two-part tariffs. Yet, the mechanisms which lead to exclusion in the two papers are very similar: both papers present issues of buyers' miscoordination, and scale economies which are created by fixed costs in their model are created instead by network effects in ours (but in the concluding section, we explain that we obtain the same results by dropping network externalities and assuming that the entrant has still to incur fixed sunk costs).

Our paper is also related to Innes and Sexton (1993, 1994), who also analyze the anticompetitive potential of discriminatory pricing. In their papers, however, they consider a very different contracting environment, strategic variables, and timing of the game. In particular, after the incumbent made its offers, they allow the buyers to contract with the entrant (or to enter themselves), so as to create countervailing power to the incumbent's. Despite all these differences, Innes and Sexton's insight that discrimination helps the incumbent to 'divide and conquer' consumers reappears in our paper, even if we also allow for the entrant to use the same discriminatory tools available to the incumbent, and even if contrary to Innes and Sexton's (1994) finding, in our case a ban on discrimination cannot prevent inefficient outcomes: in our setting, exclusion can arise also under uniform linear pricing.

The paper continues in the following way. Section 2 describes the model, Section 3 solves the model under the assumption that all pricing schemes must be linear. Three cases are analyzed: uniform pricing, explicit (or 3rd degree) price discrimination and implicit (or 2nd degree, or rebates) price discrimination. Section 4 studies some extensions of the model. First, we consider two-part tariffs; then, the possibility that firms subsidize customer's usage, i.e., can charge negative linear prices (in the base model we restrict linear prices to be non-negative); finally, we discuss the case of full (or buyer-specific) discrimination (in the base model we do not allow firms to discriminate across identical buyers). Section 5 concludes the paper.

2 The setup

Consider an industry composed of two firms, the incumbent I, and an entrant E. The incumbent supplies a network good, and has an installed consumer base of size $\beta_I > 0$. (The network good is durable: "old" buyers will continue to consume it but no longer need to buy it.) I incurs constant marginal cost $c_I \in (0, 1)$ for each unit it produces of the network good.

The entrant can supply a competing network good at marginal cost $c_E < c_I$, i.e. it is more efficient than the incumbent. E has not been active in the market so far, that is it has installed base $\beta_E = 0$, but it can start supplying the good any time; in particular, when the game starts it does not have to sink any fixed costs of entry.

The good can be sold to $m+1$ different "new" buyers, indexed by $j = 1, \ldots, m+1$. There are $m \geq 1$ identical small buyers, and 1 large buyer. Goods acquired by one buyer cannot be resold to another buyer, but they can be disposed of at no cost by the buyer who bought them (in case the latter cannot consume them). Side payments of any kind between buyers are ruled out. Define firm i's network size s_i (where $i = I, E$) as

$$s_i = \beta_i + q_i^1 + \ldots + q_i^{m+1} \tag{1}$$

i.e. the firm's installed base plus its total sales to all "new" buyers.

To simplify the analysis, we assume for now that demands are inelastic. (Section 4.2 presents the results for elastic (linear) demand functions.) The large buyer's demand for firm i's network good at unit price p_i^l is given by:[9]

$$q_i^l\left(p_i^l\right) = \begin{cases} 1-k & \begin{array}{c} \text{if } s_i \geq \bar{s} \text{ and } p_i^l \leq 1 \\ \text{or } s_i < \bar{s} \text{ and } p_i^l < 0 \end{array} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

while the typical small buyer's demand for firm i's network good at unit price p_i^s is

$$q_i^s\left(p_i^s\right) = \begin{cases} \frac{k}{m} & \begin{array}{c} \text{if } s_i \geq \bar{s} \text{ and } p_i^s \leq 1 \\ \text{or } s_i < \bar{s} \text{ and } p_i^s < 0 \end{array} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The parameter $k \in (0,1)$ is an indicator of the relative weight of the small buyers in total market size: $1-k$ measures the large buyer's market share, while k measures the market share of the group of small buyers. Assume that $1-k > k/m$, so that the large buyer's demand is always larger than a small buyer's demand (provided they both demand strictly positive quantities). Note that the assumption $1-k > k/m$ implies an upper bound on k, namely

$$k < \frac{m}{m+1} \in \left[\frac{1}{2}, 1\right) \tag{4}$$

and that total market size is fixed at 1: $m(k/m) + (1-k) = 1$.

Next, define buyer's net consumer surplus as gross consumer surplus minus

[9] These demand functions apply for general (positive or negative) p_i^l. In the base model we restrict prices to be non-negative. Section 4 considers the case where prices can be negative.

total expenditure:

$$CS_i^l\left(p_i^l\right) = \begin{cases} q_i^l(1-p_i^l) & \text{if } s_i \geq \bar{s} \text{ and } l \text{ buys } q_i^l \leq 1-k \\ (1-k) - q_i^l p_i^l & \text{if } s_i \geq \bar{s} \text{ and } l \text{ buys } q_i^l > 1-k \\ -q_i^l p_i^l & \text{if } s_i < \bar{s} \text{ and } l \text{ buys } q_i^l \leq 1-k \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$CS_i^s\left(p_i^s\right) = \begin{cases} q_i^s(1-p_i^s) & \text{if } s_i \geq \bar{s} \text{ and } s \text{ buys } q_i^s \leq \frac{k}{m} \\ \frac{k}{m} - q_i^s p_i^s & \text{if } s_i \geq \bar{s} \text{ and } s \text{ buys } q_i^s > \frac{k}{m} \\ -q_i^s p_i^s & \text{if } s_i < \bar{s} \text{ and } s \text{ buys } q_i^s \leq \frac{k}{m} \\ 0 & \text{otherwise} \end{cases}$$

The demand functions defined above can be derived from these expressions of net consumer surplus.

Since both types of buyers have the same prohibitive price $\bar{p} = 1$, a monopolist who could charge discriminatory linear prices would set a uniform unit price $p_i^m = 1$.

If firm i's network size s_i is below the threshold level $\bar{s}$, no buyer (neither large nor small) would want to buy firm i's good.[10] We assume that

$$\beta_I \geq \bar{s} \tag{6}$$

i.e. the incumbent has already reached the minimum size, while the entrant's installed base is $\beta_E = 0$. In order to operate successfully, the entrant will have to attract enough buyers to reach $\bar{s}$.[11]

Key Assumption: Neither demand of the large buyer alone, nor demand of all small buyers taken together, is sufficient for the entrant to reach the minimum size:

$$\bar{s} > \max\ \{1-k, k\}. \tag{7}$$

In other words, in order to reach the minimum size, the entrant has to serve the large buyer plus at least one (and possibly more than one) small buyer.

Note that only units which are actually consumed by a buyer count towards firm i's network size.

We also assume that the threshold level $\bar{s}$ is such that if the entrant sells to all $m+1$ buyers, then it will reach the minimum size: $\bar{s} \leq 1$.

This, together with the assumption $c_E < c_I$, implies that the social planner would want the entrant (and not the incumbent) to serve all buyers.

[10] The assumption that a buyer's demand is positive only if the network in question reaches the threshold size $\bar{s}$ is designed to capture in an admittedly simple way the presence of network effects. Rather than assuming that the utility of a consumer increases continuously with network size, we assume a discontinuous formulation; this has the advantage that the old generation of buyers can be safely ignored when studying welfare effects: since we shall assume that they have already attained the highest level of utility, new buyers' decisions will never affect old buyers' utility.

[11] Note that if the entrant manages to reach the minimum size $\bar{s}$, then consumers will consider I's and E's networks as being of homogenous quality, even if $s_I \neq s_E$.

The game. Play occurs in the following sequence: At time $t = 0$, the incumbent and the entrant simultaneously announce their prices, which will be binding in $t = 1$. At time $t = 1$, each of the $m + 1$ buyers decides whether to patronize the incumbent or the entrant.[12]

As for the prices that firms can offer in $t = 0$, in the base model (Section 3) we will restrict attention to linear pricing schemes, but we consider three different possibilities: (1) uniform prices (Section 3.1); (2) explicit (or third-degree) price discrimination (Section 3.2); and (3) the case of central interest, that is implicit (or second-degree) price discrimination, i.e. the case of standardized quantity discounts or "rebates" (Section 3.3).

Section 4 will show that the main results are robust to changes in the assumptions we make in the base model on prices. There, we shall analyze the cases where prices can be negative, where firms can set two-part tariffs rather than linear prices, and where full price discrimination is allowed, that is firms can make buyer-specific offers (in the base model, we do not allow firms to discriminate among buyers of the same type).

We also assume that offers are observable to everyone, e.g. because they have to be posted publicly. Then, when the buyers have to decide which firm to buy from, the firms' offers will be common knowledge.

3 Equilibrium solutions, under different price regimes

In this Section, we assume that firms set linear (and non-negative) prices, and we consider three different price regimes: uniform prices; explicit (3rd degree) price discrimination; implicit (2nd degree) price discrimination (i.e., rebates).

3.1 Uniform pricing

Assume that firms can only use uniform linear prices, p_i with $i = I, E$. In line with Segal and Whinston (2000), we find that our game has two types of pure-strategy Nash equilibria: one where all buyers (or sufficiently many) buy from the entrant, and one where all buyers buy from the incumbent.

The following proposition illustrates these two types of equilibria.

Proposition 1 *(equilibria under uniform linear prices) If firms can only use uniform flat prices, the following two pure-strategy Nash equilibria exist under the continuation equilibria as specified (after eliminating all equilibria where firms play weakly dominated strategies):*

(i) ***Entry equilibrium:*** *E sets $p_E = c_I$, I sets $p_I = c_I$, and all buyers, after observing $p_E \leq p_I$, buy from E.*

(ii) ***Miscoordination equilibrium:*** *I sets $p_I = p_I^m = 1$, E sets $p_E = p_E^m = 1$, and all buyers, after observing $p_I - p_E \leq 1$ buy from I.*

[12] For simplicity, we restrict attention to the case where a buyer can only buy from one of the two firms, but not from both of them simultaneously.

Proof: see Appendix A

Which type of equilibrium will eventually be played depends on the underlying continuation equilibria, i.e. on how buyers coordinate their purchasing decisions after observing the firms' offers:[13] If a buyer can rely on all other buyers patronizing E whenever E's offer is at least as good as I's, then it is perfectly rational for this buyer to buy from E as well. This, in turn, corresponds exactly to what all other buyers expected him to do, and so confirms the rationality of their own supplier choice. Under such a continuation equilibrium, the entry equilibrium of Proposition 1 (i) will arise.

If instead each buyer suspects all other buyers to patronize I even when I's price is strictly higher than E's, then no buyer will want to buy from E: Recall that no individual buyer's demand is ever sufficient for E's network to reach the minimum size $\bar{s}$. Then, being the only buyer to buy from E means ending up with a good that has zero value to that buyer (no matter how cheap it is). Hence, as long as buying from I still gives non-negative surplus, each buyer will want to buy from I, which then confirms all other buyers in their decision to buy from I as well.

Note also that under the miscoordination equilibrium, the incumbent can charge the monopoly price without losing the buyers to the entrant.[14] These equilibria are particularly troublesome, because they show that the highest possible price can persist even in the presence of an efficient competitor.

The equilibria characterized in Proposition 1 represent extreme cases, in the sense that the underlying continuation equilibria are the most favorable ones for the firm that serves the buyers in equilibrium. *These equilibria are by no means the only equilibria that can arise in our game.*

For instance, there are other equilibria where all buyers do miscoordinate on the incumbent, but the latter can at most charge some price $\tilde{p}_I < p_I^m = 1$. Such an equilibrium can be sustained by continuation equilibria where buyers buy from I as long as $p_I - p_E \leq \tilde{p}_I$, but would switch to E if the price difference exceeded $\tilde{p}_I$. Likewise, there are entry equilibria where the entrant must charge a strictly lower price than c_I to induce buyers to coordinate on E. *For the rest of the paper, we will focus on those continuation equilibria which are the most profitable ones for the firm that eventually serves the buyers.*

Finally, there can also be equilibria where both I and E offer the exact same price, and a critical number of buyers patronize E (so that E reaches the minimum size), while the remaining buyers buy from I. These equilibria can only be sustained by very specific continuation equilibria, and we will not consider them in the following sections of this paper.

[13] where "coordination" refers to the collective behavior under *individual* decision making; we do not allow buyers to meet in $t = 1$ and make a joint decision on which firm to patronize.

[14] In this situation, the entrant is indifferent among all prices $p_E \geq 0$ it could charge, and might as well offer the monopoly price, which weakly dominates all other possible equilibrium prices.

3.2 Explicit (3rd degree) discrimination

In this section, we first analyze miscoordination equilibria and then entry equilibria.

3.2.1 Miscoordination equilibria

Proposition 1 gives us the equilibrium for the case of uniform linear pricing. Assume now that the two firms can do 3rd degree (or explicit) discrimination (this is partial discrimination: firms cannot offer different prices to buyers of the same size), so each of them can offer a different price to the small and to the large buyer.

With respect to the uniform pricing case, *nothing changes in the miscoordination equilibria*, the most profitable of which is for the Incumbent still the one where $p_I^s = p_I^l = p_I^m = 1$,[15] while the entrant sets $p_E^m = 1$ and all buyers buy from I. Clearly, the incumbent would have no incentive to deviate from this solution. No buyer would deviate either: if any of them decided to accept the lower price offered by the entrant given that all others buy from the incumbent, he would have zero surplus and would reduce his utility.

Proposition 2 *Under the appropriate continuation equilibria, the miscoordination equilibrium where I charges $p_I^s = p_I^l = p_I^m = 1$, E sets $p_E^s = p_E^l = p_E^m = 1$, and all buyers buy from I, will exist for all parameter values even if firms can price-discriminate by buyer type.*

Proof: Consider the following continuation equilibrium: Following offers where either $p_I^s - p_E^s \leq 1$ or $p_I^l - p_E^l \leq 1$, buyers buy from I. Then, even if the entrant can charge different prices to both groups (where both prices may be strictly lower than I's prices), no single buyer will have an incentive to switch to the entrant as long as he expects all other buyers to buy from I: E's network cannot reach the minimum size with only one buyer, so its good gives zero utility, and as long as E charges a non-negative price for it, I's offer will (weakly) dominate E's offer. The rest of the proof is analogous to the proof of Proposition 1.□

Thus, the possibility to price discriminate does not allow the entrant to solve the miscoordination problem. Hence, miscoordination equilibria will continue to exist even if we allow for explicit price discrimination.

3.2.2 Entry equilibria

For entry equilibria, things change. To fix ideas, start with the candidate entry equilibrium where both firms charge c_I and all buyers buy from the entrant (we have seen that this is an entry equilibrium in the uniform linear pricing case). This equilibrium can be disrupted by the incumbent setting a price $c_I - \epsilon$ to one

[15] Note that in our model the monopoly price charged by a firm under explicit discrimination will be the same for all buyers. This is clearly a special feature of the model, which simplifies the analysis without losing much insight.

category of buyers and the monopoly price to the other category: the loss made on the former would be outweighed by the profits made on the latter. Indeed, under this deviation the former category strictly prefers to buy from I, which is then able to prevent entry (and the latter category would then prefer to buy from I rather than from the entrant, since they would derive zero utility from buying from E).

Therefore, an entry equilibrium can exist only if the entrant is able to match the best offer that the incumbent can make both to the small buyers and to the large buyer. Let us see it formally.

First, the best offer the incumbent can make to the small buyers is given by the solution of the following program:

$$\begin{array}{l} \max_{p_I^s, p_I^l} CS^s(p_I^s) = \frac{K(1-p_I^s)}{m}, \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \in [0,1], p_I^l \in [0,1], \end{array} \tag{8}$$

where (i) is the profitability constraint of the incumbent.

Next, note that the best offer the incumbent can make to the large buyer is given by the solution of:

$$\begin{array}{l} \max_{p_I^s, p_I^l} CS^l(p_I^l) = (1-K)(1-p_I^l), \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \in [0,1], p_I^l \in [0,1] \end{array} \tag{9}$$

We see that the best offer the incumbent can make to the small buyers is to set $p_I^l = p_I^M = 1$ and lower p_I^s as much as possible while still satisfying the profitability constraint (i); likewise, the best offer to the large buyer is obtained by setting $p_I^s = p_I^M$ and lowering p_I^l as much as allowed by (i).

The offer (p_I^s, p_I^M) to the small buyers is feasible as long as the incumbent breaks even (i.e., constraint (i) must be satisfied):

$$m\frac{k}{m}(-c_I + p_I^s) + (1-k)(1-c_I) \geq 0, \tag{10}$$

The offer (p_I^M, p_I^l) to the large buyer is feasible as long as:

$$(1-k)(-c_I + p_I^l) + m\frac{k}{m}(1-c_I) \geq 0. \tag{11}$$

Call $\widehat{p}_I^s$ and $\widehat{p}_I^l$ respectively the prices that solve the equations associated with inequalities (10) and (11) above. The best possible deviations for the incumbent are identified by respectively:

$$p_I^s = \max(\widehat{p}_I^s, 0) \quad \text{and} \quad p_I^l = \max(\widehat{p}_I^l, 0),$$

since we limit attention to non-negative prices (see below for the case where prices can be negative).

The entrant can match the incumbent's deviations if it is able to offer more surplus to the buyers, while still making profits. In other words, the entrant will be able to profitably enter at equilibrium if it can set prices (p_E^s, p_E^l) such that:

$$CS^s(p_E^s) \geq CS^s(p_I^s) \tag{12}$$

$$CS^l(p_E^l) \geq CS^l(p_I^l) \tag{13}$$

$$\pi_E(p_E^s, p_E^l) \geq 0. \tag{14}$$

After substitution, conditions (12) and (13) become:

$$\frac{k}{m}(1 - p_E^s) \geq \frac{k}{m}(1 - p_I^s) \tag{15}$$

$$(1 - k)(1 - p_E^l) \geq (1 - k)(1 - p_I^l) \tag{16}$$

Optimality requires the entrant offering the minimum necessary to satisfy these conditions, so at equilibrium they will be binding:

$$p_E^s = p_I^s;\ p_E^l = p_I^l,$$

from which condition (14) becomes:

$$k(p_E^s - c_E) + (1 - k)(p_E^l - c_E) \geq 0,$$

or:

$$\pi_E(p_I^s, p_I^l) : k(p_I^s - c_E) + (1 - k)(p_I^l - c_E) \geq 0. \tag{17}$$

We therefore have to find $\left(p_I^s, p_I^l\right)$. By solving the equalities associated with (10) and (11) above, we obtain:

$$\widehat{p}_I^s = \frac{c_I - (1 - k)}{k}; \quad \widehat{p}_I^l = \frac{c_I - k}{1 - k}.$$

Note that $\widehat{p}_I^s < c_I$ and $\widehat{p}_I^l < c_I$; also:

$$\widehat{p}_I^s \geq 0 \quad \text{if} \quad c_I \geq 1-k; \text{ and } \widehat{p}_I^l \geq 0 \quad \text{if} \quad c_I \geq k.$$

Therefore, the incumbent's optimal offer will be:

$$p_I^s = \begin{cases} \widehat{p}_I^s & \text{if } c_I \geq 1-k \\ 0 & \text{if } c_I < 1-k \end{cases} \quad p_I^l = \begin{cases} \widehat{p}_I^l & \text{if } c_I \geq k \\ 0 & \text{if } c_I < k \end{cases}$$

This identifies four regions, and for each of them we have to verify whether (cond 3) holds or not:

$$\begin{array}{rcl} \text{if } c_I & \in & [1-k, k] \text{ and } k \geq 1/2\text{:} \quad \pi_E(\widehat{p}_I^s, 0) \geq 0 \\ \text{if } c_I & \in & [k, 1-k] \text{ and } k < 1/2\text{:} \quad \pi_E(0, \widehat{p}_I^l) \geq 0 \\ \text{if } c_I < \min & & \{k, 1-k\}: \pi_E(0,0) \geq 0 \\ \textit{else} & : & \pi_E(\widehat{p}_I^s, \widehat{p}_I^l) \geq 0 \end{array}$$

After replacing, we can then find that:
(1) $\pi_E(\widehat{p}_I^s, \widehat{p}_I^l) = k\left(\frac{c_I-(1-k)}{k} - c_E\right) + (1-k)\left(\frac{c_I-k}{1-k} - c_E\right) \geq 0$,
which is satisfied for:

$$c_I \geq \frac{1+c_E}{2} \equiv \overline{c}_{I1}$$

(2) $\pi_E(0, \widehat{p}_I^l) = -c_E k + (1-k)\left(\frac{c_I-k}{1-k} - c_E\right) \geq 0$
which holds for:

$$c_I \geq k + c_E \equiv \overline{c}_{I2}$$

(3) $\pi_E(\widehat{p}_I^s, 0) = k\left(\frac{c_I-(1-k)}{k} - c_E\right) - c_E(1-k) \geq 0$
which holds for:

$$c_I \geq 1 + c_E - k \equiv \overline{c}_{I3}.$$

(4) $\pi_E(0,0) = -c_E \geq 0$,
which never holds, apart from the knife-edge case where $c_E = 0$.[16]

Conclusions Figure 1 illustrates the results of the analysis of entry equilibria (recall that miscoordination equilibria exist for all parameter values). For

[16] Since prices cannot go below zero in this basic model, the best that the incumbent can offer to buyers is to give them the good for free; but when $c_E = 0$, the entrant could match that offer without making losses, and entry equilibria would always exist. Clearly, though, this is as very special case.

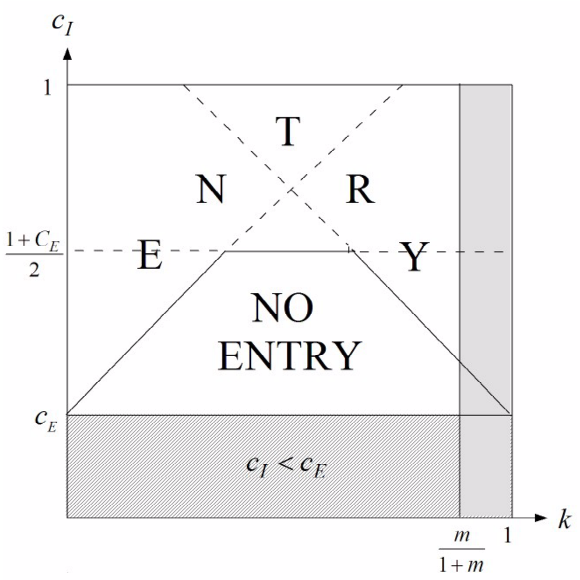

Figure 1: Regions where entry equilibria exist and do not exist under explicit price discrimination (the grey areas are outside of the parameter space)

given k, the figure shows that the larger c_I with respect to c_E, the more likely for entry to be an equilibrium of the game. The effect of k on equilibria outcomes is slightly more complex. In particular, entry is more likely at very low levels and very high levels of k. This is because the entry equilibrium may be disrupted by the incumbent's offers to the buyers; such offers are made by discriminating among the buyers, for instance by extracting rents from small buyers and offering surplus to the large (or vice versa). If for instance k is very small, the incumbent is not able to extract much surplus from the small buyers and accordingly the best offer to the large buyer will not be very attractive.[17] Instead, it could extract a lot of surplus from the large buyer and could in principle make a princely offer to the small buyers. However, small buyers account for a small proportion of demand (k very small), and the large rent earned from the large buyer can only be passed on to small buyers through price cuts on each unit they buy. But since prices are restricted to be non-negative here, the incumbent will soon hit the $p_I^s = 0$ constraint when k is small. Thus, the incumbent can only transfer a small part of the rent from large to small buyers, and so the entrant will find it easier to match the incumbent's best offers, hence entry equilibria will exist. The same argument can be used symmetrically to explain results for the case where k is close to 1. Of course, we shall see below that when prices are not restricted to be non-negative, these effects will disappear, and k will affect

[17] This can be seen by looking at the profit constraint (11). As $k \to 0$, the best offer that I can sustain is $p_I^l = c_I$.

results monotonically.

To sum up:

- Exclusionary equilibria: they always exist and take exactly the same form as under the case of uniform linear pricing.[18]
- Entry equilibria: they exist only if c_I is high enough. When they exist, note that the highest sustainable equilibrium prices are always below c_I (which is the highest sustainable equilibrium price under uniform linear pricing).
- With respect to uniform pricing, thus: price discrimination (i.e. a more aggressive pricing strategy): (a) on the one hand, makes exclusion more likely; (b) on the other hand, for given market structure, results in lower prices.[19]

3.3 Implicit (2nd degree) discrimination (or rebates)

Let us now consider the case where firms cannot condition their offers directly on the type of buyer (large or small), but have to make uniform offers to both types which may only depend on the quantity bought by buyer $j = 1, \ldots, m+1$:

$$T_i(q_i^j) = \begin{cases} p_{i,1}q_i^j & \text{if } q_i^j \leq \bar{q}_i \\ p_{i,2}q_i^j & \text{if } q_i^j > \bar{q}_i \end{cases} \tag{18}$$

Each buyer can now choose his tariff from this price menu by buying either below the sales target $\bar{q}_i$ or above it.

It is well-known that such quantity discounts or rebates, when applied to buyers who differ in size, will be a tool of (de facto) discrimination, even if the schemes as such are uniform. But to achieve discrimination, the tariffs have to be set in a way that induces buyers to self-select into the right category, with small buyers voluntarily buying below target, and the large buyer choosing to buy above it.

First, consider the large buyer $j = l$, and suppose his demand at price $p_{i,2}$ is above the threshold, i.e. $q_i^l\left(p_{i,2}\right) = 1 - k > \bar{q}_i$ (this will be the only relevant case). Then, the large buyer can either buy $q_i^l\left(p_{i,2}\right) = 1 - k$, which yields total surplus $CS_i^l\left(p_{i,2}\right) = (1-k)(1-p_{i,2})$, or he can buy at the threshold $\bar{q}_i$, i.e. $q_i^l = \bar{q}_i$ at price $p_{i,1}$, in which case his net consumer surplus is

$$CS^{l,net}\left(p_{i,1}, \bar{q}_i\right) = \begin{cases} (1-p_{i,1})\bar{q}_i & \text{if } s_i \geq \bar{s} \text{ and } \bar{q}_i < 1-k \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

[18] This is an artifact of the model. In more general models, monopoly prices will be different at the explicit discrimination equilibrium. However, the result that miscoordination equilibria will always exist and that at one of those equilibria the monopolist is able to charge monopoly prices would still be valid.

[19] Note that in this example, prices are lower only in the entry equilibria - as the exclusionary equilibrium is supported by the same equilibrium prices. We may also want to analyse in an extension a demand function where 3rd degree discrimination calls for different prices.

Next, consider a typical small buyer $j = s$, and suppose his demand at price $p_{i,2}$ is below the threshold, i.e. $q_i^s(p_{i,2}) = k/m < \bar{q}_i$. Then, a small buyer may either buy $q_i^s(p_{i,1})$, which yields total surplus $CS_i^s(p_{i,1}) = (1 - p_{i,1})(k/m)$, or he can buy the sales target $\bar{q}_i$ at price $p_{i,2}$ (i.e. a quantity which exceeds his actual demand at this price).

If $\bar{q}_i > \frac{k}{m}$, i.e. if the sales target is above the largest quantity he can consume, $q_i^s(p_i \leq 1) = k/m$, then the excess units, $\bar{q}_i - k/m$, can be disposed of at no cost.[20] Define the small buyer's net consumer surplus of buying $\bar{q}_i$ units as

$$CS^{s,net}(p_{i,2}, \bar{q}_i) = \begin{cases} \frac{K}{m} - p_{i,2}\bar{q}_i & \text{if } s_i \geq \bar{s} \text{ and } \bar{q}_i > \frac{K}{m} \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

We say that firm i's offer satisfies the *"self-selection condition"* if the large buyer prefers to buy above the threshold, and the small buyers prefer to buy below the threshold, i.e. if

$$\begin{aligned} CS_i^l(p_{i,2}) &\geq CS^{l,net}(p_{i,1}, \bar{q}_i) \\ \text{and } CS_i^s(p_{i,1}) &\geq CS^{s,net}(p_{i,2}, \bar{q}_i) \end{aligned} \tag{21}$$

For any offer that satisfies the self-selection condition, denote $(p_{i,1})$ by (p_i^s), and $(p_{i,2})$ by (p_i^l), for $i = I, E$.

3.3.1 Miscoordination equilibria

Lemma 3 *(miscoordination under rebates) Let firms use rebate tariffs as defined in (18). Under the appropriate continuation equilibria, the miscoordination equilibrium exists for all parameter values, and the highest (monopoly) prices can be sustained at equilibrium (i.e., same result as under uniform pricing and explicit discrimination)*

Proof: analogous proof as for the above cases.

3.3.2 Entry equilibria

In order to find the conditions under which entry equilibria exist, we proceed in three steps.

First, we look for the best possible offer p_I^s that the incumbent can make to the small buyers; second, we look for the best possible offer p_I^l that the incumbent can make to the large buyer; third, we see whether the entrant is able to make a profitable offer (p_E^s,p_E^l) to the small and the large buyer such that they are at least as well off as if they bought from the incumbent.[21]

[20] Recall that we excluded reselling of units between buyers (while allowing for free disposal), so the only thing a small buyer can do with units he cannot consume is to throw them away.

[21] Note that - unlike the case of explicit discrimination - *in principle* for the entrant it may not be enough to simply match the incumbent's offer, because at prices $p_E^s = p_I^s$, and p_E^l=p_I^l it may be that the self-selection constraint is not met. In other words, the self-selection condition

The incumbent's best offer to the small buyer, $\left(\widetilde{p}_I^s, \overline{p}_I^l, \overline{q}_I^s\right)$, solves Program (22):

$$\begin{array}{l} \max_{p_I^s, p_I^l} CS^s(p_I^s) = (1-p_I^s)(k/m), \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \in [0,1]\,, p_I^l \in [0,1] \\ \text{(iii) } CS^l\left(p_I^l\right) = (1-p_I^l)(1-k) \geq \overline{q}_I^s\left(1-p_I^s\right), \\ \text{(iv) } CS^s\left(p_I^s\right) \geq \overline{q}_I^s\left(1-p_I^l\right) \end{array} \tag{22}$$

where $\overline{q}_I^s = \frac{k}{m}$ and $k \in \left[0, \frac{m}{m+1}\right]$.

Remark 1 *Constraint (iv) will never be binding at any solution to Program (22).*

P roof. any such solution, either condition (iii) is *not* binding: then, $p_I^l = 1$, and $p_I^s = 0$ or some $p_I^s \in (0,1)$ that solves the break-even condition (i) with equality (these are the two solutions that can arise under *explicit* discrimination; both are possible under implicit PD as well). If instead condition (iii) is *binding*, then the incumbent will charge the p_I^l that solves (iii) given p_I^s (giving rise to two solutions specific to implicit PD). Now, for any value of p_I^s, the p_I^l that solves (iii) must be strictly larger than p_I^s (intuitively, the large buyer will be willing to pay *more* per unit if the quantity constraint $\overline{q}_I^s$ is lifted). Thus, we have that $\widetilde{p}_I^s < \overline{p}_I^l$ under all solutions to Program (22), and so the left-hand side of constraint (iv) is always larger than the right-hand side (both sides refer to the same quantity, $\overline{q}_I^s \equiv q^s\left(\widetilde{p}_I^s\right) = k/m$, but in one case the small buyer only pays $\widetilde{p}_I^s$ for this quantity, and in the other case he would have to pay $\overline{p}_I^l$). ■

Remark 2 *This rebate scheme may appear as somewhat unorthodox, since buyers are "rewarded" for buying little (and "penalized" for buying a lot). But note that: (1) this offer will never be made in equilibrium, it's just a hypothetical off-equilibrium offer. (2) A way to rationalize this scheme is to assume that each buyer is only allowed one transaction. This would rule out the possibility that a large buyer makes "multiple small purchases" so as to buy a large amount of units at the lower prices. Presumably, important transaction costs may be invoked to justify this assumption, which in a way is nothing else than the counterpart of the assumption that a small buyer cannot buy a large quantity and then resell it to others. In both cases, it is arbitrage which is prevented.*

---

on the one hand affects the incumbent by obliging it to set (weakly) higher prices but on the other hand also affects the entrant by also obliging it to set (weakly) higher prices. However, we shall prove below that that at prices $p_E^s = p_I^s$, and $p_E^l = p_I^l$ the self-selection constraints are always met.

The incumbent's best offer to the large buyer $(\overline{p}_I^s, \widetilde{p}_I^l, \overline{q}_I^l)$ solves Program (23):

$$
\begin{array}{l}
\max_{p_I^s, p_I^l} CS^l(p_I^l) = (1-k)\,(1-p_I^l), \quad \text{s.to:} \\
\text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\
\text{(ii) } p_I^s \in [0,1]\,, p_I^l \in [0,1] \\
\text{(iii) } CS^l\left(p_I^l\right) \geq CS^l\left(p_I^s\right), \\
\text{(iv) } CS_I^s\left(p_I^s\right) \geq \frac{k}{m} - p_I^l \overline{q}_I^l,
\end{array}
\tag{23}
$$

where $\overline{q}_I^l \equiv q^l\left(p_I^l\right) = (1-k)$ and $k \in \left[0, \frac{m}{m+1}\right]$.

Remark 3 *Among the two solutions under explicit discrimination, only the "second-best" solution (where the break-even-constraint is binding, so that $p_I^l = \widehat{p}_I^l > 0$) can also arise under implicit discrimination. The "first-best" solution, $p_I^l = 0$, always violates the self-sorting condition (iv), so it is not feasible under implicit discrimination. And of course, we have all the additional solutions that are specific to implicit PD (with binding self-sorting conditions).*

Remark 4 *Note that the two quantity thresholds $\overline{q}_I^s$ and $\overline{q}_I^l$ are indexed by s and l to make it clear to which of the two programs they belong.*

Remark 5 *Under any solution to Program (23), we have that $\overline{p}_I^s > \widetilde{p}_I^l$ (the reason being analogous to Program (22)).*

Remark 6 *It follows immediately that constraint (iii) will never be binding at any solution to Program (23).*

Let us now characterize the equilibrium solutions under rebates. As indicated above, we first have to identify the incumbent's best offers.

Incumbent's best offer to small buyers To find the best possible offer that the incumbent can make to the small buyers we have to solve program 22. Note first that the incumbent would like to set p_I^s as low as possible, while charging the highest possible price to the large buyer. However, I can no longer set $p_I^l = 1$ (as under explicit discrimination), because at this price the large buyer is left with zero surplus, and so his self-sorting condition can never be satisfied (he would prefer to buy even a very small quantity, k/m, at a price $p_I^s < 1$, than a large quantity $1-k$ at the prohibitive price). Hence, the large buyer's self-selection constraint will always be binding under any solution of program 22:

$$p_I^l = 1 - \frac{k(1-p_I^s)}{m(1-k)}$$

In order to satisfy the profitability constraint,the following must hold as well:

$$p_I^l \geq \frac{c_I - kp_I^s}{1-k}$$

This gives us the following solutions of the program:

$$\begin{aligned}
\widetilde{p}_I^s &= 1 - \frac{m(1-c_I)}{k(m+1)}; \quad \overline{p}_I^l = 1 - \frac{(1-c_I)}{(1-k)(m+1)}, \quad \text{if } c_I \geq 1-k-k/m \\
\widetilde{p}_I^s &= 0; \quad \overline{p}_I^l = 1 - \frac{k}{m(1-k)}, \quad \text{if } c_I < 1-k-k/m.
\end{aligned}$$

Note that there exists a region of the parameter space ($1-k-k/m < c_I < 1-k$) where under explicit discrimination the incumbent could offer a zero price to the small buyers whereas under rebates it cannot, and that $\widetilde{p}_I^s > \widehat{p}_I^s$, i.e. the best offer the incumbent can make under implicit discrimination is always less interesting than under explicit discrimination (in the limit case where $m \to \infty$, the self-selection constraint plays no role, because the large buyer will never want to behave like a small buyer whose demand is infinitely small, so that the implicit and explicit discrimination cases collapse to the same: $\lim_{m\to\infty} \widetilde{p}_I^s = \widehat{p}_I^s$, for $c_I \geq 1-k$ and $\lim_{m\to\infty} \widetilde{p}_I^s = 0$ for $c_I < 1-k$).

Incumbent's best offer to large buyer To find the best offer that the incumbent can make to the large buyer we solve program 23. The incumbent would like to set p_I^l as low as possible. (But recall from the remarks above that $p_I^l = 0$ can never satisfy the self-selection constraint of the small buyers, who would always prefer to buy a quantity $(1-k)$ at zero price - and throw away $1-k-k/m$ units - than a smaller quantity k/m at positive price.) In order to satisfy the profitability constraint and the self-selection constraints respectively the following must hold:

$$p_I^s \geq \frac{c_I - (1-k)p_I^l}{k},$$

and respectively:

$$p_I^s \leq \frac{m(1-k)p_I^l}{k}.$$

This gives us the following solutions of the program:

$$\begin{aligned}
\widetilde{p}_I^l &= \frac{c_I}{(1-k)(m+1)}; \quad \overline{p}_I^s = \frac{mc_I}{k(m+1)}, \quad \text{if } c_I < \frac{k(1+m)}{m} \\
\widetilde{p}_I^l &= \frac{c_I - k}{(1-k)}; \quad \overline{p}_I^s = 1, \quad \text{if } c_I \geq \frac{k(1+m)}{m}.
\end{aligned}$$

Note that there exists a region of the parameter space ($c_I < k$) where under explicit discrimination the incumbent could offer a zero price whereas under rebates it cannot (it offers $\widetilde{p}^l_I = \frac{c_I}{(1-k)(m+1)}$), and that $\widetilde{p}^l_I > \widehat{p}^l_I$, i.e. the best offer the incumbent can make under implicit discrimination is always less interesting than under explicit discrimination (in the limit case where $m \to \infty$, the self-selection constraint plays no role, because an infinitely small buyer will never want to behave like the large buyer, so that the implicit and explicit discrimination cases collapse to the same: $\lim_{m\to\infty} \widetilde{p}^l_I = \widehat{p}^l_I$ for $c_I > k$, and $\lim_{m\to\infty} \widetilde{p}^l_I = 0$ for $c_I \leq k$).

The entrant's profitability constraint We can now summarize the incumbent's optimal offers as follows:

$$\widetilde{p}^s_I = \begin{cases} 1 - \frac{m(1-c_I)}{k(m+1)} & \text{if } c_I \geq 1-k-k/m. \\ 0 & \text{if } c_I < 1-k-k/m. \end{cases} \quad \widetilde{p}^l_I = \begin{cases} \frac{c_I}{(1-k)(m+1)} & \text{if } c_I \geq \frac{k(1+m)}{m} \\ \frac{c_I-k}{(1-k)} & \text{if } c_I < \frac{k(1+m)}{m} \end{cases}$$

This identifies four regions, and for each of them we have to verify whether (cond 3) holds or not:

(iv) if $c_I \in \left[1-k-k/m, \frac{k(1+m)}{m}\right]$ and $k \geq \frac{m}{2(1+m)}$: $\pi_E(1 - \frac{m(1-c_I)}{k(m+1)}, \frac{c_I}{(1-k)(m+1)}) \geq 0$
(ii) if $c_I \in \left[\frac{k(1+m)}{m}, 1-k-k/m\right]$ and $k < \frac{m}{2(1+m)}$: $\pi_E(0, \frac{c_I-k}{(1-k)}) \geq 0$
(iii) if $c_I < \min\left\{\frac{k(1+m)}{m}, 1-k-k/m\right\}$: $\pi_E(0, \frac{c_I}{(1-k)(m+1)}) \geq 0$
(i) else:$\pi_E(1 - \frac{m(1-c_I)}{k(m+1)}, \frac{c_I-k}{(1-k)}) \geq 0$

After replacing, we can then find that:
(i) $\pi_E(1 - \frac{m(1-c_I)}{k(m+1)}, \frac{c_I-k}{(1-k)}) \geq 0$ is satisfied for:

$$c_I \geq \frac{m+(1+m)c_E}{1+2m}$$

(2) $\pi_E(0, \frac{c_I-k}{(1-k)}) \geq 0$ holds for:

$$c_I \geq k + c_E$$

(3) $\pi_E(0, \frac{c_I}{(1-k)(m+1)}) \geq 0$ holds for:

$$c_I \geq c_E(1+m)$$

(iv) $\pi_E(1 - \frac{m(1-c_I)}{k(m+1)}, \frac{c_I}{(1-k)(m+1)}) \geq 0$ holds for:

$$c_I \geq \frac{m}{1+m} + c_E - k \equiv \overline{c}_{I3}.$$

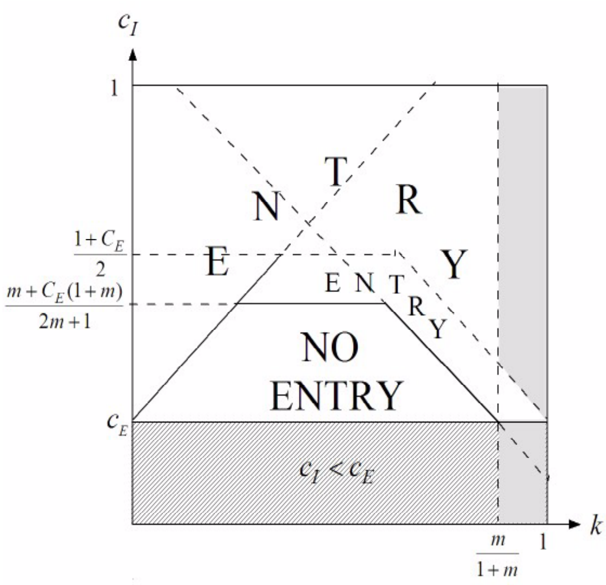

Figure 2: Regions where entry equilibria exist and do not exist under rebates (i.e. implicit price discrimination), compared to explicit discrimination

Conclusions Figure 2 illustrates the results of the analysis of entry equilibria under rebates and non-negative prices (recall that miscoordination equilibria exist for all parameter values). The main results can be summarized as follows:

- Entry equilibria exist for a larger region of the parameter space than under explicit discrimination. Indeed, if an entry equilibrium exists under rebates, it will also exist under explicit discrimination; but explicit discrimination may allow the incumbent to break some entry equilibria that would exist under rebates.
- When entry equilibria exist, the prices charged by the entrant to both groups of buyers are (weakly) higher under rebates than under explicit discrimination.

4 Extensions

In this Section, we shall deal with a number of extensions to the basic model. First, we shall analyze in Section 4.1 how results change when we consider the possibility that prices can be negative. This makes the pricing behavior of both firms more aggressive. Not surprisingly, the Incumbent will be able to exclude entry for a wider region of parameter values, but the basic trade-off between exclusion and lower prices acquires now an important dimension. Indeed, the possibility of setting negative prices, i.e. of subsidizing buyers for *using* the product, gives an important tool to the entrant to disrupt miscoordination equilibria. Contrary to the base model (where prices were constrained to be non-negative), if negative price *discriminatory* offers can be made, miscoordination equilibria do not always exist. In particular, unless the gap between incumbent and entrant's costs is sufficiently small, miscoordination equilibria do not exist, and if they exist they can be sustained only by lower than monopoly prices.

Next, Section 4.2 will deal with the case of elastic demands. So far, we have assumed that demands are inelastic for simplicity. One possible problem with these demands is that unless a productive inefficiency occurs, total welfare is the same at high or low prices. It is true that lower equilibrium prices will lead to a better social outcome apart from the limit case where consumer surplus and producer surplus have exactly the same weight in the objective function (and most antitrust authorities tend to maximize consumer welfare, not total welfare), but it is still important to look at how our results extend to a setting where demands are elastic.

It turns out that working with elastic demands allows us to uncover an interesting feature of rebates when linear prices are considered. By incorporating a quantity threshold (a certain price is offered for demand up to a certain number of units), a rebate scheme contains a de facto rationing scheme which limits the number of units that a firm has to sell at a given price. Therefore, when offering below-cost prices, a rebate allows a firm to limit losses or, which is the same, for a given amount of losses that it can sustain, it can afford offering lower prices than under an explicit discrimination scheme. This points to an interesting comparison between the relative aggressiveness of rebates v. explicit discrimination: on the one hand, the necessity to satisfy the self-selection constraints limits the aggressiveness of rebates, but on the other hand, the presence of an inherent rationing device (the quantity thresholds) allows a rebate scheme to make more aggressive offers.

Under two-part tariffs, instead, rationing is not necessary to limit losses: when a firm wants to make a generous offer to buyers, it will set the variable component of the price at marginal cost, and use the (negative) fixed fee to attract buyers. In this case, therefore, the only difference between rebates and explicit discrimination is given by the presence of the self-selection constraints under the former scheme, and the consequence is that rebates are always less aggressive (and therefore less exclusionary) than explicit discrimination.

4.1 Allowing for negative prices

In this section, we keep inelastic demands, but relax the assumption that prices must be non-negative.

4.1.1 Uniform prices

Under uniform price offers, the results are the same as in the base model. The *miscoordination equilibrium* cannot be disrupted by negative price offers, because the entrant cannot profitably offer negative prices to all buyers. For the same reason, the *entry equilibrium* will also exist for all parameter values. Therefore, *Proposition 1 still holds good.*

4.1.2 Explicit price discrimination

We consider first miscoordination equilibria and then entry equilibria.

Miscoordination equilibria The possibility to offer negative prices changes dramatically the analysis of miscoordination equilibria. Consider for instance a natural candidate equilibrium, that is the miscoordination equilibrium prevailing under uniform (non-negative) prices: $(p_I^s = 1, p_I^l = 1)$ and all buyers buy from the incumbent. Under positive prices, this miscoordination equilibrium is sustained by any continuation equilibrium where firm I sets $p_I^s = p_I^l = p_I^m = 1$, firm E sets, for instance, $p_E^l = p_E^m = 1$, $p_E^s = 0$, and all buyers buy from I. This is an equilibrium because if a small buyer, who is offered a zero price by the entrant, decided to switch to the entrant given that all others buy from the incumbent, he would get zero surplus, because the entrant does not reach critical mass and hence the utility derived from consuming the product would be zero. Therefore, the entrant would have no incentive to deviate either.

But this reasoning does not hold any longer when negative prices are admitted. Suppose that firm I sets $p_I^s = p_I^l = 1$. If firm E sets $p_E^l = p_I^l - \varepsilon = 1 - \varepsilon$ and $p_E^s < 0$, then all buyers buying from the entrant would be the only equilibrium. Indeed, by buying from the entrant each small buyer would receive a strictly positive surplus $(k/m)(-p_E^s) > 0$ even if nobody else consumed the product. Therefore, they will want to consume in order to receive the payment. But since it is a dominant strategy for the small buyers to consume the product, the large buyer will now prefer to buy from the entrant as well, since the critical network size will be met, and since $CS^l(p_E^l) = (1-k)(1-p_E^l) > CS^l(p_I^l) = 0$.

More generally, a miscoordination equilibrium with prices (p_I^s, p_I^l) will not exist if the entrant can offer a negative price $p_E^s < 0$ to the small buyers such that $CS^s(p_E^s, s_E < \bar{s}) > CS^s(p_I^s, s_I \geq \bar{s})$ while slightly undercutting the incumbent's offer to the large buyer, $p_E^l = p_I^l - \varepsilon$.[22] Now, to make it a dominant strategy for the small buyers to buy from E, E must offer a price p_E^s that yields a (weakly) higher net surplus as I's offer to the small buyers:

$$-p_E^s \frac{k}{m} \geq \frac{k}{m}(1 - p_I^s)$$

[22] In the case where $\bar{s} \leq (1-k) + \frac{k}{m}$, the entrant might as well charge a negative price to the *large* buyer, while matching I's offer to the small buyers. In this case, as soon as E attracted the large buyer, E needs just one more buyer to reach the minimum size. Thus, any small buyer will find it optimal to buy from E as well, and the miscoordination equilibrium is broken. This is not the case if $\bar{s} > (1-k) + \frac{k}{m}$, where the entrant needs more than one small buyer to reach the minimum size, so that attracting the large buyer is not sufficient to solve the coordination problem among the small buyers. For simplicity, we will focus on this "asymmetric" case here.

We see immediately that $p_E^s \leq -(1-p_I^s) < 0$ (E subsidizes small buyers' consumption of its product). If the small buyers consume E's product for sure, then the large buyer will switch to E whenever $p_E^l \leq p_I^l$. Will E be able to break-even under this optimal deviation? Inserting $p_E^s = -(1-p_I^s)$ and $p_E^l = p_I^l$ into the profit function we have that

$$-k(1-p_I^s) - c_E + p_I^l\,(1-k) \geq 0$$

Rearranging this break-even constraint, we obtain

$$p_I^s \geq 1 - \frac{1}{k}\left[p_I^l\,(1-k) - c_E\right]$$

Looking at it from the point of view of the incumbent, this means that given p_I^l, p_I^s must *not* exceed $1 - \frac{1}{k}\left[p_I^l\,(1-k) - c_E\right]$, or else I becomes vulnerable to the deviation described above. Hence, I's problem reads

$\max_{p_I^s, p_I^l} \pi_I = (p_I^s - c_I)\,k + \left(p_I^l - c_I\right)(1-k)$
s.t. (i) $p_I^l \leq 1$
(ii) $p_I^s \leq \min\left\{1 - \frac{1}{k}\left[p_I^l\,(1-k) - c_E\right], 1\right\}$

If $(1-k) - c_E < 0$, the problem is trivially solved by

$$p_I^s = p_I^l = 1$$

If instead $(1-k) - c_E \geq 0$, we can insert $p_I^s = 1 - \frac{1}{k}\left[p_I^l\,(1-k) - c_E\right]$ into the objective function to see that the choice variables drop out, so that the objective function reduces to:

$$\pi_I = k + c_E - c_I$$

Thus, I will be able to break even iff

$$c_I \leq k + c_E$$

Proposition 4 *Let $\bar{s} > (1-k) + \frac{k}{m}$. If both firms can charge negative prices, a miscoordination equilibrium will exist iff $c_I \leq k + c_E$. (i) If $c_E \leq 1-k$, the equilibrium is characterized by*

$$\begin{aligned} p_I^l &= 1,\ p_I^s = 1 - \frac{1}{k}\left[1 - k - c_E\right] \\ p_E^l &\in [0,1]\,,\ p_E^s = -\frac{1-k-c_E}{k} \end{aligned}$$

(ii) If instead $c_E > 1-k$, the equilibrium is characterized by $p_I^s = p_I^l = 1$, and $p_E^s = p_E^l = 1$.

Proof: see Appendix

Figure 3 illustrates in the space (k, c_I) the region where the miscoordination equilibrium arises, for the case $c_E < 1/2$. It shows that this equilibrium

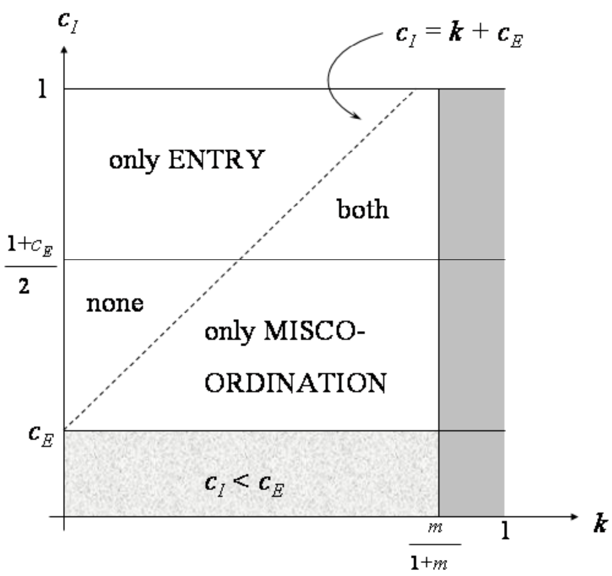

Figure 3: Regions where miscoordination equilibria and/or entry equilibria (or none) exist under negative prices, for $c_E < 1/2$

exists only if c_I is sufficiently close to c_E. The main conclusions from the analysis are that: (1) when negative prices are possible, then allowing for explicit discrimination disrupts miscoordination equilibria when c_I is sufficiently high. (2) When a miscoordination equilibrium exists under explicit discrimination (with linear prices which can be negative), the incumbent will not be able to enjoy the monopoly outcome ($p_I^s = 1, p_I^l = 1$), unless $c_E > 1 - k$; the incumbent needs to lower its prices to prevent the entrant from stealing its buyers.

Compared to uniform pricing regimes, where a miscoordination equilibrium which reproduces the monopolistic outcome is always possible, explicit discriminatory schemes have the effect of both rendering miscoordination equilibria less likely, and, where such equilibria survive, of reducing the equilibrium prices at those equilibria with. Note that in this case, p_I^s may even be below-cost, i.e. $p_I^s < c_I$!

Entry equilibria The analysis of entry equilibria when we allow for negative prices requires just a small modification of the problem already analyzed in Section 3.2 above, i.e. allowing for p_I^s and p_I^l to take negative values.

The best offer the incumbent can make to the small buyers is given by the solution of the following program:

$$\begin{array}{l}\max_{p_I^s, p_I^l} CS^s(p_I^s) = \frac{k}{m}(1 - p_I^s), \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \leq 1, p_I^l \leq 1. \end{array} \tag{24}$$

The best offer the incumbent can make to the large buyer is given by the solution of:

$$\begin{array}{l}\max_{p_I^s, p_I^l} CS^l(p_I^l) = (1-k)\,(1 - p_I^l), \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \leq 1, p_I^l \leq 1. \end{array} \tag{25}$$

By following the same steps as in Section 3.2 one can check that the incumbent's best offers are

$$\widehat{p}_I^s = \frac{c_I - (1-k)}{k}; \quad \widehat{p}_I^l = \frac{c_I - k}{1-k}.$$

An entry equilibrium will exist only if the entrant is able to profitably match simultaneously both best offers, i.e. $p_E^s = \widehat{p}_I^s$, and $p_E^l = \widehat{p}_I^l$. Therefore, such an equilibrium exists if and only if:

$$\pi_E(\widehat{p}_I^s, \widehat{p}_I^l) = k\left(\frac{c_I - (1-k)}{k} - c_E\right) + (1-k)\left(\frac{c_I - k}{1-k} - c_E\right) \geq 0,$$

which is satisfied for:

$$c_I \geq \frac{1 + c_E}{2}.$$

Figure 3 illustrates entry equilibria. The figure also shows that under explicit discrimination, there might be a situation where, for given c_E and k, for c_I sufficiently close to c_E a miscoordination equilibrium exists, for intermediate values of c_I no equilibrium in pure strategies exists, and for high values of c_I only the entry equilibrium will exist. (To be precise, such a situation exists if $c_E < 1/3$). For high values of k, there exists an area of parameter values where both miscoordination and entry equilibria will coexist.

To interpret these results, recall that under uniform pricing both entry and miscoordination equilibria exist under all parameter values. This multiplicity of equilibria in the base case makes it difficult to identify precise policy implications. However incomplete (depending on the values of c_E, there may also exist other regions where no equilibria exist under explicit discrimination, or where multiple equilibria exist also under explicit discrimination), the following Table allows to fix ideas. It shows that for relatively high efficiency gaps between incumbent and entrant, if explicit discrimination schemes are allowed consumer welfare will always be (weakly) higher than under uniform pricing (miscoordination equilibria never exist, and entry equilibria are characterized by (weakly) lower prices). For relatively low efficiency gaps between incumbent and entrant, though, the impact on consumer welfare is not unambiguous: at equilibrium, the incumbent will always serve, and the desirability of explicit discrimination schemes depends on which equilibrium would prevail under uniform pricing: if under uniform pricing a miscoordination equilibrium is played, then explicit discrimination will increase consumer welfare, but if under uniform pricing an entry equilibrium is played, then explicit discrimination leads to exclusion and higher prices. We would then find again the same tension between exclusion and low prices that we have stressed in the main Section above, although it is to be noticed that - apart from very specific cases - exclusion can be achieved by the incumbent only by decreasing equilibrium prices.

	Uniform pricing	Explicit discrim. (neg. prices)
$c_I > \max\left\{\frac{1+c_E}{2}, k+c_E\right\}$	$\begin{cases} I \text{ serves: } p_I^l = p_I^s = 1 \\ \Longrightarrow CS = 0 \\ E \text{ serves: } p_E^l = p_E^s = c_I \\ \Longrightarrow CS = 1 - c_I \end{cases}$	E serves: $\widehat{p}_E^l \leq c_I; \widehat{p}_E^s \leq c_I$ $\Longrightarrow CS \geq 1 - c_I$
$c_I < \min\left\{\frac{1+c_E}{2}, k+c_E\right\}$	$\begin{cases} I \text{ serves: } p_I^l = p_I^s = 1 \\ \Longrightarrow CS = 0 \\ E \text{ serves: } p_E^l = p_E^s = c_I \\ \Longrightarrow CS = 1 - c_I \end{cases}$	I serves: $\widehat{p}_I^l = 1; \widehat{p}_I^s \lesseqgtr c_I$ $\Longrightarrow CS \leq 1 - c_I$

4.1.3 Implicit price discrimination (rebates)

It would be tedious to characterize all the equilibrium solutions for the case of rebates as well. Like for the case of explicit discrimination, the possibility to set negative prices allows the incumbent to make more aggressive offers, eliminating entry equilibria which would have existed under uniform prices; also, and again like for explicit discrimination, it allows the entrant to subsidize a group of buyers and induce them to use the product independently of what other buyers do, thus leading to the disruption of miscoordination equilibria. The fact that the self-selection constraint needs to be satisfied does not therefore eliminate the possibility to disrupt some of the equilibria;[23] however, it does imply that competition is softer under rebates than under explicit discrimination. Even in this case, therefore, we find the result that rebates are less exclusionary than explicit discrimination, but lead to higher prices when similar equilibrium market structures are compared.

[23] At first sight, one may wonder why a buyer may want to buy at positive prices when it could mimic a buyer who is offered a negative price. But recall that a large buyer may get more surplus from buying $1-k$ units at a positive price than a smaller number of units k/m at a negative price. However, we have seen in Section 3.3 that small buyers will never be willing to buy at positive price if they have the chance to buy more units than they need at zero price. A fortiori, this is true when the price offered for a large number of units is negative.

4.2 Elastic demands

Here we relax the assumption that demands are inelastic, by assuming a simple linear demand function for the buyers. We briefly deal with two cases:

4.2.1 Linear prices

Here the interesting bit should be to show that rebates work as a rationing device, which may help the incumbent when making offers to the buyers

4.2.2 Two-part tariffs

Here the results should be similar as in the base model

4.3 Full Discrimination

TO BE DONE

5 Concluding remarks

The purpose of this exercise was to demonstrate the exclusionary potential of rebate arrangements in the presence of network externalities. Our findings are particularly interesting insofar as, in our model, the entrant is in a fairly good initial position compared to other papers on exclusionary practices: it does not have to pay any fixed cost to start operating in the industry, entrant and incumbent can approach all buyers simultaneously (i.e. the incumbent has no first-mover advantage in offering contracts to the buyers before the entrant can do so), and the entrant has the same pricing instruments at its disposal.

In the base model, we assume that firms can only charge non-negative linear prices. First of all, we find that exclusionary equilibria exist for all parameter values, and that even monopoly prices can be sustained in these exclusionary equilibria under each price regime (uniform pricing, explicit discrimination, and rebates).

Under uniform pricing, entry equilibria exist as well for all parameter values. However, under both explicit and implicit price discrimination, entry equilibria exist only if c_I is high enough. When they exist, the highest sustainable equilibrium prices are always below c_I (which is the highest sustainable equilibrium price under uniform linear pricing). Compared to uniform pricing, price discrimination (i.e. a more aggressive pricing strategy): (a) on the one hand, makes exclusion more likely; (b) on the other hand, for given market structure, results in lower prices.

We also find that under implicit discrimination, entry equilibria exist for a larger region of the parameter space than under explicit discrimination. Indeed, if an entry equilibrium exists under rebates, it will also exist under explicit discrimination; but explicit discrimination may allow the incumbent to break some entry equilibria that would exist under rebates. Moreover, when entry equilibria exist, the prices charged by the entrant to both groups of buyers are (weakly) higher under rebates than under explicit discrimination.

We also study several extensions of our base model:

(1) Negative Prices: This makes the pricing behavior of both firms more aggressive. Not surprisingly, the incumbent will be able to exclude entry for a wider region of parameter values, but the basic trade-off between exclusion and lower prices acquires now an important dimension. Indeed, the possibility of setting negative prices, i.e. of subsidizing buyers for *using* the product, gives an important tool to the entrant to disrupt miscoordination equilibria. Contrary to the base model (where prices were constrained to be non-negative), if negative price *discriminatory* offers can be made, miscoordination equilibria do not always exist. In particular, unless the gap between incumbent and entrant's costs is sufficiently small, miscoordination equilibria do not exist, and if they exist they can be sustained only by lower than monopoly prices.

(2) Elastic Demands: It turns out that working with elastic demands allows us to uncover an interesting feature of rebates when linear prices are considered. By incorporating a quantity threshold (a certain price is offered for demand up to a certain number of units), a rebate scheme contains a de facto rationing scheme which limits the number of units that a firm has to sell at a given price. Therefore, when offering below-cost prices, a rebate allows a firm to limit losses or, which is the same, for a given amount of losses that it can sustain, it can afford offering lower prices than under an explicit discrimination scheme. This points to an interesting comparison between the relative aggressiveness of rebates v. explicit discrimination: on the one hand, the necessity to satisfy the self-selection constraints limits the aggressiveness of rebates, but on the other hand, the presence of an inherent rationing device (the quantity thresholds) allows a rebate scheme to make more aggressive offers.

(3) Two-Part Tariffs: Under two-part tariffs, instead, rationing is not necessary to limit losses: when a firm wants to make a generous offer to buyers, it will set the variable component of the price at marginal cost, and use the (negative) fixed fee to attract buyers. In this case, therefore, the only difference between rebates and explicit discrimination is given by the presence of the self-selection constraints under the former scheme, and the consequence is that rebates are always less aggressive (and therefore less exclusionary) than explicit discrimination.

Interesting extensions of our model could be to allow for buyers to compete against each other downstream, to see whether the same kind of results as in Fumagalli and Motta (Forthcoming) would arise. Another issue of interest could be to allow for partial (or even full) compatibility between I's and E's network, and to introduce compatibility as a strategic choice variable.

Finally, note that we should expect very similar results in a model where there are no network externalities, but instead buyers have switching costs, and the entrant faces some fixed costs of entry. Farrell and Klemperer (forthcoming) have pointed out the analogies between network externalities and switching costs:

"Both switching costs and proprietary network effects arise when consumers value forms of *compatibility* that require otherwise separate purchases to be made from the same firm. Switching costs arise if a consumer wants a group, or especially a series, of his own purchases to be compatible with one another: this creates economies of scope among his purchases from a single firm. Network effects arise when a user wants compatibility with *other* users (or complementors), so that he can interact or trade with them, or use the same complements; this creates economies of scope between *different* users' purchases.

These economies of scope make it unhelpful to isolate a transaction: a buyer's best action depends on other, complementary transactions. When those transactions are in the future, or made simultaneously by others, his *expectations* about them are crucial. When they are in the past, they are *history* that matters to him. History also matters to a firm because established *market share* is a valuable asset: in the case of switching costs, it represents a stock of individually locked-in buyers, while in the case of network effects an installed base directly lets the firm offer more network benefits and may also boost expectations about future sales." (Farrell and Klemperer (2004): page 1 - Introduction)

References

[1] Aghion, Philippe and Patrick Bolton. 1987. "Contracts as a Barrier to Entry". *American Economic Review,* 77(3): 388 - 401.

[2] Armstrong, Mark and John Vickers. 1993. "Price Discrimination, Competition and Regulation". *Journal of Industrial Economics*, 41(4): 335-359.

[3] Bernheim, B. Douglas and Michael D. Whinston. 1998. "Exclusive Dealing", *Journal of Political Economy,* 106(1): 64 - 103.

[4] Farrell, Joe and Paul Klemperer. Forthcoming. "Coordination and Lock-In: Competition with Switching Costs and Network Effects". Mark Armstrong and Robert Porter (eds.) *Handbook of Industrial Organization Volume 3.* Amsterdam: Elsevier.

[5] Fumagalli, Chiara and Massimo Motta. 2006. "Exclusive Dealing and Entry When Buyers Compete", *American Economic Review*, 96(3): 785-795.

[6] Gyselen, Luc. 2003. "Rebates: Competition on the Merits or Exclusionary Practice?", in Claus-Dieter Ehlermann and Isabela Atanasiu (eds.) *European Competition Law Annual 2003: What Is an Abuse of a Dominant Position?*. Oxford/Portland Oregon: Hart Publishing.

[7] Innes, Robert and Richard J. Sexton. 1993. “Customer coalitions, monopoly price discrimination and generic entry deterrence”, *European Economic Review* 37: 1569 - 1597.

[8] Innes, Robert and Richard J. Sexton. 1994. “Strategic Buyers and Exclusionary Contracts”. *American Economic Review,* 84(3): 566 - 584.

[9] Kobayashi, Bruce H. 2005. “The Economics of Loyalty Discounts and Antitrust Law in the United States”. *Competition Policy International*, 1(115): Autumn 2005.

[10] Rasmusen, Eric B., J. Mark Ramseyer and John S. Wiley. 1991. “Naked Exclusion”. *American Economic Review,* 81(5): 1137 - 45.

[11] Rey, Patrick, Jordi Gual, Martin Hellwig, Anne Perrot, Michele Polo, Klaus Schmidt and Rune Stenbacka. 2005. “An Economic Approach to Article 82”. Brussels: European Commission, Competition Directorate General.

[12] Segal, Ilya R. and Michael D. Whinston. 2000. “Naked Exclusion: Comment”. *American Economic Review,* 90(1): 296 - 309.

[13] Stole, Lars. Forthcoming. “Price Discrimination and Imperfect Competition.” in Mark Armstrong and Robert Porter (eds.) *Handbook of Industrial Organization Volume 3.* Amsterdam: Elsevier.

[14] Thisse, Jacques-François, and Xavier Vives. 1988. “On the Strategic Choice of Spatial Price Policy”. *American Economic Review*, 78(1): 122 - 37.

[15] Vickers, John. 2005. “The Abuse of Market Power”. *Economic Journal,* 115(504): F244-261.

6 Appendix A - Proofs

Proof of Proposition 1:

(i) With all buyers buying from E at $p_E = c_I$, total demand is $mq_E^s(p_E) + q_E^l(p_E) = 1 \geq \bar{s}$, and so E will reach the minimum size. Thus, E's product has the exact same value to the buyers as I's, and it sells at the same price. Given that buyers coordinate on the entrant whenever E's offer is at least as good as I's, no buyer has an incentive to deviate and buy from I instead. I will not want to deviate either: To attract the buyers, I would have to set a price $p_I < c_I$, i.e. sell at a loss; and increasing p_I above c_I will not attract any buyers. E has no incentive to change anything about its price either: increasing p_E would imply losing the buyers to I, and decreasing p_E will just reduce profits.

Note that we eliminate all equilibria in weakly dominated strategies, where I sets $p_I \in [0, c_I)$ instead of $p_I = c_I$, and E sets $p_E = p_I < c_I$.

(ii) Suppose that all buyers buy from I. Then, recall that $\bar{s} > \max\{1-k, k\}$, implying that none of the individual buyers alone is sufficient for E to reach the minimum size. Thus, E's product has zero value for any single buyer, and so no buyer will want to deviate and buy from E, even if p_E were strictly lower than p_I. I sets $p_I = p_I^m$, which is the most profitable among all prices under which buyers will miscoordinate on the incumbent. Thus, I has no incentive to increase or decrease its price. Since buyers will not switch to E even if the price difference between the two firms is maximal, i.e. even if E charges $p_E = 0$ (so that $p_I - p_E = 1$), E has no incentive to decrease its price.

We eliminate all equilibria in weakly dominated strategies, where I sets $p_I = p_I^m$, and E sets $p_E < p_E^m$.$\square$

Proof of Proposition 4:

(i) Let $c_E \leq 1-k$. If the incumbent raises p_I^l above 1 (the prohibitive price), the large buyer will not buy anything. Reducing p_I^l below 1 would only reduce profits. Note that $c_E \leq 1-k$ implies that $p_I^s \leq 1$. If the incumbent raises p_I^s above $1 - \frac{1}{k}[1-k-c_E]$, the small buyers will find it individually rational to buy from E:

$$-p_E^s \frac{k}{m} = \frac{1}{m}(1-k-c_E) > \frac{k}{m}(1-\tilde{p}_I^s)$$

Reducing p_I^s below $1 - \frac{1}{k}[1-k-c_E]$ would only reduce profits.

Under this equilibrium, all buyers buy from the incumbent, so that the entrant's profits are zero. We argued before that the entrant's optimal deviation is to set $p_E^l = p_I^l = 1$, and to reduce p_E^s below $-\frac{1-k-c_E}{k}$ to attract the small buyers. But such an offer would violate the entrant's break-even condition:

$$\tilde{p}_E^s k - c_E + p_I^l(1-k) < -k(1-p_I^s) - c_E + p_I^l(1-k) = 0$$

The entrant has no incentive either to increase p_E^s above $-\frac{1-k-c_E}{k}$, as it does not make any sales in equilibrium.

Finally, no individual buyer has any incentive to deviate and buy from the entrant instead: each of the small buyers is indifferent between I's and E's offer, and the large buyer strictly prefers to buy from I than being the only buyer to buy from E.

Can there be any other miscoordination equilibrium, where I charges a lower p_I^l, namely $p_I^l < 1$, and an accordingly higher $p_I^s = 1 - \frac{1}{k}\left[p_I^l(1-k) - c_E\right]$? No, because no matter which prices E sets, I would want to increase p_I^l to 1 without changing p_I^s, thereby increasing profits without losing the large buyer to E. Therefore, such a price pair cannot sustain an equilibrium.

(ii) Let $c_E > 1-k$, and let $p_I^s = p_I^l = 1$. Clearly, the incumbent has no incentive to change its prices. Recall that under E's optimal deviation, E's break-even condition reads

$$-k(1-p_I^s) - c_E + p_I^l(1-k) \geq 0$$

Inserting $p_I^s = p_I^l = 1$, we get

$$-c_E + (1-k) \geq 0$$

This condition is always violated if $c_E > 1-k$. In other words, business-stealing by the entrant is impossible even if the incumbent charges monopoly prices to both groups of buyers. Therefore, the entrant is indifferent among all the prices it can set such that I serves the buyers: $p_E^s = p_E^l = 1$ dominates all others. The rest of the proof is analogous to the reasoning above.□