Tying and Freebie in Two-Sided Markets
Preliminary and Incomplete

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Abstract

In two-sided markets where platforms are constrained to set non-negative prices, we study the effect of tying and of pure bundling. Bundling can be deployed by platforms as a tool to introduce implicit subsidies on one side of the market in order to solve the usual coordination failure in two sided market framework. A multi-product monopoly platform uses bundles to raise participation on both sides, which benefits consumers. In a duopoly context, bundling has also a strategic effect on the level of competition. Contrary to the monopoly case, tying or bundling may be ex-post and/or ex-ante optimal for a contested platform. But the competing platform benefits from it if the equilibrium implicit subsidy is large enough. The impact on consumers surplus and total welfare depends on the extent of asymmetry in externalities between the two sides, with a negative effect if there is enough symmetry, and a positive effect with strong asymmetry.

1 Introduction

One key issue for two-sided platforms is that the need to coordinate consumers on an efficient allocation through adequate pricing may require to subsidize the participation of some consumers. Based on the initial work of (Caillaud and Jullien 2003), (Rochet and Tirole 2003) and (Armstrong 2005), the literature has emphasized the role of the price structure in solving coordination problems. (Caillaud and Jullien 2003) and (Armstrong 2005) show that platforms may be tempted to set "negative prices" on one side in order to enhance participation. In our model, a platform constrained to set non-negative price ties the sales of another good with the access to the platform as a way to relax this constraint and eventually implement the implicit discount. As emphasized in (Jullien 2005), when a two-sided platform is constrained to set non-negative prices, it may rely on bundles as a

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way to solve this coordination failure generated by the two-sided nature of market. By bundling, it can introduce implicit subsidies on one side of the market with the scope of raising participation on the targeted side, increasing membership value for the other side which will be more willing to join the platform.

We study this phenomenon in the context of a monopolistic and a duopolistic two-sided market with single-homing, and analyze the impact of mixed bundling (which reduces to tying in our model) and pure bundling by one platform on the allocation, as well as on the consumer surplus, the platforms' profits and the total welfare.

The concept of two-sided market refers to situations where platforms offer services used by two distinct groups of customers interacting. Instances involving such two-sided network externalities are numerous and include among others activities such as intermediation, credit card, medias, computer operating systems, video games, shopping malls or yellow pages.[1] In all these activities, tying is a widespread phenomenon and may take several forms. One form corresponds to the practice of offering gifts along with the service, as a magazine offering a DVD with its paper version. Another form, illustrated by the case of the Windows Media Player, consists in bundling a monopoly good with a complementary competitive two-sided good.[2] Last but not least, a widespread practice among web portals like Yahoo or Google consists in offering for free a large bundle of services to one side of the market. More importantly, addressing the fact that in several cases the bundle is offered for free to consumers can be difficult and can pose problems when judging anti-trust cases. This paper offers a framework in which this issue is addressed.

In the paper, the rationale for tying differs from entry deterrence motives emphasized by (Whinston 1990), and from price discrimination motives as in (Adams and Yellen 1976) or (Schmalensee 1984) and it is specific to two-sided markets. Indeed the rational for tying here is an attempt to stimulate demand on one side with the objective to raise the willingness to pay and, as a consequence, the price and the profit on the other side.

The model is a standard two-sided market similar to (Armstrong 2005), where a participant on one side derives a positive externality increasing with the level of participation on the other side. In the duopoly case, platforms are horizontally differentiated on both sides, and agents register to one platform only. We then assume that one platform has the possibility to offer another good to consumers alone or in a bundle.

When the platform is a monopoly on the tied good market and selling the good alone is profitable, it uses a mixed bundling strategy, selling

[1] See (Evans 2003a; Evans 2003b), and (Rochet and Tirole 2005; Rochet and Tirole 2003) for more examples.

[2] See (Choi 2004).

the good unbundled and bundled. In our model, the demand for the tied good is homogeneous among members of one side so that there is no price-discrimination possibility. The only reason to sell a bundle is to offer a participation subsidy with the scope of raising participation on one side. We show that all consumers benefit from this, since the enhanced participation of one side raises the perceived quality of the service on the other side. Another possibility is that a platform that has no market power on other markets, bundles a good with access to the platform and sell it at a price below cost. In this case the platform would sell only a pure bundle and implications for welfare are less clear.

In a duopolistic context, we account for strategic effects as the use of bundles affects the competitor's behavior. In a one-sided context, (Whinston 1990) shows that pure bundling reduces the equilibrium profit of all platforms and thus may result in entry deterrence. In his analysis, mixed bundling is never optimal while pure bundling is never ex-post optimal (once entry has occurred) but may be ex-ante optimal. In our context bundling, as in (Whinston 1990), results in a lower price of the platform service for the targeted side, but unlike the one-sided case, mixed or pure bundling may be ex-post optimal. Moreover the strategic effects differ substantially. In our model the service is provided for free on the side subject to tying and profits are obtained by charging the other side. In order to emphasize the subsidy aspect, we focus mostly on the case of mixed bundling where it is the sole effect at work. We then extend the analysis to pure bundling.

For the platform deploying bundling, enhancing participation of one side gives a "quality" advantage on the other side and affects also the intensity of price competition on this other side. A key point is that bundling occurs on the side where the platform is subsidizing participation. In this context, intuition may be misleading unless one realizes that the subsidized size should be considered only as the "input" sold to the other side. Increasing the subsidy through bundling has two effects. First it allows to raise the price for a given participation on the other side which is the rational for bundling. But a by-product is that it increases the opportunity cost of selling on the profitable side. The reason is that more sales on the profitable side implies more sales at a loss on the subsidized side. The opportunity cost of selling on the profitable side then accounts for the loss generated on the subsidized side, and increases with bundling since bundling is a way to raise the subsidy.

As a result the impact of bundling on the behaviour of the platform on the profitable side has to be analyzed as the combination of the effect of increasing the perceived quality and the effect of increasing the (opportunity) cost. We can then identify two effects: a demand shifting effect associated with higher quality, and a competition softening effect associated with higher cost. The impact on equilibrium profits is then ambiguous and due to softened competition, the competitor's profit may increase when one platform uses tying. Similarly the bundling platform may obtain less profits. We analyze these effects and the welfare implications first with mixed bundles and then with pure bundles.

The impact on consumers is also ambiguous. Consumers on the subsidized side always benefit from a larger subsidy, but consumers on the profitable side may benefit or not depending on which effect dominates. The main conclusion is that when the two sides evaluate the participation of each other in a symmetric way, total consumer surplus and total welfare decrease with bundling. When the subsidy is given to consumers who do not value the participation of the other side, then aggregate consumer surplus increases on both sides of the market. However in any case some consumers are hurt.

We then compare pure bundling and mixed bundling, where we use the usual feature that pure bundling modifies the opportunity cost of selling to the subsidized side.

Despite the importance of tying in two-sided markets, there has been little contribution to this issue. (Rochet and Tirole 2005) analyze the practice of tying credit and debit card on the merchant side of the payment card market, and show that this results in a more efficient setting of interchange-fees. They share in common with us the conclusion that tying may enhance efficiency by inducing a better coordination between the various sides. (Choi 2004) analyses a situation inspired by the Windows Media Players where one side or both sides multi-home and the tied good is essential to participation to the platform. He shows that even if foreclosure may arise the welfare implications are ambiguous. In a very preliminary work, (Farhi and Hagiu 2004) analyze the strategic implications of pure bundling focusing on the fact that it reduces the perceived marginal cost of selling to the targeted side. As in (Choi 2004), their model assumes multi-homing on one side. They find that in some cases tying may soften competition and raise profit of both platforms. We corroborate this finding for mixed and pure bundle with single homing. Although the key mechanisms are different, our paper shares the fact that bundling results in enhanced participation on one side.

The paper is organized as follows. In Section 2 we set up a two-sided monopolistic framework. We define sufficient conditions to have negative prices in equilibrium and we study the impact of relaxing the non-negativity constraint by means of mixed bundling. Section 3 extends the analysis to a duopolistic framework with mixed bundling. Section 4 compares pure bundling and mixed bundling. Section 5 discusses the case of pure bundling, allowing the good bundled to have a negative social value.

2 A monopoly platform

2.1 Two-sided market and negative prices

Consider a platform serving two groups of agents, denoted by 1 and 2, each of total size 1. The platform incurs in a cost f_i for each agent subscribing to side i. Every agent of each group cares about the total number of agents in the other group. To anticipate on the duopolistic model, agents on each side are indexed by a parameter x distributed on the unit interval and we assume a linear demand. If the platform attracts n_1 and n_2 on each side, the utility of the agents of group 1 and 2 are:

$$u_1 = v_1 - t_1 x + \alpha_1 n_2 - p_1 \text{ and } u_2 = v_2 - t_2 x + \alpha_2 n_1 - p_2,$$

where $v_i > f_i$ is the intrinsic valuation of participation, $\alpha_i \geq 0$ is the benefit of interacting with every agent belonging to the other group and x is the heterogeneity parameter (the transport cost in the Hotelling model) that is assumed to be uniformly distributed on $[0, 1]$. In the monopoly section we focus on cases where the market is not covered and demand is differentiable. Thus consumers on side i buy the service of the platform if $x < x_i$, where $0 < x_i < 1$. Defining $D_i(h) = (v_i - h)/t_i$, the numbers of agents n_1 and n_2 of each group participating is solution of the following system of equations:

$$n_1 = D_1(p_1 - \alpha_1 n_2) \text{ and } n_2 = D_2(p_2 - \alpha_2 n_1).$$

We assume in addition that:

Assumption 1 $\Delta = 4t_1 t_2 - (\alpha_1 + \alpha_2)^2 > 0$.

Let us also define $\Gamma = t_1 t_2 - \alpha_1 \alpha_2$. The assumption implies that $\Gamma > 0$.

As we say above we assume that $n_i < 1$ in all equilibria. Under these assumptions, the relationship between the prices (p_1, p_2) and the allocation (n_1, n_2) is one-to-one, and all the profit maximization programs are concave. More precisely we have:

$$n_1 = \frac{t_2(v_1 - p_1) + \alpha_1(v_2 - p_2)}{\Gamma} \text{ and } n_2 = \frac{t_1(v_2 - p_1) + \alpha_2(v_1 - p_1)}{\Gamma}. \quad (1)$$

Since our objective is to study how the platform can circumvent the impossibility to set a negative price on one side, we impose that price p_1 is non-negative, while price p_2 is free. The platform problem is then

$$\Pi^* = \max_{p_1, p_2} (p_1 - f_1)n_1 + (p_2 - f_2)n_2$$
$$s.t.\ p_1 \geq 0 \text{ and } (1).$$

Optimal pricing formulas are standard monopoly pricing formula where the relevant cost is the opportunity cost $f_i - \alpha_{-i} n_{-i}$ of attracting one agent of side i, which reflects the fact it allows raising the price by α_{-i} for the n_{-i} agents of the other side.[3] In our linear case we obtain the allocation:

$$\begin{aligned} n_1^* &= \frac{2t_2 (v_1 - f_1) + (\alpha_1 + \alpha_2)(v_2 - f_2)}{\Delta} \\ n_2^* &= \frac{2t_1 (v_2 - f_2) + (\alpha_1 + \alpha_2)(v_1 - f_1)}{\Delta} \end{aligned}$$

and the price on side 1 is

$$p_1^* = f_1 + \frac{(v_1 - f_1)(2t_1 t_2 - \alpha_2 (\alpha_1 + \alpha_2)) + t_1 (\alpha_1 - \alpha_2)(v_2 - f_2)}{\Delta} \quad (2)$$

Consumers of side 1 can be offered the service at a price below cost. The platform would (if possible) also set a negative price and thus offer a direct subsidy when the benefit enjoyed by group 2 is large. From now on we assume that this is the case:

Assumption 2 $p_1^* < 0$.

This only occurs when the side 1 is the low externality side. Indeed one may verify that assumptions 1 and 2 imply that $\alpha_1 < \alpha_2$.

In this case, the non-negativity constraint binds and the price on side 1 is $p_1 = 0$. Then the optimal price p_2 satisfies the following condition (see Appendix):

$$p_2 - f_2 + \alpha_1 (f_1 - \alpha_2 n_2) D_1' (-\alpha_1 n_2) = \frac{D_2 (p_2 - \alpha_2 n_1)}{-D_2' (p_2 - \alpha_2 n_1)},$$

The intuition is straightforward. Again $f_1 - \alpha_2 n_2$ is the opportunity cost of one more sale on side 1. Given that p_1 is fixed, attracting one more individual on side 2 now increases sales on side 1 by $-\alpha_1 D_1' (-\alpha_1 n_2)$.

Moreover the fact that the constraint binds implies that $\alpha_2 n_2 > f_1$ which expresses the fact that there must be a recoupment on side 2 for any member of side 1. Indeed there is no point of servicing side 1 at a loss unless the externality created for side 2 is larger than the cost on side 1.

For a given level of n_1, equilibrium conditions imply that the price p_2 is smaller than without the constraint, and thus that the platform sells more on side 2 if it is constrained to set $p_1 = 0$ than when p_1 is negative. The reason is that the platform accounts for the fact that an increase in side 2 sales raises the demand on side 1. When the margin is negative on side 1,

[3] See (Armstrong 2005) or (Caillaud and Jullien 2003).

this tends to limit the incentive to raise sales. Raising p_1, although it is detrimental as it reduces n_1 and forces to reduce p_2, also reduces the loss on side 1 and thus mitigates this effect.

A second effect is that as n_1 decreases the perceived quality and thus the demand on side 2 decrease.

In our linear set-up we obtain the price $p_2^0 = \frac{t_1(v_2+f_2)+\alpha_2 v_1+\alpha_1 f_1}{2t_1}$, with quantities

$$n_2^0 = \frac{t_1(v_2 - f_2) + \alpha_2 v_1 - \alpha_1 f_1}{2\Gamma} \text{ and } n_1^0 = \frac{v_1 + \alpha_1 n_2^0}{t_1}. \tag{3}$$

We can verify that the sales on side 2 have decreased compared to unconstrained case: $n_2^* > n_2^0$.

In this situation platform A offers the service for free on side 1. However, it would benefit from relaxing the zero price constraint and therefore setting negative subscription fee. Bundling is then a way to achieve this goal by implicitly allowing platforms to offer discounts.

2.2 Bundling

Now suppose the platform can sell another good. To avoid confusion in what follows, we will use the term "service" to refer to the two-sided platform, and reserve the term "good" for the good tied to the platform. Our objective is thus to understand what are the implications of various strategies involving bundling and tying the good and the service.

For conciseness, we assume that only consumers of side 1 value the good, while consumers of side 2 have no interest in it. In other words, there is perfect correlation between the demand for the good and the potential participation as a side 1 member.[4] All consumers of side 1 have the same willingness to pay θ for the good. The marginal production cost is constant at c, and the net margin on the good alone sold by the platform is $m = \theta - c$.

We assume that the value of the good is larger than its cost, $m \geq 0$, which corresponds to a situation where the platform has market power on the good.[5] In this case the platform would sell the good unbundled at price θ, making a positive profit. It can then decide to forego some profit to

[4] Relaxing this assumption raises no difficulty as long as only side 1 consumers may buy the bundle, as for instance if the two sides are two distinct populations (consumers and professional suppliers, or reader and advertisers) and the platform discriminates. Then the analysis of mixed bundling is the same, while the only difference with pure bundling is the level of sales lost compared to the unbundled case.

[5] One alternative interpretation of the model is that the good is sold on a competitive market at price θ. Then the maximal price at which consumers are willing to buy from the platform is θ. The case $m = 0$ corresponds to a situation where the platform has access to the same technology as competitive suppliers, while $m > 0$ corresponds to the case where the platform has access to a superior technology.

enhance participation to the platform by deploying a bundling strategies. The case $\theta < c$ corresponds to a situation where it is inefficient for the platform to sell the good alone and will be discussed in a separate section on pure bundling.

Following (Whinston 1990), we shall distinguish between various types of bundling strategies, but due to our assumption of inelastic demand for the good, only two cases are relevant. The reason is that if one consumer prefers to buy a bundle consisting of the good and the service instead of the service alone, all consumers do since they all attach the same value θ to the good sold by the platform. Thus the service is sold exclusively bundled when some bundling occurs. We refer to the case where only the bundle is sold as "pure bundling", and to the case the bundle and the good alone are sold as "mixed bundling". Remind that (Whinston 1990) shows that for one-sided markets, both in the monopoly case and the duopoly case, mixed bundling has no impact on the market equilibrium, while pure bundling reduces market profitability for all platforms (hence the risk of market foreclosure).

Let the bundle be sold at price $\tilde{p}$. The utility of side 1 consumers from buying the bundle is the same as if it was from buying the service at price $\tilde{p} - \theta$ and the good at the monopoly price θ. The "effective price" of the service, denoted p_1, is thus equal to $p_1 = \tilde{p} - \theta$. The demand of bundle on side 1 is then equal to $D_1(p_1 - \alpha_1 n_2)$ under pure bundling. Under mixed bundle with a price $p_u \leq \theta$ of the good unbundled, the demand for the bundle is $D_1(p_1 - \alpha_1 n_2 + \theta - p_u)$ while the demand for the good alone is $1 - D_1(p_1 - \alpha_1 n_2 + \theta - p_u)$.

It is immediate that since $m \geq 0$, it is optimal for the monopoly platform to use a mixed bundling strategy with a price $p_u = \theta$. Indeed compared to pure bundling, this leads to the same sales of services while generating a positive revenue from customers not joining the platform.

For the moment we focus on mixed bundling and set $p_u = \theta$. The profit under mixed bundle is then:

$$\begin{aligned} \Pi_{mixed} &= \max_{p_1, p_2} \ (p_1 - f_1) n_1 + (p_2 - f_2) n_2 + m \\ s.t. \ p_1 &\geq -\theta \text{ and } (1). \end{aligned}$$

Clearly, the platform prefers mixed bundling to no bundling. The reason is that mixed bundling allows subsidizing the participation to the platform activity will selling the good to non-interested consumers. Indeed when consumers have homogeneous preferences for the good, the only effect of mixed bundling is to relax the constraint and there is no cost in using mixed bundles. Notice that, albeit its effects on the two-sided service, the same is true for society.

Then first order conditions write as:

$$\begin{aligned} \frac{p_2 - f_2 + \alpha_1 (n_1 + \lambda)}{p_2} &= \frac{1}{-\frac{p_2 D_2'(p_2 - \alpha_2 n_1)}{D_2}}, \\ (f_1 - \alpha_2 n_2 - p_1) D_1' (p_1 - \alpha_1 n_2) &= n_1 + \lambda \end{aligned}$$

where λ is the multiplier of the constraint $p_1 \geq -\theta$. The interpretation is the same as before, one more unit on side 2 raises demand on side 1 by $-\alpha_1 D_1' (p_1 - \alpha_1 n_2)$ and the marginal opportunity cost on side 1 is now $f_1 - \alpha_2 n_2 - p_1$.

When the price constraint is binding we then have

$$n_1 = n_1^0 + \frac{2\Gamma + \alpha_1(\alpha_2 - \alpha_1)}{2\Gamma}\theta \tag{4a}$$

$$n_2 = n_2^0 + \frac{\alpha_2 - \alpha_1}{2\Gamma}\theta \tag{4b}$$

with the condition $\alpha_2 n_2 - \theta - f_1 \geq v_1 + \alpha_1 n_2 + \theta$.

Let us now consider the effect of bundling on participation

Proposition 1 *Suppose that the platform uses mixed bundling. The price of the bundle is* $\tilde{p} = \max\{p_1^* + \theta, 0\}$. *Participation is higher on both sides than under no bundling, maximal at* (n_1^*, n_2^*) *for* $\theta \geq -p_1^*$.

Proof. Using $\alpha_1 < \alpha_2$, n_2 increases with θ. Then n_1 increases with θ and n_2. When the constraint is not binding, the solution coincides with the unconstrained monopoly solution. ■

When the platform uses a mixed bundle, for θ small the impact is also small and the new price is still be at a zero level. The implicit subsidy is then $p_1 = -\theta$. However, when the value θ is above the optimal subsidy obtained for side 1 in the unconstrained optimum, the price of the bundle is positive and reflects any increase in θ.

2.3 Profits and welfare

Bundling raises profits by relaxing the non-negativity constraint on side 1 price and teh profit gain is increasing with θ up to $\theta = -p_1^*$, constant above $-p_1^*$.

We now want to compare the impact on the total social welfare of the decision of the platform. On the two-sided market, the consumer surplus writes as

$$CS = \int_{D_1^{-1}(n_1)}^{v_1} D_1(x)dx + \int_{D_2^{-1}(n_2)}^{v_2} D_2(y)dy,$$

where, n_1 and n_2 are the participation levels induced by the monopoly pricing strategy, and $D_1^{-1}(n_1) = p_1 - \alpha_1 n_2$ and $D_2^{-1}(n_2) = p_2 - \alpha_2 n_1$ are the "hedonic" prices on each sides Given that the good is priced at the monopoly level, consumers obtain no surplus from the good market and CS is the consumer surplus under all scenarios.

From what precedes, bundling a good with positive social value raises participation on both sides. Thus:

Proposition 2 *Consumer surplus is higher on both sides under mixed bundling than under no bundling.*

Proof. From Proposition 1 we know that the participation with bundling increases. But consumer surplus raises when equilibrium participation levels increase. ■

The total impact on the welfare is thus unambiguously positive. Notice that due to the homogeneity assumption (inelastic demand for the good), the reduction in sales on the good market doesn't causes additional welfare losses.

3 The duopoly case

Let us now extend the analysis to the duopolistic case. Again the model is the one developed by (Armstrong 2005) and (Armstrong and Wright 2004). The consumers of each side are located as before on the unit line, and x follows a uniform distribution on each side. There are two platforms, A and B competing for these agents, each located at one extreme of the unit interval, $x_A = 0,\ x_B = 1$. Agents join only one platform - they single-home - and when an agent belonging to side i located at $x \in [0,1]$ joins platform $j = A, B$ he enjoys a utility

$$u_i^j = v_i - t_i \left| x - x_j \right| + \alpha_i n_{-i}^j - p_i^j,$$

where p_i^j is the subscription fee, v_i is the intrinsic valuation which we assume to be platform independent, and n_{-i}^j is the mass of consumers on the other side joining platform j.

In all the paper we assume that both platforms are active on both sides and that the market is covered: $0 < n_i^B = 1 - n_i^A < 1$.

The implied demands of platform A on each side are:

$$n_1^A = \frac{1}{2} + \frac{\alpha_1(p_2^B - p_2^A) + t_2(p_1^B - p_1^A)}{2\Gamma} \tag{5a}$$

$$n_2^A = \frac{1}{2} + \frac{\alpha_2(p_1^B - p_1^A) + t_1(p_2^B - p_2^A)}{2\Gamma}. \tag{5b}$$

We Assumption 1 which guarantees regular demands and concavity of the platforms' problems.

As in the monopolistic case, we want to study how platforms can circumvent equilibrium negative prices by means of pure bundling or mixed bundling, and we assume that the prices on side 2 for each platform are free (or always positive) while we put a constraint on side 1 prices.

The situation with unconstrained prices is characterized in (Armstrong 2005). Symmetric equilibrium optimal unconstrained prices are:

$$p_1^{UC} = f_1 + t_1 - \alpha_2 \text{ and } p_2^{UC} = f_2 + t_2 - \alpha_1,$$

and the profit of each platform is $\Pi^{UC} = \frac{1}{2}(t_1 - \alpha_2 + t_2 - \alpha_1)$. Assumption 1 implies that this is positive. The consumer surplus with no bundle can be computed to be $S^{UC} = 2v - f_1 - f_2 + \frac{3}{2}(\alpha_1 + \alpha_2) - \frac{5}{4}(t_1 + t_2)$.

We replace Assumption 2 by the similar condition for a duopoly.

Assumption 2bis $p_1^{UC} = f_1 + t_1 - \alpha_2 < 0$.

Platforms then set a zero price on side 1 and adjust the price consequently on side 2.

We show in appendix that when the non-negativity constraint binds, the symmetric equilibrium prices are:

$$p_1^C = 0 \text{ and } p_2^C = p_2^{UC} + \frac{\alpha_1}{t_1} p_1^{UC}$$

When the externality α_2 is large, agents of side 2 attach high value to the participation of agents of side 1 and therefore platforms subsidize side 1 participation. Unconstrained platforms would set in equilibrium negative prices on side 1. In this case, the effect on the platform's profit would be a loss on side 1 equal to $\frac{t_1 - \alpha_2}{2} < 0$ and a gain on side 2 equal to $\frac{t_2 - \alpha_1}{2}$. When the platform is constrained to non-negative pricing, the non-negativity constraint implies a smaller loss equal to $-\frac{f_1}{2}$ on side 1 and a smaller gain on side 2.

The two platforms share the market equally and make the following equal profit:

$$\Pi^C = \Pi^{UC} + \frac{\alpha_1 - t_1}{2t_1} p_1^{UC}.$$

which can be verified to be non-negative. The total consumer surplus is

$$S^C = S^{UC} - \frac{\alpha_1 - t_1}{t_1} p^{UC}$$

The non-negativity price constraint can be beneficial or harmful for consumers depending on the sign of $\alpha_1 - t_1$. The total welfare is unchanged compared to the case where the non-negativity constraint doesn't bind: market shares remain unchanged and platforms readjust prices without welfare losses. Thus the impact on profits and the impact on consumers have opposite signs. One can see that the platforms' equilibrium profit is lower than the unconstrained one if $\alpha_1 > t_1$, higher if $t_1 > \alpha_1$. The reverse holds for consumers surplus.

Typically the constraint raises the prices on one side and reduces them on the other side. Profit decreases if the demand on side 1 is relatively more sensitive to side 2 participation than to an increase in the price of side 1 (α_1/t_1 large).

Since $m = \theta - c \geq 0$, platform A has incentives ex-post to sell the good both alone and bundled. The stand-alone price of the good is θ and all the agents of side 1 buy it if they don't join the platform. The total mass of agents who buy the good either in a bundle with the subscription fee, either alone, is 1. We start by analyzing this case.

3.1 Mixed Bundling

The price of the bundle is $\widetilde{p}$, and as before we denote by $p_1^A = \widetilde{p} - \theta$ the implicit subscription price in the bundle. Again mixed bundling just relaxes the non-negativity price and the platform's A problem becomes:

$$\max_{p_1^A, p_2^A} (p_1^A - f_1)n_1^A + (p_2^A - f_2)n_2^A + m$$
$$s.t.\ p_1^A \geq -\theta \ and \ (5)$$

while platform's B problem is unchanged.

The reasoning is the same as before, for low values of θ the platform sells the bundle at $\widetilde{p} = 0$. At some point the non-negativity constraint is not binding and the allocation becomes independent of θ. The critical value for this corresponds to the price that platform A would set at the equilibrium of a game where only platforms B is constrained to set a non-negative price on side 1. Due to the strategic interaction this price differs from p_1^{UC} and is computed in appendix to be $-\theta^A$, where

$$\theta^A = -\left(\frac{3\Gamma}{\Delta + 2\Gamma}\right) p_1^{UC}$$

We then obtain:

Proposition 3 *Under mixed bundling, platform A sets a price for the bundle $\tilde{p} = p_1^A + \theta$, where $p_1^A = \max\left\{-\theta, -\theta^A\right\}$. Platform B sets a zero price on side 1.*

Proof. See Appendix. ■

When the value of the good is low, platform A gives always the bundle for free. The negative subscription fee is compensated by the value of the bundled product. If Platform A disposes of a good that has high value (θ

big enough), the price of the bundle is positive since the value of the bundled good is bigger than the optimal subsidy.

On side 2, platforms optimally set their prices according to the following response functions:

$$\begin{aligned} p_2^A(p_2^B) &= \frac{1}{2}p_2^B + \frac{1}{2}p_2^C - \frac{\alpha_1+\alpha_2}{2t_1}p_1^A \\ p_2^B(p_2^A) &= \frac{1}{2}p_2^A + \frac{1}{2}p_2^C + \frac{\alpha_2}{2t_1}p_1^A \end{aligned}$$

where p_1^A is the equilibrium implicit price of registration to platform A.

The two best response functions show the standard complementarity between the prices of platform A and B in side 2. However, the standard property may not hold anymore. In two-sided markets, platforms also adjust the price in side 2 by taking into account the discount on the good offered by platform A to participants on side 1.

There are two effects associated with the fact that the implicit subscription fee p_1^A is now negative.

Demand shifting: demand on side 1 shifts toward platform A. This implies that on side 2 platform A becomes more attractive. Consequently, the best reply of platform B shifts downward while the best reply of platform 2 shifts upward.

Competition softening: since now platform A incurs in higher losses per customer on side 1, it has less to gain in raising demand on side 2 by cutting prices. As a consequence, platform A tends to set higher prices which shifts its best reply further upward.

While the demand shifting effect is clearly detrimental to platform $B's$ profit, the competition softening effect on the contrary is beneficial. We see that the implications for platform $B's$ profit are not clear cut. The combination of these two effects determines the optimal pricing strategy of the platforms in side 2. The equilibrium prices on side 2 are:

$$p_2^A = p_2^C - \frac{\alpha_2+2\alpha_1}{3t_1}p_1^A \tag{6a}$$

$$p_2^B = p_2^C + \frac{\alpha_2-\alpha_1}{3t_1}p_1^A. \tag{6b}$$

An increase in absolute term of the subscription fee p_1^A makes price p_2^A raise. Platform A subsides side 1 and recoups on side 2. Depending on the relative magnitude of externalities, the equilibrium reaction of platform B in side 2 may be to decrease or increase its price. If $\alpha_1 < \alpha_2$, see Figure 1 left-hand side graph, the "competition softening" strategic effect is not strong enough to offset the effect of the shift in demand and platform B

reduces its prices when platform A bundles. On the contrary, see Figure 1 right-hand side graph, when the externality is higher on side 1, $\alpha_1 > \alpha_2$, the "competition softening" effect dominates and all prices increase on side 2.

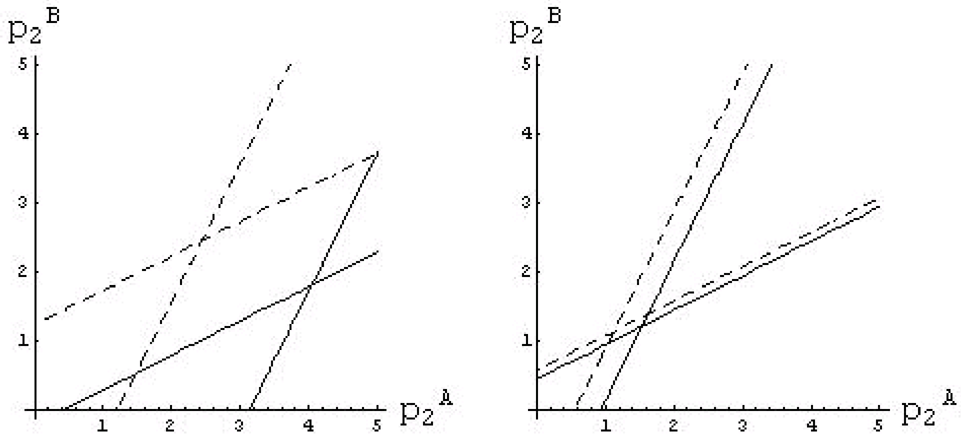

Figure 1: Platforms' best response functions.

However, platform B always sets a lower price than platform A on market 2. The equilibrium platform market shares are then asymmetric given by:

$$n_1^A = \frac{1}{2} - \left(\frac{3t_1t_2 - \alpha_1(2\alpha_2 + \alpha_1)}{6\Gamma t_1}\right) p_1^A \qquad (7a)$$

$$n_2^A = \frac{1}{2} - \left(\frac{\alpha_2 - \alpha_1}{6\Gamma}\right) p_1^A \qquad (7b)$$

The striking feature is that despite the subsidy of platform A on side 1, its market share may not increase on side 2. Indeed its market share on side 2 decreases with bundling under circumstances leading the competitor to increase its price, i.e. when the "competition softening" effect dominates ($\alpha_1 > \alpha_2$). The reason is that platform A opts for a higher mark up but lower sales on side 2 at given prices of the competitor. This rather counter-intuitive results is explained by the fact that the margin is negative on side 1.

An even more counter-intuitive result is that platform A may sell less on side 1. This occurs when α_1 is within the top range of admissible values (see Figure 2) so that $\alpha_1\alpha_2 < t_1t_2 < \alpha_1\left(\frac{2\alpha_2+\alpha_1}{3}\right)$.

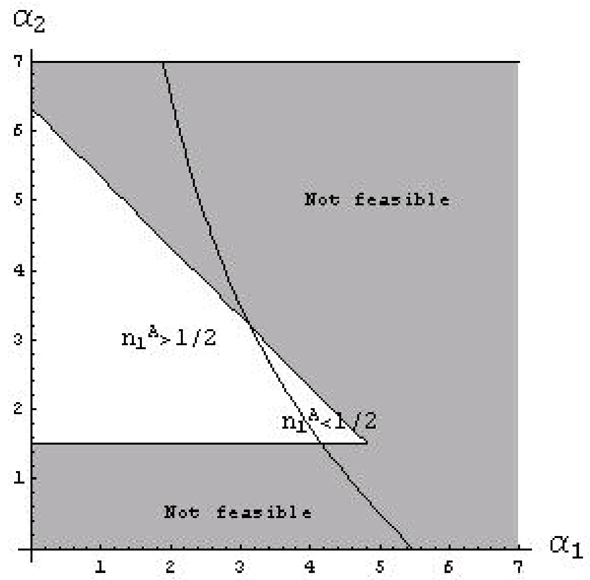

Figure 2: Platform's A side 1 market share. (Example with $t_1 = 1, t_2 = 10, f_1 = 1/2$).

3.2 Profits and welfare with mixed bundling

As pointed above, due to the strategic interaction and competition softening effects the impact of mixed bundling on the platforms is not obvious. Using the expression $p_1^A = \max\left\{-\theta, -\theta^A\right\}$, total profits are the following:

$$\begin{aligned}\Pi^A_{mixed} &= \Phi\left(p_1^A, 0\right) + m \\ \Pi^B_{mixed} &= \Phi\left(0, p_1^A\right)\end{aligned}$$

where $\Phi(x, y)$ is the profit of platform i in a game where the price of side 1 are fixed to $p_1^i = x$ and $p_1^j = y$, and the two platforms compete in price on side 2. One can show that this profit is

$$\begin{aligned}\Phi(x, y) &= \Pi^C - \frac{\Delta + 5\Gamma}{18 t_1 \Gamma} x^2 + \frac{(\alpha_2 - \alpha_1)^2}{18 t_1 \Gamma} y^2 \\ &\quad - \frac{\alpha_1 - \alpha_2 - 3 p_1^{UC}}{6 t_1} x - \frac{2(\alpha_1 - \alpha_2) + 3 f_1}{6 t_1} y + \frac{2\Delta + \Gamma}{18 t_1 \Gamma} xy.\end{aligned}$$

The effect on the profit depends on the relative levels of externalities on each side of the market, and in particular on the equilibrium reaction of the competitor.

Proposition 4 *Platform A's profit is higher under mixed bundling than under no bundling if and only if the subsidy* $\min\left\{\theta, \theta^A\right\}$ *is smaller than* $\frac{3\Gamma}{\Delta + 5\Gamma}\left(\alpha_1 - \alpha_2 - 3 p_1^{UC}\right)$.

Proof. The function $\Pi^A_{mixed} - (\Pi^C + m)$ has the same sign as

$$\alpha_1 + 2\alpha_2 - 3(t_1 + f_1) + \frac{\Delta + 5\Gamma}{3\Gamma} \max\left\{-\theta, -\theta^A\right\} > 0$$

which gives the condition. ■

The condition always holds if $\alpha_2 - \alpha_1 < \frac{2\Delta+\Gamma}{3\Gamma}\theta^A$. What appears is that platform A obtains more profits with bundling when there are large incentives to subsidize (p_1^{UC} small) or when the externality on side 1 is not too small compared to side 1. Notice that $\alpha_2 - \alpha_1$ is a measure of the intensity of the price reduction by platform B on side 2. Thus platform's A benefits from mixed bundling if platform B doesn't reduce its price too much, and in particular when it raises its price.

Notice that the fact that platform A obtains lower profit with mixed bundling does not mean that this will not occur in equilibrium. Indeed mixed bundling strategies are always optimal ex-post, once the competing platform has set its prices. The platform A may wish to commit ex-ante not to use such a strategy but may lack the credibility to do so.

Therefore there are instances in which platform A may deploy a mixed bundling strategy detrimental to its profits.

Proposition 5 *Platform $B's$ profit is higher under mixed bundling than under no bundling if and only if the subsidy* $\min\left\{\theta, \theta^A\right\}$ *is larger than* $\frac{6\Gamma}{(\alpha_2-\alpha_1)^2}\left(\alpha_2 - \alpha_1 - \frac{3}{2}f_1\right)$.

Proof. $\Pi^B_{mixed} > \Pi^C$ if

$$2(\alpha_1 - \alpha_2) + 3f_1 - \frac{(\alpha_2 - \alpha_1)^2}{3\Gamma} \max\left\{-\theta, -\theta^A\right\} > 0$$

which gives the conditions. ■

Notice that the condition is met for all θ if $\alpha_2 - \alpha_1$ is small enough. When side 1 benefits more from externalities than side 2, or slightly less, both platforms benefit from bundling. On the contrary, if the externalities are much higher on side 2, both platforms obtain lower profits when platform's A proposes a mixed bundle.

In the middle range, there may be conflicting effect as platform A gains for low value of the bundle while platform B gains for high values.

A striking feature is that for $\alpha_2 > \alpha_1$and an implicit subsidy $\min\left\{\theta, \theta^A\right\}$ large enough, platform $B's$ profit is higher and platform $A's$ profit is lower when A uses mixed bundle.

Thus when the market is conducive to high subsidies and platform A can bundle access with a high value good, is would be optimal for platform A to commit not to bundle.

A second important consequence is that mixed bundling by its competitor cannot hurt a platform unless the subsidy is small and is offered to the low externality side of the market. In particular, if $\alpha_1 \geq \alpha_2 - \frac{3}{2}f_1$, then platform $B's$ profit is larger when platform A uses a mixed bundling strategy.

Let's now analyze the impact of the pure bundling on the consumer surplus. Focusing on the total consumers surplus we find that the effect is positive when the externality perceived by consumers in side 1 is small.

Proposition 6 *If $\alpha_1 \leq t_1$ total consumer surplus if higher with mixed bundle than with no bundle, otherwise it is higher if and only if the subsidy $\min\left\{\theta, \theta^A\right\}$ is large enough.*

Proof. The consumer surplus on side i writes as:

$$S_i = v + \alpha_i n_{-i}^B - t_i n_i^B - p_i^B + t_i \frac{\left(n_i^A\right)^2 + \left(n_i^B\right)^2}{2}$$

The total change in consumer surplus on side 1 is:

$$S_1^{mixed} - S_1^C = \left(-p_1^A\right)\left(\frac{1}{2} + \frac{1}{t_1}\left(\frac{3\Gamma + \alpha_1\left(\alpha_2 - \alpha_1\right)}{6\Gamma}\right)^2\left(-p_1^A\right)\right) \quad (8)$$

On side 2 we obtain the change in surplus:

$$S_2^{mixed} - S_2^C = \left(-p_1^A\right)\left(-\frac{\alpha_1}{2t_1} + t_2\left(\frac{\alpha_2 - \alpha_1}{6\Gamma}\right)^2\left(-p_1^A\right)\right). \quad (9)$$

The total consumer surplus changes by

$$ST^{mixed} - ST^C = \left(-\frac{p_1^A}{2t_1}\right)\left(\frac{\left(3\Gamma + \alpha_1\left(\alpha_2 - \alpha_1\right)\right)^2 + t_1 t_2\left(\alpha_2 - \alpha_1\right)^2}{18\Gamma^2}\left(-p_1^A\right) + t_1 - \alpha_1\right)$$

positive if $\alpha_1 \leq t_1$, or if $\left(-p_1^A\right)$ is large. ■

Thus a small subsidy may decrease total consumer surplus.

Beyond these aggregate effects, because consumers are affected depending on their different locations, there are also distributional effects between sides and locations. Concerning sides we find:

Proposition 7 *The consumer surplus is higher on side 1 with mixed bundle than with no bundle. It is higher on side 2 if and only if the subsidy $\min\left\{\theta, \theta^A\right\}$ is larger than $\frac{9\alpha_1\Gamma^2}{t_1 t_2\left(\alpha_2 - \alpha_1\right)^2}$.*

Proof. Follows equations (8) and (9). ■

On side 1, bundling is equivalent to a reduction of prices which benefits consumers. On side 2, the effect is more ambiguous. The demand shift on side 1 raises the perceived quality of platform A but reduces it on platform B. The overall impact is thus ambiguous. Moreover the competition softening effect implies higher prices.

In the limit case where $\alpha_1 = 0$, the consumer surplus is higher on both sides. Indeed the utility obtained by the marginal consumer on side 2 (indifferent between A and B) is independent of the subsidy: the reduction in B's price is just enough to maintain at a constant level the utility of the marginal consumer (who shifts to the right) implying that the surplus increases on this side.

Consumers are also affected in a different way depending on their locations. One way to look at this issue is to measure the change in gross utility offered by each platform on each side: $\Delta u_i^l = \alpha_i n_{-i}^l - p_i^l - \left(\alpha_1 \frac{1}{2} - p_i^C\right)$.

Proposition 8 *When α_1 is small, platform A proposes a higher utility to both sides, while platform B proposes a smaller utility. The reverse is true is α_1 is large enough (and in particular above α_2).*

Proof. See Appendix. ■

Thus, provided that the externality on side 1 is not too large, the clients of platform B loose with the introduction of the bundle, while those of platform A benefit from it.

For large values of α_1, we see again that some counter-intuitive effects may arise. Figure 3 plots these changes in utility. Dashed lines depicts utilities of side 1 while the solid lines of side 2. The tick lines represent the utilities enjoyed by consumers of platform A.

Points where curves of the same side cross corresponds situation where the consumers are shared equally on that side. For low α_2 and thus high α_1, platform B sells more on both sides and customers of platform A receive a lower utility when this platform uses bundles.

Note that there is no instance where all customers benefit from bundling nor instances where they all suffer as a result of bundling. Thus there will always be some conflicts between consumers on the issue of bundling.

Total welfare changes according to the following expression:

$$W^{mixed} - W^C = (p_1^A)^2 \frac{Z}{36\Gamma^2 t_1}$$

where $Z = -9\left(t_1 t_2 - \alpha_1\alpha_2\right)^2 + t_1 t_2\left(5\alpha_2^2 - \alpha_1^2 - 4\alpha_1\alpha_2\right) - 4\alpha_1\alpha_2^3 + \alpha_1^4 + 3\alpha_1^2\alpha_2^2$. Figure 4 illustrates the behaviour of Z.

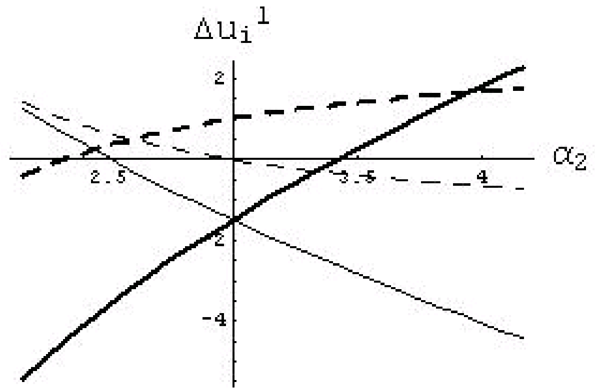

Figure 3: Consumers' utilities. (Example with $t_1 = 1$, $t_2 = 10$, $\alpha_1 \alpha_2 = 9$, $p_1^A = -1$).

As we can see from Figure 4, the effect is ambiguous. There can be instances where total welfare increases or decreases. In particular, the zones where total welfare increases are associated with high asymmetry in the membership externality.

3.2.1 Symmetric network effect ($\alpha_1 = \alpha_2$)

Assume that the externality is the same on both sides $\alpha_1 = \alpha_2 = \alpha$. Note that the assumption of unconstrained negative equilibrium prices on side 1 and regularity conditions imply that $t_1 < \alpha < t_2$. In words, the market configuration is such that side 1 is more competitive than side 2 and the network effect cannot be too small nor too big with respect the degree of competition.

Since $\Delta = 4\Gamma$, platform $A's$ optimal implicit price is now $p_1^A = \max\left\{-\theta, \frac{1}{2} p_1^{UC}\right\}$. Symmetry implies that platform A expands its market share in side 1: $n_1^A = \frac{1}{2} - \frac{p_1^A}{2t_1}$. In side 2, platform A, consistently with the general framework, charges higher prices $p_2^A = p_2^C - \frac{\alpha}{t_1} p_1^A$. However, due to the symmetry, market shares are left unchanged, $n_2^A = n_2^B = \frac{1}{2}$ and platform B maintains also its price unchanged: $p_2^B = p_2^C$. In side 2, symmetry leaves platform B neutral with respect the mixed bundling strategy which means that from the point of view of platform B the effects of demand shifting and competition softening cancel out.

Platform A finds always profitable to mixed bundle due to higher margins in side 2 which completely offsets the loss in side 1. Also platform B always benefits. The reduction of losses in side 1 due to the reduction of side 1 market share and neutrality in side 2 allows platform B to enhance its profit with respect the constrained case.

However, the effect on total consumer surplus and total welfare are negative.

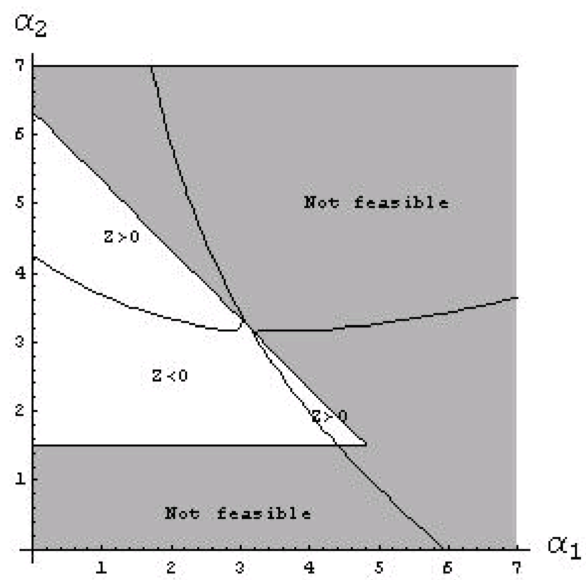

Figure 4: Evaluation of Z. (Example with $t_1 = 1, t_2 = 10, f_1 = 1/2$.)

Corollary 1 *Assume $\alpha_1 = \alpha_2$. If platform A uses a mixed bundle, then profit increases for both platforms, total consumer surplus decreases and total welfare decreases.*

Proof. $W^{mixed} - W^C$ has the same sign as $Z = -9\Gamma^2 < 0$. Total consumer surplus decrease since total profit increases. ■

Thus mixed bundling hurts consumers on side 2 in a significant manner, sufficient to offset the other effects. To understand the result on welfare, notice that when $\alpha_2 = \alpha_1$, the effect of bundling is only to shift demand toward platform A on side 1, while the two platforms continue to share the market equally on side 2. However to raise the total value of network effect it is necessary to shift demand on both sides in the same direction. Here the value of network externalities is unchanged because what is gained on side 2 by the customers of platform A is lost by the customers of platform B. Thus the only welfare effect is an increase of the total transport cost.

3.2.2 Network effects only on side 2 ($\alpha_1 = 0$)

Suppose that only side 2 cares about the participation of the other side: $\alpha_1 = 0$.

Again $\Delta = 4\Gamma$, the equilibrium prices in side 1 is $p_1^A = \max\left\{-\theta, \frac{1}{2}p_1^{UC}\right\}$ as in the example above. As in the general case, by subsiding side 1, platform A consequently charge higher prices on side 2 to recoup the losses on side 1. However in this context, by bundling, platform A increases its market shares unambiguously on both sides and platform B reacts by setting lower prices on side 2. The fact that consumers on side 1 do not benefit from participation eliminates the competition softening effect in favor of the demand shifting effect. Platform A expands the markets share on side 1 by subsiding. It becomes therefore more attractive on side 2 and it can charge higher prices while expanding its marker shares also in side 2. Mixed bundling can be profitable for both platforms when the network effect α_2 is small. In this case, platform A offsets its losses on side 1 by charging higher margins and expanding its market share in side 2. Platform B then reduces its price on side 2 as a reaction to the shift in demand.

Corollary 2 *Assume $\alpha_1 = 0$. If platform A uses a mixed bundle, then consumer surplus increases on each side. Platform A's profit and total welfare increases if α_2 is large, while platform B's profit increases and total welfare decreases when α_2 is small.*

Proof. Platform's A profit is larger with mixed bundle iff

$$\alpha_2 > \min\left\{3\left(f_1 + t_1\right), \frac{3}{2}\left(f_1 + t_1 + \theta\right)\right\}.$$

Platform's B profit is larger iff $\min\left\{\theta, -\frac{1}{2}\left(f_1 + t_1 - \alpha_2\right)\right\} > \frac{6\Gamma}{(\alpha_2)^2}\left(\alpha_2 - \frac{3}{2}f_1\right)$,in particular if $\alpha_2 < \frac{3}{2}f_1$. From consumer surplus increases. $W^{mixed} - W^C$ has the sign of $Z = t_1 t_2\left(-9t_1 t_2 + 5\alpha_2^2\right)$. ■

Mixed bundling is actually beneficial for consumers when there is no network externality on side 1. Consumers on side 1 benefit from the subsidy and consumers on side 2 on average pays lower prices due to platform B pricing strategy.

4 Pure bundling *vs* mixed bundling

Let us now turn to the case where platform A sells only the pure bundle at price $\widetilde{p} = p_1^A + \theta$. Our objective in this section is to understand whether platform A would prefer to commit to pure bundling or to mixed bundling. Such commitment may differ depending on whether platform A just maximizes its duopoly profit, or whether the strategy is used to deter platform B from entering the market, in which case A may choose to minimize B's duopoly profit. We are thus mostly interested here by the comparison between profits under mixed and pure bundling.

Under pure bundling, the behaviour of B is unchanged but the platform A's problem becomes thus:

$$\max_{p_1^A, p_2^A} \; (p_1^A - f_1 + m)n_1^A + (p_2^A - f_2)n_2^A$$
$$s.t.\; p_1^A \geq -\theta \; and \; (5)$$

where $m = \theta - c$.

With pure bundling any additional clients on side 1 generates a revenue m. As pointed by (Whinston 1990), the implications for pricing strategies are equivalent to a reduction in the marginal production cost f_1 of platform A on side 1 by an amount m in the mixed bundling model. Thus the platform prices are the same as the prices that would set a platform using a mixed bundle with a marginal cost $f_1 - m$.

We provide a full characterization of the equilibrium in appendix. Because of the complexity of the effects, we shall present them sequentially.

Notice that when $m = 0$, the equilibrium prices and profits are the same under pure and mixed bundling. In what follows, we fix all parameters except m. Starting from $m = 0$, we analyze the impact of m on the prices and profits under pure bundling. We denote $p_i^j(m)$ the equilibrium price of platform j on side i ($p_1^A(m) = \tilde{p}(m) - \theta$). The equilibrium prices under mixed bundling are thus $p_i^j(0)$.

4.1 The opportunity cost effect on side 2

Suppose that $\theta < \theta^A$ and that m is not too far apart from zero. By continuity of the equilibrium, the platform B sets a zero price on side 1, and the platform A sells the bundle at a zero price. The implicit price of the platform 1 service on side 1 is thus $p_1^A(m) = p_1^A(0) = -\theta$.

Considering side 2 the behavior of platform B is unchanged, but the price of platform A is affected by the change in the opportunity cost of selling on side 2, due to a change in the profitability of extra consumers on side 1. This leads to a smaller best reply on side 2 of A and equilibrium prices on side 2 writes:

$$
\begin{aligned}
p_2^A(m) &= p_2^A(0) - \frac{2\alpha_1}{3t_1}m, \\
p_2^B(m) &= p_2^B(0) - \frac{\alpha_1}{3t_1}m.
\end{aligned}
$$

The effect of pure bundling is thus to reduce prices on side 2. Since it raises the incentives of platform A to sell on side 1, platform A prices in a more competitive manner on side 2 and platform B follows. Consequently, platforms' market shares are:

$$
\begin{aligned}
n_1^A(m) &= n_1^A(0) + \frac{\alpha_1^2}{6\Gamma t_1}m, \\
n_2^A(m) &= n_2^A(0) + \frac{\alpha_1}{6\Gamma}m.
\end{aligned}
$$

The pure bundle direct effect increases platform A's market shares on both sides. Platform's total profits are thus:

$$\Pi^A_{pure}(m) = \Phi(p^A_1(m), 0) + \frac{\alpha_1(-6\Gamma + \alpha_1 m + 2(\alpha_2 - \alpha_1)p^A_1(m))}{18t_1\Gamma}m \quad (10a)$$

$$\Pi^B_{pure}(m) = \Phi(0, p^A_1(m)) + \frac{3\Gamma(3t_1 - \alpha_1) + \alpha_1^2 m - (7\Gamma + t_1 t_2 - 2\alpha_1^2)p^A_1(m)}{18t_1\Gamma}m \quad (10b)$$

where $\Phi(x, y)$ is defined in the section on mixed bundle.

Proposition 9 *Assume that m is positive but close to zero, $\theta < \theta^A$ (zero price of bundle). If $\alpha_2 \geq \alpha_1$, then platform A prefers mixed bundling to pure bundling. If α_1 is small enough, then platform B prefers pure bundling to mixed bundling.*

Proof. Recall that $\Pi^A_{mixed}(m) = \Phi(p^A_1(0), 0) + m$. We thus have when $p(m) = p(0)$:

$$\frac{d\left(\Pi^A_{pure}(m) - \Pi^A_{mixed}(m)\right)}{dm}|_{m=0} = \frac{\alpha_1(-6\Gamma - 2(\alpha_2 - \alpha_1)\theta)}{18t_1\Gamma} - 1$$

which is negative if $\alpha_1 \leq \alpha_2$. Recall that $\Pi^B_{mixed}(m) = \Phi(0, p^A_1(0))$. We thus have when $p(m) = p(0)$

$$\frac{d\left(\Pi^B_{pure}(m) - \Pi^B_{mixed}\right)}{dm}|_{m=0} = \frac{3\Gamma(3t_1 - \alpha_1) + (7\Gamma + t_1 t_2 - 2\alpha_1^2)\theta}{18t_1\Gamma}$$

which is positive if α_1 is small. ■

Thus when the externality on the side where bundling occurs is small, platform A would not benefit from committing to pure bundling as this would reduce the duopoly profit and may even increase the incentives of a competitor to enter compared to mixed bundling.

4.2 The opportunity cost effect on side 1

Suppose now that $\theta > \theta^A$ and that m is not too far apart from zero. Platform B sets a zero price on side 1, and platform A sells the bundle at a positive price. Since the fact that the good is only sold in the bundle modifies the opportunity cost of selling to side 1 customers, the price of the bundle is affected. It is shown in appendix that the new implicit equilibrium price is

$$p^A_1(m) = p^A_1(0) - \rho m \text{ where } \rho = \frac{3\Gamma + \alpha_1(\alpha_2 - \alpha_1)}{\Delta + 2\Gamma}$$

If we restrict to the case where $\alpha_1 \leq \alpha_2$, we find the intuitive conclusion that the price of the bundle decreases .

The analysis of the side 2 is the same as before but accounting for this change in prices. From equations 6, prices on side 2 are

$$\begin{aligned} p_2^A(m) &= p_2^A(0) + \frac{2\alpha_1+\alpha_2}{3t_1}\rho m - \frac{2\alpha_1}{3t_1}m, \\ p_2^B(m) &= p_2^B(0) - \frac{\alpha_2-\alpha_1}{3t_1}\rho m - \frac{\alpha_1}{3t_1}m. \end{aligned}$$

Consequently, platform $A's$ market shares are:

$$\begin{aligned} n_1^A(m) &= n_1^A(0) + \left(\frac{3t_1t_2-\alpha_1(2\alpha_2+\alpha_1)}{6\Gamma t_1}\right)\rho m + \frac{\alpha_1^2}{6\Gamma t_1}m \\ n_2^A(m) &= n_2^A(0) + \left(\frac{\alpha_2-\alpha_1}{6\Gamma}\right)\rho m + \frac{\alpha_1}{6\Gamma}m \end{aligned}$$

We see that the additional effect reduces the aggressiveness of platform A on side 2, while the reaction of platform B depends on $\alpha_2-\alpha_1$ with a price reduction if it is positive.

The equilibrium profits are still given by equations (10). The slopes $\frac{d\left(\Pi^j_{pure}(m)-\Pi^j_{mixed}(m)\right)}{dm}|_{m=0}$ are now augmented by $-\rho\Phi_1\left(p_1^A(0),0\right)$ for A and $-\rho\Phi_2\left(0,p_1^A(0)\right)$ for B.

We thus obtain

Proposition 10 *The conclusions of proposition 9 holds for $\theta>\theta^A$ if $\Phi_1\left(p_1^A(0),0\right)\geq 0$ (for A) and $\Phi_2\left(0,p_1^A(0)\right)\leq 0$ (for B).*

Proof. Immediate from above. ■

It is worth relating these conditions to the ones obtained in the previous section when comparing the mixed bundle profit to the situation with no bundling. To do that notice that $\Phi(x,0)$ is concave and $\Phi(0,y)$ is convex, which allows to show that

Corollary 3 *If platform A prefers no bundling to mixed bundling, it also prefers mixed bundling to pure bundling.*
If platform B prefers mixed bundling to no bundling, it also prefers pure bundling to mixed bundling.

Proof. By concavity and convexity

$$\begin{aligned} \Pi^A_{mixed}-\Pi^C-m &= \Phi\left(p_1^A(0),0\right)-\Phi(0,0)\geq\Phi_1\left(p_1^A(0),0\right)p_1^A(0) \\ \Pi^B_{mixed}-\Pi^C &= \Phi\left(0,p_1^A(0)\right)-\Phi(0,0)\leq\Phi_2\left(0,p_1^A(0)\right)p_1^A(0) \end{aligned}$$

The result then follows from $p_1^A(0)<0$. ■

Under these conditions, in particular when the subsidy with bundling is large enough, we see that platform A will have no reason to use pure bundles to deter entry, nor to enhance its duopoly profits.

4.3 Large impact on opportunity cost

When m is far apart from zero, new phenomena may arise. In particular the strategic effects may in some cases induce platform B to increase its price. Indeed we show in appendix, that on side 1, the price of the bundle and/or the price of platform 2 can be positive.

To understand this issue let us first define a notion of strategic complementarity on side 1. Define $\hat{p}_1^B\left(p_1^A, m\right)$ as the equilibrium price of platform B on side 1 in a game where the price of platform 2 is constrained to be p_1^A but the price of platform B is not constrained and can be negative. Then it is shown in appendix, equation (13), that

$$\begin{aligned}\frac{\partial \hat{p}_1^B\left(p_1^A, m\right)}{\partial p_1^A} &= \frac{\Delta-\Gamma}{\Delta+2\Gamma} \\ \frac{\partial \hat{p}_1^B\left(p_1^A, m\right)}{\partial m} &= \frac{\alpha_1\left(\alpha_2-\alpha_1\right)}{\Delta+2\Gamma}\end{aligned}$$

The first derivative shows that the prices on side 1 are "strategic complements" only if $\Delta > \Gamma$, which holds if network effects are not too high, or if the platforms are differentiated enough.[6] However if $\Delta < \Gamma$, decreasing the price of platform A on side 1 leads to an increase of the price of platform B on the same side.

The configuration $\Delta - \Gamma > 0$ characterizes a situation in which the difference in the network extenalities between the two sides is not very strong ($\alpha_2 - \alpha_1$ low), since $\Delta - \Gamma = 3\Gamma - (\alpha_2 - \alpha_1)^2$.

The second derivative shows that the effect on B's price depends on the relative intensity of the network effect on both sides. If side 2 has the highest externality, $\alpha_2 > \alpha_1$, increasing the marginal cost f_1 (reducing m) leads platform B to reduce its prices. Notice that the same condition gives the impact of mixed bundling on the price of platform B on side 2. In particular, the unconstrained price of B on side 1 decreases with m under circumstances that leads B to increase its price when mixed bundle is introduced.

One interpretation is to see that the only impact of m for a given p_1^A is that platform A reduces its price p_2^A. Thus the residual demand faced by platform B on side 2 decreases. To see the impact, we can reconsider the monopoly pricing formula (2) and notice that $\frac{\partial p_1^*}{\partial v_2} = \frac{t_1(\alpha_1-\alpha_2)}{\Delta}$. If $\alpha_2 > \alpha_1$, the platform increases its price on side 1 when its value on side 2 decreases, which is the case for the residual demand when p_2^A decreases.

In other words, the marginal benefit of raising demand on side 1 increases with the demand of the other side when side 1 is the low externality side ($\alpha_1 < \alpha_2$). Then the unconstrained price on side 1 decreases with the demand on side 2. Because pure bundling makes A more aggressive on side 2, platform B is relatively less attractive and its price increases on side 1.

[6] The exact condition is $3t_1t_2 > \alpha_1^2 + \alpha_2^2 + \alpha_1\alpha_2$.

On the opposite if $\alpha_1 > \alpha_2$, a platform facing an increase in the demand of side 2 would exploit this opportunity by simply raising its price on side 1.

Overall we see that when $\alpha_2 > \alpha_1$, increasing m raises the optimal price of B on side 1. Then for large large values of $m = \theta - c$ two new types of equilibria may arise.

First if θ is high and the cost c is small, all the prices may be positive and thus coincide with the equilibrium allocation of an unconstrained game where the cost of platform A on side 1 is $f_1 - m$.

In the case where $\Delta > \Gamma$, an additional effect is that p_1^B increases when p_1^A is still constrained. There may then be an intermediate range of values θ such that platform A gives the bundle for free while platform B chooses a positive price on side 1.

Similarly, when $\Delta < \Gamma$, for low values of θ the effect of m may dominate the effect of θ leading to a situation where platform B chooses a positive price and not platform A.

The comparison between mixed and pure bundling profit become then very complex so that we don't pursue it further (the equilibrium allocation are presented in appendix).

4.4 Symmetric network effect $(\alpha_1 = \alpha_2)$

In this example we study the case of pure bundling under the hypothesis of symmetric network externalities between the two sides ($\alpha_1 = \alpha_2 = \alpha$). In this case, it is immediate to see that $\Delta = 4\Gamma > \Gamma$ so that $\hat{p}_1^B\left(p_1^A, m\right)$ increases with p_1^A and is independent of m.Thus platform B never chooses a positive price on side 1. Platform A then charges

$$p_1^A(m) = \max\left\{-\theta, \frac{1}{2}p_1^{UC} - \frac{1}{2}m\right\}$$

while platform B is still constrained. Under symmetry there are thus only two relevant pricing regimes to study.

Demands are

$$\begin{aligned} n_1^A(m) &= \frac{1}{2} - \frac{p_1^A(m)}{2t_1} + \frac{\alpha^2}{6\Gamma t_1}m \\ n_2^A(m) &= \frac{1}{2} + \frac{\alpha}{6\Gamma}m \end{aligned}$$

Platform A sells more under pure bundling than under mixed or no bundling on both markets.

The profit is

$$\Pi^A_{pure}(m) = \Pi^C - \frac{1}{2t_1}p^A_1(m)^2 + \frac{(t_1+f_1-\alpha)}{2t_1}p^A_1(m) + \frac{\alpha(-6\Gamma+\alpha m)}{18t_1\Gamma}m$$
$$\Pi^B_{pure}(m) = \Pi^C - \frac{f_1}{2t_1}p^A_1(m) + \frac{3\Gamma(3t_1-\alpha)+\alpha^2 m-(8\Gamma-\alpha^2)p^A_1(m)}{18t_1\Gamma}m$$

We then obtain

Proposition 11 *If $\alpha_1 = \alpha_2 = \alpha$, platform A prefers mixed bundling to pure bundling, and at least for α small platform B prefers pure bundling to mixed bundling.*

Proof. For α small, the profit of platform B increases with m and decreases with p^A_1. Thus platform B prefers the situation with pure bundling ($m > 0$) to mixed bundling ($m = 0$).

Notice that interiority conditions $n^A_i < 1$ imply that $m < \min\left\{\frac{3\Gamma}{\alpha^2}\left(t_1 + p^A_1(m)\right), \frac{3\Gamma}{\alpha}\right\}$. Using $p^{UC}_1 = t_1 + f_1 - \alpha$ and $\alpha m < 3\Gamma$, we have

$$\Pi^A_{pure}(m) < \Pi^C + \frac{1}{2t_1}\left(p^{UC}_1 - p^A_1(m)\right)p^A_1(m)$$

Since $p^A_1(m) = p^A_1(0)$ if $p^A_1(0) = -\theta$, we confirm also that platform A prefers mixed bundling when it implies a free bundle. Moreover, in the case where mixed bundling implies a positive price, $p^A_1(0) = \frac{p^{UC}_1}{2}$, implying that the RHS is maximal in p^A_1. Thus platform A prefers mixed bundling. ■

5 Bundling inefficient goods: pure bundling *vs* no bundling

In what precedes we assumed that $m = \theta - c > 0$.which corresponds to the case where the firm has some market power that allows it to sell the good at a price above cost. As already pointed, when such a possibility doesn't exist, the platform may still subsidize participation by bundling a good such that $m < 0$ but $\theta > 0$ and setting a price below θ for the bundle. Of course under such a circumstance, the good will not be sold unbundled as this would involve a loss.[7] In this section we allow for $m < 0$, and extends the analysis to pure bundling.

[7] In the case $m < 0$, the platform may wish to commit to mixed bundling (in duopoly) but ex-post it could always set a price c and not sell. Thus the platform would need to commit to set a price θ for the good alone along with a mixed bundle strategy.

5.1 Monopoly

As before the bundle is sold at price $\tilde{p}$, and the "effective price" of the service is $p_1 = \tilde{p} - \theta$. In the case of a monopoly the demand of bundle on side 1 is then equal to $D_1(p_1 - \alpha_1 n_2)$, leading to a profit for the monopoly platform:

$$\begin{aligned} \Pi_{pure} &= \max_{p_1,p_2} \ (p_1 - f_1) n_1 + (p_2 - f_2) n_2 + m.n_1 \\ s.t.\ p_1 &\geq -\theta \end{aligned}$$

where the demand n_1 and n_2 are solutions of (1) as before.

As already mentioned the equilibrium prices and the equilibrium allocation are the same as with mixed bundling by a platform with a marginal cost $f_1 - m$ on side 1. The proof of proposition 1 shows that the participation levels are decreasing with the marginal cost:

Proposition 12 *Consider a monopoly platform selling a pure bundle. Then the participation levels are nondecreasing with θ and m on both sides.*

Proof. If the constraint is binding , the allocation is given by (4), evaluated for a cost $f_1 - m$ which defines increasing quantities. When the constraint is not binding:

$$\begin{aligned} n_1 &= n_1^* + \frac{2t_2}{\Delta} m \\ n_2 &= n_2^* + \frac{(\alpha_1 + \alpha_2)}{\Delta} m \end{aligned}$$

again increasing with m. Thus participation increases with θ. ■

The result implies that for socially valuable goods ($m > 0$), participation is maximal with a pure bundle. When m is negative but close to zero, a pure bundle induces participation levels close to the mixed bundle, and thus above the unbundled case. But when m is large negative relative to θ, in particular if the value θ is small and the cost c is higher, participation with a pure bundle will be smaller than with no bundle.

A direct computation shows also that the price of the pure bundle is

$$\tilde{p} = \max\left\{ p_1^* + \theta - \frac{2t_1t_2 - \alpha_1(\alpha_1 + \alpha_2)}{\Delta}(\theta - c), 0 \right\}.$$

When externalities are small, the price of the bundle is monotonic with θ. Figure 5 shows two possible configurations depending on whether c is larger or smaller than $-p_1^*$.

The case where $2t_1t_2 - \alpha_2(\alpha_1 + \alpha_2) < 0$ is more counter-intuitive as the price of the bundle is non-increasing with θ. In this case the bundle is sold at a zero price for values θ above some threshold. Figure 6 shows the two cases.

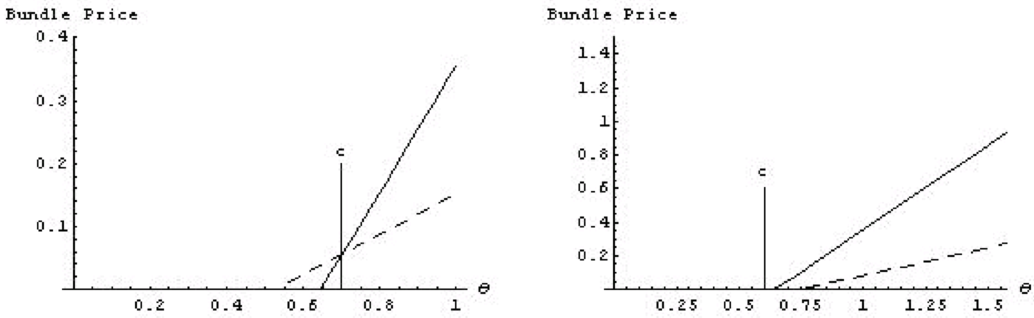

Figure 5: Bundle prices under mixed bundle (solid line) and pure bundle (dashed line)

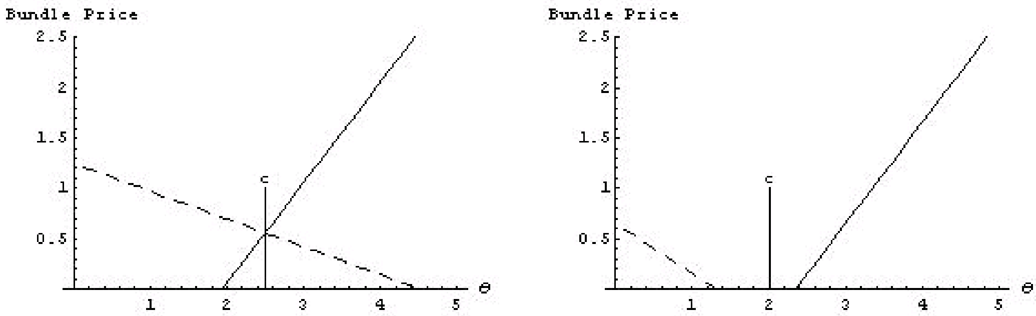

Figure 6: Bundle prices under mixed bundle (solid line) and pure bundle (dashed line)

In general pure bundling implies a gain (relaxing the no-subsidy constraint) and a loss either because the good is sold at a loss ($m < 0$) or because some profitable sales do not occur ($m > 0$). It is immediate that the pure bundling profit is increasing with θ (as long as the constraint binds) and with the margin m.

Given that pure bundling and mixed bundling yield the same positive gain in profit for a zero margin, we obtain:

Proposition 13 *For any given* θ*, there exists* $m_\theta < 0$ *such that* $\Pi_{pure} - \Pi^*$ *is positive for* $0 \geq m > m_\theta$*.*

Concerning the impact on consumer surplus we should account for the fact that the objective of the platform bundling is to raise participation which should benefit consumers.

Conjecture 1 *If platform A is a monopoly and* $\Pi_{pure} - \Pi^* > 0$*, then consumer surplus is higher on both sides under pure bundling than under no* *bundling.*

5.2 Duopoly

Considering the duopoly cases, the analysis is similar to the one developed in the preceding section except that now m can be negative. When m is close to zero, the allocation and the profits with pure bundling will be close to the allocation with mixed bundling. Therefore for m negative but close to zero the previous analysis of the comparison between mixed bundling and no bundling applies to the comparison between pure bundling and no bundling.

However the comparison between pure and mixed bundling is reversed. In particular for $\alpha_1 \leq \alpha_2$ and $m < 0$, platform A prefers pure bundling to mixed bundling.

For m large negative, the analysis parallels the analysis of the previous section (see Appendix), and any prices may become positive on side 1. To simplify matters we present the case of a symmetric network effects.

So assume that $\alpha_1 = \alpha_2 = \alpha$. Under symmetry there are only three relevant pricing regimes to study. The first case occurs when $m > p_1^{UC} + 2\theta$ and both platforms are constrained to set zero prices. In the second one, $3p_1^{UC} < m < p_1^{UC} + 2\theta$, platform A charges a positive price ($p_1^A = \frac{1}{2}p_1^{UC} - \frac{1}{2}m$) while platform B is still constrained. In the third one which occurs when $m < 3p_1^{UC}$, both platforms charge positive prices (respectively, $p_1^A = p_1^{UC} - \frac{2}{3}m$ and $p_1^B = p_1^{UC} - \frac{1}{3}m$). Therefore, under symmetry, platform A never suffers from a situation in which she is constrained and the rival isn't. Note also that the case when both platforms can charge positive prices arises only when $m < 0$.

The discussion of the profits' increment with respect the constrained case (and therefore on the incentives to pure bundle) of platforms is complex. We illustrate the effects with an example obtained by letting θ and α varying with $t_1 = 1, t_2 = 10, f_1 = 1/2, c = 1/2$. Figure 7 illustrates the results for the situation where the two firms are constrained ex-post, $m > p_1^{UC} + 2\theta$ or $0 > f_1 + t_1 - \alpha + \theta + c$ which corresponds to the light area on the graph. The graph show the zone of positivity and negativity of the profits increments of both platforms compared to no bundle.

We see that platform B always benefits unless m is large positive. Platform A benefits as long as θ is not too small, thus for m mildly negative. When m is very low, platform A would choose not to bundle the good.

The pattern for the profit gains obtained for $m < 0$ is the same for the other regimes: platform B always prefers pure bundle to no bundle, platform A also unless m is very small negative. The difference is that for m large positive the gain of platform A may be negative.

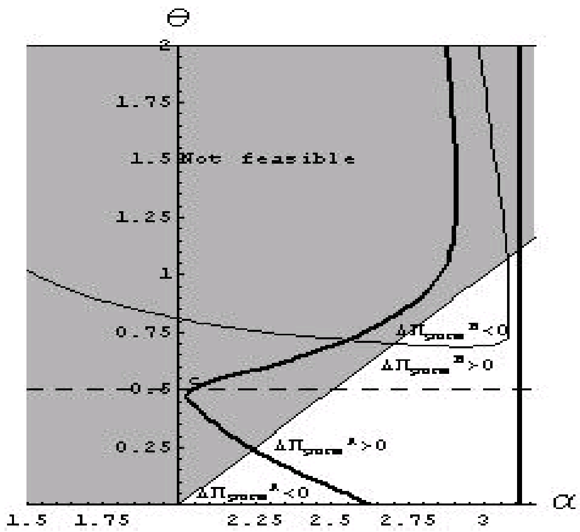

Figure 7: Evaluation of platforms' profits.

Thus low social value of the good determines a situation in which platform B benefits from bundling (competition softening effect). However, when the value of the good becomes large positive the standard foreclosing effect may arise since platform B can be hurt. This confirms the results that in a two-sided framework there could be foreclosure through bundling.

Figure 8 depicts the consumer surplus and total welfare effects under symmetry.

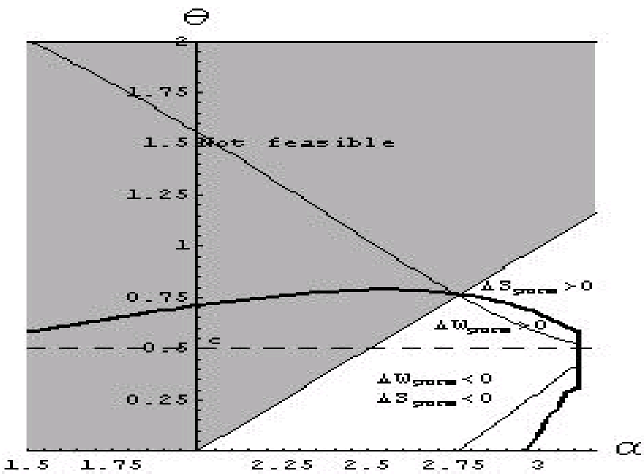

Figure 8: Evaluation of pure bundling welfare effects.

The graph shows that the conclusion that when $\alpha_1 = \alpha_2$, mixed bundling has a negative impact on consumer surplus and welfare, extends to the case of pure bundling when $m < 0$. The same conclusion applies to the other two regimes.

Notice however that pure bundle could both profitable for A and for welfare is m is positive.

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A The monopoly case

Proof. For the proofs we may rely on the inverse demand, which can be written as

$$p_i = H_i(n_i) + \alpha_i n_{-i}, \tag{11}$$

where $H_i()$ is the inverse of the demand function D_i,and will be referred to as the "hedonic price". The profit function is

$$\max_{n_1,n_2} (H_1(n_1) - f_1)n_1 + (H_2(n_2) - f_2)n_2 + (\alpha_1 + \alpha_2) n_1 n_2$$
$$s.t. \quad p_1 = H_1(n_1) + \alpha_1 n_2 \geq 0$$

The optimal quantities obtained at

$$\begin{aligned} H_1(n_1) + H_1'(n_1)n_1 + (\alpha_1 + \alpha_2) n_2 &= f_1 - \lambda H_1'(n_1) \\ H_2(n_2) + H_2'(n_2)n_2 + (\alpha_1 + \alpha_2) n_1 &= f_2 - \lambda \alpha_1 \end{aligned}$$

where λ is non negative and is the multiplier of the non-negativity constraint. The solution reduces to

$$\begin{aligned} H_2(n_2) + H_2'(n_2)n_2 + \alpha_2 n_1 + (n_1 + \lambda) \alpha_1 &= f_2 \\ \lambda (H_1(n_1) + \alpha_1 n_2) &= 0 \\ \frac{\alpha_2 n_2 - f_1}{-H_1'(n_1)} &= n_1 + \lambda \geq n_1 \end{aligned}$$

which gives the first order conditions.

When bundling is used we have to consider the program

$$\begin{aligned} \max_{n_1,n_2} \quad & (H_1(n_1) - f_1 + \gamma m)n_1 + (H_2(n_2) - f_2)n_2 + (\alpha_1 + \alpha_2) n_1 n_2 + (1 - \gamma) m \\ s.t. \quad & H_1(n_1) + \alpha_1 n_2 + \theta \geq 0 \end{aligned}$$

where $\gamma = 1$ if there is pure bundling, $\gamma = 0$ if there is mixed bundling. The solution is

$$\begin{aligned} H_1(n_1) + H_1'(n_1)n_1 + (\alpha_1 + \alpha_2) n_2 &= f_1 - \gamma m - \lambda H_1'(n_1) \\ H_2(n_2) + H_2'(n_2)n_2 + (\alpha_1 + \alpha_2) n_1 &= f_2 - \lambda \alpha_1 \\ \lambda (H_1(n_1) + \alpha_1 n_2 + \theta) &= 0 \end{aligned}$$

which yield the new first order conditions. ■

B Duopoly

B.1 Duopoly with no bundle

Proof. Denote by λ^A the multiplier associated to the non-negativity constraint. Taking derivatives of the Lagrangians we obtain the following first order conditions for platform A:

$$\begin{aligned}
0 &= \frac{1}{2}+\frac{\alpha_1(p_2^B-p_2^A)+t_2(p_1^B-p_1^A)}{2\Gamma}-\frac{\alpha_2\left(p_2^A-f_2\right)}{2\Gamma}-\frac{t_2\left(p_1^A-f_1\right)}{2\Gamma}+\lambda^A\\
0 &= \frac{1}{2}+\frac{\alpha_2(p_1^B-p_1^A)+t_1(p_2^B-p_2^A)}{2\Gamma}-\frac{t_1\left(p_2^A-f_2\right)}{2\Gamma}-\frac{\alpha_1\left(p_1^A-f_1\right)}{2\Gamma}
\end{aligned}$$

and the symmetric for B. If p_1^{UC} is negative, both the problems of the platforms are constrained. Therefore, setting $p_1^A=p_1^B=0$ and $\lambda^A=\lambda^B>0$, we obtain the following optimal response functions, for platform A:

$$\begin{aligned}
p_2^A(p_2^B) &= \frac{1}{2}p_2^B+\frac{f_2+t_2-\alpha_1}{2}+\frac{\alpha_1}{2t_1}(f_1+t_1-\alpha_2)\\
p_2^B(p_2^A) &= \frac{1}{2}p_2^A+\frac{f_2+t_2-\alpha_1}{2}+\frac{\alpha_1}{2t_1}(f_1+t_1-\alpha_2)
\end{aligned}$$

The optimal constrained symmetric equilibrium prices are:

$$\begin{aligned}
p_1^C &= 0\\
p_2^C &= f_2+t_2-\alpha_1+\frac{\alpha_1}{t_1}(f_1+t_1-\alpha_2)
\end{aligned}$$

with :

$$\lambda=-\frac{f_1+t_1-\alpha_2}{2t_1}>0.$$

Platforms share the market equally and the equilibrium profits are

$$\Pi^C=\Pi^{UC}+\frac{\alpha_1-t_1}{2t_1}(f_1+t_1-\alpha_2)$$

■

B.2 Duopoly with mixed bundle

Proof of Proposition 3. When platform A uses mixed bundles, first order conditions yield

$$\begin{aligned}
0 &= \frac{1}{2}+\frac{\alpha_1(p_2^B-p_2^A)+t_2(p_1^B-p_1^A)}{2\Gamma}-\frac{\alpha_2\left(p_2^A-f_2\right)}{2\Gamma}-\frac{t_2\left(p_1^A-f_1\right)}{2\Gamma}+\lambda^A,\\
0 &= \frac{1}{2}-\frac{\alpha_1(p_2^B-p_2^A)+t_2(p_1^B-p_1^A)}{2\Gamma}-\frac{\alpha_2\left(p_2^B-f_2\right)}{2\Gamma}-\frac{t_2\left(p_1^B-f_1\right)}{2\Gamma}+\lambda^B,
\end{aligned}$$

on side 1, and

$$\begin{aligned}
0 &= \frac{1}{2}+\frac{\alpha_2(p_1^B-p_1^A)+t_1(p_2^B-p_2^A)}{2\Gamma}-\frac{t_1\left(p_2^A-f_2\right)}{2\Gamma}-\frac{\alpha_1\left(p_1^A-f_1\right)}{2\Gamma},\\
0 &= \frac{1}{2}-\frac{\alpha_2(p_1^B-p_1^A)+t_1(p_2^B-p_2^A)}{2\Gamma}-\frac{t_1\left(p_2^B-f_2\right)}{2\Gamma}-\frac{\alpha_1\left(p_1^B-f_1\right)}{2\Gamma},
\end{aligned}$$

on side 2.

First, for given prices p_1^A and p_1^B on side 1, the optimal response functions on side 2 are:

$$\begin{aligned} p_2^A(p_2^B) &= \frac{1}{2}p_2^B + \frac{1}{2}p_2^C - \frac{\alpha_1+\alpha_2}{2t_1}p_1^A + \frac{\alpha_2}{2t_1}p_1^B, \\ p_2^B(p_2^A) &= \frac{1}{2}p_2^A + \frac{1}{2}p_2^C - \frac{\alpha_1+\alpha_2}{2t_1}p_1^B + \frac{\alpha_2}{2t_1}p_1^A. \end{aligned}$$

Solving for these prices we obtain

$$p_2^A = p_2^C - (\alpha_2+2\alpha_1)\frac{p_1^A}{3t_1} + (\alpha_2-\alpha_1)\frac{p_1^B}{3t_1} \text{ and } p_2^B = p_2^C - (\alpha_2+2\alpha_1)\frac{p_1^B}{3t_1} + (\alpha_2-\alpha_1)\frac{p_1^A}{3t_1}.$$

The first order conditions on market 1 then write:

$$\begin{aligned} \lambda^A &= \frac{(\Delta+2\Gamma)\,p_1^A - (\Delta-\Gamma)\,p_1^B}{6\Gamma t_1} - \frac{(f_1+t_1-\alpha_2)}{2t_1} \\ \lambda^B &= \frac{(\Delta+2\Gamma)\,p_1^B - (\Delta-\Gamma)\,p_1^A}{6\Gamma t_1} - \frac{(f_1+t_1-\alpha_2)}{2t_1} \end{aligned}$$

First notice that it is not possible that no constraint binds.

Now suppose that both constraints are still binding. Therefore the prices are $p_1^A = -\theta$ and $p_1^B = 0$. Observe that $\lambda^A < \lambda^B$ which implies that this is the solution if $\lambda^A > 0$ or $p_1^A = -\theta > -\theta^A = (f_1+t_1-\alpha_2)\frac{3\Gamma}{\Delta+2\Gamma}$.

Now suppose $\theta > \theta^A$. Setting $\lambda^A = 0$ and $\lambda^B > 0$, the equilibrium prices are $p_1^A = -\theta^A$ and $p_1^B = 0$.we verify that:

$$\lambda^B = -\frac{2\Delta+\Gamma}{2t_1(\Delta+2\Gamma)}(f_1+t_1-\alpha_2) > 0.$$

The prices on side 2 are then

$$\begin{aligned} p_2^A &= p_2^C - (\alpha_2+2\alpha_1)\frac{(-\theta^A)}{3t_1} \\ p_2^B &= p_2^C + (\alpha_2-\alpha_1)\frac{(-\theta^A)}{3t_1} \end{aligned}$$

■

Proof of proposition 8.

$$\begin{aligned} \alpha_1 n_2^A - p_1^A - \alpha_1\frac{1}{2} &= \left(\frac{\alpha_1(\alpha_2-\alpha_1)+6\Gamma}{6\Gamma}\right)(-p_1^A) \\ \alpha_2 n_1^A - p_2^A - \left(\alpha_2\frac{1}{2} - p_2^C\right) &= \left(\frac{t_1 t_2(\alpha_2-\alpha_1)-3\alpha_1\Gamma}{2\Gamma}\right)\left(-\frac{p_1^A}{3t_1}\right) \end{aligned}$$

$$\begin{aligned} \alpha_1 n_2^B - \alpha_1\frac{1}{2} &= \alpha_1\left(\frac{\alpha_2-\alpha_1}{6\Gamma}\right)p_1^A \\ \alpha_2 n_1^B - p_2^B - \left(\alpha_2\frac{1}{2} - p_2^C\right) &= \left(\frac{t_1 t_2(\alpha_2-\alpha_1)+3\alpha_1\Gamma}{2\Gamma}\right)\frac{p_1^A}{3t_1} \end{aligned}$$

For α_1 close to zero, the first two are positive and the last two are negative. But the reverse holds if $\alpha_1 - \alpha_2$ is large positive and Γ is small. ■

B.3 Duopoly with pure bundle

Proof. When platform A sells a pure bundle, its problem is thus:

$$\max_{p_1^A, p_2^A} (p_1^A - f_1 + m)n_1^A + (p_2^A - f_2)n_2^A$$
$$s.t.\ p_1^A \geq -\theta \ and\ (5)$$

while platform's B problem is unchanged. When platform A uses mixed bundles, first order conditions yield

$$0 = \frac{1}{2} + \frac{\alpha_1(p_2^B - p_2^A) + t_2(p_1^B - p_1^A)}{2\Gamma} - \frac{\alpha_2\left(p_2^A - f_2\right)}{2\Gamma} - \frac{t_2\left(p_1^A - f_1 + m\right)}{2\Gamma} + \lambda^A$$
$$0 = \frac{1}{2} - \frac{\alpha_1(p_2^B - p_2^A) + t_2(p_1^B - p_1^A))}{2\Gamma} - \frac{\alpha_2\left(p_2^B - f_2\right)}{2\Gamma} - \frac{t_2\left(p_1^B - f_1\right)}{2\Gamma} + \lambda^B$$

on side 1, and

$$0 = \frac{1}{2} + \frac{\alpha_2(p_1^B - p_1^A) + t_1(p_2^B - p_2^A)}{2\Gamma} - \frac{t_1\left(p_2^A - f_2\right)}{2\Gamma} - \frac{\alpha_1\left(p_1^A - f_1 + m\right)}{2\Gamma}$$
$$0 = \frac{1}{2} - \frac{\alpha_2(p_1^B - p_1^A) + t_1(p_2^B - p_2^A)}{2\Gamma} - \frac{t_1\left(p_2^B - f_2\right)}{2\Gamma} - \frac{\alpha_1\left(p_1^B - f_1\right)}{2\Gamma}$$

on side 2.

First, for given prices p_1^A and p_1^B on side 1, the optimal response functions on side 2 are:

$$p_2^A(p_2^B) = \frac{1}{2}p_2^B + \frac{1}{2}p_2^C - \frac{\alpha_1}{2t_1}m - \frac{\alpha_1 + \alpha_2}{2t_1}p_1^A + \frac{\alpha_2}{2t_1}p_1^B,$$
$$p_2^B(p_2^A) = \frac{1}{2}p_2^A + \frac{1}{2}p_2^C - \frac{\alpha_1 + \alpha_2}{2t_1}p_1^B + \frac{\alpha_2}{2t_1}p_1^A.$$

Solving for these prices we obtain

$$p_2^A = p_2^C - \frac{2\alpha_1}{3t_1}m - (\alpha_2 + 2\alpha_1)\frac{p_1^A}{3t_1} + (\alpha_2 - \alpha_1)\frac{p_1^B}{3t_1}$$
$$p_2^B = p_2^C - \frac{\alpha_1}{3t_1}m - (\alpha_2 + 2\alpha_1)\frac{p_1^B}{3t_1} + (\alpha_2 - \alpha_1)\frac{p_1^A}{3t_1}.$$

The first order conditions on market 1 then write:

$$\lambda^A = \frac{(\Delta + 2\Gamma)\,p_1^A - (\Delta - \Gamma)\,p_1^B - 3\Gamma p_1^{UC} + (3\Gamma + \alpha_1(\alpha_2 - \alpha_1))\,m}{6\Gamma t_1}$$
$$\lambda^B = \frac{(\Delta + 2\Gamma)\,p_1^B - (\Delta - \Gamma)\,p_1^A - 3\Gamma p_1^{UC} - \alpha_1(\alpha_2 - \alpha_1)\,m}{6\Gamma t_1}$$

Suppose that both non-negativity constraint bind, then $p_1^A = -\theta$, $p_1^B = 0$ and

$$\begin{aligned}\lambda^A &= \frac{-\theta\left(\Delta+2\Gamma\right) - 3\Gamma p_1^{UC} + \left(3\Gamma + \alpha_1\left(\alpha_2-\alpha_1\right)\right)m}{6\Gamma t_1} \\ \lambda^B &= \frac{\theta\left(\Delta-\Gamma\right) - 3\Gamma p_1^{UC} - \alpha_1\left(\alpha_2-\alpha_1\right)m}{6\Gamma t_1}\end{aligned}$$

This is an equilibrium if

$$\begin{aligned}\psi^B(m) &\leq \theta \leq \psi^A(m), \text{ when } \Delta-\Gamma>0 \\ \theta &\leq \min\{\psi^B(m), \psi^A(m)\}, \text{ when } \Delta-\Gamma<0 \\ \theta &\leq \psi^A(m) \quad \text{and } 3\Gamma p_1^{UC} + \alpha_1\left(\alpha_2-\alpha_1\right) < 0 \text{ when } \Delta-\Gamma=0 \\ \textit{where } \psi^A(m) &= \theta^A + \rho m, \ \textit{and } \rho = \frac{3\Gamma+\alpha_1\left(\alpha_2-\alpha_1\right)}{\Delta+2\Gamma} \\ \psi^B(m) &= \frac{3\Gamma p_1^{UC} + \alpha_1\left(\alpha_2-\alpha_1\right)m}{\Delta-\Gamma}.\end{aligned}$$

Suppose now that $\lambda^A = 0$ and $\lambda^B = 0$. Solving the system we obtain the following equilibrium prices:

$$\begin{aligned}p_1^{A*}(m) &= p_1^{UC} - \left(\frac{\alpha_1\left(\alpha_2-\alpha_1\right)+\Delta+2\Gamma}{2\Delta+\Gamma}\right)m \\ p_1^{B*}(m) &= p_1^{UC} - \left(\frac{3\Gamma-\alpha_2\left(\alpha_2-\alpha_1\right)}{2\Delta+\Gamma}\right)m\end{aligned}$$

which can be solution if

$$\theta \geq -p_1^{A*}(m), \ \textit{and } p_1^{B*}(m) \geq 0$$

Suppose now that $\lambda^A = 0$ and $\lambda^B > 0$, consequently $p_1^B = 0$. Solving the system we obtain the following equilibrium prices:

$$\begin{aligned}p_1^A &= -\psi^A(m) \\ \lambda^B &= \frac{-\left(2\Delta+\Gamma\right)p_1^{UC} + \left(3\Gamma-\alpha_2\left(\alpha_2-\alpha_1\right)\right)m}{2t_1(\Delta+2\Gamma)}\end{aligned}$$

which is an equilibrium if

$$\theta > \psi^A(m) \text{ and } 0 \geq p_1^{B*}(m).$$

Suppose now that $\lambda^A > 0$ and $\lambda^B = 0$, consequently $p_1^A = -\theta$. Solving the system we obtain the following equilibrium prices:

$$\begin{aligned}p_1^B &= \frac{3\Gamma p_1^{UC} + \left(\Delta-\Gamma\right)\left(-\theta\right) + \alpha_1\left(\alpha_2-\alpha_1\right)m}{\Delta+2\Gamma} \qquad (13) \\ \lambda^A &= \frac{-\left(2\Delta+\Gamma\right)\left(\theta+p_1^{UC}\right) + \left(\alpha_1\left(\alpha_2-\alpha_1\right)+\Delta+2\Gamma\right)m}{2t_1\left(\Delta+2\Gamma\right)}\end{aligned}$$

which is an equilibrium if:

$$\begin{aligned}
\theta &< \min\left\{\psi^B(m), -p_1^{A*}(m)\right\}, \text{ when } \Delta-\Gamma>0 \\
\psi^B(m) &< \theta < -p_1^{A*}(m), \text{ when } \Delta-\Gamma<0 \\
\theta &< -p_1^{A*}(m) \text{ and } 3\Gamma p_1^{UC} + \alpha_1(\alpha_2-\alpha_1) > 0, \text{ when } \Delta-\Gamma=0
\end{aligned}$$

Notice that:

$$\begin{aligned}
\psi^A(m) - \psi^B(m) &= -\frac{3\Gamma(\Gamma+2\Delta)}{(\Delta+2\Gamma)(\Delta-\Gamma)} p_1^{B*}(m) \\
\phi^A(m) - \psi^B(m) &= -\left(\frac{\Delta+2\Gamma}{\Delta-\Gamma}\right) p_1^{B*}(m)
\end{aligned}$$

Notice also that

$$\phi^B(m) = -p_1^{UC} + (\frac{3\Gamma - \alpha_2(\alpha_2-\alpha_1)}{2\Delta+\Gamma})m = -p_1^{UC} + (\frac{\Delta-\Gamma-\alpha_1(\alpha_2-\alpha_1)}{2\Delta+\Gamma})m..$$

Summing up, we have the following cases.

- First if $p_1^{B*}(m) > 0$ and $p_1^{A*}(m) + \theta > 0$, no price is constrained by the non-negativity constraint.

  Otherwise:

- When $\Delta - \Gamma > 0$:
  - When $p_1^{B*}(m) < 0$, the price of the bundle is zero for $\theta \leq \psi^A(m)$,and $p_1^B > 0$ if $\theta < \psi^B(m)$.
  - When $p_1^{B*}(m) > 0$, the price of the bundle is zero and $p_1^B > 0$.
- When $\Delta - \Gamma < 0$:
  - When $p_1^{B*}(m) < 0$, the price of the bundle is zero for $\theta \leq \psi^A(m)$, and $p_1^B = 0$.
  - When $p_1^{B*}(m) > 0$, the price of the bundle is zero, and $p_1^B > 0$ if $\theta > \psi^B(m)$.
- When $\Delta - \Gamma = 0$, note that $\psi^A(m) = -p_1^{A*}(m) > 0$. We thus have two cases:
  - When $p_1^{A*}(m) > -\theta$, there are two regimes according to whether $p_1^{B*}(m)$ is positive or negative.
  - When $p_1^{A*}(m) < -\theta$, there are two regimes according to whether $3\Gamma p_1^{UC} + \alpha_1(\alpha_2-\alpha_1)$ is positive or negative.

■